Insurance: Mathematics and Economics 79 (2018) 124–136

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

De-risking strategy: Longevity spread buy-in

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ARTICLE INFO

Article history:
Received January 2017
Received in revised form December 2017
Accepted 10 January 2018
Available online 31 January 2018

Keywords:
Buy-in
De-risking strategy
Longevity risk
Actuarial valuations
Survival functions

ABSTRACT

The paper proposes a demographic de-risking strategy for a pension provider, to deal with the future uncertainty in longevity over a long time horizon. The innovative idea of a longevity spread buy-in is presented. The formulae for calculating the buy-in premium are proposed in the case of pension plans. The proposal directly impacts the pension provider's risk management systems and hence can be an important part of the overall approach to risk management. The numerical results, developed under specified stochastic hypotheses for the dynamics of the underlying financial and demographic processes, show how the proposal of the paper can be practically implemented.

1. Introduction

Several approaches can be considered when dealing with longevity risk management for portfolios of life insurance and pension contracts. With regard to defined contribution pension plans, longevity risk management is treated by means of tools such as securitization and risk-sharing, as studied in Cairns et al. (2006b), Blake et al. (2006) and Cox et al. (2010). Another approach is to hedge the longevity risk by transferring it to a third party with techniques such as the longevity swap, the pension buy-in and pension buy-out. Thus, Cox et al. (2013) focus on this research agenda obtaining a type of hedge ratio when transferring longevity risk from a defined benefit pension plan. As clearly observed in Lin et al. (2015), the longevity hedge can be made by longevity swaps and longevity insurance, removing only the longevity risk.

When the pension buy-in is structured for the longevity risk transfer, the pension provider matches his future obligations or part of them, paying a premium to a third party: the pension buy-in transaction involves the trustees of a scheme purchasing an insurance policy to cover the future outflows due to the current pensioners. The pension buy-out strategy transfers obligations and assets, all or part of them, to a third party: the pension annuity buy-out removes the pension risks by transferring the accrued pension liabilities to a regulated insurance company in return for a premium. The basic difference among these last three strategies is that, while the longevity hedge transfers only the longevity risk, the pension buy-in and pension buy-out, based on the valuations of future obligations in the first case and obligations and assets in the second case, transfer also other risks such as the interest rate risk.

In recent years, the increase of buy-in and buy-out market has been significant, in particular in the UK. In fact, UK is leading the way in pensions de-risking (cf. Lin et al., 2015), but other countries also are looking to reduce pension risk increasing the employment of buy-in and buy-out techniques. At the moment, it seems that the economic crisis has had the effect of a strong growth in the volume of transactions, in particular of the buy-ins (cf. Lin et al., 2015). Looking ahead, on the basis of the turbulence of the financial markets and continuing upward trends longevity, it is likely that pension schemes will be still oriented to manage benefits and liabilities, so de-risking is expected to be at the top of insurance companies' objectives also for the next decade. Actuarial practitioners have already been predicting for a few years that annual transaction volumes could hit $25 billion by 2017 (LCP, 2012). The majority of annuity transactions have been structured as buy-in since the cost of de-risking pension scheme is often lower than the expected future cost of doing nothing (cf. Grant Thornton, 2011).

On this topic, Lin et al. (2015) develop an interesting analysis on the impact of the transaction costs, the counter-party default

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URL: http://www.legalandgeneral.com/pensionrisktransfer (P. Sagoo).

https://doi.org/10.1016/j.insmatheco.2018.01.004

probability and the underfunding ratio on the expected total pension cost, showing how they can influence any hedging choice. In particular, they show that the buyout strategy is more expensive than the buy-in one. These innovative de-risking strategies open up new opportunities for pension plans; nevertheless an acceptable and reliable method of valuing a buy-in and buy-out has yet to be developed.

One of the main challenges of a buy-in strategy is managing the longevity risk in pension annuities. If the human lifetime reveals itself longer than expected, this implies meaningful financial risk to manage and the buy-in strategy allows this risk to be transferred to insurers or reinsurers, giving rise to an asset for the pension provider. Differently from the longevity swaps, a buy-in strategy can involve and control the risks connected to longevity risk, leaving the liabilities in the pension plan. The relevance of longevity risk for annuity/pension providers depends on the specific characteristics of the particular annuity or pension portfolio. A basic problem lies in measuring the effect of mortality improvements on the present value of pensions/annuities; within this context, Khalaf-Allah et al. (2006) treat just this problem, taking into account the interactions with the age and gender of the pensioners, the evolution of interest rates and the survival trend. Such an analysis allows us to mark out the key age ranges related to higher expected costs due to future mortality improvements.

The current paper is centered on longevity risk management: the aim is hedging the longevity uncertainty in those age intervals in which the risk of underestimating future liabilities is high, as a result of higher mortality improvement than expected. Identifying the so-called "dangerous" age intervals is an important goal in the light of the performance analysis approach, in particular when measuring the impact of the longevity risk on a life annuity portfolio (cf. Di Lorenzo and Sibillo, 2002) and when quantifying additional costs due to mortality improvements (cf. Khalaf-Allah et al., 2006). The idea which we propose in the paper is connected to what we call the longevity spread, that is the spread between the number of survivors implied by the technical base the insurer chooses for the actuarial valuations and a model considered "dangerous", in the sense that it represents a more risky scenario in terms of number of survivors — a scenario which is possible with low probability. Within this framework, a particular buy-in strategy is proposed just in order to cover the risk due to an excessive number of survivors (with consequent higher future costs than expected), in the case that the dangerous survival trend comes true. The model is implemented and the empirical application is illustrated. The paper is organized as follows: in Section 2 a survey of the pension buy-in and buy-out schemes is presented; in Section 3, a description of the buy-in contracts is provided; in Section 4, the actuarial assessment of the pension plan is explained; Section 5 introduces the idea of the longevity spread and presents the buy-in strategy we propose, providing the valuation formulas. Finally, in Section 6 an empirical application of the model is shown. Section 7 concludes.

2. Buy-in and buy-out transactions: a market roadmap

The pension buy-out strategy transfers obligations and assets, all or part of them, to a third party: the pension annuity buy-out removes the pension risks by transferring the accrued pension liabilities to a regulated insurance company in return for a premium. The basic difference among these strategies is that, while the longevity hedge transfers only the longevity risk, the pension buy-in (or bulk annuity) and pension buy-out, based on the valuations of future obligations in the first case and obligations and assets in the second, transfer also other risks such as the investment risk, interest rate risk, inflation risk and in some cases operational risk.

Buy-in and buy-out transactions are extremely popular in the UK, with a steady risk transfer of over $10bn a year in 2014 and 2015, followed by around $5bn by December 2016. In 2016 the market got off to a slow start, mainly due to the headwinds of low interest rates (low interest rates make funding a buy-in transaction more expensive for unhedged pension funds because they increase the size of the pension deficit), the implementation of Solvency II and Brexit uncertainty.

Solvency II insurance regulation came into force in January 2016, and requires higher capital requirements for insurers that are offering bulk annuities. This naturally leads to a higher cost of risk transfer; however, the market remained buoyant, with steady volumes and increased competition with new entrants to the market.

In a market where funded solutions increase in price, we would expect pension funds to turn to longevity swaps as an interim solution, which prepares pension funds for a buy out at some point in the future. This is a market in which we believe that there is space for innovation and further growth.

At the current time, the longevity swap market is dominated by two distinct types of transactions:

"Traditional longevity swaps", whereby full risk transfer for longevity risk is achieved through the realized longevity of current in payment pensioners being swapped principally for fixed longevity rates plus a fee as already described in this paper. This is the preferred pension fund hedging strategy for longevity risk;

"Capital Markets Longevity Swaps" on the other hand are mainly used in the Netherlands by insurance companies to manage longevity risk. The latter swaps tend to be based on reference population based mortality statistics rather than individuals in the pension fund or annuity book. They also tend to offer out-of-the-money protection from declining mortality rates, resulting in a payment to the hedge buyer if longevity improvements exceed a certain threshold. One final feature is that they tend to be shorter dated than traditional swaps, with anywhere between 5–40 years in maturity.

We believe that various pressures could lead to the development of a "middle way" for longevity de-risking transactions which lies between "Traditional Longevity Swaps" and "Capital Market Longevity Swaps".

These pressures consist of: limited capacity in the reinsurance market which is the ultimate taker of longevity risk, the high cost of de-risking longevity in deferred liabilities and increased regulatory requirements for banks and insurance companies pertaining to longevity risk.

"Middle Way" longevity swaps would provide out-of-the-money, capital efficient transactions that could be based on individual lives instead of population based mortality rates. The use of population based transactions has been subject to increasing regulatory scrutiny recently. The scrutiny is focussed around demographic basis risk arising from the reference of the swap to population mortality rates, rather than the actual lives in the portfolio. Therefore, a 'middle way' swap, which uses capital markets features (such as shorter maturity, and having an out of the money payoff) but which references actual lives, may be a more palatable solution for market practitioners and regulators alike.

In Fig. 1 the de-risking transactions described above are presented schematically. We have used color coding to show how the features of existing transactions could combine into a new type of longevity swap. There we summarize the two principal types of longevity risk-transfer transactions that have been seen most widely executed in the longevity risk transfer market to date, along with a new type of swap, here shown as 'The Middle Way'. We believe that 'The Middle Way Swap' could form a blueprint for a new type of risk transfer transaction, which essentially bridges the features of the two most popular transactions so far, by taking the advantages for users from both, and combining them into one new instrument. The two most frequently seen transactions in the

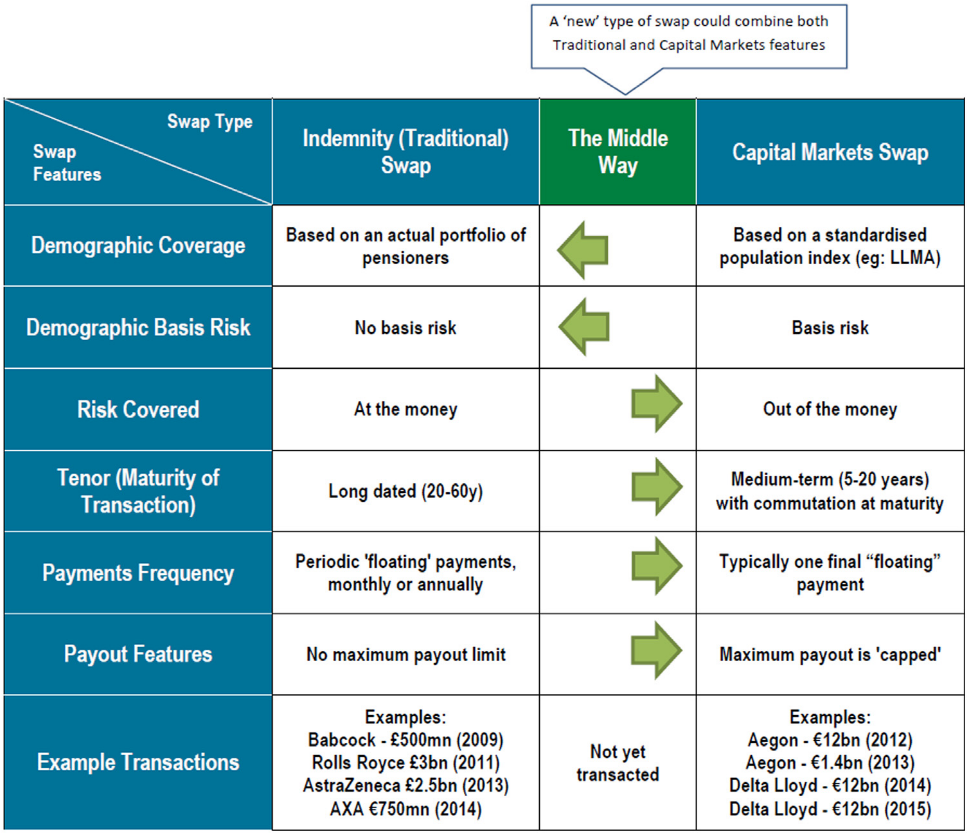

Fig. 1. A scheme of de-risking transactions.

market thus far are shown in the table as the 'Traditional Longevity Swap' and the 'Capital Markets Swap'. The former is by far the most frequently transacted type of swap to date, used mainly in the UK market by pension schemes to pass risk to the insurance and reinsurance market. This consists broadly of indemnity insurance on longevity of individual lives in a portfolio. The Capital Markets Swap on the other hand, relies on population level mortality data to determine its value, and has tenor and pay-out features that are more suited to capital markets institutions like banks and insurers. The Capital Markets Swap has largely been transacted by insurers and banks thus far, mainly for de-risking purposes by the Dutch insurance industry. 'The Middle Way' swap shown in the green column in the table represents a possible future development of the longevity risk transfer market, which removes the hurdle of basis risk, whilst maintaining the useful features of capital markets swaps such as the ability to tranche risk and minimize losses on a transaction.

3. Buy-in transactions

We are concerned with Closed Pension Plans, providing private pensions to a closed cohort of pensioners, and which may complement a public pension programme. Pension plans must meet the cost of increasing life expectancy: in other words, the liabilities for the providers increase on the basis of the unforeseen longevity trend improvements. As noted above, the institutions arranging private pension plans can operate de-risking strategies for reducing the future costs due to the longevity risk. In particular, they can transfer the risk due to the unexpected longevity to a third party by strategies as longevity hedges, such as buy-in and buy-out. The contract purchased under a "buy-in" is treated as a plan asset, so it does not affect the funding levels. Instead, a buy-out strategy removes assets and liabilities from the plan. Buy-in contracts are being widely used instead of pension annuity buy-outs, because of their affordability in respect of the cost of transferring the full amount of risk.

Most of the literature and practical applications of buy-ins, buy-outs and hedges have focused on either defined benefits (DB) pension plans or annuity portfolios held by insurance companies. Our focus is centered on defined contribution (DC) pension plans.

The technical design of a DC pension plan can be augmented by a traditional buy-in scheme where the pension plan sponsor pays an up-front premium to the insurer/reinsurer, who then makes periodic payments to the pension plan equal to those actual payments made by the sponsor to members. The bespoke buy-in contract is set up on the basis of the particular de-risking objectives of the pension plan sponsor. In this paper, we propose a tailor-made policy, addressed to segments of the pension planners for which immunization or isolation of liabilities make sense. According to a pension program, benefits must be funded through the premiums paid by members, the so-called contributions. In a pension programme, the main income consists of the contributions paid by the future pensioners during the saving (or accumulation or working) period and investment returns during the same period. So, if the contributions are underestimated in respect of the amount of liabilities, an under-funding condition develops. In the following, the buy-in strategy we are going to propose is framed within a Defined Contribution Pension Plan and provides a coverage of the longevity risk, which has a strong impact in the case of a long time horizon.

4. Actuarial description

Let us suppose that the protection is arranged in the framework of an individual pension annuity plan. The contract covers 2 periods — the first is the saving period and the second is the retirement period. We indicate by But the pension benefit the insurer or other institution (provider) pays to the insured (pensioner) at the beginning of each year while he lives, with $t = T, T+1, \ldots$ and T the retirement starting point. The pensioner pays anticipated periodic premiums Ct (contributions) during the saving period, $t = 0, 1, 2, \ldots, T-1$, to the aim of funding the benefits payable during the retirement period.

In line with a standard Defined Contribution pension plan (DC) (Olivieri and Pitacco, 2011) we will not consider flexibility in updating contributions or guarantees in the case of death or disability of the pension plan member, but fixed annual contributions (often a percentage of the members' salary) and the derived benefits amounts. Contributions and benefits are required to balance each other in a precise actuarial equilibrium assessment:

$$Contributions_{t=0}(0, T) = Benefits_{t=0}(T, \infty) \tag{1}$$

in which, on the left hand side, we have the expected present value at $t = 0$ of the contribution flow and, on the right hand side, the expected present value of the benefit flow.

The actuarial relation at time t included in the working period ($0 < t < T$), shows that the equilibrium between Contributions and Benefits, valued at that time, will be maintained thanks to the fund F_t(liabilities for the provider) accrued up until then, as follows:

$$Benefits_t(T, \infty) - Contributions_t(t, T) = V_t \tag{2}$$

whilst, for t falling during the retirement period $t \geq T$:

$$Benefits(t, T) = V_t.$$

Neglecting the expected amount of the expenses and indicating with $\delta(t-1, t)$ the rate of return arising from the investment of the fund in the period $(t-1, t)$, we can write the consistency of the fund if the person is alive at time t in the following cases:
during the working period, just after the contribution payment,

$$F_t = F_{t-1}[1 + \delta(t-1, t)] + C_t \quad t < T \tag{3}$$

with $F_0 = C_0$, and during the retirement period, just after the benefit payment,

$$F_t = F_{t-1}[1 + \delta(t-1, t)] - B_t \quad t \geq T. \tag{4}$$

We assume from now on that benefits and contributions are constants over time, having:

$$C_t = C, \quad t = 0, 1, \ldots, T-1$$

and

$$B_t = B, \quad t = T, T+1, \ldots.$$

All the actuarial calculations will be made at the issue time $t = 0$. So we adopt a forward perspective in the evaluation procedure, in the sense that the information flow necessary for valuations referred to any contractual time is that one available in $t = 0$. This assumption will be expressed, when necessary, by $''0''$ as a superscript.

The contribution amount C is given by:

$$C = \frac{B}{\ddot{a}^{(0)}_{x:T}} {}_{T/}\ddot{a}^{(0)}_x, \quad 0 \leq t < T \tag{5}$$

where $\ddot{a}^{(0)}_{x:T}$ represents the actuarial expected value of the anticipated annuity of 1 paid in case of life by the insured aged x till age $x + T$ and ${}_{T/}\ddot{a}^{(0)}_x$ is the actuarial expected present value of an anticipated annuity of one monetary amount paid in case of life to the insured surviving at time T.

As observed in Olivieri and Pitacco (2011), both in the cases of flexibility of the premiums amount and of fixed contributions as in this case, many risk management problems arise for the provider, due to the random length of the payment time horizon. The size of the fund delivering the benefits depends on the contribution level and the risk management concerns interest rate control and the forecasting of survival probabilities, during both the working and the retirement periods. The long duration of the contract implies the need for a careful risk management activity.

5. The longevity spread buy-in strategy

The proposal concerns a buy-in strategy in a longevity risk management perspective.

As argued in the literature, the systematic risk associated with the longevity phenomenon is due to the random advancement of the mode of the curve of deaths towards the ultimate lifetime (see for example Di Lorenzo and Sibillo (2002) and Pitacco (2004)). Projected tables can help in managing this risk but the choice of the correct demographic technical base among those available, that is the choice of the correct level of mortality projection, means that there is a model risk present. The approach of studying its impact on the relevant actuarial quantities was presented in Di Lorenzo and Sibillo (2002) and Coppola et al. (2011) in a context in which the uncertainty in the choice of the survival table interacts with the randomness in the financial hypotheses (i.e. the random interest rates).

We focus on the adverse implications of the choice of the technical base survival model. Specifically, we are interested in the survival model, which presents the highest deviations, in absolute value, from the chosen demographic base. In what follows, this analysis leads us to identify the most "dangerous" survival model.

Within this framework, the variance can be considered a good risk measure.

Since we observe two main risk sources (investment risk and model risk), we will use the following variance decomposition formula (cf. Knight, 2000, page 78):

$$Var[Y] = Var[E[Y|X]] + E[Var[Y|X]] \tag{6}$$

where the random variable Y is affected by two risk sources, say X and W, and has finite variance.

Now, if:

- $v(t, t+h)$ is the stochastic value in t of 1 monetary unit available in $t+h$
- $K_t(\cdot)$ is the component of the future cash flow at time t. It is the financial quantity in which we are particularly interested and it is valued at time t by means of the process $v(t, t+h)$
- c is the number of identical policies in the portfolio at issue
- M represents the random survival model used for calculations
- L_i is the random variable "curtate future lifetime" of the ith insured. The L_i's are independent of each other, conditional on any given survival function (cf. Pitacco, 2007) and are identically distributed. For this reason in the following $L_i = L$
- ${}_hp_{x+t}$ is the probability of the individual aged $x+t$ to survive to age $x+t+h$
- ω is the extreme age

from (6) we have:

$$Var[K_t(c)] = Var[E(K_t(c)|M)] + E[Var(K_t(c)|M)]. \tag{7}$$

Thus, the variance of $K_t(c)$ is split into two components, quantifying the relative weights of both the two risk sources. The first term on the right hand side of (7) provides a measure of the model risk impacting at time t on $K_t(c)$. In fact, it measures the variability of $K_t(c)$ due to the randomness of the choice of the survival model, whilst the variability of the stochastic rates of return involved in the valuation process $v(t, t+h)$ is averaged out.

Our focus is on the first term of the right hand side of formula (7):

$$Var_t\left[E\left(K_t\left(c\right)|M\right)\right]=Var_t\left[E\left(\sum_{i=1}^{c}K_t\left(i\right)|M\right)\right]$$
$$=Var_t\left\{cE\left[\sum_{h=1}^{L-t}v\left(t,t+h\right)|M\right]\right\}=$$
$$=c^2Var_t^{(M)}\sum_{h=1}^{\omega-t-1-x}{}_hp_{x+t}v\left(t,t+h\right). \quad (8)$$

Formula (8) allows for the quantification of the risk due to the uncertainty of the survival model chosen in the $K_t(\cdot)$ valuations. It is useful when risk hedging strategies are directed to specific time intervals or age intervals, which have been identified as the most unstable from the viewpoint of forecasting survival probabilities.

The mortality assumption which the provider chooses for the actuarial calculation is the result of a choice among different longevity scenarios, in the sense that he/she will model the mortality trend on the basis of the longevity behavior he/she believes the most suitable. The random nature of the longevity phenomenon implies the uncertainty in this choice, meaning that worst scenarios can be at most unlikely but possible.

As already noted in Di Lorenzo and Sibillo (2002), several studies point out the existence of precise age intervals in which the longevity model risk impact is stronger.

The aim of this paper is to develop a buy-in strategy during the retirement phase, focusing on these age intervals. The strategy will be structured as a protection against the numbers of survivals being higher than those forecast, whose realization has a small but non zero probability. In practice, there exists, on one hand, the survival model that the provider adopts and, on the other hand, the survival model that he/she considers the most "dangerous". Following this line of reasoning, the variable of interest is the spread between the numbers of survivors in the two aforementioned models hypotheses. The provider will look for the coverage of the risk arising from the exceeding number of survivors if the dangerous hypothesis will occur.

We indicate by N_j^{hP} the number of survivors at time j implied by the "dangerous model" hP, obtained by stressing the survival improvement (high Projection model). Moreover N_j^P is the number of survivors at time j forecast by the adopted demographic base P (Projected model), used for all the actuarial calculations.

We introduce the longevity spread S_j:

$$S_j=N_j^{hP}-N_j^P. \quad (9)$$

In terms of a covering strategy, the provider is able to assign the probability ρ_j – set out at the issue time $t=0$ – that the model hP will be the correct one, contrary to the forecast at time 0. In this sense ρ_j turns out to be the degree of reliability that the provider attributes to the event that the "dangerous model" should become the actual one in $t=T$, on the basis of the provider's own experience, as normally required in most of the strategic choices of the insurer.

The longevity risk that the provider aims to control by way of a hedging action is represented in formula (8). It can be transferred by an appropriate risk management activity through a buy-in transaction. The buy-in strategy's aim is to cover the eventual extra-payments due in the case the hP model should happen in place the P model. Once projecting at time 0 both the models P and hP, the pension provider can estimate the number of survivors year by year under both demographic hypotheses. The provider will focus on all the cases in which $N_j^{hP}>N_j^P$; even if hP is more projected to be higher than P, we cannot exclude the possibility that $N_j^{hP}\le N_j^P$ could sometimes happen.

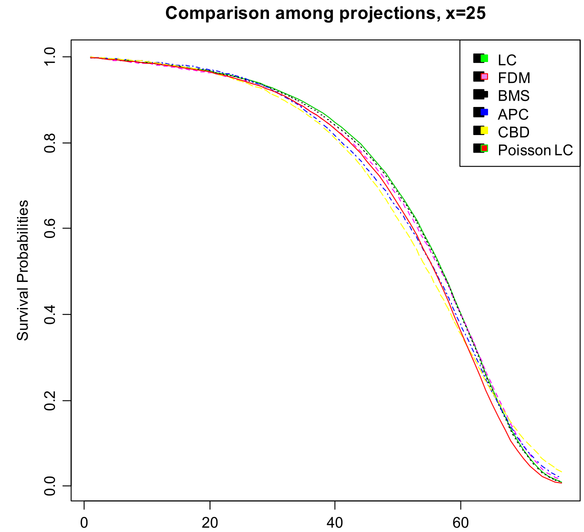

Fig. 2. Preliminary survival analysis among different stochastic models, American Male Population, $x=25$ in 2016.

If:

$$\widehat{S}_j=max\left\{N_j^{hP}-N_j^P,0\right\} \quad (10)$$

the single buy-in value $V_0^{B_i}$, valued at time 0, is the following expected value:

$$V_0^{B_i}=\sum_j\widehat{S}_j\rho_jBv'\left(0,j\right) \quad (11)$$

in which, according to formula (10), the sum is extended to those values of j verifying that $N_j^{hP}>N_j^P$ and $v'(0,j)$ is the discount factor defined according to the reinsurance policy context.

In order to realize an appropriate hedging of the risks involved, the provider has to calculate the spreads in the specific risky age intervals and assign the probabilities of the adverse scenario.

6. The longevity spread buy-in strategy and the risk control. Numerical evidence

The proposed longevity spread buy-in strategy is developed here in the case of a specific numerical example. We consider a homogeneous pension scheme of $c=1000$ plans in which all the members are US males, 25 years old at the issue time $t=0$. They pay anticipated monthly contributions during the saving period $(0\le t<T), T=40$, and receive a monthly average pension benefit $B=100/12$ during the retirement period $(t\ge T)$ until death.

We used the following R-packages: demography and StMoMo for the demographic analysis and SMIF5 for the financial one.

6.1. Demographic scenario description

In this section, we compare some of the most popular survival models in order to describe N_j^{hP} and N_j^P in formula (9). Several contributions are dedicated in the literature to this topic. In particular we recall Benchimol et al. (2016), where the authors have compared survival models by means of two model assembling approaches. Moreover, Cairns et al. (2009) have analyzed eight different stochastic mortality models, comparing their performances.

The first step consists in the comparison of different and widely used stochastic mortality models. In this exemplification we will consider the well-known traditional version of the Lee Carter — LC

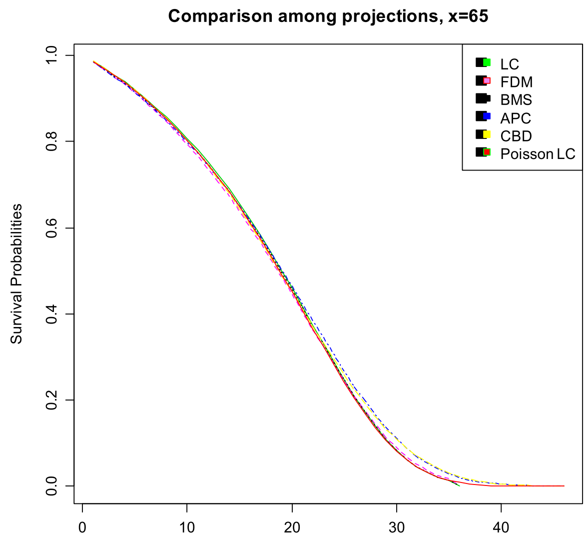

Fig. 3. Preliminary survival analysis among different stochastic models, American Male Population, $x = 65$ in 2056.

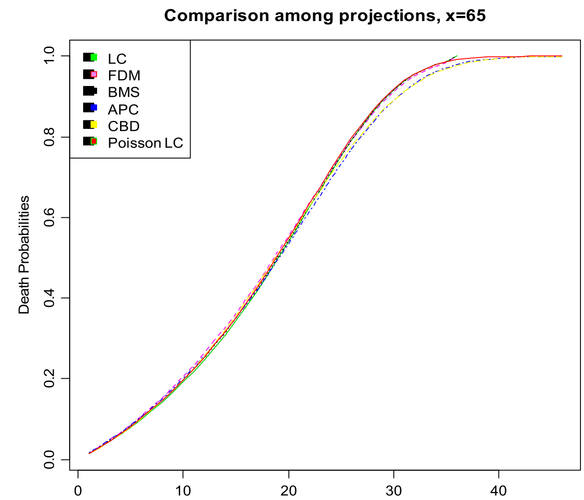

Fig. 5. Preliminary Analysis on Death Probabilities among different stochastic models, American Male Population, $x = 65$ in 2056.

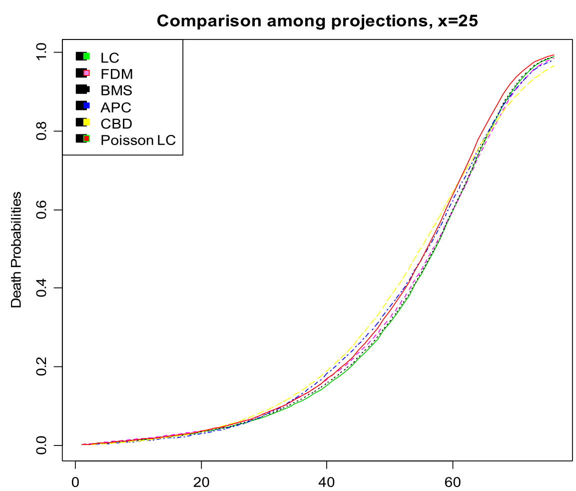

Fig. 4. Preliminary Analysis on Death Probabilities among different stochastic models, American Male Population, $x = 25$ in 2016.

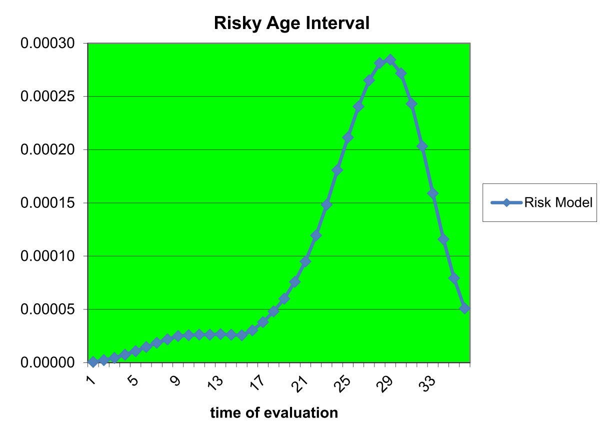

Fig. 6. The impact of the model risk on an annuity portfolio fund.

Table 1
Explained variability Poisson LC.

Error measures		
Deviance	AIC	BIC
211452.3	281848.2	283658.3

(Lee and Carter, 1992), the Booth–Maindonald and Smith model — BMS (Booth et al., 2002), the Functional Demographic model — FDM (Hyndman and Ullah, 2007), the Age–Period–Cohort model — APC (Renshaw and Haberman, 2006), the Cairns, Blake and Dowd model — CBD (Cairns et al., 2006), and the Poisson Lee Carter Model — Poisson LC (Renshaw and Haberman, 2003). The analysis is here based on data referred to the US Male population (age 0:100, 1955–2015) from the Human Mortality Database.

In Figs. 2–5 the survival probabilities and the death rates for a US male aged 25 in 2016 and aged $x = 65$ in 2056 are shown in the six cases. Within these models, we will select the most meaningful ones for the aim of our application.

On the basis of the descriptions in Figs. 2 and 4, considering the long term perspective and the prudential approach in choosing the first order demographic technical base, the insurer opts for the Poisson Lee Carter model.

Then, for what concerns the buy-in strategy, the insurer considers Figs. 3 and 5, and we assume that in his opinion the most dangerous model is the CBD one. The basic equations characterizing both the models, the Poisson Lee Carter and the CBD, are recalled in Appendix A. Throughout the application, we develop the buy-in strategy during the retirement phase and in particular refer to the specific interval in which the CBD model is perceived by the insurer as the most risky with respect to the technical base. The next step is just the definition of this interval. The trend in Fig. 6, points out the impact of the dangerous scenario, measured as in formula (8). It refers to the survivors aged 65 belonging to the initial cohort of 1000 insureds aged 25. In this application, according to Fig. 6, we consider the age interval 85–100 as the most risky. The choice of this interval has a strong impact on the annuity payments of life office pensioners, as also shown in Khalaf-Allah et al. (2006). On the basis of this evidence, we will focus the longevity-spread buy-in strategy on this particular age interval.

For the sake of completeness, in Figs. 7, 8, 9, 10, and in Figs. 11, 12, 13, 14, besides Tables 1 (Schwarz, 1978) and 2 (Akaike, 1973), we provide the graphical and numerical evidences of the fitting, residuals and forecasting analysis respectively for the Poisson LC and CBD models.

Since the survival probabilities are very small quantities, to better appraise the differences between them, we conduct an

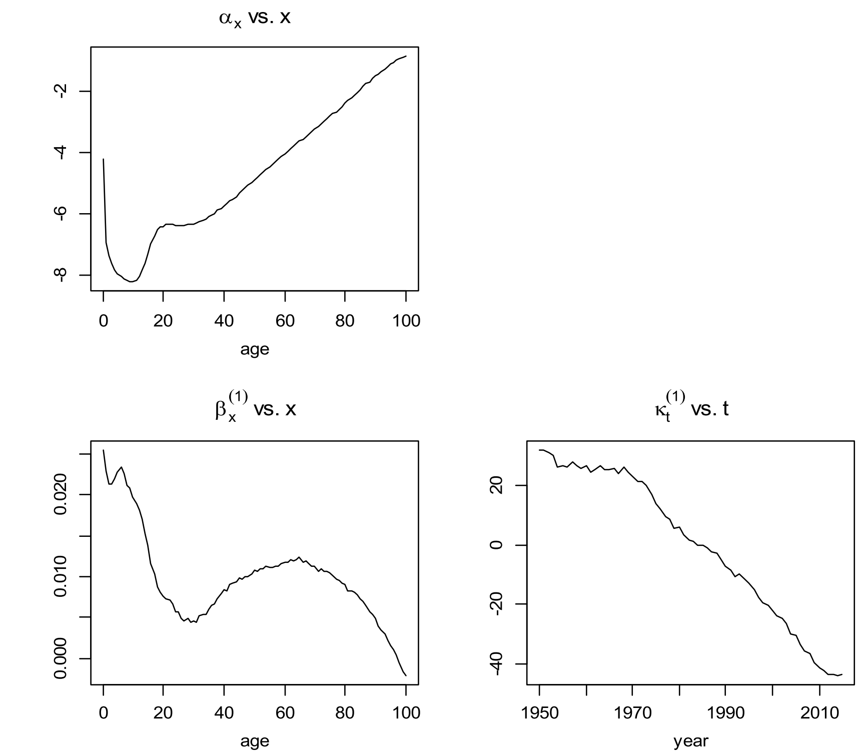

Fig. 7. Fitting of the parameters on American Male population — Poisson LC.

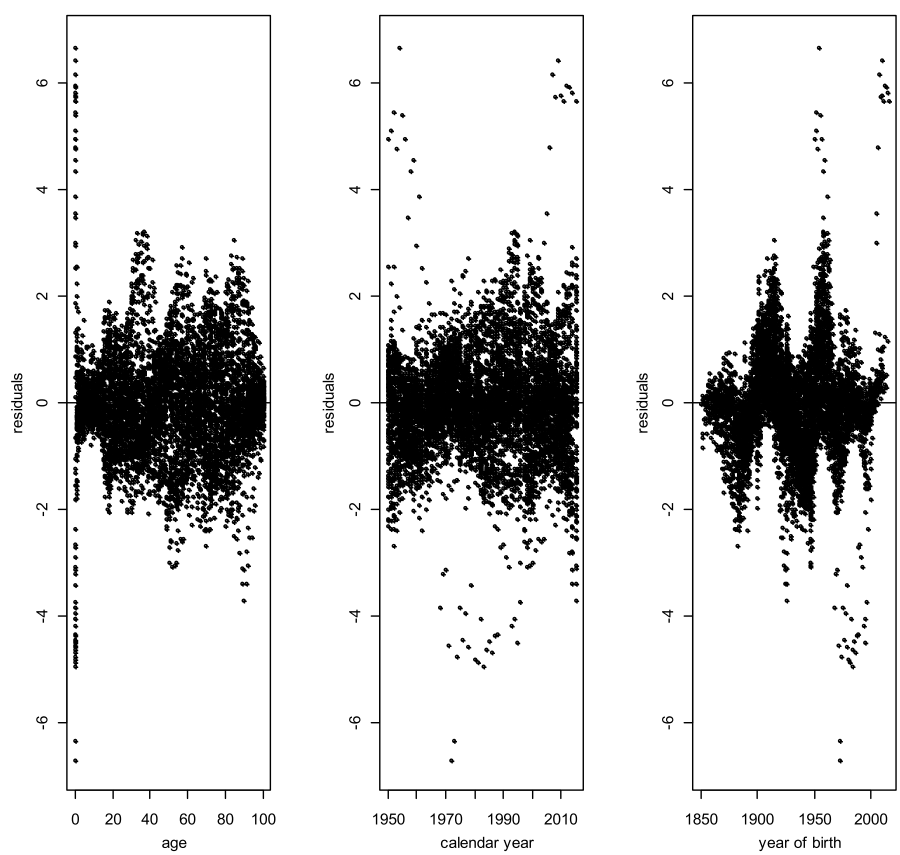

Fig. 8. Residuals vs age, calendar year, year of birth — Poisson LC.

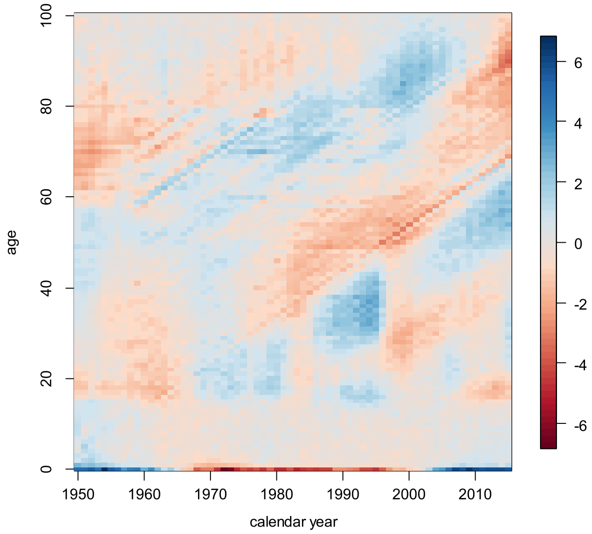

Fig. 9. Residual analysis — Poisson LC.

Table 2
Explained variability CBD.

	Error measures	
deviance	AIC	BIC
63214.36	89934.42	90696.48

exploratory data analysis by using as dissimilarity measure, the Markov operator distance $d_{MO}(X, Y)$, with X and Y being time series. Consider two regularly sampled data:

$$X_i = X(i\Delta) \text{ and } Y_i = Y(i\Delta), \quad i = 0, 1, \ldots, N$$

where $\Delta > 0$, not shrinking to 0 and such that $T = N\Delta$. The following relation holds, as in De Gregorio and Iacus (2010):

$$d_{MO}(X, Y) = \sum_{j,k \in J} \left| \left(\hat{P}_\Delta\right)_{j,k}(X) - \left(\hat{P}_\Delta\right)_{j,k}(Y) \right| \quad j = 0, 1, \ldots, N \tag{12}$$

for a given L^2-orthonormal basis $\{\varphi_j, j \in J\}$ of $L^2([a, b])$ where J is an index set, by following Gobet et al. (2004), we can obtain an estimator $\hat{P}_\Delta$ of $< P_\Delta\varphi_j, \varphi_k > \mu_{b,\sigma}$ with

$$\left(\hat{P}_\Delta\right)_{j,k}(Z) = \frac{1}{2N} \sum_{i=1}^{N} \left\{ \varphi_j(Z_{i-1}) \varphi_k(Z_i) + \varphi_k(Z_{i-1}) \varphi_j(Z_i) \right\}, \quad j, k \in J. \tag{13}$$

The terms $\left(\hat{P}_\Delta\right)_{j,k}$ are approximations of $\langle P_\Delta\varphi_j, \varphi_{jk}\rangle\mu_{b,\sigma}$, that is the action of the transition operator on the state space with respect of the unknown scalar product $\langle\cdot, \cdot\rangle\mu_{b,\sigma}$.

In particular, $\mu_{b,\sigma}$ is the unknown invariant distribution of the process depending on the unknown drift $b(\cdot)$ and diffusion $\sigma(\cdot)$ coefficients (see Appendix B). Then $\hat{P}_\Delta$ can represent a proxy of the probability structure of the model.

Figs. 15–19 explain how the different projections of the survival probabilities, death rates and central death rates ($x = 25$ and $x = 65$) for all the models initially considered, are related each other. In the Poisson Lee Carter and CBD heat maps, the gradient of colors is shown in the legends of Figs. 9 and 13.

Positive residuals are represented by a range of colors between light-blue and dark-blue, negative residuals are in different shades of red, and small or absent residuals in earth up to almost white.

In the Poisson Lee Carter setting, the residuals have a larger range of values, reflecting a poorer fitting performance. Nevertheless, several diagonals in the CBD heat map in correspondence of

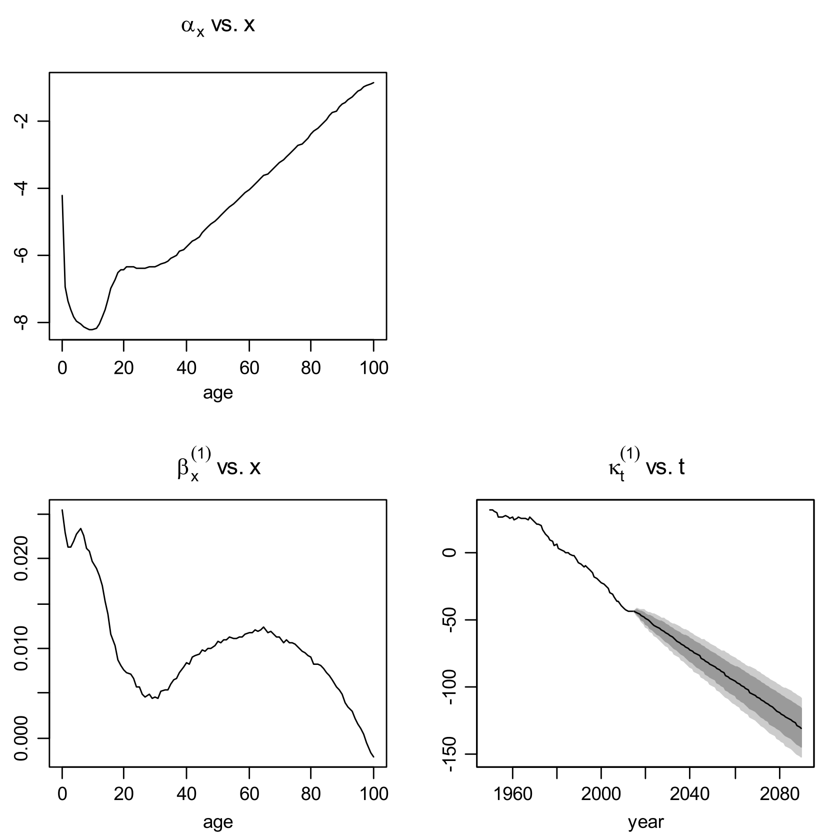

Fig. 10. Forecasting — Poisson LC.

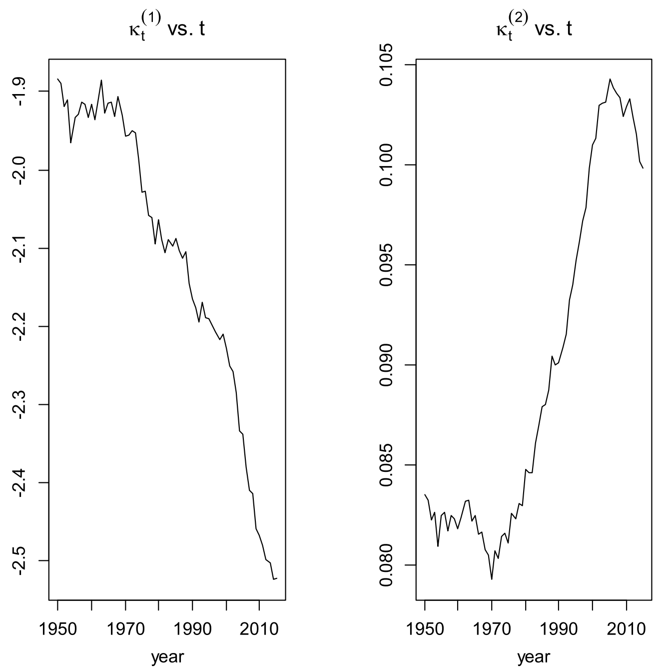

Fig. 11. Fitting of the parameters on American Male population — CBD.

different calendar years reveal a poor capacity of the model for capturing any cohort effects.

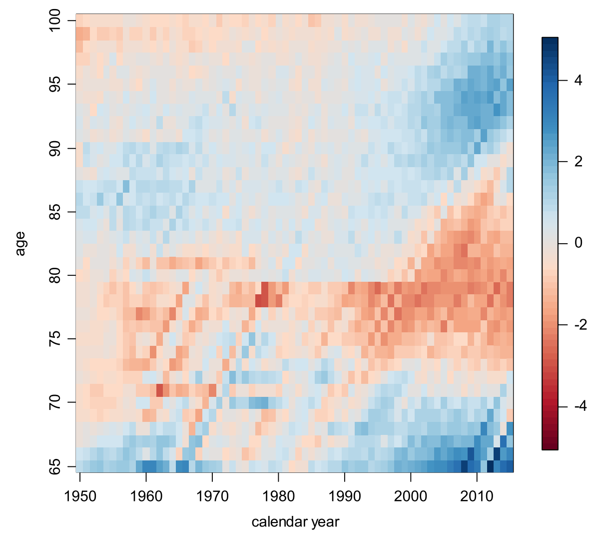

Fig. 13. Residual analysis — CBD.

Moreover, in Table 3 we report the percentage of explained variability for each of the three multidimensional scaling representations referred, respectively, to the survival probabilities and death probabilities for $x = 25$ and $x = 65$, and for the central mortality rates.

As we have highlighted in the earlier discussion, the buy-in strategy aims to hedge the uncertainty in the choice of the survival model. The uncertainty can lead to the use of the "wrong" survival model, and this can produce pernicious effects on the insurer's

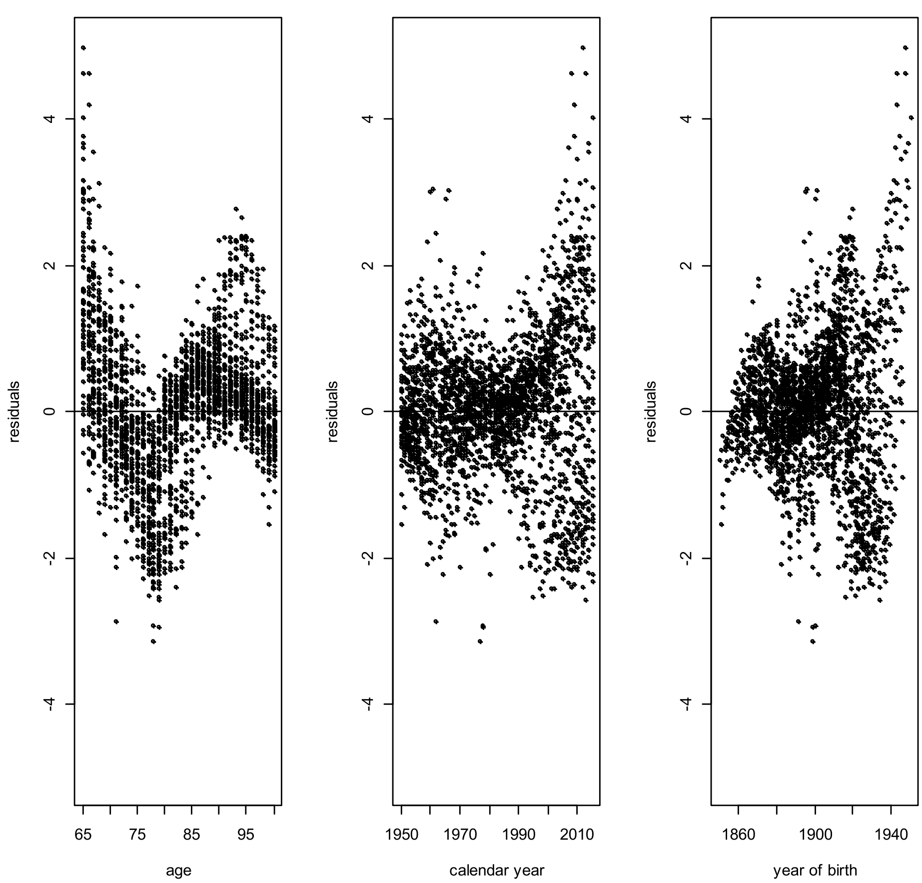

Fig. 12. Residuals vs age, calendar year, year of birth — CBD.

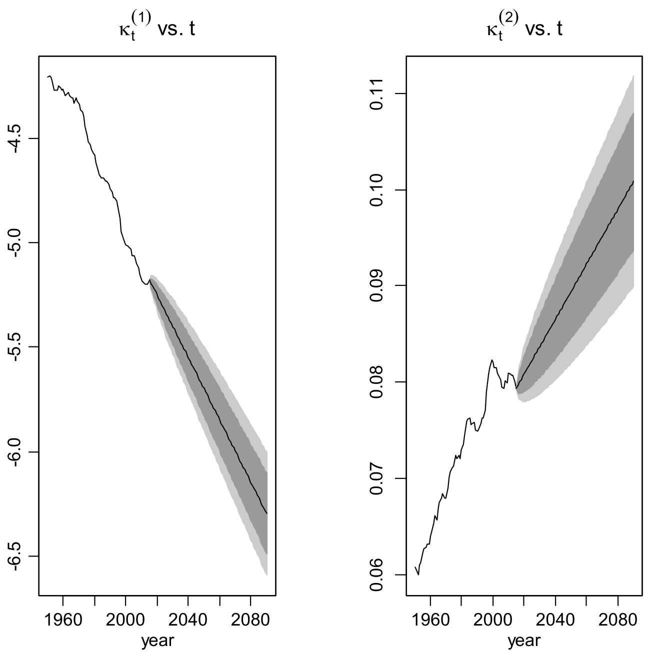

Fig. 14. Forecasting — CBD.

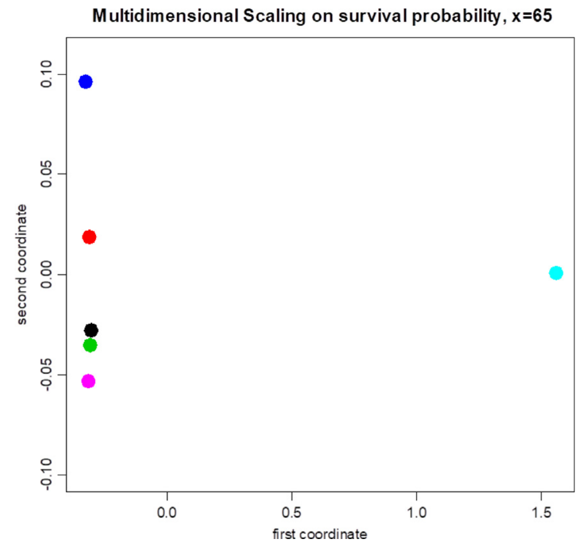

Fig. 16. Comparisons by Multidimensional Scaling among survival probabilities, insured aged $x = 65$. The turquoise, blue, red, green, black, fuchsia are respectively referred to the probabilities inferred by the models CBD, APC, FDM, BMS, LC, Poisson LC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

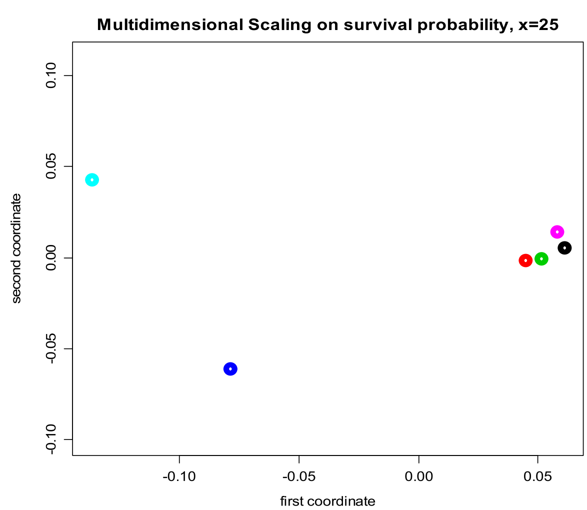

Fig. 15. Comparisons by Multidimensional Scaling among survival probabilities, insured aged $x = 25$. The turquoise, blue, red, green, black, fuchsia are respectively referred to the probabilities inferred by the models CBD, APC, FDM, BMS, LC, Poisson LC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

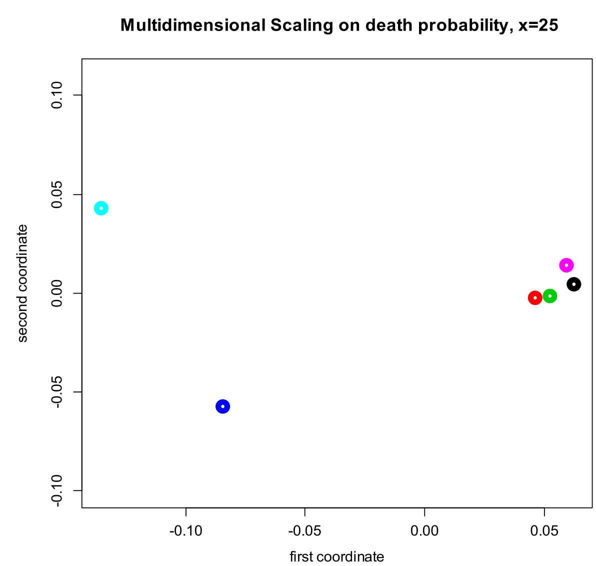

Fig. 17. Comparisons by Multidimensional Scaling among death rates, insured aged $x = 25$. The turquoise, blue, red, green, black, fuchsia are respectively referred to the probabilities inferred by the models CBD, APC, FDM, BMS, LC, Poisson LC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3
Explained variability.

Parameter	
On survival probability, $x = 25$	0.9212166
On death probability, $x = 25$	0.9212166
On survival probability, $x = 65$	0.9968947
On death probability, $x = 65$	0.9968947
On central death rate $m_{x,t}$	0.9868483

solvency. The real longevity could reveal itself to be higher than the one arising from the technical base and the longevity spread buy-in strategy aims to hedge just this situation.

The longevity spread given by Eqs. (9) and (10) between the Poisson LC and CBD models is more evident in Fig. 20, in which we show the distribution of the longevity spread between Poisson LC and CBD survivors in the age interval 85–100 referring to the buy-in strategy.

6.2. Financial scenario description

Over the past few years, many stochastic models have been developed to represent the future dynamics of interest rates on the basis of an accurate fitting of the observed yield curve both at the long and short maturities. Due to its tractability and flexibility, a single-factor model has become very popular, yielding a simple closed form solution for the bond prices: the CIR model (Cox et al., 1985). Nevertheless, as shown in Canabarro (1995), single factor models do not accurately fit the observed yield curves in respect of

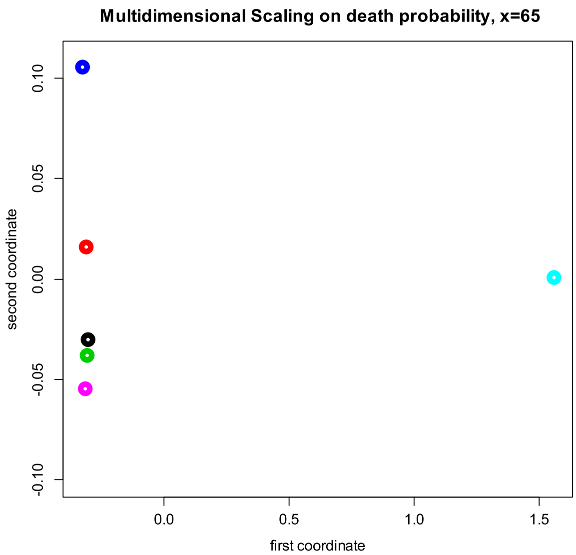

Fig. 18. Comparisons by Multidimensional Scaling among death rates, insured aged $x = 65$. The turquoise, blue, red, green, black, fuchsia are respectively referred to the probabilities inferred by the models CBD, APC, FDM, BMS, LC, Poisson LC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

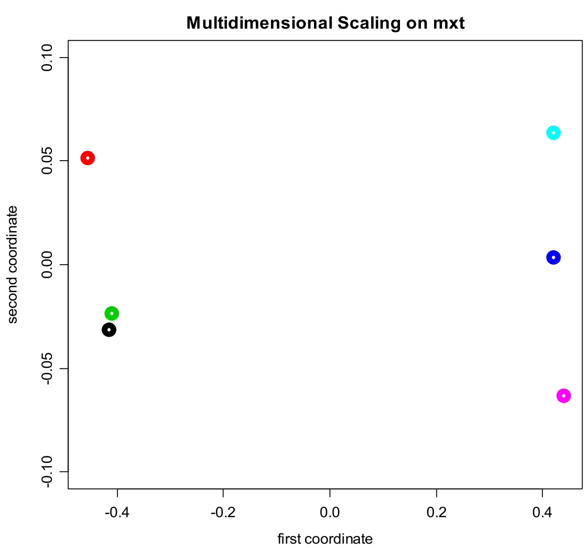

Fig. 19. Comparisons by Multidimensional Scaling among central death rates. The turquoise, blue, red, green, black, fuchsia are respectively referred to the probabilities inferred by the models CBD, APC, FDM, BMS, LC, Poisson LC. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

multifactor generalizations. In particular, the dependence on a single factor greatly limits the possible shapes of the yield curve and leads to forecasts that are not reliable in a long-term context. Indeed, the multi-factor generalization can incorporate random variations in long term rates as highlighted by Chen and Scott (2003).

In light of these considerations, multi-factor generalization appears more appropriate to represent the stochastic fluctuations of interest rates. In particular, we take into account the nominal pricing model of Cox et al. (1985) Let the instantaneous nominal interest rate r be obtained as the sum of two independent state variables, $r = Y_1 + Y_2$, both generated by square root diffusion processes:

$$dY_i\,(t) = k_i\,(\vartheta_i - Y_i\,(t)\,dt) + \sigma_i\sqrt{Y_i\,(t)}dW_i\,(t) \qquad i = 1, 2 \tag{14}$$

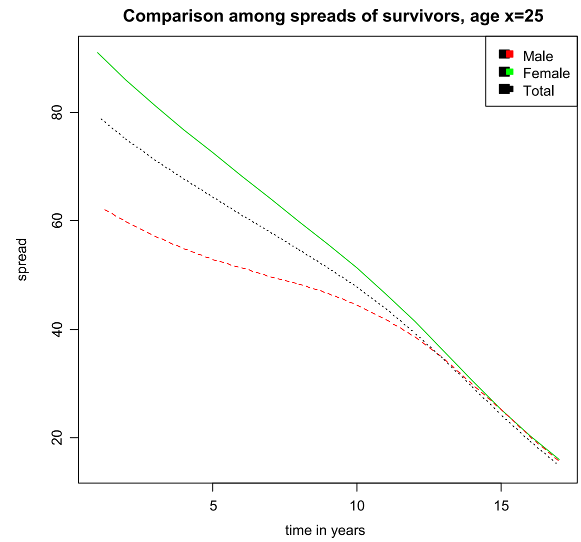

Fig. 20. Comparison among spreads of survivors, age interval 85–100.

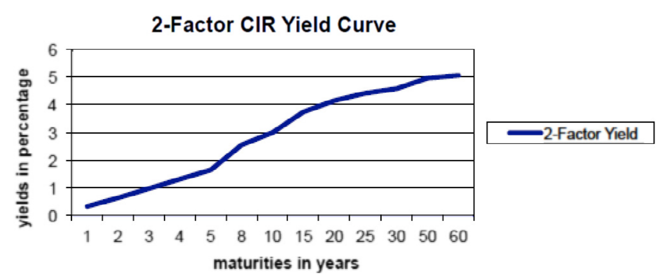

Fig. 21. Calibration of the two factor CIR model on Treasury Bonds.

Table 4
The two-factor Cox Ingersoll and Ross parameter estimation.

Parameter	
k_1	0.03300
k_2	0.05040
Θ_1	0.01532
Θ_2	0.03281
σ_1	0.80560
σ_2	0.06600

as described in (Chen and Scott, 2003) where $W_1\,(t)$ and $W_2\,(t)$ are (independent) Wiener processes, being k_1 and k_2 the speed of mean reversion for each individual process, ϑ_1 and ϑ_2 the risk neutral long run mean, σ_1 and $\sigma_2$the volatility of the factors.

A calibration of the model is here performed according to Remillard (2013) on the American Treasury bonds ranging from (2nd January) 1990 up to (15th May) 2012 for short and long maturities respectively 3-months, 10 years. The parameter estimates are given in Table 4.

Fig. 21 shows the outcome on the dataset under consideration, which determines an increasing yield curve.

On the basis of these results for the interest rates, we calculate the premium payable during the working period of the pension scheme within a fair valuation context. In a defined contribution plan, fixed contributions are paid into an individual account by employers and employees. We will calculate the expected present value of future benefits and the single buy-in value considering, as an example, the interest rate at level of 3%, coherently with the two-factor CIR yield curve in Fig. 21.

To calculate the single buy-in premium, in this application we consider a simplified case, positing in formula (10), that $\rho_j = \rho, \forall j$.

This assumption appears reasonable, considering that the aim of the application is a strategy covering infrequent but dangerous events. Hence formula (11) becomes:

$$V_0^{Bi} = \rho \sum_j \hat{S}_j B v'(0, j) \quad (15)$$

On the basis of the previous presented demographic scenarios, the buy-in value is computed, taking into account the longevity spreads expressed in terms of the potential number of survivors exceeding the expected numbers. In this example, the strategy for choosing the liabilities to be insured by the buy-in transaction involves the selection of the liabilities related to the age intervals characterized by a strong longevity improvement (ages 85 up to 100 in this case).

In our example, the single buy-in value from Eq. (11) results:

$$V_0^{B_i} = 13254.47\rho.$$

The size of the buy-in value can be expressed as a ratio of V_0^{Bi} the expected present value of future benefits:

$$\frac{13254.47}{350383.3}\rho \simeq 0.037\ \rho.$$

7. Conclusions

In the previous sections, we established a general equation from which a buy-in value can be found in the context of defined contribution pension plans. The main point of the novel approach is represented by the strategy which we propose to guarantee the affordability of the buy-in transaction taking into account the demographic model risk.

The suggested strategy involves the appropriate risky age intervals characterized by the high impact of the survival model risk, where a measure for quantifying this influence is indicated. Such a measure represents the variability due to the effect of the randomness of the projection, the effect of the other risk components (random rates of return and mortality random deviations) having been averaged out.

Following this line of argument, the pension plan sponsors can achieve a convenient buy-in strategy by hedging against the longevity spreads occurring in those age intervals where the risk of underestimating future liabilities is higher. The idea that we propose is to consider the longevity spreads between two survival models, both realistically possible: the most reliable and the most dangerous ones, valued from the point of view of the pension provider. The longevity spreads will be the risky quantities to hedge. In the previous section, an implementation of the strategy on a pension plan and in a specified demographic and financial scenario is illustrated.

By means of this contract, the pension provider covers the risk of the model which he considers to be the most dangerous, providing higher numbers of payments than expected. This is realized in accordance with a prudential perspective, allowing flexibly for the insurer's subjective perception of the risk due to the most dangerous model (and which is practically quantified by the value assigned to the degree of reliability of the dangerous model). Within this context, an eventual actual experience worse than that one due to the most dangerous model would not be transferred by the buy-in strategy that we propose; this tail risk, characterized by a low probability, would remain with the pension provider.

From the counterparty's point of view, the liability is limited to the spread up to the experience of the most dangerous model and for this he receives an anticipated single premium.

However, the above techniques can be applied also in the cases of different designs and configurations of pension plans. Future researches could be developed on more complex buy-in operations, where the de-risking strategy is also addressed to transfer the investment risk, beyond the longevity risk.

Appendix A

The Lee Carter model in the Poisson setting is characterized by the following expressions:

$$D_{xt} \approx Poisson\left(E_{xt}\mu_{xt}\right)$$

in which

$$\mu_{xt} = exp\left(\alpha_x + \beta_x k_t\right)$$

and E_{xt} being the exposures to the risk of death (in other words the number of person years from which D_{xt} occurs) and where the parameters are subjected to the constraints:

$$\sum_t k_t = 0 \qquad \sum_t \beta_x = 1.$$

The force of mortality is thus assumed to have the log-bilinear form:

$$ln\left(\mu_{xt}\right) = \alpha_x + k_t\beta_x.$$

The main characteristic of the CBD setting is that the mortality is represented by the logit of the initial death rates, according to the following equation:

$$log\frac{q_x(t)}{1-q_x(t)} = \beta_x^{(1)}k_t^{(1)} + \beta_x^{(2)}k_t^{(2).}$$

Note that $q_x(t)$ is the probability of an insured aged x of dying by t, $\beta_x^{(i)}$functions reflect the age effects, representing $k_t^{(i)}$ the period-related effects.

Appendix B

Let X_t be an ergodic diffusion process solution to

$$dX_t = b(X_t)\,dt + \sigma(X_t)\,dW_t, \qquad 0 \le t \le T.$$

Let $s(x) = \exp\left\{-2\int_{x_0}^{x}\frac{b(y)}{\sigma^2(y)}dy\right\}$ and $m(x) = \frac{1}{\sigma^2(x)s(x)}$

be the scale and the speed measures $x_0 \in [a, b]$, $[a, b]$ the state space of X.

Then the invariant measure of X is $\mu_{b,\sigma}(x) = \frac{m(x)}{M}$, with $M = \int_{x_0}^{x} m(x)$.

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