Generalized Impulse Balance: An Experimental Test for a class of 3×3 Games

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Abstract

In this paper we introduce a generalized version of impulse balance equilibrium. The stationary concept is applied to 3×3 games based on the Bailiff and Poacher Game (Selten, 1991) and its predictive success is experimentally tested against the one of Nash equilibrium. Experiments with 26 different games were conducted; 12 games with completely mixed Nash equilibria and 14 games with partially mixed Nash equilibria. In all games, generalized impulse balance yields predictions that are closer to the data than the ones of Nash equilibrium. Overall, generalized impulse balance has a significantly higher predictive success than Nash equilibrium.

JEL classification: C70, C91

Keywords: Impulse balance, Nash equilibrium, 3×3 games, stationary concepts

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1 Introduction

The predominance of behavioral concepts in terms of descriptive success over Nash equilibrium is a well documented fact. Regularly, behavioral concepts describe the data gathered in laboratory experiments much better than the Nash equilibrium does. Such behavioral concepts are, for example, new stationary concepts (e.g., McKelvey & Palfrey, 1995, 1998; Selten & Chmura, 2008; Goerg & Selten, 2009), models of level-k thinking (e.g., Nagel, 1995; Ho, Camerer, & Weigelt, 1998; Crawford & Costa-Gomes, 2006), and models describing learning processes (e.g., Erev & Roth, 1995, 1998; Ho, Camerer & Chong, 2007; Chmura, Goerg & Selten, 2012).[1]

One stationary alternative that performed significantly better in previous studies than Nash equilibrium is impulse balance equilibrium (Selten & Chmura, 2008). It is a non-parametric concept based on the idea of learning direction theory (Selten & Buchta 1999). Probabilities of decisions are modeled as behavioral tendencies and adjusted similar to the adjustments of a marksman aiming at a trunk: *"If he misses the trunk to the right, he will shift the position of the bow to the left and if he misses the trunk to the left he will shift the position of the bow to the right. The marksman looks at his experience from the last trial and adjusts his behavior [. . .]."* (p. 86, Selten & Buchta, 1999). If there is no need for further adjustments a stationary state is reached.

In previous studies impulse balance was first applied to auctions (Selten, Abbink, & Cox, 2005; Ockenfels & Selten, 2005) and later on to 2×2 games (Avrahami, Güth, & Kareev, 2005; Selten & Chmura, 2008), $2 \times 2 \times 2$ games (Avrahami, Güth, & Kareev, 2005), as well as, cyclic games (Goerg & Selten, 2009). In addition to out-perfoming Nash, impulse balance equilibrium, performed at least as good as quantal response equilibrium in 2×2 games. Although, quantal response equilibrium is a parametric concept and impulse balance

[1] In Ho, Camerer & Chong (2007) self-tuning experience-weighted attraction learning competes against quantal response equilibrium, which is a harder competitor than Nash.

equilibrium is not (Selten & Chmura, 2008; Brunner, Camerer & Goeree, 2011; and Selten, Chmura & Goerg, 2011).

Up to now impulse balance equilibrium was only applied to games in which players had two decision alternatives to choose from. In this paper now, we introduce a generalized version of impulse balance equilibrium, which can be applied to any normal form game. The predictive power of generalized impulse balance is then tested against the one of Nash equilibrium in experimental 3×3 games, which are based on the *Bailiff and Poacher Game* (Selten, 1991). In total we gathered data on 26 games, 12 games with completely mixed Nash equilibria and 14 games with partially mixed Nash equilibria. Each game was played over 200 rounds by 12 matching groups of 8 fixed players.

Over all games, impulse balance performs significantly better than Nash equilibrium. In both types of games – games with completely mixed Nash equilibria and games with partially mixed Nash equilibria – generalized impulse balance yields predictions that are closer to the data than the ones of Nash. In fact, generalized impulse balance has in all 26 games a smaller quadratic distance to the data than Nash. This difference in predictive power is significant in 24 out of the 26 games.

In the following we will describe the game structure of the *Bailiff and Poacher Game*, define the two stationary concepts and apply them to the *Bailiff and Poacher Game*. Thereafter, experimental design and procedure are presented. In the result section, we compare the performances of generalized impulse balance and Nash over all games, within the two different type of games and within the two player types. We conclude the paper with a short summary and discussion.

2 Game Structure and Stationary Concepts

In this section, we first introduce the structure of the experimentally investigated games, which are all based on the Bailiff and Poacher Game (Selten, 1991). After that, we provide the conditions for completely and partially mixed Nash equilibria. Subsequently, we define the generalized impulse balance and apply the concept to the Bailiff and Poacher Game.

2.1 The Bailiff and Poacher Game

The *Bailiff and Poacher Game* (p. 144 in Selten, 1991) is a simple 3×3 games in which a poacher tries to steal fish from three different ponds. A bailiff can secure only one of the three ponds at a time. If the poacher is caught he gets punished; if the poacher is not caught he can steel fish from the pond and receives a payoff for it. The ponds have different sizes and therefore, they yield different payoffs to the poacher if he is not caught. The game is very similar to *Rock, Paper, Scissors* with the difference that it is not a symmetric game.

Poacher

Bailiff

	q_1	q_2	q_3
p_1	$+1$, 0	0, V_2	0, V_3
p_2	0, V_1	$+1$, 0	0, V_3
p_3	0, V_1	0, V_2	$+1$, 0

Figure 1: The structure of the investigated Bailiff and Poacher Game

Figure 1 gives the payoff matrix of the *Bailiff and Poacher Game.* The bailiff receives a payoff of 1 if he catches the poacher or a payoff of 0 if he does not. If the poacher is not caught, he receives V_i which is different for each pond i. The payoffs V_i are ordered $V_1 > V_2 > V_3 > 0$ and reflect the different pond sizes. If the poacher is caught he receives a payoff of zero.[2]

[2] To avoid negative payoffs in our experiment, the punishment is set to zero. Instead one could punish the

2.2 Nash Equilibrium

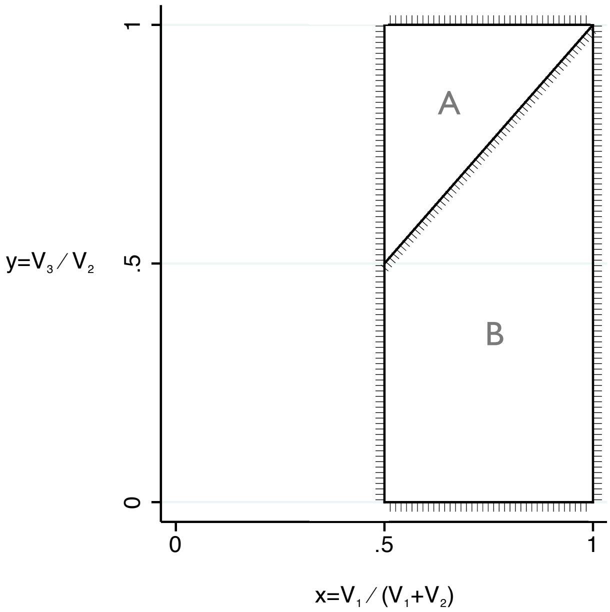

Figure 2: Payoff combinations with completely mixed equilibria (A) and partially mixed equilibria (B). x and y are auxiliary variables with $x = V_1/(V_1 + V_2)$ and $y = V_2/V_2$. If the border of an area is ruled on the outside it does not belong to this area. The border between A and B belongs not to A, but to B for $\frac{1}{2} < x < 1$.

In this paper we investigate 3×3 games with unique equilibria that are either completely mixed $\big(p_1 > p_2 > p_3 > 0$, and $q_i = \frac{1}{3}$ for $i = 1, 2, 3\big)$ or partially mixed $\big(p_1 > p_2 > p_3 = 0$, $q_i = \frac{1}{2}$ for $i = 1, 2$, and $q_3 = 0\big)$ with p_i denoting the probabilities of the row player and q_i the ones of the column player. In appendices A.1 and A.2 we show that the games have exactly one equilibrium for combinations of payoffs V_i that fulfill $V_1 > V_2 > V_3 > 0$. In appendix A.3 we determine the payoff conditions that either lead to completely or partially mixed equilibria. The following inequality 1 gives the payoff condition for a completely mixed

poacher, which would yield negative payoffs. However, this would not affect the Nash equilibrium as long as the punishment is the same for all ponds.

equilibrium and inequality 2 gives the payoff condition for a partially mixed equilibrium.

$$V_3 > \frac{V_1 V_2}{V_1 + V_2} \qquad \text{with } p_1 > p_2 > p_3 > 0, \text{ and } q_i = \tfrac{1}{3} \text{ for } i = 1, 2, 3 \tag{1}$$

$$V_3 \leq \frac{V_1 V_2}{V_1 + V_2} \qquad \text{with } p_1 > p_2 > p_3 = 0, q_i = \tfrac{1}{2} \text{ for } i = 1, 2, \text{ and } q_3 = 0 \tag{2}$$

All combinations of payoffs leading to either completely mixed or partially mixed equilibria are given in figure 2. Set A contains all combinations of V_1, V_2, V_3 with $V_1 > V_2 > V_3 > 0$ that result in completely mixed equilibria and set B contains all combinations that result in partially mixed equilibria. All investigated games have combinations of payoffs V_1, V_2, V_3 such that they are either part of set A or B. They are given in section 3.1.

Given the payoffs the resulting equilibrium probabilities for completely mixed games are then calculated as

$$p_i = 1 - \frac{\frac{2}{1+V_i}}{\frac{1}{1+V_1} + \frac{1}{1+V_2} + \frac{1}{1+V_3}}, \text{ and } q_i = \frac{1}{3}, \text{ for } i = 1, 2, 3. \tag{3}$$

and the ones in the partially mixed games with $V_1 + V_2 + V_3 = 15$ as[3]

$$p_i = \frac{V_i}{15}, \text{ for } i = 1, 2, 3, q_i = \frac{1}{2} \text{ for } i = 1, 2, \text{ and } p_3 = 0. \tag{4}$$

2.3 Generalized Impulse Balance

Generalized impulse balance assumes that a player i follows behavioral tendencies reflected in the probabilities $p_{i1}, .., p_{in}$ for selecting the choices $\phi^*_{i1}, ..., \phi^*_{in}$. These probabilities are not consciously chosen, but in line with learning direction theory (Selten & Buchta, 1999; Selten, Abbink, & Cox, 2005), the result of an adjustment process.

[3] We standardized the payoffs with 15 to obtain 14 different combinations of V_1,V_2, and V_3 (see appendix A.5). However, the Nash equilibrium is invariant to the multiplication of all payoffs with a constant, and therefore, other payoffs with sums different to 15 could be used.

Consider a situation in which a player i has chosen a pure strategy ϕ_i in the preceding period, but in the present period it turned out that another strategy ϕ_i^* would have been the better choice. We call such a situation a *foregone payoff situation.* A player will be motivated in such a situation to move towards the superior pure strategy ϕ_i^*. Thus, in response to the foregone payoff situation probabilities $p_{i1}, .., p_{in}$ will change. If no more changes of these probabilities occur impulse balance equilibrium is reached.

Selten & Chmura (2008) defined impulse balance equilibrium (Ockenfels & Selten, 2005) for completely mixed 2×2 games. Payoffs of those completely mixed 2×2 games can be parameterized as in figure 3. The field D, L player 1 experiences the forgone payoff c_L and in U, R he experiences c_R. In U, L player 2 experiences a forgone payoff d_U and in D, R he experiences d_D. Therefore, for player 1 foregone payoff situations are on the off diagonal and for player 2 on the main diagonal in figure 3. In such forgone payoff situations impulses from strategy ϕ_i to ϕ_i^* occur. If expected impulses from ϕ_{i1} to ϕ_{i2} are equal to expected impulses from ϕ_{i2} to ϕ_{i1} then impulse balance is achieved. In this case a probability vector reflecting the behavioral tendency of a player will reproduce itself in the long run.

		Player 2	
		L	R
Player 1	U	$a_L + c_L$; b_U	a_R ; $b_U + d_U$
	D	a_L ; $b_D + d_D$	$a_R + c_R$; b_D

Figure 3: Structure of 2×2-Games with mixed equilibria

In Chmura, Goerg, & Selten (2012) we demonstrated that impulse balance leads to impulse proportionality in 2×2-games.[4] In the following, we will turn our attention to the general case of the impulse balance and define the equilibrium probabilities via impulse proportionality. In the general case, impulses are calculated based on the impulse matrix, which is determined from the payoff matrix. In the following a denotes the payoffs from the matrix for the column player and b the ones of the row player. α_j is the lowest payoff for the column player in column j and β_i is the lowest payoff for the row player in row i.

$$a = (\alpha_{ij})_{n \times m} \quad \text{and} \quad b = (\beta_{ij})_{n \times m}$$
$$\alpha_j = \min_{i=1,2,3} \alpha_{ij} \quad \text{and} \quad \beta_i = \min_{j=1,2,3} \beta_{ij}$$

The impulse matrixes for column and row players are then given as:

$$A = (a_{ij}) \quad \text{with} \quad a_{ij} = \alpha_{ij} - \alpha_j$$
$$B = (b_{ij}) \quad \text{with} \quad b_{ij} = \beta_{ij} - \beta i$$

Let $p = (p_1, ..., p_n)$ and $q = (q_1, ..., q_m)$ be the mixed strategies of the two players. In the following T denotes the transpose operator for matrices, as well as, row and column vectors.

[4]In Chmura, Goerg, & Selten (2012), we introduced the concept of impulse matching learning. According to this learning algorithm, players choose actions with probabilities that are determined by the proportionality of impulses. Let us take a look at the example of the row player. The stationary point of this process is reached, i.e. the row player does no longer adjust the probabilities, if the ratio of the two probabilities for U (given as p_U) and D (given as p_D) is the same as the ratio of expected impulses for U and D: $\frac{p_U}{p_D} = \frac{q_L c_L}{q_R c_R}$. Impulses to U (given as c_L) occur when L is played by the column player (probability q_L); impulses to D (given as c_R) occur when R is played (probability q_R). The equation can be rewritten as $p_D q_L c_L = p_U q_R c_R$, which is the impulse balance equation. In impulse balance equilibrium the impulse balance equation must be fulfilled. Thus, in 2×2-games impulse balance equilibrium leads to impulse proportionality.

Then, the expected impulses $I(q)$ and $J(p)$ of the two players are given as

$$I(q) = \begin{pmatrix} I_1(q) \\ ... \\ I_n(q) \end{pmatrix} = Aq^T \quad \text{and} \quad J(p) = \Big(J_1(p), ..., J_m(p) \Big) = Bp$$

In the generalized impulse balance equilibrium, the two players play according to impulse proportionality, which is given as:

$$\begin{aligned} p_i &= \frac{I_i(q)}{I_1(q) + ... + I_n(q)} \quad \text{for} \quad i = 1, ..., n \\ q_j &= \frac{I_j(p)}{I_1(p) + ... + I_m(p)} \quad \text{for} \quad i = 1, ..., m \end{aligned}$$

Application to the Bailiff and Poacher Game: Recall figure 1 which gives the payoff matrix of the investigated games. For these games, the payoff matrix and the impulse matrix are identical. The expected impulses are then given as

$$I(q) = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \quad \text{and} \quad J(p) = \Big((1-p_1)V_1, (1-p_2)V_2, (1-p_3)V_3 \Big).$$

For generalized impulse balance the equilibrium $p_i = q_i$ for $i = 1, 2, 3$ and q_i is calculated according to the impulse proportionality:

$$q_j = \frac{(1-p_i)V_i}{(1-p_1)V_1 + (1-p_2)V_2 + (1-p_3)V_3} \quad \text{for} \quad i = 1, 2, 3 \tag{5}$$

Uniqueness: In the following we show that the investigated games have unique impulse matching equilibria. From equation 5 it follows that

$$\frac{p_i}{1-p_i} = \frac{V_i}{(1-p_1)V_1 + (1-p_2)V_2 + (1-p_3)V_3} \quad \text{for} \quad i = 1, 2, 3$$

We introduce the auxiliary variable Z as $Z = (1-p_1)V_1 + (1-p_2)V_2 + (1-p_3)V_3$, insert it into the previous equation, solve it, and simplify for p_i.

$$\begin{aligned}
\frac{p_i}{1-p_i} &= \frac{V_i}{Z} \\
p_i &= (1-p_i)\frac{V_i}{Z} \\
1-p_i &= \frac{1}{1+\frac{V_i}{Z+V_i}} \\
p_i &= \frac{V_i}{Z+V_i}
\end{aligned}$$

Given p_i we can rewrite Z as

$$\begin{aligned}
Z &= \frac{V_1}{1+\frac{V_1}{Z}} + \frac{V_2}{1+\frac{V_2}{Z}} + \frac{V_3}{1+\frac{V_3}{Z}} \\
Z &= Z\Big(\frac{V_1}{V_1+Z} + \frac{V_2}{V_2+Z} + \frac{V_3}{V_3+Z}\Big)
\end{aligned}$$

The sum of the probabilities must be $p_1 + p_2 + p_3 = 1$ and thus the following equation must be fulfilled:

$$\frac{V_1}{Z+V_1} + \frac{V_2}{Z+V_2} + \frac{V_3}{Z+V_3} = 1 \tag{6}$$

The left side of equation 6 is equal to three for $Z = 0$ and it becomes smaller with increasing

Z. The limit for $Z \to \infty$ is zero. Consequently, equation 6 has exactly one solution and this solution is positive.

3 Experimental Design

We now turn to the experimental design. We first give the parameters of the investigated games and the resulting Nash equilibria, as well as, generalized impulse balance equilibria. Afterwards we turn to the procedural details.

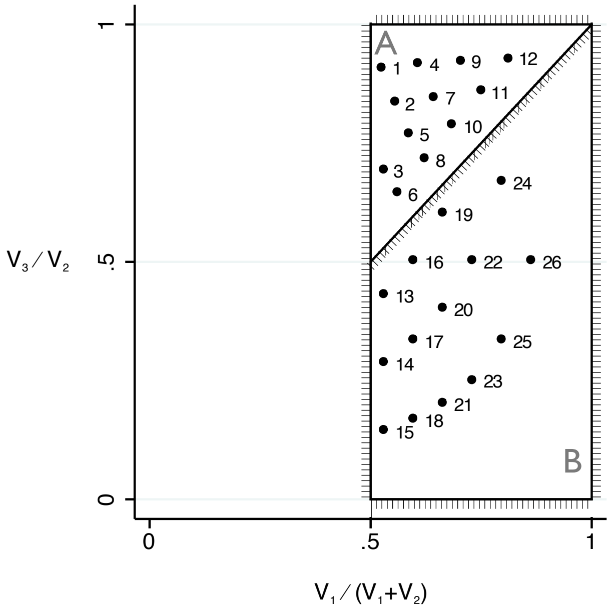

Figure 4: Actual payoff combinations for completely mixed equilibria (A) and partially mixed equilibria (B). Payoff combinations are labeled with game numbers. If the border of an area is ruled on the outside it does not belong to this area. The border between A and B belongs not to A, but to B.

Game	V_1	V_2	V_3	Nash Equilibrium						Generalized Impulse Balance					
				Row Player			Column Player			Row Player			Column Player		
				Up	Middle	Down	Left	Middle	Right	Up	Middle	Down	Left	Middle	Right
1	55	49.5	45	0.40	0.33	0.27	0.33	0.33	0.33	0.36	0.33	0.31	0.36	0.33	0.31
2	60	48	40	0.46	0.33	0.20	0.33	0.33	0.33	0.38	0.33	0.29	0.38	0.33	0.29
3	58.5	52	36	0.46	0.40	0.14	0.33	0.33	0.33	0.38	0.35	0.27	0.38	0.35	0.27
4	66	42	38.5	0.53	0.27	0.20	0.33	0.33	0.33	0.41	0.30	0.29	0.41	0.30	0.29
5	65	45.5	35	0.53	0.33	0.14	0.33	0.33	0.33	0.41	0.32	0.27	0.41	0.32	0.27
6	63	49	31.5	0.53	0.40	0.07	0.33	0.33	0.33	0.40	0.34	0.25	0.40	0.34	0.25
7	71.5	39	33	0.60	0.27	0.14	0.33	0.33	0.33	0.44	0.30	0.26	0.44	0.30	0.26
8	70	42	30	0.60	0.33	0.07	0.33	0.33	0.33	0.44	0.32	0.25	0.44	0.32	0.25
9	78	32.5	30	0.66	0.20	0.14	0.33	0.33	0.33	0.47	0.27	0.26	0.47	0.27	0.26
10	77	35	27.5	0.66	0.27	0.07	0.33	0.33	0.33	0.47	0.29	0.24	0.47	0.29	0.24
11	84	28	24	0.73	0.20	0.07	0.33	0.33	0.33	0.51	0.26	0.23	0.51	0.26	0.23
12	91	21	19.5	0.79	0.14	0.07	0.33	0.33	0.33	0.56	0.23	0.21	0.56	0.23	0.21
13	8	7	3	0.53	0.47	0.00	0.50	0.50	0.00	0.41	0.38	0.21	0.41	0.38	0.21
14	8	7	2	0.53	0.47	0.00	0.50	0.50	0.00	0.44	0.40	0.16	0.44	0.40	0.16
15	8	7	1	0.53	0.47	0.00	0.50	0.50	0.00	0.47	0.43	0.10	0.47	0.43	0.10
16	9	6	3	0.60	0.40	0.00	0.50	0.50	0.00	0.44	0.35	0.21	0.44	0.35	0.21
17	9	6	2	0.60	0.40	0.00	0.50	0.50	0.00	0.47	0.37	0.16	0.47	0.37	0.16
18	9	6	1	0.60	0.40	0.00	0.50	0.50	0.00	0.50	0.40	0.10	0.50	0.40	0.10
19	10	5	3	0.67	0.33	0.00	0.50	0.50	0.00	0.48	0.31	0.21	0.48	0.31	0.21
20	10	5	2	0.67	0.33	0.00	0.50	0.50	0.00	0.50	0.33	0.17	0.50	0.33	0.17
21	10	5	1	0.67	0.33	0.00	0.50	0.50	0.00	0.53	0.36	0.10	0.53	0.36	0.10
22	11	4	2	0.73	0.27	0.00	0.50	0.50	0.00	0.53	0.29	0.17	0.53	0.29	0.17
23	11	4	1	0.73	0.27	0.00	0.50	0.50	0.00	0.57	0.32	0.11	0.57	0.32	0.11
24	12	3	2	0.80	0.20	0.00	0.50	0.50	0.00	0.57	0.25	0.18	0.57	0.25	0.18
25	12	3	1	0.80	0.20	0.00	0.50	0.50	0.00	0.61	0.28	0.11	0.61	0.28	0.11
26	13	2	1	0.87	0.13	0.00	0.50	0.50	0.00	0.65	0.22	0.13	0.65	0.22	0.13

Table 1: Payoffs and resulting equilibria; Games 1-12 have an unique Nash equilibrium in completely mixed strategies; Games 13-26 have an unique Nash equilibrium in partially mixed strategies; Probabilities in the table are rounded to the first two numbers after the decimal point.

3.1 Experimental Games and Predictions

In total we gathered data on 26 games, 12 games with completely mixed equilibria and 14 games with partially mixed equilibria. The games were selected to cover a broad range of parameter combinations fulfilling the requirements discussed in section 2.2. Figure 4 gives the distribution of payoffs used for the completely mixed games (A) and partially mixed games (B).

Table 1 gives the actual payoffs for each game, as well as, the resulting probabilities of Nash equilibrium and generalized impulse balance. Games 1 to 12 are with completely mixed equilibria and games 13-26 are with partially mixed equilibria. We tried to find combinations (V_1, V_2, V_3) that were integer numbers or with .5 after the decimal point. Sections A.4 and A.5 in the Appendix give more details on the selection of payoffs (V_1, V_2, V_3).

3.2 Procedures

The experiments were conducted in 2007 at the Smith Experimental Economics Research Center of the Shanghai Jiao Tong University. The experiment was programmed in z-Tree (Fischbacher, 2007) and subjects were students recruited via online announcements. In total 1,246 students participated.

The experiments were conducted by local research assistants. Upon arrival, subjects were seated in cubicles, the printed instructions were distributed, and read to the subjects.[5] After all remaining questions were answered the experiment started. During each session two games were played, each of the games was played for 200 rounds. Subjects were assigned to matching groups of 8 players, 4 subjects in the role of column players and 4 subjects in the role of row players. The roles and the matching groups were fixed for the course of the whole experiment. Subjects knew that their roles were fixed, but did not know the size of

[5] An English translation of the instructions is available in Appendix B.

the matching groups and they were not informed about the second game to come.

Games 1-13					Games 14-26				
1. Game	#Obs.	2. Game	#Obs.	# Subjects	1. Game	#Obs.	2. Game	#Obs.	# Subjects
1	6	14	6	48	14	6	1	6	48
2	6	15	6	48	15	6	2	6	48
3	6	16	6	48	16	6	3	6	48
4	6	17	6	48	17	6	4	6	48
5	6	18	6	48	18	6	5	6	48
6	6	19	6	48	19	6	6	6	48
7	6	20	6	48	20	6	7	6	48
8	6	21	6	48	21	6	8	6	48
9	6	22	6	48	22	6	9	6	48
10	6	23	6	48	23	6	10	6	48
11	6	24	6	48	24	6	11	6	48
12	6	25	6	48	25	6	12	6	48
13	6	26	6	48	26	6	13	6	48

Table 2: Order of played games, number of gathered observations, and number of subjects

For each of the 26 games we gathered 12 observations. The first 6 observations are completely independent as they were gathered as first games played in each session. The additional 6 observations per game were gathered with the second games in the sessions. In total we gathered 156 completely independent observations, i.e., the games that were played first, and additional 156 observations resulting from the games that were played second in each session. Table 2 gives an overview of the order in which the games were played and the number of collected observations.

4 Results

In the following we will compare the predictive success of Nash equilibrium and generalized impulse balance equilibrium. Our measure of predictive success is the mean quadratic distance of the corresponding concept to the data. The predictive success of a stationary concept increases with a decreasing mean quadratic distance, i.e., the smaller the mean quadratic distance, the closer the predictions to the experimental data. The mean quadratic distance Q is the average quadratic distance over all 26 games and over all 12 matching groups per game. It is defined as

$$Q = \frac{1}{26}\sum_{g=1}^{26}\left(\frac{1}{12}\sum_{n=1}^{12}\left(\sum_{i=1}^{3}(q_{i,g} - f_{i,g,n}^{\text{Poacher}})^2 + \sum_{i=1}^{3}(p_{i,g} - f_{i,g,n}^{\text{Bailiff}})^2\right)\right),$$

with $f_{i,g,n}$ being the observed mean frequencies for strategy i in game g and matching group n of the poacher or the bailiff. The probabilities for the poacher and bailiff for strategy i in game g are given as $q_{i,g}$ and $p_{i,g}$, respectively.

4.1 Overall Comparison

Figure 5 gives the mean quadratic distances of Nash equilibrium and impulse balance equilibrium in the completely and partially mixed games. Table 3 gives the mean quadratic distances for each game, overall games, for the completely mixed and for the partially mixed games. In addition, p-values of the two-sided Fisher-Pitman permutation test for paired replicates are given.[6]

The quadratic distance of Nash to the data is over all games 2.2 times larger then the one of impulse balance; 1.8 times larger in the completely mixed games and 2.6 times larger in the partially mixed games. Thus, impulse balance performs significantly better then Nash equilibrium over all games and, in addition, in the subsets of completely and partially mixed games (all comparisons with $p < 0.01$, two-sided Fisher-Pitman permutation test). In fact,

[6] All tests within one game are based on all 12 observations, the tests over different games are only based on the first 6 observations. We did this to prevent testing two observations from the same matching group as independent observations. However, the order of games does not influence the behavior: comparing the mean quadratic distance to Nash in the 156 games played first with the one from the 156 played second yields no significant differences ($p = 0.868$, two-sided Fisher-Pitman permutation test). The same holds true if we compare the quadratic distance of generalized impulse balance ($p = 0.943$).

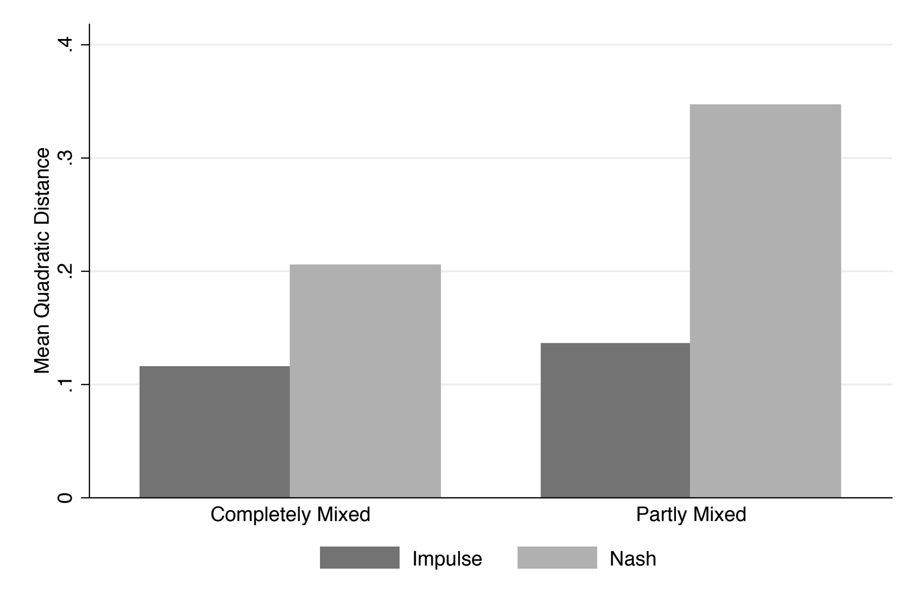

Figure 5: Mean quadratic distances of generalized impulse balance and Nash equilibrium in completely and partially mixed games

impulse balance equilibrium has in all 26 games a smaller quadratic distance to the data than the Nash equilibrium. The respective difference is for all but two games (5 and 19) significantly smaller for impulse balance.

4.2 Comparison by Player Type

In the previous section we have tested the mean quadratic distances over both player types. We now analyze the behavior of row and column players separately.

The mean quadratic distance of column players' behavior to the predictions of Nash equilibrium is 1.9 times larger than to the predictions of impulse balance; the one of the row players even 2.5 times. Thus, impulse balance yields for both player types predictions that are significantly more accurate then the ones of Nash (both $p < 0.01$, two-sided Fisher-Pitman permutation test).

	Game	Mean Quadratic Distance Nash equilibrium	Impulse balance	Test result
Completely Mixed Games	1	.0081996	.0012142	$p < 0.01$
	2	.046566	.0161672	$p < 0.01$
	3	.3925316	.3370174	$p = .024$
	4	.3533856	.3157302	$p = .069$
	5	.438024	.4311328	$p = .693$
	6	.1479702	.0436702	$p < 0.01$
	7	.0889139	.0036896	$p < 0.01$
	8	.1067579	.0035281	$p < 0.01$
	9	.2257292	.1066953	$p < 0.01$
	10	.2447339	.1094036	$p < 0.01$
	11	.1773554	.0063303	$p < 0.01$
	12	.2358219	.0106885	$p < 0.01$
Partially Mixed Games	13	.4418155	.093474	$p < 0.01$
	14	.102257	.0035309	$p < 0.01$
	15	.0538412	.0057088	$p < 0.01$
	16	.1594839	.0023688	$p < 0.01$
	17	.5023688	.1825391	$p < 0.01$
	18	.5692925	.33373	$p < 0.01$
	19	.2267045	.1659287	$p = .148$
	20	.1364765	.0043931	$p < 0.01$
	21	.6072232	.3554866	$p < 0.01$
	22	.6190886	.2515403	$p < 0.01$
	23	.1364617	.0119635	$p < 0.01$
	24	.2552776	.0126557	$p < 0.01$
	25	.7569099	.4457908	$p < 0.01$
	26	.2805532	.0257224	$p < 0.01$
Overall		.2812978	.1261577	$p < 0.01$
1-12		.2054991	.1154389	$p < 0.01$
13-26		.3462682	.1353452	$p < 0.01$

Table 3: Mean quadratic distances for Nash equilibrium and generalized impulse balance. The p-Values are for the two-sided Fisher-Pitman permutation test with paired replicates, test within games are based on all 12 observations, tests over different games are based on the first 6 independent observations

Figure 6 gives the mean quadratic distances of the two stationary concepts to each player type in completely and partially mixed games. The figure reveals that the better performance of impulse balance for column players is driven by the bad performance of Nash in partially mixed games. There, Nash has a significantly larger distance to the behavior of column

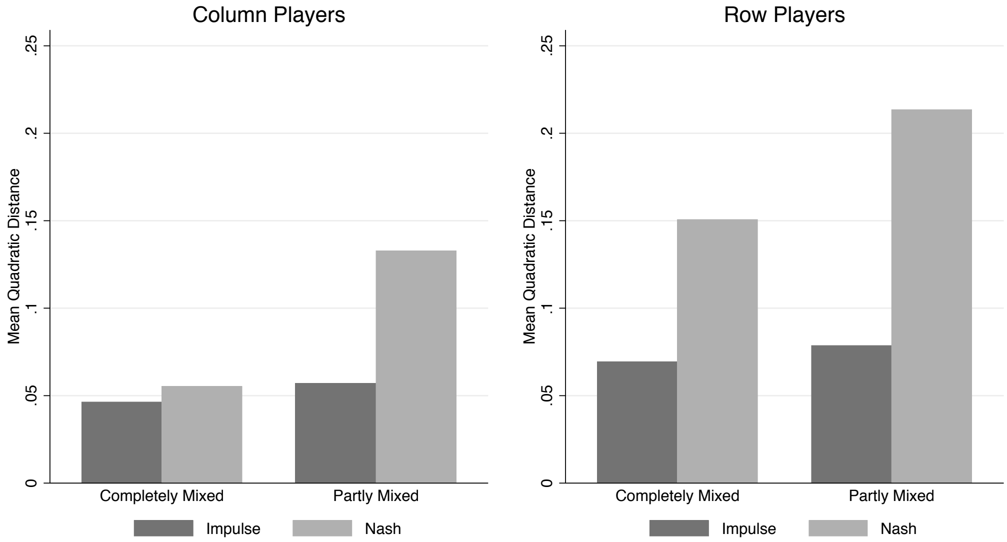

Figure 6: Mean quadratic distances of impulse balance equilibrium and Nash equilibrium in completely and partially mixed games for the two player types

players than impulse balance ($p < 0.01$, two-sided Fisher-Pitman permutation test); while in completely mixed games the difference is not significant ($p = 0.25$, two-sided Fisher-Pitman permutation test). For row players the picture looks a little bit different. There, impulse balance dominates Nash in completely, as well as, partially mixed games (both $p < 0.01$, two-sided Fisher-Pitman permutation test).

5 Summary and Discussion

In this paper we derived a generalized version of impulse balance equilibrium, which is a non-parametric stationary concept based on the idea of learning direction theory (Selten & Buchta, 1999). The predictions of generalized impulse balance were tested against the ones of Nash in 26 experimental 3×3 games, all based on the *Bailiff and Poacher Game* (Selten, 1991).

Over all games generalized impulse balance performed significantly better than Nash equilibrium: the quadratic distance of the Nash equilibrium to the data was 2.2 times larger then the one of impulse balance. Generalized impulse balance did not only perform better over all games, but in each single game of the 26 games played. In all but two games this smaller distance to the data was statistically significant with a two-sided Fisher-Pitman permutation test.

Our 3×3 data, together with the evidence gathered in 2×2-games (Avrahami, Güth, & Kareev, 2005, Selten & Chmura, 2008), $2 \times 2 \times 2$-games (Avrahami, Güth, & Kareev, 2005), as well as, cyclic games (Goerg & Selten, 2009), suggests that impulse balance is very good predictor for human behavior in games with mixed Nash equilibria. In all of these studies impulse balance outperformed the mixed predictions of Nash equilibrium.

With quantal response equilibrium (McKelvey and Palfrey, 1995) there already exists a well established concept, which usually fits empirical data better than Nash does. However, it has one free parameter to be estimated from the data, while impulse balance is parameter free. We, therefore, believe that impulse balance is a powerful alternative to Nash equilibrium for games with mixed strategies.

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A Equilibria and Payoffs

The games in our experiment have the following structure, with $V_i > V_j > V_k > 0$

Bailiff \ Poacher	i	j	k
i	$+1$, 0	0, V_j	0, V_k
j	0, V_i	$+1$, 0	0, V_k
k	0, V_i	0, V_j	$+1$, 0

Figure 7: The structure of the investigated 3×3 games

In the following sections, we first show the uniqueness of completely and partially mixed equilibria for the investigated games. Thereafter, we will determine the conditions that lead to completely mixed and partially mixed equilibria and, finally, we will show how we selected the payoffs of the 26 games investigated in this paper.

A.1 Uniqueness of the completely mixed equilibrium

The equilibrium probabilities for the bailiff are given as:

$$p_i = 1 - \frac{\frac{2}{1+V_i}}{\frac{1}{1+V_1} + \frac{1}{1+V_2} + \frac{1}{1+V_3}},$$

while the poacher plays with

$$q_i = \frac{1}{3} \text{ , for } i = 1, 2, 3.$$

The expected payoff of the bailiff is:

$$E_{\text{Bailiff}} = \frac{1}{3}$$

The expected payoff of the poacher is:

$$
\begin{aligned}
E_{\text{Poacher}} &= \frac{1}{3}[p_1(V_2+V_3)+p_2(V_1+V_3)+p_3(V_1+V_2)] \\
&= \frac{1}{3}[(1-p_1)V_1+(1-p_2)V_2+(1-p_3)V_3] \\
&= \frac{V}{\frac{1}{1-p_1}+\frac{1}{1-p_2}+\frac{1}{1-p_3}}
\end{aligned}
$$

with $V = V_1 + V_2 + V_3$.

Proposition 1. *If $p = (p_1, p_2, p_3)$ with $p_1 > p_3 > p_3 > 0$ and $p_1 + p_3 + p_3 = 1$ is an equilibrium of the game, the game has no additional equilibria.*

Proof:

1. The game has no pure strategy equilibrium. This is straight forward given the game structure in figure 7.

2. It remains to be shown that no equilibrium exists for which the pure strategy k has the probability $p_k = 0$ and the two remaining strategies i and j have positive probabilities p_i and p_j.

We consider the following part of the original game:

	i	j
i	$+1$, 0	0, V_j
j	0, V_i	$+1$, 0

If player D does not use strategy k it cannot be part of W's equilibrium strategy. Thus, an equilibrium with $p_k = 0$ must be an equilibrium of the above 2×2 game. The equilibrium probabilities of this game are given as:

$$
q_i = q_j = \frac{1}{2},\ p_i = \frac{V_i}{V_i + V_j}, \text{ and } p_j = \frac{V_j}{V_i + V_j}.
$$

Without loss of generality we assume that $V_i > V_j$, which implies $1/V_j > 1/V_i$ and thus $p_i > p_j$. Then, the resulting equilibrium payoffs in the 2×2 game are

$$E_{\text{Bailiff}} = \frac{1}{2}, \text{ and } E_{\text{Poacher}} = \frac{1}{\frac{1}{V_i} + \frac{1}{V_j}}.$$

The equilibrium of the 2×2 game is an equilibrium point of the investigated 3×3 game if

$$\frac{1}{\frac{1}{V_i} + \frac{1}{V_j}} \geq V_k$$

holds. Otherwise, V_k would be the only best response of D. Thus, the investigated 3×3 games have a completely mixed equilibrium with $p_k > 0$, if

$$\frac{1}{\frac{1}{V_i} + \frac{1}{V_j}} < V_k.$$

This corresponds to

$$\frac{1}{V_k} < \frac{1}{V_i} + \frac{1}{V_j}.$$

The left side reaches its maximum for $i = 1$ and $j = 2$ and the inequality holds for all triple with

$$\frac{1}{V_3} < \frac{1}{V_1} + \frac{1}{V_2}.$$

Assume that $p = (p_1, p_2, p_3)$ with $p_1 > p_3 > p_3 > 0$ and $p_1 + p_3 + p_3 = 1$ is a completely mixed equilibrium of the game. From the inverse formula it follows that the last inequality is equivalent to :

$$1 - p_3 < 1 - p_1 + 1 - p_2 \Leftrightarrow 0 < 1 - p_1 - p_2 + p_3 \Rightarrow 0 < 2p_3$$

Which is always true. Thus, we conclude our first result:

Result 1. *Our 3×3 game has exactly one completely mixed equilibrium $p = (p_1, p_2, p_3)$ with $p_1 > p_3 > p_3 > 0$ if conditions $\frac{1}{V_3} < \frac{1}{V_1} + \frac{1}{V_2}$ and $V_1 > V_2 > V_3 > 0$ are fulfilled. If this is the case no other equilibrium than the completely mixed one exists.*

A.2 Uniqueness of the partially mixed equilibrium

The game has a partially mixed equilibrium (p_1, p_2, p_3) with $p_1 > 0, p_2 > 0$ and $p_k = 0$ if

$$V_k \leq \frac{1}{\frac{1}{V_i} + \frac{1}{V_j}}.$$

This is the case iff

$$\frac{1}{V_k} \geq \frac{1}{V_i} + \frac{1}{V_j}$$

is true. If such an equilibrium for $k = 1, 2, 3$ exists adding the corresponding inequalities yields

$$\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} \geq 2[\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}],$$

which is obviously not true. Can two of those equilibria exist? If so, one of the equilibria must be with $p_3 = 0$. Therefore, it follows that:

$$\frac{1}{V_3} \geq \frac{1}{V_1} + \frac{1}{V_2}.$$

If the game has in addition a partially mixed equilibrium with $p_2 = 0$ it follows that:

$$\frac{1}{V_2} \geq \frac{1}{V_1} + \frac{1}{V_3}.$$

Substituting V_3 in the previous inequality leads to

$$\frac{1}{V_2} \geq \frac{1}{V_1} + \frac{1}{V_1} + \frac{1}{V_2}.$$

Which corresponds to

$$0 \geq \frac{2}{V_1}.$$

Therefore, there exists no partially mixed equilibrium with $p_2 = 0$. Analogously, one can show that the same must hold for $p_1 = 0$. Thus, in a partially mixed equilibrium (p_1, p_2, p_3) it is necessary that $p_3 = 0$. Since the game must have at least one equilibrium we have the following result.

Result 2. *If the game does not have a completely mixed equilibrium it follows that the equilibrium of the part game with strategies* 1 *and* 2 *is also the equilibrium of the whole game. No further equilibria exist for the whole* 3×3 *game.*

A.3 Payoff conditions for the different types of Equilibria

From results 1 and 2 it follows that for $V_1 > V_2 > V_3 > 0$ the following result must be true:

Result 3. *The game has exactly one equilibrium. For*

$$V_3 > \frac{V_1 V_2}{V_1 + V_2}$$

it is the completely mixed equilibrium (p, q) *with* $p_1 > p_2 > p_3 > 0$ *and* $q_i = \frac{1}{3}$ *with* $i = 1, 2, 3$. *For*

$$V_3 \leq \frac{V_1 V_2}{V_1 + V_2}$$

it is the partially mixed equilibrium (p, q) *with* $p_3 = q_3 = 0$, $p_1 > p_2 > 0$ *and* $q_i = \frac{1}{2}$ *with* $i = 1, 2$.

We will now investigate the implications of result 3 for the parameter triple (V_1, V_2, V_3). Therefore, we introduce the auxiliary variables x and y:

$$x = \frac{V_1}{V_1 + V_2} \text{ and } y = \frac{V_3}{V_2}$$

The inequality $V_3 > \frac{V_1 V_2}{V_1 + V_2}$ can be rewritten as $\frac{V_3}{V_2} > \frac{V_1}{V_1 + V_2}$, which means that

$$y > x.$$

Because of $V_1 > V_2$ we know that $x > \frac{1}{2}$ and because of $V_3 < V_2$ we know that $y < 1$. Therefore, the equilibrium is completely mixed iff the inequality

$$\frac{1}{2} < x < y < 1 \tag{7}$$

holds. Likewise, the equilibrium is partially mixed iff the inequality

$$\frac{1}{2} < x \leq y < 1 \tag{8}$$

holds. We now determine for each given pair (x, y) with $\frac{1}{2} < x < 1$ and $\frac{1}{2} < y < 1$ and a given $V = V_1 + V_2 + V_3$ the corresponding parameters V_1, V_2, and V_3.

The parameters V_i as functions of x, y, and V

From $x(V_1 + V_2) = V_1$ it follows that $V_1 = \frac{1}{1-x} V_2$. In addition, $V_3 = yV$ holds. Thus,

$$V = (\frac{1}{1-x} + 1 + y) V_2.$$

The inverse of the coefficient of V_2 can be simplified as:

$$\frac{1}{\frac{1}{1-x} + 1 + y} = \frac{1-x}{2 - x + y(1-x)}$$

We now introduce a always positive auxiliary variable

$$z = 2 - x + y(1-x)$$

and get

$$V_1 = \frac{V}{z}, V_2 = (1-x)\frac{V}{z}, \text{ and } V_3 = y(1-x)\frac{V}{z}.$$

From this, one can see that $V_1 > V_2 > V_3 > 0$ as well as $x = \frac{V_1}{V_1 + V_2}$ and $y = \frac{V_3}{V_2}$ hold for the values of V_i.

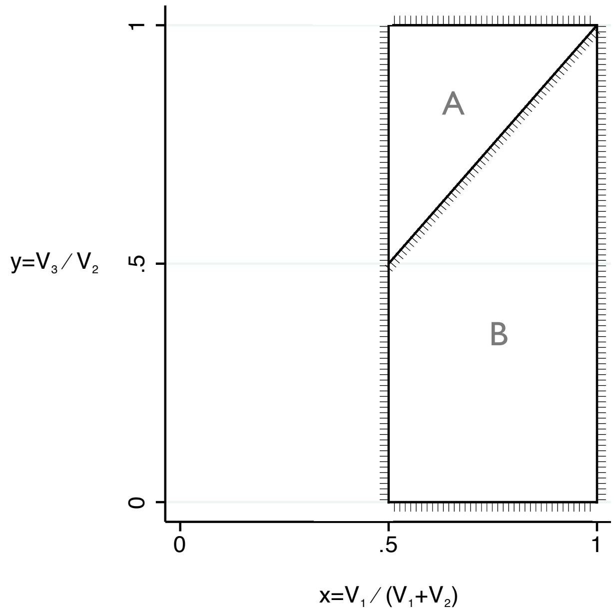

Figure 8: Payoff combinations with completely mixed equilibria (A) and partially mixed equilibria (B). x and y are auxiliary variables with $x = V_1/(V_1 + V_2)$ and $y = V_2/V_2$. If the border of an area is ruled on the outside it does not belong to this area. The border between A and B belongs not to A, but to B for $\frac{1}{2} < x < 1$.

Figure 8 gives the combination of parameters that lead to completely and partially mixed equilibria. If the border of an area is ruled on the outside it does not belong to this area. The border between A and B belongs not to A, but to B for $\frac{1}{2} < x < 1$.

For a given V it holds that $V_1 = \frac{V}{z}$, $V_2 = (1 - x)\frac{V}{z}$ and $V_3 = y(1 - x)\frac{V}{z}$ with z=2-x+y(1-x). The set $A \cup B$ contains all (x, y) which results in the triples (V_1, V_2, V_3) with $V_1 > V_2 > V_3 > 0$. Points (x, y) in A lead to triples (V_1, V_2, V_3) with completely mixed equilibria ($p_1 > p_2 > p_3 > 0$) and points (x, y) in B lead to triples (V_1, V_2, V_3) with partially mixed equilibria ($p_1 > p_2 > p_3 = 0$).

A.4 Payoffs for the completely mixed games

In our experiments, we want to investigate 12 games with completely mixed equilibria. We search for twelve parameter combinations that are relatively uniformly distributed in the space (p_1, p_2, p_3) with $p_1 > p_2 > p_3 > 0$ and $p_1 + p_3 + p_3 = 1$. Therefore, we consider all games with equilibria

Game	k_1	k_2	k_3	m_1	m_2	m_3	P	M_1	M_2	M_3	V_1	V_2	V_3
1	6	5	4	9	10	11	990	110	99	90	55	49.5	45
2	7	5	3	8	10	12	960	120	96	80	60	48	40
3	7	6	2	8	9	13	936	117	104	72	58.5	52	36
4	8	4	3	7	11	12	924	132	84	77	66	42	38.5
5	8	5	2	7	10	13	910	130	91	70	65	45.5	35
6	8	6	1	7	9	14	882	126	98	63	63	49	31.5
7	9	4	2	7	11	13	858	143	78	66	71.5	39	33
8	9	5	1	6	10	14	840	140	84	60	70	42	30
9	10	3	2	5	12	13	780	156	65	60	78	32.5	30
10	10	4	1	5	11	14	770	154	70	55	77	35	27.5
11	11	3	1	4	12	14	672	168	56	48	84	28	24
12	12	2	1	3	13	14	546	182	42	39	91	21	19.5

Table 4: Parameters and payoffs for the completely mixed games

$p = (p_1, p_2, p_3)$ for which

$$p_i = \frac{k_1}{15} \text{ for i=1,2,3}$$

with $k_1 > k_2 > k_3$ and $k_1 + k_2 + k_3 = 15$ apply. The payoffs are calculated with the help of the inverse formula with a preference for small integer numbers. We now introduce the following notations:

$$m_i = 15 - k_i, \text{ for } i = 1, 2, 3$$

$$P = m_1 m_2 m_3$$

$$M_i = \frac{P}{m_i}.$$

It follows from the inverse formula that

$$V_1 : V_2 : V_3 = \frac{1}{1 - p_1} : \frac{1}{1 - p_2} : \frac{1}{1 - p_3}$$

and therefore it holds that

$$V_1 : V_2 : V_3 = \frac{15}{m_1} : \frac{15}{m_2} : \frac{15}{m_3} = \frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}.$$

Multiplying the right side with P yields

$$V_1 : V_2 : V_3 = M_1 : M_2 : M_3$$

Table 4 give the 12 combinations of k_i, m_i, P, and M_i investigated in our experiment. V_i can be fixed as $V_i = M_i$ for $i = 1, 2, 3$. If M_1, M_2, and M_3 have a common divisor K one can instead fix V_i as $V_i = \frac{M_i}{K}$.

A.5 Payoffs for the partially mixed games

In our experiments, we want to investigate 14 games with partially mixed equilibria. We search for fourteen parameter combinations that are relatively uniformly distributed in the space $(p1, p2, p3)$ with $p1 > p3 > p3 = 0$ and $p1 + p3 + p3 = 1$.

Game	V_1	V_2	V_3
13	8	7	3
14	8	7	2
15	8	7	1
16	9	6	3
17	9	6	2
18	9	6	1
19	10	5	3
20	10	5	2
21	10	5	1
22	11	4	2
23	11	4	1
24	12	3	2
25	12	3	1
26	13	2	1

Table 5: Payoffs for the partially mixed games

Let $p = (p_1, p_2, p_3)$ be a strategy of D with $p_1 + p_2 = 1$ and $p_3 = 0$. If p is together with $q = (\frac{1}{2}, \frac{1}{2}, 0)$ an equilibrium of a game with positive payoffs V_1, V_2, V_3 and $V_1 > V_2 > V_3 > 0$ then the following holds:

$$p_1 = \frac{V_1}{V_1 + V_2}, p_2 = \frac{V_2}{V_1 + V_2}, \text{ and } V_3 \leq \frac{V_1 V_2}{V_1 + V_2}.$$

Thus,

$$V_1 = p_1(V_1 + V_2), V_2 = p_2(V_1 + V_2), \text{ and } V_3 = p_1 p_2(V_1 + V_2)$$

For every fixed $V = V_1 + V_2 + V_3$ and every partially mixed equilibrium $p = (p_1, p_2, 0)$ has the game with the parameters V_1, V_2, and V_3 exactly this equilibrium if

$$V_1 = p_1(V_1 + V_2), V_2 = p_2(V_1 + V_2), \text{ and } 0 < V_3 = p_1 p_2(V_1 + V_2)$$

holds. This condition describes the combination of parameters (V_1, V_2, V_3) with $V_1 > V_2 > V_3 > 0$ that belong to games with equilibria $(p_1, p_2, 0)$. In equilibrium, the probability p_i is given as

$$p_i = \frac{k_i}{15}, \text{ with } V_i = k_i \text{ for } i = 1, 2.$$

Table 5 gives all combinations (V_1, V_2, V_3) with $V_1 > V_2 > V_3 > 0$ for which $p_3 = 0$ and integer k_i and V_3.

B Instructions

Welcome to this experiment!

General information

This experiment gives you the opportunity to earn money with your decisions. The size of your earnings depends on your own decisions and on the decisions of the other participants of the experiment. You will receive a show-up fee irrespective of the result of the experiment. Please read the explanations of the experiment carefully. All participants receive identical explanations. We would like to ask you not to communicate with the other participants from now on. If you have any questions, please feel free to ask us. All decisions are taken anonymously. You will shortly draw a random number. This number corresponds to the number of the booth in the laboratory.

The course of the experiment

24 people in this room will participate at this experiment. The duration of this experiment will be 200 periods. Please take a look at the figures on page 2. A similar table will be shown to you during the experiment. You will be either a row player or a column player. In each round of the experiment you will be asked to choose a row or a column, respectively. The player type will be assigned to you randomly. You will be this player type till the end of the experiment. In each round, you will be matched to a different player of the other type. Neither you will know who the other player is nor if you have been playing with him before.

The numbers shown in the table indicate the points you can earn during the experiment. The points shown in the upper left corner of each cell are the payoffs for the row player; the points in the lower right corner are the payoffs for the column player. To avoid confusion, in the experiment your payoffs will be written in blue while the other player's payoffs will be shown in red. To choose a row or a column, you just have to press the according button.

After both players have taken their decision, the resulting cell and your payoff will be shown to you. This procedure will be repeated for 200 periods. From round 2 on, you can see the sum of points you earned so far in a box on your screen.

End of the Experiment and Payments

At the end of the experiment, you will be told your Final payoff, i.e. the sum of the payoffs of each round. Afterwards, a short questionnaire will appear on your screen. Please answer the questions as carefully as possible. The points will be transferred to RMB by a fixed exchange rate.