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Rethinking teaching strategies : a framework and demonstration through augmenting Maple

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Rethinking Teaching Strategies: A Framework and Demonstration through **Augmenting Maple**

Iraklis Paraskakis

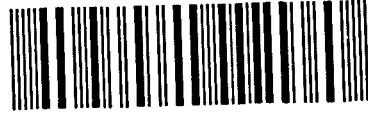# Rethinking Teaching Strategies: A Framework and Demonstration through **Augmenting Maple**

Iraklis Paraskakis

Abstract

In this work, an interdisciplinary approach has been adopted for the study of

- teaching strategies of an Intelligent Tutoring System, in the paradigm of multiple teaching strategies, and

- the use of Computer Algebra Systems (CAS) in teaching problem solving in university mathematics .

As a result, the *SIMTA* (Styles Implemented by Methods Tactics Actions) theoretical framework has been developed to support and sustain teaching strategies in the paradigm of multiple teaching strategies. TeLoDe (TEaching Linear Ordinary Differential Equations), is a prototype Intelligent Tutoring System, teaching the solution of linear second order differential equations with constant coefficients in a novel way. This novel way, which has been empirically tested, has been achieved by augmenting Maple and represents an alternative use of CASs where the human lecturer and Maple are interlocked in a symbiotic and interdependent manner.

In SIMTA, the contemporary concept of *teaching strategy* is rethought and proposed to be viewed at two fundamental levels:

- the *organisational level*

- and the *operational level*.

The *organisational level* deals with the structure of the teaching strategy whereas the *operational level* deals with the manifestation of that structure.

In SIMTA the organisational level is represented by a triple generic structure, *method*, *tactic*(s), *action(s)*. A *method* is a mechanism for structuring the subject matter (e.g. analogy, examples, generalisation, specialisation). Likewise, a *tactic* is a mechanism for facilitating the interaction (e.g. explicit interaction, implicit interaction). An *action* is a low level activity such as display this message, ask this question.

In SIMTA, the exact manifestation of the above generic structures (analogies, examples, implicit interaction, explicit interaction) depends on the concept of *style*: different styles

result in different manifestations of the same generic structures. Thus, in SIMTA the concept of multiple teaching strategies is seen as merely a collection of teaching strategies manifested under the *same* style. These strategies operate with the aim of offering alternative representations of the same task at hand and ensuring that the learner is active by *activating*, *directing* and *maintaining* exploration.

To help demonstrate the feasibility of SIMTA, two styles, the *expository style* and the *guided discovery style* have been formed. The expository style draws on Ausubel's theory of meaningful learning, whereas, the guided discovery style draws on Bruner's work. These styles have been implemented in *TeLoDe*.

TeLoDe, incorporates a teaching strategy module, based on a style, and declarative knowledge. Its purpose is threefold:

(i) to serve as a research tool for the SIMTA framework,

(ii) to serve as a prototype, demonstrating clearly how a 'second generation' CAS which undertakes the procedural aspect of mathematics allowing the human tutor to concentrate on its conceptual aspect, could be developed,

(iii) to demonstrate how Maple and human lecturers are given clear roles which are, nevertheless, interdependent in carrying out the teaching of university mathematics.

Two small-scale empirical studies were carried out in order to test SIMTA and TeLoDe respectively. The first study involved lecturers whereas the second study was carried out in a classroom environment. The results found from these studies demonstrate that TeLoDe has a potential as a teaching tool for problem solving in university mathematics in a novel way.

Dedication

Στούς γονείς μου καί τόν αδελφό μου

Acknowledgements

I would like to express my gratitude to my internal supervisors Tim O'Shea and Robin Mason, who have guided me during the last year of my study. This thesis would have never been materialised if it was not for them.

I am very grateful to Dr. Mark Elsom-Cook and Dr. Nick Alexandrou who as internal (and later external) and external supervisors respectively have contributed considerably with their ideas and encouragement, throughout this research. I would also like to acknowledge the support that I received from Dr. Ann Jones and Dr. Laurence Alpay during the middle years of my research.

Special thanks to Prof. David Hawkridge and to my friend Judith Daniels for commenting on the full draft of this thesis. I would also like to thank Prof. Diana Laurillard, Dr Eileen Scanlon, Dr. Teresa de Soldato, Prof. John Mason and Sue Marr who have generously commented on parts of this work.

I would like to thank all the subjects that participated in my empirical studies and in particular Mr Tim Yeboats, Lecturer from Lambeth College, who kindly agreed to use one of his classes to carry out the second phase of the empirical studies.

A special thank you goes to my good friend and colleague Maria Yannissi-Denazi and her family. I would like to thank Maria for all her help, encouragement and discussions and for introducing me to the 'strange' world of Educational Psychology.

Warmest thanks to Dr. Jane Barnard and Dr. Erica Morris for their friendship and encouragement as well as for constructive comments and proof reading of my thesis. Also I would like to thank Dr. George Karakitsos for helping me to sort out the implementation issues for my thesis. A special thanks to Olwyn Wilson, Pauline Adams, Hansa Solanki, Di Mason and Pat Cross for being always ready to help.

I would like to thank the Greek Community in Mitlon Keynes, and especially Mrs Niki Beales and her family as well as Dr. Stylianos Hatzipanagos, Harry Benetatos for their friendship.

Finally, I would like to express my deepest gratitude to my family who have unquestionably supported me throughout my studies.

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Chapter 1

Chapter 1: Introduction

This thesis is an interdisciplinary work drawing from the fields of Mathematics, Intelligent Tutoring Systems (ITS), Educational Psychology and Educational Technology. This research represents some steps towards creating an Intelligent Tutoring System for problem solving in undergraduate university mathematics using Computer Algebra Systems (CAS). To accomplish this task, two 'independent' subjects, CAS in education and teaching strategies, are brought together to underpin the ITS.

The contributions of this work can be divided into two parts, the first concerning the issue of teaching strategies and the second concerning the issue of CAS in education. The framework SIMTA (Styles Implemented by Method Tactics Actions) embodies the contributions regarding teaching strategies, whereas the prototype TeLoDe (TEaching Linear Ordinary Differential Equations) embodies the contributions regarding CAS in education. SIMTA and TeLoDe have both been developed during the course of this thesis and have been presented in several relevant international conferences [Paraskakis, 1997a, 1997b, 1997c].

SIMTA is a theoretical framework, providing the foundations for an underpinning educational model upon which a teaching strategy module for an ITS could be defined.

TeLoDe is a prototype Intelligent Tutoring System, teaching the solution of linear second order ordinary differential equations with constant coefficients. TeLoDe, is also used as a vehicle to demonstrate that a teaching strategies module for such an ITS could draw from the theoretical framework of SIMTA. TeLoDe is an augmentation of Maple which enables it to be of potential use in teaching problem solving in university mathematics in a novel way.

This chapter introduces the thesis, examining firstly its background, secondly its aims, and finally examining its structure.

1.1 Background to the thesis

This section is divided into two parts. The first focuses on the issue of teaching strategies, demonstrating the need for further research in the area and giving a brief preview of how SIMTA proposes to tackle the problem. The second focuses on the potential impact that an augmented CAS could have in teaching problem solving in mathematics at university level.

1.1.1 The issue of teaching strategies

In the field of Artificial Intelligence in education a key objective has been to identify and formalise teaching strategies [Collins, 1980], [Elsom-Cook, 1991], [Ohlsson, 1986, 1991], in such a way that they can be applied across different domains. A number of systems have contributed to the development of understanding in this field, (e.g. see SCHOLAR, WHY, Collins et al., [1974], Stevens et al., [1982], DOMINIE, Elsom-Cook [1991], MENO-TUTOR, Woolf & McDonald [1984]).

These systems have been hampered for two main reasons. Firstly, they have not been motivated by a consistent underpinning educational model. Secondly, there has been a confusion of terms both in the way the terminology is used by the different systems and in the way this terminology fails to relate consistently to the use of similar terms in the fields of education, instructional science, cognitive science etc.

The primary reason why these systems were hampered is a result of the diverse objectives pursued in each system. For example, in the case of SCHOLAR (second version) the concern was with the use of heuristics; in the case of WHY the objective was to demonstrate context dependent teaching strategies; in the case of MENO-TUTOR the objective was how to engage the knowledgeable student in a fundamentally different way than that of a confused student; and finally in the case of DOMINIE the important concept of multiple teaching strategies was the objective.

Furthermore, the interest behind the research on each system was different. For example, in the case of WHY it was a cognitive one, e.g., deduction of a causal model, making inferences from incomplete knowledge. In the case of MENO-TUTOR, the interest was in providing a computational model through the use of ATN (Automated Transition Networks). In the case of DOMNIE the objective was to provide a computer model for multiple teaching strategies, i.e., prove the feasibility of creating such a model.

As a result of these diversities, the concept of teaching strategies is seen, firstly in the case of WHY, in the form of a *method* which embodies a number of *strategies* under it. So the Socratic method is expressed in terms of an identification strategy, an entrapment strategy, a counterexample strategy and so on. Secondly, in the case of

MENO-TUTOR the concept of teaching strategies is broken down into three states, the pedagogic state, the strategic state and the tactical state. A teaching strategy initiates at the pedagogic state and is refined as it moves down the states. Thirdly, in the case of DOMINIE, where the concept of multiple teaching strategies is explored, the terms teaching strategy and teaching style are used in a synonymous fashion. Multiple teaching strategies are seen as a collection of a number of autonomous teaching strategies or styles that are brought together to offer more than one way of interaction between the subject matter and the student.

From a purely educational perspective, it would be ideal to combine these three approaches to teaching. That is to refine the student causal model through Socratic dialogues, engage the knowledgeable student and confused student in fundamentally different ways and to be able to employ a multiple teaching strategies model. However, the above models were not designed as general frameworks, but were purpose built models with different structures, according to their particular needs. This means that combining them would give rise to the same problem as would defining teaching strategies in a generic way.

The problem of teaching strategies still warrants investigation, especially as the design of a teaching strategy module could inform not only traditional ITSs, but could also be used to inform the design of a teaching strategy module that uses the latest communication technologies, such as the Web [Woolf, 1996], [Eklund & Brusilovsky, 1998]. The use of such technologies has recently been encouraged, especially in tertiary education [Dearing, 1997]. Moreover, the problem of multiple teaching strategies is an acute one and still puzzles researchers in the area [Beck et al., 1996].

The concept of multiple teaching strategies is not a new one; educators have discussed this concept since the 70's e.g. Mosston, [1972], Brady [1985], Joyce and Weil [1982], Eggen et al., [1979]. Here the concept of multiple teaching strategies, or to use their definitions, styles or models, is seen as offering the student alternative ways of dealing with different goals, and of mirroring effective teaching which is described by Eggen et al., [1979] as requiring

> alternative strategies to accomplish different goals. (p. 3)

However, regarding the issue of bringing together these strategies or models or styles, Eggen et al., [1979] in their last chapter, 'Creativity in Teaching: Synthesising the Models', state:

> We want to emphasise that the models are means to ends and not ends in themselves, and they should be modified if, in the teacher's judgement, a modification would be more effective in a given situation. To illustrate this point we will consider a teacher using parts of different models to achieve related teaching goals within a week's unit (p. 353) [Eggen et al., 1979].

Consequently the question of how multiple teaching strategies are formed and operate is an issue that is not resolved. The approach adopted in DOMINIE in attempting this question, i.e., simply collating a number of different teaching strategies under a loose principle, as discussed in Chapter 3, raises a number of questions. There appears to be

- no explicit formal definition of multiple teaching strategies,

- no explicit definition of their role and objectives

- no consideration of factors for combining all these teaching strategies.

As a result, as discussed in detail in Chapters 3 and 4, DOMINIE could switch from a rather protective strategy, such as cognitive apprenticeship, where the task is broken into simpler tasks and demonstrated clearly before the student is asked to repeat anything, to a highly discovery based strategy, where a problem is presented and the student is asked to solve it. Unless the switch is planned and carefully controlled, it could result in the alienation of the student. Making this point should not be interpreted as a rejection of grouping polarised strategies in multiple teaching strategies, but merely emphasises the need for some sort of theoretical consideration when developing/grouping the teaching strategies.

Moreover, the interest in this thesis in multiple teaching strategies lies not only in finding alternative teaching strategies for different goals, but also in the identification of alternatives for the *same* goal. To include this latter aspect, when reference is made to the concept of multiple teaching strategies in the rest of this thesis, it will be referred to as the "paradigm of multiple teaching strategies". By paradigm, it is meant that for a

teaching strategy to be considered as eligible for operating in the paradigm of multiple teaching strategies, the basic principle is to offer *alternative* ways of viewing the *same* task at hand.

SIMTA is a theoretical framework supporting teaching strategies operating in the paradigm of multiple teaching strategies. SIMTA is capable of informing the structure of a teaching strategy module of an ITS. SIMTA tackles directly all the issues raised above by

- being grounded in a detailed educational model rather than just applying aspects of an existing one

- providing a set of terminology to describe the teaching process

- instantiating these terms and their relationship through the definition of the SIMTA framework as well as by implementing them

- demonstrating in practice in a mathematical domain -linear second order ordinary differential equations with constant coefficients- that the SIMTA framework is capable of informing and sustaining a teaching strategy module such as TeLoDe.

For the development of SIMTA the fields of Intelligent Tutoring Systems and related fields such as Educational Psychology and Cognitive Science are drawn together to underpin SIMTA.

1.1.2 The issue of CAS in education

Computer Algebra Systems (CASs) are very powerful mathematical solving engines capable of tackling a wide range of symbolic problems both in Calculus and Algebra. CASs were created as powerful symbolic calculators to enable scientists to carry out tedious long calculations [Harper et al., 1991]. Thus, it is fair to point out that educational considerations were not an integral part of the developing process (this point is further analysed in Chapter 2) but there is great potential in harnessing and exploiting these powerful solving mathematical engines.

Despite this inherent shortcoming their potential is obvious and their use in education has been widespread e.g. TRANSMATH (Leeds University), MathWise (consortium of UK Universities), METRIC (Imperial College) to mention a few projects. However, in these projects CASs are used in an ancillary fashion, that is current teaching practices are not altered as a result of using CASs (this point is discussed in §2.3). To provide a context for this argument, consider the following scenario about teaching the solution of linear second order ordinary differential equations with constant coefficients. This is based on my experience as a student being taught these equations.

After a short and quick definition of the differential equation, followed by an example of a relevant natural phenomenon (e.g. the motion of a mass vibrating up and down at the end of a spring), the emphasis turned on solving the differential equation. The lecturer proceeded to state that to solve the differential equation, you substitute m^2 for $\dfrac{d^2y(x)}{dx^2}$, m for $\dfrac{dy(x)}{dx}$ and 1 for $y(x)$. Thus, a quadratic equation is derived called the Auxiliary Equation. That equation is solved and depending on the nature of the roots the general solution of the differential equation looks like:

If the roots, m_1, m_2 are *real and distinct* then the general solution, in the case of the homogenous (or complementary function in the case of the non-homogenous) equation, is of the form

$$y(x) = A*e^{m_1*x} + B*e^{m_2*x} \text{ where A, B are arbitrary constants}$$

If the roots, m_1, m_2, are real and equal the general solution is of the form

$$y(x) = (A + B*x)e^{m_1*x} \text{ where A, B are arbitrary constants}$$

If the roots are complex numbers then the general solution is of the form

$$y(x) = e^k (C_1*\cos(n*x) + C_2*\sin(n*x)) \text{ where } C_1, C_2 \text{ are arbitrary constants}$$

In the case of the complex numbers the formula was worked out from the formulae of the real and distinct case.

From the above scenario it is apparent that the teaching of that particular type of differential equation was focused on the solution of the equation. Furthermore, there is a well established procedure for attaining the solution, and this, in conjunction with the fact that CASs are capable of directly solving these types of equations, means that the use of CASs in an ancillary fashion poses some questions. For example, to devise a computer based tutoring system that helps the student through the steps (auxiliary equation, complementary function, general solution), as in Paraskakis [1989] or as is proposed in TRANSMATH or MathWise, is not the most efficient use of technology nor is it necessarily most helpful for the student. For example, my reaction as a student was confusion on several points as follows:

- Firstly and most obviously, where did the exponential function come from?

- Secondly, why is the formula different in the case of the equal roots?

- Thirdly, where did that m come from?

Laurillard, [1993], records a similar experience as a student in her first lecture and also suggests that this may have been her own students' experience when she first became a university lecturer. Furthermore, as reported in Chapter 10 some university lecturers still use the teaching described in the scenario above. Whilst the scenario from my personal experience and Laurillard's accounts should not be considered necessarily as an indication of a problem in undergraduate mathematics (as this is a question of an empirical study beyond the scope of this research) it nevertheless should encourage a debate on the use of CASs.

This debate resembles that concerning the use of calculators in the 70's in the sense that the main themes are:

- Should CASs be used in an ancillary fashion where there is an under utilisation of their potential

- or should their potential in solving equations be fully exploited and our teaching practices be adopted around this premise?

Particular questions to assist this debate could be as follows:

- Is the knowledge involved in solving the given type of differential equation now *redundant*?

- Should it be taught, or should CASs be used to provide the solution?

- If it is not taught then what is to be taught instead?

- Can Computer Algebra Systems always be trusted to provide the correct solution to these complex problems? What if an answer is wrong? (see §2.4)

TeLoDe is an Intelligent Tutoring System which is based on an augmentation of Maple, a CAS, teaching the solution of linear second order differential equations with constant coefficients. TeLoDe represents an alternative use of CASs in teaching university mathematics where a novel approach in teaching these differential equations is proposed. This alternative use of CASs and the novel approach in teaching draws from the second theme.

Moreover, TeLoDe demonstrates that this novel approach in teaching could also be captured within an ITS and thus be automated. SIMTA, which informed the teaching strategy module, and the incorporation of declarative knowledge component in Maple, has been instrumental in developing TeLoDe. Consequently, TeLoDe could serve as a basis for developing a second generation of CAS where pedagogical considerations are central to their development.

1.2 Aims of the thesis

Within this background the work described in this thesis examines the issue of teaching strategies with the aim of developing the SIMTA framework. The workability of SIMTA will be demonstrated by informing the structure of the teaching strategy module in TeLoDe.

The main research question, regarding the issue of teaching strategies, is:

How are teaching strategies formed and how do they operate in the paradigm of multiple teaching strategies?

In answering this question the following questions will also be addressed:

What is a teaching strategy?

How could it operate?

What could be the constituent parts of a teaching strategy?

What could be the factors that influence the decisions of teaching strategies?

Many of these issues will be explored in the framework of the three standard questions of ITS work on teaching strategies, i.e., *what to say next?*, *when to say it?*, *how to say it?* (e.g., see Sleeman and Brown, [1982]).

As a secondary issue the thesis will demonstrate the potential application of Computer Algebra Systems in an educational environment, notably:

What is their impact in education?

How could one harness their powerful mathematical engines and address their inherent pedagogical weaknesses?

1.3 Structure of the thesis

The thesis is structured as follows:

Chapter 2: *A review of Computer Algebra Systems*. This chapter begins with some historic and background information on CASs. It then reviews Maple, using some topics from the Open University Maths Foundation course (M101) [M101, 1987] for context. In this review references to other computer algebra systems such as Derive and Mathematica are also made.

The use of Computer Algebra Systems in universities is examined through three projects, METRIC, TRANSMATH and MathWise. Through this demonstration the current trend of using Computer Algebra Systems is projected and acts as precursor for the augmentation argument which is presented in the final section of Chapter 2. The objective of this alternative is twofold: to force a rethinking of traditional mathematical practices given the capabilities of CASs and to equip the students so that they can be in control of CASs and can check the results produced by them.

Chapter 3: *Review of teaching strategies in ITSs*. This chapter reviews four ITSs: SCHOLAR (second version), WHY, MENO-TUTOR and DOMINE. The aim of this review is to identify whether there is a model that informs the structure of a teaching strategy module, central to the augmentation of a CAS. The review shows that despite the inspiring results of these systems and later works (also examined), no such model is present. It is suggested that in creating such a model some design aspects of teaching strategies need to be sought from the ITS field and related fields of Educational Psychology and Cognitive Science.

Chapter 4: *Design aspects of a teaching strategy*. This chapter puts forward some design aspects of a teaching strategy that would form the building blocks of SIMTA. Some of these design aspects - what governs verbal explanations and the issue of alternatives in explanations as well as the issue of grouping teaching strategies together - originate from work in ITSs (as reviewed in Chapter 3). Other design aspects regarding the role and objective of a teaching strategy, factors affecting a teaching strategy, as well as an insight as to what could be meant by multiple teaching strategies, are gathered from reviewing work in the fields of Educational Psychology and Cognitive Science.

Chapter 5: *The SIMTA Framework*. This chapter begins with the rationale of SIMTA, where it is shown explicitly how the design aspects of a teaching strategy from Chapter 4, fit in and are accommodated in SIMTA. It then moves on to describe how SIMTA and its elements, *style, method, tactic* and *action* are chosen and organised in such a way as to reflect SIMTA's rationale. The chapter concludes by discussing how SIMTA advances our understanding of teaching strategies operating in the paradigm of multiple teaching strategies by comparing SIMTA with the four ITSs reviewed in Chapter 3.

Chapter 6: *The SIMTA Framework in practice: creating a style and its implications*. This chapter describes how to create a style, its implications on the other elements of SIMTA, method, tactic, action and the structure of the knowledge base of an ITS. This is explored with the aid of the *expository style*, which is also implemented in TeLoDe, and it draws from Ausubel's [Ausubel et al., 1978] work on meaningful reception learning. To demonstrate that a different definition of a style will result in a different manifestation of methods, tactics and actions another style, the *guided discovery style* is also defined. This style draws from Bruner's work [Bruner, 1966] Bruner, 1971] [Bruner, 1977]. The chapter concludes with a discussion as to why the two styles defined are distinct.

Chapter 7: *Analysis of knowledge content of TeLoDe*. This chapter describes how the subject matter of linear second order differential equations with constant coefficients is analysed with the Maple system and the expository style in mind. As a result a novel approach in teaching this highly procedural topic is proposed where Maple is an integral part of this process.

Chapter 8: *Definition of knowledge representation and methods in TeLoDe*. This chapter describes how the theoretical analysis of the differential equation is to be implemented. It therefore serves as a definition of the knowledge representation and methods to be used in TeLoDe. However, the analysis presented here could also inform the way in which the teaching of the topic could occur in a classroom with a human teacher.

Chapter 9: *Implementation of TeLoDe*. This chapter presents the implementation of TeLoDe. TeLoDe's main purpose is to demonstrate the feasibility of SIMTA, primarily through the implementation of the expository style. To demonstrate that *distinct styles* result in different manifestation of methods, tactics and actions, a caricature of the guided discovery style is also implemented. TeLoDe's secondary aim is to show how CASs could be developed incorporating pedagogical considerations. TeLoDe is run under both styles and its output is annotated.

Chapter 10: *Empirical test of TeLoDe*. This chapter reports on two small empirical studies. The first study was carried out to disseminate the principles of SIMTA, to demonstrate how CASs could be used in education and to obtain feedback on TeLoDe's potential as an educational tool from professionals in the field. The second study was

carried out in a classroom where TeLoDe was used by the lecturer to teach students linear second order ordinary differential equations with constant coefficients.

Chapter 11: *Conclusions*. This chapter draws together the threads from previous chapters. It highlights the contributions of the research and discusses the implications of SIMTA for current thinking about teaching strategies. Some directions for further research, in relation to SIMTA and TeLoDe, are proposed.

CHAPTER 2

Chapter 2: A Review of Computer Algebra Systems

This chapter introduces Computer Algebra Systems (CASs), presents an account of their current use in tertiary education, and proposes an alternative use for CASs which leads up to the case for augmentation of CASs.

In §2.1 some background and general information regarding the genesis of CASs as well as some educational possibilities for CASs are presented. In §2.2 an insight into idiosyncrasies of CASs is presented. Although these idiosyncrasies deal mainly with the interface, they also demonstrate cases where CASs simply provide the wrong answer to a problem. For this demonstration Maple is used and to provide context for the discussion, the topics of *functions and graphs, inequalities, differentiation and integration*, from the Open University Mathematics foundation course are used. To give a wider picture of how CASs handle these topics, a quick account from other systems such as Mathematica and Derive is also given here.

An account demonstrating the current trend of use of CASs in education is examined in §2.3. In this account three projects, TRANSMATH (Leeds University), METRIC (Imperial College) and MathWise (consortium of UK Universities) are examined. In this section it is shown that the current trend has not brought any changes in traditional mathematical practices.

The case for an alternative use of CASs which leads to the case for augmentation is presented in §2.4. The objectives of this alternative are as follows:

- to force a rethinking of traditional mathematical practices given the capabilities of CASs

- to equip the student to be in control of CASs and to be able to check their results

- to promote the development of second generation CASs, where education, and not powerful symbolic calculators, is their primary concern.

2.1 Background information and educational possibilities for Computer Algebra Systems

Computer Algebra Systems enable the exact solution of problems in symbolic form. This contrasts with the numerical analysis approach used in conventional computer languages such as FORTRAN or BASIC, where a numerical approximation is obtained. CASs are interactive and allow the user to define an expression, apply an operation

and manipulate the output. Standard operations include algebraic simplification, calculus, (i.e., integration, differentiation, power series etc.), algebra, systems of equations, differential equations as well as the use of the system as arbitrary-precision desk calculators [Hodgkinson, 1987]. They typically provide the following facilities:

- Manipulation of power series and expressions involving functions such as sine, logarithm and exponential.
- Symbolic differentiation and integration of a large number of functions.
- Algebraic solutions of :

  . Polynomial equations up to 4th order.
  . Sets of simultaneous linear equations, including over-determined systems.
  . Some sets of simultaneous non-linear equations.
  . Some ordinary differential equations.

- Manipulation of matrices whose components may be algebraic expressions. This includes symbolic matrix inversion.
- Vector algebra and calculus in general, orthogonal curvilinear co-ordinate systems.
- Arbitrary-precision arithmetic, integer and floating-point.
- Number theoretic calculations.
- The ability to save the results of calculations for subsequent use.
- Facilities for plotting functions in 2 and 3 dimensions [Harper et al., 1991].

These systems are further equipped with facilities for manipulation of expressions such as editing, selecting parts of an expression, etc. Moreover, in cases where a facility (either manipulative or operational) is not available, the user is given the option of creating such a facility, via the high-level programming capabilities, by writing procedures that can serve as extensions of the system (e.g., see Paraskakis [1988], Paraskakis [1989]).

CASs were originally available only on powerful mainframes but in 1983 muMATH became the first CAS to be available on a PC [Stoutemyer & Rich, 1983]. In the late eighties Maple [Char et al., 1988] followed by Mathematica [Wolfram, 1991] became available on PCs. However, they required very powerful PCs and their functionality

was limited compared to their mainframe editions. Other packages like REDUCE [MacCallum, 1991] and MACSYMA (see [Poutney, 1997], [Hodgkinson, 1997]), despite the fact that they were one of the early CASs on mainframes did not make an appearance on PC platforms until the early and later nineties respectively.

The fact that CASs were developed with the consideration of producing more powerful and robust algorithms, that address the issue of providing powerful symbolic calculators, is evident in two ways. First their lack of interface, and second, but most important, the fact that the knowledge contained in all these packages is in the form of a black-box [du Boulay et al., 1981]. That is, the steps inscribed in algorithms for solving a problem do not necessarily correspond to the steps that would be taught to a student for solving the same problem. However, all is not lost since the provision of a high level language enables the creation of a glass-box, i.e., the creation of the steps that correspond to those used in teaching (e.g., see Paraskakis [1988], Paraskakis [1989]).

Figure 2.1 A screen download of muMATH and Maple

The screen downloads of muMATH and Maple, see figure 2.1, show the screens that face the users. These two screen downloads have been placed next to each other to demonstrate how similar the two interfaces are, even though muMATH was developed in 1983 and Maple V Release 2 was developed in early 1990's.

In both cases the user enters an interpreter environment, where the user is expected to issue a command which the software will execute. This may be acceptable to a user concerned only with the provision of a fast and correct answer to a problem that would otherwise take a considerable amount of time if done by hand. However, if the CAS is to be used in an educational environment then the lack of an interface may present a

problem. In fact, as early as 1985, [Stoutemyer, 1985], the team behind muMATH became aware of the interface issue and as a result created Derive [Rich et al., 1989] which is a menu-based system (see figure 2.2).

Figure 2.2: A screen download of DERIVE

However, the provision of a menu-based system came with a price since in Derive the source language, muSIMP, which was available in muMATH, is no longer available to the user. Whilst Derive cannot be augmented, the provision of the menu has given it an edge on ease of use and it has become very popular in the teaching of mathematics for A-level and first year undergraduate courses (e.g. see [Watkins, 1990]).

The above observations regarding the black-box, glass-box issue and the interface, do not render CASs incapable of use in education. In fact as early as 1986, Small et al., [1986] listed five potential benefits from the use of CASs in the classroom:

1. Change in students' attitudes. That is many students regard the mastery of algorithms as the primary aim of a mathematics course.

2. Hope to get the students more actively involved with mathematics. Currently, students regard the subject as a body of facts that they have to memorise. Consequently, they do not perceive themselves as active participants, but merely as recipients. Thus, usage of CAS, will relieve and thus allow and encourage the student to be more active and become more explorative.

3. The use of CAS will allow the students to devote more time in organising their thoughts. That is CAS will take away the burden of computation. Consequently, students will have the opportunity to think the problem through and reflect on their thoughts, try them out and redefine them as and if necessary.

4. The use of CAS will allow the students to deal with more realistic examples and thus enable them to appreciate the interplay of mathematics and other subjects.

5. It is hoped that CAS will offer more advantages to the mathematics curriculum than the calculators did as they are capable of symbolic manipulation and arbitrary precision arithmetic is relevant to greater portions of the mathematics curriculum. Moreover, CAS are also equipped with numerical and approximation techniques which can be used in conjunction with the symbolic ones and thus CAS offer a full range of computer usage in the mathematics curriculum.

In order to investigate the benefits stated above, as well as to grasp a better understanding of the facilities offered by the CASs, Maple is reviewed in the next section. To provide context for this review, three topics from the Open University Mathematics foundation course are used [M101, 1987], [M203, 1979]. These topics are *functions and graphs, inequalities, differentiation and integration*. To provide a wider picture of CASs' capabilities, as well as limitations, a quick account from other systems such as Mathematica and Derive is also given here.

2.2 The Maple package

Maple [Char et al., 1990a, 1990b] is the product of Waterloo University in Canada. It was one of the first Computer Algebra Systems. Like the other packages it is a general system for mathematical computations. Its code is considered to be one of the most proven and with the latest version it also provides 2 and 3-D graphics. Maple is organised into libraries and thus the user has to load the appropriate library for the corresponding application. However, in the case of Mathematica [Wolfram, 1991] the libraries that are loaded by default are very simple. So in the case of integration, if requested to integrate anything other than a constant or a straightforward one, Mathematica prompts that the appropriate library needs to be loaded.

The help offered is an on-line manual describing the purpose of the function and how it can be called. It also explains the nature of the arguments and their position. Furthermore, there are a number of examples which can be executed to help the user to understand the command syntax.

Maple is case sensitive. All Maple commands must be issued in lower case. This can be very annoying because the system, for example, does not understand the complex notation i for imaginary numbers in lower case, requiring it to be in upper case only. In the case of Mathematica, the first letter of every command needs to be in upper case. Moreover, Mathematica maintains a clear distinction between capital and small case letters. So, for example, if the following command is issued

Integrate [X^3, x] Mathematica returns $X^3 x$

whereas

Integrate [x^3, x] Mathematica returns $\dfrac{x^4}{4}$

Maple is more robust than Mathematica from the mathematical point of view [Harper et al., 1991]. According to a recent review it retains that edge [PCW, 1997] whereas Mathematica has an edge on graphics. Maple is better equipped with functions for carrying out mathematics, and the algorithms used are more reliable compared to Mathematica. Although it is able to produce 3-D graphs, the facilities (e.g. animation, rotation) are not as extended as in Mathematica. In some cases facilities such as animation do not even exist, although later versions are very likely to provide them.

Maple also provides a number of functions that do not automatically solve the problem and produce the answer. These functions enable the student to perform operations in steps, as a human would, and these steps are displayed. In some cases these functions are extensions of what similar, or even the same, commands in Maple can do. Use of these functions, for example, enable the student to complete the square in an expression, carry out integration by parts in an overt manner and so forth.

Maple, like Mathematica, has its own programming language. It will enable the user to enhance the facilities provided by Maple, or in some cases modify the definition of a number of built-in commands. However, its construction is not well founded, at least from a programming point of view. Mathematica's own language is superior and better founded. In fact in later versions of Maple the language structure has moved away from a functional approach to that of a procedural one. For example in Maple Rel. 5, it is a requirement to declare explicitly all local and global variables.

On the other hand whilst earlier versions, like Maple Rel 2 used in this research, did not support the space as multiplication Maple V Rel. 5 supports it.

2.2.1 Functions and graphs

Maple is equipped with the ability, at least in principle, to deal with 2-D graphs. In the case of 2-D plot, the command, as described in Char et al., [1990a], is

$$plot(f, h, v, options);$$

f is the expression. (Note that a relation is *not* valid. For example, $x^2 + 2x + 5 = 0$ is not a valid input but $x^2 + 2x + 5$ is a *valid* input.)

h is the range for the x-axis (optional).

v is the range for the y-axis (optional).

options include titles, colours, steps on the axes, etc.

It is advisable that when a plot is requested, both x and y axis ranges are specified, as Maple's choice is not always optimal and may not suit the specific case. The default value for the x-axis appears to be 5 whereas for the y-axis there seems to be none. As a consequence of this, when cubic graphs are sketched it is not clear where they cross the x-axis, especially when the roots are close. Furthermore, the quality of the 2-D graph (e.g., in the line and axes display) is not as satisfactory as the 3-D graphs.

It supports multiple plotting and the command is

$$plot (\{f1,f2..\}, h, v, options);$$

Each graph will be displayed using a different colour. The graphs are plotted according to their order in the function definition, i.e. f1,f2 etc.

As mentioned above, Maple cannot plot relations. If an expression is part of a relation then the user has to select the expression explicitly and request its plot. The selection can be done either by using the selector functions of Maple or by editing the relation itself, as that is supported by Maple.

Functions in Maple can be declared, although the notation is not as straightforward as in Mathematica. The notation that has to be followed here is as follows:

$$f := x \to x^2 + 2x$$

Afterwards, the function f can be called by calling the function name and the value of the variable in brackets, i.e. $f(1), f(2)$, etc. Functions of two variables can also be declared and the declaration is as follows:

$$g := (x,y) \to x^2 + y^3$$

In this case g has to be called with two values, one for x and one for y.

Mathematica is well placed to deal with graphs, since as mentioned earlier it has better graphics. The command for drawing a graph in Mathematica is as follows:

>   Plot [expr, {var, xmin, xmax}, Options]

where

>   **expr** is either the function itself or the name of the function
>   **var** is the dependent variable
>   **xmin** is the minimum value for the x-axis
>   **xmax** is the maximum value for the x-axis

and **options** corresponds to a number of control variables, e.g., **PlotColor**, which sets the colour **True** or **False**, **PlotRange** which specifies the values for the y-axis, that is it changes the default values, **PlotLabel** which labels the graph, etc. [Wolfram, 1991].

To perform zooming, the easiest way is to select the graph and then use the mouse to enlarge it or reduce it. For scaling, the option **Range** has to be used, where the user can specify explicitly the scale for the axes. The command **Show** enables the user to show graphs previously displayed and for which the whole process has been assigned to a function name. For example, suppose a user wants to draw the graphs of $x^2 + 2$ and x^3 and then combine the two graphs. The process is as follows:

The user assigns each plot operation to a function name

>   g1 = Plot[x^2+2,{x,0,3}]
>   g2 = Plot[x^3,{x,0,3}]

and presses **ENTER** and Mathematica will display the two graphs on two different systems of axes. To combine the displays all that has to be done is to issue the command **Show [g1,g2]** and Mathematica will display both graphs on the same system of axes.

To declare a function in Mathematica is straightforward. Assume that the user wants to declare the function f to hold the expression $x^2 + 3x + 2$. The command issued here is

$$f(x) = x^2 + 3x + 2$$

However, there is a peculiarity when using the $f(x)$ notation for differentiation or integration. In this case instead of the $f(x)$ notation, the notation $f(x\_)$ must be used as that enables Mathematica to refer to the expression assigned to $f(x)$.

Derive is very well equipped in handling functions and graphs. Using the **Declare** option from the menu the user is prompted to enter the name of the function and then to enter the rule of the function. Once this is accomplished the function can be differentiated by using the command **Differentiate** from the menu, or plotted using the command **Plot**. In this case Derive automatically evaluates if the graph is a 2-D or a 3-D one. In the case of 2-D the user can **Zoom In** or **Out** with respect to **X** or **Y** axes or **Both**. The **Scale** command can be used to set the scale exactly to the value that the user wants. The effect of scale is like zooming in or out although finer control in scaling is achieved in this way. Using the command **Center** the centre of the 2-D plot window can be set. To position the window over an interesting part of a plot, all that the user has to do is move the cross to the point of interest and then issue a **Center** command.

From the above descriptions it becomes apparent why Derive has an edge on ease of use. Being menu driven it prompts the user step by step. Thus one is not required to remember complex syntax rules. There is no need to improvise (as in Maple for declaration of functions) or use the underscore notation (as in Mathematica) to perform any operations on the function. However, the lack of the programming language in the case of Derive is not outweighed by the provision of a user friendly interface.

2.2.2 Inequalities

Inequalities is a relatively hard concept to grasp, particularly if the inequality given to a student is a complicated one, as in the problem given below.

The question: find all the values of x that satisfy the inequality.

$$\frac{x^3 + 2x^2 + 2x - 2}{x^2 + 2x + 2} \geq 1$$

There are several different ways in which to tackle such problems. They can be solved graphically, algebraically or by a combination of the two.

In the case of a graphical approach, if done by hand, students are likely to be put off by just looking at the left-hand side of the above inequality: it would be time-consuming to solve in this way because of its complicated form and students may not have much confidence in their answers. Use of the graphic facilities of a CAS could remove this problem for the student. The student would be able to draw the graph as many times as required, using different scales, zooming in, zooming out, viewing it from different angles, drawing graphs together. All these operations can be carried out without distracting the student from the actual problem.

Algebraic approaches can also prove difficult for the student, especially in cases where "hard" cubics or higher order expressions are involved. Such difficulties will, at least, occupy the student well before the student advances to the actual problem, the solution of inequalities. In some cases, a small algebraic mistake can alter the expression in such a way that it becomes impossible to solve the inequality. The use of a CAS to perform the algebraic manipulation would offer both speed and reliability, enabling the student to focus on the inequality itself. Even after the solution to the problem has been obtained, it can be verified graphically, or algebraically, in order to understand why certain values are excluded while some others are included.

This topic is handled by Maple better than any other package. By default Maple is designed to handle inequalities. Indeed when asked to solve

$$x^2 + 3x + 2 \geq 2$$

it displayed $x \leq -3$ and $x \geq 0$ which are the correct answers.

When harder problems were given to Maple it was still able to solve them. It was able to find the ranges of values of x. It could also handle the cases where no denominator was involved and the equal cases. However, in cases, like the ones below, the equal

sign was displayed on the values that x could take *regardless* of the fact some of them were *roots of the denominator.*

Hard inputs included forms such as

$$\frac{x^3 + 4x^2 - x - 6}{x^2 - 2x - 3} \geq 2$$

$$\frac{x^3 + 2x^2 - 2x - 6}{x^2 + x - 2} \geq 2$$

The command to carry out the solution of the inequality is the same as for the equation. The system senses that it is dealing with inequalities rather than with equalities and reacts accordingly.

It is still, however, possible to assist the student in solving the inequalities by either carrying out all the algebraic calculations, factorising, completing the squares, etc., or by plotting the graph. The latter provides a means for searching for intervals where the solutions may lie, or in case where the solutions have been found, to verify that they are correct.

Mathematica (version 2) is not able to solve inequalities directly although version 3 is able to solve them. However, in this instance there are three possible ways around this.

One is to create an application that would solve the inequalities. This is where the provision of a high level language is of enormous importance. It allows expansion or tailoring of a system's capabilities to suit ones' objectives.

The other option is to solve the problem graphically. Mathematica allows a number of expressions to be plotted together and so the student may be able to approximate the roots and find the range of x.

Another way, is, of course, the algebraic one, where algebraic manipulations are carried out to reduce the expression in a form from which conclusions can be easily drawn. Mathematica has built in functions that can expand or factorise either the numerator or the denominator or both. A sample of these functions is: **Expand**, which multiplies out products and powers; (in cases where a denominator exists it is not expanded); **ExpandAll**, which expands even the denominator; **Together**, which collects

all terms together over a common denominator and finally, **Apart**, which breaks an expression involving denominators into terms with simple denominators.

In the case of *Derive*, the version that was tried could not solve inequalities and given the lack of a language there was nothing that could be done about this. The only option would have been that of a graphical approach. Whilst it is most certain that later versions of Derive will be capable of solving inequalities, the point raised here that is restricted by the considerations and views of Derive's authors. If the authors failed to consider a point when developing their code, then it may be that the only option is to wait for the next version.

2.2.3 Differentiation and integration

Differentiation and integration are handled by CASs in a rather satisfactory manner and in fact CASs are capable of carrying out complete solutions to most problems encountered.

A use of CASs, in cases where the complete solution can be given, as suggested by Watkins [1990], is to give a number of functions to the students, get the students to differentiate them using a package and ask the students to find the rule by observing the result. Such a suggestion could cover the product or quotient rule in differentiation.

CASs can also offer help in identifying the connection between integration and differentiation. However, the examples set have to be chosen very carefully as the packages tend to "forget" about the constant of integration. For example, differentiating $x^2 + 1$ would result in $2x$. Integrating through a package, the result x^2 would be displayed, rather than $x^2 + c$.

Some integrals are not dealt with satisfactorily by all packages. These shortcomings could themselves be used to provoke the student to consider why this happens, for example, because of singularities, as in this example, $\int_0^5 \frac{1}{1-x} dx$.

Maple fully supports differentiation and integration. It copes especially well with all cases of integration, by parts, partial fractions, substitution etc.

The command for carrying out differentiation is

$$diff(f, x_1, x_2, x_3..x_n)$$

where f stands for the expression and $x_1...x_n$ are the variables. If the user wants the third derivative of x^3 then the command $diff(x\wedge3, x\$3)$; has to be issued. The dollar sign represents power and in this case it means the third derivative of the function x^3. Such a notation is used to denote differential equations.

The command for integration is

$$int(f, x) \text{ or}$$

$$int(f, x = a..b) \text{ or}$$

$$int(f, x = a..b, continuous).$$

The first command is used to find an indefinite integral whereas the last two are used for finite integrals. The last command could be used to "foul" Maple to carry out the finite integral although the function is not a continuous one. By default Maple carries out a test for continuity and if it is found to be false the evaluation is abandoned. It is for this reason that when asked to evaluate the following integral $\int_0^5 \frac{1}{1-x} dx$ it is returned unevaluated.

Mathematica, although quite capable in differentiation and integration has a number of hiccups regarding the syntax of its commands. Mathematica is the only package to offer the dash facility for denoting differentiation of a function, but, (as discussed in §2.2.1), the variable must be followed by an underscore.

The commands for differentiation are as follows:

D [expr, var] or
F'[var]

where F[var] is the name of the function and the user has declared it using the underscore after the variable.

For integration the commands are as follows:

for the indefinite integral

Integrate [expr, var]

and for the definite integral

Integrate [expr, { var, xmin, xmax}]

Finally, Mathematica was not able to calculate the **definite** integral $\int_0^5 \frac{1}{1-x}dx$ properly as it was obvious that the function mentioned has a singularity at $x = 1$. The above problem was tried for the values of 0 and 5. The answer was **-Log[-4]**. Obviously, this is not the correct answer as the function $\frac{1}{1-x}$ is not continuous, i.e., it has a singularity at $x = 1$. Mathematica ignores this fact and carries out a straightforward substitution and subtraction of the upper and lower values. In fact the approach should be to break the integral around the point of singularity and use limits, but Mathematica does not have this kind of knowledge. However serious this point may be, the existence of the high level language can be used to overcome the problem, by forcing Mathematica to carry out a singularity test before carrying out the definite integral.

Derive can handle both differentiation and integration to a satisfactory level. Again the use of the menu makes it very easy.

Differentiation in Derive is carried out by selecting the commands **Calculus** and **Differentiate**. Derive first prompts for the expression then for the dependent variable and finally the order. Issuing the command **Simplify**, the derivative of the expression is presented to the user.

To integrate, the commands **Calculus** and **Integration** have to be issued. As in differentiation, the expression is requested followed by the variable and the values for lower and upper limits. If the values of the lower and upper limits are left empty (simply by pressing return) then Derive calculates the indefinite integral. If values are entered then the value of the definite integral is displayed.

It must be noted that Derive failed to provide the correct answer to the $\int_0^5 \frac{1}{1-x} dx$ problem. In this case the reply was a complex number. Unlike Mathematica where this could be rectified, here there is nothing that can be done.

2.3 Current use of Computer Algebra Systems in education

Demonstrating the commands and their syntax of Maple and other CASs, in handling the topics of *functions and graphs, inequalities* and *differentiation and integration* provided a concrete example of some of the strengths and weaknesses of CASs. Their weaknesses can be summarised as follows:

1. CASs are very powerful mathematical engines and are bound to increase their solving ability with time. However, even Maple and Mathematica, undoubtedly the field's leaders, were reported in 1999 [Foster & Bau, 1999] as having problems with simplification (a major obstacle in CAS) and integration amongst other topics.

2. CASs lack interface facilities. That is, users are not aware of the range of the commands available, nor are the commands intuitive. Moreover, their syntax is not straightforward nor is it universal for all commands. For example in Maple all commands have to be in small case whereas in Mathematica the first letter of a command has to be a capital. Moreover, the number of parameters that each command accepts is not known nor is it known how many of these parameters are optional.

3. What does one do in the case where the wrong result is produced by the CAS?

The lack of interface facilities was reported as early as 1987 by Karian & Stterrett, [1987] based on their experiences of using Maple in teaching undergraduates mathematics. They found, contrary to their expectations, that students would become self-starters, that students were frustrated by the format of the commands and that they required a lot of instruction on using Maple. With the exception of DERIVE, all known CASs suffer from this problem. The problem is partly due to the design of CASs as they were not originally meant to be used for teaching purposes. The objective was to provide a fast, efficient, accurate and reliable tool for experts who knew mathematics but wanted to carry out long and tedious calculations, such as integrals, solutions of differential equations, etc.

CAS developers tried to address these inherent interface problems by the introduction of Notebooks. Notebooks are environments that allow one to mix text, graphs and commands, and thus overcome the interpreter environment that created some of the problems reported in 1987. Mathematica was first to introduce Notebooks followed closely by Maple. For more details on Notebooks and their use see Kelmanson et al., [1993], Kent et al., [1994], Templer et al., [1996].

Despite the Notebook addition, interface problems persisted, although in a slightly different form, as reported by Kelmanson et al., [1993], Cheng et al., [1995], as part of the TRANSMATH project. TRANSMATH is a computer based learning package designed to help students in numerate disciplines overcome the difficult transition from pre-university mathematics to that encountered at first year undergraduate level [Kelly et al., 1994]. TRANSMATH courseware, developed at Leeds University, accesses the powerful mathematical engine of Mathematica, but substitutes an alternative hypermedia front-end created using Toolbook (HyperCard-type software for the PC) [Kelly et al., 1994]. The rationale here was:

> that the underlying preface to the design style is that if the courseware is at all difficult to follow, students will stop using it ... give up on mathematics altogether [Kelly et al., 1994].

Furthermore, to free the students from Mathematica's syntax, a keypad was developed where the students could enter expressions in the order that they would write them down. What was effectively created here amounts to a glass-box application that enables the students enter expressions in the order that they would write them down. To the TRANSMATH team the benefits of Toolbook as well as the development of the keypad became quite clear when another tutor, Matrix-Theory Tutor, based on Mathematica's Notebook, was criticised mainly on the Notebook front-end [Cheng et al., 1995].

Mathematica is now reported to have addressed Notebooks' shortcomings in its latest version, version 3. As reported by Ramsden, [1997], hypertext functionality is given to Mathematica, and a number of interface issues are addressed. However, these developments in interfacing and functionality do not seem to impact on what can be seen as the heart of the issue of the use of CASs as teaching tools; that is, the impact of CASs on the way in which mathematics is taught at university level.

The remainder of this chapter addresses this issue by looking at recent case-studies. The projects mentioned here do not represent a comprehensive survey of the CAS that are used for teaching but rather are intended to provide a flavour of the current use. Derive, for example has been used as part of the foundation course in Plymouth University since 1990, see Watkins [1990], but it is not examined here. Because of its lack of a programming language, it could not be considered for augmenting Computer Algebra Systems.

In the Leeds project, [Cheng et al., 1995], Mathematica is seen as an accessory, as a tool that helps teaching a number of topics. Mathematica assists the teaching process by allowing animation, different plotting and enabling the user to see the effect of varying parameters. For example, TRANSMATH leads the student step by step in finding the integral of a fraction, by pointing to long division of polynomials and guiding the student through to find the quotient and so forth [Cheng et al., 1995]. The topics covered by TRANSMATH are:

- Introduction to Differentiation

- Techniques of Differentiation

- Indefinite Integration

- Definite Integration

- Ordinary Differential Equations

Concurrently with TRANSMATH the Transitional Mathematics Project (TMP) was run at Imperial College (IC) where according to Noss [1995], in his evaluation study of the two projects, the rationale was different. Again in TMP Mathematica was used but this time with the Notebook's front-end. The rationale in TRANSMATH was that students should not be bothered with the workings of Mathematica, hence the use of Toolbook to overcome interface problems and the Keypad to ensure that the input of mathematical notations in Mathematica should reflect human notation and not Mathematica's. By contrast, at Imperial College they actively encouraged students to "play" with Mathematica, that is, to learn to use the commands of Mathematica (At the University of North London a course entitled Computer Algebra exists which teaches students about Maple, its commands and how to use the package). This difference does

not affect the overall role of CAS but rather the interaction between the student and Mathematica; the difference can be summarised as *learn to program Mathematica* as opposed to *learn to program the keypad* [Noss, 1995].

MathWise is another computer based learning package covering quite a range of topics from complex numbers to calculus [Harding et al., 1996]. MathWise is a product of the UK Mathematics Courseware Consortium, a project in the Teaching and Learning Technology Programme. It produces computer based learning modules for mathematics, particularly for science and engineering students. A number of universities collaborate in this project. Again Mathematica is the CAS upon which these modules are based. The aim in MathWise is the same as that at Imperial College and Leeds, to provide a tool to help in the teaching of the modules.

2.4 The case for augmenting CASs

As the projects cited above indicate, CASs are well established in the teaching of mathematics but have limitations. The teaching of mathematics topics is much the same it was before CASs, although they provide the ability to plot graphs, animate, and so on.

I would like to propose an alternative use of CASs in education, based on the unquestionable benefit of CASs' powerful solving engines. If the attainment of the solution of a problem could be entrusted to a CAS, then procedures such as transposition of a matrix, or how the quotient rule and the product rule in differentiation operate, or how the general solution of a differential equation is achieved, could be examined and assessed. The assessment of these procedures will consider the following two options:

1. Is it safe to entrust the whole process to a CAS?

2. How can the process be revised so that

2a. the process is seen as a result of conceptual knowledge

2b. there is a logical sequence between the steps in the process.

As a result more attention could be concentrated on the conceptual side of the topic and thus provide the student with a richer picture. This will allow rethinking of what is

taught and how it is taught. At the same time the purpose of CASs is also rethought, since the role that they are now asked to play is more central than that defined in the projects mentioned previously.

To encompass this modification, CASs are required to "know" about the topics that they are required to cover in their procedural aspects. From the description of CASs, §2.2 to §2.5, it is apparent that the knowledge currently possessed by them is only procedural. Moreover, since the procedural knowledge of CASs is in the form of a black-box [du Boulay et al., 1981], a glass-box is required that would provide us with the option to follow the process of attaining a solution. This point is further discussed in Chapter 7, where the solution of the linear second order differential equation with constant coefficients is considered with the help of Maple and where the procedures for attaining the general solution are reassessed.

This augmentation of a CAS amounts to its transformation from an inherently non-pedagogical package to one offering pedagogical opportunities. Incorporating these considerations involves two tasks. First, CAS knowledge must be enriched by creating a glass-box for its procedural knowledge and by associating these procedures with declarative knowledge (see Chapters 7 and 8). Second, teaching strategies have to be built into the CAS.

Augmentation of a CAS is needed to avoid the pitfalls experienced in the use of calculators. Expectations that calculators would allow students to develop deeper conceptual understanding and would change mathematical teaching practices, were not realised [Hembree & Dessart, 1986]. Hembree and Dessart carried out a meta-analysis of 79 research studies which indicated that the use of calculators improves the average student's basic skills with paper and pencil too, both in working exercises and in problem-solving. Their use also improves attitudes towards mathematics. Where such tools are accepted, however, their use has usually been confined to traditional mathematical practices. The same fate is feared by Ruthven [1994] and Hennessy [1999] for the graphics calculator. In the case of the calculators, it was argued that such a fate was attributable largely to the failure to alter the curriculum to reflect significantly and appropriately the impact of calculators in teaching of mathematics [Shumway, 1988]. More recently, reference to the use of calculators in the UK National Curriculum has been made [DFE/WO, 1995] but the advice is not detailed enough to be considered helpful:

Information Technology

Pupils should be given opportunities, where appropriate, to develop and apply their information technology (IT) capability in their study of mathematics (p. 1)

...

use calculators both as a means to explore number and as a tool for calculating with realistic data, *e.g. numbers with several digits* (p. 3)

CHAPTER 3

Chapter 3: Review of teaching strategies in ITSs

This chapter reviews the teaching strategies module of SCHOLAR, WHY, MENO-TUTOR and DOMINIE. These systems are examined in detail since they share the same objective as this thesis, i.e., to derive a generic model for teaching strategies. Moreover, the systems examined here are considered to be classics in the study of teaching strategies in ITSs. Examination of these systems provides a spectrum of how the problem of teaching strategies has been approached.

This chapter describes the first version of SCHOLAR [Carbonell, 1970], where facts from the knowledge base were reproduced, and the second version of SCHOLAR [Collins et al., 1974], where the use of heuristics in informing the teaching strategies module emerges for the first time, see §3.1. The second version of SCHOLAR formed the foundation for the widely acclaimed Socratic method as manifested in WHY [Stevens & Collins, 1977], [Collins & Stevens, 1980], described in §3.2. The objective within ITSs now leaped from that of reproducing facts from the knowledge base, to that of encouraging the student to form and test hypotheses and, in general, discover knowledge. This approach was further expanded by MENO-TUTOR [Woolf & McDonald, 1984], [Woolf, 1984] where a model for engaging the knowledgeable and confused student in fundamentally different ways emerged, as described in §3.3. The last system to be reviewed in detail is DOMINIE, [Elsom-Cook et al., 1988], [Elsom-Cook, 1991], which deals with the paradigm of multiple teaching strategies, as described in §3.4.

The review concludes with a discussion, see §3.5, where the findings from each system are drawn together. In this discussion the need for further research becomes apparent, especially since systems such as EXPLAIN [Wood & Wood, 1996], [Reichgelt, et al., 1993] and SONATA [Angelides & Tong, 1995], do not consider the issues outlined above. A brief overview of these systems is also given in §3.5.

ITSs such as the Anderson Tutors (Lisp Tutor [Anderson et al., 1984] and Geometry Tutor [Anderson et al., 1985a]), and the GUIDON system [Clancey, 1982, 1983] are not included, since these do not consider the problem of teaching strategies at a fundamental level. The strategies incorporated in the Anderson Tutors, according to Wenger [1987],

> include reminding or statements of facts, analogy and decomposition or simplification of problems. Their actions, however, are usually indirect, in the form of hints. (p. 291)

Of course the Anderson Tutors, were based on Anderson's ACT* theory [Anderson et al., 1985b] which provides a context for their strategies and actions. However, the objective in this thesis is not to provide a simple explanation from a particular perspective but rather to aim for a more abstract, general and inclusive approach. Analogous comments apply to the GUIDON system, which expresses its teaching strategies in the form of T-rules and bear a resemblance to those used in the WHY system. The discussion about the WHY system therefore applies to GUIDON as well.

COCA [Major, 1993] is a tutoring system whose objective was to create an authoring shell for teaching strategies. COCA was based on DOMINIE and looked at the meta-level of teaching strategies, that is what governs the changes between the different strategies. Since our objective is focused at the foundations of teaching strategies, COCA is not relevant to this study.

In order to identify whether a model of teaching strategies exists or to identify the factors that influence or should influence the decision of teaching strategies, the following framework is used:

> What to say next?

> When to say it?

> How to say it?

The question, *what to say next*, will help identify the factors that influence the selection of sequence between topics, such as what is next in the curriculum. In the case of *when to say it*, the aim is to identify the factors that influence the decision about whether to provide the answer to the problem at hand or to give a hint or further elaboration. In the case of *how to say it*, reference is made to factors that affect the choice between a statement, a question or a counterexample (see §3.2 for explanation of counterexample).

In deciding *what to say next*, the interest is in the form of the knowledge representation and the student model and the extent to which they are used to affect the teaching decision. The internal structure and implementation of these components are not relevant to this discussion.

In deciding *when to say it*, the interest is in the form of knowledge representation, the tutor model (or teaching strategies model) and the student model. Whilst the interest in student model is again confined to its effect on the teaching decision, the structure of the knowledge base is now important in the search for elaboration and alternatives in expressing the same problem. The structure of the tutor model is of interest in the search for factors/principles that affect the choice between alternatives from the knowledge representation.

Finally, in deciding *how to say it*, the interest is again in the structure of all three components of an ITS: the knowledge representation, the tutor model and the student model. The knowledge representation and the student model are viewed to see how they affect the teaching decision. In the tutor model the interest is in its structures: mainly in the factors/principles that are encompassed to enable the system to express a statement or a question or a counterexample in one form or another.

3.1 The SCHOLAR system

In this section, both versions of the SCHOLAR system are reviewed. Reference to the first version is made simply to demonstrate the evolution in the study of the problem of teaching strategies by the field of ITS, since SCHOLAR, in its first version, is considered to be the first ITS. Furthermore, it acts as benchmark regarding Collins' team's (also responsible for the WHY systems as examined in §3.2) progress in the study of teaching strategies through the field of ITSs.

SCHOLAR, in its first version, has a knowledge base consisting of facts about the geography of South America [Carbonell, 1970]. An *importance tag* is attached to every fact to help determine the importance of the fact. The values for these tags vary from *0* to *6*. The most important pieces of information are marked as *0*, while the least important are marked *6*. For example, if we examine the property *location* in Brazil, then the information *Northeastern South America* is assigned the value 0, since this is the most important fact. The information *Bordering-Countries*, which gives only the names of these countries, is rated 1, whereas further details on each bordering country are rated 2 [Carbonell & Collins, 1973].

The teaching strategy in the first version of SCHOLAR relies heavily on the use of these importance tags. In conducting a dialogue, SCHOLAR is guided in its traversal of the knowledge base by an internal dynamic agenda. The agenda, similar in its operation to

a pushdown list, contains information that the system considers relevant and important for the chosen topic. SCHOLAR bases its choice of topic on a random biased switch although the user is allowed to make a choice. Through the use of the importance tags the contents of the agenda are generated. To avoid flooding of the agenda a *cut-off* function is employed and it relies on the use of importance tags. This way information whose importance tag is above a certain value is left out of the agenda. The information that SCHOLAR considers important is presented to the student without further delay (Line numbers 2-5), see figure 3.1. Upon request, less important information is presented without any further hold-up (Line numbers 7-8) as shown in figure 3.1.

TELL ME ABOUT THE AMAZON	(1)
THE AMAZON IS NAVIGABLE.	(2)
IT IS A RIVER.	(3)
IT IS IN BRAZIL AND PERU.	(4)
THE LENGTH IS 4,000 MILES.	(5)
PLEASE TELL ME MORE ABOUT THE AMAZON	(6)
THE MOUTH IS IN THE ATLANTIC.	(7)
THE TRIBUTARIES ARE THE MARANON, THE RIO NEGRO, THE JURUA, THE PURUS, THE TAPAJOZ, THE MADEIRA, AND THE TOCANTINS.	(8)

Figure 3.1: Illustrative dialogue between SCHOLAR and a student. (The student's questions are enclosed in asterisks)

It should also be noted that the teaching strategy in this first version of SCHOLAR did not build up its dialogue from the student answers. SCHOLAR failed to accumulate information about the state of the student and react accordingly. If a question was wrong then SCHOLAR acknowledged that the answer was wrong and the correct answer was provided.

This lack of reaction to student's answers and the primitive state of the teaching strategies module, prompted Collins and his team to carry out further research specifically on the teaching strategies module. Collins et al. [1974], reports that the questions that generated further research were:

> how the {human} tutor related his teaching to student's prior knowledge and in how the tutor corrected student's errors. (p. 14)

However, their research eventually included questions to do with:

topic selection,

the interweaving of questioning and presentation,

reviewing by the tutor,

the use of hints,

the tutor's *response to errors (p. 14)* [Collins et al., 1974].

Examination of protocols showed that human tutors *select the next topic* mainly according to importance. It also showed that *context* influences their selection mechanism. For example, if the topic under exploration is *major river systems* then it is noted that the tutor carries on talking about the differences between estuaries and deltas. Such a 'diversion' was followed because the student was not able to discriminate between the two successfully. When the topic *geographical features* was introduced, the fact that the student mentioned Cape Horn triggered a lengthy discussion about Cape Horn. Therefore, an improvement can be seen in the sense that students' answers will play a constructive role in the selection of the next topic to be discussed. Nevertheless, it must be pointed out that such an influence occurs if and only if the student's contribution is correct and relevant to the overall topic [Collins et al., 1974].

The analysis of these protocols provided Collins and his colleagues with a number of heuristics that could be used to direct the topic selection. They are:

1. When the topic is an attribute (e.g. geographical features), *select the most important unused value under the current topic.* When the topic is a value (e.g. South America), *select the most important attribute and value under the current topic.* (Context affects this selection by temporarily increasing the importance of topics that are related to the previous topic discussed.)

2. If the attribute and value selected are below some criterion level of importance, which indicates that all the important information under the current topic has been exhausted, then pop up from the current topic to the previous topic in the pushdown list of topics, and start again at rule 1. (The criterion level appears to depend on some combination of importance weighted by the time available.)

3. The attribute and value selected are above the required level of importance, so formulate a question about the value of the attribute, or present the attribute and value to the student.

4. Add new topics to the pushdown list of topics. (This is the major way context affects the selection of topics.) When the current topic is an attribute, the new value is added to the top of the pushdown list. When the topic is a value, first the new attribute and then the new value are added. If the student gives an unexpected correct answer, his value is used instead of the value from the data base in adding to the pushdown list. If an answer is incorrect an error correction strategy takes over temporarily.

5. The top item on the pushdown list of topics becomes the next topic.

[Rules from Collins et al., 1974, (p. 29-31)].

Deduction of the above rules could be perceived as an indication that, if implemented, SCHOLAR would move from having a random selection mechanism to having a more coherent selection mechanism. However, this is not the case, as the analysis of the heuristics in the following sections demonstrates.

With respect to *interweaving questions and presentations* the mechanisms are extremely primitive because they are based on the following:

> As an explicit theory of the interweaving of questioning and presentation, we would argue that when the tutor thinks the student may know the answer, he introduces the topic as a question rather than presenting the information (p. 37-38) [Collins et al., 1974].

This extract does not provide sufficient evidence about when to proceed with a question or with a presentation. In the human tutor-student dialogues, students are evaluated about whether they are likely to know the answer based on the number of right and wrong answers that they have already given, in relation to the importance of the questions asked [Collins et al., 1974]. If the student appears to have answered top-level questions correctly in the past then a question is proposed, otherwise a presentation is chosen. But there is no guarantee that a piece of information, whose value appears to be above the level that the tutors believe the student can command, would be unknown to the student. Furthermore, questions may produce information about students' misunderstandings. Such information about the student's

misunderstandings could direct the flow of the lesson, whilst presentation would hardly offer such an opportunity.

Although *reviewing* was found to be of great importance [Collins et al., 1974], there is no specific strategy added to SCHOLAR for this purpose. The topics tend to reappear in different contexts. Close examination of the dialogues shows that information that has been missed by the student in the first pass is brought forward by the system during subsequent passes. However, at these later passes the system asks the questions and if the student misses them again then they are presented.

A *review* in SCHOLAR seems to lose its essential function, that is, to go through the most important information that the student should assimilate. Moreover, the human tutor concentrates on information that is really important and holds the key for better and more effective assimilation. Time is not wasted covering pieces of information that the student has missed, especially if that information appears to be a simple detail. These considerations are not taken into account. SCHOLAR simply traverses the knowledge base and if a piece of information is not marked as known, then a question is presented on the screen. Although *the use of hints*, was part of the research questions, there appears to be no information regarding hints and no strategies were implemented to tackle their use.

On the issue of *the tutor's response to errors*, an important extension has been added to SCHOLAR. This is its partial ability to parse and discuss errors. For example, the dialogue would be handled as shown in figure 3.2.

> NAME THE CAPITAL OF BRAZIL
> *RIO DE JANEIRO*
> WRONG. THE CAPITAL OF BRAZIL IS BRASILIA   (first version)
>
> NAME THE CAPITAL OF BRAZIL
> *RIO DE JANEIRO*
> RIO DE JANEIRO WAS THE CAPITAL OF BRAZIL. THE CAPITAL NOW IS
> BRASILIA   (second version)

Figure 3.2: Comparison between the first and second SCHOLAR with respect to their handling and utilisation of wrong answers.

In this new version, factors that distinguish the student's answer from the correct one are reported. To avoid redundancies SCHOLAR simply ticks the parts of the answer that are correct so that they can be skipped. Lack of a student model does not allow full utilisation of the student's errors which in turn could play a dramatic role in the decisions of the teaching strategy.

3.1.1 What to say next?

With reference to, *what to say next*, a major improvement is seen in the evolution of SCHOLAR. The system dropped its random-based selection and moved towards a heuristic based mechanism. However, hidden randomness may still exist in the second version of SCHOLAR. Although the agenda, which operates under certain heuristic rules, now plays an important role in the unfolding dialogue, there are no clear rules that govern when the tutor should expand on a topic, or when the system should take into account the student's correct answer. The rules appear to be concerned with ensuring that topics related to a topic being discussed, or being mentioned by the student, are entered into the agenda.

3.1.2 When to say it?

With respect to the question, *when to say it*, SCHOLAR in either version relies heavily on the importance tag values. Facts are presented according to their values in conjunction with the cut-off function. Factors such as weakness demonstrated by the student, or the need for coherent sequencing of information, are not considered. Probably the fact that neither version keeps a proper student model, according to Wenger [1987], could explain why SCHOLAR was not able to adopt a more sophisticated mechanism.

3.1.3 How to say it?

On the question, *how to say it*, the findings and their implementation are very simple in the second version (the first does not deal with this question at all). The alternatives considered here are either *presentation* or *question*. Facts are asked for or presented in a final form.

SCHOLAR always starts off with a question and goes on deeper and deeper into the knowledge base until either a criterion for pop up is met or the student has answered something wrongly. In the latter case, the system offers some information that is

embedded further down and then pops up. Such an a priori estimate is based on the value importance tags for which the student has answered or failed to answer a question. If the information to be presented happens to be in the interval of these values then SCHOLAR proceeds with the question, otherwise a presentation is offered. Clearly, again, such considerations are vague and do not provide a basis for selection between the alternatives, question or presentation. Therefore it can be concluded that in fact SCHOLAR is not dealing with the choice of *presentation* or *question*.

Although it is important to establish when a question or a presentation is best suited, I believe that the *context* of the selected mode should also be examined. For example, even though the use of hints was studied in the analysis of the human tutor's dialogues, a specific strategy to enable such a facility was not included in SCHOLAR. Also it is important to consider the motivation that lies behind a desire to question the student: *what is the student going to gain by attempting to answer this question?* Moreover there should be consideration as to whether all information should be presented or whether the student should be called upon to work from a subset of the information.

3.1.4 Summary

The review of both versions of SCHOLAR shows the transition from almost a non-existent teaching strategy module to a sketchy teaching strategy module that is capable of more than simply reproducing facts from the knowledge base. It is sketchy because, despite the use of heuristics, it reacts to information gathered by the student model and the knowledge base as opposed to having a direction which is shaped by the information supplied by the student model and knowledge base. Nevertheless it is the first time that heuristics derived from the analysis of human tutor dialogues, were used as the backbone of such a module.

Even though the teaching strategy module of the second version of SCHOLAR is sketchy it nevertheless has formed the founding stone for the important and widely acclaimed Socratic method. This was researched and implemented in WHY which is examined in the next section.

3.2 The WHY system

WHY could be considered as a cornerstone in the study of teaching strategies in Intelligent Tutoring Systems for three reasons:

- First, it marks the departure from *retrieval-oriented* systems (such as SCHOLAR, SOPHIE, WEST) to *context-dependent* systems [Woolf & McDonald, 1984]. The emphasis in the former is on retrieving the correct answer whereas in the latter is on eliciting the answer and guiding the student in general.

- Second, for the first time the form of the structure of the knowledge base is influenced by the form of the teaching strategy module (see figure 3.3).

- Third, WHY's teaching strategy module is the first one which is 'substantial' i.e., it has content, even though its content relies on heuristics. It contains a number of strategies, all of which form the Socratic method, which are selected and executed according to the needs of the individual learner. In this sense the teaching strategy module is not merely reacting to information gathered by the student model, as in SCHOLAR, but rather shapes or fine tunes its actions according to the needs of the learner.

WHY's domain, in its first version, is concerned with the causes of rainfall whereas in its second version (not implemented) the domain is the growing of rice. To obtain information on the content of the teaching strategy module Collins and his team again analysed human tutors' dialogues with students [Stevens & Collins, 1977]. The main difference here is that the emphasis is on elicitation of knowledge, guiding the student and not in retrieving the correct answer from the knowledge base.
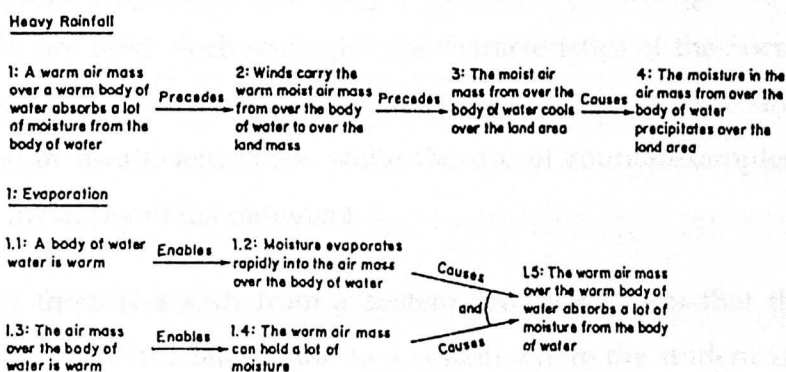
Figure 3.3: The WHY system script for heavy rainfall and the subscript for evaporation. [Stevens et al., 1982]

To achieve elicitation and guidance the knowledge representation in WHY is not in the form of a semantic network. It is structured in a hierarchical fashion in the form of

scripts and *subscripts* [Stevens & Collins, 1977]. Furthermore, the authors argue that the phenomenon of rainfall could be viewed as a temporally-ordered linear sequence of events [Stevens et al., 1982]; an illustration of this linearity as represented in WHY is given in figure 3.3.

The WHY system is capable of carrying on a dialogue about factors influencing rainfall by presenting the student with different cases, asking for predictions, probing for relevant factors, entrapping the student when not all necessary factors have been identified and making use of counterexamples [Stevens & Collins, 1977].

The discussion is usually initiated by WHY suggesting a case. If the case is new, then it prompts the student to make a prediction. The answer provided by the student is analysed to verify if it contains all the steps as described in the knowledge representation of WHY. If some step(s) in the student's answer have been missed out or others added, then WHY employs its other strategies for remedial actions. The remedial actions depend on the position of the missing/added step.

If a step is missing then it is the case of *prior* factor(s). In this case the system enquires through questioning about the missing step(s). If the questioning process is not successful, in the sense that the student is not able to name a factor, then the system suggests a factor and then in turn questions its necessity or sufficiency. The subsequent questions are triggered in order to identify *intermediate* or *subsequent* causes.

To establish if a missing step is necessary or sufficient the strategies of *entrapment* and *counterexamples* are used. Such strategies are characteristics of the Socratic method, of eliciting knowledge [Stevens & Collins, 1977]. In the former case, the student is trapped into accepting an insufficient factor while the use of counterexamples forces him to recognise the invalidity of his statement.

It is clear that there is a shift from a system producing facts that the student was expected to reproduce at a later point, to a system where the student is encouraged to *discover* the knowledge and develop a thinking process. However, WHY lacks the ability to conduct dialogues with reference to overall directions or as they call them high-order goals [Stevens & Collins, 1977], [Collins, 1977], [Resnick, 1977].

To address this issue Collins and his team returned to the human tutors. However, this time the human tutors were asked to explain their actions on the basis of what they

thought the student knew or did not know on the basis of the student's response, and to explain why they responded to the student in the way they did [Stevens & Collins, 1977].

From this analysis Stevens and Collins [1977] concluded that the *top level* goals of a Socratic tutor are to:

1. Refine the student's causal model.
2. Refine the student's procedures for applying the model.(p. 6)

Furthermore, they argued that:

these goals directly govern the selection of cases. As the student's knowledge becomes more refined, moving from an understanding of first-order factors to high-order factors, cases are selected which are exemplary of the factors the tutor is trying to teach. As the student's predictive ability becomes refined, cases are selected which are progressively more novel and complex, thus taxing predictive ability more and more ... (p. 6) [Stevens & Collins, 1977].

To achieve these top level goals, they claimed that achievement of the subgoals *diagnosis* and *correction* was required. These subgoals are responsible for the selection of their basic strategies, such as use of prediction rules, entrapment rules, inform-student rules, insufficient-factor rules, etc. Each of these basic strategies often serves several purposes.

In the case of *prediction* the student is forced to make a prediction about a carefully selected case. This strategy serves two purposes; firstly to identifying a bug, a missing or an added step, and secondly to encourage the student to make a guess and thus enable the dialogue to continue.

With respect to *entrapment*, the scope of this strategy is to pose a misleading question in order to trap the student into accepting an *unnecessary* or *insufficient* factor. It acts as the precursor of the *inform-student* or *probe reasoning strategy*.

The *inform-student* strategy is used to present the correct information to the student. It can be either in the form of a correct fact or relationship or to point out a necessary factor or a sufficient factor [Collins, 1977].

Finally, through the strategy of *insufficient factors*, WHY can "surround" an insufficient factor via the formation of a general rule about it, picking a counterexample for it, probing or pointing for a necessary factor and finally probing for similarities in two cases so that the student is able to recognise the insufficiency of the factor, and thus correct the bug.

The diagnosed bugs were classified and dealt with accordingly. They were organised as *factual, outside-domain, overgeneralisation, overdifferentiation* and *reasoning strategy* bugs. The priority assigned to correcting these bugs was governed by the following heuristics [Stevens & Collins, 1977]:

1. Errors before omissions.
2. Prior steps before later steps.
3. Shorter fixes before longer fixes.
4. Lower-order bugs before higher-order bugs.

The above constraints imply that the bugs are dealt with in the order in which they have been listed above. When more than one instance of a bug is diagnosed it is placed on an *agenda* according to its priority. The agenda is updated dynamically. That is, while a bug is being pursued to be fixed, should another bug with a higher priority surface then the attempt to fix the current bug is temporarily postponed until the bug with the higher priority is resolved.

Despite the refinement of the strategies that formed the Socratic method, Collins and his team were concerned with the lack of explicit rules specifying the conditions to be satisfied in order to execute a particular strategy. At the same time they raised the stakes of their objective by pursuing a further goal, the highest level goal:

To teach students how to derive a new theory for a domain of knowledge
[Collins & Stevens, 1980].

To achieve their objectives, WHY underwent a second refinement, although it must be stressed here that this refinement was never implemented. As a result of the refinement, 60 production rules were produced and these were classified under four strategies, according to the nature of their function. The strategies are *Case Selection Strategies (CSS), Entrapment Strategies, Identification Strategies* and *Evaluation Strategies.* Now these strategies formed the Socratic method.

The important point about this refinement, is that in order to achieve their objectives Collin and his team had to completely rethink the structure of WHY. In particular the knowledge representation had to be reorganised so that the conditions for satisfying the productions rules were explicitly stated. This is a further evidence of the impact that the structure of the teaching strategy component has on the organisation of the knowledge structure. This is a point of great importance and explored in this thesis as well (see Chapter 6).
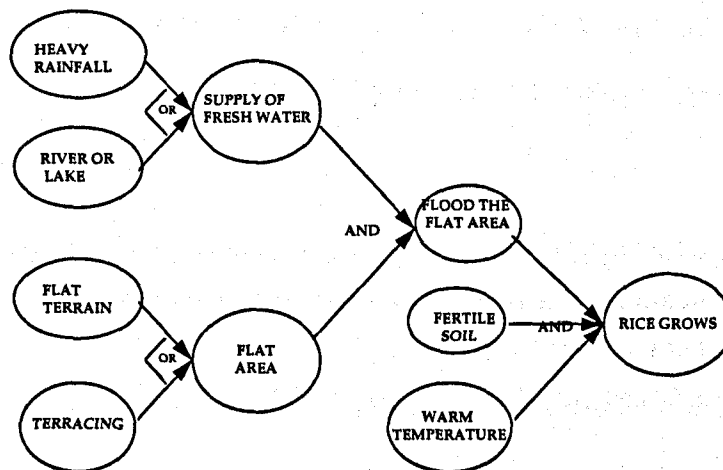
Figure 3.4 : The knowledge representation in the last version of WHY [Collins & Stevens, 1980]

Although the knowledge representation is still based on the principle of *scripts* and *subscripts*, the relationship between scripts and subscripts is not a linear one but rather that of an n-ary tree where the nodes are connected by *and/or* links. Figure 3.4 illustrates the new structure of the knowledge base by referring to the conditions required for growing rice.

The *and* link indicates the necessity for all conditions to exist and that a subset of them is not sufficient. On the other hand, the *or* link indicates that even one condition is sufficient and that none is necessary. For example, either *heavy rainfall* or a *river* or a *lake* is a sufficient source for *fresh water* but neither is necessary [Collins & Stevens, 1980], [Stevens & Collins, 1980]. In the case of *fertile soil*, it is a necessary factor for the rice to grow, but it is not sufficient as *flood the flat area* and *warm temperature* are also required. This is indicated by use of the *and* connector. Here, the function of the knowledge base is upgraded as it provides more information. These go beyond the traditional knowledge bases as they indicate what conditions *must* exist for rice to grow.

In the next three subsections the WHY system is analysed using the framework outlined at the beginning of this chapter, that is with regards to questions, *What to say next?*, *When to say it?*, *How to say it?*.

3.2.1 What to say next?

With reference to the first question, *what to say next*, it is clear that WHY is making its decisions on the following factors:

1. The student's response.
2. The top-level goal that it is pursuing.
3. The way the knowledge base is structured.
4. The agenda.

The student's responses are evaluated before a decision is made. However, in its effort to consider these responses WHY has moved to the other extreme of the spectrum from SCHOLAR, i.e., from too deterministic to too opportunistic. That is WHY is too eager to adapt to student's responses. In fact WHY is so opportunistic that it is capable of creating, what I would call, a *never-ending cycle*. In this cycle WHY could go round and round a given insufficient/unnecessary factor for ever and there is no rescue mechanism for getting out of this loop. This is because, although the student response is supposed to be only one of the four factors influencing the decision on *what to say next*, close examination of the dialogues between students and WHY reveals that in fact the student response is the dominant, if not the only, factor.

The system in that sense is considered to be driven by what could appear to be student needs. However, as Wenger [1987] reports WHY, like SCHOLAR, does not keep a proper student model. If this fact is coupled with the limitations of a student model, as reported by Laurillard [1988], further questions arise about whether the needs that WHY tries to adapt are those of the student.

Another reason for this opportunism, could be the fact that the higher level goal(s) of WHY, are in fact limited, to refine the student's causal model. To that extend WHY succeeds as it adapts to the current state of the student's model and tries to rectify it. But if this approach is seen in the wider perspective of education where refinement of the student's causality is only one factor then clearly the WHY approach requires revision.

Such a revision could be based on the concept of what I would call *pluralism*. This concept implies that a number of factors, such as the *tutor*, the *student*, *the student model* and the *knowledge base*, are involved in the decision process. Moreover pluralism implies that there is a *symbiosis* between these factors and that all factors are equally weighted in bidding in the decision making process.

Are these factors considered in WHY's decision making list, above? The answer, on the surface, is yes, with one exception, the *student*. Factors no. 1 and no. 4 represent the student model, no. 2 represents the tutor, no. 3 represents the knowledge representation. Nevertheless, as has been mentioned the factors do not possess equal weighting in bidding for a particular topic.

3.2.2 When to say it?

With reference to the second question, *when to say it*, WHY considers the following factors:

1. The top-level goal currently pursued.
2. The classification of the bugs.

These factors provide the necessary information for the system to decide when is the best time to introduce a particular topic into its tutoring capacity.

With reference to the first factor the system's reactions are based on heuristics. If the top-level goal explored is *refinement of the student's causal model* then topics are presented in a *breadth-first* manner. If the second high-level goal is being pursued then the topics are presented in order to enhance a deep understanding of the subject, or as Collins and Stevens [1977] put it, *depth-first*.

So the knowledge base in this instance is conceived of as playing an important role in the decision about what new topic should be introduced to the student. Despite this the knowledge representation does not possess the equality that we are searching for in a *pluralistic* tutoring system. It is merely carrying the wishes, in this case, of the teaching strategy. The only condition that it imposes upon the conduct of the dialogue is its structure, which will always be observed.

Bugs are dealt with according to their position on the agenda. Their position, as mentioned above, is decided by a number of heuristics. The tutor in all cases seems to

have overall control of the situation and the student has no means of expressing his preferences. In such cases, there are questions about the sort of adaptability that a tutoring system is supposed to provide. Does the system actually adapt to the needs of the student or is the student forced to adapt to the way that the tutor has conceived the subject matter?

3.2.3 How to say it?

With reference to the third question, *how to say it*, the decision is again based on a number of factors. They are:

1. The student's response.
2. The subgoal that the tutor currently pursues.
3. The agenda.

In this case WHY opens new horizons. It is considered to be the first system that attempts to seriously consider this problem. However, this attempt is only at a surface level. WHY's repertoire is based on different instantiations of the simple forms of questions and presentation. Their form is decided according to the nature of the subgoal currently pursued by the tutor. If the goal is *diagnosis* then, depending on the nature of the bug, either a *factor-rule* or *prediction-rule* could be activated. Again, when the goal is *correction*, the appropriate rules are fired.

Clear distinctions concerning *how to say it* are achieved as every rule contains in its action part a particular set of actions and these sets are distinctive for every rule. However, a rule is always executed in the same fashion: *pluralism* in this respect is only resolved at a surface level. For example, the system as designed has no ability to present an entrapment rule or a prediction rule in more than one way.

To illustrate the point further, consider the shape of the entrapment rule.

Entrapment

    Pose a misleading question.
    Form a general rule for an insufficient factor.
    Form a general rule for an unnecessary factor.

Figure 3.5: The actions to be carried out in the case of the entrapment rule [Collins & Stevens, 1977].

It is clear from figure 3.5 that whenever the entrapment rule is activated these three actions will be carried out in the same order with the same content. If the student fails to adapt his thinking process to that of the tutor then an artificial bug may be created and pursued by the tutor. Such a bug is created solely because of the tutor's inability to handle an alternative presentation at a deeper level.

3.2.4 Summary

The WHY system is seminal and classic in the study of teaching strategies, in the sense that it is the first ITS that has a proper teaching strategies module, despite its limitations. A teaching strategy module containing a number of, what Collins and his team call, strategies required for the Socratic method. The teaching strategy in WHY is not simply reacting to the knowledge base or student but rather acts on their input. Moreover, it is the first time that the knowledge base of a tutoring system is influenced by the contents of a teaching strategy module, even though in this case it dominated the structure.

WHY is the first system that moved from a retrieval-oriented approach to a context-dependent one. Consequently, the student is actively engaged in participating in the evolution of the lesson. The student is not told facts; instead the student is directed/guided to *discover* them. The student is encouraged to form hypotheses and test them.

Also WHY has provided a computer model that supports the demanding Socratic method and indeed other ITS's, DOMINIE [Elsom-Cook et al., 1988], COCA [Major & Reichgelt, 1992] Major [1993], used WHY as the foundation for their version of the Socratic method.

However a number of issues that require further attention have been evident in WHY. First of all is the use of heuristics which are rather limited when it comes to looking for principles in designing a teaching strategies model for an ITS. Whilst heuristics may form an initial model, their very ad-hoc nature, even if they are derived through an analysis of human tutor dialogues, compromises and limits them.

Also issues such as lack of pluralism, the 'never ending cycle' and the lack of progress on the question *how to say it*, were identified in the design of WHY and are important issues. Consideration of the first two issues becomes more important when the design

of a teaching strategy module for an ITS is seen from a wider educational perspective, as adopted in this thesis, and not from a focused perspective, however important, such as refinement of learner's causal model.

MENO-TUTOR [Woolf & McDonald, 1984] is reviewed in the next section to examine what progress has been made regarding the issues raised in the review of WHY. There is an interest in the issue of heuristics, to see if any principles for teaching strategies are identified, but the focus is on any progress made regarding the question *how to say it* as MENO-TUTOR is able to engage the knowledgeable student in a way that is fundamentally different from the way that it engages the confused student.

3.3 The MENO-TUTOR system

> MENO-TUTOR is an example of a machine tutor that uses intelligence within the tutoring components. It has the ability to examine earlier discourses with a student and adapt its discourse appropriately; for instance, it will engage the knowledgeable student in a way that is fundamentally different from the way it engages the confused student. We call this kind of system "context-dependent" and contrast it with what we call "retrieval-oriented" systems, such as SOPHIE and WEST. Note that while we have emphasised guiding the learner based on what the tutor knows about him, other systems have placed their emphasis on retrieving the correct answer [Woolf & McDonald, 1984].

To achieve the above claims, MENO-TUTOR's discourse component architecture was organised in two levels: the *tutoring* which is responsible for planning the discourse and the *surface language generator* which is responsible for producing the natural-language output [Woolf & McDonald, 1984]. The tutoring component makes decisions about what discourse transitions to make and what information to convey or query. The construction of the discourse component was implemented in a way that enables MENO-TUTOR to be independent of the subject residing in the knowledge base [Woolf & McDonald, 1984].

To bring out its teaching strategies the focus will be on the first component of MENO-TUTOR, the tutoring component. The tutoring component is broken into three levels: the *pedagogic state*, the *strategic state* and the *tactical state*.
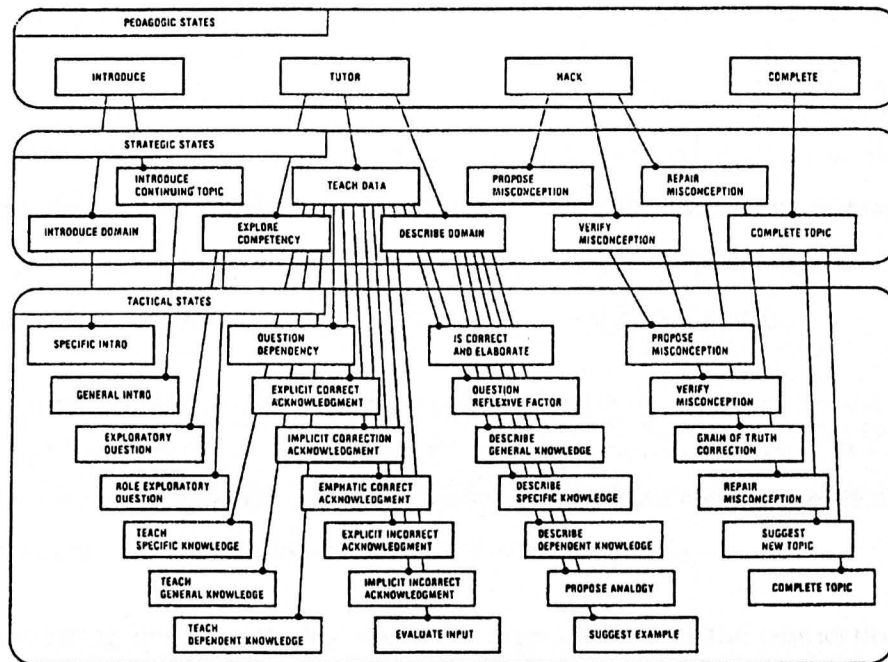
Figure 3.6: The discourse management network used by the tutoring component of MENO-TUTOR [Woolf & McDonald, 1984]

At the highest level, the pedagogic state, the system is able to select one of the following four options: introduce, tutor, hack and complete. With respect to the decisions taken at this stage, Woolf [1984] states:

> At the highest level planning states constrain the discourse to a specific tutoring pedagogy that determines, for instance, how often the system will interrupt the student or how often it will probe him about misconceptions. At this level a choice is made between approaches such as diagnosing student knowledge or introducing a new topic. (p. 86)

How the choice is made between the alternatives is not clear. There are no references to what factors to consider or how they influence the decisions.

In the strategic state level the alternatives vary from introducing a topic, to repairing a misconception and completing a topic. These alternatives bridge the various options in the pedagogic state with their corresponding ones in the tactical state. They are considered to be refinements of the pedagogic options. Woolf [1984], states that

> at the second level, the planning states further refined the pedagogic plan into a strategy, which specifies a schematic script for the response. For instance, the choice,

here, might be between questioning the student or providing a series of examples. (p. 86)

The third state, the tactical state, contains the most refined actions in the MENO-TUTOR architecture. Examples of them are: teach specific knowledge, teach general knowledge, explicit correct acknowledgement, implicit correct acknowledgement, explicit/implicit incorrect acknowledgement, etc. Woolf [1984] states:

> At the lowest level, a tactic is selected to implement the strategy. For instance, if the strategy had been to question the student, the system can choose from half a dozen ways to formulate that question: e.g. it can question the student about a specific topic, or the dependency between topics, or the role of a subtopic. (p. 86)

What is interesting here is how the states are organised, how the connection between them, at the various levels, is achieved and which principles are illustrated by the current implementation of MENO-TUTOR. The states that constitute the tutoring component are similar to an augmented transition network (ATN).

> Each state is organised as a LISP structure with slots for functions that are run when the state is evaluated. The slots define such things as the specification of the text to be uttered, the next state to go to, or how to update the student and discourse models [Woolf & McDonald, 1984].

The tutoring component's operation is based on the usage of default paths and meta-level rules. There are two instantiations of default paths. In the first instance they draw the connections between the states among the various levels. The second instance exists only on the strategic and tactical level. In this case the paths indicate the flow between the various states on the same level. This point is best illustrated in figure 3.7.

S1-EXPLORE-a Strategic Meta-Rule
> From: teach-data
> To: explore-competency

Description: Moves the tutor to begin a series of shallow questions about a variety of topics

Activation: The present topic is complete and the tutor has little confidence in its assessment of the student's knowledge.

Behaviour: Generates an expository shift from detailed examination of a single topic to a shallow examination of a variety of topics on the threshold of the student's knowledge.

T6-A. IMPLICITY-a Tactical Meta-Rule

> From: explicit-incorrect-acknowledgement
>
> To: implicit-incorrect-acknowledgement

Description: Moves the tutor to utter a brief acknowledgement of an incorrect answer.

Activation: The wrong answer threshold has been reached and the student seems confused.

Behaviour: Shifts the discourse from an explicit correction of the student's answer to a response that recognises, but does not dwell on, the incorrect answer.

Figure 3.7: Default paths on the two instances. [Woolf & McDonald, 1984]

However, the innovative point about its control structure is that paths are not fixed; each default path can be pre-empted at any time by a meta-level rule that moves MENO-TUTOR onto a new path, which is ostensibly more in keeping with student history or discourse history [Woolf & McDonald, 1984].

The meta-level rules operate on the strategic and tactical level. Their information is gathered from the knowledge base, the domain model and the student model [Woolf & McDonald, 1984]. Before MENO-TUTOR is analysed in terms of the framework, as discussed at the beginning of this chapter, it is of great interest to examine the principles upon which the tutoring component of MENO-TUTOR was designed.

Woolf [1984] defined tutoring as follows:

> Tutoring is a linguistic exchange whose goal is to clarify a body of knowledge to which the student has already been exposed (e.g. through lectures or reading) (p. 6).

Woolf attempts to create a theory of tutoring by analysing portions of 12 human dialogues with the following intention:

> ... my goal was to recognise speech patterns used by the speaker and to correlate these with what I inferred to be the intention of the speaker or the assumptions made by him about the listener (p. 58) [Woolf, 1984].

Through the analysis of these protocols she arrived at a taxonomy in which she recognised the two most potent distinctions which are: the *guidance discourse* and the *reconstruction discourse*. Under these two headings Woolf created three columns which

correspond to the following classification: *speaker's goal, assumptions about the listener* and *speech patterns* [Woolf, 1984].

In that taxonomy Woolf recognised that under *guidance discourse* when the speaker's goal was to *accomplish a task* , the assumptions about the listener were *minimal experience* and the speech patterns used were *provide instructions* and *correct the listener's words or actions*. When the speaker's goal was to *explore knowledge*, the assumptions about the listener were *incomplete knowledge* and the speech patterns *provide instructions and interrogate*.

Under *reconstruction discourse*, when the speaker's goal was *rebuild listener's cognitive model* the assumptions were *confused knowledge* and the speech patterns *interrogate and correct*. When the speaker's goal was *change the listener's assumptions* the assumptions were *wrong knowledge* and the speech patterns *interrogate and provide reasoned arguments*.

What is rather troubling here is the extensive use of heuristics; instead of moving to a more principled approach where principles could be derived from related fields such as, Educational Psychology, Cognitive Science, Instructional Design or Instructional Psychology, the whole design of MENO-TUTOR appears to be deeply embedded in heuristics. Coupled with this is the fact that the research question is of qualitative nature, and the whole design of MENO-TUTOR appears more ad-hoc and unprincipled. Consequently, questions regarding the whole organisation arise.

For example, it is not apparent how Woolf arrived at the most potent distinctions. In particular, why is the speaker stripped of the opportunity to accomplish a task under *reconstruction discourse*, and only given the choices of either *rebuild listener's cognitive model* or *change the listener's assumptions*? The last two options, *rebuild listener's cognitive model* and *change the listener's assumptions*, appear as a subset of *accomplish a task* where the overall task is to *accomplish a task*, the task being either to *rebuild listener's cognitive model* or *change the listener's assumptions*. Woolf's findings in the taxonomy, appear to be not independent, but rather interrelated.

Sometimes, heuristics appear to have been deduced in a process that contradicts a fundamental principle, which is to build on what the learner's experience, a fundamental principle in Educational Psychology (e.g. see Ausubel et al., [1978]). For example consider the following excerpt:

The most potent distinction for my purposes is between *guidance discourse* and *reconstruction discourse*. In *guidance discourse* the speaker's goal is to accomplish a task; the speaker directs the listener to largely new information without concern for what <u>he</u> <u>already knows.</u> (p. 59) [Woolf, 1984] (my underline).

In the next three subsections the MENO-TUTOR system is analysed using the framework outlined at the beginning of this chapter, that is with regard to questions, *What to say next?*, *When to say it?*, *How to say it?*.

3.3.1 What to say next?

On the question of *what to say next*, MENO-TUTOR is influenced by the default path and the meta-level rules. Derivation of either of these is ad-hoc, and based on the analysis of human tutor's dialogues. How these rules are conceived and how they are attached to the architecture of MENO-TUTOR is not clear. That is demonstrated clearly in the following:

In the present design, there is no default path out of **explicit-incorrect-acknowledgement** at the tactical level. With a different set of rules, the tutor might, for example, continue speaking ... [Woolf & McDonald, 1984].

Moreover, how the states that make up paths are formed or how conditions for the meta level rules have been derived, is not clear. There is no explicit justification for the inclusion or exclusion of any particular state. For example, when a meta-rule is triggered because two questions have been answered wrongly this forces MENO-TUTOR to pre-empt the default path. Why was the deciding factor two questions and not three or one? What is observed here is a lack of principles that could address such demanding qualitative questions.

However, the inclusion of meta-level rules assists MENO-TUTOR in being less opportunistic than WHY, even if it is temporary. That is the case as again MENO-TUTOR lacks a global goal. Consequently, the operation of default paths guiding MENO-TUTOR, in what could be seen as a *plan-based* approach, is devastated by the over eagerness of the meta-level rules to step in.

Again, the lack of pluralism is evident since the meta-level rules seem to operate on the information provided by the student model, which in MENO-TUTOR is not more sophisticated than that of WHY and thus prone to the same pitfalls.

3.3.2 When to say it?

As far as the question *when to say it* is concerned, MENO-TUTOR is influenced by the response of the student since one of the objectives for the tutor was to be *sensitive and responsive to student's knowledge level* [Woolf & McDonald, 1984]. The meta-level rules also influence when to say it.

However, it is not clear at all how meta-rules exercise influence. Moreover meta-rules actions rely exclusively on the student model. Here the role of knowledge base is limited and superficial and the student is not allowed any influence either.

3.3.3 How to say it?

With respect to the question, *how to say it*, MENO-TUTOR considers this question at the highest level. Its decision can be *tutor*, *introduce*, etc. [Woolf & McDonald, 1984]. Woolf [1984] perceives these choices as different expository styles. However, after the introduction has been done tutoring will follow and if a need for hacking arises then hack will become active and so on. Therefore, the options that are offered at the pedagogic level consist of a selection of serial stages that one has to go through in order to tutor; they are not alternatives.

The factors that appear to influence MENO-TUTOR's decisions, in the case of *how to say it*, are the student's response, the meta-level rules and the default paths. As the system exists, it appears at surface level to offer alternative ways of handling situations. These are facilitated by the variety of tactical options offered and by the fact that meta-level rules can be altered in order to handle a situation differently. However, as the tutor stands it can offer only one option at a time. To achieve the alternative options the system has to be programmed, and even then only one option can be utilised. Consequently, in real terms there exists only one way of handling a situation at a time. But even if these options could coexist and interrupt the student after one mistake instead of two, see figure 3.8, the question that is of interest here is the following: What is the rule/principle that oversees such a modification?

> Tutor: Do you know what the climate is like in Washington and Oregon?
> Student: Is it cold?
> Tutor: No, it is rather mild. Can you guess about the rainfall there?
> Student: Normal, I guess.

Tutor: Well, the Japan Current, which starts in the Southeastern Pacific, goes along the coast of Japan and across the North Pacific, ending up off the coast of Washington and Oregon. How do you think that current affects the climate there?

Student: It's probably rainy.

Tutor: It is indeed very wet; there are rain forests in both Washington and Oregon. What does that tell you about the temperature of the Japan Current?

Student: It's warm.   Example 1

Tutor: Do you think it rains much in Oregon?

Student: No.

Tutor: Why do you think it doesn't rain much in Oregon?

Student: Because the neighbouring states are dry.

Tutor: The neighbouring states are dry, but you can not generalise from that. In fact it does rain a lot in Oregon. Can you guess what causes the rain there?

Student: Well, there are high mountains in the region and the states border the Pacific Ocean.

Tutor: Yes. The Pacific borders Oregon. How do you think it is involved in the heavy rainfall there?

Student: Ocean air rises and cools upon reaching the land.

Tutor: That is right.   Example 6

Figure 3.8: Two examples demonstrating the effect of changing the meta-rules on the behaviour of MENO-TUTOR [Woolf, 1984]

It is not clear and this is evident in the following quotation:

> Two meta-rules from Example 1 (see figure 3.8) were modified to achieve this discourse in Example 6 (see figure 3.8) The first rule as modified, caused the tutor to change its tutoring strategy. In the earlier discourse, this rule was used conservatively; the transition was made only after topics were completely discussed and the tutor had some confidence about the student's knowledge (or lack of it!). In this discourse, however, the modified rule was applied after a single incorrect answer, thus shifting the focus of the discourse very abruptly at its very beginning.

> The second modified rule caused the tutor to question the student about misconceptions. Typically this rule is triggered after all topics are complete, either by the questions about them having been answered correctly or by the student having been corrected by the tutor. In the second discourse (Example 6), however, the rule was modified to eliminate that requirement. [Woolf & McDonald, 1984]

Again, with respect to the factors involved in decision making, the knowledge base seems to play no part, even though it could offer advice about how a topic should be covered, because of possible complexities within the topic itself. Therefore, it can be concluded that there exists no *pluralism* in MENO-TUTOR.

3.3.4 Summary

MENO-TUTOR, despite its limitations, mainly on the extensive issue of heuristics, especially in a qualitative problem, has a number of important contributions.

The concept of meta-level rules is used for the first time, in an explicit manner, to alter the behaviour of the tutor. Another important feature of the MENO-TUTOR is that of breaking the teaching strategy into three stages and that the teaching strategy is being refined at each stage. That is, the teaching strategy starts with a high level objective which is further refined at the lower stages.

However, it is not clear in MENO-TUTOR how this breaking of teaching strategies was arrived at. Again, here is the issue of using heuristics. They may be plausible and intuitive but they lack any proper and argued foundation, even if these heuristics were based on analysis of human tutor dialogues, despite the fact that in the analysis certain educational principles were ignored, as it was in the case of MENO-TUTOR.

Despite the issues raised in the review of MENO-TUTOR, it must be acknowledged that it was able to present a computer model capable of engaging the knowledgeable student in a way that is fundamentally different from the way it engages the confused student.

3.4 The DOMINIE system

DOMINIE is a procedural skills tutor and it is designed to be domain-independent. The innovative feature of DOMINIE is that of *multiple teaching strategies* [Elsom-Cook et al., 1988]. DOMINIE is the first tutoring system that offers multiple teaching strategies even though these were used for teaching procedural skills. DOMINIE took the trend, started by MENO-TUTOR a step further; i.e., it involved breaking the teaching strategies in stages, where a strategy is refined down the stages. This break down provides clarity regarding the conditions under which a specific strategy was to operate and regarding how to handle the subject matter.

The fact that DOMINIE was designed as a procedural skill tutor, thus could not be used for teaching a subject, acts as a barrier in gathering information about the structure of teaching strategies. Thus, this review is confined to the feature of multiple teaching strategies. The aim is to draw any principles or considerations regarding the selection and grouping of teaching strategies to form multiple teaching strategies.

Even though some of the strategies in DOMINIE drew from work in other fields (Cognitive Science and Education), the presence of heuristics and lack of pluralism is present in DOMINIE and as such the criticisms that were made about MENO-TUTOR apply here as well. Moreover, there appears to be no consideration of the grouping of strategies, instead the criterion seems to be that the strategies must appear as distinct to the user as possible.

The strategies incorporated in DOMINIE are:

> **Cognitive Apprenticeship**
> **Successive Refinement**
> **Discovery Learning**
> **Discovery Assessment**
> **Abstraction**
> **Socratic Diagnosis**
> **Practice**
> **Direct Assessment**

To achieve the effect of multiple teaching strategies the system has an overall strategy that controls the selection of the most appropriate strategy given the circumstances. The factors that influence the decision of this overall strategy are :

> achieving a balance between teaching and assessment; the appropriateness of the strategy to a given area; the student's prior success with the different strategies and the student's personal preferences [Elsom-Cook et al., 1988].

To give a flavour of the construction and the effect of the strategies employed by DOMINIE here is a summary of three strategies: *cognitive apprenticeship, successive refinement* and *discovery learning*.

The main idea behind the strategy of *cognitive apprenticeship* is that

cognitive skills can be learnt in the way that crafts were learnt from an expert in that craft. The apprentice commences by watching the expert in action and asking questions. As time passes the expert allows the apprentice to perform small parts of the whole task...(p. 66) [Elsom-Cook, 1991].

To achieve this effect the system selects a topic for which all sub-sub-goals are known to the user but <u>not</u> the majority of the sub-goals [Elsom-Cook et al., 1988]. The chosen topic is then taught using a mixture of demonstrations and exercises for the user.

The *successive refinement* strategy is a top-down approach. The basic principle, according to Elsom-Cook et al., [1988], is that

a good teacher should be able to explain a topic at a number of levels of detail, always providing an interaction at a level which is meaningful to the pupil in her current state.

To accomplish its objectives the strategy selects the highest level goal not already taught. This is then thoroughly taught while its sub-goals and actions involved in achieving that goal are briefly summarised. The whole strategy evolves in a recursive manner where lower parts of the plan (i.e. sub-goals) are thoroughly taught when they become the goals.

Discovery learning as embodied in DOMINIE occurs in a specially constructed environment. The student is assessed and then a task which is slightly beyond his state of knowledge is selected. The student is then invited to explore the chosen topic. In discovery learning, the strategy selects topics that are considered to be analogical to what the student already knows. Two topics are considered to be analogical if they involve the same procedure in different problem contexts with different parameters [Elsom-Cook, 1991]. Should the student face difficulties then the tutor offers help in a progressive manner in conjunction with the current state of the student.

This description of the three strategies shows that the system is able to present a topic in several distinct ways. In each theory there are explicit rules about *what to teach* and *how to teach* under this strategy.

DOMINIE is considered at a generalised level as each teaching strategy reacts in a distinctive way at the detailed level. The same problems occur here as in the other systems with respect to pluralism and the student's active involvement in the evolution of the teaching process.

3.4.1 What to say next?

With respect to the question, *what to say next* the following factors have been identified as affecting the process:

> The rules as laid down by the strategy.
> The student model.

For example, in the case of cognitive apprenticeship, what goal to teach next is determined by the philosophy of the strategy which requires that the next topic should satisfy the criterion: not all sub-goals are known by the student. In this case although there is involvement of the knowledge base, its role could only be described as a secondary one. It is simply traversed in order to satisfy the criterion as set by the strategy. In an active role the knowledge base could provide information about which topic should be introduced next because, for example, it serves as a prerequisite of the topic currently explored. Such information could conflict with the beliefs of the strategy employed.

Again the student has no means of expressing his own wish about what information should be presented next. His involvement in the whole process is a passive one. He will have to comply with what is on offer, i.e., what the system thinks are his needs.

3.4.2 When to say it?

With respect to the question, *when to say it*, the factors that influence the decisions are:

> Student's progress.
> Depends on the strategy employed.

In the case of the second factor, in some strategies there exist certain rules which, in a manner of speaking, govern the suitability of *when to say it*. For example, in the case of discovery learning, a topic is considered only if the student possesses analogous knowledge. That is an important criterion of that strategy. Again, in the case of cognitive apprenticeship a topic is chosen only if there exist sub-goals not known to the student and all sub-sub-goals are known to the student. Once again, the system dominates the selection of the most suitable time to act; the user is not actively involved in the process. It could be said that the user is indirectly involved through the student model. But the system tries to serve the student's needs as expressed by the tutor, not the student. Given the limited ability of the current techniques employed in

the student model it is obvious that these perceived needs may not reflect the student's actual needs.

3.4.3 How to say it?

Finally, *how to say it*, concerns the following two factors

> The student's progress.
> As defined explicitly by the strategy employed.

In this instance the knowledge base is quite rich in information. It could for example, provide information about the complexity of the chosen topic and consequently provide the system with information that could question the appropriateness of the strategy employed. DOMINIE fails to capitalise on such opportunities.

3.4.4 Summary

It has to be acknowledged that DOMINIE is a pioneer in multiple teaching strategies. Moreover, it provides an opportunity to consider these multiple teaching strategies at a deeper level. However, a number of issues have arisen as a result of DOMINIE'S implementation.

First, as pointed out by Elsom-Cook [1990, 1991], DOMINIE is very eager to change from the current strategy to another one. He correctly argues that:

> It is apparent that human teachers do not switch between styles {strategies} in the extreme manner of DOMINIE. Rather they pursue one style and try to repair the failures. (p. 71)[Elsom-Cook, 1991].

Elsom-Cook [1991] calls for further research into multiple teaching strategies and advocates that DOMINIE requires modification so that selection between strategies appears to be in a more reasoned manner.

However, a number of issues are brought to attention through DOMINIE: first is the issue of grouping between the different strategies. The problem, as suggested above does not seem to lie, in the meta-level strategy but is rather more fundamental. Thus, it is imperative to examine and establish an educational model that will underpin the foundation of a multiple teaching strategies model. This point is further discussed in §4.1.

Another issue is the fact that the terms 'teaching strategy' and 'teaching style' are used as synonyms in the case of DOMINIE; moreover, neither of them appears to be defined. One would expect that a system developed in the late eighties would have adopted a more systematic approach by defining terms and stating their relationships very clearly and explicitly. Moreover, one would expect to see that the model of multiple teaching strategies draws from a theoretical foundation, instead it appears that no such theoretical foundation exists. The emphasis in DOMINIE, thus, appears to be on the development of a computer model.

3.5 Discussion

The review demonstrated that in SCHOLAR the teaching strategy was concerned with retrieval and communication of information, in WHY with elicitation and repair of knowledge. In MENO-TUTOR the teaching strategy was context-dependent and DOMINIE provides multiple teaching strategies. Consequently, in WHY the emphasis is on providing a causal model for the learner or deductions from incomplete knowledge, whereas in MENO-TUTOR it is on providing tutoring that responds differently to the knowledgeable student and the confused one. In DOMINIE the emphasis is on exploring a model of multiple teaching strategies. Therefore, each study of teaching strategies reflects its goal, its original aim.

Consequently, in the authors' accounts of their work various pedagogic terms are introduced, e.g., teaching strategy, teaching style, pedagogic state, strategic state, tactical state, entrapment strategies, counterexample strategies, identification strategies, methods and actions. In some cases, terms such as teaching strategy and teaching style are used in a synonymous manner (as in DOMINIE). The terms are often not defined. There is an implicit understanding of the definition as well as the relationship between them.

The structure of teaching strategies is also different. For example, in WHY, the main element is the Socratic method which is realised by a number of strategies. In MENO-TUTOR, the teaching strategy is seen as made up of elements from the three states. At the same time different executions of the rules resulted in different teaching styles. Such confusion of terms becomes apparent in the case of DOMINIE where Elsom-Cook [1990, 1991] calls for clarification between teaching strategy and teaching style.

There is a possible explanation for this apparent confusion and this is that ITS is a relatively new field, and an interdisciplinary one. Consequently, different objectives are pursued by different researchers. So for example, Woolf [1990, 1992], believes that the objective of ITS is to be used in classrooms now. On the other hand Ohlsson [1991a], advocates that ITS, in their current state are research tools and only suitable for the laboratory. Thus, there is no clear line for the field. The systems reviewed here, appear to focus on AI issues rather than educational issues.

However, looking at the problem of teaching strategies from a purely educational perspective, as is the case in this thesis, it would be ideal to combine all the works. That is, refine the student causal model, engage the knowledgeable and confused student in fundamentally different ways and moreover be able to employ a multiple teaching strategies model. However, the fact that the above models were not instances of a more general framework, according to their objective, but rather purpose built models with different structures, according to their particular needs, means that a combination of them would give rise to the same problem of defining teaching strategies in a generic way.

Moreover, the fact that these designs were based more on heuristics, rather than on principles does not assist in solving the problem, see also Ohlsson [1982]. It was this lack of principles that prompted Ohlsson's classic paper [Ohlsson, 1986] entitled, "Some principles of Tutoring". In this paper Ohlsson, proposes a number of principles that are derived from work in Educational Psychology, Cognitive Science and other related fields. Ohlsson argues that design of teaching strategies should be based on these principles. So for example, he points out that background knowledge is a principle that needs to be considered. He proposes that background knowledge could be used in the form of 'precursors', to make a connection between what is known and what is to be learned.

Thus the problem of developing a model that draws from educational principles and is able to inform the structure of a teaching strategy module of an ITS is still an issue that requires attention. In fact, Beck et al., [1996] state that the problem of multiple teaching strategies is still an open research question.

Later work in ITSs, EXPLAIN [Wood & Wood, 1996] [Reichgelt, et al., 1993] and SONATA [Angelides & Tong, 1995] have not considered these issues. EXPLAIN deals

with the issue of basing teaching strategies on an empirically validated theory of instruction, whereas SONATA is an intelligent tutoring system for music learning, offering multiple tutoring strategies grouped in a manner similar to that in DOMINIE.

The teaching strategies in EXPLAIN are based on contingent learning [Wood & Wood, 1996]. This theory of learning is empirically validated [Reichgelt, et al., 1993] and is based on the notion that when the learner is ready to learn then instruction should occur at that moment. Wood et al., [1992] states:

> ... However, the scaffolding metaphor fails to address the issue of how the instructional process should be organised in time to locate the upper bounds of a learner's competence. When, for example, should the tutor give a verbal instruction or demonstrate an operation, describe a task-critical feature, or choose relevant from currently irrelevant material? (p. 13)

> Specification of instructional contingency is straightforward. Where a learner fails to understand or comply with a preceding instruction n then more help should be given on instruction $n+1$. Conversely, where a learner succeeds, then any help offered should exert less control than n. (p. 14)

EXPLAIN, as an ITS, moves away from basing its teaching strategies on heuristics and analysis of human tutor dialogues, to basing them on empirically valid instruction [Reichgelt, et al., 1993]. For example, it provides a way of deciding whether the tutor should give a verbal instruction or demonstrate an operation. This may resemble the "provide a generalisation" or "provide a counterexample" choice as seen in WHY, but they are not analogous. In WHY the choice is made to point out a gap in the student's knowledge, whereas in EXPLAIN the choice is made to instruct the student. In EXPLAIN the principle is more general than in WHY. However, EXPLAIN's approach does not add to knowledge about teaching strategies because it neither defines a teaching strategy nor provides a model for its structure and operation.

In SONATA [Angelides & Tong, 1995], the strategies offered are:

> learning through exploration,
>
> practice with a hint,
>
> multiple choice,
>
> strict question and answering.

The emphasis in SONATA is not on how the strategies select the material but rather on how the selection between the strategies takes place. To that end Angelides & Tong [1995] state:

> ... For each activity, SONATA's choice of strategy depends on the student's prior success with the strategies. A strategy is regarded as successful if the student provides the correct solution in a problem-solving activity which employs that strategy. This factor affects SONATA's strategy selection at two levels. At a local level, the decision depends on the performance of the student in the previous activity. ... At the higher level, SONATA's strategy selection is influenced by the overall success ratings of the strategies. (p.58)

Therefore in its selection of strategies SONATA is more simplistic than, and fails to draw any lessons from, DOMINIE. Consequently, it fails to address the problems of multiple teaching strategies, as discussed in §3.4. Moreover, the criteria for selecting the next strategy are similar to those used by the self-improving quadratic tutor [O'Shea, 1979], whereas at least DOMINIE operated on an overall strategy of decreasing intervention.

Looking at the problem of teaching strategies from an educational perspective and with the objective of laying the foundation for identifying and formalising teaching strategies, some of the principles that would be of interest are:

- what could be the constituent parts of a teaching strategy,

- how do they interrelate,

- how could a teaching strategy operate?

Answers to these questions, which are part of the research question stated in Chapter 1, would enable a better understanding of the role and objectives of a teaching strategy. Thus, by forming a better understanding of the above questions, then this understanding could be used as the cornerstone upon which to develop a framework that would allow the formalisation of teaching strategies in such a way that they can be used across different domains.

Since this work is interdisciplinary, I propose to look for the answer to these questions in work in related fields such as Educational Psychology and Cognitive Science, where

the practitioners are also interested in these questions. A further benefit of looking at work in these fields and trying to link it with work in ITSs, is the potential of a framework that would enable all the enormous wealth of knowledge concerning education in Educational Psychology, to be tapped into and utilised in a productive manner for ITSs and education.

CHAPTER 4

Chapter 4: Design Aspects of a Teaching Strategy

TeLoDe, a prototype Intelligent Tutoring System for solving linear second order ordinary differential equations with constant coefficients, represents a novel use of Computer Algebra Systems in the teaching of university mathematics where current practices are altered and adapted to optimise the benefits of technology, as discussed in Chapter 2. To create TeLoDe, Maple was augmented by incorporating declarative knowledge and a teaching strategies module. To inform the design of the latter, a review of teaching strategies in ITSs was carried out, as reported in Chapter 3, with the objective being to establish whether there existed a model.

The review showed that despite the impressive and inspiring results of research into teaching strategies, no overall model exists. Instead, there are available models for conducting Socratic dialogues, engaging knowledgeable and confused students in fundamentally different ways as well as a model for supporting multiple teaching strategies. Thus a model consolidating on previous ITS work on teaching strategies but being based on educational considerations was sought.

This chapter puts forward considerations that ought to be central to such a model - in this case SIMTA. Some of these considerations became apparent in the course of the review of teaching strategies, from the ITS perspective, whereas others are the result of examining work from the related fields of educational and cognitive psychology. However, it should be pointed out that reference to work from educational and cognitive psychology in this chapter should not be viewed as an exhaustive account of the work carried out in these fields. Instead, work of eminent, widely acclaimed and accepted scholars in these field was reviewed with the sole purpose to provide a further insight into the complex and dynamic problem of research into teaching strategies.

The design aspects of teaching strategies, examined in this chapter, will further our understanding of some of the research sub-questions posed in Chapter 1, see §1.2. These questions are:

- What is a teaching strategy?

- How could it operate?

- What could be the factors that influence the decisions of the teaching strategy?

In §4.1 issues from the ITS perspective, as raised in the review of teaching strategies, are discussed. These issues deal with the question *how to say it?* as well as the grouping of teaching strategies operating in the paradigm of multiple teaching strategies. Section 4.1 concludes by examining some theoretical considerations concerning the objective of a teaching strategy. In §4.2 the cognitive and educational psychology perspectives are discussed. This discussion provides a valuable insight into teaching strategies. From the cognitive perspective, a number of factors, such as the student, the lesson structure and the subject matter are considered to be primary factors affecting teaching. From the educational psychology perspective, an understanding of the objective of a teaching strategy emerges. Bruner [1966] proposed that the objective of a teaching strategy is to encourage the exploration of alternatives. The alternatives are of the form: *enactive*, *iconic* and *symbolic*. A further role of the teaching strategy should be that of *activating* the exploration, as well as *maintaining* and *directing* it.

Section 4.2 concludes by exploring the issue of grouping/developing teaching strategies operating in the paradigm of multiple teaching strategies. This discussion is made with reference to Eggen et al.'s, [1979] models of teaching that have been developed for human teachers or as part of teacher training courses. As a result, an innovative view of a teaching strategy is proposed. It is proposed that a teaching strategy could be viewed at two fundamental levels:

- the *operational level*

- and the *organisation level*.

The *organisational level* deals with the structure of the teaching strategy whereas the *operational level* deals with the manifestation of that structure.

4.1 The ITS perspective

This section will point out design aspects of a teaching strategy from an ITS perspective. In particular, issues, dealing with the question *how to say it?*, such as

- what governs verbal explanations and

- how to form alternatives in explanations

are explored in §4.1.1. The issues of incompatibility of teaching strategies, lack of a mechanism to select a strategy and a rationale for switching to a different strategy is explored in §4.1.2 and in §4.1.3 some theoretical deliberations, from the ITS perspective, on the aims of a teaching strategy are presented.

4.1.1 What governs verbal explanations and the issue of alternatives in explanations

As stated in §3.5, the question *how to say it?* is the one that has received the least attention in the systems reviewed. Furthermore, in the case of WHY, two forms were proposed: presentation or question. Even DOMINIE which claimed to have explicit rules, on close examination revealed this not to be the case.

For example, the criteria for selecting DOMINIE's cognitive apprenticeship strategy included the facts that all sub-sub goals of a topic were known to the student, but not the majority of the sub-goals and consequently the topic was taught using a mixture of demonstrations and exercises [Elsom-Cook et al., 1988]. This raises the question of how these demonstrations and exercises take place. For example, is the demonstration interactive or presented by the tutor? If it is presented by the tutor, is it done using an analogous situation, which is known to the student, and thus the student has to work out similarities and differences, or is it done in the form of a statement? According to Elsom-Cook et al., [1988], the way that the material was delivered was based on the explicit definitions provided by the strategy chosen. However, the explicit definitions refer only to whether the subject matter should be handled in a top-down or bottom-up approach: there is no explicit definition of the actual presentation. For example, Spensley and Elsom-Cook, [1988] described the actions of cognitive apprenticeship as follows: once cognitive apprenticeship is activated, DOMINIE presents the superordinate goal to the one to be taught, in order to provide a context for the action. Then it will present the relevant sub-steps one by one, by stepping through a demonstration of the actions with associated verbal explanations, and then asking the student to repeat the actions. Once all the sub-steps of the goal initially demonstrated have been performed, the student will be asked to perform a higher goal. The approach adopted here is bottom-up.

From this description of cognitive apprenticeship, it is clear that there is no information about exactly how the steps are performed by DOMINIE and moreover, what governs the chosen verbal explanations; and it is this which is of interest. It

appears that the way of carrying out the steps is fixed and ad-hoc. There appear to be no criteria for the evolution of the teaching strategy regarding how to present the material to the student. Instead, the emphasis appears to be on the criteria for selecting a teaching strategy and not on how it is executed and why.

DO YOU THINK THAT ANY PLACE WITH MOUNTAINS HAS HEAVY RAINFALL? (RULE 6: FORM A GENERAL RULE FOR INSUFFICIENT CAUSE)

YES (STUDENT RESPONSE)

SOUTHERN CALIFORNIA HAS MOUNTAINS. WHY DOESN'T SOUTHERN CALIFORNIA HAVE HEAVY RAINFALL? (RULE 7 : PICK-UP A COUNTER-EXAMPLE FOR AN INSUFFICIENT CAUSE, AND RULE 2: ASK FOR PRIOR CAUSES)

Figure 4.1: A counter example for causes of rainfall [Collins & Stevens, 1980]

In considering SCHOLAR and WHY, which might claim to have conditions for a counterexample strategy as well as for an entrapment strategy or an identification strategy, the review demonstrated that these mechanisms existed at a very superficial level. This is because the counterexample or entrapment or identification strategy will always be presented in the same manner and there are no explicit rules governing the presentation. For example, consider the dialogue in figure 4.1 where WHY presents the student with a counterexample to demonstrate the insufficiency of the cause.

This counterexample, could be presented as follows: WHAT ABOUT SOUTHERN CALIFORNIA? In this case the counterexample is not obvious; it is an implicit one. The student is required to assert that Southern California, although mountainous, does not have heavy rainfall, before a reply to the question can be given. Alternatively, the counterexample could have been given by stating that Southern California although mountainous does not have heavy rainfall and give reasons. This counterexample could be considered as an explicit one. In SIMTA these three counterexamples are considered as alternatives.

Although it might be argued that the only difference between the three alternative forms of the counterexample, presented earlier, is wording, the fact is that they place a different demand on student and tutor. In the explicit counterexample, the student is not as active as the tutor. In the implicit counterexample, the tutor is providing the minimal information whereas the student is very active.

The presence of these alternatives could have partially enabled WHY to avoid the problem of the tutor getting stuck in a 'never ending cycle', see §3.3. A 'never ending cycle' is where the student gives, again, an insufficient cause in his answer to a question as in fig. 4.1, WHY triggers a new counterexample strategy and the pattern is repeated and the tutor therefore becomes locked in this 'never ending cycle'. Such a problem could be because the student has not understood the question properly. Using alternatives for the same topic could have enabled WHY to overcome this potential danger.

4.1.2 The issue of grouping teaching strategies together

The review of DOMINIE revealed the lack of any theoretical considerations in bringing all these strategies together. This lack of an underlying theory has lead to the potential of grouping incompatible strategies. To demonstrate this point let us recall two strategies from DOMINIE, cognitive apprenticeship and discovery learning.

The cognitive apprenticeship strategy operates on the same principles as a traditional apprenticeship. The student watches and imitates an expert performing the task. The task is broken down into steps and sub-steps that are to be used in a bottom-up fashion. In DOMINIE, cognitive apprenticeship will begin by presenting the superordinate goal to the one to be taught in order to provide a context for the task. It will then present the relevant sub-steps one by one. It will perform a step and then ask the student to repeat the action. This is achieved through the strategy of practice assessment.

On the other hand, discovery learning involves putting the student in a situation in which he does not actually know the correct form of behaviour and helping him to find the appropriate behaviour for himself [Spensley & Elsom-Cook, 1988]. In this case analogies between tasks are sought to enable people to learn to generalise by placing them in new situations which are analogous to ones which they have met before. This strategy is always followed by discovery assessment, which monitors a student's solution attempts and if the analogy is not immediately apparent, guides the student towards a solution by giving him hints based on the known analogous procedure [Spensley & Elsom-Cook, 1988].

The problem here is because of the very different nature of the two strategies. One is overprotective as it breaks down the task into sub-steps which are first taught and then

imitated. On the other hand, in the discovery learning strategy, the student is exposed to an analogous situation and is expected to make generalisations and thus be able to see the similarities and differences between the two situations. DOMINIE could potentially choose cognitive apprenticeship for a task and, should that strategy fail, then select discovery learning as its next strategy. In this case the student is suddenly moved from an overprotective environment where his effort is minimal, to an environment where he is exposed and abandoned to use his own devices.

DOMINIE's behaviour therefore raises a number of issues. First, the selection of the next strategy is random. Of course, selection is based on whether the current topic can be analysed in a top-down or bottom-up fashion, or whether analogous tasks can be identified. However, such considerations cannot be allowed to play a primary role; the aim is not to satisfy the subject matter but the cognitive needs of the student. Thus, the objective should be to identify the correct manifestation of the subject matter that will satisfy the cognitive needs of the student.

Second, the discrepancy between the cognitive apprenticeship and discovery learning strategies indicates that the strategies were developed independently. They were not developed as complementary to each other, so that when one failed the other would come in and carry on from there. This is further demonstrated by the fact that the selection of the next strategy is based on whether it can handle the task at hand in the prescribed manner, i.e., in a top-down or bottom-up fashion.

The switch from an overprotective strategy, where the cognitive demands on the student are 'minimal', to an open ended strategy where the cognitive demands on the student are high, is in danger of alienating the student. Bruner [1966] stated that alternative approaches to a particular problem should be sought in order to *encourage*, *direct* and *maintain* exploration. Thus, in a tutoring system this should be a primary consideration.

The argument being made here should not be seen as an outright rejection of grouping polarised teaching strategies, but rather as a need to develop an explicit rationale which will provide a principled approach to grouping/developing any kind of teaching strategy that is to operate in the paradigm of multiple teaching strategies. Under this rationale polarised teaching strategies developed under a common set of

principles could be grouped together and thus avoid the potential alienation of the learner.

4.1.3 A theoretical consideration of teaching strategies

Ohlsson [1991] outlined three 'theories' of teaching that are relevant for Intelligent Tutoring Systems research. According to his descriptions the requirements of each theory are different as are the roles that the tutors are expected to play. The first theory, which he calls the *traditional* view, sees teaching as the *communication of subject matter*. The first ITS, SCHOLAR [Carbonell & Collins, 1973] is a clear subscriber to this view. The second theory views teaching as the remediation of incomplete or incorrect mental representations. Examples of this type of teaching strategy can be seen in WHY [Collins & Stevens, 1980] and in the second version of SCHOLAR [Collins et al., 1974]. Ohlsson called this the *current view*. Finally, the third theory views teaching as the facilitation of knowledge construction. For Ohlsson, this is the *future view* and the one that promises a radical improvement in instruction as the research needed for constructing such systems is not research in intelligent tutoring systems per se, but research in learning [Ohlsson, 1991].

Moreover, Ohlsson [1991a, 1992] has accused scientists from other fields relevant to ITS, (mainly educational psychology and education), of not offering something pragmatic that an ITS person could use. Instead, according to Ohlsson [1992], these scientists are more concerned with high-level ideas, such as discovery vs. exposition, meaningful vs. rote learning etc.

In contrast, Goodyear [1991], being an educationalist, claims that ITS designers reject much of what educationalists have to offer for constructing teaching strategies for ITSs. He argues that ITS scientists hold the following four misconceptions:

>Human teachers are poor models,
>They don't carry out individualised teaching,
>There is a lack of formal accounts of education and finally,
>There is a lack of expert models.

Goodyear [1991], believes that their lack of consideration of work in related fields is based on these misconceptions, which he sets out to clarify.

One of Goodyear's points about the first issue, using human teachers as models, is that this is a long term endeavour. He argues that if ITS research is viewed as a long term investigation into the fundamental nature of teaching then human teachers are not poor models and could assist ITS researchers. Goodyear [Ibid.] claims that

> An intelligent teacher should be able to reason about his own pedagogic limitations, capabilities, and resources. (p. 12)

Goodyear [ibid] has argued that the knowledge structures and methods of reasoning that are currently used by ITSs are quite limited and relatively straightforward and will therefore be inadequate in supporting a complex and demanding activity like teaching. Instead ITSs will require complex knowledge structures and methods of reasoning, which can draw from modelling human teachers. Whilst it is true that human tutors may be poor models, from an ITS perspective, they nevertheless represent a resourceful pool of information for ITS design. Consequently, as Goodyear [Ibid] argues, human tutors could become *initial models* (my italics) for structuring teaching strategies. That is, human teachers would act as an *initiation* to the complex and dynamic world of education and teaching. Moreover, human teachers and other related fields could help ITS design in the long term, e.g. through the empirical validation of hypotheses formed in ITSs.

With respect to the second issue, that teachers do not carry out individualised teaching, Goodyear [1991] has dismissed this issue on the grounds that although human teachers teach whole classes, they are also involved in one to one situations. He has further argued that teaching is not restricted to classroom teachers and that there are many people that engage in one-to-one teaching.

Regarding the third issue, the lack of formal accounts of education, Goodyear [Ibid] accepts that although educational psychology has no computational account of teaching this may be due to the fact that the techniques used so far are not good enough to extract all the valuable information. However, as a matter of course in this thesis it will be demonstrated that looking at the problem of teaching strategies from the combined perspective of ITSs, educational psychology, cognitive psychology is the key to the rethinking of teaching strategies that is proposed in the conclusions chapter. In fact, in Chapter 5, it is explicitly demonstrated how work from educational psychology helped in defining *style*, the crucial element of SIMTA as well as how

theories of learning could inform the structure of a teaching strategy module of an ITS (see Chapter 6).

Finally, in response to the concern about a lack of expert models, Goodyear [Ibid] offers an alternative approach. He argues that ITS research is not in need of optimal ways of teaching, but rather is in need of knowledge on how to do any kind of teaching. In other words, his argument is that ITS researchers are over-ambitious and are trying to do research for which there is little background, fundamental work. This criticism seems valid since ITS researchers may know relatively little about teaching. Consequently, it would not be sensible to look for optimal ways of teaching when ITS experts are only starting to research the topic. There is a sense that ITS researchers are trying to run before they can walk.

Laurillard [1993] expresses her reservations about the relationship between learning and teaching, but acknowledges the fact that basing teaching on learning requires a principled approach to the problem of teaching strategies. Such a principled approach to the problem of teaching strategies will yield two benefits: first, answers to one main criticism made about ITSs, which is that AI does not draw on theoretical and empirical work from relevant disciplines (e.g. SCHOLAR). Second, it will provide the opportunity of minimising the use of heuristics in explaining the actions of a teaching strategy.

4.2 An Educational and Cognitive Psychology perspective

The ITS perspective has been very useful in forming a better understanding of what is involved in the question *how to say it?* as well as explaining the issue of grouping teaching strategies. Furthermore, although it alerted us to three 'theories of teaching', which would have a direct effect on the structure of teaching strategies, no information was offered on the role, nature and objective of a teaching strategy. Such information will be offered from an analysis of an educational and cognitive perspective on teaching strategies.

4.2.1 A cognitive view of teaching strategies

In this sub-section, one cognitive analysis of the skill of teaching is examined to see what factors can be found to affect teaching strategies. Such an analysis could also inform the structure of a computer tutor. This analysis, by Leinhardt and Greeno

[1986], is examined here because the skill of teaching has been analysed in terms that could be useful to an ITS design.

Teaching has been characterised as a complex cognitive skill [Leinhardt & Greeno, 1986], because it requires the construction of plans, the making of rapid on-line decisions and occurs in a relatively ill-structured, dynamic environment. Leinhardt and Greeno [1986] have identified two systems of knowledge upon which the skill of teaching rests: *lesson structure* and *subject matter*. The *lesson structure*, is required to construct and conduct a lesson. It depends partially on the *subject matter* and is constrained by the unique circumstance or set of students. However, I would like to suggest a further factor that, I believe, is central to the lesson structure. This factor has been suggested as early as 1979 by Mason. He called it the tutor since the tutor is essential to how the lesson evolves. But what does the concept of tutor represent; it appears to be a composite one and it would be of enormous help, especially for a computer tutor, if it could be broken down into basic elements. These could then be used as a starting point in addressing the difficult and complex question *how to say it?*, as discussed in § 4.1.1 as well as the issue of grouping the strategies together, as discussed in §4.1.2.

Henry St. Maurice [1991] offers an insight in our effort to unpack this complex concept of tutor. He argues that commonplaces could explain how to analyse educators' discourse. That is, he was able to highlight the assumed concepts values and principles within educators discourses, policies and practices. Such concepts, values and principles could represent one's beliefs regarding how learning is best achieved, what is education, what is, for example, mathematics or any other subject to them. Consequently, one could see how these concepts, values and principles could assist in answering the question how statements/questions are couched, or how alternatives are formed and selected. Answers to these questions could contribute to a better understanding of the overall question *how to say it?*. Furthermore, strategies operating in the paradigm of multiple teaching strategies, however polarised, could be formed in ways that are congruent to the same set of such beliefs. Now that the factors from this analysis have been identified and discussed let us turn our attention to the analysis of how to structure a computer tutor.

Leinhardt and Greeno, [1986], suggest that a teacher has a complex knowledge structure which is composed of interrelated sets of organised actions, (or schemata)

which are applied flexibly and with little cognitive effort in circumstances that arise in the classroom [Ibid]. Schemata depict activities, both large and small, such as homework checking, distributing the papers in the classroom etc. All the actions required to perform the activities are represented in the schemata. These schemata are drawn from Sacerdoti's idea [Sacerdoti, 1977], that knowledge for skilled performance consists of schemata at different levels. Sacerdoti's [1977] analysis showed how the structure of schemata at different levels of generality provides a basis for performance in a complex cognitive task involving integration of high-level goals and actions with their lower level components. Leinhardt and Greeno [1986] report that Sacerdoti's framework, has been useful in analyses of cognitive processes in solving high school geometry, medical diagnosis etc.

Leinhardt and Greeno [1986], analysed teaching as a set of schemata at different levels, in accordance with Sacerdoti's [1977] idea. Low level activities, which are those that require no significant diversion of mental resources and hence reduce cognitive load, are called routines. Routines are defined as small, socially scripted pieces of behaviour that are known by both students and teachers. Examples of low level activity are distributing homework and collecting homework. Distribution of the homework is initiated by the teacher walking across the front row of the room with a pad of paper and giving each child in the front row several sheets. Then the first child in each column takes one and passes the rest back through the column.

Leinhardt and Greeno [1986] hypothesise that the conduct of the lesson is based on an agenda which includes activity structure plans and operational routines that are specific versions of schemata in the teacher's general knowledge base. Main segments of a lesson are referred to as activity structures. Examples of activity structures are checking homework, presenting new material and setting problems on the board. Through planning nets, which represent structures of actions and goals that are generated by the knowledge base, they attempt to show how teaching occurs.

The problem with this analysis of the skill of teaching appears to be at a fundamental level. Leinhardt and Greeno's [1986] emphasis is on analysing the skill from a management or organisational point of view. In developing SIMTA the interest is in the formation of the components found in the schemata, such as concrete examples, in how the material that is to be used in explaining mathematical concepts is selected and more importantly, how it is to be used in order to be beneficial to the student.

4.2.2 An educational psychology view of teaching strategies

In this sub-section some concepts concerning the nature as well as aspects of a theory of instruction are presented. These concepts are employed to draw some inferences regarding the goal and design considerations of teaching strategies.

Bruner [1966], describes a theory of instruction as *prescriptive*, in the sense that it sets forth rules concerning the most effective way of achieving knowledge or skill as well as *normative* in the sense that it sets up criteria and states the conditions for meeting them. Furthermore, in distinguishing between a theory of instruction and theories of learning and development, Bruner [1966] draws the conclusion that theories of learning and development are not irrelevant to a theory of instruction.

> In fact, a theory of instruction must be concerned with both learning and development and must be congruent with those theories of learning and development to which it subscribes (p. 40).

Bruner [1966] identifies four major aspects that a theory of instruction must have:

> 1. A theory of instruction should specify the experiences which most effectively implant in the individual a predisposition towards learning-learning in general or a particular type of learning.

> 2. A theory of instruction must specify the ways in which a body of knowledge should be structured so that it can be most readily grasped by the learner.

> 3. A theory of instruction should specify the most effective sequences in which to present the material that it is to be learned.

> 4. A theory of instruction should specify the nature and pacing of rewards and punishments in the process of learning and teaching.

> [Bruner, 1966]

With respect to the first condition, it is noted that instruction must create an environment that is encouraging for the student, an environment that will make the learner not only willing but also able to learn. Bruner [1966] is very specific when he refers to an environment; it is not just any environment; it is one, from a cognitive perspective, that encourages the predisposition to explore alternatives. He offers three general forms of alternatives, the *enactive*, the *iconic* and the *symbolic*. In the first case,

the child is active, touching objects, playing with them and thus forming an understanding of the 'outside world'. In the second case, the child is not as physically active so he can now think of icons, pictures and manipulate them mentally. In the last case, the child is in a position to use symbols to represent concepts and objects, thus, all manipulations are done through this new representation. Although Bruner's propositions are relevant to primary school children, the essence of the idea, the need for the provision of alternatives, seems applicable to other levels of schooling, as supported by Mason [1979]. Consequently, there is a need for an environment that supports and promotes alternatives, as learning and problem solving depend upon the exploration of alternatives.

However, a learning environment also needs to regulate the exploration on the part of the learner. Such regulation can be achieved by observing the three phases: *activation, maintenance and direction* [Bruner, 1966]. If the learner is required to explore alternatives then it is necessary to provide circumstances such that an exploration can be initiated. Obviously, it is important that appropriate situations are created so that the exploration is maintained. However, if the exploration is to be fruitful, it is imperative that it is not random.

The second of Bruner's four major aspects of a theory of instruction is concerned with the structure of a body of knowledge. Bruner refers to an optimal structure which is most suitable to the status and what he calls the 'gifts' of a learner.

> Optimal structure is defined as a set of propositions from which a larger body of knowledge can be generated, and it is characteristic that the formulation of such structure depends upon the state of advance of a particular field of knowledge. (p. 41) [Bruner, 1966]

The strength of the structure relies on

> ... simplifying information, for generating new propositions and for increasing the manipulability of a body of knowledge (p. 41) [Bruner, 1966].

The structure, however, must always be related to the status and 'gifts' of the learner. Thus, the optimal structure is not absolute but relative [Bruner, 1966]. To illustrate this point, I will quote part of Ausubel's summary of educational psychology:

> ... Ascertain what the learner knows and teach him accordingly (p. iv) [Ausubel et al.,
> 1978].

Again, here, the importance of the subject matter or knowledge representation is indicated alongside some characteristics that the subject matter, presented to the learner, has to possess if instruction is to be successful. The most important condition of all is that the optimal structure is not absolute but relative. It is also one of the basic premises of ITSs that each learner has different needs. Starting from this premise, it is obvious that simplification of information, the generation of new propositions and manipulability of a body of knowledge should be consistent with the status and abilities of the learner.

Bruner's third condition refers to effective sequencing. This should be a sequence that is obviously best suited to the individual, a sequence that supports the optimal structure of the knowledge, but most importantly, a sequence that will implant a predisposition towards learning. This can be achieved by activating an exploration, maintaining it and directing it. As mentioned earlier this kind of management can be achieved through alternatives. This is best illustrated by citing the following example from Bruner [1966]:

> Given, for example, that one wishes to teach the structure of modern physical theory,
> how does one proceed? Does one present concrete materials first in such a way to elicit
> questions about recurrent regularities? Or does one begin with a formalised
> mathematical notation that makes it simpler to represent regularities later encountered?
> (p. 41)

In this example it is clear that there is no one sequence for all learners, and the optimum in any particular case will depend upon a variety of factors such as background knowledge, stage of development and nature of the material. Moreover, it is clear that the way that the material is presented to the learner influences the exploration of alternatives.

4.2.3 Viewing multiple teaching strategies from an educational models perspective

In this sub-section the issue of developing/grouping teaching strategies operating in the paradigm of multiple teaching strategies is examined through some educational models. Although the terminology used here is about *models* and not teaching

strategies it nevertheless encompasses the same ideas that are of interest to us in this thesis. These educational models are aimed at human teachers and form part of teacher's education courses. The aim is to further our understanding of this rather fundamental point surrounding the issue of developing/grouping of teaching strategies operating in the paradigm of multiple teaching strategies.

A review of these models may show any theoretical considerations used in the development and organisation of these educational models or at least throw some light on developing/grouping teaching strategies operating in the paradigm of multiple teaching strategies. If any theoretical considerations were found then these would be adopted and used in SIMTA. This is because any such considerations would have been made with human tutors in mind, whereas SIMTA is considered with computer tutors in mind and thus the restrictions of implementation apply here.

Early models of teaching as seen in Mosston [1972], Lapp et al. [1975] and Stallings [1977], tend to propose a continuum of teaching strategies that range from teacher-centred to pupil-centred. Moreover, these models appear to be caught in the flurry of educational changes in the 1960s with the advances of constructivism and the battles with behaviourism. Later models, e.g., Brady [1985] and Dean [1982] appear to be concerned with structuring the lesson and the activities in the classroom, defining the roles of the teacher and the pupil. Consequently, whilst this research is legitimate and unquestionably part of classroom teaching it nevertheless is not directly relevant to the research described in this thesis.

However, there is one work, that of Eggen et al. [1979] which is of particular interest because

- it promotes the idea that *models* are associated with *specific* teaching goals in a classroom and

- all proposed models are structured in the paradigm of constructivism.

The models described in this work are not examined along with the teacher-centred pupil-centred continuum nor are they concerned with issues relating to classroom management. The models developed here are:

The Inductive model,

The Deductive model,

The Concept attainment model,

The Taba model,

The Ausubel model,

The Suchman inquiry model.

The models of Inductive and Deductive reasoning are almost self explanatory, as they promote inductive and deductive reasoning respectively. In the case of deductive reasoning, the pattern is that from a major premise and a minor premise the student should be able to draw a conclusion. For example, 'All men are mortal (major premise) Socrates is a man (minor premise) therefore, Socrates is a mortal' [Eggen et al., 1979]. Conversely, in the case of Induction, thinking proceeds from the specific to the general. In this case the individual makes observations which are then generalised.

The Concept Attainment model was originally designed by Joyce and Weil [1982] and is based on the research of Jerome Bruner [Eggen et al., 1979]. It is an inductive model designed to teach one form of content, concepts. The Taba model, is again an inductive model designed to teach generalisations, whereas the Ausubelian model is a deductive model designed to teach interrelated bodies of content. Finally, the Suchman model [Eggen et al., 1979], is called an inquiry model, because it employs both reasoning techniques, and according to Eggen et al., [1979] the major difference between Suchman's model and other inquiry models, is that the problem is carefully posed by the teacher and is carefully designed to motivate the student. Another distinguishing characteristic of Suchman's model is that students gather data in a simulated process through questioning rather than actual implementation of data.

There are two main issues that are apparent from these models. The first one is that in a classroom situation all these goals, attainment of concepts, use of deduction and induction, explore relationships between concepts and so forth, are necessary elements. Thus, these models are not offering alternatives to the same problem as they may all be required in the course of one lesson. In this sense Eggen's models resemble the organisation of DOMINIE, where some of the strategies there were developed on the top-down or bottom-up fashion.

The second issue surrounds the fact that each model draws from a different theory. Combine this with the necessity, as discussed above, that some or all of these models

may be required in the course of a lesson and then you potentially face a problem equivalent to that of polarised teaching strategies, as discussed in §4.1.2.

If a human teacher is to conduct a lesson requiring attainment of concepts and exploration of their relationships then according to Eggen et al. [1979] that teacher should adopt a model that draws from Bruner's work, for the attainment of concepts, a model that draws from Ausubel's work, for the exploration of relationships between concepts. To describe this in St. Maurice's [1991] words, as discussed in §4.2.1, the teacher is asked to adopt the assumed concepts, principles and values of Bruner followed by the assumed concepts, principles and values of Ausubel. No consideration is given here to the fact that there exists a conflict between these two theories. Ausubel is a firm advocate of expository learning, in fact he considers exposition as the mechanism par excellence for learning, [Ausubel et al., 1978], whereas Bruner is an advocate of discovery learning.

To take this point further, imagine that Ausubel and Bruner were to teach a lesson involving concept attainment and exploration between concepts. I find it hard to believe that they would adopt each other's theories for teaching the specific goal. The implication would have been that their own theories are not capable of handling these teaching goals, a fundamental contradiction to the generality of their theories.

I would like to suggest that if all six models, as described by Eggen et al., [1979], were to be used by Ausubel then he would have manifested them in a manner that is in complete agreement with his theory of learning. The same would hold for Bruner as well. Consequently, all models of Eggen et al., [1979], may be manifested in accordance with Ausubel's theory or Bruner's theory. In either case, the objective of each of the models is being preserved, but its manifestation is done in accordance with the subscribed theory, Ausubel's or Bruner's. It is these theories that would enable one to distinguish between the two manifestations of the models.

This deduction gives rise to a novel idea in approaching the issues

- of developing/grouping teaching strategies operating in the paradigm of multiple teaching strategies

- and grouping of polarised teaching strategies.

This idea involves the unpacking of the concept of a teaching strategy and viewing it at a further level of detail where a teaching strategy is seen as a composite structure made up of essentially two fundamental levels:

- the *organisational level*

- and the *operational level*.

At the *organisational level*, the concern is with the objective of the teaching strategy, for example, concept attainment, exploration of relationships between concepts and so forth. At the *operational level*, the concern is with the implementation of the organisational level of the teaching strategy, e.g., Ausubel's theory, Bruner's theory, Suchman theory and so forth.

This idea, of viewing a teaching strategy at two levels, is to be considered as the fundamental cornerstone upon which the SIMTA framework was defined. It provided SIMTA with the unique opportunity of offering a novel way of understanding the issues surrounding teaching strategies.

4.3 Summary

In this chapter some design aspects of teaching strategies have been presented from three perspectives: the ITS and the fields of education and cognitive psychology.

From the ITS perspective, it was possible to explicitly demonstrate which are the factors influencing the question *how to say it?*. This was accomplished with the aid of a strategy from DOMINIE which claimed that it had explicit rules for this question. However, the in-depth analysis showed that the rules were concerned with a top-down or a bottom-up approach. There were no rules regarding how a demonstration or how an example was pursued.

How examples or demonstrations could be pursued was illustrated through examples from the WHY system. Taking one of the counterexamples, it was shown that there were two alternative ways of forming this counterexample. Consequently, this gave rise to the question of which factors influence the formation of these alternatives as well as the factors for selecting between them.

Finally, from the ITS perspective, the issue of grouping teaching strategies that operate in the paradigm of multiple teaching strategies was explored. By analysing cognitive apprenticeship and discovery learning strategies it was argued that there was the potential of alienating the student should cognitive apprenticeship be followed by discovery learning. This potential alienation was attributed to the lack of a theoretical model considering any parameters in grouping/developing teaching strategies that are to operate in the paradigm of multiple teaching strategies.

Such a theoretical model was sought in theoretical deliberations by Ohlsson [1991, 1992] and Goodyear [1991]. In these considerations Ohlsson [1991] outlines three 'theories of teaching' relevant for the design of Intelligent Tutoring Systems. The first one, which he calls the traditional view, sees teaching as the communication of subject matter. The current view, the second theory, perceives teaching as remediation of incomplete or incorrect mental representations. Finally, the third theory, the future view, views teaching as facilitating knowledge construction. It is apparent, from this outline, that the structure of a teaching strategy reflects the view that is adopted by the computer tutor.

Ohlsson [1991a, 1992], furthermore, accuses scientists of related fields to ITS of not offering something pragmatic that an ITS person could use and claims that teachers and studies of teachers have nothing to offer. Goodyear [1991] rejects these claims, by pointing out that ITSs are in need of any help offered to them and that such systems are in need of basic and fundamental work, which related fields have the potential to contribute to.

Viewing teaching strategies from the cognitive perspective confirmed that the subject matter and student were factors affecting teaching strategies (these factors were also pointed out in the review of the educational psychology perspective). More importantly, it was possible to identify and analyse a third factor, called tutor. Analysis of this factor offers the promise of being able to address the factors affecting the question *how to say it?* as well as putting together some theoretical considerations for grouping/developing teaching strategies that operate in the paradigm of multiple teaching strategies.

A further insight into the issue of grouping teaching strategies operating in the paradigm of multiple teaching strategies is provided from the analysis of the

educational psychology perspective. Here, the role of a teaching strategy is defined as encouraging the predisposition to explore alternatives. Such exploration, however, requires regulation. This in turn is achieved by observing three phases: *activation*, *maintenance* and *direction*. Furthermore, three general forms for the alternatives are offered; these are *enactive*, *iconic* and *symbolic*. These forms could potentially inform the structure of teaching strategies.

However, the real breakthrough on the issue of developing/grouping teaching strategies operating in the paradigm of multiple teaching strategies is provided from the analysis of the Eggen et al., models of teaching. Here, the concept of teaching strategy is proposed to be further broken down and be viewed at two fundamental levels:

- the *organisational level*

- and the *operational level*.

Uncoupling the *organisation* of a teaching strategy from the way this organisation is manifested, i.e., the *operation* of the teaching strategy, offers the unique opportunity of having more than one manifestation of a teaching strategy. For example, as discussed in §4.2.3, the concept attainment model can be manifested under Ausubel's theory or Bruner's theory or Suchman's theory. Because each theory is different each manifestation of the attainment concept model, for example, is different. If you apply the same exercise to the other five models, described by Eggen et al., and you collate all the models that are manifested under the same theory then that could form the basis of a novel and innovative way of developing/grouping teaching strategies operating in the paradigm of multiple teaching strategies. In fact, this idea is considered to be the fundamental cornerstone of SIMTA.

However, there are also a number of design aspects, as summarised earlier in this section, that need to be accommodated in this idea. All these design aspects of a teaching strategy are brought together in Chapter 5 to define SIMTA, our theoretical framework, which will in turn inform and support, in a principled manner, the structure of a teaching strategy module of an ITS.

CHAPTER 5

Chapter 5: The SIMTA Framework

In the last chapter, the idea of unpacking the concept of a teaching strategy was proposed. This consideration is central and fundamental in defining SIMTA (Styles Implemented by Methods Tactics Actions). According to this idea it is proposed that the concept of a teaching strategy be viewed at two levels:

- the *organisational level*

- and the *operational level*.

The *organisational level* is concerned with the structure of the teaching strategy whereas the *operational level* is concerned with the manifestation of this structure.

Another design aspect, which is also central to SIMTA is to view teaching strategies as offering alternative representations of the same task. One form of this idea was demonstrated with the aid of a counterexample where it was shown that there are three ways of expressing the same counterexample.

This chapter demonstrates how the SIMTA framework is underpinned by these two fundamental ideas as well as how the rest of the design aspects, discussed in Chapter 4, are accommodated. SIMTA is a framework that supports and sustains teaching strategies operating in the paradigm of multiple teaching strategies. SIMTA attempts to lay down the foundations for an educational model that informs the structure of a teaching strategy module of an ITS. Through the definition of SIMTA the main research question as well as the research sub-questions dealing with the issue of teaching strategies will be addressed. The main research question is:

How could teaching strategies be formed and how could they operate in the paradigm of multiple teaching strategies?

The sub-questions are:

What is a teaching strategy?

How could it operate?

What could be the constituent parts of a teaching strategy?

What could be the factors influencing the decisions of teaching strategies?

In §5.1 the rationale of SIMTA is presented. The underlying principles of SIMTA are also presented and discussed in relation to the novel view of a teaching strategy operating in the paradigm of multiple teaching strategies. Here, the influence of design aspects of a teaching strategy, as discussed in Chapter 4, is explicitly shown. This view of a teaching strategy operating in the paradigm of multiple teaching strategies is presented in §5.2. A résumé of the overall organisation of SIMTA is presented, before each element of SIMTA, namely the *style*, *methods*, *tactics* and *actions*, is further explored. In the résumé it is explained how the underlying principles and conditions of SIMTA, as discussed in §5.1 are reflected in the organisation of the SIMTA framework. In §5.3 the way in which the SIMTA framework advances our understanding of the complex concept of a teaching strategy is discussed.

5.1 The rationale of SIMTA

The rationale of SIMTA is to develop a theoretical framework that can inform the structure of a teaching strategy module in an ITS. Through SIMTA, a new view regarding the concept of teaching strategy operating in a multiple teaching strategies paradigm is put forward. This view is underpinned by the following three premises:

- The *development/grouping* of teaching strategies operating in the multiple teaching strategies paradigm is *congruent* to a *common set of beliefs*[1]

- the current concept of a teaching strategy is uncoupled and revisited with the notion that a teaching strategy can be viewed at two levels, the *operational level* and the *organisational level*

- the *role* of a teaching strategy operating in the paradigm of multiple teaching strategies is to offer *alternative* representations of the *same* topic.

The first premise marks a departure from viewing multiple teaching strategies as a collection of autonomous teaching strategies, as demonstrated by DOMINIE [Elsom-

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[1] The term *common set of beliefs* is equivalent to what St. Maurice [1991] refers to as the assumed concepts, values and principles within educators' discourses, policies and practices. Such concepts, values and principles could represent one's beliefs regarding how learning is best achieved, what is education etc. (see also §4.2.1).

Cook, 1988]. By autonomous it is meant that teaching strategies were developed in isolation to each other and then brought together under some loose principle which outlined a direction in alternating between them such as 'decrease teacher intervention'.

Conversely, in SIMTA the development/grouping of teaching strategies is *underpinned* by the notion of being congruent to a common set of beliefs. Consequently, viewing these assumed concepts, values and principles as central and crucial to the development/grouping of teaching strategies makes them in turn instrumental in underpinning our understanding of the question *how to say it?*.

Moreover, stipulating that any development/grouping of teaching strategies, operating in the paradigm of multiple teaching strategies, is *congruent* to a common set of beliefs offers a way forward in resolving the issues of:

- enabling a number of distinctive manifestations of a teaching strategy

- differentiating between distinctive manifestations of a teaching strategy

- grouping of teaching strategies, even polarised ones.

However, resolution of the above issues will only be possible because of the second underlying principle (uncoupling the contemporary concept of a teaching strategy). SIMTA, in fact, takes the notion of viewing the concept of a teaching strategy at two levels, the *operational* and *organisational*, a step further and proposes a separation of these two levels, and that these two levels become entities. Such a separation is seen as extracting from the teaching strategy the elements responsible for driving the teaching strategy, i.e., separating what 'drives' a teaching strategy from what 'makes' or 'is' a teaching strategy. Consequently, the separate entities need only to come together when the teaching strategy is in action, or when, for example, they are observed in a classroom, or in a tutoring system.

One entity is what 'drives' the teaching strategy or the common set of beliefs which in St. Maurice's [1991] terminology encapsulates the assumed concepts, principles and values that drive educators' discourse, policies and practices. The other entity is the teaching strategy itself free from the elements that 'drive' it. Therefore, it is possible to

have a number of teaching strategies, however diverse or polarised, developed, differentiated and grouped together in accordance with the common set of beliefs.

The last underlying premise, stipulating that the role of a teaching strategy is to promote alternative representations of the same task, can provide an initial insight to what could be a diverse or polarised teaching strategy. A teaching strategy in SIMTA is not associated with a specific teaching goal, as was the case with Eggen et al.'s, [1979] models of teaching. The association with a specific teaching goal, such as attainment of a concept, contradicts the underlying premise of offering alternative representations of the same task. Instead, a teaching strategy in SIMTA is associated with offering the learner alternative representations of a goal, such as attainment of a concept. This is in line with the idea of Bruner [1966] of offering enactive, iconic and symbolic representations of the same task.

Associating the role of a teaching strategy with offering alternative representations of a specific teaching goal, as opposed to the specific teaching goal, further facilitates developing/grouping of diverse or even polarised teaching strategies. The other factor, as discussed earlier, is that of developing/grouping teaching strategies *congruent* to a common set of beliefs.

Having set the broad picture of SIMTA by outlining the nature, role and operation of a teaching strategy, our attention can now turn to the factors that affect the decisions and actions of a teaching strategy. These factors, among others, as identified in §4.2, should be considered when attempting to define a teaching strategy. They are:

the subject matter

the student

the tutor.

It is not suggested that the above list is exhaustive, but SIMTA's attempt to define teaching strategies was confined to this list. Thus, for example, factors, such as motivation and social context were not considered in this thesis research. Another aspect of the rationale of SIMTA was to demonstrate clearly and openly how these factors interact to generate a teaching strategy given a topic and a specific situation.

Since the SIMTA framework operates in the paradigm of multiple teaching strategies, where teaching strategies offer alternative representations of the same task, the question that arises here is that of alternating between the strategies. It must be stressed that the pitfall observed in DOMINIE where teaching strategies were changing so readily was avoided when dealing with this question in SIMTA. As discussed in Chapter 3, this pitfall of DOMINIE was attributed to the fact that there was no overall structure overseeing the development/grouping of teaching strategies. The approach adopted in SIMTA is supported by an observation study of teachers in a classroom, carried out by Douglas [1991]. Douglas states that teachers have a preferred teaching strategy, which if they sense has failed, then they try remedial action. Only if the remedial action fails will the teachers change to another teaching strategy.

To summarise this section, SIMTA draws from the fields of Intelligent Tutoring Systems, Educational and Cognitive Psychology in order to propose a novel view of teaching strategies operating in the paradigm of multiple teaching strategies. The following section demonstrates how the SIMTA framework is organised to reflect the rationale discussed in this section.

5.2 Organisation of SIMTA

SIMTA is a two dimensional framework which is organised in a hierarchical fashion and supports the three factors, subject matter, student and tutor, as stated in §5.1. SIMTA consists of four elements: *style*, *methods*, *tactics* and *actions*. Central to the organisation of SIMTA are the beliefs that

- a teaching strategy is 'freed' from the elements that are responsible for 'driving' the teaching strategy

- and that developing/grouping of teaching strategies operating in the paradigm of multiple teaching strategies is congruent to a common set of beliefs, which is responsible for 'driving' the teaching strategies.

A *teaching strategy* in SIMTA is defined in terms of a triple generic structure, namely *methods*, *tactics* and *actions* and operates under a *style*. The *style* encapsulates the common set of beliefs. The generic structure of a teaching strategy satisfies the premises

- that a teaching strategy must offer *alternative* representations of the *same* task at hand

- a teaching strategy is *repaired* before it is changed.

Consequently, a *teaching strategy* is concerned with the structure of the subject matter and the interaction of that structure with the student. The *method* is a mechanism for structuring the subject matter, the *tactic* is a mechanism for controlling the interaction between the tutor and the student, whereas the actions are low level mechanisms facilitating the interaction.

A *method* is the primary element of a teaching strategy in SIMTA and in fact acts as an identifier for the teaching strategy. The methods represent all possible ways of structuring a subject matter. Examples of methods are analogy, examples, generalisations, specialisations and investigations. Consequently, given a specific task a number of methods can be identified for that task and thus offer a number of *alternative* structures supporting the task at hand.

The *tactic* is a mechanism for facilitating the interaction of *a* method with the learner. Tactics represent generic dialogue structures and their spectrum ranges from *implicit* to *explicit* dialogue structures. Thus, a given method uses a number of tactics to facilitate interaction with the learner in more than one way, i.e., implicitly, explicitly or anything in between. Consequently, for a given task and a given method, a number of *alternative* interactions for *that* method with the learner are possible.

The *action* is a low level mechanism facilitating the tactic and is necessary for a computer tutor. Examples of an action are: display a message, pick an example, etc.

From the above description it is shown that a teaching strategy as defined in SIMTA, adheres to the principle of offering alternative representations for a given task at two levels:

- the method level

- and the tactic level.

Alternatives at the level of tactic are considered to be *variations* of essentially the same teaching strategy, since the method is the primary element characterising the teaching

strategy. However, alternatives at the method level are considered to be *different* teaching strategies because the methods that characterise the teaching strategies represent different structures, i.e., a change from the method of analogy to that of example. Figure 5.1 shows how two styles S1 and S2 are defined in the SIMTA framework.
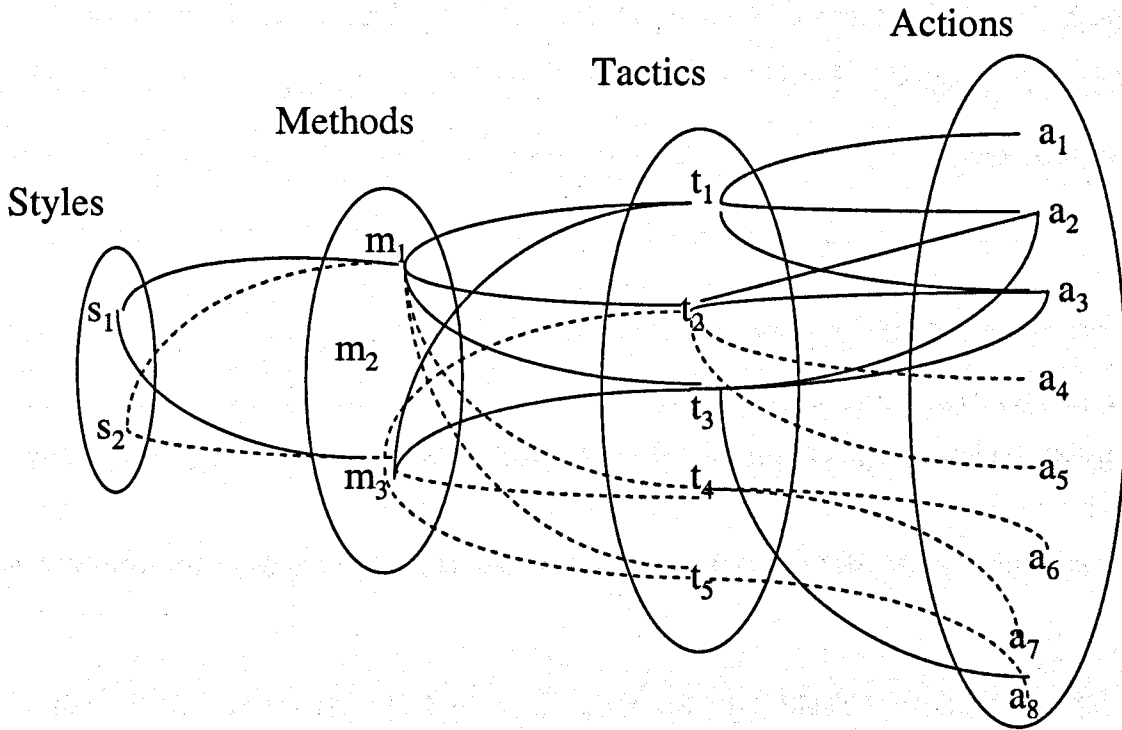
Figure 5.1: Representation of multiple teaching strategies in a style (S1 is represented by the whole line whereas S2 is represented by the dashed line)

As depicted in figure 5.1, under S_1 (*style* 1) the *methods* m_1 and m_3 are selected. In turn the *methods* m_1 and m_3 under S_1 use *tactics* t_1, t_2, t_3 and t_1 and t_3 respectively. Under the style S_1 the *tactic* t_1 uses actions a_1, a_2, and a_3 tactic t_2 uses a_2 and a_3 whereas t_3 uses a_2, a_3 and a_8. Again as depicted in figure 5.1, under S_2 (style 2) the methods m_1 and m_3 are selected. This time the methods m_1 and m_3 use different tactics to facilitate interaction of these methods and that reflects the fact that methods m_1 and m_3 operate under S_2 (style 2). The choice of actions is also different and these are a_4 and a_5 for t_2, a_6 and a_7 for t_4 and a_8 for t_5.

Therefore, one can see how these different combinations of methods and tactics, at both levels, result in a number of teaching strategies. However, given a specific task there is more than one manifestation both at the method level and at the tactic level. That is, there is more than one analogy or example and more than one implicit or

explicit tactic. Thus, the question is how one selects between these different analogies or examples and the implicit or explicit tactics. The answer to this question is that selection depends on the *style*. The style ensures that only the congruent manifestations between these different methods and their tactics are grouped together.

The *style* is the crucial element of SIMTA which encapsulates the common set of beliefs and thus ensures that all teaching strategies that operate under that style are congruent to that common set of beliefs that define that style. Therefore, changing that common set of beliefs will result in a grouping of different manifestations between these different methods and their tactics. However, even though in both cases what we essentially have are *distinct* teaching strategies, since the common set of beliefs are distinct, the teaching strategies draw on the same generic structure which is manifested in distinct ways to reflect that common set of beliefs defining the style at that time. Therefore, SIMTA is organised in such a way as to reflect one of the underlying premises, as discussed in §5.1, that a teaching strategy can be represented as a *generic structure, method, tactic, action*, which is a separate entity from what 'drives' that teaching strategy, the *style*. It is self evident that a teaching strategy cannot operate without the presence of a style.

The next task in describing the organisation of the SIMTA framework, is to demonstrate how the organisation supports the notion that a teaching strategy is being *repaired* before it is replaced by another. The remedial action of a teaching strategy occurs at the level of *tactic*, because as discussed earlier, a change at that level results in a *variation* of what is essentially the same teaching strategy as it is still based on the same method. However, if all tactics under that method have been exhausted then the alternation occurs at the level of *methods* and this results in a *different* teaching strategy as a new method is now used. Thus, an alternation of teaching strategies in SIMTA occurs at two levels, the method and action result either in a different teaching strategy or in a variation of the same teaching strategy respectively.

The following four subsections examine the elements of the SIMTA framework, namely style, method, tactic and action in detail, aiming to set the foundations for formal definitions of these elements.

5.2.1 Style

The style is the most crucial element of SIMTA and represents the third factor, tutor, influencing teaching strategies, as discussed in §5.1. This style is the element that 'drives' the generic structure of teaching strategies in SIMTA. This element encapsulates the common set of beliefs which guarantees that only congruent manifestations of different teaching strategies, based on different methods, are grouped together under that style. The style resolves the issue of grouping polarised teaching strategies, as discussed in §4.1.2, since their manifestation is congruent to the same set of common beliefs. That is because teaching strategies in SIMTA have a generic structure which is developed in accordance with the common set of beliefs or what St. Maurice [1991] described as, the assumed concepts, values and principles within educators' discourses, policies and practices. Consequently, diverse teaching strategies, such as those that are protective and guide the student very clearly and those that simply set the problem for the student and provide little guidance, are manifested in ways reflecting the common set of beliefs.

The question of interest is, how is a style defined to represent these common set of beliefs? The definition of style should support and sustain teaching strategies which have a generic structure. Moreover, the teaching strategies informed by SIMTA aim to promote Socratic dialogues, engaging the knowledgeable student in a way that is fundamentally different from that of a confused one, as well as supporting teaching strategies operating in the paradigm of multiple teaching strategies.

To achieve this definition, the field of Educational Psychology provided a very helpful insight. An analogous problem was faced by Ausubel in the 1960s [Ausubel, 1968] where he wanted to demonstrate that reception learning is not invariably rote and discovery learning is not necessarily meaningful. He therefore had to demonstrate that principal kinds of learning, such as concept formation and problem solving, could occur under either reception or discovery learning and could be, again, either meaningful or rote learning. That is, Ausubel [1968] found a way of representing all kinds of learning that occur in the classroom. The situation with the definition of a style is analogous but this time the emphasis is on teaching rather than learning. Moreover, the definition of SIMTA must provide

- guidance as to the nature of the common set of beliefs,

- how one can identify them

- and give a clear indication of how the common set of beliefs affects the manifestation of the generic structure of a teaching strategy.

To reflect the above premises, a style is defined as a set consisting of two elements, *type of learning* and *principles* (see fig. 5.2).

$$\text{STYLE} = \begin{cases} \text{TYPE OF LEARNING} \\ \quad\quad + \\ \text{PRINCIPLES} \end{cases}$$

Figure 5.2: Definition of a style

The *type of learning* could be reception or discovery learning. Consequently, the type of learning sets the scene as it provides some direction as to what kind of learning teaching is to support. However, that is insufficient because Ausubel et al. [1978] have demonstrated that both reception and discovery learning, under certain conditions, may lead to either meaningful or rote learning. This is where the other element, *principles*, in the definition of style comes in. It is the *principles* that will

- ensure whether the learning that occurs is meaningful or rote

- provide the details required in the manifestation of generic teaching strategies.

For example, in the case of providing the details required for manifestation, it is the *principles* that will explicitly specify how statements or questions are couched, whether a bottom-up or a top-down or both approaches are supported and under what circumstances. In addition, the principles will specify whether there are any conditions set either on the student's background knowledge or on structuring the subject matter that is to be learned or which concepts are best suited for introducing new concepts and so forth.

One characteristic of a style is that once it has been defined it is then *distinct*. That does not mean that the definition of a style cannot change or that if you have defined two styles then their intersection is necessarily the empty set. In fact, to claim that the

definition of a style cannot change is a contradiction of what the style is supposed to represent.

Obviously, if the type of learning between two styles is different, then it is obvious that these two styles are then distinct. Likewise, if the principles between two styles are different, then the styles are distinct. However, if the types of learning are the same but the principles differ then the styles are also distinct.

5.2.2 Methods

The methods in SIMTA deal with the first factor, the *subject matter*, as identified in §5.1. The methods are mechanisms responsible for structuring the subject matter. Examples of methods are:

Analogy

Investigation

Examples

Generalisation

Specialisation

Concept Definition

This list of methods is not exhaustive, but rather indicative of the spectrum that is available. These methods have been shown to be possible ways of learning in Machine Learning (e.g., see [Devi, 1991], [Michalski et al., 1986]) and such ways of structuring the subject matter have also being suggested by educationalists, (e.g., Eggen et al., [1979]).

The methods are considered to be the primary identifier of a teaching strategy informed by SIMTA. The methods must satisfy the criterion of offering alternative representations of a specific task. So, for example, given a specific problem, the role of the methods is to establish if an analogy or an example or a concept definition can be used to tackle the task at hand. The conditions for establishing which methods are chosen depend on the definition of the style that is currently operative. For example, while a concept may exist to form the basis of an analogy if that concept does meet certain criteria then it cannot be selected and thus the method of analogy is not

available. The remainder of this sub-section outlines the main generic features of the methods.

The method of analogy will usually draw on background knowledge known to the student. This is done so as to present the student with a situation that has a number of commonalities with the situation that he is currently engaged in. Analogies help the student to be active and tend to keep the tutor in the background.

In the method of investigation, the student usually draws on the material to be learned by examining the relationships between concepts and their attributes and how this relates to other concepts and their respective attributes. For example, if concept A depends on concept B, which in turn depends on concept C, then the method of investigation is used to either elicit these dependencies from the student or make the student aware of these dependencies before the implications of these are explored.

The method of examples is used for both cases of straightforward examples and also where it is appropriate to use a counterexample. In the case of a straightforward example the student is presented with examples which he can work out in order to elicit the necessary information. For example, in introducing the concept of differential equations, an example of a differential equation could be presented to the student. This is because the concept of equations is known as well as the fact that depending on the form of the variable, the name of the equation alters, (e.g., logarithmic equation when the variables are logarithms and trigonometric when trigonometric functions are involved) and the student is made aware of the concept of derivative. By studying the example, the student should be able to conclude that an equation that involves derivatives is a differential equation.

The method of generalisation usually draws on background as well as on knowledge to be taught and steers the student to form hypotheses, test them and from these deduce a generalisation. Conversely, the method of specialisation behaves in an almost identical manner to that of generalisation, but here the student is steered to deduce a specialisation. Examples of both methods could be: establish whether the relationship between two concepts is subordinate or superordinate.

The method of concept definition is used to define concepts. This method provides the student with the answers should all other alternatives fail to reach the goal. In SIMTA this is the method which has been called the safety device. The answer will be given to

the student in accordance with the active style. This is the method where the student is least active and the tutor is most active.

5.2.3 Tactics

A tactic is the mechanism responsible for facilitating the interaction of the subject matter, as structured in the methods, with the student. While the methods may be the backbone of teaching strategies informed by SIMTA, because teaching strategies are recognised as based on analogy or examples or investigation and so on, the tactics nevertheless are the interface of these teaching strategies. Tactics work within the structures supported by the specific method and their objective is to facilitate the interaction of this structure with the student.

This leads to the question of determining the necessary and sufficient amount of information that is given to the student, as well as the way that the information is presented to the student. As discussed in §5.2, the tactics must also offer alternative representations of the task within the limitations imposed by the method under which a tactic is active. Consequently, each tactic must be capable of conclusively resolving the task at hand. An alternative tactic is sought if and only if the task has not been resolved.

The spectrum of tactics range from the explicit to the implicit. In the case of the former all the information that is represented in the method is provided to the student, whereas in the case of the latter, the absolute minimal but sufficient amount of information is presented to the student. In both cases, the presentation occurs in accordance with the definition of the style under which the teaching strategy has been manifested.

To be more concrete, let us consider the case of the counterexample from WHY, as discussed in §4.1.1. In this instance, the method is examples, instantiated in the form of a counterexample and the tactic is implicit, i.e., minimal information. Consequently, the question posed to the student would be in the form WHAT ABOUT SOUTHERN CALIFORNIA? The information that is provided to the student is minimal; the student is required to assert that although Southern California is mountainous it does not have heavy rainfall, before the student is expected to reply to the question. If the tactic was explicit, then it would have been stated that Southern California although mountainous, does not have heavy rainfall and the reasons why would be stated. A

tactic asking, why, although Southern California is mountainous, does it not have heavy rainfalls (this is the way that the WHY system handles the counterexample), should be considered as one positioned between the implicit and explicit tactics described earlier.

It is important to note that the transition of tactics is from implicit to explicit, to ensure that the student is active. However, no compromise is made if it becomes apparent that the student is in need of assistance, thus complying with the principle of directing and maintaining exploration. As long as there is room for the student to be active then that venue will be pursued, but should that prove counterproductive then the safety device (being explicit) is activated.

5.2.4 Actions

An *action* is a low level activity and is used by the tactic to, for example, display messages, pick examples and ask questions. There exist two basic types of actions in SIMTA, the statement type and the question type. These two types are defined in a generic manner to enable the teaching strategies to carry out their task. An action is required by SIMTA for two reasons: because SIMTA will inform the teaching strategy module of an ITS and it also represents the lowest level of activity in which a teaching strategy can be analysed. Examples of actions are:

Display **the** message

Ask **the** question

Pick **this** example.

5.3 Summary and discussion

SIMTA is a theoretical framework that supports and sustains teaching strategies in the paradigm of multiple teaching strategies. SIMTA attempts to lay down the foundations for a consistent underpinning educational model informing the structure of a teaching strategy module of an ITS. A number of research questions were instrumental in defining SIMTA.

For reasons of clarity, the sub questions of the research will be answered first followed by the main research question.

With respect to the question, What is a teaching strategy?, the answer is:

SIMTA's teaching strategy is concerned with

- structuring the subject matter in terms of analogies, examples, generalisations, concepts definitions etc.,

- and facilitating the interaction of these structures with the learner.

With respect to the question "How could it operate?", the answer is:

SIMTA's teaching strategy operates with the aim

- of offering alternative representations of the same task

- and by *activating*, *directing* and *maintaining* exploration to ensure that the learner is active.

With respect to the question "What could be the constituent parts of a teaching strategy?", the answer is:

A SIMTA teaching strategy is defined in terms of a triple generic structure, namely a *method, tactic(s)* and *action(s)*.

A *method* is concerned with structuring the subject matter in a way that these structures offer alternative representations of the same task. A *tactic* is concerned with facilitating the interaction of the structures with the student in a way that offers, again, alternative ways of facilitating the interaction of these structures with the student. *Action* is a low level mechanism required by a computer tutor and used by a tactic to carry out the interaction with the student.

With respect to the question "What could be the factors influencing the decisions of teaching strategies?", the answer is:

Three factors were identified which are neither the only ones nor do they represent a comprehensive list of factors affecting the decisions of a teaching strategy. However, these factors are the minimal fundamental set of factors that ought to be considered in the decisions of a teaching strategy. These are:

- Tutor

- Subject matter

- Student.

In SIMTA the first factor, tutor, is represented by the *style* element which ensures that the manifestation of all different teaching strategies are congruent to that style. The second factor, the subject matter, is represented in SIMTA by the *method* element. It encapsulates not only the material that is to be learned, or core knowledge, but also the background knowledge relevant to the core knowledge. The third factor, the student, is not represented directly in SIMTA, but every care is taken to ensure that the background knowledge of the student is fully utilised when formulating the tactics and the methods. Therefore, the concepts, from the background knowledge, used for analogies or examples are checked to be familiar to the student.

With respect to the main research question, "How could teaching strategies be formed and how could they operate in the paradigm of multiple teaching strategies?", the answer is:

Given a specific task to be taught then first and foremost a *style* is required to be present. A style will set out exactly how the methods and tactics are to be manifested.

At the method level, the knowledge base is searched to identify, for example, analogies, examples, or concept definitions, that can be used as alternative representations of the task. At the tactics level the methods, identified for the task at hand, are examined to see what kind of interactions can be supported, i.e., are there any implicit or explicit or in between tactics for presenting the method to the student. Since the actions are considered to be low-level activities which are directly tied to tactics no further analysis is required to specify them.

Once all possible methods and tactics for a given task have been identified it is then considered that all possible teaching strategies for that specific task have been *formed*. If more than one method has been identified, then it is said that a number of *different* teaching strategies have been *formed* and if more than one tactic for each method has been identified then it is said that a number of *variations* of these different teaching strategies have been *formed*.

With respect to the *operation* of these different teaching strategies and their variations, it is important to note that the role of a teaching strategy is to offer alternative representations of the same task.

This is achieved at two levels:

- the method level

- the tactic level.

When all tactics, under a chosen method, are exhausted and have not been successful, then another method is chosen and the cycle, as described above, is repeated. This cycle of alternating between the methods is repeated until either all methods are exhausted or the task at hand has been resolved successfully. By alternating between the teaching strategies, either at the level of method or of tactic, the student is encouraged to explore and the alternation ensures that such exploration is activated, directed and maintained.

So, having explicitly answered the research questions posed in this thesis with respect to the issue of teaching strategies, the next question is what are the advantages over previous work?

Through SIMTA, it will be possible to inform teaching strategies where the use of heuristics, as in the case of SCHOLAR, is minimised. So, for example, issues such as *reviewing* and *use of hints*, that were touched upon in the review of SCHOLAR, can now be formulated in more than one way and consequently, be seen as essential parts of activating, maintaining or directing the exploration. Especially as SIMTA offers the option of selecting from a number of different methods and within them from a number of different tactics.

With respect to WHY, two of its major limitations, the relentless pursuit of elicitation and the inability to express its strategies in more than one way, which were demonstrated in the case of the counterexample (see §4.1.1), are addressed in SIMTA. Through the tactics element, an example or counterexample can now be presented in more than one way. Consequently, a counterexample or entrapment strategy, which might be considered to be the best strategy for the task, can be used by selecting the appropriate tactic for the student and thus still benefit from the use of the counterexample or the entrapment strategy.

In the case of MENO-TUTOR, the breaking of the teaching strategy into three states, pedagogical, strategic and tactical, however intuitive and plausible, lacked proper and argued foundation. On the other hand, SIMTA's structure was argued and derived from theoretical considerations, as discussed in Chapter 4. Moreover, each level of SIMTA is defined in a *descriptive* rather than a *prescriptive* fashion. That is, the definition of each level describes the objective and the characteristics of the elements that can be accommodated at that level, rather than specifying the elements themselves. For example, the method level is defined as identifying all possible structures from the knowledge base, rather than specify that the elements in the method level are: analogy, examples, investigation, etc.

Finally, with respect to DOMINIE, there is a fundamental difference in the way that the issue of multiple teaching strategies has been tackled in DOMINIE and in SIMTA. In the case of DOMINIE, the concept of a teaching strategy and that of multiple teaching strategies are viewed as autonomous entities and thus forming the relationship of subordinate and superordinate concept respectively. Furthermore, in DOMINIE the methodology adopted in tackling the issue suggests that to form multiple teaching strategies all that is required is to collate a number of *different* teaching strategies together. However, when it comes to defining *different* then there is no clear direction or principle.

In direct contrast, in SIMTA the concept of multiple teaching strategies is viewed simply as another way of expressing the fact that a *number* of teaching strategies operate under a style. These teaching strategies that operate under a style have been created with the aim of offering *alternative* representations of the *same* task. However, it is not possible to carry out teaching with just one teaching strategy based on analogy or examples, whereas cognitive apprenticeship may be able to support teaching on its own. This is one cardinal difference. Another fundamental difference is the fact that the teaching strategies in SIMTA are grouped together in a way that ensures congruence with the definition of that style. This resolves the issue of alienating the student by switching between an overprotective strategy to an open ended one. Finally, the relationship between a teaching strategy and style in SIMTA is that of a subordinate and superordinate concept.

Apart from the advantages of SIMTA's framework over individual work, SIMTA offers a new approach in tackling the study of teaching strategies. First, there is a shift of

attention from the standard framework of *What to say next?*, *When to say it?* and *How to say it?*, to mechanisms that drive the teaching strategy, i.e., the *style*, and to the structures that make up the teaching strategy, i.e., the triple generic structure *method, tactic(s), action(s)*. It is merely implied that the framework takes a secondary importance in the study of teaching strategies. Moreover, the fact that SIMTA draws from a number of interrelated fields, Intelligent Tutoring Systems, Educational Psychology and Cognitive Science should be seen as an indication that to study the complex and dynamic concept of teaching strategies one field is not sufficient, but that an interdisciplinary approach is a necessity.

One of the advantages of an interdisciplinary approach, is that channels are forged between the different disciplines through which the research from these fields can flow and be used in further defining and refining SIMTA. Looking at research from other fields through the SIMTA framework could provide a number of principles that would further enhance our understanding of teaching strategies as these works are looked at with specific cases in mind. For example, principles in forming analogies or principles in defining explicit or implicit tactics might be sought. These principles will also help in answering the charge laid at the door of AI and Computers in Education "Why does your system do that?" which in some cases used to be followed by the answer "I thought of that" (see Ohlsson, [1982]).

The description of the SIMTA framework in this chapter concludes the analytical approach adopted in this thesis in relation to the issue of teaching strategies. The remainder of the thesis embarks on a synthesis approach leading to the implementation of TeLoDe.

In the next chapter two styles, drawing from the work of Ausubel et al., [1978] and Bruner [1966] are defined. The definition of these styles will help to demonstrate that the SIMTA framework is capable of informing a teaching strategy module of an ITS whose teaching strategies:

- operate in the paradigm of multiple teaching strategies

- adhere to the generic triple structure, of *method, tactic(s)* and *action(s)* and are manifested in a manner *congruent* to a defined *style*

- are manifested distinctly under two distinct styles.

Chapter 6

Chapter 6: The SIMTA model in practice: creating a Style and its implications

This chapter further elaborates on the concept of *style*. As stated in Chapter 4, the style is the most crucial element of the SIMTA framework, since it is the style, that controls the manifestation of methods and tactics. Here two styles are created, the expository style and the guided discovery style. The rationale, in creating two styles, is to demonstrate

- how the definition of a style is informed by a learning theory

- how two styles are distinct

- how the definition of a style affects the manifestation of the generic structure of methods and tactics.

Informing the definition of a style through an analysis of a theory of learning has a number of benefits as it:

- provides explicit principles for the manifestation of methods and tactics

- throws a new light in the use of learning theories as the basis for deriving teaching strategies

- brings into the forefront again the relationship between teaching strategies and a theory of learning

- makes some steps in answering the criticism that ITS models are conceptions of the researcher (as discussed in Chapter 3).

The principles that are used for the manifestation of the methods and tactics, can be consequently used as the basis for answering questions such as, *what to say next?*, *when to say it?*, and *how to say it?*. Furthermore, basing the implementation of the methods and tactics explicitly on the theory of learning not only provides a vehicle for identifying the elements lacking from a theory of learning and required by teaching, but also provides another way of looking at what elements a theory of learning ought to cover.

The expository style draws on Ausubel's theory of meaningful learning and is the style that is primarily implemented in TeLoDe. The expository style was chosen to be the

style primarily implemented because Ausubel's theory, on which it is based, is a very well structured learning theory. For this reason the implications of the expository style on structuring the knowledge base of TeLoDe are examined in some detail here as well. Bruner's ideas on instruction inform the guided discovery style.

In §6.1 a brief analysis of Ausubel's theory of meaningful reception learning is presented. Following this analysis, the definition of the expository style is presented in §6.2. The implications of the expository style in the methods, tactics and actions as well as on the structure of the knowledge base are reported in §6.3. Bruner's work is analysed in §6.4 and the guided discovery style is defined in §6.5. The implications of the guided discovery style on methods, tactics and actions are presented in §6.6. Finally, how and why the expository and guided discovery styles are distinct is presented in §6.7.

6.1 Description of Ausubel's theory of meaningful reception learning

This section presents a condensed summary of Ausubel's theory of meaningful reception learning. This summary contains only the elements that are directly related to the formation of the expository style, as set out in §6.2.

6.1.1 Reception v discovery and rote v meaningful learning

David Ausubel is a strong advocate of *reception learning* which he believed to be a 'par excellence' mechanism for learning especially compared to discovery learning. To support his argument Ausubel first had to address, what he called a confusion, the belief that reception learning is invariably rote and that discovery learning is inherently and necessarily meaningful [Ausubel et al., 1978].

Analysing school learning along two dimensions, see figure 6.1, the rote-meaningful dimension and the reception-discovery dimension, Ausubel showed that both discovery and reception could result into either rote or meaningful learning. Thus, there are further factors that affect the learning outcome which is not simply dependent on the choice between reception and discovery learning. So Ausubel referred to his proposals as *meaningful reception learning* to ensure that these proposals for reception learning are not confused with rote learning.
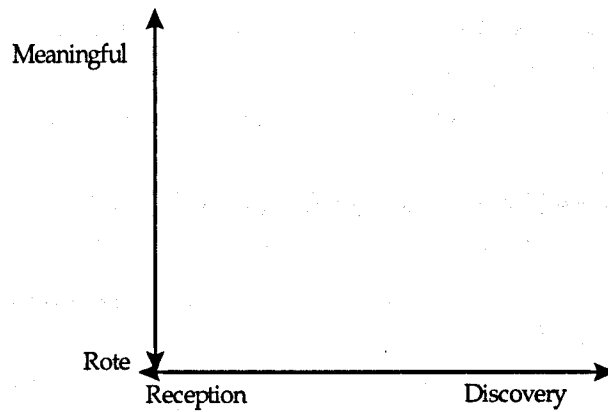
Figure 6.1: The independent dimensions of school learning [Ausubel et al., 1978]

Given a certain material to be learned, the amount that is presented to the student determines whether the learning will be by reception, discovery or by some combinations of the two, e.g., guided discovery. For *reception learning* (rote or meaningful) the material is presented to the student in final form and the student's task is to assimilate it. For *discovery learning*, the principal content of the material to be learned is withheld. According to Ausubel, the student's initial task is to discover what is to be learned before the student assimilates it. However, the most important issue for Ausubel is how meaningful learning occurs.

According to Ausubel et al. [1978], *meaningful learning* occurs when the learner consciously tries to relate new knowledge in a substantive and non arbitrary way to relevant concepts in the existing cognitive structure. In fact Ausubel states that a learner's existing cognitive structure provides the stability, clarity and organisation of the learner's knowledge in a given discipline, [Ausubel et al., 1978]. *Rote learning* takes place when the existing knowledge interacts with the new knowledge in an arbitrary and verbatim knowledge. Rote learning also occurs when no relevant concepts are available in the learner's existing cognitive structure.

6.1.2 Meaningful learning

According to Ausubel et al., [1978], for meaningful learning to occur a number of conditions must be satisfied; these conditions are centred around

- the *material* to be learned

- and the *student*.

With respect to the *student* factor, reference is made to the following two elements,

- the student's *existing cognitive structure*

- and that the student manifests a *meaningful learning set*.

Ausubel et al., [1978] defines *meaningful learning set* as

> A *disposition* on the part of the learner to relate a learning task non arbitrarily and substantively to relevant aspects of his/her cognitive structure. (my italics) (p. 41).

Placing this emphasis on the student's part, Ausubel further demonstrates his belief that how the material is presented and how the material is internalised by the student are two separate processes. For meaningful learning to occur the student has a very important role to play: the student has to integrate the new material with related material in the existing cognitive structure. In fact Ausubel believes so strongly in the student's role in meaningful learning, that he makes a distinction between the learning of meaningful material and meaningful learning.

The term *existing cognitive structure*, refers to the background knowledge, or prerequisite knowledge, that the student is required to possess in order to be able to understand the new material as it is linked, or used, or that builds upon existing knowledge. This factor is of paramount importance to Ausubel who states:

> If I had to reduce all of educational psychology to just one principle, I would say this: The most important factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly. (p. iv) [Ibid, 1978]

It is worth noting that the mere presence of existing cognitive structure in meaningful reception learning is not sufficient. Close examination of the theory of meaningful reception learning, in conjunction with one of its conditions, the meaningful learning set, presents a number of conditions that Ausubel attaches to the organisation of the existing cognitive structure. These conditions include aspects such as links, establishment of the relationships in a substantive, non-arbitrary and non-random manner. In other words, the background knowledge has to be the result of meaningful learning and not simply an accumulation of unrelated and unconnected facts.

Regarding the *material* factor, Ausubel proposes two steps in structuring the material. In the first step, the material is to be analysed and structured in such a way that all

possible connections of the material are realised. In Ausubel's terminology, this step is described as making the material *logically meaningful*. The next step builds on the previous one and stipulates that the material must now be made *potentially meaningful* to the particular student. To accomplish this, the particular learner's *existing cognitive structure* must be taken into account. This ensures that only connections/concepts, which are established in the logically meaningful process, and which can be related to the particular student's existing cognitive structure, are selected.

According to Ausubel et al. [1978], organising the material in a logically meaningful way, implies that the material is analysed and structured in such a way that all possible links, with correspondingly existing relevant ideas, are identified and drawn. These links should enable the student to learn the material in a *non-arbitrary* and *substantive* manner.

The term *non-arbitrary* means that there exists an adequate and almost self-evident basis of relating the new ideas with existing ones in a way that humans are capable of learning [Ausubel et al., 1978]. These non-arbitrary links could be seen as describing analogous ideas, examples, special cases, generalisations, etc.

The term *substantive*, indicates that the material is learned by the student in such a way that it can be expressed in different terms to those learned. For example, "the sum of internal angles of a triangle is equal to 180 degrees" and "the sum of the internal angles of a triangle is a straight line" should mean the same to a mathematics student.

To illustrate how meaningful learning could occur, consider the case of defining the steps to be performed in solving a linear second order ordinary differential equation with constant coefficients.

Introducing the student to terms like *auxiliary equation, complementary function, particular solution* and *general solution* without explaining their origin or justifying them will not lead to meaningful learning.

To facilitate meaningful learning the material should conform to the criteria of being logically meaningful and then potentially meaningful. To make the material logically meaningful, the following links have to be established.

The term auxiliary equation could be explained as a step in reducing an unknown problem to a known one. This problem solving technique is known to the student. It has been used in reducing a quadrant equation to a quadratic one. Another example is reducing a high order polynomial equation, by trying to guess a solution and then divide to reduce the order of the original equation. Even factorisation could be seen as a way of breaking the problem into parts that are known. By the same token, the auxiliary equation reduces the solution of a differential equation into that of a quadratic equation.

The concept of complementary function could be seen as mere continuation of trying to find the solutions of the differential equation based on the solutions of the auxiliary equation. This again is the next step involved in solving the quadrant. Once the solutions of the quadratic have been found then they are substituted to find the solution to the original problem.

Establishing these connections concludes the first step in making the material *logically meaningful*. The next step is to pick the connections that are relevant to the particular student. So, for example, if the student has never done quadrant equations then there is no point in picking this as a way of explaining the purpose as well as how the auxiliary equation is derived. In this case either the factorisation or the polynomial links could be used. Selecting the appropriate links makes the material potentially meaningful to that particular student.

6.1.3 Progressive differentiation

During the process of progressive differentiation, the material is programmed in such a way that the most inclusive ideas are presented first. Ausubel et al., [1978] argues that concept development proceeds best when the most inclusive and general ideas are established in the learner's cognitive structure and then the ideas are progressively differentiated as relevant new information, more detailed and specific is introduced.

To illustrate this point consider the following example. In introducing the concept of differential equations the more inclusive concept of equations is used firstly. Only then the newly established concept of differential equations can be used in introducing the less inclusive cases of different forms of differential equations (ordinary, non-ordinary, homogenous, non-homogenous, etc.). This is achieved by progressively differentiating from the more inclusive concept of differential equations. In this case the material can

be clearly related to the more inclusive ideas, subsumers, in the cognitive structure and thus the new concept is less ambiguous and less subject to being forgotten.

6.1.4 Integrative reconciliation

The principle of integrative reconciliation refers to the explicit attempt to point out significant similarities and differences and reconcile real or apparent inconsistencies between related ideas. In such instances the learner is faced with cognitive dissonance which is resolved through integrative reconciliation [Ausubel & Robinson, 1969].

To illustrate the principle of integrative reconciliation consider an example from the linear second order ordinary differential equation with constant coefficients, particularly its form of solution. Having established the relationship with the superordinate concept of equation, and in particular with the concept of algebraic equation, through the process of integrative reconciliation it is established that in this case the form of the solution is that of a function rather than a number or a constant.

6.1.5 The use of advance organisers in meaningful learning

Ausubel suggests that meaningful reception learning could be enhanced by the use of advance organisers. These are introductory, very abstract, general and inclusive concepts, ideas or propositions which are familiar to the learner; thus they serve to link what the learner already knows with the material that is to be learned [Ausubel et al., 1978].

Ausubel defines two roles that advance organisers can play and thus distinguishes between expository and comparative organisers. Expository organisers arrange a subject matter conceptually. With this approach key concepts and terms are introduced in such a way that the body of information that will be addressed is distinguished from that which has or will be studied at a future time. For example, during an introduction to real numbers an expository organiser could take the following form:

"We have talked about natural numbers, integers, fractional numbers, decimal numbers, their operations (+, -, * , /) and the properties of their operations. Now we are talking about a new number system, the system of real numbers. Real numbers are:

> all natural numbers
> all negative numbers

all fractional numbers

all decimal numbers.

We will see later that we can perform all the operations (+, -, *, /) we know on this system and that all properties of operations are retained."

On the other hand comparative organisers serve to show similarities and differences between what the learner knows and the new material to be studied. This approach uses analogies to facilitate assimilation of new material into the existing cognitive structure. For example, an interesting way to introduce the concept of relations in mathematics is through describing relationships of everyday life such as:

'is sister of'

'is taller than'

'is in the same school as'

'is two years older than'.

6.1.6 Anchorage of new information to existing cognitive structure

Most meaningful learning is substantially what Ausubel defines as assimilation of new information. Thus, according to his assimilation theory of learning, relevant concepts of the learner's cognitive structure serve as anchorage of new information. The new information interacts with pre-existing ideas to form a more highly differentiated cognitive structure. The relationship between the existing ideas and the new ones characterises the type of learning that takes place. The type of learning is either *subordinate* or *superordinate* or *combinatorial*.

Subordinate learning occurs when new information is less inclusive and general than ideas in the learner's cognitive structure. New information is related to an established idea either as an extension or qualification of it (*correlative subsumption*) or as another case of it (*derivative subsumption*).

In *superordinate learning*, new information is more inclusive and general than relevant established ideas. In this instance the established ideas are subsumed and become instances of the newly established ideas. Eventually, when assimilation has taken place, the less inclusive idea(s) become less and less available until they cannot be recalled. This is called *obliterative subsumption*.

Combinatorial learning takes place when new information cannot either be subsumed or cannot itself subsume established ideas as both new and established ideas are at about the same level of inclusiveness and generality.

6.2 Defining the expository style

The expository style draws from Ausubel's theory of meaningful reception learning. To define the expository style, in accordance with the style definition (see §5.2.1), its type of learning and principles have to be defined. Therefore, the type of learning is *reception learning* and its principles are:

- Principal content/final form,

- Meaningful learning (potentially and logically meaningful),

- Existing cognitive structure,

- Advance organisers,

- Progressive differentiation,

- Integrative reconciliation,

- Assimilation (subordinate, superordinate, combinatorial learning).

These principles will provide directions in manifesting the generic structure of methods, tactics and actions in such a way that when TeLoDe is activated under the expository style the teaching strategies will reflect the Ausubelian roots of the expository style.

The above principles do not however, provide directions for the sequence of the strategies. Given that Ausubel's theory is developed in the paradigm of constructivism the strategies will move from implicit to explicit. In an implicit strategy, the tutor will provide the least possible information to the learner, whereas in an explicit strategy, the learner is given all the information. It has to stressed that in both cases, the statements will be couched in accordance with the principles of the expository style. Also, a strategy will only change its method when all its tactics have been exhausted. Likewise the methods will move from analogy to investigation to examples and finally to concept definition.

6.3 Implications of the expository style

In this section, the implications of the expository style to the generic structures of methods, tactics and actions are explored. As argued in Chapter 5, these generic structures are manifested distinctively reflecting the definition of the chosen style.

These implications as well as the implications of the structure of the knowledge representation of TeLoDe are examined with the aid of figure 6.3. The diagram represents a high level overview of the components that are affected by the expository style. To make the relationships between the theory and the application explicit each of the three components will be analysed in turn.
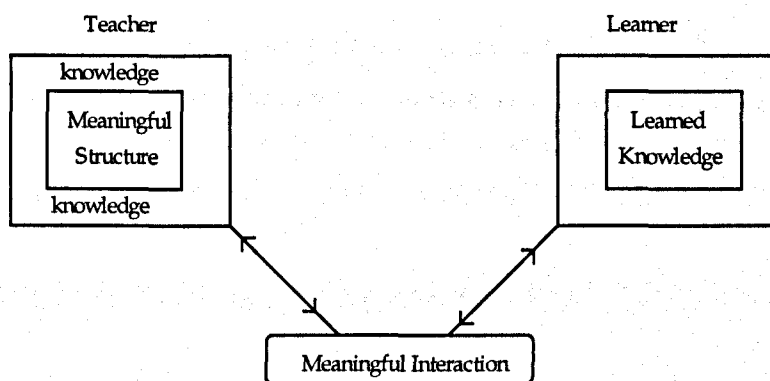
Figure 6.3: The impact of the expository style

The first one, meaningful structure, is concerned with the form of the knowledge representation; the second one, meaningful interaction, is concerned with the teaching process or meaningful interaction, and the third one is concerned with the cognitive structure of the learner and the learner's intention to acquire the new information meaningfully. Thus, there are two components concerned with structures and properties of the knowledge and one concerned with the process. The process is responsible for the transfer of content from the meaningful structure to meaningful learning according to the principles of the expository style. It is also responsible for the appropriate selection of subject matter from the meaningful structure which will enable the process of assimilation in learned knowledge. Furthermore, it is also responsible for encouraging the very existence of the meaningful learning set as required by the expository style.

Meaningful interaction falls within the remit of teaching strategies and thus analysis of this structure will reveal how the methods, tactics and actions should be manifested to

reflect the expository style. The discussion about meaningful structure serves as a precursor to the analysis of the subject matter, see Chapter 7, and to the analysis of knowledge contents and methods in TeLoDe, see Chapter 8 and TeLoDe's implementation, see Chapter 9.

6.3.1 Meaningful structure

The expository style will have both a qualitative and quantitative impact on the structure of the knowledge base. A quantitative impact means that the knowledge base will contain not only what is to be learned (core knowledge) but also background knowledge. The term qualitative refers to the nature of the links that will have to exist in the knowledge base, linking not only the background knowledge with the core knowledge but also providing links within the core knowledge. The links will describe the relationships between the nodes which will assist meaningful interaction in being carried out successfully. The qualitative and quantitative aspects will now be covered in greater detail.

6.3.1.1 Quantitative aspect of the knowledge base

The knowledge base can be viewed as containing three different kinds of knowledge. These are: *task requirements knowledge, relevant background knowledge* and *relevant missing knowledge.*

Task requirements knowledge is that part of the core knowledge which is currently the focus of the interaction. It is the knowledge that has to be integrated by the student. Defining the contents of this knowledge determines the contents of the relevant background knowledge. The term *relevant background knowledge* indicates the background knowledge that is *relevant* and *necessary* to the *task requirement knowledge* and to which it must be related non randomly and substantively. Background knowledge is the knowledge that the student has to *know* in order to be able to learn the task requirements knowledge in a meaningful manner. The term *relevant missing knowledge* indicates the gap that exists between the existing cognitive structure of the learner and the relevant background knowledge. It contains knowledge that is *relevant* and *necessary* to the task requirement knowledge.

For example, in the case of solving a linear second order ordinary differential equation with constant coefficients, the knowledge representation will be as follows: all

knowledge associated with the specific differential equations will be the core knowledge. Thus, the *tasks requirements knowledge* will contain knowledge about the four steps involved in the process of solving differential equations, along with concepts facilitating meaningful learning of the procedures involved by explaining the reasoning behind the steps performed. The concepts attached to the procedure will also form part of *the tasks requirements knowledge*. When TeLoDe concentrates on each step then that step becomes the core knowledge.

Initially a system like TeLoDe could introduce and explain the term differential equation, *tasks requirements knowledge*, and be able to demonstrate the different qualifications of the general case of differential equation. To make the structure meaningful the appropriate background knowledge needs to be recalled which could be knowledge of algebraic equations and their attributes, namely variables, order, solutions and coefficients. This is necessary in order to demonstrate the similarities/differences between the task requirement and the relevant knowledge in the existing cognitive structure (*relevant background knowledge*). In this particular step the introduction and explanation of the concept of differential equation was the core knowledge.

If there is *relevant missing knowledge* then using relevant background knowledge (that is algebraic equations) could help to bridge the gap, according to meaningful interaction, and so the learner can identify the similarities and the differences between the task requirement and the relevant background knowledge. Thus the learner can appreciate that differential equations represent another instance of the general concept of equations.

In the case of auxiliary equation, the overall *tasks requirements knowledge* is that the solution is of the form of the exponential function. It is therefore necessary that TeLoDe is equipped with background concepts such as solution, function, linear first order differential equations with constant coefficients and other forms of differential equations.

In the case of the complementary function, the *tasks requirements knowledge* is forming the solution of the homogeneous equation based on the finding (see auxiliary equation above) that it is of the exponential form. The relevant background knowledge is the realisation of the term linearity, second order derivative and integration. All this

background knowledge will enable the learner to understand the final form of the *complementary function*.

This quantitative analysis of the knowledge base could enable TeLoDe to pinpoint the relationship between the tasks requirement knowledge, and the relevant background knowledge. Consequently any relevant background knowledge that is not present will be considered as relevant missing knowledge.

6.3.1.2 Qualitative aspect of the knowledge base

To demonstrate the qualitative effect in the knowledge representation, the key is in the term meaningful learning. The learner is required to relate in some sensible and plausible and substantive way the new ideas to those relevant in the existing cognitive structure. Consequently, links that will enable subordinate, superordinate and combinatorial learning, have to be present in the knowledge base of TeLoDe. If this is not the case then TeLoDe may not be in a position to assist the process of meaningful learning. Furthermore, through the use of these links TeLoDe could provide explanations regarding the kind of learning that is promoted and why.

To illustrate the issues raised above, consider the case of the linear second order ordinary differential equation with constant coefficients. To introduce the concept of differential equation, the superordinate concept of equations was recalled, thus demonstrating that differential equations are in fact instances of equations with a number of different attributes (form of variables, form of solution, form of coefficients etc.). Therefore, the knowledge base will have to contain links as it is important to know whether the relationship between the core knowledge and the background knowledge is generalisation (subordinate), or instantiation (superordinate) or just relevant in explaining the behaviour (combinatorial). Such links are vital as the links between the core and the background knowledge will have to be made explicit and also presented to the student in its final form.

In the example above, reference was made to the attributes that will have to be present in the knowledge representation. Their presence is vital, as the expository style clearly states that the student is not expected to discover the principal content, instead it will have to be presented to him. To illustrate this point consider the analogy between the algebraic equations and the differential equations. In asking the students to find the similarities and the differences between the two forms of equations, the questions

should specify that the students will have to look for the variables, the degrees, the form of the solution etc. This is done as the expository style states clearly that the principal content has to be presented to the student.

In the case of the auxiliary equation, the first task is that of establishing that the solution is of the form of a function. In this instance the student is prompted with a concept from the background knowledge. If the concept is that of linear first order ordinary differential equation with constant coefficients then the type of learning is a combinatorial one since both the first order and the second are seen as instances of the linear differential equation, whereas if the concept used is that of linear ordinary differential equation then the type of learning promoted is a subordinate one.

Another aspect of the qualitative impact, is the establishment of relationships that may not appear to be apparent. For example, consider the case of the auxiliary where it has been established that the form of the solution is that of a function and that its property, the function and its derivatives differ only by a constant. The objective here, is to establish which function is the one that satisfies this property. Here the concepts of function and solution are connected in the context of the linear second order ordinary differential equations with constant coefficients.

6.3.1.3 Focus in knowledge representation

By focus, reference is made to a miniaturised version of the meaningful structure. That is given a specific task, then for that task the core knowledge as well as its corresponding background knowledge are selected and used in the meaningful interaction. For example, consider the introduction of the concept of differential equations. Here, the concept of the differential equation and its examples and attributes, such as unknown, coefficients order and so on as well as concepts from background knowledge, are selected from the meaningful structure to form the methods that could then be used by meaningful interaction to carry out the instruction.

If the background required is not present then it is considered to be *relevant missing knowledge* and acted upon as if it was an original task. Here, the focus is on the background and, upon satisfactory completion, the focus reverts back to the original problem. If however the background knowledge was assumed to be present but is later discovered to be absent then the approach adopted is as follows: the focus firstly turns

to other methods that can enable a 'quick' recovery, but failing that the problem is then treated as if the background knowledge is *relevant missing knowledge*.

It should be noted that the focus in knowledge representation is a two pass process. In the first pass, the principle of logically meaningful is operative whereas in the second pass the principle of potentially meaningful is operative and the focus in knowledge representation reflects the particular student needs.

As a final note it must be stressed that when the focus for a task is structured, the conditions that are placed upon the background knowledge are extensive. In the case of the potentially meaningful stage, given a concept and a relationship then every attribute has to be known, and the relationship established. That is a direct result of the conditions placed upon the existing cognitive structure of the learner, i.e., being non-random, non-arbitrary and substantive.

6.3.2 Meaningful interaction

Meaningful interaction refers to the actions that will have to be taken by the tutoring component of TeLoDe, in order to facilitate the expository style.

For example, consider an analogy between the differential equation and the algebraic equation, in trying to define the differential equation.

$$a \, \frac{d^2 y(x)}{dx^2} + b \, \frac{dy(x)}{dx} + c \, y(x) = 0$$

$$ax^2 + bx + c = 0$$

Under the expository style and in order to present the material in final form, the question to the learner could be phrased as follows:

> What are the similarities and differences between the differential and the algebraic equations with respect to variables, coefficients, etc.?

The principal content refers to the fact that the learner should focus on the variables of these equations and look out for similarities and differences. The principal of final form is the one which stipulates that the principal content must be disclosed to the student. The inclusion of the principal content is very important in the expository style as it

facilitates the understanding of the relationships/links between the core and the background knowledge. If the principal content were not included in the question, then the principle of reception, would have been violated as the students would have had two tasks to perform; firstly to identify the principal content and then perform the comparison. Moreover, the principle of making the connections explicit between the new and existing knowledge would also have been dishonoured.

Another principle in interacting under the expository style, is that an overview/introduction should be provided to help the learner as, firstly, it can be used as a reference point, and, secondly, enabling the student to recall relevant background knowledge. To perform such an interaction Ausubel proposes the use of advance organisers which have two functions: to introduce the material and cover any relevant missing knowledge. Thus the learner could always be in a position to follow the new material as introduced by the advance organiser.

For example, in the case of the method of analogy, the specific background knowledge chosen is usually a superordinate concept of the new idea. Thus the learner is helped to view the new idea as an application of the general concept. In this case the principle of interaction is active as, provided that the principal content is given in final form, the student needs to identify the similarities and the differences between the new and the existing idea. For example, in introducing differential equations, the scenario could be as follows: a linear differential equation is like an algebraic equation (connection with existing knowledge). Given examples, the learner is then asked to identify similarities and differences between the two concepts by specifying which are the attributes of the two concepts that will enable the student to discriminate between them.

In the method of concept definition, the new concept is described in terms of its attributes, according to the mechanism of concept assimilation, i.e., by referring to the relevant background knowledge and attributes concerned. For example, in defining the differential equation, the definition could be: a differential equation is an equation whose variables are derivatives of one or more variables with respect to one or more functions, the order is ..., their solution is .... Here, as expected, reference is made to the background knowledge and once the connection is established then the next task is to define the new concept in such a way that the learner is helped to see the similarities and the differences. This then facilitates meaningful learning.

Finally, another aspect that affects the organisation of the interaction is the fact that Ausubel supports a top-down approach (progressive differentiation). It is not the only approach but wherever possible it should be observed. Otherwise the structure should follow the principle of integrative reconciliation.

In the SIMTA framework the tactics are responsible for facilitating the interaction of the material to be learned with the student. Under the expository style, the tactics will alternate from implicit to explicit, as discussed in §6.2, where the principles of progressive differentiation and integrative reconciliation are instrumental in deciding the content of the tactics.

6.3.3 Meaningful learning set

The existence and the operation of a meaningful learning set denotes the responsibilities of the learner. While the organisation of the material and the efforts of the teacher help the learner to learn in a meaningful manner, there are no provisions for ensuring this. Furthermore, trying to ensure that the learner possesses a meaningful learning set leads us to a problem -teaching someone how to learn- which is beyond the scope of this work.

Although the presence of a meaningful learning set is one of Ausubel's two conditions for meaningful learning, there are no references in his work about how it can be achieved. Since a meaningful learning set in TeLoDe cannot be engendered, it will be assumed that in the interaction under the expository style the learner possesses such a set.

6.4 Description of Bruner's ideas on instruction

Bruner's notes on a theory of instruction tend to view the topic from a higher perspective than Ausubel's. For, example, Bruner [1966], states that his intentions are to

> ... attempt to develop a few simple theorems about the nature of instruction. ...The plan is as follows: first some characteristics of a theory of instruction will be set forth, followed by a statement of some highly general theorems about the instructional process. (p. 39)

From the above one could reasonably conclude that Bruner's approach to a theory of instruction could be compared to that of a curriculum developer, focusing at a general,

abstract and inclusive level with illustrations of his points. As a consequence of this, interpretations of Bruner's work could lie on a broad spectrum and still be valid, that is at curriculum or at lesson levels.

For example, Bruner calls for *optimal structure*. Optimal structure could be interpreted as relating to structuring the knowledge base, what was referred to as meaningful structure (see §6.3.1). However, it could also refer to specific aspects of the knowledge base that will interact with the student, what was referred to as focus in knowledge representation (see §6.3.1.3). The difference between the two applications is at the mechanistic level. At the level of structuring the knowledge base a global/epistemological perspective should be adopted that reflects the principles and the fundamentals of the subject matter, whereas in the case of a specific aspect of the subject matter being pursued, then, an approach which is more relative to conditions must be employed.

Bruner's theory is not as detailed as that of Ausubel's, even at a descriptive level. For example, Bruner [1966], states that optimal structure must be related to the status and the gifts of the learner and consequently the optimal structure of a body of knowledge is not absolute but relative. But what are the principles that one should look for in order to achieve this relative optimal structure for a learner? In contrast, Ausubel does specify how one could achieve the meaningful structure, namely through the process of logically meaningful and potentially meaningful (see §6.1.2).

6.4.1 Discovery learning v guided discovery learning

Bruner is a proponent of *discovery learning*, since the learner is inherently curious and is attracted to what is uncertain, unfinished or unclear. Learning occurs by exploring alternatives and consequently, according to Bruner, [1966], instruction must facilitate and regulate such an exploration.

> Since learning and problem solving depend upon the exploration of alternatives, instruction must facilitate and regulate the exploration of alternatives on the part of the learner (p. 43) [Bruner, 1966].

Through the term *alternatives*, Bruner expects the learner to form a number of hypotheses, test these hypotheses and look at the problem from a number of perspectives, before a solution to the problem is achieved. It is clear that Bruner calls

for the provision of a rich environment where the student is able to act, form hypotheses and test them. This is indicated by the term *facilitate*. However, Bruner goes one step further by stating that through instruction such an exploration is *regulated*, by *activating*, *maintaining* an *directing* it.

This differentiates Bruner's discovery environment from others such as Papert's which is described in his book 'Mindstorms: Children, Computes and Powerful Ideas', [Papert, 1980]. According to Elsom-Cook [1990], the environment that Papert described in his book is considered a 'pure' discovery environment, where the tools (the turtle) provided to the learner are sufficient for carrying out explorations, forming and testing hypotheses and thus facilitating learning. According to Elsom-Cook [1990], the responsibility of the teacher would be the conception and setting up of the environment and its tools.

By contrast, Bruner believes that the teacher has a very important role to play, or as Mason [1979] puts it "both are learning (that is the teacher and the student), though at different levels". Here the teacher is not only charged with the responsibility of setting up the environment and choosing the right tools, but also is charged with the responsibility of *regulating* this exploration. This is why the environment in the case of Bruner is that of *guided discovery*.

The next sections continue the analysis of Bruner's work in order to demonstrate the principles of the *guided discovery style*.

6.4.2 The principle of 'honest form'

The concept of 'honest form' was used by Bruner to promote his, then, controversial hypothesis that any subject can be taught to a child [Bruner, 1977]. By 'honest form' Bruner merely calls for the subject matter to be organised in a way that it can be related to the learner. Therefore, great importance is attached to the current state of the learner and this must be examined and respected.

One must take this into account and arrange the material at one level as well as the interaction in such a way that the process is best suited for the individual learner. This brings out two arguments, the one for *predisposition*, in which Bruner refers to the inherent willingness, curiosity and eagerness of the student to learn. However, for this

to be capitalised upon the teacher must arrange the material and approach on what Bruner describes as the *'child's way of viewing things'*.

6.4.3 The principles of activation, maintenance and direction

In order to harness and cultivate this inherent predisposition to learn, Bruner calls for *regulation*, so that the predisposition is sustained and encouraged. To achieve this, the concepts of *activation, maintenance* and *direction* are introduced to assist the process of regulation. These concepts will serve as principles in the guided discovery style since they could provide some direction in the sequence of the strategies operating under a guided discovery style. But for these principles to work effectively the organisation of the subject matter must now be considered in order to identify the structure that will best sustain these principles.

6.4.4 The principle of optimal structure

Bruner refers to the *optimal structure* of the learning material. The concept of optimal structure (as explained earlier) is a packed concept that can be applied at various levels. At the top level it could be interpreted as referring to the whole structure of the subject matter, as Bruner [1977] stresses the importance of structure at a high level:

> ... school curricula and methods of teaching should be geared to the teaching of fundamental ideas in whatever subject is being taught. ... The problem is twofold: how to have the basic subjects rewritten and their teaching materials revamped in such a way that the pervading and powerful ideas and attitudes relating to them are given a central role; ... (p. 18)

The structure of the subject matter is clearly important and this is reflected in this research, (see Chapter 7), where the topic of linear second order ordinary differential equations with constant coefficients is viewed from different perspectives so that the knowledge base reflects our perceptions of the topic, the principles and fundamentals that are to be conveyed to the students.

Bruner does not provide any general indicative principles (such as Ausubel's non-randomness and substantiveness) for arranging the subject matter. While the principles of non-randomness and substantiveness are high level concepts, they are at a lower level than that of optimal structure, and can therefore be used in designing the knowledge base.

Regarding the second level of the concept of optimal structure, that is the structure required to teach certain aspects of the knowledge base, Bruner calls for that structure to be relative and not absolute. Although the analogous concept from Ausubel here is that of potentially meaningful, the background knowledge is placed under different conditions. Consider the example of teaching the form of the solution in the case of the differential equations. In the expository style (see §6.3.1), to utilise another type of equation from background knowledge as potentially meaningful, both the concept of the background equation and the attribute form of the solution for that type of equation need to be known. For Bruner, the attribute for the specific concept need not be known as, according to him it is curiosity and uncertainty that triggers exploration and thus learning.

6.4.5 The principle of the three cognitive representations: *enactive iconic symbolic*

But how does learning takes place? Bruner does not specify the type of learning that could take place, i.e., subordinate or superordinate learning, representational learning etc. Moreover, principles for meaningful learning such as progressive differentiation and integrative reconciliation are not present. That is they are not present at the level that these Ausubelian principles seems to be. However, Bruner implies that the learning should consider the three cognitive representations that he has identified [Bruner, 1966, Bruner et al., 1966] namely, *enactive, iconic* and *symbolic*.

These concepts are composite ones and as such influence both methods and the level of tactics. Firstly, to explain the terms enactive, iconic and symbolic and through this explanation their influence to *methods* and *tactics*, consider John Mason's [1979] problem: Arrange four matchboxes so that each touches every other along a surface (not just a corner). According to him:

> most people reach for some matchboxes to try it out. Even one box helps them to see the
> problem. Other people think about it or even draw a few pictures. The desire to have a
> physical, manipulative model is characteristic of what Bruner calls an enactive
> representation of the problem. For many people the problem is intractable unless and
> until they obtain matchboxes or some surrogate which they can concretely manipulate.
> ... Alternatively, working with a diagram, or thinking about the problem either visually
> or from a sense of pattern is characteristic of what Bruner calls an iconic representation.
> ... The matchbox problem illustrates two of the modes of representation but not the
> third, the symbolic. This is the act of naming things, and is distinguished from the iconic

in that the latter 'looks' like the elements of the problem (matchboxes or near equivalents) whereas the symbols or names need have no pictorial resemblance. Thus the name 'pied wagtail' looks nothing like the birds to which we apply this name. If someone says to me that they saw a pied wagtail, a picture immediately pops into my mind. I am present at the place where I first saw one. An experienced bird watcher is likely to treat it in a more matter of fact manner and various facts and features of the bird are likely to arise in his mind. This is an example of working in the symbolic mode whereas I was in the iconic.

To provide context for this research, the linear second order ordinary differential equation with constant coefficients could be viewed from the three cognitive stages as follows:

If one requires an example of the differential equation, then it could be said that one is in the enactive stage. If, however, the definition of the differential equation is required then it could be said that one is in the iconic stage whereas, finally, if only mention of the name of the differential equation is sufficient without trying to visualise it then it could be said that one is in the symbolic stage.

6.4.6 The principles of economy and power

Bruner provides some directions for the content of the tactics in his illustration of two of the four major features of a theory of instruction, i.e., predisposition and structure and form of the knowledge. In the case of the former Bruner argues for the direction of the exploration to be made known to the learner in some approximate fashion as well as to be expressed at the right level. In the latter he argues for the structure to conform to the principles of *economy* and *power*; once again this emphasises the need for information to be expressed at the right level. As an example of the difference between power and economy, Bruner states that referring to the American civil war as a clash between humanity and slavery is a very powerful argument. However, referring to slavery as a struggle between two economic systems, although more economical is a less powerful argument.

An example from our case of differential equations to demonstrate the principles of economy and power could be as follows: if the linear second order ordinary differential equation with constant coefficients is defined in terms of the linear ordinary differential equation then this is quite powerful. If however, the definition uses the concept of equations then whilst it would be economical it is not very powerful. This is

because in the first instance the new equation is seen as a further elaboration of the immediately higher one, whereas in the latter case the gap between the two forms of equations is quite a big one since so many attributes are involved.

6.5 The guided discovery style

Summarising, the guided discovery style, based on Bruner's work, can be called a type of discovery learning where the tutor has a more active role than just that of setting up activities that will deduce discovery. As directed by the principles of activation, direction and maintenance, the tutor is charged with the responsibility of guiding the exploration.

While Bruner's ideas about how knowledge is learned are not as detailed as those of Ausubel, there are nevertheless principles that can be used in forming a style. This is especially so if we were to use the experience of forming the expository style. For example, Ausubel suggests that the contents of any method should conform to the principles of logically meaningful and potentially meaningful. Now in the case of Bruner, the only references are that of "honest form" and that of optimal structure being relative and not absolute. This is the point where experiences could be transferred and algorithms devised that deliver the contents of the methods in accordance with the principles of the style. The guided discovery style is therefore defined as:

> The type of learning is that of *discovery*

> The principles of the guided discovery style are as follows:

- Elicit

- Activate, direct and maintain exploration

- Provide a direction in some approximate form

- Honest form (predisposition, background knowledge),

- Optimal structure,

- Economy, power

- Enactive, iconic, symbolic

In the next section the implications of some of the principles of the guided discovery style on the methods and tactics are considered. It has to be stressed that the implementation of the guided discovery style in TeLoDe is only a skeleton one.

6.6 Implications of the guided discovery style

In this section the implications of the guided discovery style on methods and tactics are considered. The implications are examined in the context of meaningful structure and meaningful interaction, as discussed in §6.3. The principles are optimal structure; enactive, iconic and symbolic; power/economy, elicit, activate, maintain and direct.

Optimal structure

The principles that affect the meaningful structure are *optimal structure, honest form, enactive, iconic* and *symbolic*.

In §6.4.1.2, it was argued that the principle of the optimal structure is not as detailed as Ausubel's non-randomness and substantiveness. However, since both Ausubel's and Bruner's work stem from the paradigm of constructivism it would not be unreasonable to assume that these principles, non-random and substantiveness, could also be used to apply to Bruner's ideas. These concepts are in complete harmony with Bruner's guidance for

> "teaching in some intellectually honest form ... representing the structure of that subject in terms of that child's way of viewing things. (p. 33) [Bruner, 1977].

Finally, part of the rationale of analysing Bruner alongside Ausubel is that of relatability , i.e., identify any commonalities as well as differences which should be reflected in any teaching that is based on either theory. Having said that I propose that the knowledge base, used by TeLoDe, will be the same, as proposed in section §6.3.1, for both the expository style and the guided discovery style.

Enactive, Iconic and Symbolic

The implication of the three alternative representations is concerned with the sequence of the methods, rather than the contents. That is, the methods could be sequenced to reflect the principles of the alternative representations. In a somewhat simplified way,

as defined in §6.2, the sequence of methods in TeLoDe is linear and fixed: the analogy comes first, followed by that of investigation, then by the examples and finally by the definition. It has to be noted that the sequence is affected in a somewhat simplified way since the three alternative representations imply a dynamic sequence. Here, in a rather simplified manner, the analogy could play the role of the symbolic representation; the investigation plays the role of the iconic whereas the method of examples plays the role of the enactive.

In the case of the tactics, the implications are twofold: the contents as well as the sequence. Regarding the contents, a first attempt is being explored using the concept of congruent classes (this is further discussed in §9.4.2). It is at the level of tactics that the learner is either offered means of manipulating the matchboxes or encouraged to work with the iconic or symbolic representation. Another example is the three alternative ways of representing the counterexample in WHY (see §4.2). This is further argued later in this section where further principles affecting content are discussed.

Ordering the tactics as proposed in §6.2, is an initial and somewhat simplified step in establishing a way of representing the essence that the concepts enactive, iconic and symbolic convey. This is further supported by Bruner as he states that

> If it is true that the usual course of intellectual development moves from enactive through iconic to symbolic representation of the world, it is likely that an optimal sequence will progress in the same direction. (p. 49) [Bruner, 1966]

However, the implications of the alternatives go beyond the methods and the tactics and extend into the knowledge base. Here they are twofold. In the first case they concern the structure of the knowledge base in that a concept's properties and attributes should encompass all three, if possible, representations. However, it could be argued that the principle of *optimal structure* should have covered it in which case the three alternative representations should be seen as qualifiers of optimal structure. Therefore the main implication here is a peripheral one to the knowledge base. The alternative representations could be interpreted as a need to have traversal functions, or rather selector functions, that reflect them. That means that for every request that is made to the knowledge base the selector functions should be able to identify an enactive, iconic and symbolic representation. This means that the tactics then are able to make use of the three representations on the implicit to explicit scale.

Another principle that should affect the sequence as well as the contents of the tactics is that of expressing a task at the right level. Bruner states,

> A cut-and-dried routine task provides little exploration; one that is too uncertain may arouse confusion and anxiety, with the effect of reducing exploration (p. 43) [Bruner, 1966]

The sequence of the tactics has to attract, not alienate the learner, i.e., the right balance has to be struck between providing the learner with a structure that is both intriguing and supportive thus nurturing exploration. SIMTA's role in structuring and sequencing methods and tactics should provide such a structure.

6.7 The differences between the expository style and the guided discovery style

While Ausubel's and Bruner's work stem from the paradigm of constructivism, it is interesting to observe that they advocate "qualitatively discontinuous" ways of teaching. The difference between the two styles is deep and rests mainly on the different cognitive demands that are made on the teacher and the student.

The type of learning is the first difference. Consequently, the role of the teacher is also different. In the case of the expository style, the role of the teacher is to expose the material in a non-random, non-arbitrary and substantive manner to the student. In the case of the guided discovery style, the role of the teacher is to activate, maintain and regulate exploration.

Exposition of the material in the expository style occurs under certain directions which are: principal content has to be identified and then be given in final form. In the case of Bruner there exists no explicit and precise direction. However, Ausubel states that in the case of discovery the principal content is withheld from the student. But to activate the exploration, Bruner stipulates that the objective must be given in somewhat loose form. This is the second difference regarding the role of the teacher.

Regarding the role of the student, the terms predisposition and disposition provide the appropriate clues. Both Bruner and Ausubel place great emphasis, in principle, on *predisposition*; it is in principle, since there appears to be a subtle difference between the two. This difference is of great importance and further supports the thesis for the

distinctiveness of the style. On the surface, it appears that both Ausubel and Bruner agree on the importance of an inclination and Ausubel even proposes that one of the roles of the advance organiser is to create a disposition towards meaningful learning. But the difference is around the two terms *predisposition* and *disposition* and in particular the philosophy surrounding these terms. In the case of Bruner the predisposition in a learner *exists*, and all that instruction has to do is activate, direct and maintain it. Bruner even talks of the will to learn as being inherent in humans. Ausubel, on the other hand, while he agrees with the importance of predisposition, appears to be less convinced of its inherent presence. That is why Ausubel defines one of the roles of the advance organiser as being to create disposition. Ausubel goes further and requires that the learner exhibits a meaningful learning set. Such a concept is not present in Bruner's work, and would be redundant here. This is because Bruner perceives the student as being inherently willing, eager and curious.

Finally, the difference also extends to the conditions that are placed on structuring the methods. That is because different conditions apply on the use and status of the background knowledge. As stated in §6.3.1, in the case of the expository style, for a given concept, all its attributes must also be known if it is to be considered as a candidate. This is not the case for the guided discovery style since Bruner stipulates that a degree of uncertainty has to exist to provoke curiosity.

Summarising the comparison between the two styles defined here, it is worth bearing in mind that whilst the styles are *qualitatively discontinuous* they are not completely *dichotomous* (terminology used in Ausubel et al., [1978]) and this is reflected in the implementation of both styles (see Chapter 9). However much or little overlapping may be between styles, once a style is defined, then, it is *distinct*.

6.8 Summary

This chapter described how two styles, the *expository style* and the *guided discovery style*, were defined. The expository style draws on Ausubel's theory of meaningful reception learning whilst the guided discovery style draws on Bruner's work. Such a description will assist in demonstrating:

- how a learning theory could be used as a basis for defining a style

- how the definition of a style affects the generic structure of SIMTA's teaching strategy, i.e., the method, tactic and actions and the contents of the knowledge base

- how two styles are distinct.

The definition of the *expository style* is not only required for the implementation of the teaching strategy module in TeLoDe but is also a factor in the analysis of the topic to be used as the knowledge content for TeLoDe. The topic is linear second order ordinary differential equations with constant coefficients and its analysis is described in the next chapter.

The definition of the expository style also has implications for analysing the linear second order ordinary differential equations with constant coefficients, the knowledge content of TeLoDe. This analysis is presented in the next chapter.

CHAPTER 7

Chapter 7: Analysis of Knowledge Content of TeLoDe

In the last chapter, the expository style was defined. The expository style will inform the teaching strategy module of the TeLoDe prototype. In developing TeLoDe, another element that is required, as discussed in Chapter 2, is the incorporation of declarative knowledge. The topic of linear second order ordinary differential equations with constant coefficients is analysed to inform the *declarative knowledge*. One crucial factor in this analysis is the stipulation that Maple, and its capability of fully solving these type of equations, must be fully exploited and used. This is of particular interest when it comes to teaching the solution of these type of equations. The other crucial factor is that the principles of the expository style are observed and adhered to in this analysis.

The aim of this chapter is to explore alternatives in teaching the solution of linear second order ordinary differential equations with constant coefficients where the use of a Computer Algebra System such as Maple is central by providing the answer to the problem, i.e., the solution. The topic of linear second order ordinary differential equation with constant coefficients has been selected mainly for the following reasons:

- The topic is a highly procedural one and as such reflects the question about 'redundant' knowledge as discussed in Chapters 1 and 2.

- Through this alternative teaching it will be possible to suggest a shift of emphasis from the procedures involved in solving the differential equation to the concepts that require/explain the process of attaining the solution.

Other criteria for selecting this topic are as follows:

- Whilst the topic is not simplistic, its complexity is not a hindrance

- The topic is rich in background and core knowledge

- The topic's structure is very clear.

In §7.1, a range of possible ways for teaching the solution of linear second order ordinary differential equations with constant coefficients, as exemplified in a number of books is presented. These are the *top-down*, *bottom-up* and *declarative* approaches. In §7.2, the top-down approach is described followed by the reasons why it is rejected. Likewise the bottom-up approach is described in §7.3, followed by the reasons for its rejection. The declarative approach, the one that is adopted here is described in §7.4.

7.1 Teaching the solution of linear second order ordinary differential equations with constant coefficients

There are a number of ways of teaching this topic, (e.g. [Coddington, 1962], [Stroud, 1970]). The most appropriate way will vary depending on the objectives and/or the perspectives of those involved. For example, if the topic is to be taught to an *'engineering student'*, then the emphasis is different from when the audience is a *'mathematics student'*. Indeed such a difference in the approach is obvious if one browses through respective textbooks.

In the case of *engineering students* the emphasis is primarily on the procedural aspect of the topic because, for them, the equation is seen as 'means to an end', i.e., solution of such equations allows them to proceed with their engineering issues. Students that fall into this category are taught how to

  a) form the auxiliary equation through the substitution of $\dfrac{d^2y(x)}{dx^2}$ by m^2, $\dfrac{dy(x)}{dx}$ by m and $y(x)$ by 1

  b) solve the quadratic equation and

  c) then, depending on the nature of its roots, find the appropriate form of the complementary function to be selected [Stroud, 1970], [MST204, 1981].

This approach is particularly procedural since no attempt is made to reason about the steps involved in attaining the solution. Instead, the students are provided with rules and algorithms, which will enable them to solve the differential equation. The definition for the term 'procedural' comes from Hiebert and Lefevre, who state:

> It is the clearly sequential nature of the procedures that probably sets them most apart from other forms of knowledge". (p. 6) [Hiebert & Lefevre, 1986].

On the other hand, in the case of the *mathematics student*, the emphasis is on the existence and nature of solutions and an examination of the properties of linearity. Hence the nature and conditions for the formulation of the general solution are examined, that is, the number of arbitrary constants, the linear independence of solutions, the fundamental set of solutions etc. This approach could be characterised

more as a theoretical one where the emphasis is more on the understanding of the concepts involved, and less emphasis on the actual process of the solution [Ince, 1926], [Coddington, 1962].

Given that the aim here is to provide a meaningful environment for teaching the process of solving the differential equation, and that we want to use conceptual knowledge, the *declarative approach*, see §7.4, could be seen as an attempt to bridge the two previous approaches by promoting a form of declarative knowledge where the procedural knowledge is backed up by the conceptual knowledge.

The overall aim could be seen as the same as the *'engineering student'* approach, in that there must be a solution to the problem, but the teaching has more in common with the *'mathematics student'* approach. Whilst there is a desire to make use of the conceptual knowledge, trying to construct a computer tutor imposes a number of limitations on representing and handling concepts at such a high level.

Here three alternatives approaches will be presented. The first two are descriptions of approaches as exemplified by textbooks [Ince, 1926, 1956], [Burkill, 1962], [Reuter, 1958], [Wylie, 1979], Coddington [1962], whilst the last approach is the one developed in this thesis and it is influenced by the SIMTA model as discussed in Chapters 5 and 6.

The first approach, the top-down approach, is geared towards the *'mathematics student'*, whilst the second approach, or the bottom-up approach, is designed to suit the *'engineering student'*. The approach adopted here, the declarative one, could be considered as an attempt to bridge the other two.

7.2 The top-down approach

Here the *'mathematics student'* is taught to investigate the equation, its solutions, if any exist, and the conditions under which one could say whether the general solution has been obtained. The emphasis is on a conceptual understanding of the subject matter. This involves understanding what a differential equation means from an analytical and geometrical point of view, appreciating the geometric significance of its solutions, and realising that it is not always possible to find an exact solution for certain types of differential equations. Thus, there are exact methods which yield exact solutions in some cases, and wherever these cannot be used, there is no alternative but to use an approximate method for an approximation to the solution [Ross, 1964]. Nevertheless,

the students, at this point, are taught the theorem of the existence of solution, which guarantees them that there is a solution and also that the solution of the differential equation is unique [Ross, 1964], [Coddington, 1962], [Kaplan, 1964]. Given the nature of the differential equation and the fact that attainment of the solution is not always feasible, it is very understandable that one should concentrate on ensuring that a solution does exist and try to investigate its nature.

In turn, the concept of the general solution is explored in order to identify the conditions that have to be satisfied. According to Ince [1926],

> A solution which involves a number of essentially distinct arbitrary constants equal to the order of the equation is known as the *general solution*.(p. 5)

Again, as one would expect, when the concept of linear differential equation is introduced the principles of superposition and supposition are defined, along with the concepts of families of solutions, linear independence of solutions and fundamental set of solutions.

Consequently, when the problem of solving our type of equation (that is the linear second order ordinary differential equation with constant coefficients) is encountered, the students are in a position to follow the procedure meaningfully and also to see the application of the theory in this specific type of differential equation. Therefore, the students will appreciate why the solutions of the linear second order differential equation have to be linearly independent and why, in the case of linear dependent solutions (when there is a double real root from the auxiliary equation), a linearly independent solution has to be found (loss of second arbitrary constant), etc. Moreover, the students could appreciate the exact method of attaining the solutions of this type of differential equation as it is not always possible to find the exact solution.

7.2.1 Why is this approach not useful ?

From the description above, it is clear that the procedural aspect is secondary to the conceptual one in this approach, and that the procedural aspect is an application of the theoretical considerations. The emulation of such an approach in an computer environment is quite a difficult one and would have been a project on its own. However, the teaching of the solution of the given differential equation is only an illustration of the principles of the SIMTA framework, and, as such, of secondary

importance. Moreover, such a theoretical approach inherently imposes restrictions on alternatives. The restrictions exist because the topic is being perceived from a highly theoretical point of view and thus there is a shift of attention from that of learning to solve the differential equation to that of understanding the concepts involved and how they relate to each other. Of course, once the concepts have been explored the problem of solving the differential equation becomes an application of the theory.

7.3 The bottom-up approach

This approach is used if the objective is to obtain the solution to the problem of linear second order ordinary differential equations with constant coefficients. This will be referred to as the *'engineering student'* approach. Here, the process of acquiring the solution is an intermediate step in the whole process of solving a physics problem. Powered by such a motive, the focus is, unfortunately, on the very basic necessary knowledge that will enable the student to solve the problem and get the correct answer in order to carry on with the question at hand. Little effort is made to impart the procedure of solving this specific type of differential equation in a meaningful manner.

The approach in this case is centred around the procedure of achieving the solution of the differential equation. Given the differential equation, one could either propose that $\frac{d^2 y(x)}{dx^2}$ is substituted for m^2, $\frac{dy(x)}{dx}$ for m and $y(x)$ for 1 [Stroud, 1970], or at best that the student should recall the first order linear differential equation [MST204, 1981, 1992] and state that since the e^{ax} is its solution then, it would not be unreasonable to expect the solution to be of that form. Either way, the auxiliary equation is generated and upon finding the two roots of the quadratic equation, the appropriate form of the complementary function, depending on the nature of the roots, is then considered.

The rationale on which the form of the complementary function depends is not addressed at all. Furthermore, no effort is made to establish the relationship that exists between the number of solutions of the differential equation and the number of arbitrary constants and their relationship to the general solution. Consequently, concepts such as: linearly independent solutions, families of solutions and formation of the fundamental set are not explored and thus cannot be used in helping to make the process of achieving the solution a meaningful one for the student.

7.3.1 Why is this approach not useful ?

Such an approach is extremely procedural and also arbitrary. It is arbitrary because no reason is offered for the substitution of $\dfrac{dy(x)}{dx}$ by m^2 and so on, as well as why, in the case of a double real root, the form of the complementary function is different. The aim is focused on achieving the solution, even if this promotes rote learning. Additionally, the fact that there exists a mathematical package (Maple), which is capable of finding the solution of the differential equation accurately and efficiently, makes the whole process an obsolete one in this case.

Since the approach is so arbitrary, there exists an explicit conflict between this approach and our approach, that of the expository style. The phenomena of arbitrariness and unrelatability have been identified in the expository style and are viewed as problematic, see Chapter 6. These should be avoided under any circumstances.

Although our approach has a similar overall aim, to teach the process of solving our type of differential equation, it will be different since the bottom-up approach seems to have nothing constructive to offer to the student.

7.4 The declarative approach

This approach has been influenced by both the expository style as asserted by Ausubel et al., [1978], Ausubel and Robinson [1969] and SIMTA, our model of multiple teaching strategies. The aim is to present the process of solving the linear second order differential equation with constant coefficients in a meaningful manner. In other words, we are concerned with the creation of a meaningful framework which will offer the student the opportunity to integrate the conceptual aspect of the knowledge with its procedural aspect. In order to achieve this aim our objectives are as follows:

1. that the flow of teaching process is continuous, without any logical gaps. In other words, our objective is to divide the whole teaching process into interrelated steps such that the previous step acts as a precursor of the step that is to follow.

2. that the number of heuristics is kept as small as possible, since extensive use of them, especially in between steps, disrupts the logical flow of teaching as defined

earlier. Nevertheless it has to be acknowledged that in mathematics heuristics do exist and, in some cases, are inevitable.

3. that there should be explicit use of background knowledge, wherever possible, to introduce new knowledge.

This approach could be perceived as a bridging between the conceptual approach of the *'mathematics student'*, see §7.2, and the procedural approach of the *'engineering student'*, see §7.3. The knowledge that will be promoted in this approach could be described as declarative knowledge; a mixture of conceptual and procedural. The purpose of the conceptual knowledge is to provide an introduction to the topic and also to facilitate navigation by using concepts from both core and background knowledge on the domain of the subject matter.

The problem, as in the case of the bottom-up approach, is broken down into three stages; the stages used in solving the equation. These are: the stage of the auxiliary equation, the complementary function and that of the particular integral.

7.4.1 The formation of the auxiliary equation

At this point the procedural objective is to form the auxiliary equation. Conceptually, the objective is to provide a framework to support the flow of teaching the procedural objective so there will be no breaks in the process of teaching. This will involve providing adequate reasoning why the solution, in a linear second order ordinary differential equation with constant coefficients, is a function; followed by the identification of the function that would be the solution.

By combining knowledge of fundamental concepts from

        a) the core knowledge,

        b) the use of examples that require only background knowledge and

        c) analogies with background knowledge,

the student's understanding of the formation of the auxiliary equation will be furthered.

The procedural objective of forming the auxiliary equation falls within the global objective which is to find the general solution of the given differential equation. This procedural objective is a local long-term objective, since realisation of such an objective requires the successful completion of intermediate objectives. The first intermediate objective involves understanding the fact that the *form of solution*, for this type of equation, is in the *form of a function* of the *independent variable*.

This first step is the result of a situation when asked to solve an equation, about which very little is known. Recalling analogous situations from background knowledge, the first intermediate objective is established. For example, recalling the case of the quadrant equation which is then reduced to that of the quadratic equation through a substitution. This is an established problem solving technique, reduction of order which has also been seen in other topics. Consider polynomial functions. Looking for the solutions of a fifth order polynomial, the initial aim is to find a first solution and, then using that solution, reduce the order of the polynomial to one that can be solved using established methods. To find that solution required close inspection of the polynomial in order to find properties of the solution, constraints etc. For example, if the polynomial is of the homogeneous form then that means one solution is $x = 0$. If a term contains a square root, then only positive discriminants can be accepted as solutions of the problem.

To achieve the intermediate objective the following three methods could be utilised:

1. The use of examples that utilise only knowledge of integration and differentiation. For example, by offering the student examples of simple forms of differential equations, either general or specific ones, the student should be in a position to realise that the form of solution in the case of the differential equation is that of a function.

2. Another method involves realising that the form of the solution is dependent upon the form of the variable. The variable in our case is a function and its derivatives. Derivatives are operators, which operate on functions and result in functions; thus our solution is of the form of function.

3. Finally, should any of the above methods fail to deliver the expected result we are left only with the method of definition where the answer is explained to the student.

Establishing that the solution is a function is not sufficient in forming the general solution. Thus, further information is required which leads to the next intermediate objective: to find out which function is the one that satisfies the particular type of differential equation. The objective here is to look into the equation and see if there are any clues that would reveal any more information about the identity of the function. For this two methods are available.

a) Using appropriate examples that draw only from background knowledge, the student could identify that the function has the property of itself and its derivatives differ only by a constant. Obviously, the factor that yields the property of the solution is the operator derivative. To draw the student's attention to this factor the method of analogy is used by giving appropriate examples from the background knowledge. For example, the student could be given a logarithmic equation for which the values of the unknown can be only positive numbers since the operator is the logarithm which is only defined in R^+. From this example, the relevance of the operator which operates on the unknown is drawn to the student's attention. Consequently, by looking for the operator in the differential equation, which is the derivative, further properties of the solution could be established.

Having established that the key to the problem is a property of the function regarding differentiation, use of appropriate examples of the differential equation will be used to reveal the property of the function. These examples will only require knowledge that the student already possesses.

b) Alternatively, if the examples prove inadequate for the particular student to discover the property of the solution, the logic of the steps described above could be explicitly told to the student.

Having established that the solution is of the *form of a function* and its property is that *itself and its derivatives differ only by constant multiples*, the final intermediate objective is to identify the function. There exist four types of functions: *polynomial, trigonometric, exponential* and *logarithmic*. Of these, only one satisfies the property. The task is now reduced to that of identifying that function i.e., the exponential function.

This objective can be satisfied using two methods: the investigative method and the definition method.

a) In the case of the investigative method the student will be called to identify all known functions and subsequently to identify the one that satisfies the condition.

b) If the above method proves not so successful then, again, the answer will be offered alongside its justification.

From that point, up to the stage of complementary function, the whole problem becomes a procedural one, where all that it is required is to substitute the exponential function into the differential equation, carry out algebraic operations and put the result into the quadratic equation. All these operations can be very well performed by Maple.

7.4.2 The problem of complementary function

The auxiliary equation, which is a quadratic equation in x, is obtained by substituting the exponential function, e^{ax}, into the given differential equation. The auxiliary equation has two roots for x which in turn means that there are two solutions for the differential equation, Y_1 and Y_2. Following our pattern of investigation, the current step in relation to the overall objective is compared. In other words, it is necessary to investigate whether the two solutions, Y_1 and Y_2, mean that the general solution has been found. This question leads to the next step, which in the case of the homogeneous form of the equation is also the final one, the complementary function. The short term objective, in this case coincides with the overall objective i.e., to find the complementary function.

One problem here is that it is not always possible to provide a link between core and background knowledge. That is, the concepts in the core knowledge cannot be introduced as different instances of existing concepts in the background knowledge, (progressive differentiation), nor can they be viewed as augmentations of them (integrative reconciliation). The procedures involved in linking core to background knowledge will be discussed later. In this case, as it will be demonstrated, there is a need to introduce new concepts that are relevant to the topic of differential equations.

An account is given below of the analyses of the topic of complementary function and an explanation of why the first two analyses had to be rejected. The first analysis could be seen as that favoured by the *engineering student* (see §7.1) which, nevertheless failed to support the expository style. Although the second satisfied the teaching aspect it

failed on the mathematical aspect. Thus it was rejected. Finally, the third attempt should be perceived as a bridging effort in satisfying both teaching and the mathematical aspects.

7.4.2.1 The *reverse engineering* approach

This approach could be characterised as a *reverse engineering* approach since it derives from *engineering student* teaching. It is one step short of the *engineering student* teaching in the sense that the final form of the complementary function has to be worked out rather than given. In the *engineering student* approach see §7.1, all that is required is to select the appropriate form of the complementary function based on the nature of the roots of the auxiliary equation.

If the main objective in this case is to work out the form of the complementary function the following could be set as objectives:

> 1. ensure that the roots are distinct
>
> 2. ensure that the roots are real
>
> 3. form the general solution.

The first two objectives form the necessary and sufficient conditions that have to be satisfied if two solutions of a differential equation of our type could be considered as forming a fundamental set and thus be able to form a general solution (see §7.2).

The first two objectives are to be carried out by the procedure *Derive two different and real solutions* (p13), whilst the last objective will be carried out by the procedure *Form Complementary Function* (p42). In this case all the procedures are carried from left to right, see figure 7.1.

The procedure *Derive second solution* (p22), see figure 7.1, ensures that if we get two solutions that are not distinct then through procedure (p23) to (p31), see figure 7.1, we do get two distinct solutions. Likewise, in the case where the solutions are not real, the procedures (p32) to (p41) are activated to ensure that the solutions of the differential equation are of real nature, see figure 7.1.

Find Complementary Function (p12)

Derive two different & real solutions (p13)

Form Complementary Function (p42)

Form Y_1 & Y_2 (p14)

Test Y_1,Y_2 (p17)

Test if different (p18)

Apply properties of solution (p43) to Y_1 & Y_2

Subs a_1 into $y=e^{ax}$ (p15)

Subs a_2 into $y=e^{ax}$ (p16)

Examine if $\dfrac{Y_1}{Y_2} \neq constant$ (p19)

Apply r_1 of linearity (p44)

Apply r_2 of linearity (p48)

Assume a^*y_1 is a solution (p45)

Test (p46) assumption

Assume $a^*y_1 + b^*y_2$ (p49) is solution

Test (p50) assumption

Test if Y_1, Y_2 are real (p20)

Derive second real solution (p22)

Examine if I not present (p21)

Determine real solution (p32)

Assume $y=e^{(ax+h)}$ (p23)

Determine second solution (p24)

Substitute assumption into equation (p47)

Substitute assumption into equation (p51)

Combine solutions (p33)

Apply properties (p25)

Determine y_2 (p27)

Find sol1 (p34)

Find sol2 (p38)

Apply r_2 of linearity ($y_1 \cdot y_2$) (p26)

Expand exponential (p28)

Convert into Taylor series (p29)

Simplify (p30)

Apply r_2 of linearity ($y_1 + y_2$) (p35)

Apply r_1 of linearity (p36)

Apply r_2 of linearity ($y_1 \cdot y_2$) (p39)

Apply r_1 of linearity (p40)

Take limit (p31)

Multiply (p36) by $\dfrac{1}{2}$ (p37)

Multiply (p40) by $\dfrac{1}{2*i}$ (p41)

Figure 7.1 : A complete analysis of the steps involved in attainment of the complementary function

As stated earlier, the way these procedures (see figure 7.1) were originally thought of reflected an analysis of the way an expert would solve our type of differential equation. However, it is impossible to support the flow of teaching for these procedures as in the case of the auxiliary equation. This is because the knowledge that could justify the existence and the sequence of these procedures is associated with core knowledge, i.e., linear second order ordinary differential equations with constant coefficients. For example, concepts such as the linear independence of solutions and formation of fundamental set, are not encountered in background knowledge, that is algebraic, trigonometric or logarithmic equations. In fact they are not even encountered in first order differential equations since there is only one solution in this case. These concepts will be used in cases such as third order or higher order differential equations where the solutions of the equation are many and thus require conditions for forming the general solution. To demonstrate our point further let us select a procedure and try to present the flow of teaching.

Consider the procedure (p18), *Test if different*, see figure 7.1. The objective of this procedure is to ensure that the two solutions of the differential equation are linearly

independent, otherwise they cannot form the general solution. In this case, the students only know that the solutions of the differential equation *must* be *distinct*. They are not, however, aware that we want to extend the definition of *'distinct'* into a stricter one, that of *'linearly independent'* (p19). Justifications for such a need appear because we will not be able to form the general solution. The number of arbitrary constants will be reduced to one, which is contrary to our expectations. Second order implies the need to integrate twice and therefore the presence of two arbitrary constants. If the test fails, i.e., the solutions are not linearly independent, the procedure, *Derive second real root*, (p22), see figure 7.1, is activated in order to find the second linearly independent solution.

Unfortunately, because of the way the material is structured, the explanations mentioned cannot be used as the student knows nothing about the form of the general solution or about arbitrary constants. In other words, to use these justifications one has to be aware of the theoretical considerations, that is, that the form of the general solution in the case of the differential equation involves two arbitrary constants. But following the way the procedures are laid out, that will not occur until the procedure *Form complementary function* (p40) is reached, see figure 7.1. Thus, to explain the current situation it is necessary to utilise a result of a procedure that will happen at a later time.

This being the case, it is apparent that our model of teaching is not satisfied for two reasons. Firstly, the flow of teaching is not a continuous one; previous procedures do not act as precursors of the forthcoming ones. There exist gaps and unanswered questions, and moreover no background knowledge is used. The gaps/discontinuous flow means that, while looking for the general solution, the student's attention suddenly turns to the properties of the two newly found solutions concerning the general solution. That would not appear so strange if something were known about the form and nature of the general solution of the given differential equation. However, here the student is told that the solutions of the differential equation have to be linearly independent or even distinct. According to a student's experience, in a quadratic equation the roots (2,2) count as two roots. Why not here? And why linear independent as well?

This problem unfortunately, cannot be addressed if the procedures in figure 7.1 are to be used. Moreover, a fundamental point in our model of teaching, use of alternative strategies, cannot be satisfied if the knowledge is to be arranged in such a manner. In

particular, the methods of examples, analogy and, possibly, that of investigation, are not used by default. In fact the only method that can be used is that of definition, where the students are either asked or told.

7.4.2.2 A revised approach

From the above approach, then, two limitations have been identified. Firstly, limited utilisation of the background knowledge with the result that the students are not as active as preferred, and, secondly, lack of support for multiple teaching strategies. In order to overcome these two problems the following approach is proposed.

Since the difficulties arose from the problem of requiring information obtained at a later stage in order to support current activities, the idea was to reverse the activities. Consequently, it was anticipated that the conflict would be resolved. Using the form of the general solution to demonstrate and explain the need for the solutions Y_1 and Y_2 to be linearly independent, required finding the means of making the student aware of the form of the general solution and the relation between the order of the differential equation and the number of arbitrary constants. Based upon that, it would be possible to justify our actions concerning the concept of linearly independent solutions. To achieve these the following two examples were proposed:

$$\frac{d^2 y(x)}{dx^2} = 0$$

and

$$\frac{d^2 y(x)}{dx^2} = \frac{dy(x)}{dx}$$

The student would be able to solve the first example by utilising background knowledge, that is integration. All that is required is to integrate twice. Thus the answer of this question is $C_1 + C_2 x$. For the second example, the student again can produce a solution by utilising background knowledge, that is integration and first order differential equations. By integrating both sides, the equation is reduced to a first order one which the student knows, and thus can be solved using the appropriate techniques. In this case the answer is $C_1 + C_2 e^x$.

Consequently, there were two examples of a linear second order ordinary differential equation with constant coefficients and the students were in a position to find the general solution utilising only background knowledge. The first objective was to establish the relationship between the order of the differential equation and the number of arbitrary constants present. The second objective would then have been to establish the connection between the solutions Y_1 and Y_2 and their role in the general solution.

For the first objective, the first example would be used in the first instance, as it is the easiest form of our differential equation. This would be done to draw the student's attention initially and demonstrate it, the first objective, again by utilising the second example. Also first order differential equations (background knowledge), could be recalled to show that in this case the number of arbitrary constants is one. Moreover, the first objective could be further expanded by using one term differential equations of a higher order, e.g., $\dfrac{d^3y(x)}{dx^3}=0$, $\dfrac{d^4y(x)}{dx^4}=0$. In this case the number of arbitrary constants would have been 3 and 4 respectively.

For the second objective, Y_1 and Y_2 being present in the general solution, it was planned to utilise only the second example because in the first there were problems relating it to the solutions Y_1 and Y_2. Even in the second case only one of the two solutions is present. In this case, the student could be asked to solve the equation (or alternatively the system could solve the equation) and then ask the student to find Y_1 and Y_2. Thus, the student could 'see' that Y_1 and Y_2 are present in the general solution.

The advantages of this approach is achievement of the teaching objectives:

    a) the students are involved in the evolution of the lesson and thus draw their own conjectures,

    b) a wider range of methods to employ and finally,

    c) utilisation of background knowledge.

However, as Alexandrou [1992] claims, the examples are not enough, especially for the second objective. That is, these examples are only special forms of the differential

equation in question, thus it is potentially dangerous to base a whole generalisation on the basis of only one example. Also, the property of linearity is not mentioned at all, which Alexandrou [1992] believes is a drawback. Moreover, if the example $\dfrac{d^2 y(x)}{dx^2} = 0$ is to be used, its solution is, $C_1 + C_2 x$; on the face of it this appears like a polynomial and not of the exponential form as it was assumed at the beginning.

Finally, by using these two examples and in general this approach, there seems to be a discontinuity; that is, two solutions Y_1 and Y_2 had been found and then turned up to look upon special forms of examples to establish the form of general solution. This is a clear case of a breaking the flow of teaching, which again is not desirable, as stated in §7.3.4. Although there is a connection between the steps, this only emerges at a later point, thus this approach appears to fall into the same trap as the previous approach.

7.4.2.3 A declarative approach in teaching the complementary function

The aim of this final approach to structuring the knowledge is twofold. First, to overcome the problem with the mathematical aspect of the work as stated in the previous approach. Second, to support the continuous flow of teaching as well as to demonstrate explicitly the relationship between the partial solutions Y_1 and Y_2 and the general solution, i.e., to demonstrate why the general solution is a linear combination of Y_1 and Y_2. These objective should be considered in addition to ones stated earlier, such as, support of the SIMTA model, involvement of the student and utilisation of background knowledge. The following three assumptions about the student's knowledge of the topic are required for this approach to operate smoothly:

1. The term linear should be familiar to the student and consequently that the properties of linearity hold for the solutions of the homogeneous equation of this form of differential equation.

2. The students are aware of the term, families of solutions (if that is not the case then it can demonstrated from one of the properties of linearity or recalling first order differential equations.).

3. The students are also aware that the solutions of the differential equation have to be distinct. The solutions are distinct in the sense that they are unequal. However, even this assumption can be justified using the concept of families, and since the objective

is to find all possible solutions of the differential equation, once a member of a family is found, there is no further interest in that family.

Given the above assumptions an analysis of this approach may be stated here. Two solutions Y_1 and Y_2 have been found. Since they are solutions of a linear differential equation, there are certain conditions that apply to them. The students are also aware of their properties, which are that since Y_1 is a solution then $A\,Y_1$ is also a solution and also that since Y_1 and Y_2 are solutions, then so is $A\,Y_1 + B\,Y_2$. In other words, a way of combining the two solutions has been established.

The next step is to see if any of the solutions is also the general solution. That is achieved by looking at the definition of the general solution, which was also assumed to be known to the student upon introduction of the differential equation in general. Being satisfied that none of the solutions is the general solution, the next task is to find it. However, by analogy, an equation of second order has exactly two solutions. In this case there already exist two solutions which are distinct. Therefore, the general solution is not another independent solution but a combination of the existing solutions. In fact it is a linear combination of the existing solutions.

Finally, it is important that the student knows that the solutions forming the general solution need to be both distinct and linearly independent. This goal can be achieved by the method of definition. Such a need can be demonstrated by offering the student an example (see b2 in §7.3.4.3) where the roots of the auxiliary equation are equal and thus the solutions linearly dependent. However, by solving the example using a direct double integration, a solution different to the expected one is obtained. Alternatively, using the concept of a family of solutions in conjunction with the linear properties it can be deduced that the solutions have to belong to different families of solutions. Otherwise, the second condition of linearity which states that if there are two solutions then any linear combination of these is also a solution would be reduced to the first condition. This states that if there is a solution then any constant times the solution is also a solution. For example, if $Y_1 = 2\,x$ and $Y_2 = 3\,x$ then, by the second condition, any linear combination of them is also a solution, i.e., $5x$ is also a solution. But that could be achieved via the first condition where the constant for Y_1 would have been $\dfrac{5}{2}$

and for the second one $\dfrac{5}{3}$. Thus, the second condition becomes redundant, which cannot be true as the conditions are independent.

7.5 Summary

In this chapter the topic of solving linear second order ordinary differential equation with constant coefficients was analysed with two factors in mind: that Maple is capable of solving the equation and the fact that the teaching sequence had to support the expository style.

Through this analysis, it has been shown how it is possible to teach a topic, which is inherently procedural, in a declarative way using a Computer Algebra System such as Maple. Moreover, the Computer Algebra System is central to such teaching. Here there is a shift of emphasis from

1. understanding the procedure involved to understanding the concepts that drive the procedure

2. and demonstrating how problem solving techniques that were used in background knowledge are still applicable in this new piece of knowledge.

To achieve this objective, the solution of linear second order ordinary differential equation with constant coefficients was analysed from a number of different perspectives,

- the *mathematics student,*

- the *engineering student,*

- and the *declarative approach.*

The first *two* approaches were rejected. The first one was rejected on the basis that it was highly conceptual and moreover, there was a shift of attention from that of learning to solve the differential equation to that of understanding the concepts and how they relate to each other. Therefore the objective of a such an approach is different from the one adopted here.

The second approach was rejected, although had a similar overall aim to ours, as it was shown that it was highly procedural and thus it was deduced that it had nothing constructive to offer to the student.

The third approach, the declarative one, was the one that is adopted here as it satisfied all the criteria, understanding the concepts that drive the procedure, use of Maple and use of background knowledge. It is this approach that combines both procedural and conceptual aspects in such a way that Maple can be used in teaching the solution of the differential equations in a novel way.

Finally, this analysis has identified the concepts from both core and background knowledge that have to be included into the declarative knowledge of TeLoDe so that the expository style can be supported. Chapter 8, drawing on the results of this analysis will formalise the structure and links between these concepts and demonstrates how that formalism will support the function of the teaching strategy module that is to be included in TeLoDe.

CHAPTER 8

Chapter 8: Definition of Knowledge Representation and Methods in TeLoDe

This chapter puts forward the structure that the knowledge representation ought to have in order to support the teaching of the linear second order ordinary differential equations with constant coefficients, as discussed in Chapter 7. The concepts from both core and background knowledge as well as their links and relationships are identified. This explicit description will help to identify and formalise the methods, to be implemented in TeLoDe, that will support the teaching of the solution of the aforementioned equations.

In §8.1 an outline of the structure of the knowledge base where the terminology used in describing the elements of the knowledge representation and their links is given. In §8.2 the case of the auxiliary equation is examined to identify the procedures which are associated with declarative knowledge. For each of these procedures, its educational objective is stated, the concepts from both core and background knowledge are identified and the methods that could be used in teaching are also identified and described. In §8.3, the case of the complementary function is analysed in the same way as for the auxiliary equation. Again the procedures associated with declarative knowledge are further analysed to identify the concepts and the methods that could be used in supporting the teaching. A summary of this chapter is given in §8.4.

8.1 An outline of the structure of the knowledge base

The rationale of the knowledge base is to structure a network so that it will facilitate the teaching of solving linear second order ordinary differential equations with constant coefficients in a meaningful manner, as discussed in Chapter 7. Achievement of this objective depends on the use of *conceptual* knowledge. Recapitulating, the approach taken is to view the procedural aspect as a by-product of *understanding* the problem and the concepts associated with it. The emphasis, therefore, is placed on the conceptual understanding of the topic and how these concepts are to be used in order to make, first, the existence and second, the sequence of the procedures meaningful and logical.

Before describing the structure of the knowledge base and the reasons for the adopted structure, it is important to state some definitions of terms used for the categorisation of concepts and their relationships. (examples are given later, §8.2.1)

1. A *procedure* is an activity that has to be performed as part of a solution of the problem.

2. A *primary concept* is a concept which is directly relevant to a particular procedure.

3. A *kind* is an attribute which expresses what aspect of the concept is relevant to the procedure in the particular case.

4. A *secondary concept* is a concept which is not directly linked to a procedure but which has links to primary and other secondary concepts. (Their relevance is associated with the teaching of the procedure.)

5. A *concept-to-concept link* represents a particular type of relationship (e.g., generalisation, specialisation, depends_on, examples) and has a kind which expresses the context in which that relationship is relevant.

6. A *filter* is a concept that provides the procedure with a context.

The overall approach is to view the process of solving the equation as having three components: *auxiliary equation, complementary function,* and *particular integral* (in the case of the non-homogeneous problem). Figure 8.1 shows the procedures and their order.
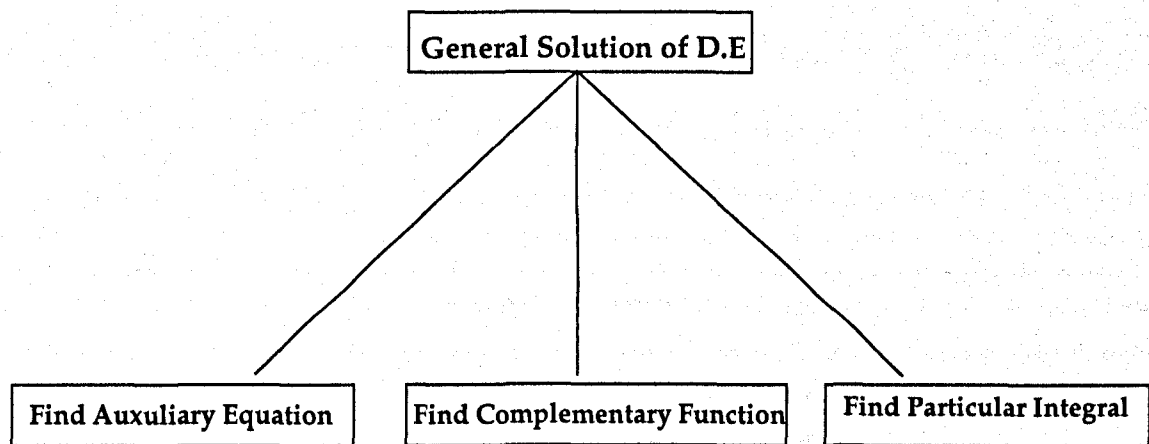
Figure 8.1: The procedures involved in attaining the solution of a differential equation

Although figure 8.1 shows the procedural analysis of the problem, there are associated educational objectives with each procedure. In the case of the auxiliary equation, the educational objective is to explore the new form of equation and gather as much information about it as possible and so be able to determine the general solution. Indeed, the organisation of the procedures involved in the case of the auxiliary equation reflects exactly that objective (see figure 8.2). It should be noted that the

educational objective is broken down into sub-objectives which are attached to sub-procedures.

In the case of the complementary function, the educational objective is to explore the concept of the general solution. That is, having formed, solved and obtained two solutions, there needs to be further analysis of the behaviour of these two solutions, their properties, their relationship with the general solution and the condition that has to be fulfilled if the relationship between the two solutions and the general solution is to hold.

Finally, it should be noted that the procedure *Find Particular Integral* is not explored here, because the focus is on the homogeneous form of the differential equation. This involves the *Find Auxiliary Equation* and the *Find Complementary Function*.

8.2 The case of the auxiliary equation

In the case of the auxiliary equation, the concern is twofold: the formation of the auxiliary equation as required by the procedural aspect of the knowledge and, moreover, the understanding of the genesis of this equation and its relation to the differential equation.

Figure 8.2 is an overview of the tasks that will have to be accomplished in order to find the auxiliary equation. It depicts the arrangement of the tasks as indicated by the declarative approach taken.
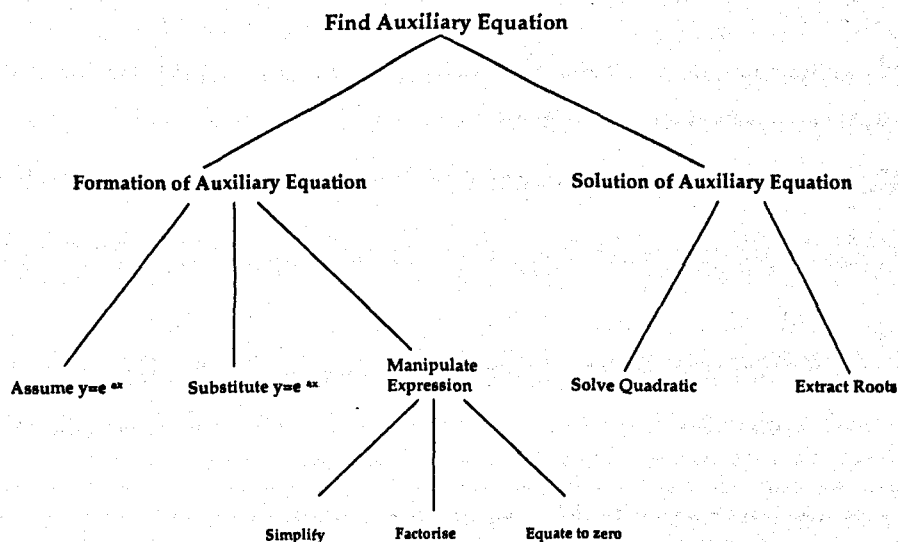
Figure 8.2: A breakdown of tasks for attaining *Find Auxiliary Equation*

In the following sections each of the above tasks including their further subdivisions (where applicable), will be examined in turn in order to distinguish between those which are clearly procedurally-oriented and those that are declaratively-oriented. Concepts used in the declaratively-oriented tasks will be categorised in accordance with the definitions given earlier in this section.

The procedure Find Auxiliary Equation is initially divided into two parts: *Formation of Auxiliary Equation* and *Solution of Auxiliary Equation* (see figure 8.2). The first part deals with the question of genesis of the auxiliary equation, whereas the second part addresses the procedural problem of solving the auxiliary (quadratic) equation. It is in the first part that a declarative approach to this problem is possible. As far as the second part is concerned, the approach adopted is very procedural.

Achieving the first part, *Formation of Auxiliary Equation*, requires its sub-procedures to be accomplished. The overall educational objective here is to identify the form of the solution, or, in other words, to enable the student to have a feeling for what the solution looks like, as discussed in Chapter 7. The next three sub-sections describe the concepts and the structures necessary to achieve this educational objective.

8.2.1 What is the form of the solution?

The objective at this stage is to investigate the *form of the solution* and, in order to achieve this, three methods are used: the *method of examples, investigation* and *definition*. However, the problem is not merely confined to investigating the form of solution, it extends to ensuring the student can comprehend the aim. To this end, the system has two methods for coping; that of analogy and definition. (Note, the term 'definition' in this instance is used in a rather broader sense. In this case the meaning of the definition is to provide the answer to the student in a meaningful manner as stipulated by the expository style.)

When the student is given a linear second order ordinary differential equation with constant coefficients and is asked to find its form of solution, the following concepts could be triggered:

a) the concepts that are immediately associated with the specific form of equation, as well as

b) concepts that are associated with generalisations of this specific equation and, furthermore,

c) the generalised concept with other specific forms which are, nevertheless, potentially meaningful to the student.

For example, in linear second order ordinary differential equation with constant coefficients, a general concept could be the abstract concept of equation. Through the generalised concept, the student could then turn to other specific forms of equations such as algebraic equations, trigonometric ones, etc.

A selection of concepts that could be utilised for executing this procedure is as follows:

> *linear second order ordinary differential equation with constant coefficients,*
>
> *derivative,*
>
> *solution,*
>
> *integration,*
>
> *function.*

The first concept is considered to be a core knowledge concept, whilst the others are considered to be background concepts.

To explain the rationale for the structure of the knowledge base as depicted in figure 8.4, a brief overview of the methods and their aims is presented: Through the *method of definition* the student will just be told that the form of the solution is that of the function. Utilising the *method of investigation*, the first objective is to establish that the form of solution depends on the form of the unknown. The rationale of this approach can be elicited from the student via the *method of analogy*. That is, presenting the students with background knowledge that is potentially meaningful to them, e.g. a linear algebraic equation in which the students are asked to identify the form of the solution. Since the type of this equation is known to the student, the form of the solution is also known to the student, as well as the fact that there exists a dependence between the form of the solution and the form of the unknown. Finally, using the *method of examples*, the student is presented with an example of a simpler form of the differential equation in hand and is asked to solve it in order to establish that the form of the solution in this type of equation is that of a function.

From the above description it is clearly necessary to structure the knowledge to support the primary concept of solution, as seen through the perspective of form. The structure should be flexible enough to assist teaching in both core and background knowledge. In this case support of the background is in the form of being able to shift into another form of equation which is potentially meaningful to the student.

Form of solution of the differential equation is: <u>function</u>
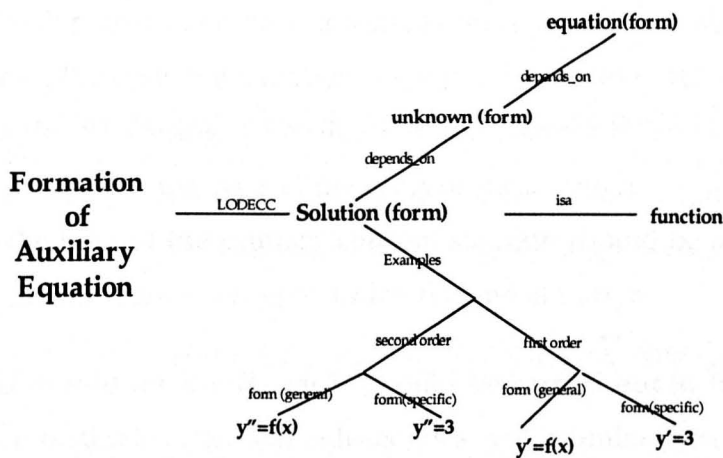
Figure 8.3: Declarative knowledge associated with procedure *Formation of Auxiliary Equation*

Moreover, it is imperative that once in the background knowledge, the system would be in the same position as if it were dealing with a core knowledge problem. In other words it would have access to a definition, examples and means of logically arguing for its value of definition. Given these considerations, figure 8.3 represents the structure of knowledge needed to support the procedure *Formation of Auxiliary Equation*.

This procedure has to be performed, as part of the solution is *Formation of Auxiliary Equation*. The primary concept is *solution* and the kind is *form*. The secondary concepts are : *unknown* and *equation*. Their respective primary to secondary concept links are: for unknown the link is *form* and for equation the link is *form* again. The values of the whole structure depend on the value of the filter. The filter in this case is *LODECC* (Linear Ordinary Differential Equation with Constant Coefficients), which indicates

the type of equation and thus provides the appropriate values for the concepts. Specialisation and/or generalisation is achieved by simply changing the value of the filter.

In the case of TeLoDe, however, when the concept of linear second order differential equation with constant coefficient is represented in the knowledge base, the form of the solution is that of the exponential. Thus, to reach the higher level, that of a function, it is imperative that a further attribute is attached to the concept of solution, as an attribute to the attribute of form. That is the attribute *type*. Although, type is anticipated to cause problems, its addition is deemed necessary if the generic nature of TeLoDe is to be demonstrated when questions from TeLoDe are automatically generated. In fact, during the system evaluation, such problems were caused and are further discussed in Chapter 10. During the evaluation, see Chapter 10, some of the subjects queried which level "what is the type of the form of the solution ..." was referring to. Thus in figure 8.3 the kind of the primary concept solution should be form(type) and the same holds for the secondary concepts, unknown and equation.

This procedure, see figure 8.3, could be carried out in three different ways, using the three methods. The next sub-sections will examine how the methods are represented and which concepts are used to support that particular method.

8.2.1.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship between the concepts. In Chapter 7, it was shown that the form of the solution depends on the form of the unknown, which in turn depends on the form of the equation. The latter is required to assist TeLoDe in providing an explanation of the rationale of the structure supported in the investigation method. That is, given an algebraic equation, for example, for which the solution is known, the student could be asked, step by step, to identify the form of the equation, then the form of the unknown and thus identify the form of the solution which is known to him. Consequently, the student could be in a position to draw the link between the form of the solution and the form of the unknown. This structure is of great importance, especially when needed to demonstrate the rationale of the investigative method, by analogy, through the use of an example from background knowledge.

8.2.1.2 The method of examples

The method of examples is denoted by the examples link which links the primary concept with the examples that exist under the particular filter. In this case, the examples are further divided based on the form that is requested. If a general form is requested then for the case of the linear second order ordinary differential equation with constant coefficients and for the procedure Formation of auxiliary equation, the example $\frac{d^2y(x)}{dx^2} = f(x)$ will be selected. If a specific form is requested then $\frac{d^2y(x)}{dx^2} = 3$ will be selected. If it is required to use background knowledge, say first order differential equations, then by requesting a general form $\frac{dy(x)}{dx} = f(x)$ will be chosen while requesting a specific form $\frac{dy(x)}{dx} = 3$ will be chosen.

8.2.1.3 The method of definition

The method of definition is indicated by an *isa* relationship. It results in TeLoDe explicitly giving the answer to the problem. Usually, this method will be accompanied by a canned text formulated in order to meet the criteria of our model of teaching.

8.2.2 Which are the properties/constraints of the solution?

Having established the solution is of the form of function, the immediate step is to check with the global objective: find the general solution. In this case, the information known is assessed to see if it is sufficient to achieve the objective. In our case, the fact that the form of solution is that of a function is not sufficient as there are a number of legitimate candidates, such as the exponential, polynomial, trigonometric or logarithmic. Since the information gathered is inconclusive, it is imperative that more information is collected in order to identify the solution.

This objective is addressed by the following two educational objectives: *properties/constraints of the solution* and *identify solution*, which are attached to procedure *Assume* $y = e^{ax}$ (see figure 8.4). Procedure *Assume* $y = e^{ax}$ is a sub-procedure of the procedure *Formation of Auxiliary Equation*, alongside sub-procedures *Substitute* $y = e^{ax}$ and *Manipulate Expression*. These sub-procedures represent the procedural break-down of the problem. Of the three sub-procedures only the procedure *Assume* $y = e^{ax}$ has

declarative significance. As far as the other two procedures are concerned the tasks involved are clearly procedural and, as such, will not be dealt with.

A selection of concepts used for the execution of this procedure is as follows:

linear second order differential equation with constant coefficients,

solution,

derivative,

function,

differentiation,

operators.

The first concept is considered to be from the core knowledge whilst the rest are assumed to be from the background knowledge.

This section will deal with the first educational objective: *properties/constraints of solution.* At this stage it is known that the solution for the given form of differential equation is a function. Since this is not conclusive, further investigation is needed to help in identifying the solution.
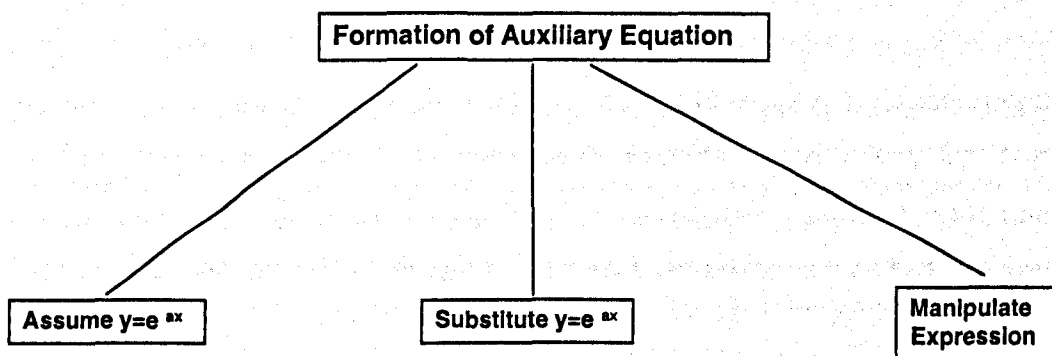
Figure 8.4: A second level analysis of the procedure *Find Auxiliary equation*

To accomplish the objective the following three methods are used: by *examples*, by *investigation* and by *definition*. The objective of structuring the knowledge representation is not confined to supporting the methods aforementioned, it extends into supporting the rationale for such an approach by either telling the student explicitly (*method of definition*) or by providing a potentially meaningful environment for the student (*method of analogy*) where this approach is familiar.

To demonstrate the rationale of the structure of the knowledge representation for the objective, *properties/constraints of solution*, a brief overview of the methods and their aims is presented. Under the *method of definition*, the student will be told that the property that the function has to satisfy is that of itself and its derivatives being the same function. Under the *method of investigation*, the first objective is to establish that the property of the solution depends on the properties of the operator, in this case the derivative, where the property, in turn, depends on the form of the equation. The sequence of this dependency can be demonstrated to the student via the *method of analogy*. The student is reminded of a potentially meaningful case, e.g. the logarithmic equation, and is asked to identify the operator which operates on the unknown and then state the constraints that are inherited by that operator. The operator in the case of the logarithmic equation is the logarithm and the constraint is that the argument must be a positive number. Consequently, in the case of the logarithmic equation all negative numbers and 0 are excluded from the search for a solution. Finally, using the *method of examples*, the student is presented with an example of a simpler form of a differential equation in hand and is asked to rearrange it in such a way that the property is apparent, i.e., that the function and its derivatives are the same.

Given these considerations figure 8.5 represents the structure of the knowledge that is required by the computer tutor in order to support the procedure *Assume* $y = e^{ax}$.

The structure presented in figure 8.5 is flexible enough to support teaching in both core and background knowledge. As in the case of *Formation of Auxiliary Equation*, it is imperative that teaching in the background knowledge is equipped with the same facilities as teaching in core knowledge. That is, the system has access to equivalent concepts, examples, and is able to argue logically for its conclusions.

In the case of the procedure *Assume* $y = e^{ax}$, the primary concept is *solution* and the kind is *property*. The secondary concepts are compound, *operator* and *equation*. Their respective primary to secondary links are as follows: for compound the link is *property*, for operator the link is *property* and finally, the link for equation is *form*. The filter in this case is *LODE* (Linear Ordinary Differential Equation). The concept compound, denotes the combination of equation and form which operate on the operator.

Property of solution (function) of the differential equation is:
its derivatives are equal
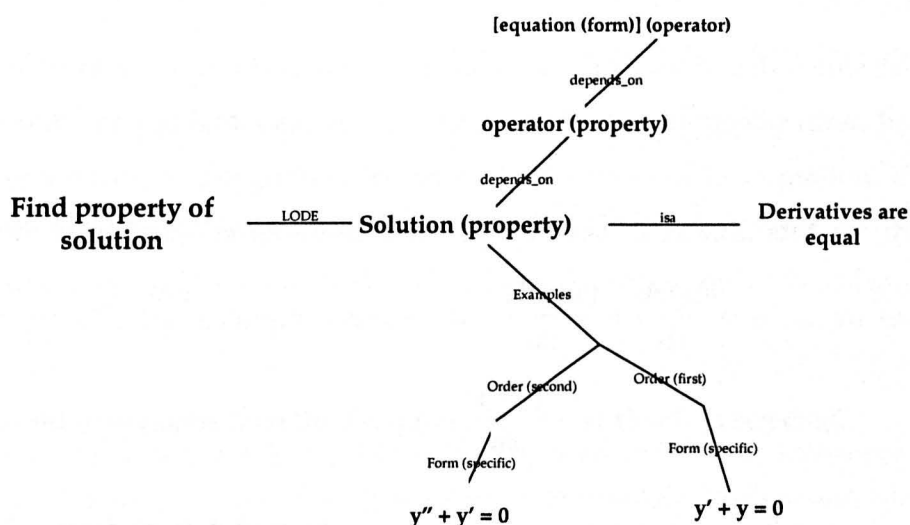
Figure 8.5: Declarative knowledge associated with the procedure *Find Property of Solution*

The procedure could be carried out in three different ways, using the three methods. An analysis of ways for representing the methods will be presented as well as concepts that support a particular method.

8.2.2.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship between the concepts. As discussed in Chapter 7, the properties/constraints of the solution depend on the operator. For the argument to be complete it is necessary to add the step of property of the operator. This is because the answer would only have been, in the case of the differential equation, a derivative if the operator dependency had been included. But what is expected in this case is the property of the derivative in this specific form of equation, which is that all derivatives of the function are equal. The same holds in the case of a potentially meaningful example, the logarithmic equation, where the operator is the logarithm, but the only way to get access to its property is via the step operator(property). Moreover, such an arrangement of the structure is of extreme importance when needed to demonstrate the rationale of the method of investigation. Using the *method of analogy,* and the example of the logarithmic equation, the student could become aware of the operator and its property and will consequently be in a

position to step backwards and identify the fact that properties of solution depends on the operator of the equation.

8.2.2.2 The method of examples

The method of examples is denoted by the examples link which connects the primary concept with the particular examples existing under the particular filter. In this case the examples are further categorised according to the order of the equation. If an example from core knowledge is required then for the case of LODE, and for the procedure *Assume* $y = e^{\alpha x}$, the example chosen is $\dfrac{d^2 y(x)}{dx^2} + \dfrac{dy(x)}{dx} = 0$. If an example from background is required then the example $\dfrac{dy(x)}{dx} + y(x) = 0$ is selected.

8.2.2.3 The method of definition

The method of definition is indicated by the *isa* relationship. It results in an explicit answer to the problem. In this case it will state that the property that the function has to satisfy is that its derivatives and itself are the same functions.

8.2.3 Which function is our solution?

In this section an analysis of the educational objective, identify solution, attached to sub-procedure *Assume* $y = e^{\alpha x}$ is presented. This represents the next logical step, following the educational objective: gather information about the solution.

To achieve the educational objective, the *method of investigation* and the *method of definition* are used. In this case the problem of rationale for the course of action taken does not exist as it is a consequence of the previous steps. However, the need for the student to comprehend is still present and, in this case, the system uses the *method of analogy*. The information gathered suggests that the solution is of the form of function and its property is concerned with differentiation. A selection of concepts that could be used to execute this procedure is as follows:

solution,

derivatives,

exponential,

properties,

functions.

All the items above are background concepts.

This information and the concepts encompassed are then represented in the knowledge representation. Through the *method of investigation* the objective is to identify the function that satisfies the condition set. Using the *method of definition,* the student will be told of the function that satisfies the condition. Given these considerations, figure 8.6 represents the structure of the knowledge that is required by the computer in order to support the procedure *Assume* $y = e^{ax}$ under the second educational objective.

Solution is: <u>exponential</u>
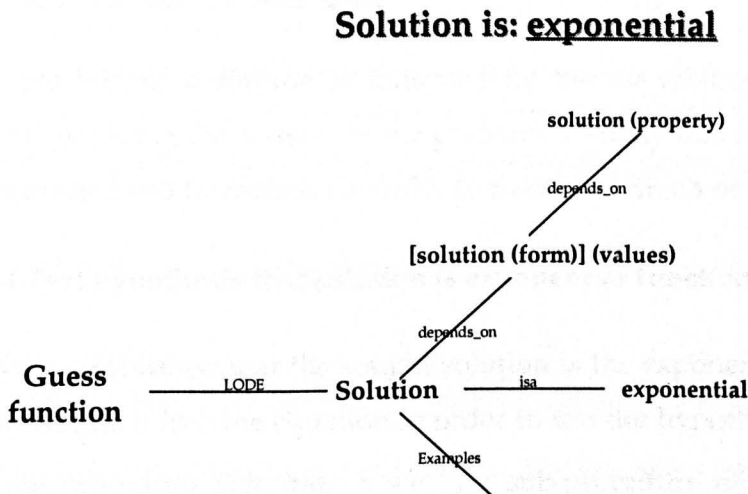
Figure 8.6: Declarative knowledge associated with the procedure *Assume* $y = e^{ax}$ (*Guess Function*)

The primary concept is *solution* and its kind is left blank as, this time, the search is for the solution rather than information. The secondary concepts are *solution(form)* and *solution*. Their main secondary links are, for the first one, *values*, and for the second *property*. Given these considerations figure 8.6 represents the structure of the knowledge required by the procedure in order to accomplish its task.

8.2.3.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship between concepts. The mathematical analysis, see §6.2, revealed how the solution had to satisfy the condition as set by the previous educational objective: derivatives are equal. In this case the problem is confined to trying to match the properties of the functions against

the required property. Whichever function satisfies the condition has to be the solution. To understand what is expected of this approach the *method of analogy* is used. The way the knowledge is represented allows the tutor to move into a potentially meaningful environment, e.g. an equation involving square roots. In this case the quantity within the root has to be positive if reference is made to the set of real numbers. Consequently the student can see the analogy between a known case, the square roots, and an unknown case, the differential equation. The concepts needed here are ones that denote the condition as well as the one that holds all possible candidates. In our case the condition is noted by the concept solution under the link property and the pool of candidates by solution(form) under the link values.

8.2.3.2 The method of definition

The method of definition is indicated by the *isa* relationship. It results in TeLoDe explicitly giving the answer to the problem. Usually this method will be accompanied by a canned text formulated in order to meet the criteria of our model of teaching.

8.2.4 Test hypothesis that solution is exponential function

Having established that the sought solution is the exponential function, it is necessary to substitute it into the equation in order to test the hypothesis. This task is carried out by the procedure *Substitute* $y = e^{ax}$, a sub-procedure of the procedure *Formation of Auxiliary Equation* (see figure 8.3). Substituting the $y = e^{ax}$ into the differential equation and carrying out the differentiations as indicated by the operators, yields an expression not containing derivatives. This expression is algebraic and can be dealt with since it involves knowledge from the background. These tasks are carried out by the procedure *Carry out Operations*, a sub-procedure of *Manipulate Expression* (see figure 8.7).

Manipulate Expression

Simplify   Factorise   Equate to zero
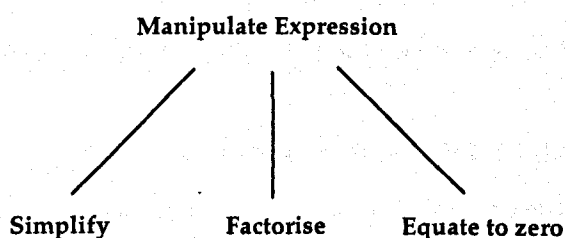
Figure 8.7: A third level analysis of the procedure **Find Auxiliary equation**

8.2.5 Solving the auxiliary equation

Successful completion of the procedure formation of Auxiliary Equation, allows the system to proceed to the next procedure, *Solution of Auxiliary Equation*.
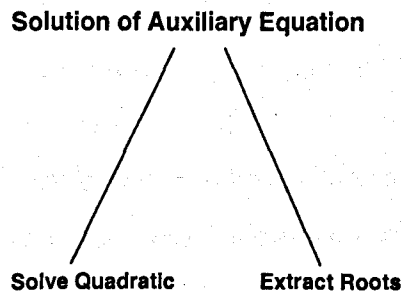
Figure 8.8: Analysis of the procedure Solution of Auxiliary Equation

The objective of this procedure is to solve the quadratic equation and find the partial solutions y_1 and y_2. Solution of the Auxiliary Equation is handled by the sub-procedure *Solve Quadratic*, (see figure 8.8) whilst the formation of the partial solutions is managed by the sub-procedure *Extract Roots* (see figure 8.8).

8.3. The case of the complementary function

Having found two solutions by completing the procedure *Find Auxiliary Equation*, the process of obtaining the general solution is halfway through. All that is necessary now, is to complete the final procedure, *Find Complementary Function*. As mentioned earlier, the educational objective of this procedure is to investigate the conditions and the properties of the general solution, i.e. to identify what is needed in order to form the general solution of the given differential equation. Consequently, the procedures requiring completion to attain the general solution are as follows:
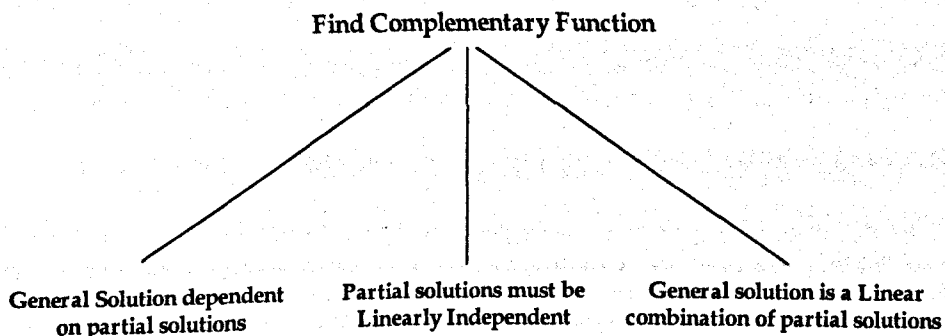
Figure 8.9: Procedures involved in attainment of **Complementary Function**

The educational objectives attached to the procedures are as follows. For the procedure *General Solution dependent on partial solutions*, the objective is to realise that, since there are 2 distinct solutions and the order of the equation is 2 (second order differential equation), then by analogy with the fundamental theorem of algebra there can be no more independent solutions. Consequently any more solutions that exist have to be expressed in terms of the existing ones. In the case of the procedure *Partial solutions must be Linearly Independent*, the educational objective is to find the condition which existing solutions have to satisfy in order to be chosen to form the general solution. In fact, the initial condition for the partial solutions was to be distinct but through this procedure it becomes apparent that this condition is not sufficient. Finally, in the case of the procedure *General solution is a Linear combination of partial solutions*, the educational objective is to establish the form of dependency between the general solution and the partial solutions.

It should be noted that all procedures depicted in figure 8.9 are declarative. Given these procedures and the aforementioned educational objectives, each procedure will be examined in order to demonstrate how the educational objectives are to be met and the concepts involved will be categorised in accordance with the definitions stated earlier in this section.

Before exploring the procedures, it is important to bear in mind the problem. The problem is to find the general solution of the second order differential equation with constant coefficients. In the first part, the auxiliary equation, it was hypothesised that the solution is of the form of a function. This had to satisfy the property that the function and its derivatives differ only by a constant and the solution was found to be the exponential function. Substituting into the differential equation and carrying out the appropriate algebraic calculations two solutions were found. Both of them were tested to see if they were the general solution. Since they were not, as they could not satisfy the criteria of the general solution, the problem of finding the general solution remained. This problem is tackled by the three procedures mentioned earlier. The first procedure to be performed is *General Solution dependent on partial solutions*, to which the educational objective of *finding the status of the general solution* is attached.

8.3.1 General solution dependent on partial solutions

The methods for accomplishing this procedure are: the *method of examples*, the *method of definition* and the *method of investigation*. Again, as in the case of the procedure *Find Auxiliary Equation*, the mechanism used to represent the knowledge has to be such that it could also be utilised for referring back to background knowledge as well as being able to explain the task. A selection of concepts which could be used to execute this procedure is as follows:

> *general solution*
>
> *linear second order differential equation with constant coefficients*
>
> *solution*
>
> *equation*
>
> *order.*

The first two concepts may be considered core knowledge whilst the others are considered background knowledge.

To explain the rationale for the structure of the knowledge representation as depicted in figure 8.10, a brief overview of the methods involved will be presented. Through the *method of definition*, the student will be told that the general solution is not a new solution and that it depends on the existing partial solutions. Using the *method of investigation*, the first objective is to establish that the general solution is a combination of the existing ones and not a new solution. The rationale of this approach can be elicited from the student via the *method of analogy*. That is, potentially meaningful knowledge from the background knowledge is used, e.g. a quadratic equation, and the student is then asked how many solutions exist for this equation. Since the student is familiar with this type of equation, the student knows it can only have two solutions and, should anymore exist, then they have to be expressed in terms of the existing solutions. Finally, using the *method of examples*, the student is presented with a simpler form of a differential equation, which the student is then asked to solve in order to establish that the general solution depends on the partial solutions.

equation (order)
depends_on

solutions (max_no)
depends_on

solutions (found)
depends_on

solution (status)
depends_on

General Solution
dependent on partial ——LODE—— general solution————isa———— dependent on existing ones
solutions   (status)

Examples

$$\frac{d^2y(x)}{dx^2} + \frac{dy(x)}{dx} = 0$$
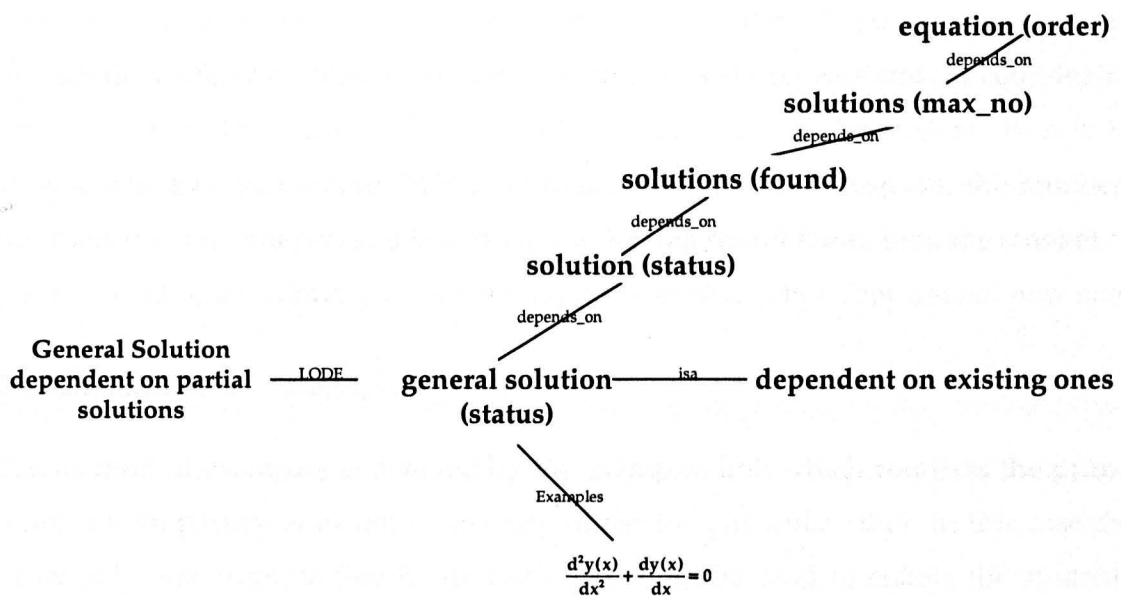
Figure 8.10: The concepts and methods required in teaching the procedure *General Solution dependent on partial solutions*

From the above description it is clear there is a need to structure the knowledge base so it will support the primary concept general solution, as seen through the perspective of status. The structure would be flexible enough to assist teaching in both core and background knowledge, as shown earlier in the case where it is needed to explain the rationale of the action taken. Given these considerations figure 8.10 represents the structure of knowledge needed to support the procedure *General Solution dependent on partial solutions*. The primary concept in this case is *general solution* and the kind is *status*. The secondary concepts are *solution* (three times) and *equation*. In the first instance of the concept solution the primary to secondary concept link is *status*. In the second instance the link is *found* and in the final case the link is *max_no*. The filter in this case is *LODE*.

The procedure could be carried out in three different ways: by the method of investigation, the method examples and finally by the method of definition. An analysis for representing the methods as well as the concepts involved will be presented.

8.3.1.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship which exists between the concepts. In this case, the status of the general solution depends on the number of allowed solutions and the number of found solutions. If the number of

solutions found is less than the solutions allowed then there is room for more independent solutions, otherwise any other solutions are repeats and/or combinations of existing ones. The rationale for this method is demonstrated by analogy. By selecting an algebraic equation where, by the fundamental theorem of Algebra, the number of solutions that are independent is known, and having found them, then the student will know that all other solutions are combinations of existing ones; they are not new ones.

<u>8.3.1.2 The method of examples</u>

The method of examples is denoted by the examples link which connects the primary concept with particular examples existing under the particular filter. In this case there exists only one example (see figure 8.10) that could be used to enable the student to deduce that the general solution is dependent on the existing distinct partial solutions.

<u>8.3.1.3 The method of definition</u>

The method of definition is indicated by the *isa* relationship. It results in TeLoDe explicitly giving the answer to the problem. Usually this method will be accompanied by a canned text formulated in order to meet the criteria of our model of teaching.

8.3.2 Partial solutions must be linearly independent

Having established that the general solution is not new but a combination of the existing solutions, it is imperative that whenever there are two distinct solutions of the differential equation they can form the complementary function, i.e. the general solution. This aim leads to the second sub-procedure, which must be performed as part of the *Find Complementary Function* procedure. A selection of concepts that could be utilised for the execution of this procedure is given below:

> *linearity,*
>
> *partial solution,*
>
> *linearly independent,*
>
> *equation.*

The first three concepts are from core knowledge whereas the last one is from background knowledge.

To explain the rationale for the structure of the knowledge representation shown in figure 8.11, a brief description of the methods will be presented: Through the *method of definition* the student will be told that to ensure the solution believed to be the general solution is, indeed, the general solution, it is necessary that the partial solutions forming the general solution are linearly independent. Using the *method of investigation*, the first objective is to establish that the partial solutions chosen for forming the general solution are linearly independent. The rationale of this approach relies on the second property of linearity. Failure on the part of the partial solutions to satisfy the condition implies that the second property of linearity (if there are two linearly independent solutions then their sum is also another solution) does not hold for these solutions.

It can be seen that it is necessary to structure the knowledge representation so that it will support the primary concept partial solutions as seen through the perspective of status. Given these considerations figure 8.11 represents the structure of knowledge needed to support the teaching of this procedure.
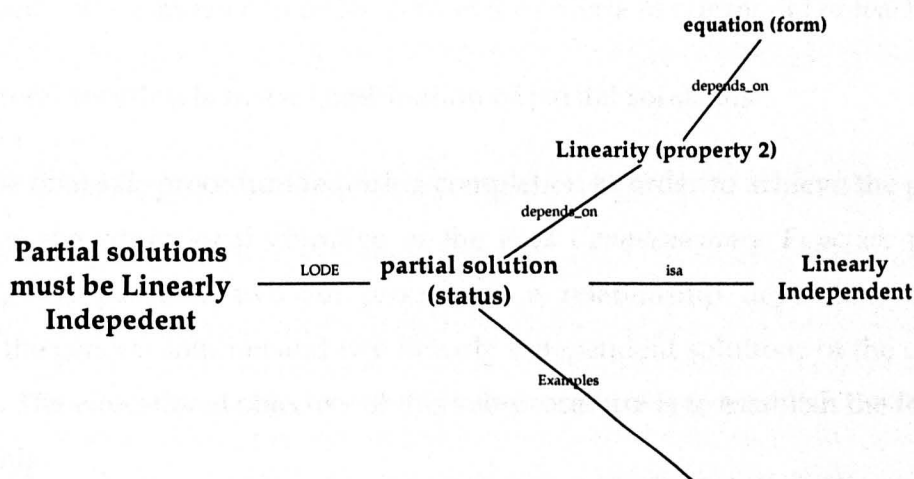
Figure 8.11: The concepts and methods required in teaching the procedure *Partial solutions must be Linearly Independent*

The primary concept in this case is that of *partial solution* and its kind is *status*. The secondary concepts are *linearity* and *equation*. Their primary to secondary concept links are for linearity *property 2* and for equation the link is *form*. The filter in this case is, again, *LODE*.

A detailed analysis of the different ways to carry out the procedure is presented. This analysis will also refer to the concepts involved in carrying out the methods.

8.3.2.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship between the concepts. The partial solutions have to be linearly independent since being solutions of the linear differential equation implies they must satisfy both properties of linearity. If one of the two partial solutions can be expressed in terms of the other solution then property 2 of linearity is reduced to property 1 (see Chapter 7) which is not permitted. Linking the primary concept with the property 2 ensures that this property has to be satisfied if the partial solutions are to be linearly independent. Although there is no need to use background knowledge, as the structure stands, it is possible to use background knowledge where there are similar conditions regarding the solutions.

8.3.2.2 The method of definition

The method of definition is indicated by the *isa* relationship. It results in TeLoDe explicitly giving the answer to the problem. Usually this method will be accompanied by a canned text formulated in order to meet the criteria of our model of teaching.

8.3.3 General solution is linear combination of partial solutions

This is the final sub-procedure requiring completion in order to achieve the procedural as well as the educational objective of the *Find Complementary Function* procedure. Following the previous two sub-procedures a relationship dependency has been between the general solution and two linearly independent solutions of the differential equation. The educational objective of this sub-procedure is to establish the form of this relationship.

To achieve this objective the method of definition and the method of investigation can be used. It has to be noted that in this case there is no need to refer back to background knowledge. The knowledge involved at this stage is mainly from the core knowledge. A selection of concepts utilised for the execution of this procedure is as follows:

partial solutions,

general solution,

linearity,

equation.

The first three concepts are from core knowledge and only the last one is from the background knowledge.

To explain the rationale for the structure of the knowledge representation as depicted in figure 8.12, a brief overview of the methods and their aims will be presented. Through the *method of investigation* the objective is to identify the property allowing existing solutions of the differential equation to be combined such that they form a new solution. By the *method of definition*, the student will be told how the two linearly independent partial solutions can be combined to form the general solution, because of the first property of linearity.

The structure of the knowledge representation here is quite straightforward. The requirement of supporting the primary concept general solution, as seen through the perspective of form, does not involve links with background knowledge. It only involves the property of linearity which leads to the result.
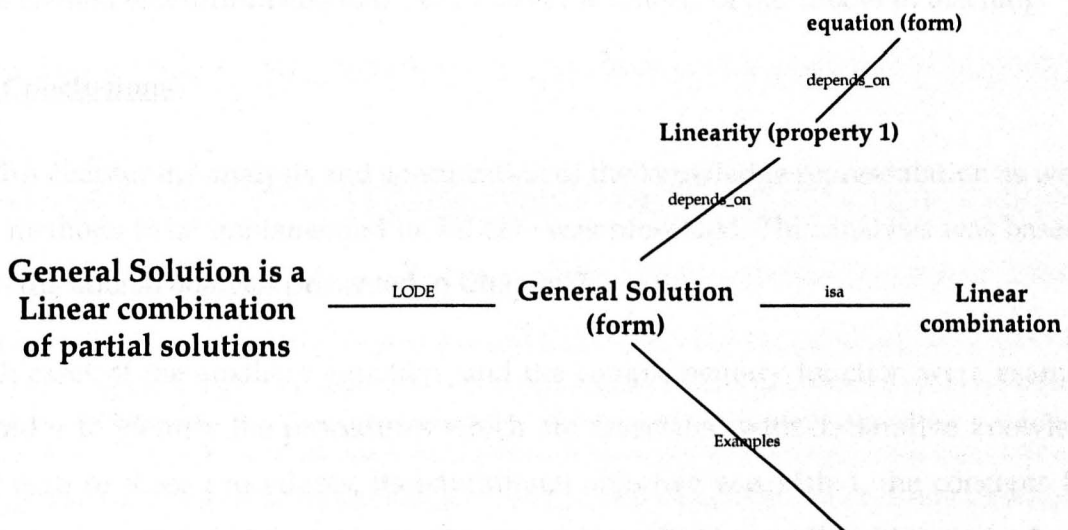
Figure 8.12: The concepts and methods required in teaching the procedure *General solution is a linear combination of partial solutions*

Given these considerations, figure 8.12 represents the structure of knowledge to support the teaching of the procedure *General solution is a linear combination of partial solutions*. The primary concept is *general solution* and its kind is *form*. The secondary concepts are *linearity* and *equation*. Their respective primary to secondary links are as follows: for the linearity the link is *property 1* whilst for the equation the link is *form*. The filter in this case is also *LODE*.

A detailed analysis of the different ways possible to carry out the procedure is presented. This analysis will also refer to the concepts involved in carrying out the methods.

8.3.3.1 The method of investigation

The method of investigation is denoted by the *depends_on* relationship between the concepts. The form of the general solution depends on the two partial solutions being linearly combined. This is precisely the case as linearity, under property 1, enables two linearly independent solutions of a differential equation to be combined in order to form another solution.

8.3.3.2 The method of definition

The method of definition is indicated by the *isa* relationship. It results in TeLoDe explicitly giving the answer to the problem. Usually this method will be accompanied by a canned text formulated in order to meet the criteria of our model of teaching.

8.4 Conclusions

In this chapter the analysis and specification of the knowledge representation as well as the methods to be implemented in TeLoDe was presented. This analysis was based on the educational analysis presented in Chapter 7.

Both cases of the auxiliary equation and the complementary function were examined in order to identify the procedures which are associated with declarative knowledge. For each of these procedures, its educational objective was stated, the concepts from both background and core knowledge were identified, as well as the methods, that could be used in teaching them, were also identified and described.

In particular, the structures requiring representation, facilitating the methods and tactics, in the knowledge base were identified as well as arguing for their provision. The necessary concepts and their attributes were also identified, as well as the methods, their mode of operation and the reasoning behind the input requirements.

Chapter 9 will describe how these structures are to be implemented in Maple's own programming language.

CHAPTER 9

Chapter 9: Implementation of TeLoDe

This chapter describes the implementation of the prototype TeLoDe. The two main elements of TeLoDe described here are the knowledge representation and the teaching strategy module. The structure of the knowledge representation has been influenced by the analysis of the topic of linear second order ordinary differential equations with constant coefficients as discussed in Chapters 7, 8 as well as by the definition of the expository style as discussed in Chapter 6.

The implementation of the teaching strategy module reflects the definition of the expository style, as discussed in Chapter 6. Furthermore, implementing the expository style in TeLoDe, will demonstrate that the theoretical framework SIMTA can inform the structure of a teaching strategy module of an ITS prototype. Implementing a fragment of the guided discovery style will further assist in demonstrating how two distinct styles lead in different manifestations of the same generic structures (methods, tactics actions) and thus producing distinct teaching strategies.

In §9.1 the specifications of the knowledge base incorporated in TeLoDe is presented. In §9.2 the specifications of the teaching strategy module, reflecting the expository style is presented whilst in §9.3 the implementation of the module is described. Annotated excerpts of TeLoDe's output, using both the expository and guided discovery styles is given in §9.4.

9.1 Specification of the Knowledge Base

The knowledge base incorporated in Maple could be viewed at two levels: In the first level Maple has been enriched by declarative knowledge describing concepts, their links as well as their attributes. In the second level the knowledge that is incorporated in Maple is a form of a glass-box as discussed in Chapter 2. The glass-box provides the steps, that are familiar to a human, in solving a problem. The problem in this case is that of linear second order ordinary differential equation with constant coefficients.

Provisions of the glass-box and the declarative knowledge will transform Maple from a powerful symbolic calculator into a potential teaching tool where problem solving is taught in a rather novel way as described in Chapter 7, reflecting the alternative approach of using CASs in the teaching of problem solving, as discussed in Chapter 2.

This section is divided into two sub-sections: the first subsection (§9.1.1) examines how the glass-box is represented in TeLoDe whereas the second subsection (§9.1.2)

examines how the concepts, their interconnections and attributes are represented to achieve the objectives of the analysis of the linear second order ordinary differential equation with constant coefficients, discussed in Chapter 7, and support the structure of the methods as outlined in Chapter 8.

9.1.1 The glass-box element of the knowledge representation in TeLoDe

The procedures reflect the steps described in Chapter 7 that are followed in order to achieve the solution of the equation. These steps are categorised into procedural ones and non-procedural ones or declaratives.

Formation of Auxiliary Equation

Assume y=e^{ax} Substitute y=e^{ax} Manipulate Expression

Identify Form   Find function   Simplify   Factorise   Equate to zero

Find Properties
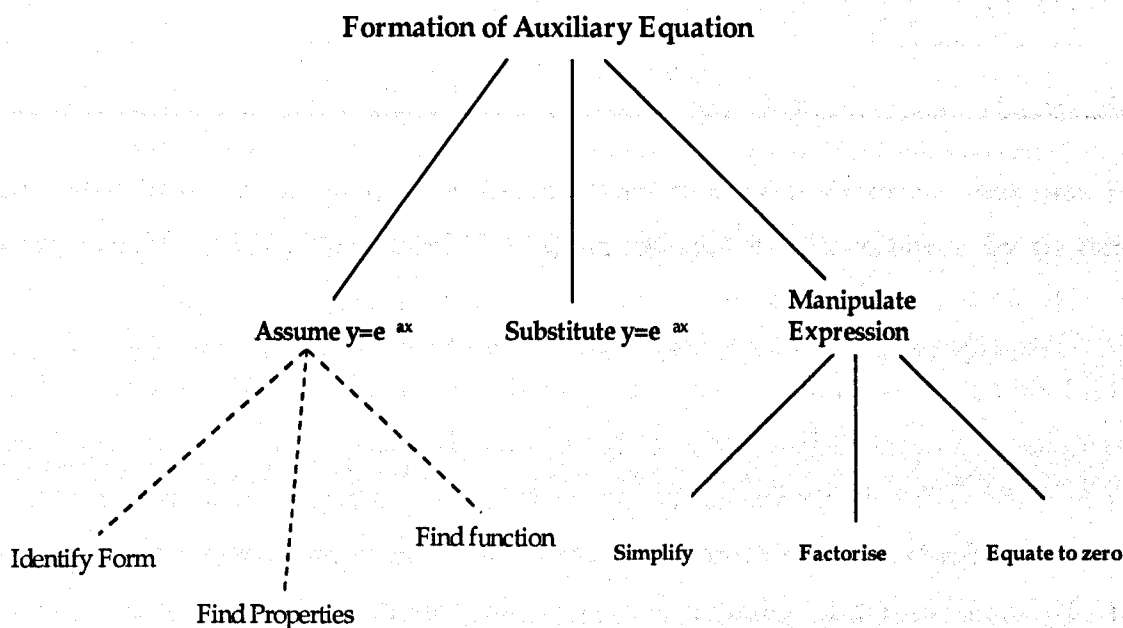
Figure 9.1 : A procedural breakdown of tasks for attaining **Find Auxiliary Equation** including both procedural and declarative procedures (declaratives are denoted by the dashed line)

The declarative steps are associated with conceptual knowledge. For example, as shown in figure 9.1, the step ASSUME y=e^{ax} is further analysed to include the three declarative steps, Identify Form, Find Properties and Find Function. These steps are associated with conceptual knowledge that would enable TeLoDe to teach the student that the solution in the case of linear second order ordinary differential equation with constant coefficients is in the form of the exponential function. From this deduction, by simply substituting into the differential equation the auxiliary equation is derived.

All the other steps shown in figure 9.1 are procedural ones as they only involve manipulation of expressions, such as substitution, carry out an evaluation, simplify etc.

These are considered basic operations and within the capability of Maple are to be carried by the user using Maple's instructions where it is necessary.

As the tree structure of figure 9.1 indicates, the step Assume $y=e^{ax}$ is complete only when the three steps below it (Identify form, Find property and Find function) are also complete. To represent this condition the program code in TeLoDe is arranged in the following way: the step number, the name of the steps and the associated sub-steps with a keyword denoting the association with the procedure, as is shown in the template below.

Step number	Step name	Associated sub-procedures and keyword	
p1	Find AE	AND	p2, p11

For a step to be carried out, the sub-steps need to have been carried out first. For example, FIND AUXILIARY EQUATION, represented by p1, followed by its name FIND AE, followed by the sub-steps p2 and p11, as shown are associated with p1.

The keyword `AND` is used to denote that both sub-steps need to be carried out for the step to have been completed. Such a keyword is used to distinguish between cases where it is sufficient to carry out one or the other sub-step, according to conditions. For example, forming the complementary function requires to establish if the solutions Y_1 and Y_2, are real and distinct, real and equal, or complex and thus branch to the appropriate steps. In this case the keyword is IF is used instead of the AND keyword.

To inform TeLoDe which steps are associated with declarative knowledge and which are not, the procedure LINKS is used. LINKS contains the links between the procedure and the *primary concept*, its *kind* and *filter*, as discussed in Chapter 8.

Procedure section	Text section	Concept section
pc1, p1	The function that we are seeking must satisfy the equation	[lodecc, [solution, form, type]]

The template (shown above) adopted in LINKS is as follows: There are three sections, the procedure section, the text section and concept section. The procedure section contains, a unique identifier for this link and the step. The text section contains some

canned that could be used to explain the link (not used in this version of TeLoDe) and finally the concept section contains the primary concept, its kind and filter.

The information in the template is interpreted as follows: The step p1 is assigned to [lodecc, [solution, form, type]]. The filter is *lodecc*, i.e., linear second order ordinary differential equation with constant coefficients, the primary concept is *solution* and the kind is *type of form*.

If a step is simply procedural, i.e., no concepts are attached to that step, then the concept section contains the empty list ([]) and this is how TeLoDe is able to differentiate between the two type of steps.

9.1.2 The declarative part

In the declarative part, the concepts, their attributes, their interconnections, examples and instances of them are represented. Thus every concept representation consists of four parts:

- The first part refers to the *definition of the concept*. This defines the concept in terms of its parent as well as stating the attributes that make it distinct from its parent.

- The second part refers to the *attributes of that concept*. Concepts could also play the role of attributes for other concepts.

- The instances section consists of *instances of a concept*, i.e., an example of what a quadratic equation looks like. These examples are not suitable for manipulation purposes.

- The fourth part refers to the *examples section*, where examples of a concept are stored. These examples are used as problems or are manipulated to demonstrate certain attributes of a concept.

9.1.2.1 The definition section

The rationale of the definition section of the knowledge base is to enable links between the concepts. It is therefore necessary to form the knowledge base in a way that any given concept is defined in terms of its parent and the constituent parts that differentiate that concept from its parent. For example, a linear ordinary differential

equation is an ordinary differential equation for whose solutions the property of linearity applies. Another example is that of differentiating between the concept of a chair and a reclining one. The reclining one has the property of its back being able to recline as opposed to the ordinary chair whose back is fixed. See figure 9.2.

```
[lodecc,
                # definition of lodecc
        [

                [ [isa, lodecc, lode],
                [ [[exists, y], [isa, y, derivative], [contains, lodecc, y],
                [has_prop, y, second_order]],
                [[forall, z], [isa, z, coefficient], [contains, lodecc, z],
                [has_prop, z, constant]]]]
        ],
        ......
] # end of lodecc
```

Figure 9.2. A representation of the definition section for the second order ordinary differential equation with constant coefficients

Through the definition section, parent-child relationships are established, all children of a given concept can be identified, as well as establish a relationship that may exist between any two given concepts. All information gathered from the definition is used, partially to establish methods such as analogy, but mainly to inform tactics of the relationship/difference between two concepts as well as to describe how a concept is defined.

The definition section is further divided into two main sections: the parent-child section, and the section that sets apart the child from the parent, which is called the *properties* section. The parent-child relationship is realised via the link *isa*. The link is followed by the concept followed by the parent (as shown below). This piece of code reads as follows: the linear ordinary differential equation with constant coefficients is a child of linear ordinary differential equation

[isa, lodecc, lode]

In the properties section, the links, *contains, exist, forall, has_prop* are used to define the properties that set the new concept apart from its parent. For example, in the case of the linear second order ordinary differential equation with constant coefficients, the properties that set it apart from its parent, that of the linear ordinary differential equation are:

- there is at least one second order derivative

- all coefficients are constants.

The property of second order is represented, as depicted in figure 9.2, by

[[exists, y], [isa, y, derivative], [contains, lodecc, y] ,[has_prop, y, second_order]].

This is translated to there *exists* y, y is a derivative, y is to be found in the linear second order ordinary differential equation with constant coefficients and its property is that it is second order. The keyword here is *exists*, which implies that a linear ordinary differential equation is second order *if and only if* the second order derivative term is present. Thus *exists* indicates the necessity of at *least one* term that satisfies the property.

The property of all coefficients of the aforementioned equation being constants, as depicted in figure 9.2, is represented as follows:

[[forall, z], [isa, z, coefficient], [contains, lodecc, z], [has_prop, z, constant]].

That is translated to: for all z, z is a coefficient, z is to be found on the given differential equation and the property is that z is a constant. The keyword here is *forall*, which indicates that *all* coefficients that exist in the linear second order ordinary differential equation with constant coefficients are constants. If even one coefficient is not a constant then the equation is not a linear second order ordinary differential equation with constant coefficients but a linear second order ordinary differential equation with variable coefficients. Thus *forall* indicates the need that all terms must satisfy the condition. This is in contrast to *exists* where one term satisfying the property is necessary and sufficient.

Such keywords are used by functions traversing the knowledge base and selecting information. Examples are the functions that find the difference between a parent and a child or between relatives. The results of this search are then utilised by the tactics to define a concept in terms of its parent followed by the extra properties that have to be satisfied.

9.1.2.2 The attributes or associated concepts section

In this section the rationale of the structure for the attributes is examined. It is important to draw a distinction between the *attributes/associated concepts* section and the

properties sub-section in the definition section. Whilst both enable definition and study of the concept, in the case of the properties section the entries are used to distinguish the concept from its parent, whereas entries in the section of attributes will help us to view the concept from a number of different perspectives, in other words to unpack the concept and look at one section at a time. Therefore, the *properties sub-section* is used to establish the concept whereas the *attributes/associated concepts* is used to examine it further.

To illustrate for the properties sub-section, consider the following example. The concept of solution is a concept in its own right. So for example, when solving $x + 1 = 0$, one facet of the concept of solution here is that the emphasis is on getting an understanding of what the solution looks like, finding its form and identifying any properties constraints that apply to the solution. This is the role that the attributes section is called to fulfil.

To that end the concept of solution, as seen in figure 9.3, has the attribute *form*. Attributes like *form* are designed to assist in collecting information in situations where, whilst the focus is on the number itself, the answer is not that obvious. To further assist in pursuing the attribute *form* the attributes *type, has_prop* and *value* are introduced, as also discussed in Chapter 8. That makes *form* an associated concept and *type, has_prop* and *value* its attributes. It is imperative, therefore, that TeLoDe is able to distinguish between the associated concepts and the attributes.

In Chapter 8 it was stated that the *type of the form of the solution* depends on the *type of the form of the unknown* which in turn depends on the *type of the form of the equation*. Such relationships/links are entered as part of the structure of the concept itself and are carried forward when the concept plays the role of an associated concept. In the case of the solution, its associated concept is *form* and the attributes of the associated concept are *type , has_prop,* and *value*. The *depends_on* relationship is defined in the attributes/associated concepts section, see figure 9.3.

There are cases, however, where the *depends on* relationship is operational under specific cases i.e., it requires a context. This applies, for example, in the case of the concept solution, see figure 9.3, where the value of its form is only applicable in cases of specific equations. This special case is denoted by a double square bracket on either

side of the entry. Such notation is used by the traversal function and ensures that an endless recursion is avoided.

```
[solution,
            # definition of concept
      [
         [ [isa, solution, expression],
         [ [[forall, y], [isa, y, objective_of_equation_solving],\
                             [has_prop, y, to_satisfy_equation_unknown]]]]
      ],
            # attributes or associated concepts
      [
         [ form, [
                  [type, depends_on, [unknown, form, type]],
                  [has_prop, depends_on, [operator,has_prop]],
                  [value, depends_on, [child,[[solution, form, type]]]]
                  ]
         ]
      ],
            # instances of concept
[instances,
         [
         [type, [nill]],
         [examples, [nill]]
         ]
      ]
], # end of solution
```

Figure 9.3: A representation of the concept of solution, including its definition section as well as the attributes and the instances section

As pointed out earlier in this chapter, there are cases where concepts exist in the attributes sections of other concepts. For example, the concept of solution exists as an attribute of a number of equations, see figure 9.4. As stated, the whole structure of an associated concept is brought forward, but this time the entries of the attributes are either *nil* or have a specific value true to the specific equation. The value of *nil* is entered in case of general and abstract concepts. This is illustrated in figure 9.4. It is worth noting that values of attributes of associated concepts, as in this case, may still contain relationship pointers.

Another instance where specific attributes are included is that of *status_is* which is meaningful to specific cases. The field *status_is* is used to indicate if a solution for that equation has been found or not.

```
[lodecc,
    ..
    ..
    ..
# attributes or associated concepts
        [
                    [solution,

                    # attributes of associated concept

                        [
                        [form,
                                    [
                                    [type, [function,\
                                                [depends_on, [[solution, form, has_prop]]],
                                                [isa, definition_of_type_of_form_of_solution]]],
                                    [has_prop, der_equal],
                                    [value, exponential]
                                    ]
                        ],

                    [status_is, nill]

                    ]
        ],
```

Figure 9.4: A representation of the attributes/ associated concepts section for the second order ordinary differential equation with constant coefficients

9.1.2.3 The instances section

The instances section incorporates examples/instances of a concept. These are to be utilised by the system to offer concrete instances of a concept and in particular to illustrate the properties of a concept as detailed in the properties sub-section of the definition section. For example, consider the concept of a quadratic equation. The instances section enables TeLoDe to pick a concrete instance of the quadratic equation. The organisation of this section is such that the chosen instances suit the needs of the teaching strategy being pursued. For example, if the system is concerned with linear ordinary differential equations, and wants to demonstrate that coefficients can be either constants or variables, then it could either pick a general instance of the concept or pick two specific instances, one exhibiting constant coefficients and the other exhibiting variable coefficients.

As depicted in figure 9.5, the instances for the concept of linear first order ordinary differential equation with constant coefficients are categorised in a number of terms

that are needed to define the equation. The rationale is that for an equation to be classified as a first order ordinary differential equation then the term involving the first derivative is enough and sufficient to define the equation. The term involving the function does not have to exist. The actual handling of the selection occurs at the tactics level.

```
                              # instances of concept

                    [instances,
                        [
                         [type,
                          [ two_term,
                           [ diff( y(x),x) +2*y(x) = 3,
                                   2*diff( y(x),x) + 2*y(x) = 4
                           ]
                          ],
                          [ one_term,
                           [ diff( y(x),x) = y(x),
                                   diff( y(x),x) = 3
                           ]
                          ]
                         ]
                        ]
                    ],
```

Figure 9.5. A representation of the instances section for the second order ordinary differential equation with constant coefficients

9.1.2.4 The examples section

The examples section consists of examples that have been selected for educational purposes and is used to demonstrate elements in the attributes or the associated concepts section, see figure 9.6. For example to demonstrate that the type of the form of the solution, in the case of a linear second order ordinary differential equation with constant coefficients, is a function, a one term specific example could be used. This example could be presented to the student, ask the student to solve it and finally ask the student to identify what is the form of the solution. Of course all, or some, of these steps could be performed by TeLoDe, and this decided at the tactics levels.

To pick a specific example, the following parameters have to be given: the concept, its associated concept and attributes have to be known. For example, to pick a differential equation, the concept of differential equation has to be passed as a parameter. This will provide access to all examples held under that concept. To access examples with particular objective in mind, for example to demonstrate that the form of solution of a differential equation is a function, the associated concept and its attributes have to be

specified. In this case the associated concept is *solution* and the attributes are *form* and *type*. This will pinpoint the specific examples. The result of this search is passed to the tactics which then select the appropriate example.

```
         # examples of concept

    [examples,
          [
          [solution,
          [
          [form,
                [
                [type,
                [
                [general,
                    [
                    [terms,
                       [1, [diff(y(x),x$2)=f(x)]],
                       [2, [diff(y(x),x$2)+y(x)=f(x),diff(y(x),x$2)+ diff(y(x),x)=f(x)]],
                ....
                [specific,
                [
                [terms,
                   [1, [diff(y(x),x$2)=3,diff(y(x),x$2)=5]],
                   [2, [diff(y(x),x$2)+y(x)=3,diff(y(x),x$2)+ diff(y(x),x)=5]],
                   [3, [diff(y(x),x$2)+diff(y(x),x)+y(x)=5]]
                   ]
                ...
    ], # end of examples
```

Figure 9.6: A representation of the examples section for the second order ordinary differential equation with constant coefficients

9.2 Specifications of the teaching strategies module in TeLoDe

The overall organisation of the teaching strategies module implemented in TeLoDe is best described with the aid of figure 9.7 where the structure of TeLoDe is laid out. Once a style has been defined and a task has been set, the procedures, pursued in an exhaustive manner to carry out the required task, are selected.

When a declarative procedure is encountered all methods that can be sustained are identified, thus forming a meaningful structure. The tactics are then selected and once the methods and the tactics have been prioritised, in accordance with the principles of the style, selecting and executing a tactic TeLoDe is ready to interact.

Once a tactic is activated, if it is not successful, another tactic is selected until all the tactics of that method are exhausted. Then the next method is activated and again the

same cycle is repeated until either a tactic is successful or the method of definition has been invoked. However, the above description is only the case if the knowledge involved is considered to be core knowledge. If there is a problem with background knowledge then the current task is suspended until the issue related to background knowledge is resolved. The background problem is resolved in two ways: either by using the "auxiliary way" or it is treated like any other problem presented to TeLoDe for the first time.
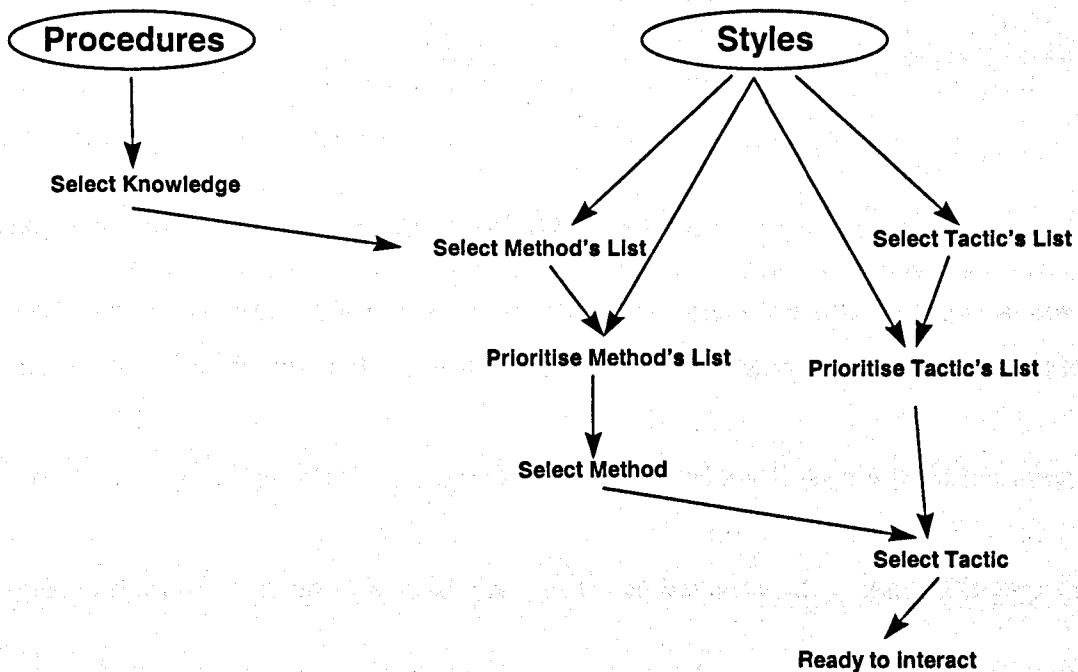
Figure 9.7 : The architecture of TeLoDe

In the first instance, in accordance with Douglas' [1991] observations, TeLoDe tries to rectify the problem by offering alternative ways of looking at the problem. Consequently, wrong answers are not treated as problems, nor is the problem expressed in terms of background knowledge. However, if the "auxiliary way" fails to resolve the problem, then TeLoDe treats the problem as if it were a new problem.

In this instance the problem is expressed in terms of background knowledge. The whole process is repeated until no more background knowledge, deemed appropriate for the student and the problem, exists. In this instance the method of definition comes into effect.

Once a concept has triggered the method of definition, that concept is noted and TeLoDe does not make use of that concept as background knowledge. Instead, TeLoDe

triggers the method of definition for any other concept that points to the noted concept as background knowledge. This is how TeLoDe avoids the WHY pitfall and also resolves the problem that all other methods have failed to resolve.

Finally, if the student is unsure of the terminology used, as discussed in Chapter 8, an explanation can be requested. In this instance, the task at hand is suspended and TeLoDe tries to establish methods and tactics that will enable it to provide the requested explanation. Again, the emphasis is on providing as much information as possible without contravening the principles of the style. In this instance the method of analogy is used. In fact TeLoDe makes a point of picking the "easiest" concept that will assist in the explanation process.

9.2.1 Specification of the methods in TeLoDe

The methods are responsible for bringing about the meaningful structure. A method organises the subject matter at two levels, the general level and the style specific level, as discussed in Chapters 5 and 6. For example, in the case of analogy, at the first level the structure is generic i.e., the concept and its attributes are linked with other concepts regardless of whether these are considered background or forward knowledge (forward refers to concepts which have not yet been taught). Accordingly, every concept in the knowledge base that has these attributes is linked.

At the second level, the style principles (potential meaningful, optimal structure) are applied to ensure that only concepts satisfying these principles are allowed to form the meaningful structure. At this stage the knowledge base informs TeLoDe of any forward knowledge and the student model comes into being, thus allowing only background knowledge which is known to the student to be selected for the meaningful structure.

The concepts and their attributes that qualify for the meaningful structure will assist in meaningful interaction by reflecting on the principles of progressive differentiation, integrative reconciliation, subordinate, superordinate and combinatorial learning (see Chapter 6), as well as reflecting on the three alternative representations, i.e., enactive, iconic and symbolic.

Each method is associated with a number of tactics, which usually are considered to be sufficient to see the task through. At this stage the tactics are attached to a method and

do not change. Methods are also associated with actions but the link is not a fixed one, see §9.3.3 and §9.5.

Each method is capable of resolving the problem at hand. A method is pursued until either the task has been resolved, or all of its tactics have been exhausted, or a problem arises with the background knowledge. If the latter is the case then the current problem is paused and the background problem is pursued. If the problem is that the tactics have been exhausted then the next method takes over to carry out the task. If all methods fail there is always the method of definition which acts as a safety net to enable TeLoDe to move forward, thus avoiding the 'never ending cycle' of the WHY system, (see Chapters 3 and 4).

9.2.2 Specifications of the tactics in TeLoDe

The tactics are responsible for carrying out the meaningful interaction, as explained in §6.3.2, in accordance with the principles of the style selected. In this implementation the principles of the style are rather embedded in the tactics, and not as clear as in the case of the methods. This is a property of the current implementation and does not cause a problem in principle.

The tactics are classified into two categories, those for teaching and those for explaining. This is done for programming reasons, to speed up TeLoDe and again, as before, this choice does not go against the theoretical considerations of the SIMTA framework nor does it compromise its general and abstract nature.

Tactics are checked thrice. The first check ensures that any tactics not belonging to the chosen style are dropped. The second check is made after the methods for a given problem have been established. In this instance, a number of tactics require the structure of more than one method. Therefore, if and only if the required methods are active, then that tactic is allowed to move forward, otherwise it is dropped. However, even after this stage, when the tactic is active and has been selected, may be dropped if a final condition is not satisfied. For example, a tactic may require a specific relationship between two concepts and if the concepts, from core and background, do not satisfy the relationship, then that tactic is dropped. The important point here is that the tactics are dynamic. There may be circumstances where a tactic is active but a change in the student model requires that tactic to be dropped.

Tactics call upon actions in a dynamic fashion. That is, the tactics indicate if the action required is a question or a statement and the appropriate action is selected.

Every tactic is designed with the same principle as that of the methods, that is, to be able to resolve the task at hand. If it fails then it informs the method which takes the appropriate action. Whether a tactic fails or not depends on the result that is brought by the action, or actions, as executed by the tactic to achieve the desired effect. Upon failure it returns the concept and its attributes that caused the problem.

9.2.3 Specification of the actions in TeLoDe

Whilst actions are the lowest level of activities of the SIMTA model, these are further divided into two classes, primary and secondary. The roles of the secondary actions are simply computing actions, like printing on the screen, analysing the input so that it can be understood by the parser, arranging for the information so that it is displayed in a meaningful manner and so on. In short the role of the secondary actions amounts to that of interface manipulator. The role of the primary actions is to group the secondary ones in such a way as to have the desired effect, that is to enable the implicit question, as requested by the tactic, to be effective.

There are two types of primary action, a question and statement. The function of the actions is to extract from the methods the information in accordance with the instructions received from the tactic, and arrange it for display. In a number of instances information from different sources has to be collated in order to achieve the desired effect. Upon execution of an action the result is transferred to the tactic, which decides what needs to be done next.

Actions are attached as default to certain methods. However, when a method changes, the actions that were currently active remain still active for the sake of coherence. In this case the default actions are off-loaded.

9.3 Implementation of the teaching strategy module in TeLoDe

In this section, the implementation of the constituent elements of SIMTA is presented as well as the structure that controls the flow of the information between them. The methods of analogy, investigation, examples and definition are presented along with 18 tactics and 9 actions.

In order to understand how TeLoDe works it is necessary to describe the high level functions. Such a description will assist in forming an understanding as well as some guidance about implementation whilst avoiding bombardment with minute details of code.

9.3.1 Implementing the methods in TeLoDe

In TeLoDe four methods were developed, namely analogy, investigation, examples and definition. The next sections discuss how these methods have been implemented. Describing the implementation of each method it is assumed that the *procedure* and its associated *primary concept*, along with its *kind* and the *filter*, are the input to the prototype.

9.3.1.1 The method of analogy

The method of analogy accepts the *primary concept*, its *kind* and *filter* and tries to identify analogical structures in two passes. In its first pass the principles of logically meaningful and that of optimal structure, at the absolute level, are operational, and as such all concepts that exhibit the *primary concept* and its *kind* (as discussed in Chapter 8) are selected. For example, if the *filter* is the linear second order ordinary differential equation with constant coefficients, the *primary concept* is solution and its *kind* is type of form, then other types of equations that exhibit this structure are selected at this stage as analogical structures. It is imperative to note that at this stage the actual value of the *type of the form of the solution* in these analogical structures has no significance. For example, in the case of a quadratic equation the value of the *type of the form of the solution* is *numeric*, whereas in the case of a linear first order ordinary differential equation it is *function*, the same as in the linear second order ordinary differential equation with constant coefficients. However, at this stage both the quadratic and the linear first order ordinary differential equation are seen as analogous structures. The only requirement at this stage is that the concepts selected have a value and that is not *nil*. A concept that exhibits the structure with the *nil* value is not selected. For example, in the case where the *primary concept* is *solution* and the *kind* is *property of the form*, then not all equations, even differential equations, have this property. However, as indicated above, to preserve uniformity of the structure these attributed are entered into the knowledge base and their value is assigned to *nil*.

Having completed the first pass, the method of analogy then reviews the concepts that have been selected in the first pass so that the successful concepts are:

a) known to the student

b) the primary concept and its kinds acting as attributes to the filter (these are also known as attributes to the concepts already selected) share the same value with that of the filter with respect to the primary concept and its kinds.

If there exist any concepts that satisfy the conditions set above, then these form a list of potentially meaningful concepts or form the optimal structure at the relative level. At this level the method of analogy is defined at the style dependent level.

9.3.1.2 The method of investigation

The method of investigation concentrates on the core knowledge, that is

- the filter,

- the primary concept

- and its kinds.

In this case the objective of the method of investigation is to identify any relationships that exists for the primary concept and its kinds that is to identify if there exist any other concepts that could be examined to reveal the value of the primary concept and its kinds in the case of the filter. For example, in the case of solution, the method of investigation establishes the relationship that exists between the equation, the unknown and the solution. For the method of investigation to become active the criteria are quite strict, especially under the expository style. Again the method of investigation operates at two levels: style dependent and independent.

Once the relationship has been established then the style independent role has been fulfilled. Whether this will become operational depends on the conditions that the style places upon the constituent elements. For example, in the case of the *type of the form of the solution*, the method of investigation returns the following relationship:

the type of the form of the solution *depends on* the type of the form of the unknown and that in turn *depends on* the type of the form of the equation.

The relationship has to be known to the student in this abstract form as do the concepts of unknown, and equation; additionally there has to exist at least an equation from the background knowledge for which this relationship holds and which is also known to the student. In short, in the case of the expository style the primary concept and its kinds have to be both logically and potentially meaningful.

9.3.1.3 The method of examples

The method of examples ensures that the examples for the given concept and its attributes exist. If that is the case then it means that the *depends_on* relationship exists and is potentially meaningful to the student. That is necessary as, in our definition of examples, the *depends_on* relationship is used as a means to guide the student through the example in a meaningful manner.

If the conditions placed above are met, then all examples, general and specific are passed to the tactics that are activated under the method of examples. These tactics make the final choice, i.e., whether the example is to be specific or general.

9.3.1.4 The method of definition

The method of definition is present at all times. Its presence ensures that if TeLoDe has run out of all other methods, or if there is no method active, the task at hand can still be addressed. The rationale for the method of definition rests on the assumption that there should be a cut off point where it may be counterproductive to push the student to solve the task at hand, and, where it may be better to provide the answer in a meaningful way. In short the method of definition could be considered as a safety net.

9.3.2 Implementation of tactics in TeLoDe

In TeLoDe 18 tactics have been implemented in total and are responsible for facilitating the interaction of the meaningful structure with the student. The tactics are associated with the methods and operate on an implicit to explicit scale. On the implicit end of the scale, the prototype will always ask questions with minimal information and will only advance up the scale towards the explicit end if all its implicit tactics have been exhausted. This reflects the theoretical considerations of the expository style as discussed in §6.2 and §6.3.

To facilitate the change between the different tactics as described above, each tactic has been allocated an index indicating its position on the implicit to explicit scale. The change between the tactics of a method is linear. The most explicit tactic is indexed *EXPL+6* whilst the most implicit tactic is indexed *IMPL+1.* Table 9.1 shows all the tactics and their index on the scale of implicit to explicit.

Index of tactic	EXPL + 6	EXPL + 5	EXPL + 4
Name of tactic	TACTI_15, TACTI_17	TACTIC_3, TACTI_16	TACTI_14, DO_EXAMP
Index of tactic	EXPL +3	EXPL + 2	EXPL + 1
Name of tactic	TACTIC_5,	TACTIC_4, TACTIC_8	TACTIC_2, TACTIC_6, TACTI_10
Index of tactic	EXPL	EXPL - 1	IMPL + 2
Name of tactic	TACTIC_1, TACTIC_9	TACTIC_14	TACTI_13
Index of tactic	IMPL + 1		
Name of tactic	TACTI_12		

Table 9.1: A table of all tactics used in TeLoDe arranged in descending order of their associated index of priority

There are eight tactics associated with the method of analogy, four with the method of investigation and four with the method of examples. Although tactics operate under a method, some tactics may also require information from the knowledge base which is extracted by a different method. For example, as shown in figure 9.8, TACTIC_3 is used by the method of analogy. However, as it explores analogies of *depends on* type of relationships, the method of investigation also has to be present.

```
tactic := proc()
    tact_lis := [
    [ `tactic_1`, [analogy], [ `teach`, expl]],
    [ `tactic_2`, [analogy], [ `teach`,expl + 1]],
    [ `tactic_3`, [[investigation, 2]], [`explain`, expl + 5]],
    [ `tactic_4`, [analogy, [investigation, 1]], [ `teach`, expl + 2]],
    [ `tactic_5`, [analogy], [ `teach`, expl+3]],
    [ `tactic_6`, [[investigation, 2]], [ `teach`, expl]],
    [ `tactic_7`, [[investigation, 2]], [ `teach`, expl + 1]],
    [ `tactic_8`, [[investigation, 2]], [ `teach`, expl + 2]],
    [ `tactic_9`, [examples], [ `teach`, expl]],
    [ `tacti_10`, [examples, [investigation, 2]], [ `teach`, expl + 1]],
    [ `tacti_11`, [analogy, [investigation, 1]], [ `teach`, expl + 4]],
    [ `tacti_12`, [analogy], [ `teach`, impl + 1]],
    [ `tacti_13`, [analogy], [ `teach`, impl + 2]],
```

. . .

Figure 9.8: A section of the procedure **TACTICS**, containing information regarding the order and association of tactics.

All tactics accept four parameters as input; these are

a) the *filter*

b) the primary concept and its attributes

c) the analogous concept in the case of an analogy, the *depends_on* relationship in the case of investigation and the examples in the case of examples

d) the appropriate list of actions.

Once a tactic is passed the appropriate information, through its parameters, an action is displayed. Depending on the student's input, the tactic will return either a true value or a false value. In both cases the problem that was tackled by the tactic is returned as well. If the return value is true, then the control passes to the method. If the return value is false and the problem is with background knowledge, then again control passes to the methods, but if the problem is with core knowledge then the next available tactic is sought. If there is one then it is activated, otherwise control passes back to the methods.

9.3.3 Implementing the actions in TeLoDe

The actions are low level activities, as described in Chapter 5, and are used to carry out the objectives of the tactics. Nine (9) actions have been developed in TeLoDe and are associated with methods, as shown in table 9.3. The actions in TeLoDe are classified as either QUEST or STAT because their main functions are either to ask a question or make a statement.

	ANALOGY	INVESTIGATION	EXAMPLES
ASK_DEP		✓	
ST_DEP		✓	✓
ST_DEP_1		✓	✓
ST_DEP_2		✓	✓
WHAT_IS	✓	✓	✓
ANS_IS	✓	✓	✓
AL_FIELD			✓
CAN_YOU	✓		
PRES_EXA		✓	✓

Table 9.3: Association between the actions and the methods in TeLoDe

The tactics are placed on a scale from implicit to explicit. For example, consider the following *depends_on* relationship: the type of the form of the solution depends on the type of the form of the unknown which in turn depends on the type of the form of the equation. The functions ST_DEP_1 and ST_DEP_2, which are STAT type, are used in TeLoDe to state these relationships. To differentiate between the two, since the first function states only the relationship between the solution and the unknown, whereas the latter function states all three relationships, the indices STAT, STAT+1, STAT-1 is

used to differentiate between them, like in the case of tactics. The same holds for the QUEST type actions.

Actions that are of QUEST type are also classified and fall into the following four categories: WHAT, WHY, FIELD, and PRESE. This classification not only assists in selecting the appropriate question but also enables TeLoDe to work out the correct answer. For example the action WHAT_IS asks questions of the form "What is", whereas the ASK_DEP asks questions of the form "What does ... depend on".

Even though the actions, like the tactics, are associated with the methods, see table 9.3 in this case for the sake of dialogue coherence tactics will temporarily switch methods. For example, if the method of investigation is active and the student is asked about a *depends_on* relationship with the use of ASK_DEP (see table 9.3), if the method of analogy is required, then the action, ASK_DEP will be used by the method of analogy despite that it is not attached to the method of analogy (see table 9.3).

However, even if the actions are considered low level in SIMTA, in TeLoDe they use even lower level actions to achieve their task. For example, the code [lodecc, [solution, form, type], isa, function] is translated by PRE_ANS as " the type of the form of the solution is that of a function". PRINTING is responsible for the output, given to a certain peculiarity of Maple's interface printing functions. GET_VAL is used to extract the answers from the knowledge base, while PICK_EXA is used to extract a specific example.

9.3.4 The interface and the student model

Maple's programming language was not designed as a fully high level programming language. As such its handling of input is rather limited. To overcome this inherent difficulty a simple parser was developed that enabled TeLoDe to accept both Maple's commands as well as text. To that end the function PARSING is used. If the input happens to be a Maple command then control reverts to Maple's interpreter, otherwise the function PIECES is used to break up the text into words. The result of PIECES is then passed as a list which is manipulated by TeLoDe according to circumstances, i.e., where there is a request for explanation, to see if the answer matches the one derived from the knowledge base.

The student model is based on the overlay model. The role of the student model in TeLoDe is that of confirming whether a concept and its attributes are known. There is no provision for updating the student model. Thus the student model plays a consultative role. This is achieved by the function STUD_KNO, which operates on the procedure STUD_REC which holds the details of what concepts and attributes of that concept are known.

9.4 Description of TeLoDe's output

In this section, excerpts from TeLoDe runs are annotated in order to illustrate this implementation. In §9.4.1 the output, produced by TeLoDe, is under the expository style whereas in §9.4.2 the output is under the guided discovery style.

The italic styled text indicates the output as produced by TeLoDe and bold text indicates the difference between a tactic and its predecessor. The sign > is used by Maple to indicate that it is ready to accept the student's input. Wherever there is nothing after the sign that should be interpreted that no answer has been given to the system, this is treated by TeLoDe as a "wrong" answer.

9.4.1 TeLoDe using the expository style

>*main(prob, expl+6, exp, tact_lis, actions);*

Prob indicates the problem, in our case linear second order ordinary differential equation with constant coefficients, *expl+6, expl* are the parameters that are set by the style and assist in the selection of the tactics that comply with the expository style. *Tact_lis* represents all the tactics that are available to the system and *actions* all the actions available.

The system is now active and the first procedure with no associated declarative knowledge is displayed as follows:

The procedure that we are dealing with now is Formation of Auxiliary Equation
Press return to continue

The problem here is to establish that the type of form of solution is that of a function.

in tactic 1

What is the type of the form of the solution in the linear first order ordinary differential equation
    with constant coefficients?
>explain

The method above is that of analogy. In this case the system has found that the first
order linear differential equation with constant coefficients is the most appropriate
concept from the background knowledge to be used for introducing the problem. The
above concept is known to the student and the question asked complies with the
principle `final form` in that, it informs the student directly of the objective of the
question; that is, the objective is the type of form of the solution. Furthermore, the
principle of integrative reconciliation is also applied here as the tactic points out a
connection between the problem and a known concept from the student's background.
Note that the system has started in the mode where the learner is most active.

The subject was not familiar with the meaning of "type of form of the solution" and
asks the system for an explanation.

dealing with explanation
in tactic_3
The type of the form of the solution in the quadratic depends on the type of the form of the unknown
and
The type of the form of the unknown depends on the type of the form of the equation
What is the type of the form of the unknown in the quadratic?
>

The system pauses the problem at hand, and generates a problem for the "type of form
of the solution". In this instance the system looks at the background to identify
concepts known to the user that could be used to demonstrate the meaning of the "type
of form of the solution". The system is now in implicit mode and that implies that the
system is inactive and the student is active. The concept that is chosen to demonstrate
the problem, possess the attributes the "type of form of the solution" but its value is
different from that of the originating concept. At the same time the concept that is
chosen is the simplest possible that exists. That is in direct contrast to the case where
the method of analogy is used in "teach" mode. In this case, dealing with the
explanation, the concept that satisfies these conditions is that of the quadratic. The
method used here is that of the analogy.

in tactic_15
The type of the form of the unknown in the quadratic is letter
and since

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>

In this instance, given the failure of the previous tactic, the system moves along the tactics to one less implicit. It states the value of the variable as that of letter and that given the relationship stated in the previous tactic, the value of the solution is a numeric.

in do examples
in req_scra
Here is an example of quadratic:

$$x^2+2x+1=0$$

What is the type of the form of the solution in the quadratic?
>

What is the solution of the quadratic?
>

In this instance, the tactic is executed at two levels. At the first level, the method of analogy is now replaced by that of examples. An example of a quadratic is presented to the subject and the question concerning the "type of form of the solution" is asked. Again, the rationale is to engage the student while the system remains as inactive as possible. As the question is not answered, the tactic makes an effort to approach the problem in a step by step manner. It asks first for the solution, and then for its "type of form". The second question is not asked as the first answer was not given.

in tactic 16
This is an example of quadratic: •

$$x^2+2x+1=0$$

The solution of the quadratic is -1-1
What is the type of the form of the solution in the quadratic?
>

Following from the previous question and with the system moving towards being explicit, the answers are offered and their type of form asserted.

in tactic 17
This is an example of quadratic:

$$x^2+2x+1=0$$

The solution of the quadratic is -1 -,1
Thus,
The type of the form of the solution in the quadratic is numeric

The system is now at its most explicit tactic and offers the answer to the student in a meaningful manner, in accordance with the principles of the expository style. This concludes its attempt to explain the term "type of form of the solution". It then moves back to the question that initiated the explain facility.

What is the type of the form of the solution in the case of first order linear differential equation with constant coefficients?
>

The method above is that of analogy. In this case the system has found that the first order linear differential equation with constant coefficients is the most appropriate concept from the background knowledge to be used for introducing the problem. The above concept is known to the student and the question asked complies with the principle `final form` i.e., it informs the student directly of the objective of the question; that is, to find the type of form of the solution. The principle of integrative reconciliation is also applied here as the tactic points out a connection between the problem and a known concept from the student's background. Note that the system has started in the mode where the learner is most active.

Assume that the student does not know the answer. Since this is a question that, according to the student model, should have been known, the system tries a corrective action by again applying the principle of the expository style and offering alternatives. The system here will remain in the mode where the student is most active.

What does the type of the form of the solution in the case of first order linear differential equation with constant coefficients depend on?
>

The method here is that of investigation. Here the system is trying to explore the relationship between the solution and the unknown. If the learner offers no answer, then the system will increase the flow of information to the student; that is, it will move towards more explicit information.

The type of form of solution in the case of first order linear differential equation with constant coefficients depends on the type of form of the unknown. What does the type of form of the unknown in the case of first order linear differential equation with constant coefficients depend on?
>

The method here is also that of investigation. In this case the system has given the answer and tries to continue along the same line of thinking by trying to explore the

relationship between the form of the equation and the form of the unknown. Again if the student does not answer, then the system will try to employ a further tactic. If this is not possible, it will try to employ another method to resolve the background problem.

What is the type of form of solution in the linear ordinary differential equations with constant coefficients?
> The type of form of solution in the case of the linear ordinary differential equation with constant coefficients is that of a function.

The method above is that of analogy. As the system was not able to continue the above corrective actions to rectify the background problem, the program suspends the question of linear second order ordinary differential equation with constant coefficients and generates a new problem in the face of first order linear differential equation with constant coefficients. In this case the background concept is the general form concept of linear ordinary differential equations with constant coefficients. The student answers and the system acts as follows.

What is the type of form of solution in the case of first order linear differential equation with constant coefficients?
>

The student does not reply. However, since the question related to the problem of a lack of background, the system is now in a position to increase the amount of information by changing to a less implicit tactic.

The type of form of solution in the case of the linear ordinary differential equation with constant coefficients is that of a function. What is the type of form of solution in the case of first order linear ordinary differential equations with constant coefficients?
>

The method used is again that of analogy. The system will now search to see if there exists another tactic with more information. The system moves along the line from implicit to explicit.

The type of form of solution in the case of the linear ordinary differential equation with constant coefficients is that of a function. The linear ordinary differential equation with constant coefficients is related to that of first order linear differential equation with constant coefficients. What is the type of form of solution in the case of first order linear differential equation with constant coefficients?
>

If there is no answer, the system will try to increase the information available.

The type of form of solution in the case of the linear ordinary differential equation with constant coefficients is that of a function. The linear ordinary differential equation with constant coefficients is the father to that of first order linear differential equation with constant coefficients. What is the type of form of solution in the case of first order linear differential equation with constant coefficients?

> The type of form of solution in the case of first order linear differential equation with constant coefficients is that of a function.

The problem with the background knowledge that was supposed to be known to the student has now been resolved and thus the system is able to come back to the original problem, that of linear second order ordinary differential equation with constant coefficients.

The type of form of solution in the case of first order linear differential equation with constant coefficients is that of a function. What is the type of form of solution in the case of linear second order ordinary differential equation with constant coefficients?
>

The system came back to the method of analogy and moved to the tactic of presenting the answer to background concept by asking the question for the core knowledge.

What does the type of the form of the solution in the case of second order linear differential equation with constant coefficients depend on?
>

The method now is that of investigation. The mode is that of implicit information since the system is confident that the student should be able to work it out. The reason why the system moved to investigation without first trying the extra two tactics under the method of analogy (as demonstrated in the case of first order linear differential equation with constant coefficients) is because there is no parent/child relationship between the concepts of core and background knowledge. Thus the only action the system could take was to move to another method to explore an alternative way of viewing the problem.

What does the type of the form of the solution in the case of first order linear differential equation with constant coefficients depend on?
>

The above method is that of analogy. Note that the action now asks for a dependency rather than a value; this is for homogeneity as the previous question was about dependency. As the student did not answer, had the system continued along the same line, it would have had no option but to explicitly increase the amount of information going to the learner. However, by posing the same question for a background concept, the program provides the opportunity for the learner to actively engage and produce the answer. At the same time it adheres to the principle of the student being active, of integrative reconciliation and of using an existing cognitive structure.

The type of form of solution in the case of first order linear differential equation with constant coefficients depends on the type of form of the unknown. What does the type of form of the solution in the case of second order linear differential equation with constant coefficients depend on?
>

Again the system has kept in the method of analogy by increasing the information available to the student.

Here is an example of a second order linear differential equation with constant coefficients

$$\frac{d^2y(x)}{dx^2} + 2\frac{dy(x)}{dx} + 2\,y(x) = 0$$

What does the type of form of solution depends on?

> The type of form of solution depends on the type of form of the unknown

In this case as there are no other tactics that can be employed to elicit the answer from the student, the system moves on to the next available method. The system now uses the method of examples and the tactic employed utilises minimal information.

The type of form of solution in the case of second order linear differential equation with constant coefficients depends on the type of form of the unknown. What does the type of form of the unknown in the case of second order linear differential equation with constant coefficients depend on?
>

As the student answers correctly, the system comes back to the method of investigation and uses the tactic that displays the answer to the background concept by asking for the concept in the core knowledge.

Here is an example of a second order linear differential equation with constant coefficients

$$\frac{d^2y(x)}{dx^2} + 2\frac{dy(x)}{dx} + y(x) = 0$$

What is the type of form of solution in the case of second order linear differential equation with constant coefficients?
> The type of form of solution in the case of second order linear differential equation with constant coefficients is that of a function.

The system uses the method of examples because the investigation method has no more tactics to use. The prototype continues until it is established that the solution of the second order linear differential equation with constant coefficients is of the form of the exponential function. If the student had not answered the question correctly, the prototype would had to use the method of definition since there were no more tactics to be used under the method of examples.

9.4.2 TeLoDe using the guided discovery style

The output annotated in this section has been produced by TeLoDe running under the guided expository style. The motivation in implementing the guided discovery style was to offer a "hands-on" experience of how the guided discovery and the expository style would treat the same task in distinct ways. Consequently, a skeleton functionality of the guided discovery style has been implemented with the sole purpose of achieving the above objective.

>*main(prob, expl, impl, tact_lis, actions);*

In this instance, TeLoDe is under the influence of the guided discovery style. In this instantiation, the methods behave exactly as in the case of the expository style, thus the meaningful structure is the same. This will make the contrast between the two styles even more apparent. That is, as will be demonstrated, how the strategies are influenced by the different styles at the tactics level. The problem is the same, the number of tactics are the same and the number of actions are the same. The only difference in the call of *main*, is the range that defines the upper and lower limit of tactics. Thus, in the case of the guided discovery style, tactics above EXPL and below IMPL are excluded, see table 9.1.

The procedure that we are dealing with now is, FIND AE
Press return to continue

in tact 13

What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?

In this instance, TeLoDe, in accordance with the principles of the guided discovery style, provides a direction to the objective. Thus it asks the question for the type of the form of the solution in the case of the linear second order ordinary differential equation with constant coefficients. The method here is that of analogy.

in tacti 12
Can you think of another linear ordinary differential equation
whose only one derivative is first_order for which the question
What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?
is of similar value ?

As the student does not answer, the method of analogy in this instance TeLoDe, that is under the guided discovery style, utilises the background knowledge. However, in direct contrast with the expository style, here TeLoDe requires the student to identify the background concept. To that end, it describes the background concept in terms of its parent and the properties sub-section.

in 14
Can you think of another linear ordinary differential equation
whose instance is $\dfrac{d(y(x))}{dx} = y(x)$ for which the question

What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?

is of similar value ?

As the background concept has not been identified, TeLoDe moves on the scale from implicit to explicit and will offer more information regarding the background concept, while staying short of naming the concept. This tactic will use the parent of the concept alongside an instance of the concept. Both in this tactic and the previous one the effect is achieved by the use of the concept of congruence classes, i.e., when TeLoDe is given a concept it automatically creates a congruence class for that concept.

in tactic 1
What is the type of the form of the solution in the linear first order ordinary differential equation with constant coefficients ?
>function

Having failed on the second attempt to elicit the name of the background concept, TeLoDe has no more tactics for elicitation and thus the next tactic is the one that initiates the expository style. In this case the background concept is named and the question regarding the type of the form of the solution is formed.

What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?
>function

Since the question has been answered, TeLoDe moves to the core knowledge concept and asks the same question, as that at the beginning.

finished, i.e., true answer
The procedure that we are dealing with now is, assume y=exp
Press return to continue

The procedure that we are dealing with now is, Identify the form of the solution
Press return to continue

in tact 13

What is the property of the form of the solution in the linear second order ordinary differential equation
with constant coefficients ?
>der_equal

Again, as in the case of the type of the form of the solution case, TeLoDe asks the question directly for the property of the form of the solution for the concept of the core knowledge.

finished, i.e., true answer

in tact 13

What is the value of the form of the solution in the linear second order ordinary differential equation with
constant coefficients ?
>exponential

Again, as in the case of the type of the form of the solution case, TeLoDe asks the question directly for the value of the form of the solution for the concept of the core knowledge.

finished, i.e, true answer
The procedure that we are dealing with now is, substitute y=e(a*x) into DE
Press return to continue

9.5 Summary

In this chapter, the implementation of the prototype TeLoDe was described. The description of the implementation was divided into two categories:

- the knowledge base

- and the teaching strategy module.

In the case of the knowledge base, the rationale of structuring the knowledge base was presented. The provision of knowledge base was seen here as a way of providing a glass-box element to Maple. Consequently, the structure of the knowledge base reflected the structures described in Chapters 7 and 8.

The knowledge base, at the highest level could be seen as a collection of two sub-bases:

- the *procedural* one

- and the *declarative* one.

In the first one, all the procedures required for attaining the solution of the linear second order ordinary differential equations with constant coefficients were included reflecting the hierarchy as well as the dependency between different procedures and corresponding sub-procedures. Here also procedures associated with declarative knowledge were denoted.

In the declarative part of the knowledge base, all the concepts from both background and core knowledge were included. For each concept, three parts were used. The first part was used to define the concept in terms of its parent as well as stating the properties that make this concept distinct from its parent. The second part referred to attributes of that concept, whereas the third part consisted of instances of that concept. Finally, in the last part examples of that concept were incorporated. These examples could be used/manipulated to demonstrate certain aspects of that concept.

Before the implementation of the teaching strategies module was described, the overall structure of TeLoDe was presented followed by the specification for the methods, tactics and actions. The description of the implementation of the teaching strategy module was kept at a high level by explaining in non-technical terms how the code achieved the specifications.

Finally, example outputs of TeLoDe, under both the expository and the guided discovery style, were annotated and presented.

CHAPTER 10

Chapter 10: Empirical Test of TeLoDe

This chapter reports on the exposure of TeLoDe to the 'outside' world, which was carried out with professionals (university lecturers) and students. Two small scale formative evaluations were carried out. TeLoDe was primarily implemented to convey the design principles of the SIMTA framework and secondarily the alternative use of CASs in problem solving. This is reflected in the aims of the study carried out with the professionals. However, to carry out the evaluation with the students, a lecturer(s) was approached and asked how in their opinion TeLoDe could be used in classroom.

In §10.1, the aims of the first study are stated and in §10.2 the criteria and prerequisites in selecting the participants are presented. In §10.3 the rationale of the questionnaire and the questions asked are given, and §10.4 gives a brief account of the experiment. In §10.5 the data of the first study is presented. The analysis of the data, related to the aims as stated in §10.1, is presented in §10.6.

In §10.7. the aims of the second study are stated, followed by a description of the first attempt in University of Manchester, see §10.8. The experiment in Lambeth College in London is described in §10.9 and the results of these two experiments for this second study are analysed in §10.10. Some conclusions from both studies are drawn in §10.11.

10.1 Aim of the evaluation

The aims of this small scale formative evaluation study with professionals in the field were to:

a) gain an insight about the SIMTA framework

b) explore whether TeLoDe might be effective as an educational tool.

In order to achieve these aims, it was necessary to investigate whether

1.a) the *teaching styles* informed by the SIMTA framework are recognisably distinct

1.b) the *expository style* can reason about its behaviour

1.c) the *expository style* can handle different topics from other areas of mathematics.

In order to evaluate TeLoDe as an educational tool it was necessary to examine:

2.a) the style's reflection of the underlying theory of learning

2.b) the selection of material of the subject matter

2.c) TeLoDe's adaptability to student needs

2.d) the usability of TeLoDe

2.e) whether the user is alienated by the change of the strategies

2.f) whether changes in altering of a strategy were beneficial or not to the user.

10.2 Criteria and prerequisites in selecting participants

Given the interdisciplinary nature of the project, the lecturers who were to participate in the evaluation needed to satisfy certain criteria:

1) They had to be familiar with the topic of linear second order ordinary differential equations with constant coefficients.

2) They had to be aware of the Computer Algebra Systems and their capabilities as well as their limitations.

These criteria were set because TeLoDe was developed as a research tool, not for use with students. For example, expressions such as 'type of form of solution' although not standard mathematics, were introduced because they were necessary from an implementation perspective, see §8.3.1. Moreover, it was imperative that participants were knowledgeable about the topic in order to evaluate the novel approach adopted (see Chapter 7).

The participants were drawn from 4 fields of expertise:

(1) lecturers teaching differential equations (3 participants),

(2) lecturers who are teaching differential equations or are aware of differential equations and are interested in mathematical packages and in particular in Maple (2 participants),

(3) lecturers who are interested in educational technology with an understanding of differential equations (1 participant)

(4) mathematics education lecturers who are interested/aware of mathematical packages (2 participants).

The aim was to gather distinct information from each specific category as outlined below:

Category (1)   feedback on the analysis of the topic of differential equations from a declarative perspective.

Category (2)   comments on the utilisation of the use of Maple: in particular whether there is a potential for Maple in its augmented form (coupled with the teaching strategies) as an educational tool and whether the analysis of the differential equations and the shift of attention (from procedural to declarative, see §7.2, §6.3) is appropriate and balanced (neither too ambiguous, nor too simplistic).

Category (3)   information about teaching strategies, that is the plausibility of the model of multiple teaching strategies, its representation (as perceived through interaction) and its potential interaction with the student. From the participant's interaction with TeLoDe the aim was to gather information about the coherence of the model, the smooth evolution of the alternative teaching strategies and whether it was evident that the system had changed teaching strategy.

Category (4)   information about the formation of teaching strategies, and the potential of TeLoDe as well as an assessment of the proposed approach for the teaching of the solution of the differential equation.

The information and comments discussed above are not exclusive to each category but indicative of the main input of that category according to their expertise.

10.3 The rationale of the interview questions

Given the classification of the participants and their discrete expertise, each category was given different questions. For example, the lecturer with experience in teaching differential equations can comment on the potential of the novel approach more readily than an educational technologist with less experience in this domain.

The questions below provide an interview outline but the actual questions asked varied according to participants' expertise. The questions asked relate to the aims outlined in §10.1. The identity of each aim is shown in brackets after each question. The generalised questions are as follows:

1) How do you evaluate the SIMTA model in TeLoDe, in terms of its coherence? (2a, 2c, 2e).

2) How do you evaluate the explanations provided by the system regarding

   a) how subject matter is selected (1c, 2b)

   b) its manner of presentation to the student (1c, 2e, 2f)

   c) the form in which questions or statements are couched? (1c, 2a, 2e, 2f).

3) What do you think about TeLoDe as a teaching tool in general? Suggestions for improvement? (1a, 1b).

4) Does TeLoDe adapt to the students' individual needs and if so, in what ways? (2c).

5) How did you find the interaction with the system? (2d).

10.4 The procedure

The participants were sent a document in advance setting out a resume of the SIMTA framework as well as explaining the expository style and providing a very brief explanation of the interaction. During their use of TeLoDe, the participants were asked if they had any queries about the document or any other questions. The objective of this document was to ensure that they had an understanding of the expository style and thus understood the way TeLoDe was behaving. Audio recordings were made during the whole guided tour of the program as well as during the question time. All computer interactions were also saved. The sessions lasted from 40 min to 2 hours.

Although TeLoDe has a limited natural language interface the researcher acted as a guide to TeLoDe, in order to allow the participants to focus more on the expository style (and thus on SIMTA) and TeLoDe and less on the interface itself. Moreover, given the idiosyncratic nature of the questions, because they were automatically generated, it was imperative that the experimenter offered explanations when required. Despite these actions, interface problems were not avoided. In fact, this issue was raised a number of times during the experiments (see §10.5.1).

For example, TeLoDe, in the expository style, asks the first question:

What is the type of the form of the solution in the case of the linear first order ordinary differential equation with constant coefficients?

The problem concerns second order differential equation, but according to the expository style there is always a connection with the background knowledge. Therefore, TeLoDe asks a question on first order differential equation before it asks a question on second order equations; this might be confusing to a first time user. Also, the expression, 'type of form of solution' is not a known term in mathematics, see §8.3.1.

A typical run of TeLoDe can be seen in §9.6.1 for the expository style and in §9.6.2 for the guided discovery style.

10.5 Presentation of data of the first study

The results of the study are presented under general headings, reflecting the areas that interested the participants.

10.5.1 Interacting with TeLoDe

All participants expressed some concerns about the interface. There were two main problems. Firstly the idiosyncratic nature of the phraseology generated by TeLoDe for the questions, and the kind of input that TeLoDe would understand. The more 'vocal' participants were the lecturers who taught differential equations (category 1), because they tried to visualise TeLoDe in classroom situation.

One participant (category 2) commented:

What do you mean by type of form of solution? what is this?

This participant was alienated by the use of this idiosyncratic terminology (type of form of solution) and the whole session was affected by this. The participant returned to this issue repeatedly.

Two participants (category 1) took this issue a step further and commented on the 'side effects' of the phraseology. One commented:

> The student is required to have understood what is meant by the type of the form of the solution, if one is expected to answer the question correctly. If the answer is wrong then the situation could be explained as follows: it might mean that the student did not know the answer and thus answered wrongly. It is equally plausible to assume, though, that the student did not understand what the question was asking and thus replied wrongly.

I explained that TeLoDe is equipped to provide explanations using the EXPLAIN facility, see §9.2.4, where 'the type of the form of the solution' is explained using the concept of the quadratic equation (background knowledge) The participant commented further that:

> I believe such a jump is a big one! I am not so sure that the student will be able to connect that in the case of the quadratic equation the answer is numeric and thus in the case of the differential equation the answer is function. In fact it would be very interesting to see whether a student would make this jump.

The second participant referred to the questions as a:

> 'mind reading game' ... I don't like this game at all; I prefer to be able to ask the system than the other way around.

However, two other participants (category 4) found the question : 'What is the type of the form of the solution ...' very interesting. The first participant said:

> Am I being asked to think at the general solution level or am I being asked at the component level; i.e., the exponential level.

This comment suggests that the use of the idiosyncratic terminology inspired the subject to ask what was meant. This participant was able to discriminate between the

levels of a solution of a differential equation. That was the whole objective of the inclusion of this term.

The second participant said:

> I like the fact that the system takes this step by step approach, I mean breaking up the concept into steps and searching one step at a time.

Another comment concerns the repeated use of the full name of concepts and lack of anaphora to them (see example):

What does the type of the form of the solution in the case of first order linear differential equation with constant coefficients depend on?
>

...

The type of form of solution in the case of first order linear differential equation with constant coefficients depends on the type of form of the unknown. What does the type of form of the solution in the case of second order linear differential equation with constant coefficients depend on?
>

According to the participant

> the names of first order linear differential equation (and others) are always repeated, thus space is taken on the screen and tires people.

Finally three participants (2 from category 1, 1 from category 4) raised the issue that the whole interface was text based. The two from category 1 noted:

> Reading from the screen is not easy.

> I would have liked to have a graph/tree structure to denote in which concept in the background TeLoDe is now in.

The participant (from category 4) noted:

> ... use of verbal description could inhibit students as they find it difficult to place it. If however, they had an instance of what is a second order differential equation-in this case it is easier for them to place a finger, thus they are in a better position to understand. ... Also by just adding a word 'linear' after the differential equation the whole concept changes. What if it is not noted in all of this text? I wonder how a student would notice these things!

The comments about the input that TeLoDe was expecting, included the following:

> I am not so sure that the system is able to accommodate more than one type of answer; that is if my first answer is typed wrongly then the system takes it as the answer and acts accordingly

> The answer here is function; What if I type f(x)? will it understand?

Multiple choices were suggested as a possible way of minimising the problem of 'mind reading'. Three participants (2 from category 1, 1 from category 2) suggested the use of pull down menus with a number of possible answers. The pull-down menus would have provided a hint as to what the answer might look like. In fact one participant (category 1) was hostile and concerned about the 'lack' of TeLoDe's discrimination between answers.

The interface was viewed as especially problematic by those who viewed the system from a student's perspective. However, demonstrating the automatic generation capability of TeLoDe was a priority (see §9.1.2, §9.1.3), even if that meant problems for the interface. Again, the emphasis was on substantiating the working of the SIMTA framework and its qualities.

The concerns about the interface are legitimate. It is acknowledged that the interface requires more work, and that as the system stands it is not ready for evaluation by students.

Multiple choices or pull down menus could be considered as future development. TeLoDe does provide an explanation facility already, e.g. to explain what is meant by 'type of form of solution' (see §9.5.1). The objective of such a facility was to demonstrate the versatility of TeLoDe in essentially using the teaching mechanisms for explaining its actions. The explanation tactics, like the teaching tactics, were developed in accordance with the style, expository or guided discovery.

It may be true that the explanation is not straightforward to follow and requires a 'big jump'. However, it is not central to this research, because no claim for effective teaching strategies is being made. At this stage the objective was to identify what constitutes a teaching strategy and how to define a framework for supporting teaching strategies in the paradigm of multiple teaching strategies.

The concerns about the heavy use of text are, again, legitimate but not an issue at this stage as TeLoDe is a prototype. A number of suggestions to get around this text issue included the use of graphs or examples.

10.5.2 The issue of evolution/coherence of strategies

The evolution and coherence between alternative strategies is interlinked to the overseeing component, style. All participants were shown both the expository and the guided discovery styles, §9.5.1 and §9.5.2, and afterwards asked if there was an observable difference between the two styles. The answer was a unanimous yes.

When asked about the way teaching strategies evolved, two of the participants (category 1, category 2) were uneasy with the style that was used, as they would have preferred an inductive/exploring style. The strategies reflected the expository style (based on Ausubel). They identified themselves more with the guided discovery style (based on Bruner).

They also appeared to dislike the structure that the expository style imposed. It is true to say that the expository style is based on Ausubel's theory of learning which is organised and well structured. However, it should be remembered that the implementation took place in order to demonstrate the principle that the style affects the strategies. As stated earlier, no pedagogical claim is being made here.

The first one (category 2) stated:

> The system seems to be in total control, you are the possessor of knowledge and it is provided in a stepped fashion. I guess I am more an inductive person. I would have liked to see examples and be able to deduce from there certain characteristics.

It is no surprise that this participant identified more with the demonstration of the guided discovery style and felt more at ease once examples under the expository style were presented.

The second one (category 1) stated:

> The control is too rigid; it is not student centred; the system does not try to identify what I know of the subject and try to build on that.

The educational technologist noted that at some points the evolution of the strategies was not as smooth as it could have been. This is especially obvious when coming back from the explanation mode to the teaching mode and also when moving from the analogy method to that of the investigation, (this point was also made by one of the first category participants see §10.5.1).

The participant stated:

> There appears to be a small jump from one mode to the other. Sometimes even between the different methods, but I understand this is not a big issue.

One of the mathematics educators stated:

> As your system stands it appears to be moving on a linear mode; in its strategies one is followed by the other. I am not so sure that this is the case in real situations. ... I understand that once a method in your system is activated the system sticks with it until all of its tactics have been exhausted. Humans are not like that; humans are different. They do not always want to wait until the conditions are satisfied. (This point is further taken up in Chapter 11).

On the other hand two participants (category 1 and 2) did like the fact that the problem of solving the linear second order ordinary differential equation with constant coefficients was broken down into steps (see Chapters 7, 8). They also like the way that TeLoDe moved from a point of implicitness to a point of explicitness. They claimed that their experiences as teachers were reflected in the structure of the system. Moreover, one participant, (category 1), liked the idea of the method of definition, which ensures the answer is provided so that the student can carry on. Again, according to the participant, it reflected a realistic classroom situation, i.e. the teacher tries to help the student reach the answer but, unfortunately there needs to be a cut-off point where the answer is provided.

That participant stated:

> I like the idea of the evolution between strategies in order to maintain student interest. However, I would have liked
>
> 1) a dynamic evolution rather than linear

2) the student to have been able to select a strategy that suits him and he is confident with (the subject refers to strategies based on analogy or investigation or examples).

> I like the way multiple teaching strategies are realised in analogy, investigation and examples. I very much like the idea of the definition method, since in my experience once I have exhausted all other methods I have no option but to give the answer and carry on!

It is worth noting that another participant (the one who claimed to be more inductive) had the completely opposite idea regarding the method of definition.

This person stated:

> How do we know that the student does in fact learn after the definition method is being activated. We don't know do we?

Finally, the majority (6 out of 8 and the other two were unhappy because of the expository style rather than the strategies, see beginning of this section) appeared to be happy that the strategies were coherent. Their only concern was the problem of phraseology, as discussed in §10.5.1. It is imperative that a distinction is drawn between the phraseology used and the 'final form', as discussed in §6.3.2.

10.5.3 TeLoDe as potential teaching tool

Seven of the eight participants viewed TeLoDe in a positive manner but one participant (category 2) appeared to be quite concerned with the lack of interface and the inability of TeLoDe to provide help or to accept more than one answer, see §10.5.1. Even the inductive person was quite enthusiastic with the concept of TeLoDe and commented:

> I have not used Maple, although I have used other packages and I am very much interested in programming and creating programs that assist in teaching. I am very interested in going away and doing some programming in Maple myself. I like the potential of TeLoDe. In particular I can see that the system is able to accommodate a number of other equations and I find this quite good.

This person stated further:

I like the alternative approach to teaching the solution of the differential equation, different from the norm that follows the traditional procedural way. However, I am not convinced how well that approach has been realised by the Ausubelian model.

Another participant (category 1) was interested in how TeLoDe operates, i.e., the internals. The subject stated:

I quite like the idea that it looks at these attributes and the relationships between the concepts. I would like to see how this is achieved.

I demonstrated explicitly all the steps that are automatically performed by TeLoDe, i.e., how the methods are formed, how TeLoDe is able to differentiate between the concepts and its attributes, see §9.1.2.2, §9.1.3, and, once this is done, how tactics operate. (This was also requested by one of the maths educators).

The participant made the following comment about the potential of TeLoDe:

I think that augmented Maple (TeLoDe) is good, in fact it is very good. As these systems stand (CAS) they just give you the answer; they teach you nothing. ... It is very good that the system is able to accommodate two different styles and your approach to the teaching of the differential equations is a novel one. ... The questions that the system asks are meaningful, but in retrospect, possibly if there was a smoother introduction then these problems may go away. ... I like very much the structure of TeLoDe and I think that the model (SIMTA) could lend itself very well to the problem of drawing graphs. So if your model (SIMTA) can support a graph drawing then it is definitely a good model.

Another participant from the same category stated:

It is very important to draw a distinction between Maple and your system. The objectives of the two are quite different and there is no comparison between the two. Maple is primarily designed to deliver a result; your system is geared for teaching. I think that as it stands, provided the interface issue is addressed (see §10.5.1), it is sufficient to deliver the teaching as planned, i.e., through methods tactics actions. I am also quite happy that we could generalise for other knowledge bases apart from differential equations and be able to apply to accommodate a larger class of problems.

Both the educators and the educational technologist were happy with the concept of TeLoDe. One of the educators stated:

> I think this is a fascinating tool; not just for its novelty for the teaching of the solution of the differential equation but, to me more interestingly, it is a tool that allows us to explore how different styles, as you call them, operate. It is fascinating how by the change of tactics you can have such a different behaviour. The structure of your model (SIMTA) allows you to analyse a style into its components and vice versa, it points out what is required to form a style. It is fascinating. I really want to see how the different components operate and interact as I think this is quite difficult. I am not saying that your system is not doing it, it just seems quite difficult to manifest because in any of these theories (Ausubel and Bruner) there is a long way between theory and practice; that is the tinkering that one does inside the detail.

Here the participant pointed out that linearity, in terms of sequences of strategies, is a problem (see §10.5.2) and also that TeLoDe relies on the procedural aspect to move it from one aspect of the problem to the other.

Finally, the same participant concluded that there is tremendous potential in TeLoDe but a lot of work is required to realise this. The participant stated:

> It would be very interesting to see how easy it is to represent a human-like dialogue. It is interesting how many tactics that would require and how complex or easy their structure would have to be. I think TeLoDe is a good tool to explore this.

From the above quotations four main points were raised, which will be addressed in the rest of this section. These points are: can TeLoDe's structure accommodate more than one subject matter, the structure and complexity of SIMTA's tactics, the 'linear' evolution of strategies in TeLoDe and the fact that it relies on procedural knowledge.

For the first point, it is encouraging that for the first time a participant is able to explicitly suggest a different topic that in their opinion can be supported by SIMTA (as realised by the expository style in TeLoDe). However, one of the maths educators was not so convinced of the unqualified ability of TeLoDe to accommodate all problems (see §10.6).

The second point about the structure and complexity of SIMTA's tactics is further discussed in Chapter 11.

In response to the third point about linearity, I explained how the system builds up its methods, its tactics and how it moves along the implicit to explicit path. This is an issue that requires further consideration and there is a model of motivation [del Soldato & du Boulay, 1995] which could in theory enable a more dynamic approach. This point is discussed further in Chapter 11.

The fourth point concerns the use of procedures. Procedures need to be used since part of the research question (see Chapters 1 and 2) is to explore how CASs could be transformed to take on the procedural aspect of mathematics. This point is also discussed further in Chapter 11.

10.5.4 Adapting to the student

The participants were all asked if the TeLoDe adapts to student's individual needs and if so in what ways? To this question all participants agreed that TeLoDe adapted to student's replies.

According to the educational technologist, TeLoDe is adaptable, as this term is understood in ITSs.

Another participant (category 1) pointed out that although the system adapts to the student responses it only adapts within its given tree structure. This point was also made by one of the educators. Another participant (category 1) thought that the student did not have much control. The participant stated:

> The student does not have as much control as I would have liked and the student is concentrating on your teaching rather than their own learning.

It is worth noting that this comment was made when TeLoDe was using the expository style. The participant then referred to the interface problem and said that if there was a multiple choice then things could have looked better. Again, the interface issue appears to dominate. The participant did however acknowledge that the system reacts to the replies and tries to provide assistance within the definition of the chosen style. TeLoDe does not possess a student model in terms of analysing the reply and trying to point

where the problem may lie. Instead the student model (see §9.4) acts as a check to see whether something is known.

Another point that was raised in the study, was the 'inability' of TeLoDe to understand different forms of replies, see §10.5.1. Although this is an interesting point, it is outside the scope of this work.

10.5.5 Selection of the subject matter in TeLoDe

In TeLoDe use of background knowledge is central, in both expository and guided discovery styles. The first step under both styles involves the selection of the linear first order ordinary differential equation with constant coefficients as a background concept.

All participants agreed that the selection of background concepts in TeLoDe was correct in terms of mathematics. One participant (category 1) was very positive and pleased with the selection of the concepts whilst another one from the same category expressed a concern from a pedagogical perspective. Again two participants were still concerned with the interface issue (see their comments below).

The participants' comments (in the order summarised above) were as follows:

The 'positive' participant's comments were:

> This is very interesting, this is the only thing that the two concepts (first and second order) have in common. It is very good that your system has picked that up.

The participant with pedagogical concerns commented:

> I am quite happy that your system goes from the second order to the first order; this is good as it would give an understanding and a pattern for the second order differential equation. However, going back to the linear ordinary differential equation may be just about OK; but beyond that I believe it may prove counter-productive; counter-productive in the sense that the student may find it quite hard to deduce that the answer in the case of type of the form of the solution is 'function'.

The same participant commented on the choice of quadratic equation as used by the explanation facility of TeLoDe:

I am not so sure that the student will draw the analogy and deduce that if in the case of the quadratic the answer is numeric then in the case of the linear first order differential equation with constant coefficients, the answer is function (see excerpt of actual run).

in tactic 17
This is an example of quadratic:
$$x^2+2x+1=0$$
The solution of the quadratic is -1 -,1
Thus,
The type of the form of the solution in the quadratic is numeric

finished with the explanation

What is the type of the form of the solution in the case of first order linear differential equation with constant coefficients?

>

The participant's point is that there should have been another tactic drawing attention to the fact that the answer in the quadratic is numeric as the unknown is x, which is alphanumeric. In the differential equation the unknown is a function and its derivatives and thus the type of the form of the solution is a function.

For the choice of quadratic equation, one of the educators' comment was:

> I don't want to be asked what is the type of the form of the solution in the case of the quadratic equation; I am dealing with differential equations and I want to know what the type of the form of the solution is in this case. ...

Finally, there were comments of the two participants still concerned with the interface issue:

The participant from category 2 commented:

> I am still concerned with the way the questions are formed here. ... I suppose it looked all right!

The participant from category 1 refers to the 'mind reading' issue, see §10.5.1,

> but otherwise I feel that the system did choose correctly the concepts from the background knowledge.

The pedagogical concerns raised here, may be valid, but they are outside the scope of this work. In particular, in its choice of the quadratic equation TeLoDe was deliberately selecting this concept as it was operating in an implicit mode, thus the lack of the tactic mentioned above. Whilst TeLoDe operated in the implicit mode, the fact that it was in 'explain' mode also influenced the choice of the background concept. The selected background concept had to be the 'easiest' ones and its value different from the core concept thus promoting integrative reconciliation learning (see §6.2, §6.3, §9.5.1).

10.6 Analysis of results of the first study

The results of this empirical study showed that participants were positive about many aspects of both TeLoDe and SIMTA. It needs to be stressed that this was a small scale formative evaluation. However, some interesting information have emerged which relates to the initial aims, see §10.1.

		1a	1b	1c	2a	2b	2c	2d	2e, 2f
category 1	KFS	P	NE	NE	KFS	KFS	NE	KFS
	KFS	P	NE	NE	KFS	KFS	NE	P
	KFS	P	NE	NE	Po	NE	NE	NE
category 2	KFS	P	NE	NE	P	Po	NE	P
	KFS	P	NE	NE	P	Po	NE	P
category 3	KFS	P	P	P	P	KFS	NE	P
category 4	KFS	P	P	P	P	KFS	NE	P
	KFS	P	P	P	P	KFS	NE	P

Table 10.1: A summary of the results of the first empirical study

Table 10.1, represents the overall results which are analysed subsequently, where at a glance it is possible to see the results of this study. The coding used to represents the answers of the participant is as follows:

- KFS (Know For Sure) indicates that the participant is positive about the question.

- P (Plausible) indicates that the participant can see the logic/principle, but uncertain for implementation.

- Po (Possible) indicates that the participant has reservations for both the principle and implementation.

- NE (no-evidence) Participants are either negative or posses no expertise and thus cannot answer question.

To see if the system is recognisably different in different teaching styles (1a)

The answer to this question is a qualified yes. All participants were able to observe the change in a different teaching style. This was even more apparent as the participants were able to compare the two results, i.e. participants were able to compare the 'handling' of the same problem under the expository and the guided discovery style.

To see if SIMTA has the potential to handle the different topics in mathematics (1b)

Whilst participants from all the categories acknowledged this, it is important to note that one of the maths educators pointed out that there may be restrictions on the topics that can be accommodated. One subject from the first category pointed out that drawing graphs could be effectively covered by SIMTA. All the other participants acknowledged that a number of equations could easily be accommodated by the SIMTA framework. However, the maths educator pointed that in a topic like probability where the structure is not that hierarchical, the accommodation could prove problematic.

To see if SIMTA can provide reasoning concerning its behaviour (1c)

There was not a conclusive response to this question. SIMTA's reasoning behaviour was noticed by the educators and the educational technologist but only after I drew

their attention to it. Consequently, I propose that it should be a topic for further evaluation studies.

To see if the style reflects its underlying theory of learning (2a)

As stated before, this was only noticed by the educators. They were able to 'see' that the expository style drew from Ausubel's work whilst the guided discovery style drew from Bruner's.

Selection of the subject matter (2b)

The results here were in general positive since all participants agreed (from a mathematical perspective) that the concepts selected were appropriate. One commented positively on the fact that TeLoDe was able to pinpoint first order linear differential equation with constant coefficients as a background for the second order linear differential equation with constant coefficients under the attributes of the type of the form of the solution. The two concepts do not have the same general solution. However, another subject was concerned about the pedagogical validity of choosing general concepts such as ordinary differential equations, see §10.5.5.

Adaptability (2c)

For this issue, although there was agreement that TeLoDe did adapt, there was also a different understanding of what adapting meant. Some of the participants understood this to mean that the system should have been able to accept a number of inputs as replies and felt that TeLoDe was generally too teacher centred.

Usability (2d)

The answer to this issue is that there is no evidence. Although, all lecturers acknowledged that potential exists much work is required. In fact a lecturer commented

... it would be very interesting to see how students would react to TeLoDe.

The remaining sections of this chapter report on a small scale formative study carried out with students.

Teaching strategies- are these beneficial to the student?(2e, 2f)

Comments on this covered a whole spectrum. Some of the first category thought that the fact that TeLoDe was changing strategies (both at the level of methods and tactics) would help to maintain student interest. Another felt that too much control was exercised and would have preferred a more inductive process.

10.7 Aims of the second empirical study

The objective of this empirical study was:

1.   to explore the potential use of TeLoDe in a real classroom situation when

         i)   incorporated in a teaching session by a lecturer

         ii)   used as a learning tool by the students

2.   to gain an insight into how students might react to the alternative use of CASs in problem solving as encapsulated in TeLoDe. Particular interests included:

         i)   if TeLoDe adapts to the student's needs

         ii)   the usability of TeLoDe

         iii)   whether the user is alienated by the change of strategies

         iv)   whether changes in altering of a strategy were beneficial or not to the user

3.   to explore students' reaction to the novel way proposed for teaching the solution of the linear second order ordinary differential equations with constant coefficients.

Given TeLoDe's idiosyncratic terminology and interface this study required that the lecturer acted as a mediator between TeLoDe and the students.

The participants in this study were:

- a lecturer with experience/interest in the use of CAS in classrooms

- students who possessed minimum knowledge of linear first order ordinary differential equations.

A number of lecturers from different universities/colleges were approached, but the majority of them declined due to pressure of duties. The first two successful encounters were:

with the Pure Mathematics department of the University of Manchester

and the Mathematics department of Lambeth College in London.

From Manchester two senior members expressed an interest/willingness in participating in the study. They had recently started using MACSYMA, another CAS. From London, there was an interest in exploring the introduction of CASs in the teaching, as the lecturer was an experienced user of CASs. The first two lecturers suggested that their first year students could be subjects for this study. The third lecturer offered one of his BTEC classes. In both cases the students were exposed to the topic of linear second order ordinary differential equations with constant coefficients.

It was anticipated that this study would require three sessions:

- Session 1: This is an introductory session during which the researcher would present:

    - the rationale of TeLoDe

    - present TeLoDe itself

    - explain the aims of the study.

- Session 2: After some thinking time the lecturers would come up with ideas/proposals as to how TeLoDe could be used with their students.

- Session 3: The actual experiment takes place.

In the following sections the results from Manchester and Lambeth College are presented and analysed.

10.8 The experiment in Manchester

Following session 1, the two lecturers from Manchester said that they could not participate in the experiment. Their reasons were:

- TeLoDe was not what they expected (they thought that the prototype was another CAS such as Maple or Mathematica)

- their intention was to install TeLoDe in a computer lab with 70 computers where the students could try it out

- TeLoDe's declarative approach to a procedural problem.

From the above points, it seems that the lecturers at Manchester had different expectations. The fact that TeLoDe was not another Computer Algebra System but had adopted a declarative approach to a highly procedural problem was their first point. Moreover, one of the lecturers commented:

> we have found that our students are very good with algorithmic knowledge, but when it comes to conceptual knowledge and you try to teach them something it is very difficult. They do not seem able to follow you and require substantial and constant backtracking.

As TeLoDe tried to adopt a declarative approach, and had idiosyncratic phraseology made, they considered TeLoDe unusable by their students. Moreover, as TeLoDe was using a kind of 'Socratic' approach, in their opinion, further complicated the way that TeLoDe could be used in classroom situation, let alone in a computer lab. One of the lecturers, said

> I would have no idea how to use your system; it seems to restrict the way that I would like to teach.

The lecturers showed the researcher a booklet, that was devised by another member of staff, with exercises written for the *Macsyma* system. In that booklet a number of exercises were devised demonstrating the capabilities of *Macsyma* as well as encouraging the student to make deductions by trying out problems and observe, generalise from the answers provided (this use of CASs has been discussed in Chapter 2). That was how a CAS was used by them and whilst they would like a system that provides more than just an answer, TeLoDe was not something that they could see as beneficial for their students. To prove their point, a third year student was fetched and the researcher was challenged to use TeLoDe with that student.

The researcher explained to the student that the objective of TeLoDe was to offer an alternative way to use of CAS in teaching problem solving. This alternative use aimed at exploring how the auxiliary equation is derived. This exploration relies on concepts from background knowledge as well techniques that have been used in previous topics in mathematics.

After this explanation, TeLoDe was initiated with the researcher acting as the mediator between the student and TeLoDe. As had been anticipated, and also pointed out in the first study, the idiosyncratic terminology was a problem. The student said:

> Why is it (TeLoDe) asking me what is the type of form of the solution? What does the type form of the solution mean? I have never seen this expression before!

I explained that the prototype has an explanation facility which I invoked for the student. Again the student seemed uneasy about the terminology used as well as by the answers accepted by TeLoDe. During the explanation, the quadratic equation was used to demonstrate what the type of form of solution means. When the researcher typed the answer *numeric* which is the answer that the prototype accepts as correct, the student asked:

> How does it know that it is numeric? How do I know what is the answer that I am expected to type?

As the system turned back to the original question, after completion of the explanation, the prototype asked the same question which had originated the explanation. As the student did not answer the question, the prototype kept asking questions by moving further back into background knowledge and as the student did not answer, TeLoDe was either moving further into background or using different methods. The student, who by now expected replies from the prototype and not continued questions said:

> Why does your program keep asking me questions? Why does it keep asking all these questions? Why can it not give me the answer? Can I ask it a question?

The researcher explained that TeLoDe is designed to exhaust all its implicit strategies before it moves to explicit ones. At this point the prototype was still asking for the type of form of solution, which the student could not give and thus the researcher explained

that in the case of the differential equation the answer was function as in the quadratic equation it was numeric.

After this the student was able to answer the second question. This was achieved, first, with the use of the explanation facility provided in TeLoDe, where it was pointed out that in the case of logarithmic functions the argument must be a positive number. Once this was established, then solving simple examples of differential equations that the researcher gave on paper for the student to solve, the student was able to deduce that the function and its derivatives differ only by a constant. The last question, that the only function satisfying the above property is the exponential function, was answered straight away without any prompting.

After the interaction with TeLoDe was complete, the student commented:

> I really like the motivation behind this system but I do not like the control that it exercises over me. It asks all the questions; the terms are difficult and I really like to be able to be in control. How was I supposed to know that the answer to this question is function or to that is der_equal and so on? I like the combination of Maple and this system; it is just the terms and the control I do not like. If you could fix the terms and reverse the control then I think it would be a good program.

As can be seen from these comments, which appear to be in line with those of the first study, the student is estranged by the terms used as well as by TeLoDe's control. Nevertheless, despite these shortcomings the student is able to 'see through' and appreciate the 'philosophy' of TeLoDe. This is an encouraging result. The lecturers, however, did not change their minds. This could be attributed to the given reason, that the teaching approach of TeLoDe was not congruent with how they teach differential equations.

10.9 The experiment in Lambeth College, London

Following session 1, a meeting was arranged a week later to discuss ideas and plan the way that the lecturer would use TeLoDe in the classroom. In this case, the lecturer had decided to adopt a teacher centred approach. The reasons were twofold:

- his students were inexperienced in the use of CAS

- the idiosyncratic interface/terminology.

The lecturer had further decided that in this case a computer, running TeLoDe, would be linked to an overhead projector so that it can be seen by the whole class. The class size was 12 students. The researcher played the role of the assistant, controlling the computer.

The lecturer further commented, regarding his decision to adopt a teacher centred approach:

> I like the way the system seems to use background knowledge and tries to jog the student's memory. It reminds them that any problem in mathematics is jogging the memory and you have to think what you know and how that can help you to solve the new problem. However, as your system seems, to me, to combine more than one 'new' factor (Maple, TeLoDe, novel approach in problem solving) and its idiosyncratic terminology, it seems only prudent to me to limit some of these factors. In my opinion this way (teacher-centred approach) will help the students to concentrate on TeLoDe and its novel approach in the teaching of differential equations.

10.9.1 The experiment

On the day of the experiment, after the initial set-up was completed, the lecturer gave a small introduction to TeLoDe explaining its rationale and objectives. He explained that TeLoDe is trying to teach the solution of linear second order ordinary differential equations with constant coefficients in an alternative way.

TeLoDe was initiated by the researcher and the lecturer started the lesson. At this stage the students appeared to be taken by surprise by the terminology. (TeLoDe was using the method of analogy and asking the question for a background concept, linear first order differential equation with constant coefficients). The students asked the lecturer what is meant by the term 'type of form of solution' and why they had not seen a differential equation yet? One student said:

> Why does it not show a differential equation?

The lecturer pointed out that TeLoDe would try and explain the terms used. (The researcher here entered the command 'explain'). Despite prompting from the lecturer it

was not possible to make the students deduce that in the case of a quadratic equation the answer is a number and thus could be said that it is of numeric type. Having finished with the explanation, TeLoDe came back to the original question. The students were not able to answer the question.

The researcher pressed return (indicating no answer). TeLoDe then moves into 'auxiliary mode' trying to resolve the problem with background knowledge. In this case TeLoDe asks for the relationship between the unknown and the solution. Here the lecturer, wrote on the board a quadratic equation and next to it a simple linear first order ordinary differential equation with constant coefficients. The lecturer carried on to ask explicitly what is the unknown in the quadratic, what is the solution and if a relationship can be drawn. After some prompting, some students were able to say that the solution depends on the unknown (some other students appeared bemused), but they could not answer the question for the differential equation. As such TeLoDe was forced to exhaust all its methods and tactics for this problem and reveal that the answer here is function.

Following this TeLoDe moved to the next declarative procedure, where the objective was to identify that the function and its derivatives differ only by a constant. Of course the terms property of the form of solution did not help. However, the 'explain' facility did help. Invoking the explain facility, the analogy is drawn between the logarithmic function and its arguments (must be positive). Most of the students acknowledged this and when the lecturer asked why, one of the students said:

> Well if you draw the graph of the logarithmic function you will see it.

another student said:

> If you ask in the calculator the logarithm of -2 it says ERROR; you cannot find it!

However, when TeLoDe returned back the students could not immediately spot that in this case the derivative operator was the key to identifying the property of the function. When the problem was broken down into smaller pieces by the lecturer with suitable examples then one or two students started to notice a pattern, but still could not verbalise it. When this pattern was verbalised for them by the lecturer those students appeared to move their heads in recognition. The lecturer asked the researcher to enter the answer der_equal and TeLoDe moved to the last question, which is the value of the

type of form of solution. Again the terminology was strange to the students, but the lecturer drew student's attention to the overall problem. He stated:

> So we want to solve a differential equation; we have found out that the solution looks like a function and has a specific property; itself and its derivatives differ only by a constant. How many functions do we know?

To which the majority of the students replied:

> Polynomials, trigonometric, logarithmic and exponential function.

So, the lecturer continued

> Let's see which function is our function?

The lecturer continued to try one by one the functions to see for which function the property was true. The students participated in this exploration and all students agreed that it was only the exponential function that satisfied the property.

That indicated the end of the experiment and the lecturer asked the students what was their opinion of TeLoDe as well as whether they would have preferred one that emulated the traditional way of teaching?

One student replied:

> Well it is different from how I was used to do it. When do you solve the equation?

Another one said:

> I would prefer the system that does it the traditional way because it breaks down the problem for you.

Another student said:

> Well it is more difficult this way; I would prefer a system doing it the way we had done it before. It is easier that way.

However, another student said:

Doing it this way (with TeLoDe) could be difficult at first, but I think it is very nice because it helps you to make a connection between the quadratic equation (here the student refers to the auxiliary equation) and the differential equation. In the past we were given a formula and we did not even know how it was derived. This program helps you to understand how it is done and also helps you with other background knowledge. So it is OK, yeah it is nice.

10.10 Analysis of the results of the second empirical study

The results of the second small scale formative evaluation overall are in accordance with the results of the first empirical study. Whilst anticipated problems with the interface and terminology used were anticipated, the participants were, nevertheless, able to 'see through' and understand the rationale of TeLoDe.

Also, this study demonstrated that it was possible to incorporate TeLoDe in a classroom situation where some discussion and exploration was be generated. Of course the known idiosyncrasies of TeLoDe prevented it from being used by students directly as a learning tool. However, even in a teacher centred approach it was able to stimulate some discussion and make students revisit background knowledge and view it from different perspectives. In that sense, as the lecturer said, TeLoDe helped to 'jog' students' memory and make them think.

Regarding students' reaction to TeLoDe (second aim of this study), the results confirmed again the idiosyncratic terminology and interface. The result from Manchester clearly demonstrated that TeLoDe as it stands requires a mediator and that was the overall result from London as well. Moreover, since TeLoDe was used by the lecturer in a teacher-centred approach, it did not really provide any answers to this issue. As such there is no evidence about TeLoDe's adaptability or whether the changes between strategies either alienated the students or were beneficial to the students.

However, what is very encouraging is that in both cases, Manchester and London, the students were able to understand and appreciate TeLoDe's rationale (third aim of this study). All students acknowledged that the approach in teaching adopted was a novel one and was appreciated once the interface issues were overcome.

10.11 Conclusions

This chapter presented the aims and results of two small scale formative evaluations. In the first study, the participants were professionals (university lecturers) in related fields whereas in the second study, TeLoDe was tried out with students.

In particular, in the first study the aims were to:

- gain an insight about the SIMTA framework

- explore whether TeLoDe could be effective as an educational tool.

The subjects of this study were the university lectures and their comments were considered overall as positive on both accounts. However, some concerns were expressed about the interface and the terminology used in TeLoDe.

The aims of the second study were to:

- explore the potential use of TeLoDe in a real classroom situation

- gain an insight into how students might react to the alternative use of CASs in problem solving as encapsulated by TeLoDe

- explore students' reaction to the novel way proposed for teaching the solution of linear second order ordinary differential equations with constant coefficients.

The results of this study demonstrated that users had some difficulty interacting with TeLoDe. However, since TeLoDe was a prototype its interface was not appropriate for normal student and lecturer use. Despite these difficulties, the study clearly demonstrated proof of concept and confirmed TeLoDe's potential as a teaching tool for problem solving in university mathematics in a novel way.

CHAPTER 11

Chapter 11: Conclusions

This final chapter presents a summary of the thesis and the main outcomes of the research. This research has touched upon two subjects: teaching strategies and Computer Algebra Systems (CAS) in teaching problem solving in university mathematics. The two subjects are brought together to underpin TeLoDe, an Intelligent Tutoring System for problem solving in university mathematics.

The aims in developing TeLoDe are as follows:

- to demonstrate the feasibility of SIMTA, i.e., that the teaching strategy module of an Intelligent Tutoring System such as TeLoDe can be informed by SIMTA

- to demonstrate a novel approach of using Computer Algebra Systems in teaching of undergraduate mathematics.

This chapter is divided into four sections. A summary of the thesis is given in §11.1, followed by a detailed analysis of the contributions in §11.2. In §11.3 ideas for further work are outlined. Some of these ideas draw on limitations illustrated by the implementation of TeLoDe. Section §11.4 is an account of the summing up of this research where the researcher's view is put forward.

11.1 Summary of thesis

This thesis has approached the issue of teaching strategies in a bottom-up fashion. That is, having reviewed appropriate work from the field of Intelligent Tutoring Systems and then branched out to the related fields of Educational Psychology and Cognitive Science, a number of design considerations were identified upon which the theoretical framework SIMTA was based. In this bottom-up approach all components of SIMTA, *style, method, tactic* and *actions* have been given definitions, and their inter-relationships as well as their relationship to the contemporary concept of a teaching strategy have been stated.

SIMTA's teaching strategy is defined in terms of a generic triple structure, *method, tactic(s), action(s)* and, for the teaching strategy to operate, a *style* has to be defined. A method is responsible for structuring the subject matter, a tactic is responsible for controlling the interaction with the student and actions are low-level activities required for computer tutors. The role of SIMTA's teaching strategy is to offer alternative

representations of the *same* topic and thus encourage learning by *activating, directing* and *maintaining* exploration as advocated by Bruner [1966]. The objective of the teaching strategy here is merely concerned with structuring the subject matter and controlling its interaction with the student at a high level. The exact manifestation of both methods and tactics depends on the *style*.

To demonstrate that the SIMTA framework is capable of informing a teaching strategy module, two styles were defined. The *expository style*, which draws on Ausubel's work and the *guided discovery style* which draws on Bruner's work. Furthermore, the definition of two styles demonstrated how two styles are *distinct* and the impact of a style in the manifestation of the methods and tactics.

One of TeLoDe's objectives is to define another potential role for Computer Algebra Systems in university mathematics. TeLoDe is the augmentation of Maple with declarative knowledge and a teaching strategy module. The topic selected as a case study is that of solving linear second order ordinary differential equations with constant coefficients. The teaching proposed considers Maple's solving capabilities to be central and focuses on the declarative aspects of the solution as well as explaining the logic and the reasoning involved in achieving the solution.

A small scale empirical study with lecturers of Mathematics, Mathematics Education and Educational Technology was carried out. The objective of this study was to demonstrate the SIMTA framework as well as the alternative role that Maple was given in TeLoDe. Since the prototype TeLoDe is ultimately aimed for use with undergraduate students, a further study was carried out. In this study, given the prototype's idiosyncratic phraseology, a lecturer acted as a mediator between TeLoDe and the students.

11.2 Contributions

Overall, this thesis has

- provided a deeper understanding of the complex and dynamic concept of teaching strategy and its relationship to multiple teaching strategies, and

- has demonstrated an alternative use of Computer Algebra Systems in university mathematics.

The contributions to teaching strategies are a result of the process involved in designing and defining SIMTA, its elements and their relationship as well as their instantiation through TeLoDe.

In the case of the Computer Algebra Systems, the contributions are a result of the process involved in developing and implementing the novel approach for teaching the solution of linear second order ordinary differential equations with constant coefficients. This approach assumes that the solution is provided by the CAS and, thus, its purpose is to propose alternative objectives involved in teaching the solution of the given topic as well as topics with a similar structure to the given topic.

The research on teaching strategies, which are examined through the SIMTA framework, have contributed to two main research fields. Firstly there have been four contributions to the field of Intelligent Tutoring Systems. These are:

- Proposes a vocabulary describing the operation and organisation of teaching strategies in the paradigm of multiple teaching strategies

- Proposes a formalisation of teaching strategies

- Clarifies the paradigm of multiple teaching strategies

- Provides an explanation for the *how to say it?* question.

Secondly, there have been two contributions to the field of Educational Psychology. These are:

- Provides a methodology for the analysis of teaching styles

- Provides a methodology for the synthesis of teaching styles

Each of these contributions is now discussed in details below.

Proposes a vocabulary describing operation and organisation of teaching strategies in the paradigm of multiple teaching strategies

In SIMTA the *method* is concerned with the structure of the subject matter to form a meaningful structure; the *tactic* is concerned with the interaction of the meaningful structure with the student which is achieved by the *actions*. The exact form of the

meaningful structure and the meaningful interaction is determined by the *style*. A *method*, a *tactic* and its *actions* form a *teaching strategy* whose operation is determined by the *style*. Change at either level of methods or tactics results in a different strategy, thus resulting in a number of *different* teaching strategies or *variations* of the same teaching strategy respectively. Thus *multiple teaching strategies* in SIMTA merely implies that there is more than one strategy.

The *style* is *distinct* and is comprised of a type of learning and a set of principles. It is the *style* that makes the *manifestation* of the *teaching strategies*, operating under that style, *distinct*. It is the *style* that ensures that a teaching strategy based on the method of analogy under *style_1* is different to that under *style_2*. In other words, the framework shifts the emphasis from how one arrives at an analogy, to how one uses that analogy. For example, consider the teaching strategies, based on the method of analogy, for the type of the form of the solution in the case of the linear second ordinary differential equations with constant coefficients, under the expository and guided discovery styles (see figures 11.1 and 11.2).

in tactic 1
What is the type of the form of the solution in the linear first order ordinary differential equation with constant coefficients?

Figure 11.1: An excerpt from TeLoDe's output under the expository style

in tact 13
What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?

in tacti 12
Can you think of another linear ordinary differential equation
whose only one derivative is first_order for which the question
What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?
is of similar value ?

Figure 11.2: An excerpt from TeLoDe's output under the guided discovery style

In the case of the *expository style*, the analogy points to the linear first order differential equation with constant coefficients. The question asked identifies the analogous concept explicitly. However, in the case of guided discovery style, the student is expected to discover the analogical concept first before there is any further interaction.

Proposes a formalisation of teaching strategies

The SIMTA framework provides a formalisation, in terms of a description, for both the *role* and *structure* of teaching strategies. As a result of SIMTA, teaching strategies can be viewed at three levels: the *abstract*, the *manifested* and the *efficacy* ones. The *abstract level* deals with the definition of a teaching strategy, the *manifestation level* deals with the implementation of a teaching strategy whereas the *efficacy level* deals with the effectiveness of the teaching strategy (this level has not been considered in this work).

At the *abstract* level, a teaching strategy is said to have been defined *if and only if* a *method* for handling the subject matter and a *tactic* handling the interaction have been selected. (As the actions are considered low level mechanisms they are not important at this level). This level is independent of the *style*. For example, in the case of methods, the options are in the form of analogies, examples, definitions, generalisations, and so on. The interaction can be viewed from an implicit viewpoint, from where minimal information inferences can be drawn, to an explicit viewpoint, where inferences are drawn and explained, or in between. Therefore, if analogy is the preferred method and implicit is the preferred tactic then it is said that a teaching strategy has been defined.

At the *manifestation* level, the teaching strategy, has to provide directions on how exactly the method and the tactic are to be implemented. This level is *style* dependent, i.e. a teaching strategy is differently manifested under different styles. For example, the conditions for an analogy under the expository style are different to those under the guided discovery style. Likewise for the tactics. Moreover, at this level it has been demonstrated whether all teaching strategies (i.e., methods, tactics and actions) defined at the abstract level can actually be implemented.

At the *efficacy level*, the teaching strategies that have been implemented, should be examined for their pedagogical effectiveness. This level requires extensive field work, involving systems such as TeLoDe being used in real classroom situations.

Clarifies the paradigm of multiple teaching strategies

SIMTA's teaching strategies are by no means autonomous ones. So for example, a teaching strategy based on the method of analogy cannot be used all the time to carry out teaching in the classroom, since there will be cases where an analogy simply does not exist. The same is true for all teaching strategies informed by SIMTA that are based on any method. Consequently these teaching strategies are not multiple ones at the level of classroom teaching. In fact at the level of classroom teaching, SIMTA's teaching

strategies should be viewed as *complementary* ones, in the sense that not all of them may be defined all the time and also if one fails then another one takes over. In other words, to carry out whole classroom teaching, strategies based on methods of analogy, example, investigation, generalisation, specialisation will be required.

However, if teaching is examined at the tasks level, then SIMTA's teaching strategies could be seen as multiple teaching strategies since, for each task, a number of different teaching strategies, based on methods, may be formed. The emphasis is on 'could', since there exists a number of design considerations for SIMTA. First, the *role* of a teaching strategy, informed by SIMTA, is to offer alternative representations of the *same* task at hand and its *objective* is to *activate*, *direct* and *maintain* exploration. Second, given the teaching strategy's generic structure (method, tactic, action) which is manifested according to the definition of a style, the emphasis is on identifying what *drives* various manifestations of what is essentially seen as the same strategy. Furthermore, the objective is extended by incorporating under a style, strategies that were considered to be diverse ones, such as student-centred or teacher-centred or discovery-based or expository-based. This is achieved by grouping only compatible or congruent manifestations of these diverse strategies.

The most significant implication of the SIMTA framework, therefore, is not only that it appears to, at least, question the DOMINIE implementation but that it appears to describe how the one 'all powerful' teaching strategy may be structured and operate. If this is the case, then that strategy is not a monolithic one and thus in need of accommodating more than one of them. That strategy operates in the paradigm of multiple teaching strategies and also combines diverse strategies.

Moreover, SIMTA's structure appears to be able to accommodate Douglas' results from an empirical study [Douglas, 1991]. For example, if the preferred strategy of a teacher is based on the method of analogy, and the teacher always tries to make the student try out hypotheses, then an implicit tactic could be selected. *Repair of that strategy will* happen by using more tactics that provide more information to the students and if that strategy fails, then another, different strategy, based on the method of examples, could be used.

Provides an explanation for the how to say it? question.

SIMTA's teaching strategy is represented by a generic structure which is manifested in accordance with the style under which it operates. Moreover, given that the style is defined in terms of a *type of learning* and *principles* which, in turn, are used to manifest the methods and the tactics, then, by simply back-tracing these principles an explanation for the behaviour of teaching strategies is achieved. Thus it is possible to reason why this concept was picked up for an analogy, as opposed to another one, but more importantly for the first time it is possible to reason for the *how to say it?* question. For example consider the excerpt below from TeLoDe running under the expository style.

in tactic 1
What is the type of the form of the solution in the linear first order ordinary differential equation with constant coefficients?

The question is to identify the type of form of solution for the linear second order ordinary differential equation with constant coefficients. TeLoDe under the expository style selects the concept of linear first order ordinary differential equation with constant coefficients since that was the closest concept that was known to the student and which shared the same answer as the equation in question. As in the expository style, it is stipulated that the information to the student must be in final form, as discussed in Chapter 6, it is pointed out to the student explicitly that the objective is the 'type of form of solution' and the concept from the background knowledge is also given. In other words, the student's attention is drawn on the concept and its required attributes. By contrast, in the excerpt below, where TeLoDe was executed under the guided discovery style, the concept from the background knowledge is not given to the student but rather the student is invited to deduce it.

in tacti 12
Can you think of another linear ordinary differential equation whose only one derivative is first_order for which the question
What is the type of the form of the solution in the linear second order ordinary differential equation with constant coefficients ?
is of similar value ?

In both cases the learning theories that form the backbone of the respective style provide the explanations for TeLoDe's action.

The SIMTA framework provides a methodology for the analysis of teaching styles. Such an analysis would assist in forming teaching strategies that adhere to that style. SIMTA would assist in identifying the type of learning as well as the operational principles (i.e., affecting the way that the methods and tactics operate) and thus form teaching strategies that operate in that style. For example, style_1 could prescribe that, for an analogy to be formed, the analogical concept must satisfy criteria cr_1, cr_2, cr_3 and so on. However, under style_2, the criteria for selecting an analogical concept may be just cr_1 and cr_2. Thus, concepts that were rejected under style_1 are now valid under style_2. Likewise, the workings of the tactics are affected.

If a teaching style is to be based on a specific learning theory, then the definition of two styles can be used as a methodology that would enable one to 'read' the theory with certain issues in mind. These could include principles for structuring the subject matter, principles for motivation, principles for sequencing teaching strategies and so forth.

Provides a methodology for the synthesis of teaching styles

In classroom teaching, the style that is being pursued is not visible. What is visible are the teaching strategies that have been manifested under that style. Using the SIMTA framework, it is possible to analyse classroom teaching and, thus provide a style that could produce the teaching strategies observed. Such an exercise is useful if one would like to make a comparison study on classroom teaching and where teachers are reluctant to box themselves in a particular stereotype for whatever reasons. Moreover, such an exercise could prove beneficial in teacher training where the teaching strategies are analysed to form a style which is then input to a computer tutor for the trainee to observe and comment on the analysis of their style.

Having outlined the contributions within the theme of teaching strategies using the SIMTA framework, we will now turn to the contributions within the theme of CAS using the TeLoDe system. There are three contributions to the fields of Educational Technology and Mathematics Education. These are:

- Augmentation of Maple

- A novel approach to the use of Information Technology in teaching problem solving in undergraduate university mathematics

- Novel approach to teaching the solution of the linear second order ordinary differential equations with constant coefficients.

The first one, is a contribution to the field of Educational Technology whereas the second one is a contribution of Educational Technology in Mathematics Education. The final one is a contribution to the field of Mathematics Education. These contributions are discussed in detail in turn below.

Augmentation of Maple

The TeLoDe system provides an example of how Maple and similar packages could be augmented to address the pedagogical limitations that were pointed out in the review of these packages.

TeLoDe is an example of how Computer Algebra Systems could be transformed, not only providing answers to problems, but also teaching how the solution is to be achieved. As a result of the augmentation Maple is now potentially capable of actively interacting with the student and providing information about how the solution of the problem is approached. The design considerations of TeLoDe go some way towards addressing the inherent pedagogical limitations of Computer Algebra Systems. The efficacy of such augmentation is, however, beyond the scope of this research and requires a lot of background work before we are in a position to answer it confidently and conclusively.

A *form* of the TeLoDe implementation could be seen as a "blue print" for a second generation Computer Algebra Systems, with pedagogical considerations in mind. The emphasis is on the form, since the current form of TeLoDe could only be seen as rigid and not a flexible system.

A novel approach to the use of Information Technology in teaching problem solving in undergraduate university mathematics

The research proposes a novel approach for teaching solution of linear second order ordinary differential equations with constant coefficients. The proposal views the human teacher and the computer as complementary. This is because the role of TeLoDe

is not to emulate current approaches for the solution of these equations, but rather to augment Maple's ability by incorporating declarative knowledge as well as a teaching strategy module informed by SIMTA. Moreover, the approach proposed here could be carried by the human tutor who will rely on Maple to provide the answers to the problems. The analogy here is that of the use of calculators where the students are taught the principles of addition, subtraction, etc., but the operations are left to the calculator. In the case of Maple and other Computer Algebra Systems, there is a further need, that of ensuring that the solution given by the package is the correct one. Given the complexity of the problems that are tackled by Maple and other CAS, it is not unusual to have mistakes. Thus, it is imperative that students are equipped to use the package as a tool which they control, not as a tool they depend on.

> *Novel approach of teaching the solution of the linear second order differential equations with constant coefficients*

The novel approach proposed for teaching the solution of these differential equations, was warmly received by experts during the empirical test of TeLoDe.

Whilst the emphasis on teaching these differential equations was on how to solve them, the objectives were to recognise the type of the differential equation and apply a certain formula/procedure to achieve the solution.

By contrast, the rationale for the approach implemented in TeLoDe, reflects the concern of justifying the steps involved in the solution and also shows how the concepts could be used to explain these procedures. The teaching on which TeLoDe was developed, involves justifications at every step and a logical case is presented for the process involved in reaching the solution. Coupled with the fact that Maple is capable of achieving the solution, the emphasis of this novel approach is to explain the presence of the "exponential" and thus provide a "check" mechanism to Maple's answer. In other words, the emphasis is not on what the answer is but rather on why this is the answer to the problem.

The proposed approach could be carried out without TeLoDe and just with Maple as it currently exists. Then the role of the human tutor would be to emphasise the form of the solution and Maple would provide the answer, which the students could then check to see if the exponential function is present or not. Thus, the role of the human tutor and the CAS are seen as complementary.

11.3 Further work

In this section some ideas for further work are outlined. These ideas for further work could be classified under two main headings: Artificial Intelligence issues and Teaching Strategies issues. Some of these ideas have been touched upon already in the development of the prototype TeLoDe, whereas, others can be seen as a reflection following the development. However, before these ideas are discussed, it is important to outline how TeLoDe is to be transformed from a prototype to an educational tool.

11.3.1 Transforming TeLoDe

First, the rest of the steps required for attaining the general solution of a linear second order ordinary differential equation with constant coefficients will have to be implemented. These are:

- forming the complementary function and

- finding the particular integral.

All the background work required for these steps, i.e., the primary concepts, their kinds, secondary concepts, has been completed, as discussed in Chapters 7 and 8. The steps required for this task are as follows:

- expand the knowledge base by incorporating relevant concepts and all their links,

- add the appropriate *filter*, *primary concept* and *kind* to the relevant declarative steps

- produce the code for carrying out these steps

- debug TeLoDe to ensure that new code has not introduced any bugs to existing code.

Successful completion of all the above steps means that TeLoDe will be able to handle completely the problem of solving linear second order ordinary differential equations with constant coefficients. However, as both empirical studies have showed there is a need to resolve the phraseology used in TeLoDe.

Resolution of this issue requires collaborative work between an educational technologist and a mathematician (lecturer in mathematics). The educational technologist must have a mathematics background as well as sound programming knowledge. The problem stems from the fact that even during the empirical study with the lecturers, it was necessary to explain the terminology. The objective of this collaboration will be to produce a terminology for this novel approach of teaching the solution of the differential equations. Such terminology would not only appear natural but would also adhere to Artificial Intelligence principles, as far as programming is concerned. For example, the term 'type of form of solution', reflects the need to prompt the user of TeLoDe to reach a higher level of abstraction, by characterising the solution of the differential equation.

If such a set of terminology is produced that satisfies both the educational technologist and the mathematician, this terminology will have to be used in a classroom situation to see what implications this might have for students. This would be an indicative test as to whether the terminology issue has been resolved or not.

11.3.2 Artificial Intelligence related issues

Define a neighbourhood

During the empirical test, one of the subjects commented that he would have liked to be able to ask TeLoDe an example of the concept that he was asked about. Whilst this request is a straightforward one, it gives rise to a more interesting issue. At any moment during the interaction, TeLoDe deals with a certain concept. At that point, the user should be able to ask questions and help. The problem posed here is how to define the space where the user is allowed to ask questions and request hints. For example, dealing with the concept of quadratic, all attributes of that concept should be accessible to the user e.g. coefficients, unknown, solution, order. However, these are also concepts and, in turn, have attributes and so on. If the rule adopted here is that for every concept its attributes must also be present, then that extends to cover a substantial part of the knowledge base.

The issue here is, given a topic or concept, what constitutes immediately relevant information, less immediate relevant information, and so forth. What are the principles that direct this operation? As the title suggests one possible way of approaching this

problem is the concept of neighbourhood in limits. That is, by approaching the limit from above or below a neighbourhood can be defined.

Issues of tautologies (a possible solution could be congruence classes)

Another issue that was partially addressed during the development of tactics for the guided discovery style, is establishing a number of equivalent ways for referring to a concept without naming it. Currently, the solution is 'hardwired' in the sense that given a concept, it could be defined in terms of its parent with qualifications, or its parent with qualifications followed by an instance or by referring to that concept itself. However, the objective is to establish principles that oversee the structure of the congruence classes, i.e., these principles should be derived instead of being hardwired.

11.3.3 Teaching strategies issues

Sequencing the teaching strategies in SIMTA

SIMTA currently has a rather simple approach to sequencing teaching strategies, that is, they are linear. The problem of sequencing, whilst known to us, was also picked up during the empirical test of TeLoDe. The sequence of teaching strategies should, without any doubt, be a dynamic one. One way of moving towards a more dynamic approach to the problem of sequencing the teaching strategies is the adaptation of a motivational model. A model for motivation as suggested by del Soldato [1994], [del Soldato & Du Boulay, 1995] could be used to provide a dynamic perspective at both the level of methods and tactics.

However, one has to be careful, since overconfidence, as pointed out by Mason [1979] is also a factor to consider and address. By the same token one must be aware of another problem. According to Holt [1969], a teacher during a maths lesson, initially tries to address Ruth's 'I do not understand'. However, the teacher, after a while, realises that Ruth's motive is to reduce the problem to a 'yes' or a 'no' situation, where her effort would be minimal. This case will be referred to as the 'lazy' case since Ruth's impression that she does not understand is not a genuine one.

Consequently, there is a need for significant further research in this area, given the qualitative aspect that is addressed, if there is to be a successful distinction between the overconfident, lazy and, more importantly, the struggling states.

As TeLoDe stands, it moves from one topic to another via the tree structure built around the procedures and sub-procedures required to resolve the task at hand. Whilst this may be sufficient in the case of problem solving, if SIMTA is to be used in areas where there is no problem solving, then, a content planner is required. The content planner will provide SIMTA with the concept and the attributes that are required to initiate the process. PEPE, a planner that distinguishes between content and delivery has been proposed by Wasson [1990, 1992]. It has to be pointed out that in the case of SIMTA PEPE would only be used as a high level planner, since SIMTA, once activated, is able to select the concepts required to attain the task at hand. However, even in that case SIMTA could operate in conjunction with the content planner of PEPE and SIMTA in fact restrict itself to being the delivery planner of PEPE.

Extending tactics' functionality in TeLoDe

Artificial intelligence techniques in the implementation of tactics in TeLoDe are at the minimal level. The benefit of the SIMTA framework is its clarity in its structure and its ability to demonstrate how both methods and tactics are affected by the style. In the case of methods, one can easily distinguish between the generic level of the analogy method and the style influenced level of the analogy where, in the case of the expository style, the principles of logically meaningful and potentially meaningful are applied. This is not so clear in the case of tactics. The tactics as exist in TeLoDe, are the first of three levels of complexity. These levels can be defined as follows:

1) A one dimensional categorisation related to a particular style

2) A multidimensional categorisation which is utilised by style related rules

3) A semantically tagged model manipulated by rules for individual styles.

In the first case the system has an explicit model based on expl ... expl+6 (expository style), impl ... to expl (guided discovery style) which are defined in the context of a given style. This categorisation is therefore different for the expository and guided discovery styles. This is currently implemented in TeLoDe.

In the second case, a similar categorisation (as above) is used upon multiple dimensions. For example, impl $\rightarrow$ expl, concrete $\rightarrow$ abstract. This categorisation is then

used by style dependent rules. But the categorisation itself does not need to be changed for different styles. For example, in this case pairs such as (impl, concrete), (impl, abstract) ... (expl, concrete) and so on would be sufficient to identify the tactic that is required.

In the third case, each tactic, whether canned or generative, has an associated representation of its semantic content and its communicative intent. A set of rules uses this information in conjunction with the principles of the style to select a tactic appropriate for the style and current context. For example, in a discovery style, rules may select a tactic which identifies a counterexample intended to show the students a condition that has been omitted in their reasoning. If the same situation arose in the expository style then the tactic would identify a counterexample intended to show the condition as well as to point out the condition and how it relates. The tactics in either case would have looked as follows:

tactic counter-example	tactic info_state
principle: do not give answer	principle: final form
principle: relate information	principle: disclose
current_example: X - omitted	current_example: X - omitted
new_example: X +omitted	new_example: X + omitted
style(omitted)	*style(omitted)*

The left hand-side represents the discovery style whereas the right hand-side represents the expository style. To select either tactic the style has to provide the *principles* on which it operates and thus the appropriate tactic is selected to satisfy the needs of the task under the principles of the style. This is how the tactics in TeLoDe could be developed to reflect the generic and style-dependent states of the tactics.

SIMTA provides a theoretical basis for further studies of teaching strategies

SIMTA brings together previous work carried out in Intelligent Tutoring Systems and related disciplines, reorganising it into a framework which enables us to more

accurately define the nature of teaching strategies. As a consequence of this, a base for a principled and systematic study is provided. This enables moving forward and making progress in an area of research where development has been restricted. SIMTA proposes certain theoretical issues that need to be examined, firstly at an *abstract* level, then at a *manifestation* level and finally, at an *efficacy* level.

At the *abstract* level, a deeper and more precise definition of the elements of SIMTA (style, methods, tactics and actions) will be sought. For example, methods such as analogy, generalisation, specialisation etc. will need to be researched to investigate their nature as well as the conditions that a subject matter has to satisfy for any of these methods to be defined. Regarding the nature of the methods, research could concentrate on identifying whether analogies, examples, etc., are primitive structures which can not be analysed further or, otherwise, identify these basic structures that make up an analogy, example, etc. Likewise, research might reveal that methods such as generalisation or specialisation rely on the use of other methods, such as analogies, examples, definitions which are sequenced in such a way as to achieve the desired effect. Further research on tactics will yield a more precise definition of an explicit or an implicit tactic as well as precise conditions that need to be satisfied for the operation of tactics.

At the *manifestation* level, research would concentrate on the use of existing Artificial Intelligence techniques, or will propose new ones, that are required to implement the style, methods, tactics and actions. This implementation will have to preserve the generic nature of the SIMTA framework as well as demonstrating how easy or difficult it is to emulate human-tutor-like dialogues.

At the *efficacy* level, extensive empirical studies will be carried out to identify which teaching strategies are found effective as well as the conditions under which these strategies are successful. Such results could be used to fine tune a model of motivation used in deciding the sequence between the alternative teaching strategies as well as identifying optimal teaching strategies for particular tasks.

The above research would contribute to a better understanding of the complex nature of teaching strategies but would also provide clear guidelines for the implementation of an authoring system where the user will be guided into creating teaching strategies that could be used by a computer tutor. This authoring tool would guide the user in

defining a style, selecting methods, tactics as well as informing them on the precise form that the knowledge base must take in order to support the chosen style, methods and tactics.

11.4 Researcher's view

My original intention was to build an Intelligent Tutoring System, utilising CAS as an integral part of its knowledge base. An essential part of an Intelligent Tutoring System is its teaching strategy module. My review of teaching strategies in existing Intelligent Tutoring Systems showed a lack of an overall framework that I could use for my own system. Therefore, I was obliged to research the topic of teaching strategies and to develop such a framework (SIMTA) before I could begin to build my own system.

My achievements in this study have primarily concentrated on developing a theoretical framework, but I have also implemented a prototype and tested it empirically. My most significant contribution is in the end, not the Intelligent Tutoring System I originally planned, but the unpacking of the heretofore monolithic concept of a teaching strategy. I defined two fundamental levels: operational (represented by the style) and organisational (represented by the triple generic structure, method(s), tactic(s) and action(s)). I also demonstrated conclusively that the concept of multiple teaching strategies is not tenable, and that the monolithic concept of a teaching strategy is in fact made up of a number of teaching strategies which are congruent to the style.

I have proposed an alternative use of Computer Algebra Systems which exploits their full potential and require a rethinking of what and how mathematics is taught to students. While it is known that Computer Algebra Systems' knowledge is limited to procedural knowledge, I am in no way implying that procedural knowledge is abandoned, but I am implying that the benefits for the student of being exposed to procedural knowledge are rethought and focused with the use of Computer Algebra Systems.

The primary aim of the implementation through TeLoDe was no longer to build and test a usable system, but to help me clarify my understanding of teaching strategies as well as the structure of mathematical knowledge required to support the alternative use of Computer Algebra Systems.

References

Alexandrou, N., *Personal Communication*, University of North London, 1992

Anderson, J. R., Farrell, R., Sauers R., Learning to Program in Lisp, *Cognitive Science*, Vol. 8, pp. 87-129, 1984

Anderson J. R., Reiser B. J., It approaches the effectiveness of human tutor, *Byte*, April 1985a

Anderson J. R., Boyle C. F., Reiser B. J., Intelligent Tutoring Systems, *Science*, No: 228, 1985b

Angelides M. C., Tong A. K. Y., Implementing multiple tutoring strategies in an intelligent tutoring system for music learning, *Journal of Information Technology*, Vol. 10, No: 1, pp. 52-62, 1995

Ausubel D. P, *The psychology of meaningful verbal learning*, Grune & Stratton. Inc., 1968

Ausubel D. P., Novak J. D., Hanesian H., *Educational Psychology: A cognitive view* 2nd edition, 1978

Ausubel, D. P., Robinson F. G., *Readings in school learning*, New York: Holt, Rinehart and Winston, Inc., 1969

Beck, J., Stern, M., Haugsjaa, E., Applications of AI in Education, *ACM Crossroads*, September 1996

Blandford, A., *Design Decisions and Dialogues*, Unpublished PhD Thesis, CITE, The Open University, 1991

Brady L., *Models and Methods of Teaching*, Prentice Hall, Australia, 1985

Bruner S. J., *Towards a Theory of Instruction*, Harvard University Press, 1966

Bruner S. J., Olver R. R., Greenfield P. M., *Studies in Cognitive Growth*, Wiley 1966

Bruner S. J., *The Process of Education*, Harvard University Press, 1977

Bruner S. J., *The Relevance of Education*, Redwood Press Ltd, 1971

Burkill C. J., *The Theory of Ordinary Differential Equations*, Oliver & Boyd, 1962

Carbonell R. J., AI in CAI: An Artificial-Intelligence Approach to Computer-Assisted Instruction, *IEEE Transactions on Man-Machine Systems*, Vol. 11, No: 4, 1970

Carbonell R. J., Collins A. M., Natural Semantics in Artificial Intelligence, in *Proceedings of 3rd International Joint Conference on Artificial Intelligence*, Stanford, California, pp. 344-351, 1973

Char W. B., Geddes O. K., Gonnet H. G., Monagan B. M., Watt M. S., *Maple: First Leaves, A Tutorial Introduction to Maple*, 2nd Edition, Waterloo Maple Publishing, 1988

Char W. B., Geddes O. K., Gonnet H. G., Monagan B. M., Watt M. S., *Maple Reference Manual*, 5th Edition, Waterloo Maple Publishing, 1990a

Char W. B., Geddes O. K., Gonnet H. G., Monagan B. M., Watt M. S., *Maple: First Leaves, A Tutorial Introduction to Maple*, 3rd Edition, Waterloo Maple Publishing, 1990b

Cheng. S. Y., Kelly A. D., Maunder S. B., TRANSMATH- From Conception to Delivery, *Proceedings of International Conference in Hypermedia*, Sheffield, July 3-5 1995

Clancey W., Tutoring Rules for guiding a case method dialogue, in *Intelligent Tutoring Systems*, Sleeman H. D., Brown S. J., (eds.), Academic Press, New York, 1982

Clancey W., GUIDON, *Journal of Computer-Based Instruction*, Vol. 10 pp. 8-15, 1983

Coddington A. E., *An introduction to Ordinary Differential Equations*, Prentice-Hall, 1962

Collins A., Processes in Acquiring Knowledge, in *Schooling and the Acquisition of Knowledge*, Anderson R. C., Spiro R. J., Montague W. E., (eds.), Lawrence Elbraum Associates, 1977

Collins A., Stevens A. L., *Goals and strategies of interactive teachers*, Bolt Beranek and Newman Inc. Cambridge Massachusetts, Report No. 4345, 1980

Collins A., Warnock H. E., Passafiume J. J., *Analysis and Synthesis of Tutorial Dialogues*, Bolt Beranek and Newman Inc. Cambridge Massachusetts, Report No. 2789, 1974

Dean, G. P., *Teaching and Learning Mathematics*, The Woburn Press, 1982

del Soldato T., du Boulay B., Implementation of Motivational Tactics in Tutoring Systems, *Journal of Artificial Intelligence in Education*, Vol. 6 No: 4, 337-378, 1995

del Soldato T., *Motivation in Tutoring Systems*, Unpublished PhD thesis, School of Cognitive Science and Computing Science, The University of Sussex, 1994

Dearing, Sir Ron, *Higher Education in the Learning Society*, National Committee of Inquiry into Higher Education, HMSO, London, 1997

Department for Education and the Welsh Office (DFE/WO), *Mathematics in the National Curriculum*, HMSO, London, 1995

Devi, R., *Modelling Arithmetic Strategies*, Unpublished PhD Thesis, CITE, The Open University, 1991

Douglas S., Tutoring as interaction: Detection and repairing tutoring failures, in *Teaching Knowledge and Intelligent Tutoring*, Goodyear P.,(ed.), Ablex Publishing Corp., 1991

du Boulay B., O'Shea T., Monk J., The Glass box inside the Black box: Presenting Computing Concepts to Novices, *International Journal of Man-Machine Studies*, Vol. 14, 1981

Eggen P. D., Kauchack D. P., Harder R. J., *Strategies for teachers, Information processing models in the classroom*, Prentice Hall, Inc., 1979

Eklund J., Brusilovsky, P., Individualising Instruction in Web-based Instructional Systems in Higher Education, in *Proceedings of The Apple University Consortium's Academic Conference*, pp. 27-30, Melbourne, Australia, September 27-30 1998

Elsom-Cook M. T., (ed.), *Guided Discovery Tutoring: A framework for ICAI research*, Paul Chapman, 1990

Elsom-Cook M. T., Dialogue and Teaching Styles, in *Teaching Knowledge and Intelligent Tutoring*, Goodyear P., (ed.), Ablex Publishing Corp., 1991

Elsom-Cook M. T., Spensley F., Byerley P., Brooks P., Mhende M., Federeci M., Scaroni C., Using Multiple Teaching Strategies in an ITS, in *Proceedings of ITS-88*, Montreal, 1988

Foster K. R., Bau H. H., Symbolic Manipulations Programs for the Personal Computer, *Science*, Vol. 243, 1989

Goodyear P., Research in Teaching and The Design of Intelligent Tutoring Systems, in *Teaching Knowledge and Intelligent Tutoring*, Goodyear P., (ed.), Ablex Publishing Corp., 1991

Harding R. D., Lay S., Moule H., Co-operative cross-platform development, *Association for Learning Technology Journal, (ALT-J)*, Vol. 4, No. 1, 1996

Harper D., Woof C., Hodgkinson D., *A Guide to Computer Algebra Systems*, Wiley, 1991

Hembree R., Dessart D. J., Effects of hand-held calculators in pre-college mathematics education: a meta-analysis, *Journal for Research in Mathematics Education*, Vol. 17 No. 2, 1986

Hennessy S., *The Potential Of Portable Technologies For Supporting Graphing Investigations: Review Of The Literature*, CITE Technical Report, 1999

Hierbert J., Lefevre P., Conceptual and Procedural Knowledge in Mathematics, An Introductory Analysis, in *Conceptual and procedural knowledge: The case of Mathematics*, Hierbert J., (ed.), Lawrence Erlbaum Associates, 1986

Hodgkinson E. D., *Algebraic Computing in Education*, Dept. of Applied Mathematics and Theoretical Physics, University of Liverpool, (Also presented at the Conference: Mathematics and Statistics Curricula in Higher Education for the 1990's), 1987

Hodgkinson E. D., Macsyma 2.2: New Features, *Maths & Stats*, CTI Centre for Mathematics and Statistics, Vol. 8, No: 2, 1997

Holt, J., *How Children Fail*, Penguin Books, 1969

Hunt, E. D., *Matching models in Education: the co-ordination of teaching methods with student characteristics*, Ontario Institute for Studies in Education, 1971.

Ince L. E., *Integration of Ordinary Differential Equations*, Oliver and Boyd, 1956

Ince L. E., *Ordinary Differential Equations*, Dover Publications, Inc., 1926

Joyce B., Weil M., *Models of teaching*, Englewood Clifs, N.J: Prentice Hall, 1980

Kaplan, W., *Elements of Differential Equations*, Addison-Wesley Publishing Company, 1964

Karian A. Z., Sterrett A., Getting started with a Computer Algebra System, *The Mathematical Intelligencer*, Vol. 9, No. 3, 1987

Kelly A., Maunder S., Cheng S., TRANSMATH, CTICMS, *Maths & Stats* Newsletter, Vol. 5, No. 3, August 1994

Kelmanson M. A., Maunder S. B., Cheng S. Y., A Mathematica-based CAL Matrix - Theory Tutor for Scientists and Engineers, *Association for Learning Technology Journal, (ALT-J)* Vol. 1, No. 2, 1993

Kent P., Ramsden P., Wood J., *Multimedia for Teaching and Learning*, Times Higher Education Supplement pp. x-xi (London), 13 May 1994

Lapp D., Bender H., Ellenwood S., John M., *Teaching and Learning, Philosophical Psychological Curricular Applications*, Macmillan New York, 1975

Laurillard D., *Rethinking University teaching: A Framework for the Effective Use of Educational Technology*, Routledge, London, 1993

Laurillard, D., The pedagogical limitations of generative student models, *Instructional Science*, Vol. 18, pp. 235-250, 1988

Leinhardt G., Greeno G. J., The cognitive skill of teaching, *Journal of Educational Psychology*, Vol. 78, No: 2, 75-95, 1986

M101, *Units: Blocks II, III & IV, Maths Foundation Course*, The Open University, 1987

M203, *Introduction to Pure Mathematics, Unit 4: Differential equations and flows*, The Open University, 1979

MacCallum, M. A. H., REDUCE and its use in teaching, *Maths & Stats*, CTI Centre for Mathematics and Statistics, August 91

Major N., Reconstructing Teaching Strategies with COCA, in *Proceedings of AI-ED' 93*, Brna P., Ohlsson S., Pain H., Association for the Advancement of Computing in Education, 1993

Major N., Reichgelt H., COCA: A shell for Intelligent Tutoring Systems, in *Intelligent Tutoring Systems*, Frasson C., Gauthier G., McCalla G. I., (eds.) Springer-Verlag, 1992

Mason J., Which medium which message?, *Visual Education*, 29-33, 1979

Maurice St. H., A guide to commonplaces: on the use of loci in educator's discourse, Journal of Curriculum Studies, Vol. 23 No:1, 41-53, 1991

Michalski S. R., Carbonell, G. J., Mitchell, M. T., *Machine Learning: An artificial intelligence approach Vol. II*, Morgan Kaufmann, 1986

Mosston M., *Teaching: From Command to Discovery*, Wadsworth Publishing Company Inc., 1972

MST204, *Mathematical Models and Methods, Unit 06: Differential equations II*, The Open University, 1981

MST204, *Mathematical models and methods, Unit 02: Differential equations I*, Open University 1992

MST204, *Mathematical models and methods, Unit 03: Animal Populations: their growth and exploitation*, Open University 1992

MST204, *Mathematical models and methods. Unit 04: Newtonian mechanics in one dimension*, Open University 1992

Noss R., Final report of the Transitional Mathematics project/TRANSMATH evaluation, on WWW at ULR: http://othello.ma.ic.ac.uk/articles/Evaluation Report, 1995

O'Shea T., A self-improving quadratic tutor, *International Man-Machine Studies*, Vol. 11, pp. 97-124, 1979

Ohlsson S., Some principles of Intelligent Tutoring, *Instructional Science*, Vol. 14, 293-326, 1986

Ohlsson, Knowledge requirements for teaching: The case for fractions, in *Teaching Knowledge and Intelligent Tutoring*, Goodyear P.,(Ed), Ablex Publishing Corp., 1991

Ohlsson, S., System Hacking Meets learning theory: reflections on the goals and standards of research in Artificial Intelligence and Education., *Journal of Artificial Intelligence in Education*, Vol. 2 (3), 1991a

Ohlsson, S., Tell me your problems: A Psychologists visits AAAI82, in *AISB Quarterly*, 1982

Ohlsson, Towards Intelligent Tutoring systems that teach knowledge rather than skills: Five research questions, in *New Directions in Educational Technology*, Scanlon E., O'Shea T., (eds.), NATO ASI Series, Springer-Verlag London, 1992

PCW, A review of Mathematical Software, in *Personal Computer World (PCW)*, February 1997

Paraskakis I., *A user guide to muMATH and muSIMP. A package to solve second order differential equations with constant coefficients*, BSc project, The Polytechnic of North London, June 1988

Paraskakis I., *Computer Based Tutoring Systems, Theory and Applications (The case of second order differential equations with constant coefficients)*, MSc project, The London School of Economics and Political Science, September 1989

Paraskakis I., Transforming a Computer Algebra Package into a Teaching Tool, *Proceedings of 3rd Panhellenic Conference with International participation in Didactic of Mathematics and Informatics in Education*, University of Patras, Patras, Greece, 1997a

Paraskakis I., Design and Implementation of a Multiple Teaching Strategies module into Maple, a Computer Algebra System, *Proceedings of CAL-97*, University of Exeter, 1997b

Paraskakis I., Designing a multiple teaching strategies for transforming Maple into a teaching tool for Advanced Mathematics, *Proceedings of EARLI-97*, University of Athens, Athens, 1997c

Pountney D., Teaching with Macsyma 2.2 for Windows, *Maths & Stats*, CTI Centre for Mathematics and Statistics, Vol. 8, No: 3, 1997

Ramsden P., Mathematica in Education: Old Wine in New Bottles or a Whole new Vineyard, in *Innovations in Mathematics: Proceedings of the Second International Mathematica Symposium* (Rovamiemi, Finalnad, June 1997). Southampton UK, Boston MA: Computational Mechanics Publications, 1997

Reichgelt H., Shadbolt N., Paskiewicz T., Wood D., Wood H., EXPLAIN: On implementing more effective tutoring systems in *Proceedings of AISB93*, 1993

Resnick L., Holding an instructional conversation: Comments on chapter 10 by Collins, in *Schooling and the Acquisition of Knowledge*, Anderson R. C., Spiro R. J., Montague W. E., (eds.), Lawrence Elbraum Associates, 1977

Reuter E. H. G., *Elementary Differential Equations and Operators*, Routledge and Kegan Paul Ltd, 1958

Rich A., Rich J., Stoutemyer D., *DERIVE, A mathematical Assistant for your Personal Computer*, 3rd Edition, Soft Warehouse Inc., 1989

Ross L. S., *Differential Equations*, Blaisdell Publishing Company, 1964

Ruthven, K., Super calculators and the secondary mathematics curriculum, in *Teaching Mathematics*, Selinger, M.,(ed.), Routledge, 1994

Sacerdoti E. D., *A Structure for Plans and Behavior*, New York: Elsevier-North Holland, 1977

Shumway R. J., Technology, Mathematics, and the International Congress, Special report on ICME-6, Budapest, Hungary, 1988

Sleeman D., Brown S. J., *Intelligent Tutoring Systems*, Academic Press, 1982

Small D., Hosack J., Lane K., Computer Algebra Systems in Undergraduate Instruction, *The College Mathematics Journal*, Vol. 17, No. 5, 1986

Spensley F., Elsom-Cook T. M., DOMINIE: *Teaching and assessment strategies*, CAL Research Group Technical Report No: 74, The Open University, 1988

Stallings J. A., *Learning to look A handbook on classroom observation and teaching models*, Wadsworth Belmont California, 1977

Stevens A. L., Collins A., Goldin, E. S., Misconceptions in students' understanding, in *Intelligent Tutoring Systems*, Sleeman D., Brown, S. J. (eds.), Academic Press, 1982

Stevens A. L., Collins A., *The Goal Structure of a Socratic Tutor*, BBN Report No:3518, 1977

Stevens A. L., Collins A., Multiple conceptual models of a complex system, in *Aptitude, Learning & Instruction, Vol. 2: Cognitive Processes Analyses of Learning & Problem Solving*, Snow R., Federico P., Montague W (eds.), LEA, 1980

Stoutemyer R. D., The influence of computer and informatics on mathematics and its teaching, *Proceedings ICMA*, 1985

Stoutemyer, R. D., Rich A., *muMATH-83 Reference manual*, The Soft Warehouse, 1983

Stroud A. K., *Engineering Mathematics*, Macmillan & Co Ltd, 1970

Templer R., Klug D., Gould I., Kent P., Ramsden P., James M., Mathematics Laboratories for Science Undergraduates, in *Mathematics Education in the 21st Century*, Hoyles C., Morgan C., Woodhouse G., (eds.), London: Falmer Press (in press)

Wasson B. J., PEPE: A computational framework for a content planner. In Instructional Models in *Computer-Based Learning Environments*, Dijkstra S. A., Krammer H. P. M., van Merrienboer J. J. G., (eds.), NATO ASI Series F, Vol. 104, 153-170, New York: Springer-Verlag, 1992

Wasson, B. J., *Determining the focus of Instruction: Content Planning for Intelligent Tutoring Systems*, Unpublished Ph.D. Dissertation, Research Report 90-5, Department Of Computational Science, University of Saskatchewan, 1990

Watkins A., *A guide to using DERIVE*, Department of Mathematics and Statistics, Polytechnic of South West, Plymouth, 1990

Wenger E., *Artificial Intelligence and Tutoring Systems, Computational and Cognitive Approaches to the Communication of Knowledge*, Morgan Kaufmann Publishers, INC, 1987

Wolfram S., *Mathematica: A System for Doing Mathematics by Computer*, Addison Wesley Publishing Company Inc., 1991

Wood D., Wood H., Commentary Contingency in tutoring and learning, *Learning and Instruction*, Vol. 6., No. 4 pp. 391-397, 1996

Wood D., Shandbolt N., Reichgelt H., Wood H., Paskiewicz T., EXPLAIN: Experiments in planning and Instruction, *AISB Quarterly*, No:81, pp. 13-16, 1992

Woolf, P. B., Intelligent Multimedia Computer Tutors, *Communications of the ACM*, Vol. 34, 4, April 1996

Woolf B., 20 Years in the Trenches: What have we learned? In *Intelligent Tutoring Systems: At the crossroads of Artificial Intelligence and Education*, Frasson G., Gauthier G., (eds.), Norwood, N. J.: Ablex, 1990

Woolf P. B., Towards a computational model of tutoring, in *Adaptive Learning Environments*, Jones M., Winne P. H., (eds.) Springer-Verlag Berlin Heidelberg, 1992

Woolf P. B., *Context dependent planning in a Machine Tutor*, Unpublished PhD thesis, University of Massachusetts, 1984

Woolf B., McDonald D., Building a Computer Tutor: Design Issues, *IEEE Computer*, Vol. 17 No:9, 1984

Wylie R. C., *Differential Equations*, McGaw-Hill Inc., 1979

Glossary

Teaching Style

Teaching style is defined as a set of a type of learning and principles. Examples of types of learning are reception learning and discovery learning. The principles provide directions how to carry out the teaching. The style defines an environment that is principled and allows only compatible and of uniform structure strategies to coexist, thus forming multiple teaching strategies.

Teaching method

A teaching method is used to structure the subject matter. Examples are analogy, examples, investigation, etc. These have been shown in machine learning to be ways that humans learn.

Teaching tactic

A teaching tactic is responsible for carrying out the interaction between the teacher and the student, or in our case the system and the user. The interaction is based on the methods that have been identified and the tactics control the manner as well as the amount of information that are given to the student.

Teaching action

Given a style, a method, and a tactic, the next step for the system is to act. That is expressed by an action. A teaching action is a low level activity.

Teaching Strategy

A teaching strategy is a collection of a method, a tactic and action(s). If all these have been defined then it is said that the system has a teaching strategy. A teaching strategy is an abstract concept. For the teaching strategy to become concrete a style needs to be defined.

Procedure

A procedure is an activity that has to be performed as part of a solution of the problem.

Primary Concept

A primary concept is a concept which is directly relevant to a particular procedure.

Kind

A kind is an attribute which expresses what aspect of the concept is relevant to the procedure in the particular case.

Secondary Concept

A secondary concept is a concept which is not directly linked to a procedure but which has links to primary and other secondary concepts. (Their relevance is associated with the teaching of the procedure.)

Concept-to-Concept Link

A concept-to-concept link represents a particular type of relationship (e.g., generalisation, specialisation, depends_on, examples) and has a kind which expresses the context in which that relationship is relevant.

Filter

A filter is a concept that provides the procedure with a context.