An investigation of some differences in A-level mathematics syllabuses in England and Wales

Thesis

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An investigation of Some Differences in A-level Mathematics Syllabuses in England and Wales.

Ph.D. Thesis

Institute Of Educational Technology

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Tables in Main Text.

Abstract

At the time when this study began, there were nine boards offering Advanced Level Mathematics syllabuses. Some of the boards offered three, and sometimes four versions of A-level Mathematics. The study looks at these various forms from a number of different standpoints.

The first of these is a consideration of the 'readability' of the question papers themselves, using the Cloze Procedure. The data is analysed by a three-way, fully crossed, analysis of variance.

The work then moves on to consider the structure of the various papers. It then proceeds to analyse questions from the papers under various headings. The method is a substantial modification of a method used by the GCE examination boards in cross-moderation studies.

A questionnaire was developed to explore the opinions of sixth form teachers, regarding the various versions of A-level Mathematics. The opinions of university staff were also sought. The question as to whether some A-level courses are better preparation for university mathematics courses is addressed. Results of students at A-level and in the first year university mathematics examinations are compared. The students who participated in the 'readability' exercise were also interviewed, after looking at further questions from a selection of A-level Mathematics papers.

During the course of this study, a number of boards have started to offer modular, or unit based, courses. This significant development is considered towards the end of the study. Two schools and a sixth form college were visited, each one using a different modular A-level syllabus. An account of the observations is given.

The study closes with a discussion of the findings from the various themes and makes suggestions for possible improvements.

1. Introduction

Each year in England and Wales, substantial numbers of students sit GCE A-level Mathematics examinations. The Government Statistical Services (1994) show that in 1993-4 there were a total of 60,419 entries for mathematical subjects in England. As this figure is the number of entries, we must allow for the fact that some students take two A-levels in mathematics and some students re-take the subject, hence the total number of students, as opposed to entries is, probably, around 50-55 thousand each year.

Since A-level Mathematics is of direct relevance to this sort of number, who are sitting examinations each year, and also to employers and institutions of higher education, who are interested in their results, the subject is, surely, deserving of investigation at more than a superficial level. The author's experience, in looking for previous research in this area, reveals that incredibly little has been written concerning A-level Mathematics over the years. The present study is, therefore, in part, a small step towards rectifying this situation.

In recent years, there has been a decline in the number of students sitting A-level Mathematics examinations. Cockcroft (1982) shows that in 1979 there were 47,020 entries from school pupils. The Government Statistical Services (op.cit.) shows the figures for 1993-4 were 36,124. It should be stressed that these are school entries and do not include Further Education entries, or mature students. In part the reduction is due to demographic trends. The 1993-4 cohort is much smaller than that in 1979, but nevertheless, there has been a substantial reduction in the numbers taking A-level Mathematics over the years. If this decline is to be reversed, then it is important that the 'product', viz. GCE A-level Mathematics, should be as satisfactory as we can make it. The unattractiveness of the subject is reported in the media from time to time, but seldom are the underlying reasons explored.

It is hoped that the findings from this study may throw some light on this area, by way of a bi-product, althought it is not the main point of this work.

The author has more than twenty-five years experience as a sixth-form teacher, during which time he has encountered syllabuses from four of the examining boards. He also has nearly twenty years experience as an A-level Mathematics examiner, the last seven of which have been as an assistant chief examiner. During this time, the number of A-level syllabuses available in Mathematics, at any one time, has ebbed and flowed. Malvern (1977) in a study on the sequencing of A-level mathematics teaching, in relation to physics, reported 47 syllabuses at that time. In those days it was common for boards to have a 'modern','traditional' and a 'compromise' syllabus. More recently, the 'modern' and 'traditional' have merged into the 'compromise', with an ensuing reduction in the total number of syllabuses. Now, however, we are witnessing a further expansion in that total number, with the arrival of modular syllabuses.

With so many versions, it seems likely that the playing field is not exactly level. There are likely to be variations in both the content and the style of the papers which are set. Only a certain amount of material in pure mathematics, comprising about 40% of the syllabus, is common to all the versions, hence there is plenty of scope for variation in the other 60%. Since different boards, and different chief examiners within each board, set the papers, there will be considerable variations in use of language, notation and emphasis, according to the setter's particular view of the subject.

As well as the diversity of material and style of examination, there may well be differences in the grading of these examinations. How can we be sure that the grades awarded by one board are awarded in the same way that another examination board awards them ? To put this another way, if a student was prepared equally for two boards (this itself is probably difficult to achieve), would that student receive the same grade at the end of the course, from each board ? These questions are difficult to answer, as examination boards are protective of their data and procedures. From the authors limited experience of double-entering, i.e. students sitting examinations from two different boards in the same session, doubts were raised in his mind about the comparability of grades. This double entering was some years ago and is not possible today, as examinations on the various boards are synchronised. In cases where some pupils had been double-entered at the author's school, the difference in grades averaged more than a grade per pupil.

In earlier work on the school/university interface in mathematics, Jennings (1985) found a suggestion of differences in the 'readability' of A-level Mathematics questions from different GCE boards. This is the starting point for this study. We have attempted to find a more rigorous method of measuring 'readability' of such questions, in order to assess if there are significant differences between the various examination boards in this particular area.

The work has been extended to analyse papers and questions in different ways. We have also taken into account the views of A-level teachers and pupils concerning the various A-level courses. Staff from several university mathematics departments have been able to express their views of A-level Mathematics. Furthermore, we have been able to gather data from a number of university mathematics departments, in an attempt to see if some A-level courses offer better preparation for university work than others. Finally, during the course of this study, we have encountered the development of modular A-level syllabuses. This has been an important new development and some time has been devoted to assessing the impact that has been made by this development in several schools, each using a different form of modular course.

We have attempted to explore the differences between the various examination boards in a number of ways and using a variety of techniques. This cannot hope to be an exhaustive process. The limitations of time, finance and manpower are considerable. Nevertheless, it is hoped that the work may shed light on an area of vital importance, which affects many students of mathematics each year.

2. Literature Review.

In the survey which follows, of relevant literature, the sequence is largely chronological. This will enable us more easily to compare findings with contemporary events such as the introduction of 'modern mathematics' into school syllabuses.

2.1 Arrival of 'Modern' Mathematics to Cockcroft.

Early in 1961 the Supply Committee of the National Advisory Council for the Training and Supply of Teachers established a working party to investigate the special problem of the shortage of teachers of mathematics in schools. Arising from the findings of this working party was a pamphlet, edited by Rollett (1963), which exhibited those honours degree courses in which mathematics constituted at least half the work.

The introduction to the pamphlet has a number of remarks which impinge on the present study. Thus we read:

"Mathematics is an exacting subject and many students find after a year (or less) in a university that they have insufficient aptitude for the work."

The pamphlet highlights the difference in content between school work and university work. e.g. Between school calculus (relying heavily on intuition) and rigorous analysis or between vectors in school work, which often have concrete geometrical applications in 2-D or 3-D, and vectors at university which are intensely abstract.

To remedy the situation it is suggested that:

(i) A more critical and scholarly attitude to the subject is developed in the sixth form.

(ii) More attention is drawn to the logical structure of mathematics via Boolean Algebra, Matrices or Group Theory

(iii) Pupils can be guided to appreciate the possibility of giving concise and precise statements.

Against this backcloth, at the start of the 1960's The Schools Mathematics Project (SMP) came into being. This arose when school teachers became far more clearly aware of changes which were taking place in university mathematics. The most obvious effect was that the number of A-level mathematics syllabuses doubled, since examination boards offered a 'modern' and a 'traditional' syllabus. Commenting on the proliferation of syllabuses at an SMP conference, Quadling (1980) says that although it may be a bad thing, it has usually arisen as a response to a genuinely felt need. He thinks that an imposed uniformity might have far worse consequences. He points out that when the SMP syllabuses were established, there was considerable collaboration between interested parties, otherwise the proliferation might have been far greater.

It should also be noted that in the early 1960's the Mathematics in Education and Industry (MEI) project was launched by London day schools. Furthermore, statistics began to be recognised as an alternative application of mathematics at A-level. Prior to this applied mathematics had been solely mechanics. Thus in 1962 The Oxford and Cambridge Schools Examination Board ran two separate A-levels, Mathematics and Mathematics with Statistics. Paper 1 was pure mathematics and Paper 2 was either a paper in mechanics or a paper in statistics.

James (1968), a Northumberland headmaster, produced a study of the interdependence of sixth form mathematics and courses of higher education. A key theme of his work is the idea that A-level mathematical syllabuses are extremely diverse. Hence there is no agreed body of knowledge common to the various syllabuses. This is contrary to the view held by many outside the field of mathematical education. This situation is somewhat different today as A-level syllabuses embody the Standing Conference on University Entrance (SCUE) proposals for what has come to be termed the 'common core'. This, however, only relates to the pure mathematics content of single-subject A-level mathematics syllabuses.

James made a comprehensive table of the contents of various A-level mathematics syllabuses to demonstrate his point.

The results of such diversity James says is that:

(i) Universities find it hard to select candidates.

(ii) Having selected candidates, it is difficult to design first year courses.

James calls for reform and the implementation of the SCUE proposals is clearly an answer to this and a step in the right direction. It is the contention of this study, however, that this is only a beginning and much more needs to be done in this area. The SCUE proposals, as we have said, refer only to pure mathematics content of single-subject syllabuses. They are certainly beneficial to engineers, physicists and other users of mathematics. Their usefulness for honours mathematics courses is less, as many of the students will have taken two mathematics A-levels.

James thinks that one of the tangible results of reforming the sixth form syllabus would be that more time becomes available for problem papers, project work, open-ended investigations or directed assignment of reading leading to a mathematical essay. This, he thinks, is a better diet than the usual one of exposition and practice of routine examples.

It is interesting to note that these conclusions embody much of current thinking about A-level, where The School's Examination and Assessment Council (SEAC) is aiming for less syllabuses and the introduction of coursework at A-level, so that A-level becomes a more natural extension of GCSE.

Neill (1976) produced a summary of sixth form mathematics syllabuses. The object of his summary was to help planners of tertiary courses which have a substantial mathematical content. He points out a number of difficulties which are peculiar to mathematics:

(i) Mathematics can be studied as a subject in its own right , but the vast majority of A-level students are studying it as a service subject to help with science subjects, economics, geography etc. When schools have small numbers in the sixth form, or limited teaching resources, or both, then both these types of pupil are in the same class. The course, therefore, has to be appropriate to a wide cross-section of needs. The examination syllabus will therefore have to reflect this divergence of needs.

(ii) Mathematics at A-level can be studied as a single or double subject. Whereas in the past university mathematics departments could assume students had taken two A-levels in mathematics, this was not now the case. Tertiary education departments, therefore, needed to know what was the minimum amount of syllabus they could count on all their students having studied at school.

(iii) The changes brought about by modern mathematics have meant that there has been a vast increase in the number of syllabuses available. In 1976 Neill reported over 50 variations of mathematics syllabuses at A-level.

Neill's survey was restricted to single-subject syllabuses and ignored special papers. Furthermore it did not include Scottish syllabuses.

The problems of analysis for each syllabus include:
(i) The information used was from the printed syllabus, but this gives little guidance as to the depth of study intended, or to the weighting attached to using specific techniques. (Neill mentions here the importance of looking at

the papers set - but his summary does not attempt to do so. This will be one of the aims of the present study.

(ii) The variety of formula sheets available. Some give very detailed information and some hardly any. Some syllabuses had no formula sheet at all.

(iii) The meaning of words in the syllabus. The example he quotes is from trigonometry. Does "addition theorems" mean sinA + sinB as well as sin(A+B) ?

(iv) Omission of information, e.g. some syllabuses will mention 'equating real and imaginary parts' with reference to complex numbers and some will not. He notes that it is difficult to study complex numbers without this technique!

(v) Options. These fall into two categories:

a) Syllabuses with a number of optional topics from which candidates select a specific number of topics.
b) Syllabuses with an optional paper, usually Paper 2, where there is a choice between mechanics and statistics.

(vi) Syllabuses which are constantly changing.

Because different examinations have a different size of entry, Neill attempted to estimate what percentage of the total 1977 Mathematics entry would have studied a particular topic, by giving weightings according to the 1976 entry for the various examinations. Thus, because Oxford had 10% of the 1976 single-subject entry and JMB 13%, it was estimated that 23% of the 1977 entry would have some knowledge of roots and coefficients of polynomials. (Since Oxford and JMB are the only Boards which have this topic at the single-subject level.)

Holland (1979) carried out a similar exercise for AEB, in preparation for the SCUE common core to be introduced.

Pitt (1976), vice-chancellor of Reading University, drew attention to the number of syllabuses mentioned in Neill's study. He asked to what extent they were either necessary or desirable. His contention was that the common core of these syllabuses was clearly identifiable and had not changed greatly from the days when he had sat Higher School Certificate some 40 years earlier. He urged that a firmer definition of the requirements of mathematical education for able 18+ students be made.

Craggs (1976), a member of the Southampton University Engineering Mathematics Department, writing about the N and F proposals for 18+ students, as a replacement for A-level, set out what he considered to be the essential core of 18+ mathematics. He believed this to be a necessary prerequisite for entry into a three year degree course in engineering or science. This was to be an influential document in the formation of the SCUE proposals.

Crank (1976), talking about first year undergraduates, points out that although a candidate may be successful at A-level, one cannot guarantee that he/she has mastered a specific topic in the syllabus that they have studied. A situation can arise where two candidates with A-level from the same board have almost no knowledge in common. Crank identifies the advent of SMP as the major cause of the diversity and mentions that other project examinations have added to the problem. He, like Craggs, gives what he thinks should be in the common core.

Pitt (1978) gives details of the SCUE common core. This has now been incorporated into the pure mathematics part of all A-level single-subject Mathematics syllabuses of all the GCE boards.

The appearance of the common core, however, did not reduce the number of syllabuses, nor did it rationalise the content, as examination boards still continued to offer their various syllabuses, which had to include the common core, but other topics were added to give each syllabus its own distinctive flavour.

Malvern (1979), in a study on the sequencing of A-level Mathematics in relation to A-level Physics, circulated over 500 schools and found that, from 316 respondents, 46 different A-level mathematics syllabuses were being used (and 105 different textbooks). From this he concluded it would be difficult to sequence sixth form mathematics teaching in an order beneficial to

A-level physics courses. It does, however, illustrate the diversity that exists in A-level mathematics.

The references made so far, especially those to Neill and Holland, concentrate on the printed syllabus and not to the papers themselves, a point noted by Quadling (1980) at the Stoke-Rochford Conference on A-level mathematics.

At the same conference, Chatwin (1980) (currently a chief examiner for an A-level applied mathematics paper), refers to some of the differences in papers. He refers to 'style', pointing out that the 'modern' syllabuses arose because of the need to introduce topics like statistics, numerical mathematics and modelling. He also pointed out that 'traditional' syllabuses did not altogether reflect needs at the time when 'modern' syllabuses were introduced. He gives, in his view, two powerful criticisms of the 'modern' syllabuses and papers. Firstly, as a wide variety of topics are covered they can only be done so superficially. Secondly, the mathematical models which appear in textbooks and papers have little or no value either because they are bad models or because the students do not possess the techniques or knowledge to examine how they could be improved. He suggests two things:

(i) Students should be required to achieve mastery of a smaller body of techniques than is currently required in A-level examinations.

(ii) Students should only be expected to analyse mathematical models whose relation to the real world they can properly understand.

Chatwin also refers to 'difficulty'. The examination has to separate candidates right across the ability range. Where there is a choice of questions, they must be of equal difficulty, as far as can be judged by those setting the paper. Questions should neither be very easy nor very demanding. He gives an important principle:

"It is important that well-prepared candidates of moderate competence can find questions on the examination paper which they can understand and work at profitably."

He thinks that providing the above conditions are fulfilled it does not matter if there is some variation in the mean mark on the examination from year to year.

He makes the point that 'difficulty' is often confused with 'unfamiliarity'. In his view when examination papers become stereotyped then they cease to be beneficial. He regards analysing unfamiliar situations with familiar methods as an important mathematical ability. He is convinced, therefore, that A-level students and their teachers should be prepared for questions with some degree of originality.

A-level examinations, he says, are intermediate examinations and should take into account future development of material at university and elsewhere; e.g. it would be useful if A-level students were aware of how conservation laws of energy and momentum may be derived from Newton's laws of motion.

Chatwin gives examples of A-level questions which show differing levels of complexity, firstly a set of three questions on approximate integration and secondly a set of three questions on the use of vector methods. (See fig. 1)

Both 1 and 2 he notes have several levels of difficulty whereas 3 is straightforward throughout.

Questions 4 and 6 do not show, in his view, the need to use vectors in many problems and question 5 has a dubious model for the launching process. Again question 6 is more straightforward than 4 or 5. This shows a certain difference in philosophy between the boards concerned.

The Red discussion group at the Stoke-Rochford Conference (1980) commented that SMP examinations might be less daunting if the examination questions were more stereotyped. The view expressed was that candidates score low marks and leave the examination room feeling that they have failed to show their worth. The language problem in applied mathematics, it was felt, was another layer of difficulty; a certain amount of difficult translation work had to take place when doing the questions. It was the view of the group that this translation was a skill worth teaching. Nevertheless one must ask if it is fair that some examination boards will set papers where such skill is tested and others do not. (This is evident in the examples given by Chatwin.)

Figure 1: SMP A-level Questions.

1 JMB Syllabus B Paper 2 June 1975 Question D11

Given that $x \geq 4$, show that $e^{-\frac{1}{2}x^2} \leq e^{-2x}$ and hence show that

$$\int_4^8 e^{-\frac{1}{2}x^2}\,dx < 0.0002.$$

[Take e^{-8} to be 0.0003.]

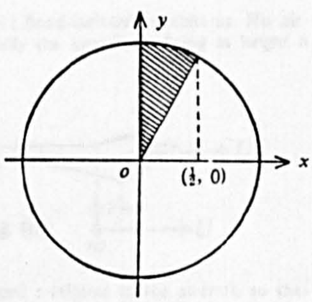
Use Simpson's rule with 5 ordinates to estimate the value of $\int_0^4 e^{-\frac{1}{2}x^2}\,dx$ and hence obtain an estimate of $\int_{-8}^{8} e^{-\frac{1}{2}x^2}\,dx$.

2 SMP Mathematics Paper 2 June 1977 Question 11

The following steps give a method for finding a numerical approximation to π. You are asked to carry them out.

(i) The diagram represents a circle of unit radius.

Show that the shaded area is $\frac{1}{12}\pi$ square units.



(ii) Deduce that

$$\pi = 12\int_0^{\frac{1}{2}} \sqrt{(1 - x^2)}\,dx - \frac{1}{2}\sqrt{27}.$$

(iii) Give a Taylor approximation for $\sqrt{(1 + z)}$, neglecting powers of z above the second. Deduce an approximation for $\sqrt{(1 - x^2)}$, neglecting powers of x above the fourth.

(iv) Use your approximation to estimate both $\int_0^{\frac{1}{2}} \sqrt{(1 - x^2)}dx$ and $\sqrt{(25 + 2)}$.

(v) Hence estimate π, giving your answer to 3 significant figures.

3 London Syllabus C Paper 2 June 1978 Question 6

Evaluate $\int_0^1 \frac{4}{1 + x^2}\,dx$ using

(a) Simpson's rule for 4 intervals,

(b) the trapezium rule for 4 intervals,

(c) definite integration,

and give four decimal places in each of your answers.

(All steps of your working must be shown.)

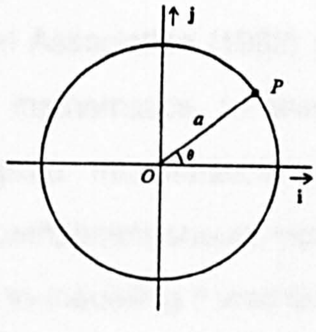
Figure 1 (Continued).

4 JMB Syllabus B Paper 2 June 1975 Question D12

As shown in the diagram a particle P of mass m is attached to one end of a light rod whose other end is attached at O. The position vector of P relative to O is

$$\mathbf{r} = a\cos\theta\ \mathbf{i} + a\sin\theta\ \mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and upward vertical unit vectors respectively, and P is constrained to move in a circle in a vertical plane with constant angular velocity $\omega = \dfrac{d\theta}{dt}$.



Determine the magnitude of the velocity of P for given θ and describe its direction.

At $\theta = \pi/4$ the particle P is released from the rod and is acted on by only the gravitational force $-mg\mathbf{j}$. Show that at time t after the instant when the particle is released, the position vector of P relative to O is

$$\mathbf{r} = \frac{a}{\sqrt{2}}(1 - \omega t)\mathbf{i} + \left\{\frac{a}{\sqrt{2}}(1 + \omega t) - \tfrac{1}{2}gt^2\right\}\mathbf{j}.$$

Determine the velocity of P when it attains its maximum height, and the value of t at which it reaches the horizontal plane through O.

5 SMP Mathematics Paper 2 June 1977 Question 9

A flying-doctor launches a supply-filled projectile to aid flood-isolated inhabitants. His aircraft has mass M and the projectile has mass m. Initially the aircraft is flying at height h horizontally with speed U (as in Fig. (i)).

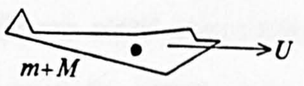


Fig. (i)

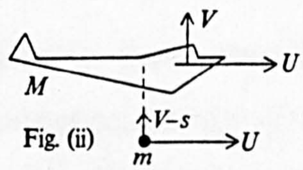


Fig. (ii)

The projectile is launched vertically downwards with speed s relative to the aircraft, so that immediately after launch the velocities of the aircraft and projectile are as in Fig. (ii). By considering momentum, find V in terms of s, m and M. Hence, taking unit vectors $\mathbf{i}$ and $\mathbf{j}$ in the horizontal and upward vertical directions respectively, show that the initial velocity of the projectile can be written in the form $\begin{bmatrix} U \\ -\alpha s \end{bmatrix}$, and state the value of α.

Ignoring any braking mechanism and neglecting air resistance, show by considering energy that the projectile will hit the water at speed

$$(U^2 + \alpha^2 s^2 + 2gh)^{\frac{1}{2}}.$$

Also, by integrating the equation

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix},$$

find at what horizontal distance before spashdown the projectile must be released.

6 London Syllabus C Paper 2 June 1977 Question 7

In this question the units of length, time and force are the metre, second and newton respectively.

State the centre and radius of the circle which has vector equation

$$\mathbf{r} = 6\mathbf{i} + 8\mathbf{j} + 6(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta).$$

A particle P of mass 2 kg moves on this circle with constant angular velocity $\pi/12$ radians per second. Write down the position vector of P at time t given that, at $t = 0$, P is at the point corresponding to $\theta = 0$.

Calculate the components parallel to $\mathbf{i}$ and $\mathbf{j}$ of the resultant vector force on the particle when $t = 4$.

Find the position vector of the particle when $t = 8$ given that the forces on the particle cease to act when $t = 4$.

(Your answers may be left in terms of π).

2.2 Cockcroft and Beyond.

The Stoke-Rochford Conference was a response to the SCUE common core. Likewise The Mathematical Association (1982) published a report on the applied content of A-level mathematics. In this they argued that a compulsory component of applied mathematics should appear in all single-subject syllabuses. This component should include some mechanics and some statistics. With regard to modelling it was felt that, because of the constraints of time, the first priority should be to build on experiences that students already have, using established models in the most positive way possible - underlining the assumptions and limits of validity. The content they suggest is shown below. (See fig.2)

Some eight years later, however, these recommendations have not been substantially acted upon. Examination boards still set papers on mechanics as alternatives to papers on statistics for A-level Mathematics, as well as papers containing a mixture of mechanics and statistics. This is despite having undertaken a certain amount of syllabus revision in this period, with the introduction of 'compromise' syllabuses, whereby they have ceased to offer 'modern' and 'traditional' syllabuses and have just offered a single syllabus.

Our brief was to examine the Applied content of A-level, but the following proposal is a complete course because we feel that applied and pure cannot be separated: one motivates the other, the latter provides the tools for the former.

Those parts of the course which are our extended developments of mechanics and statistics are in sections 7 and 8.

Although we have not mentioned computers, nor the use of calculators explicitly, we intend the flavour of the whole course to be affected by their presence: the availability of quick generation of numerical results both directly and using iterative methods, and the use of graph plotters and graphical displays can add to understanding and appreciation. Equally, the process of writing programs can in many topics give new insight into the mathematics involved.

Topic	Notes
1.1 Modelling and functions	A limited survey, confined to looking at data from a variety of contexts, and including the use of tables, graphs and algebraic formulae.
.. developing from this	
1.2 Function: domain and range; illustrations by mapping diagram and graph. Particular functions and their graphs: algebraic, circular, log and exponential. Graph sketching; simple transformations of graphs; symmetry; odd, even and periodic; composite and inverse.	Understanding of the relationship between a graph and the assoc. algebraic relation. In particular, ability to sketch $y = x^n$ for integral and simple rational n, $ax + by = c$, $x^2/a^2 + y^2/b^2 = 1$. Geometrical properties of the ellipse, parabola and hyperbola excluded. Transformations $y = af(x)$, $y = f(x) + a$, $y = f(x-a)$, $y = f(ax)$.
.. also	
1.3 Trigonometric functions of angles of any magnitude. Circular measure, $s = r\theta$, $A = \frac{1}{2}r^2\theta$. The addition formulae, double angle, factor formulae. Expression of $a\cos\theta + b\sin\theta$ in form $r\sin(\theta+\alpha)$. Period, amplitude, phase. Solution of simple equations, inverse trig. functions.	
.. and still related as far as possible to useful models	
2. Addition, subtraction, multiplication and division of polynomials, including addition of rational forms; factor theorem for polynomials. The quadratic function, quadratic equations and inequalities; solution of other simple polynomials; partial fractions.	Mention of complex numbers but little development (*see Common Core proposals*).

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Topic	Notes
Situations where limits, sequences and series crop up, leading to	
3. Inductive definition of sequence. Sigma notation; summation of simple series (AP and GP; use of formulae for $\sum_{1}^{n} i^k$ for k=1,2,3). Sum to infinity of GP.	
Rates of change in models from 1, leading to	
4.1 Derivatives of functions of one variable; differentiation of products, quotients, composite functions and functions defined implicitly. Application of differentiation to gradients of graphs, maxima and minima. Second and higher derivatives	Derivatives of x^n, sinx, cosx, tanx, $\sin^{-1}x$, $\tan^{-1}x$, e^x, a^x, ln x.
.. and	
4.2 Approximation: tangents to a curve and their equation, Taylor approximation to simple functions; Newton-Raphson method of solution of equations.	
Situations where area under a graph is of interest, leading to	
5.1 Integration: as anti-differentiation - standard integrals, integrations by parts, simple substitution. Integration as summation; application to area, volume, centre of mass. Numerical methods: trapezium and Simpson's rules	Integrals of x^n, sinx, cosx, e^x, 1/x, $1/(1+x^2)$, $1/\sqrt{(1-x^2)}$. The relation with corresponding techniques of differentiation should be explicit.
.. or where, given the rate function f'(x), we wish to find f(x)	
5.2 Differential equations: simple step-by-step solution of $dy/dx = f(x,y)$; families of solution curves. Explicit solution of first-order equations with separable variables.	

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Figure 2: Mathematical Association's Applied Content.

Simple experiments involving moving bodies ....

6.1 The idea of a position vector; position vectors as functions of time: $\underline{r} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ in 2D	eg. One ball bearing dropped vertically as another is projected horizontally from the same height (listen as they hit the floor together; take multiflash photo to analyse).
Numerical investigation of average velocities, leading to	
6.2 $\underline{v} = \frac{d\underline{r}}{dt} = \begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix}$, $\underline{a} = \frac{d\underline{v}}{dt}$ for the motion of particles in the plane (among other paths, motion in a circle and under constant acceleration are required).	Where $\underline{r}$, $\underline{v}$, $\underline{a}$ are expressed as functions of time in component form.
6.3 Position vectors, the basic trinagle $\overline{AB} = b + {}^{-}a$; the ratio theorem; vector equation of the line $\underline{r} = \underline{a} + k(\underline{b}-\underline{a})$.	All this to be done on plain paper with arbitrary vectors, and on squared paper with unit vectors $\underline{i}$ and $\underline{j}$ and columns $r = \begin{pmatrix} x \\ y \end{pmatrix}$, to get the student used to the ideas by encouraging him to move from point to point on the paper.
6.4 Scalar product; angles between lines.	$\begin{pmatrix} a \\ b \end{pmatrix}$ perpendicular to $ax + by = c$

7. Mechanics

The mathematical work on mechanics is restricted to those topics for which pure mathematical tools are available (or will be developed) for their discussion. This means that the models are very simple. Nevertheless, using only particles, and restricting motion to two dimensions, there is a rich supply of mechanical phenomena which can be developed and illustrated in the classroom. The syllabus introduces concepts which are demonstrably useful when simple experiments and problems are discussed and have obvious importance outside the classroom.

Discussion of the conditions and limitations of validity is considered very important.

Topic	Notes	Links
7.1 Newton's laws of motion for a particle.	One or 2D only; the second law as $\frac{d}{dt}(m\underline{v}) = \underline{F}$ $\Sigma\underline{F} = m\underline{a}$ problems solved by drawing vector polygons and in components; simple experiments.	Vectors and their derivatives.
Statics of particles.	As a particular case of motion.	
Laws of friction; Hooke's law; particle attached to an elastic string (in 1D); small oscillations.	Experimental evidence; discussion of validity. Use of energy. S.H.M.; the equation $\ddot{x} + w^2x = 0$	Integration of $\ddot{x} + w^2x = 0$ as solution of two first order differential equations.
Motion of a particle under gravity.	2D motion may be assumed.	Integration; energy.
Motion of a particle in a vertical circle.	Use of Conservation of energy.	
7.2 Conservation of mechanical energy.	Restricted to 2D conservative forces and derived by integration of Newton's 2nd law; discussion of situations where the principle does not apply.	Scalar product; integration.
7.3 Conservation of linear momentum for two particles in direct impact; concept of impulse; restitution.	Derivation by integration of Newton's 2nd law. Use of simple experiments.	Integration.

8. Probability and Statistics

In this section of the syllabus we are attempting to convey the notions of modelling non-deterministic situations and of using data to reflect on the model (early stages of inference). We aim to expose pupils to practical problems and data involving random variation and uncertainty, to introduce probability as a measure of uncertainty and probability theory as the mathematical model for combining uncertainties. Sampling comes in as a means of both obtaining information about a practical problem and, by use of probability theory, as information from which one can draw inferences.

Since the syllabus is part of a mathematics A-level it concentrates on the mathematical aspects of elementary probability and statistics. This does not lessen the importance of doing practical and project work, but it does explain the omission of such topics as practical problems of data collection, carrying out surveys, reading tables, simple aspects of experimental design.

Figure 2 (continued).

elementary time series, moving averages, index numbers, birth and death rates, data sources and many other items that might be included if the syllabus were to be seen largely as a service to other subjects such as economics, geography, biology and chemistry.

Candidates should appreciate the aims, scope and limitations of statistical procedures. Questions may involve a discussion of the practical implications of statistical conclusions.

Topic	Notes	Links
8.1 *Descriptive Statistics*		
Frequency and relative frequency distributions. Histograms. Sample mean, median, percentiles, variance and standard deviation.	To illustrate variation in practical problems. Calculated for grouped and un-grouped data.	Use of the Σ notation, algebraic manipulation, linear interpolation.
8.2 *Probability*		
Relative frequency of an event and its relationship to probability.	Probability as a measure of uncertainty.	The idea of a limit.
Sample space. An event as a subset of the sample space.		
Equally likely events.	Including simple combinatorial methods.	The notation $\binom{n}{r}$. Permutations and combinations.
The addition law for probabilities.		
Conditional probability and the multiplication law; independent events.		Sets and set notation. Union and intersection of Sets. Simple Boolean algebra.
Applications of these laws to two or more events.	Use of appropriate diagrams such as Venn diagrams, tree diagrams and Karnaugh diagrams.	
8.3 *Probability distributions*		
Discrete and continuous random variables.		A random variable as a function.
Probability functions, probability density functions and distribution functions.	As models to describe patterns of variability.	Standard mathematical functions. Sums of series, including sums to infinity.
Distribution parameters such as the mean, median and variance.		Sums of series and integration.

Topic	Notes	Links
Discretė uniform distributions.	To model random choice and link with random number tables.	
Binomial distribution. The distribution of a proportion.	With emphasis on the assumptions necessary to make these useful models. Pascal's triangle; development of binomial theorem for positive integral index.	Link with Taylor series for $(1+x)^n$ for n rational or negative: general binomial theorem.
Continuous uniform distribution.		
Other simple continuous models.	Use of integration to evaluate probabilities and find values of distribution parameters.	Standard functions such as exp x, sin x, polynomials.
The normal distribution as an approximation to the binomial. Use of normal tables.	Including the use of the continuity correction.	Areas under curves.
Expectation. Expectation of a function of a random variable.		The Σ notation and integrals of standard functions used above, including integration by parts.
Expectation of the sum of independent random variables.	Intuitive treatment only.	
8.4 *Sampling, estimation and hypothesis testing*		
Population and sample. Practical sampling. Random samples.	Use and implications of the terms 'with replacement' and 'without replacement'.	
Modelling a population by a probability distribution.		
Using sample statistics to estimate distribution parameters. Unbiased estimators of the mean and proportion.		See 'Expectations' in §8.3.
Hypothesis testing using the binomial distribution as a model (or the normal approximation where appropriate).	Includes the use of the sign test.	Binomial theorem.
Null and alternative hypotheses, critical region, the two types of error and significance level.		Areas under curves. Summation and integration.

Reference

(1) A minimal core syllabus for A-level mathematics - a joint statement by the Standing Conference on University Entrance (SCUE) and the Council for National Academic Awards (CNAA).

Figure 2 (continued).

Backhouse et al. (1980) examined the "Choice of Mathematics for Study at School and University". In the course of their survey of schools in Oxfordshire, they interviewed heads of department. This revealed that schools taking MEI and SMP, the latter especially, felt that the grades awarded were lower than they might be on other examinations. SMP was felt to be difficult, but one enthusiast thought that it should still be taught as the mathematical content was so good. It would be interesting to know whether his pupils would rather have good course content or higher grades. Unfortunately we are not told. The study did reveal that whatever syllabus was being taught, the school genuinely believed that universities wanted what they were teaching.

The difference in grades awarded by different GCE boards in A-level Mathematics had previously been noted by Scott (1975) in a study of three different A-level Mathematics examinations. In the study, schools were asked to use A-level papers from another board in their mock A-level examination in the spring term. These were then marked and graded by the examiners from that board, according to the official mark scheme for the paper. The results were compared with the summer results from the candidates' own board. Allowance was made for the difference in candidates' performances in a mock examination in the spring and in the A-level examination in the following June. This was done by allowing candidates to work a mock exmination from their own board. The evidence from 500+ candidates from 36 schools showed that one board was consistently more generous in its grading, particularly at grade C and above.

Cockcroft (1982) referred to the variety of A-level Mathematics syllabuses. He points out that these have arisen chiefly because of (a) the number of boards, (b) the need for mechanics and statistics to be examined and (c) the 'modern' and 'traditional' examinations. He points out that 'compromise' syllabuses will mean that (c) will become less influential, so that there will be a decrease in the number of syllabuses. (This is something which we can now see has occurred.)

Cockcroft did not feel that there was a strong argument for an "arbitrary reduction in the number of syllabuses". Indeed he thought that it was important for the sake of curriculum development that changes to syllabuses should occur from time to time and new syllabuses should be produced.

He also thought that there should be careful monitoring of A-level syllabuses. In 1982 this scrutiny was performed by the A-level sub-committee of the Mathematics Committee of the Schools' Council. This body has been superseded by the Secondary Examinations Council (SEC) and more latterly by the Secondary Examinations and Assessment Council (SEAC), who still undertake scrutiny of all boards' A-level examinations. Each year at least two boards' papers are looked at and SEAC reviews and approves new syllabuses and schemes of examination.

An interesting point made by Cockcroft is that differences in the ways in which A-level courses are taught may result in greater differences in the performances of students who have followed the same course than result from differences of content between syllabuses.

It is noticeable that no reference is made to differences in examination papers and grading which results from the examination.

HMI (1982) in a report on "Mathematics in the Sixth Form" refer to the examination boards aiming to reduce the number of A-level mathematics syllabuses available by producing 'compromise' syllabuses, whereby 'modern' and 'traditional' syllabuses are combined. It is interesting to note that such an authoritative body does not consider it worth noting any differences between the various alternatives available. They merely point out that there is a general pattern of sixth form colleges offering a 'modern'/'compromise' syllabus and a 'traditional' syllabus, whereas schools, in general, only offer one type of syllabus.

Some teachers, they point out, feel that traditional A-level syllabuses were more acceptable in higher/further education, particularly for the physical sciences. The rationale for this was that they provided a better opportunity for the weak candidate to succeed. The report simply says that such arguments may or may not be valid, but makes no attempt to address the matter in any depth.

The report makes a number of points about mechanics and statistics:

(i) Statistics is more attractive to girls.

(ii) Numbers taking mathematics in the sixth form will be higher if there is a statistics component in the A-level.

(iii) Where a statistics component is available, the combinations of A-level subjects are more varied.

This 'broadening' of sixth form studies has been the focus of much thought in recent years, with the publication of the, now rejected, Higginson Report and the introduction of AS-levels (equivalent to half an A-level in standard and content).

The Institute of Mathematics and its Applications (1987) made the following observations to Higginson :

(i) Too few students participate in 16+ mathematics courses to satisfy the demands of science and technology for the national welfare.

(ii) Insufficient students take mathematics degrees to perpetuate a supply of qualified mathematics teachers.

(iii) The amount of mathematics required by some undergraduate courses will fall short of a full A-level Mathematics.

(iv) Many AS courses are simply subsets of A-level. The opportunity to widen the intake base should be grasped. Syllabuses should be developed to lock onto Intermediate GCSE Mathematics.

(v) The availability of appropriate courses at A or AS level could help to lower the current 30% failure rate at A-level.

(vi) Girls should be encouraged to participate more in 16+ mathematics courses. At present they are under-represented.

The IMA suggest two possible ways forward:

a) Two different base levels. e.g. There could be a properly structured AS programme alongside A-levels or two alternative A-level curricula, one being less demanding in curriculum content.

b) The introduction of a modular approach to sixth form studies, enabling students to tailor their study around specified modules.

The latter option the Institute thinks would avoid a discontinuous change in the sixth form curriculum and would allow for a smoother transition to a broadened curriculum.

The Institute points out that assessment procedures at A-level must have maximum credibility, because of the key role played by A-level grades in selection for Higher Education. This should be carefully considered in any changes that might be made. Furthermore, attention should be given to core studies in any future design. At present the diverse provision by the various examination boards in A-level Mathematics leads to much unnecessary duplication in undergraduate mathematical studies.

Willmore (1987) thought it was a pity, to some extent, that so many AS-level courses were available in Mathematics. Much time had been spent developing a common core in Mathematics. This diversity of AS courses seemed to be a possible contradiction. He regarded A-level Mathematics as the minimum requirement for the study of Mathematics, Physics or Engineering at University. He was happy if students had 2 A-levels and 1or 2 AS-levels on arrival at University, but he thought that 5 or 6 AS -levels, with no A-levels, would lead to longer degree courses. Otherwise degree standards would inevitably be lowered, especially in a highly structured subject such as Mathematics.

In 1988 the Department of Education and Science (DES) set up a small committee, under the chairmanship of Dr. G.R. Higginson to review A-levels. The terms of reference of the Higginson Committee were:

"To recommend the principles that should govern GCE A-level syllabuses and their assessment, so that consistency in the essential content and assessment of subjects is secured; to set out a plan of action for the subsequent detailed professional work required to give effect to these recommendations."

Among the findings of the committee were that AS-levels should be developed and fully integrated into the post 16+ system. Existing A-levels should be replaced by trimmer and modified A-levels, enabling a conventional candidate to offer 5 A-levels instead of the present 3, thus broadening his/her educational base into a form similar to the French

Baccalaureate or the German Abitur.

The Secretary of State for Education, however, whilst welcoming the proposal for the development of the AS-level as a supporting qualification, turned down the new ideas for A-level on the grounds that the academic breadth and integrity of existing A-levels should not be put at risk.

Accepting that A-levels were to remain, substantially in their present form, Hart (1989), a member of SEAC, speaking at a symposium on Mathematics Education post 16, gave an outline of a consultative document that SEAC would be producing. The issues to be addressed would include: curriculum breadth, the nature and range of AS syllabuses, the desirability of continuing with common cores at A-level, the rationalisation of A-level syllabuses and the establishment of general principles and subject-specific criteria to govern both A and AS syllabuses. He thought that A-level criteria should take account of not only the mechanics of the assessment process, but also the aims and characteristics of courses.

Responding to the SEAC consultative document, the Institute of Mathematics and its Applications (1990), felt that "breadth" should be obtained within subjects. Thus "extra-curricular" skills, such as communication, analysis and synthesis, should be developed in the mathematics course, both orally and in written work. This would provide a more natural progression from GCSE.

Regarding A-level in particular, the Institute felt that the main priority was to reduce the size of A-level Mathematics syllabuses. It was felt that less attention should be focussed on memory and more on understanding and skills such as communication and analysis. More rigour should be introduced and the subject should be more relevant to employment, since two-thirds of A-level candidates do not enter Higher Education.

The institute felt there was "little justification for the great number of syllabuses available and the disparity of content between them". It argued for a major simplification and rationalisation, with common cores being reduced in size so that they did not act like a straitjacket. This rationalisation could be achieved if SEAC did not give approval to a new syllabus which was virtually the same as an existing one - examination boards should be encouraged to share syllabuses. General principles and criteria for A-level should be established by SEAC as these are vital for coherence, effectiveness and acceptability.

One of the prime reasons for this present study is the perception that there is a considerable disparity at present between the various versions of A-level Mathematics.

The Institute further point out that approximately 30% of A-level candidates do not secure an E grade and so have nothing to show for two years' work. Many of those who get a D or E grade leave the examination room feeling that they have "failed" in the parts of the papers that they have attempted. Effort should be put into making A-level Mathematics a more rewarding course for those who take it. Furthermore, encouragement should be given to showing the relevance of mathematics to other subjects; the idea of "mathematics across the curriculum" should be promoted in all A-level Mathematics courses.

Because of the large number of versions of A-level mathematics, the GCSE examination boards make considerable efforts to ensure uniformity of standards. The Standing Research Advisory Committee of the GCE Examining Boards (1990) produced a report on a cross-moderation exercise, based on scripts from the June 1987 examinations in advanced level mathematics.

The study was carried out by senior examiners from each of the boards, who scrutinised scripts taken from all the examinations (except their own) . They attempted to reach conclusions about comparative grading standards applied at the B/C and E/N boundaries. They attempted to scrutinise the largest feasible number of scripts in the time available (two days) and to remove as many potential sources of bias and distortion from the procedures employed. 1987 was chosen as it was the first year that the Inter-Board Common Core was fully operational.

Two examiners from each of the 11 examination boards participated. A month before the meeting they were supplied with 20 scripts at the E/N and 20 scripts at the B/C borderline region from each of the other examinations. Comparison of standards at the meeting was done by four working groups. Each working group had 5 of the 20 scripts available. Examiners were asked to state **in their opinion**, if a script was, or was not, a true borderline script and, if not, on which side of the borderline it should have been placed. Once individual judgements had been made, working groups decided whether or not it was possible to state a group consensus judgement in relation to the standard observed in the five scripts from a particular examination.

The total number of individual judgements at each borderline was therefore 20 X 11 X 5 = 1100.

A marked bias was detected in examiners' judgements in that judges tended to regard other examinations as being leniently rather than severely graded. This bias was more notable at the B/C boundary rather than the E/N boundary. After allowing for this bias, the JMB examination was thought to be leniently graded at the E/N boundary and the Oxford and Cambridge examination severely graded at the E/N boundary. At the B/C boundary it was considered that the SMP examination had been leniently graded and the London, Northern Ireland and Oxford and Cambridge examinations had been severely graded.

Notable correlation was found between the rank order of judgements and the rank order of grade boundary marks at both the E/N and B/C boundaries. In both cases the correlation was significant at the 1% level. A similar finding had been detected in a similar exercise on A-level chemistry grades. It is thought that the examiners are influenced by the level of marks at the grade boundary. For the examinations in the study the E/N boundary varied between 30.6% (Oxford) and 40.8% (Oxford and Cambridge) and the B/C boundary varied between 54.1% (Oxford) and 68.0% (London). There was a tendency for examinations with low grade boundaries to veer towards leniency and those with high grade boundaries to veer towards severity in marking.

Where the grade boundaries were low some examiners felt candidates could not show what they could do. The examiners suggested that there was considerable variation in the difficulty of questions on the various papers. Other examinations, by contrast, aimed at "positive achievement', clearly influenced by Cockcroft's ideal of candidates being able to demonstrate what they "know, understand and can do". The author's awareness of this situation is one of the reasons for this present study.

The SMP examiners participating in the study pointed out that different criteria applied to the SMP course and examinations. They felt that they were applying SMP criteria to the other examinations when making their judgements. Conversely they felt that the other examiners were not fully aware of the aims of SMP when making their judgements of the SMP scripts. No statistical evidence, however, emerged from the study to either support, or refute, these statements.

Candidate exposure to questions based on the common core material in the various examinations would be dependent upon either the examination structure or on the candidate's choice of question. In both cases there were examinations where the normal exposure to such questions was substantially below the minimum specified in the introduction to the common core. This has considerable implications for planners of tertiary courses. They expect a sound knowledge of core topics, but it would seem that this may not be the case, particularly if the shrewd candidate has studied past examination papers and knows the style of questions to expect.

One of the factors which it was impossible to legislate for in the study was the degree to which various boards' examinations had become stereotyped. It was felt by those taking part that some boards' examinations were quite stereotyped, whereas other boards went to some length to be innovative and avoid stereotyped questions.

In making their judgements, the examiners had three options:

(i) Rely entirely on the criteria which they use in their own examination in order to award grades.

(ii) Rely entirely on a commonly agreed set of explicit criteria.

(iii) Make use of their own criteria supplemented by a commonly agreed framework.

The GCE boards are presently trying to produce grade criteria at A, B and E grades for A-level mathematics. The examiners participating in the study were supplied with the draft grade criteria so far produced for grades B and E. These draft criteria were validated on the 1986 A-level mathematics examinations. Further validation will have taken place on scripts drawn from the 1988 examinations and those of later years. In this study, the examiners hopefully were employing option (iii), because, as yet, no sets of explicit criteria exist to exercise option (ii). Some would suggest that option (ii) is an "unattainable ideal". In practice, (note the comments of the SMP examiners) option (i) was probably being used to a large extent.

A notable feature to emerge from the study concerns the diverse balance of pure mathematics and mechanics in the various examinations. The study had, as far as possible, tried to avoid using statistics questions, but in the case of SMP and MEI examinations, statistics is a compulsory part of the examination. It was noted in the samples considered that candidates performed somewhat better on pure mathematics than on mechanics in a high proportion of the examinations. Consequently, in examinations where there were more pure mathematics questions available, candidates might be able to gain an advantage.

A companion study, of a statistical nature, was also undertaken by the Standing Research Advisory Committee of the GCE Examining Boards. Again it focussed on the June 1987 A-level single-subject mathematics examinations. Candidates were excluded if they had taken a second mathematics examination or if mathematics was the only subject taken with that particular board at that sitting. The entry was studied and there was a noticeable preponderance of male candidates. There were relatively few young and adult candidates. It was noted that the syllabuses differed significantly in the proportions of candidates entering from different types of centre. AEB examinations, for example, had a high proportion of Further

Education candidates, whilst Oxford and Cambridge and MEI had a high proportion of candidates from selective and independent secondary schools. Regarding the breakdown of entry by gender, it was noted that the proportion of females rose if the board offered mathematics with statistics as an alternative to pure mathematics and mechanics.

When the performance data was studied, it was noted that females performed better than males on average. There were also substantial differences among the examinations in terms of cumulative proportions of candidates in grades and of mean grades. This latter point was established by assigning values to grades:

A=7, B=6, C=5, D=4, E=3, N=2, U=1

The following results were obtained:

Syllabus		Mean Grade	Syllabus		Mean Grade
OCSEB	M	4.56	ULSEB	PM	3.72
MEI	M	4.48	JMB	M(PwM)	3.68
UODLE	M	4.28	JMB	M(P&A)	3.67
UCLES	M(A)	4.20	ULSEB	PMSt	3.59
UCLES	MP(AM)	4.18	JMB	M(P)	3.55
UCLES	M(C)	4.16	SUJB	PMSt	3.53
WJEC	M(C)	4.14	WJEC	M(St)	3.50
NISEC	M	4.07	JMB	M(PwSt)	3.47
JMB	St	3.91	AEB	PMSt	3.22
SMP	M	3.85	AEB	PAM	3.21
SUJB	M	3.81	AEB	M	2.79
ULSEB	M(B)	3.75			

The clustering effect of the boards in this table is quite marked, indicating a 'board effect' rather than a 'syllabus effect'.

There was some evidence that candidates from selective and independent schools and sixth form colleges performed rather better than candidates from comprehensive schools. However, the disparity in syllabuses, in terms of entry breakdown, meant that direct comparisons of examination outcome had to be viewed with caution. For this reason a 'Delta Index' was computed, in order to compare examinations, whilst allowing for the different entry profiles. From this statistic it appeared, at board level, Oxford and possibly Cambridge and the Welsh boards had been lenient at grade B, while the Welsh board only had been lenient at grade E.

Performance of candidates in pairs of subjects could be obtained using the 'Delta Index', where mathematics was one of the two subjects to be compared. Grades in biology, Nuffield biology, computer science and economics were generally better than those in mathematics. Grades in physics, chemistry and their Nuffield equivalent were much closer to the performance in mathematics.

From the subject pairs comparisons and from the mathematics, physics, chemistry triad, the Oxford Board was was considered to be lenient at grade B and AEB severe at both grade B and E.

The study points out that these findings were not supported by the Cross-Moderation exercise. In fact it is noted that the Cross-Moderation study had suggested that the JMB syllabus was leniently graded, whereas the 'Delta Index' suggested "severity if anything".

In this literature review we have traced the devlopment of A-level mathematics over the last 25 years in some detail. The most recent studies, which we have just discussed, highlight the need for consistency between the various versions that are available. In the ensuing chapters we will consider the differences that one can detect in the style and structure of what has been available in England and Wales. We shall try to assess the extent of some of the differences which we may detect.

3. Research Design and Methodology of the Survey.

This chapter includes the following sections:

1: Formulation and statement of hypotheses.

2: Discussion of the methods and instruments for testing the hypotheses.

3: Discussion of the populations and samples which are used in the survey.

4: An appraisal of the type of data which can be collected and how it can be processed.

3.1 Formulation and Statement of Hypotheses.

Ideally we require to be able to express our hypotheses in terms of the variables that are capable of being measured. One of the chief causes of concern in any social investigation is that the variables we wish to investigate are difficult to measure. This inevitably means that the best that can be achieved is to obtain measures of the variable as a function of the available resources for assessing the variable concerned.

Travers (1969) suggested five criteria in the formulation of hypotheses.
(i) Hypotheses should be stated clearly and in correct terminology.
(ii) Hypotheses should be testable.
(This is important because the outcome of the study will be determined by the extent to which it has been possible to support or refute the hypotheses as a result of the use of our various instruments.)

(iii) Hypotheses should state the relationship between the variables.

(iv) Hypotheses should be limited in scope.

(The more specific and simple a hypothesis is, the greater our chance of being able to test it. Vague generalisations, clearly, would be difficult, if not impossible, to test.)

(v) Hypotheses should not be inconsistent with most known facts.

(In our case they should be consistent with the findings from studies which we have reviewed in the previous chapter. It is undoubtedly true that some studies disagree or give contradictory findings, hence the use of the phrase "most known facts". Clearly it would be impossible to be consistent with all known facts.)

The Hypotheses

1) The 'readability' of the language used in setting papers in A-level mathematics by the various GCE examination boards is not of a uniform standard.

2) The differences in 'readability' are experienced by students of all levels of ability in A-level Mathematics.

3) There are significant differences in the construction of A-level mathematics papers by the various GCE examination boards.

4) The standard of difficulty of A-level mathematics questions set by the various GCE examination boards varies over a range of categories (as specified in chapter 6).

5) Teachers perceive that the GCE A-level Mathematics syllabuses are inconsistent with respect to:

a) content, b) level of difficulty of questions, c) grading.

6) University admissions tutors perceive that GCE A-level Mathematics syllabuses are equivalent with respect to:

a) content, b) level of difficulty of questions, c) grading .

7) Some A-level Mathematics syllabuses provide a better preparation for the study of mathematics at university than others.

8) Sixth Form students view the variety of A-level Mathematics syllabuses as bewildering and unfair.

9) Sixth Form students see that there are significant differences between the questions that can be set on the same topic by the various GCE examination boards.

10) Modular Mathematics syllabuses are a more appropriate way of approaching the study of A-level Mathematics than their competitors and previous A-level Mathematics syllabuses.

3.2 Methods and Instruments for Testing the Hypotheses.

Much of what the study is able to achieve is limited by availability of time, finance and manpower. As the author is carrying out the research single-handed and on a part-time basis, being in full-time employment as a schoolmaster at a boys independent secondary school, there were very obvious limitations on the time that was available.

There are seven GCE examination boards in England and Wales, now that the Southern Universities Joint Board has disappeared. Each sets its own A-level Mathematics examinations. The Oxford and Cambridge Schools Examination Board has two 'satellites', SMP and MEI These are project examinations, which the O&C Board administer, although candidates may enter for the examinations through any of the GCE examination boards. Hence there are effectively nine distinctly different providers of A-level Mathematics in England and Wales. We have excluded Northern Ireland from our discussion because of the practical difficulties in obtaining information from 'across the water'. The system of examinations in Scotland is different to other parts of the United Kingdom, so no consideration was afforded to matters 'north of the border' either. This left us to consider the nine sources already mentioned.

The majority of the nine boards have more than one variation of A-level Mathematics, depending whether candidates are offering (i) Pure Mathematics, (ii) Pure Mathematics and Mechanics, (iii) Pure Mathematics and Statistics or (iv) Pure Mathematics,Mechanics and Statistics.

Readability Comparison

This study has its origins in an earlier piece of work, Jennings (1985). In that work, an initial comparison of the 'readability' of questions from the various boards, using a modification of Gunnings FOG formula, suggested that there might be significant differences in the readability of questions produced by the different boards. In order to investigate this assertion further, it was decided to use the Cloze Procedure. A description of the technique follows in Chapter 4. Reasons are given there for preferring this measure of 'readability' to other measures which are available.

Ideally one would like to produce test materials for the Cloze Procedure covering several years A-level papers and numerous topics from them, covering all areas of A-level mathematics. These should be administered to a sample of subjects which included all elements of the A-level entry : male and female students, state and independent students, school, sixth form college and further education college students. It should also include a cross-section of the ability range.

As mentioned above, the work was being done single-handed. The construction of Cloze Procedure tests is time consuming in itself, so the materials were limited to a single year's papers, covering all nine boards. The author's experience of A-level papers, as a teacher, an examiner and latterly as assistant chief examiner, suggests that, although they must vary from year to year, the essential style of each board's papers is tolerably constant. Hence a 'snapshot' view would have some value. It was important to have tests which covered both pure and applied topics, with each having a 50% share, largely in line with the structure of A-level papers. (The exact structure of A-level papers can be seen in Chapter 5.) The applied tests should include mechanics and statistics in equal proportion, again a feature of those A-level papers which contain mechanics and statistics on the same paper.

In order to achieve this, it was decided to select two pure mathematics questions, one on calculus and one on vectors from each board's 1989 papers, and also one projectiles question and one probability question from the same set of papers. Each of the questions would appear in five forms. (Because every fifth symbol is deleted, one can start deletions on the 1st, 2nd, 3rd symbol etc.). The reasons for having these different versions are explained in Chapter 4. Hence there were 9X4X5 = 180 Cloze tests to produce.

The author was able to enlist the assistance of 60 pupils at his school. This gave a good spread of ability, from the boy who was to study Mathematics at Oxbridge to those who would struggle to get a grade E at A-level Mathematics. This sample obviously suffered in that it was made up entirely from boys from an independent school, a fact that was forced on the study for reasons of time, cost and access to other students. Fortunately it did cover students from a wide variety of interests and backgrounds, including overseas students (all of whose English was sound).

Paper Comparison

The relevant year's papers were readily available from the various examination boards, as were the corresponding syllabuses. Neill (1976) and Holland (1977) had drawn up matrices which noted which topics occurred in various syllabuses. There seemed little point in duplicating their work as the situation had changed little in the last 15 years. Instead it was decided to concentrate on the papers themselves. The task was essentially one of description, classification and collation. An account of the findings follows in chapter 5.

Question Comparison

'Readability' is just one aspect of an A-level Mathematics question. Other features such as the level of abstraction in the question, or the demands it makes on mathematical manipulation or logical thinking can all vary. It was decided to widen the scope of the study to encompass features such as those we have just mentioned. The author was aware that McLone and Griffiths had developed a matrix, mentioned above, with headings such as these above in one direction and a measure of question ease/difficulty in the other direction, to compare questions in Southampton University mathematics examinations. McLone had subsequently moved to the University of Cambridge Local Examinations Syndicate and a form of this matrix had been used in a cross-moderation exercise by the GCE examination boards, see McLone (1990). The results of this exercise seemed somewhat inconclusive. The scales of measurement of each aspect of a question appeared to lack uniformity in that some went from easy to hard and others from hard to easy. The conclusions were also vague, for while a scale of measurement was referred to for the judges to use, the final report did not include any of the data for the judgements that had been made. Indeed we find that some judges had withdrawn from the study, so that it was incomplete!

The headings for assessing questions were modified slightly and the scales for judging each heading ran from 0 (easy) to 3 (hard). A full description of the modified matrix appears in Chapter 6.

Having seen the difficulties experienced by the GCE boards in their cross-moderation study, where some judges failed to complete the tasks which had been assigned to them, the author decided that he would make all the judgements himself. This had the advantages that (i) the work would be completed in the available time and (ii) there would be some degree of consistency that would not be present if the judgements were made by several people. One then had to ask if the judgements made by the author were reasonable.

In order to attempt an answer to this problem, the help of a fellow teacher was enlisted. A sample of the questions, covering a number of topics and all the examination boards involved in the study, were given to the fellow teacher. Both he and the author made their judgements for these questions, independently, according to the modified matrix. This procedure enabled a check to be made, as to whether, or not, the author's judgements were reasonable. Having ascertained some degree of 'reasonableness', the author could then proceed to make judgements on all the questions, from all the boards, for the 1989 papers in A-level Mathematics.

The basic material necessary for both the Cloze experiment and the Question Analysis matrix was the set of A-level question papers for the particular year being studied. In addition to study of the question papers, the views of people would be sought concerning the variety of A-level syllabuses on offer. There were certainly three identifiable groups that would be approached : (i) teachers, (ii) students and (iii) university admissions tutors. Each of these groups would view A-level syllabuses and papers from different standpoints; for each group the papers would have important but different functions. For the teacher, the syllabuses and papers set on them dictate, to a large extent, how he or she will approach the subject in the two years leading up to the examination. For the student, it is the yardstick to measure how they have mastered the material over the two years and the key to the next stage in their career. For the university people, A-level lays the foundation for university work and also provides a vehicle for selection, in order to determine who will proceed to the university course.

Possibly, one might argue that there was a fourth group, viz. Chief Examiners. Contact with this group would be problematic, as they could only be contacted via the examination board. They are not allowed, by the examination board, to comment on the examinations they set, apart from in the official Chief Examiner's Report. They are principally concerned with the syllabus for their own examination board, so views about the standards between boards might be limited and possibly prejudiced. In the report of the Inter-board Cross Moderation Study, referred to above, we read that the SMP Chief Examiners felt that they viewed A-level Mathematics in a different way to the Chief Examiners from the other boards, so that it created difficulties when they had to grade work from other boards and Chief Examiners from other boards had to grade work on questions they had set for SMP examinations.

It was decided, therefore, to limit our enquiry to the three groups listed above. In order to assess the opinions of these groups, a mixture of questionnaires and interviews would be used.

Teachers' Questionnaire

A well worn cliche is that 'no survey is better than its questionnaire'. It is of paramount importance to avoid ambiguous, leading and vague questions. In constructing the questionnaire for teachers, it was necessary to exercise care. Clearly, as the author is a practising sixth form teacher, then some of the questions were easy to phrase in language that would be understood by his counterparts in other schools. However, it was necessary, firstly, to define the problems that the questionnaire was designed to address. These are embodied in the hypotheses which we have already stated and are concerned, chiefly, with the differences which may exist, or which teachers and/or other people perceive to exist, between the various versions of A-level Mathematics that are currently available.

Schoolteachers are busy people and so it was decided to keep the length of the questionnaire to reasonable proportions. A long questionnaire, it was thought, would produce a poor response rate. To encourage a response, a stamped addressed envelope was included for the reply.

Payne (1951) identifies both factual and opinion questions. The type of information that was:

a) Factual material which required clear-cut answers,

e.g. (i) Numbers taking A-level Mathematics.

(ii) Which type of course. (Mechanics, Statistics or a mixture)

(iii) Board used.

b) Opinions.

e.g. Teachers views on :

(i) Differences between boards, papers and syllabuses

(ii) Consistency of grades awarded by different boards.

(iii) Modular courses.

Teachers were invited to add any further comments, which were relevant to the study, at the end of the questionnaire.

The factual questions were easy to understand. The main problem was possibly the question of 'prestige'. Some respondents might exaggerate the number of pupils taking their subject, for example, or pretend not to experience difficulties. In general, however, it was felt that most respondents would give fair answers and that, consequently, this would not constitute a serious problem.

The opinion questions pose greater worries. Oppenheim (1966) suggests that the followingconsiderations are worth noting :

(i) Has the respondent ever thought about the issue before encountering the questionnaire?

(ii) The respondents opinion may vary according to the circumstances, so that there is no one correct answer that he/she can give.

(iii) Answers were more likely to vary according to the phrasing of the question than they were with factual questions.

In connection with the wording of the questions, Moser and Kalton (1971) think one should:

(i) Avoid making the questions insufficiently specific.

(ii) Use simple language. or that, which from experience, one feels the respondents will understand.

(iii) Avoid ambiguities.

(iv) Avoid vague words.

(v) Avoid leading questions, i.e. ones which begged a certain response.

(vi) Avoid factual questions which rely on the respondent's capacity to recall past events.

There are few precautions one can take to counteract the respondent's frame of mind. Hence the first two difficulties are not easily allowed for. They must, however, be borne in mind when interpreting the results.

Having obtained questions, in the light of the above points it was necessary to plan, carefully, the order in which they were to appear on the final questionnaire. At the beginning of the questionnaire the author decided to put straightforward and factual questions, in order to build up the respondent's confidence. Sensitive issues were avoided in order to encourage him to continue with the questionnaire. The author also endeavoured to use as logical a structure as was possible, so that there was an air of continuity. Questions were ordered in such a way that preceding answers, it was hoped, would not influence later ones.

A preliminary attempt at the questionnaire was sent to teachers at three schools, asking them to criticise the questionnaire and comment on the wording of questions, including the ease/difficulty of understanding them. On the basis of their replies, some questions were reworded and others rewritten. A copy of the final questionnaire and accompanying letter is included in Appendix A at the end of the study.

Admissions Tutors' Interviews.

The purpose of these interviews was to hear admissions tutors' opinions of the A-level courses that students had pursued prior to coming to university. In particular, did the A-level course prepare the student well for higher education? Were students as well prepared today as they were some years ago, or was the preparation better or worse? Did the preparation vary according to which A-level syllabus was being used? Did the absence of Mechanics and/or Statistics in the Sixth Form course cause significant difficulties for the tertiary stage of education? The interviews also provided an opportunity to see if the particular course being followed at school had any influence on the offer made to a prospective student. It was also possible to show the tutors examples of A-level papers and the A-level syllabuses and invite comments upon them.

Moser and Kalton (1971) refer to the different types of interview which are possible. They list, firstly, the formal interview, in which set questions are asked and the answers are recorded in a standardised form, possibly against some ckecklist. Secondly, the interview may take a form in which the interviewer is at liberty to vary the sequence of the questions, to explain meaning, to add additional questions and even to change the wording. Thirdly, they list a still less formal approach, where no set questionnaire is used, but the interview is built about a number of key points.

It was decided that the latter case was more appropriate to the subject matter of our present study. The first approach would achieve little more than a questionnaire. It would gain on the questionnaire in that response was assured, because one had a captive audience, but would be superior in few other respects. The third approach would provide a contrast to this. It would allow the interviewees to have freedom to express their personal views at whatever length they felt necessary, within reason. Depending on their first few thoughts, the direction that one interview took could be quite different from that taken in another. For example, one tutor might be concerned about the lack, or otherwise, of traditional calculus skills, while another would be more concerned about the shortcomings, or otherwise, of modular courses at A-level.

There appear to be four pre-requisites for a successful interview:

(i) Accessibility.

This concerns the accessibility of the respondent to the information required by the interviewer. The information required here concerned A-level mathematics across the spectrum of examination boards. As most universities visited in the study drew students from a wide variety of backgrounds, it was likely that the admissions tutors would have some experience of the variety of courses available. The question of lack of accessibility did not, therefore, seem to constitute a particular problem.

(ii) Cognition.

This refers to the tutor's capacity to understand what is required of him/her. The respondent had to realise what was relevant information, how completely to answer questions and how to phrase answers to questions. It was the task of the interviewer to help the tutor understand what was expected by the way the responses were treated, in particular, by probing deeper when answers were either not full enough or misdirected. The informal approach adopted for these interviews assisted this process. The interviewees were, by the nature of their position, highly articulate and knowledgable of the subject matter, which meant that very few responses were lacking in detail and the tutors had a good idea of what information was being sought.

(iii) Motivation.

The respondent had to want to take part in the interview. This includes the initial decision to participate and the subsequent decision to continue. In a sense the audience was captive, as the tutors had agreed to a meeting, but they were all enthusiasts for their subject and were keen to participate in a process which might give them some new insights. Indeed several remarked that they had learned some things about A-level Mathematics courses. This, they felt, would be useful to them in their work in future. There were obviously some guarded responses to the outside investigator, but these seemed to be minimal and, in general, there was a pleasing frankness and co-operation on the part of the tutors. One might suppose that this was because the subject matter of the interview was germane to their function as the admissions tutor for their department. This, consequently, would engender a good level of motivation.

(iv) Interaction.

An interview is a social process involving an exchange between two people. To achieve some degree of success, some degree of rapport needs to be established between interviewer and interviewee. The fact that the author was engaged in Sixth Form teaching of students who might want to study mathematics at their department, was useful in the setting up of dialogue between the two parties. Furthermore, the author's experience as an A-level examiner was useful in establishing a platform on which to build the interview. An important factor in improving the interaction was for the interviewer to refrain from expressing his own views, or to pose questions in such a way that he obtained the desired response. The admissions tutors were able to express their views, with only the occasional nudge to keep them to the point. One difficulty encountered, from time to time, was when the tutor got on to a particular hobby horse. After allowing a certain amount of time on this topic, the course of the interview had to be re-directed, as no useful further information was being gained.

Kahn and Cannell (1957) distinguish between five types of inadequate response : (i) partial response (ii) non-response (iii) irrelevant response (iv) inaccurate response (v) verbalised response problem - when the respondent says he doesn't understand.

A crucial part of the interaction process is to keep the interview going smoothly. To elicit further information to an inadequate response a number of techniques are useful. One way is to use an expectant pause, not too long, or it may produce a negative effect. Expectant glances are also useful. So that the subject is not misdirected, one can use a neutral question such as :

"How do you mean?"

"Can you explain a little further?"

Sometimes repeating the question will have the desired effect, or using the respondent's own words in a questioning manner can produce a further response.

Sixth form Student Interviews.

The considerations enunciated already pertaining to the conduct of interviews are still valid. Here it was decided to employ the second type of interview, which we have mentioned above, viz. one in which there were some key themes, but still an informal approach. It was felt that, in an informal manner, one was more likely to receive a useful amount of information from the students. However, the sixth form students were probably less able than university lecturers to give full responses, without some direction or indication as to what was required. The interviewer, therefore, needed to direct the course of the conversation so that some indication was given of the response, which was required.

Furthermore, the content of the interview would remain strictly relevant. This was important in order to keep the time for the interview to a minimum. The school day is very busy and there is not any facility for long, one to one interviews. The interviews varied somewhat in time, as some students had little to say, and some were interested in the study and spoke at greater length. The length varied from about 5 minutes to a little over 10 minutes.

The students taking part in the interviews were taken from the author's own school, for a number of reasons :

(i) It would be difficult to get permission to enter another school to talk to students, as there would be objections from staff and parents to such a procedure. In these days of 'league tables', staff and parents are very wary of the school's standing. However well-intentioned a social researcher from another school might be, there would inevitably be suspicions of hidden agendas.

(ii) The study, which is being conducted single-handed on a part-time basis, has strict limitations on time, manpower and finance.

(iii) The students were well-known to the interviewer and so the discussion could get under way speedily.

(iv) Interviews could easily be arranged at a mutually convenient time for interviewer and interviewee.

(v) Preparatory work to be done before the interview could be accomplished without unnecessary disruption of the students' normal teaching.

Prior to the interview the students were given questions, from each of the examination boards under consideration, on a particular topic, for them to work through, as an exercise on that particular topic.

The general structure of interviews with the sixth form students was as follows:

(i) A brief outline of the purpose of the study was given. It was pointed out that each of the questions had been set by a different examination board.

(ii) Students were asked if they had detected any differences in the questions (unprompted).

(iii) If the reply to (ii) was unforthcoming, they were prompted to consider such things as:

(a) Wording.

(b) Structure, or lack of it, in the question.

(c) Difficulty of the question.

(d) Overall length of the question.

(iv)The students were asked their opinions about the variety of different versions of A-level Mathematics and how that variety might affect them at the end of their sixth form course.

In the interviews conducted with both sixth form students and university lecturers, the author kept notes of the proceedings and these are presented in a later section of the study.

First Year University Results.

At most of the universities, one of the lecturers in the Mathematics department acts as an admissions tutor. With his head of department's permission, he/she was asked if a list could be provided, showing for each student : A-level subjects and grades, examination board for A-level mathematical subjects and the results of first year university examinations in Mathematics, taken at the end of the first year course at university.

In some universities a unit system operated and the mark for that unit was the one used for our analysis.

In order to preserve anonymity, and to encourage participation, the admissions tutor was asked to remove any names and only to provide marks.

The results are shown in Chapter 8.

Modular Mathematics Investigation.

By 1996 all the Examination Boards will have revised syllabuses in operation. One of the principal reasons for this is to ease the transition from GCSE (Key Stage 4) to A-level. At the time of writing three of the Examination Boards have a modular syllabus under way, viz. London, SMP 16-19 and MEI.

It was decided to visit a school or college, close to Bedford, which was using each of these modular schemes. Details of the schemes are to be found in Appendix F. The purpose of the visit was to find out (i) why the institution was using this modular scheme and (ii) how it compared with (a) what they used to do and (b) other schemes that were available. (What were the advantages and disadvantages of the modular scheme that the particular school or college was using ?)

A description of the case studies follows in Chapter 10.

3.3 Populations and Samples which are used in the Survey.

The number of GCE Examination Boards is not great. It was therefore possible to use syllabuses and papers from all of the seven English and Welsh GCE Examination Boards. The Scottish system of examinations is different and was not considered. It was thought unlikely that many Scottish pupils would apply to English universities which were involved in our study. Although using GCE, again it was thought that few Irish students would feature in the work, hence the Northern Ireland Examination Board was not included. (In fact, hardly any Scottish or Irish students were encountered.)

Clearly a good deal of personal cost is involved in preparing and sending out a mail questionnaire. The cost for a large sample was going to prove prohibitive. Naturally, the larger the sample one could take, the more reliable and informative the results would be. The geographic area of Bedfordshire, in which the author lives, seemed a sensible choice, initially. This provided an area whose schools are of various types; urban and rural, comprehensive and selective, single-sexed and mixed, maintained and independent. There are, in addition, sixth form colleges and a college of further education, which were included. It was considered wise to include the independent schools, as the percentage of students coming from independent schools at university is higher, and very considerably higher at some universities, than the percentage of secondary pupils at independent schools. As we are partly concerned with some A-level students who go on to university, it is sensible to include all of the different possible backgrounds in our work.

If our population was defined as 'secondary schools in England' then this would be what Bennett (1973) calls an 'opportunity sample'; i.e. one in which the researcher has deliberately picked the elements in the population which he wishes to study. On the other hand, if the population is defined as secondary schools in Bedfordshire, then the population is of a suitable size so that no further sampling is required. In this case it would not be possible to generalise about the rest of the country, simply from the evidence obtained from this small geographical area (with the exception of the metropolitan counties, the smallest, in area, of the English counties). Such generalisation, however, would be beyond the scope of this study. What we would have is some 'hard evidence' obtained about a limited area. In the end it is for the reader to assess which is the more preferable; detailed knowledge of a small area, or vague generalisations about a larger one.

In the event, because the schools of Bedfordshire predominantly used the Cambridge Board's A-level Mathematics syllabus, the questionnaire was circulated to teachers at a sixth form teachers conference at York University. Because it has a strong reputation for Mathematics, the conference participants came from all over the country. The respondents to the questionnaire came from a wide range of types of school and used a variety of examination boards. As the conference was residential, it meant that the response rate to the questionnaire was good, as there was a captive audience.

The selection of universities was governed, to some extent, by the need to choose universities which were popular among the students at the author's own school. The school was generously providing the funds for the study, and it was to be hoped that findings would be of some general benefit to pupils at the school. University lecturers would also be keen to participate, when they already had some students coming from the author's own school. Furthermore, they were likely to be able to attract more such pupils. The universities participating include Birmingham, Leeds, Loughborough, Reading and Southampton. Communications with the above mentioned universities are good via the motorways M1, M3, and M4, as are rail connections. In today's highly mobile society this is an important consideration, after the obvious academic ones. Loughborough is a 'new' technological university and provides a contrast to the more traditional 'redbrick' nature of the others. Interview material was also gathered at some Cambridge and London colleges. Cambridge is a rather special case in that all its Mathematics students will have the highest grade at A-level. London colleges, because of accommodation difficulties and the high proportion of overseas students has a somewhat different student body to the other universities.

The group of universities constitutes around 10% of all English and Welsh universities (before the re-naming process), and again is no more than an opportunity sample. By studying, in some depth, a small group of universities, it was hoped to be able to gather meaningful information about the universities' views of the various versions of A-level Mathematics syllabuses. The views expressed would certainly be of interest to the author's own school and presumably other schools in the same geographical area could find these opinions of a similar value.

The fact that the universities are readily accessible to Bedford meant that the author could spend more time on a visit than if he went to Exeter, Lancaster or Newcastle.

The recent introduction of Modular A-level Mathematics syllabuses meant that there were few schools operating them in the locality. Schools using the MEI and SMP schemes were able to be located from the respective headquarters of these projects, and the author was able to find users of the London syllabus from his experience as an examiner. The restrictions outlined above, of time, finance and manpower, still applied to the selection of schools in this category.

3.4 An Appraisal of the Type of Data which can be collected and how it can be processed.

From our 'Readability" experiment we would have numerical data. This would essentially be percentage completion rates of the various Cloze passages. The Cloze passages would be on various topics taken from A-level papers, produced by each of the nine examination boards. Such data would lend itself most readily to an analysis of variance procedure, the full details of which are included in the results section of Chapter 4 of the study. In this experiment we would also have the performance of the students on their trial examination. Correlation, if any, between the performance on the Cloze passages and performance in the trial examination would be investigated.

The review of A-level papers, which was to be carried out, would be mostly descriptive and largely self-explanatory. Certain numerical comparisons could readily be drawn between the proportion of marks, or time, allowed for pure mathematics, mechanics or statistics.

From the Question Comparison exercise we would have rankings, for each of a range of categories, for a particular question. Questions would be selected from papers from each of the examination boards. Again an analysis of variance procedure would be appropriate, but because we were dealing with rankings, rather than percentages, then we would require non-parametric techniques to analyse the data.

From the Sixth Form Teachers Questionnaire there would be little numerical data and certainly not enough to submit it to statistical analysis. This numerical data would be derived mainly from the number of respondents in various groups; e.g. how many respondents used JMB examinations and how many used London. There were a number of questions, early on in the questionnaire, relating to factual questions, which would produce some numerical data, such as how many students were taking mechanics as part of the course and how many were taking statistics. Again the numbers were so small that they did not lend themselves to statistical analysis.

The bulk of the material gathered from the questionnaire concerned teachers' opinions about the various versions of A-level Mathematics syllabuses. This data would have to be handled descriptively as well. In assessing the opinions from the questionnaire, one would have to rely on such factors as the volume of opinion expressed on a certain point, or the rationality of the arguments put forward by respondents, in order that we can develop explanations and conclusions. This would also apply to the data derived from interviews.

Further descriptive data would be obtained from the interviews with university lecturers, sixth form students and the schools participating in the Modular A-level schemes.

Other numerical data was to be obtained from the results of the students in A-level and university examinations at the end of their first year at university. It might be possible to assess if the courses of some boards at A-level produced students whose performance was significantly better in the first year at university. This would have to be viewed with caution, as any differences in university performance could well be attributable to factors other than the course followed in the sixth form. Some of the data obtained from universities could be checked against national figures, such as those found in Cockcroft (1982) and HMI (1982). For example, we might be able to check the proportion of mathematics students who had taken two mathematical A-level subjects in our data, against the proportion in the national figures. We might then be able to assess, to some extent, if our sample was typical or atypical.

4. 'Readability' and A-level Mathematics Papers.

In our literature review, we noted that most previous attempts to compare A-level Mathematics syllabuses have resulted in merely a summary of the statements made in the syllabus document. This does not take into account the depth of treatment of topics, nor the complexity of the questions asked about the topics. Croasdale (1991) in a study of written examination papers in mathematics at, or near, the level of British General Certificate of Education Advanced Level, using a cluster analysis procedure, makes this very point. When looking at some of the shortcomings of his study he notes:

" We have attempted to compare only the context of papers. We have not addressed, for example, the testing of understanding of concepts, as distinct from facts or skills, which a given paper may achieve. Nor has the level of difficulty of papers been considered......."

One aspect of the difficulty of papers is the 'readability' of the questions contained in the papers. Dale and Chall (1948) defined 'readability' in the following way:

"In the broadest sense, readability is the sum total (including interactions) of all these elements within a given piece of printed material that affects the success which a group of readers have with it. The success is the extent to which they understand it, read it at optimum speed and find it interesting."

4.1 What is 'Readability'?

In trying to describe 'readability', we are seeking a procedure which would enable us to predict, in a reliable way, how a particular set of readers would cope with a certain passage of text.

As a starting point, one can begin by thinking about what can be measured, also what factors make a text difficult. There are clearly many imponderables. However, Harrison (1980) lists the following as influential factors:

(i) legibility of print
(ii) illustration and colour
(iii) vocabulary
(iv) conceptual difficulty
(v) syntax
(vi) organisation.

Legibility should not be confused with readability. The latter covers the entire spectrum of factors. Legibility refers to things such as height of print, fonts (whether with or without serifs etc.), roman or italic lettering, spacing between lines, justification and other similar features.

Vocabulary from the above list is clearly an important factor determining the difficulty of text. The complexity of the vocabulary, to some extent, can be measured by (i) word length, often measured by the number of letters or syllables and (ii) the frequency of usage of various words. These items are clearly able to be measured. For this reason they have been incorporated into various readability formulae.

The reading ability associated with a particular age of reader is sometimes referred to as the 'reading age' or 'reading quotient'. The function of a readability formula is to assign to a passage of text a predicted reading level of x. This means that the passage should be appropriate for the average x year-old reader.

It should be noted that 'word frequency', by which we mean the extent to which a word occurs in everyday use is an indirect measure of abstraction. Word frequency can be assessed in the form of a ratio; e.g. once per n thousand words, or by some sort of ordinal list; e.g. cat more frequent than giraffe, which is more frequent than diplodocus. Care, however, must be exercised, because, suppose we were investigating the reading matter of a group of palaeontologists, the diplodocus could well feature more prominently than the cat!

Conceptual difficulty, in contrast, cannot be measured reliably, yet it may have a profound influence on the extent to which words, or longer passages of text can be understood. It, too, may depend on the nature of the readers being investigated. For example, the word 'limit' is a reasonably common word in everyday use. We come across its use for speed limits, or boundaries of one sort or another. However, it would have a more precise meaning, in the context of calculus, to the sixth form reader, than to someone in the second or third form; i.e. the quantity to which a function, or the sum of a series, can be made to approach as closely as desired.

Syntax difficulties may arise in a variety of ways: (i) The use of 'the active voice' is more easily understood than the use of 'the passive voice' ; e.g.'The weight stretched the string' (easier), rather than 'The string was stretched by the weight' (harder). (ii) The use of active verbs is more easily understood than the abstract noun formed from the verb. e.g. 'The reduction in the length of the string will produce an increase in the speed of the pendulum' (harder), compared with 'If you reduce the length of the string, you will increase the speed of the pendulum (easier). (iii) The use of Modal Verbs, incorporating the use of 'might','could','may' and 'should' cause increased difficulty. (iv) The number of clauses per sentence is related directly to the difficulty of the sentence. (v) Compression and/or substitution can also lead to confusion. e.g. 'The race I timed was fast' (confused), compared with 'The race, which I timed, was fast' (clearer).

Organisation of text, by use of features such as bold type, underlining, italicising, introductory paragraphs and summaries etc., can help to improve the reader's understanding.

It should be noted that 'readability' is an attribute of texts, whereas 'comprehension' is an attribute of readers. Readability incorporates those aspects of texts which make it easy for a reader to understand.

Klare (1963) was a consistent advocate of pooled subjective opinions of text difficulty. From our list above one can see that many of the features are not able to be measured. For this reason we shall be looking, later, at the 'Cloze Procedure', which is an attempt to address this difficulty. Readability formulae are predictive measures. They do not measure text difficulty itself. They use generally established connections between certain text factors and actual difficulty as a means of predicting difficulty in other passages for readers of a certain age. The factors are largely those of word frequency and sentence length, which we have discussed above. Care should be taken not to infer a causal relationship from the observed correlations. The critical factors in constructing a good readability formula are those of validity, reliability and ease of application.

Since 1948, when Dale and Chall, et al., raised this issue , various attempts have been made to assess the difficulty of a reader in understanding a piece of text. Much of the effort has centred on developing 'readability' formulae, whereby account is taken of sentence length, number of syllables in words, frequency of word usage etc., in order to calculate an index of 'difficulty' for a given piece of text. This index was often expressed in terms of a 'reading age'. E.g. If the index of a passage, when the formula was applied, came to 12, say, then the passage was deemed to be understandable by 'the average twelve year-old reader'.

Most readability formulae have been developed by studying continuous English prose. Furthermore, in nearly all cases they have been based on American English usage of words. When studying technical writing such as that used in mathematics, one will not encounter continuous English prose, as it will be punctuated by formulae, graphs, diagrams and various symbolic material. In our particular study of British A-level mathematics, the usage of words will be quite different, in many cases, to the usage of the same words in American continuous prose. Kane (1967) introduced the terms "Ordinary English" and "Mathematical English". The latter he describes as a hybrid language, composed of Ordinary English inter-twined with various brands of highly stylised formal symbol systems. The application of readability formulae to mathematical text must, therefore, be viewed with some caution.

4.2 'Readability' Formulae.

For completeness we list several of the readability formulae that have commonly been used.

The Dale and Chall (1948) Formula.

This formula, developed in 1948, depends on a list of 3000 familiar words drawn up by Dale, as well as sentence length. The US school grade C(50) is given by:

$$C(50) = 0.1579X + 0.0496W + 3.6365 \quad \text{where}$$

X = percentage of words not on the Dale list of 3000 words

W = average number of words per sentence

C(50) = the reading grade score of a pupil who could answer correctly 50 of the questions on a reading comprehension test covering the passage.

After some experience of using the formula it was felt that reading age was being underestimated at the higher levels. Consequently a reading grade of 8-9 giving a reading age of 13-14 was revised so that 8-9 corresponded to a reading age of 16-17.

(The original scheme was Reading Age = Reading Grade + 5 .)

The Flesch (1948) Reading Ease Formula.

This formula is quick and easy to apply and simple in nature. It uses the lengths of words and sentences. The original idea behind it was one of 'thought units' rather than sentences, but this idea had to be abandoned as agreement as to what constituted a 'thought unit' could not be established. The Reading Ease index is a number between 0 (difficult) and 100 (easy). The formula is:

Reading Ease = 206.835 - 0.846S - 1.015W where
S = average number of syllables per 100 words
W = average number of words per sentence.

Farr and Jenkins (1949) table will give the Reading Ease index. This can be converted to a reading age.

e.g. Reading Age = 5 - (Reading Ease - 150)/10

(for values of Reading Ease > 70)

What Reading Ease represents can be seen from the following table:

Table 4.1. Reading Ease Table.

R.E.	Description of style	Typical syllable number	Typical sentence length	Reading Age
0-30	v.difficult	192 or more syllables	29 or more words	college
30-50	difficult	167	25	17-18
50-60	q.difficult	155	21	14-16
60-70	standard	147	17	12-13
70-80	q.easy	139	14	11
80-90	easy	131	11	10
90-100	v.easy	123 or less syllables	8 or less words	9

The variables S and W can also be used with the Fry (1969) Readability Graph.

Gunning (1952) FOG formula.

This is an extremely popular formula as it is easy to use.

The formula is:

$$F = 0.4(W+P)$$

where

W = the average number of words per sentence.

P = the percentage of words containing 3 or more syllables (excluding those ending in -ed or -ing).

F = US grade level (hence Reading Age = F + 5)

The acronym FOG stands for 'frequency of gobbledegook' and the formula, as well as being easy to use, does not depend on the American usage of words.

Jennings (1985) attempted to compare A-level Further Mathematics questions using the FOG formula. The analysis of the questions was made difficult by the fact that a good deal of the material, which involved signs, symbols, diagrams and so forth, was being ignored in applying the formula.

4.3 Assessment of 'Readability' Formulae.

With a variety of formulae for measuring 'readability' there will undoubtedly be variation in the scores obtained. Shuard and Rothery (1984) tested a passage with each of the formulae mentioned above. For this particular passage, Fry gave a reading age of 16 and FOG a reading age of 22, with Dale-Chall and Flesch giving reading ages of 18 and 'college level' respectively. ('College' here refers to the American system.) Thus for the same passage the results from different formulae range considerably. This would seem to suggest, using the categories of measurement listed in Siegel (1956) viz. Nominal, Ordinal, Interval and Ratio, that the readability formulae certainly provide Ordinal measurement and possibly Interval measurement, but fall short of Ratio measurement.

The majority of readability formulae originate in the United States of America and rely heavily on lists of well known words based on American usage. The relevance of such formulae in Britain is therefore likely to be diminished.

Formulae use only features such as word length, sentence length etc. which can be quantified. However, there are features of text such as idiom, semantic content etc. which it is difficult, perhaps impossible, to measure and yet have a profound influence on readability. Furthermore, there is a general premise built into the formulae that 'shorter' means 'easier', yet elaboration or amplification can often improve the reader's understanding.

Printing styles and page layout etc., will also have an effect on readability. (Consider, for example, the difference between tabloid and broadsheet newspapers.) This is a feature that will not be taken into account by the formulae.

The motivation of the reader must also be considered. An interested reader will be likely to understand the text better than a bored reader. The interested reader may also be able to understand text of a more complex nature than one might consider him or her capable of understanding.

These readability formulae have been mainly applied to pure textual material. In mathematical writing one has symbols, numbers, diagrams etc. interspersed throughout the material, all of which contribute to the reader's understanding. Measurement of the 'readability' of mathematical text is therefore quite problematical.

Essentially words in written mathematical text will fall into one of three categories:

a) words which have the same meaning in Mathematical English as in Ordinary English, e.g. cylinder, ladder, rainfall etc.

b) words which have a meaning only in Mathematical English, e.g. polynomial, integer, logarithm etc.

c) words which occur in Mathematical English and Ordinary English, but have different meanings in each, e.g. root, differentiate, argument etc.

The meaning of words in category b) must be learnt either from the teacher or from books. Category b) words often have Greek origins, which would be unfamiliar to most readers. Frequently these words with specific meanings are crucial in understanding a passage or a question that is being asked.

The author, for example, has encountered polynomial equations which a single-subject A-level student could readily solve, where the students have been put off by being asked to find all the real roots of the equation. These students have not reached the stage where they are aware of complex numbers, so the phrase 'real roots' is confusing and off-putting.

When a word has several meanings, then its meaning very often has to be established from the context. In Mathematical English the number of context clues is small. This makes understanding difficult. The words in category c) fall into two groups:

(i) those whose mathematical meaning is close to the ordinary meaning, e.g. gradient, average and reflection.

(ii) those whose mathematical meaning is/can be quite different from the ordinary meaning, e.g. index, root and product.

Where the meanings are different then there is little help to enable the reader to determine the correct meaning.

Some words will have two different meanings in Mathematical English. For example, modulus can mean either the absolute value or the constant of elasticity in Hooke's Law. The letter e will be used in several different ways at A-level (the base of natural logarithms, the eccentricity of a conic section, or the coefficient of restitution). In such cases the context will inevitably be required to determine the meaning.

4.4 The Cloze Procedure.

In order to attempt measurement of 'readability' and allow for some of the factors which are not taken into consideration by the 'readability formulae', a technique known as the Cloze Procedure has been adopted. This has its origins in the Gestalt School of Psychology. One of the important ideas in this interpretation of human perception is the tendency of the subject to see things as 'whole' or 'complete' items. Hence, if part of a text is missing, the reader will tend to replace the missing letter, or word, etc, in order to gain a complete piece of writing. This tendency is termed 'closure' by the gestalt psychologist, whence the word 'Cloze', introduced by Taylor (1953), for a technique to measure the readability of text.

The measure depends, in part, on the notion of 'redundancy'. To the information theorist, 'redundant' information refers to information that contains elements of prior knowledge and which is, therefore, predictable to some extent. A redundant message will, therefore, contain some old information. Frequently language is highly redundant. 'Redundancy' does not necessarily mean that the language is unnecessary, nor does it reasonably mean that there are extra words that that could have been avoided. Redundancy is often a type of reinforcement so that the author ensures that the message is getting across to the reader. So, if a reader misses a meaning, the passage will be self-correcting and he can still interpret the sense of the passage. "Redundancy" can therefore be used to measure comprehensibility and readability.

One should note, however, that not all researchers think that Cloze techniques are a valid measure of comprehension. Weaver and Kingston (1963) examined the relationship of Cloze tests and standard tests of reading, listening and language-symbolising ability. They found that the relationship varied from little to moderate. Other critics of Cloze techniques include MacGintie (1966) and Mosberg and Coleman in De Landsheere (1973).

On the contrary however, Bormuth (1967) studied Cloze tests and multiple-choice comprehension tests as a means of measuring various comprehension skills. Bormuth found only a single factor which accounted for 77% of the variation in the correlation matrix. The loadings of all the tests on this factor approached the maximum correlations possible for these tests. Bormuth concluded that Cloze tests are a valid measure of comprehension ability.

Klare (1977) considers the Cloze technique to be a useful method for assessing the readability of mathematical and scientific text. Gagatsis (1984) used the technique to compare comprehension of mathematical text and general reading ability for any text with 16-18 year-old Greek mathematics students. Jones (1974) investigated the readability of mathematics texts in use in junior schools in Sheffield. 45 texts were scored using the Fry Readability Graph. The Cloze procedure was used on passages from the six most commonly used mathematics texts. 310 children completed three 100 word passages from the book they were using in class. Each seventh word was deleted (avoiding digits and symbols). The omitted words were mainly general words, not technical words specific to mathematics. The results seemed to suggest that the Fry Readability Graph was an accurate measure of mathematical texts. One can argue here, however, that because the words were mainly general and symbols and digits were not deleted, that the readability of mathematical text was not being measured. The Fry Readability Graph has often been thought to give high reading ages for passages, but the evidence of Jones' study suggests that this is not the case for junior mathematical texts.

When using the Cloze procedure, one should only accept as correct the identical word or symbol that was originally in the passage and has been replaced by a blank.

When deleting every fifth word or symbol, it is possible to produce five different Cloze passages from one piece of text. If one were to use only one passage, one might inadvertently delete a rather large number of words for which the necessary background information doesn't exist. Therefore such a passage would seem more difficult. By using a variety of passages one gets more reliable results. It also means that, if a number of subjects are doing the test simultaneously, they will be writing different responses, so that there is less chance of cheating occurring.

When administering the test it is best not to give the subjects any information about whether they should guess or not guess the blanks. Some subjects like to take risks and others are more cautious. If a 'don't guess' instruction is given then it is open to different interpretations by the subjects. By not giving instructions all the subjects start on an equal footing.

One should note that, as well as the readability of the text, one is measuring the skill of the subject in filling in the blanks. A relationship has been found between Cloze tests and intelligence test score. For example, Taylor (1957) obtained a correlation of 0.73 and Jenkinson (in Rankin (1970)) obtained a correlation of 0.69. These correlations were obtained when all types of word were deleted. Taylor found lower correlation of 0.46 - 0.59 when only nouns, verbs and adverbs were deleted.

Hater (1969) modified the Cloze procedure so that it could be applied to mathematical material. The idea she used was one of 'tokens'. Words are a single token and expressions such as x - y would be three tokens (x, minus and y). Similarly ln(x) would be two tokens, log and x. The Cloze passage would have every fifth token deleted. If the missing token was a word a long line would be used. If a mathematical token was missing a short line would be used. The subject could then tell if a word or a mathematical symbol had to be inserted in the spaces when every fifth token was deleted.

4.5 The Cloze Experiment.

The above technique used by Hater has formed the basis on which the passages have been prepared for our Cloze experiment in the present study. A description of this experiment now follows.

One of the objects of the Cloze experiment was to compare the difficulty, with regard to readability, of the questions in the papers set by the English and Welsh GCE examination boards in A-level Mathematics. Typically these papers contain Pure Mathematics, Mechanics and Statistics questions. Pure mathematics usually accounts for 50% (sometimes more) of the questions available to candidates. It was decided, therefore, to select 4 questions from each board's 1989 papers. Two of the questions were on Pure Mathematics topics (vectors and calculus), one was on a Statistics topic (probability) and one on a Mechanics topic (projectiles). The concept of 'tokens', as used by Hater, described earlier in this chapter, was employed. Every fifth token was deleted. It was, therefore, possible to obtain five sets of deletions for each question, by beginning deletion on the 1st, 2nd, 3rd, 4th or 5th token.. The subjects were arranged in groups of 10, so that within each group of 10, two people only were working on the same material. The tokens were blanked out. Long and short lines were not used. The subjects were told that they had to supply a response for the blanks, which appeared at intervals of five tokens. It was explained to the subjects that a token was a mathematical sign or symbol, or a word or a number. Examples of the questions, with deletions, are provided in Appendix B. The questions appear in the original typescript used by the examination board, as layout of questions should be considered when making comparisons.

The subjects were all male pupils studying Mathematics at A-level. There were six sets of 10 pupils. One of the sets was studying Mathematics and Further Mathematics. One set was a mixed ability set and the other four sets were setted according to mathematical ability. The Mathematics and Further Mathematics set can generally be considered to be of superior mathematical ability to the other five sets. The word 'generally' is used, because within the top set of the other four sets and the mixed ability set, there were some pupils who could potentially have taken Further Mathematics A-level, very successfully, but happened to choose a combination of A-level subjects which did not include Further Mathematics.

The score of each subject in their mock A-level Mathematics examination was also recorded. This would give an indication of a traditional measure of mathematical ability. This measure should, naturally, be viewed with some caution for a variety of reasons. Anomalies are likely to occur because bright students will often do little revision for the 'mock', as the real thing is some months away. In addition, students will only just have completed the syllabus and will have had little practice at past A-level papers at this stage.

The questions used in the experiment varied in length. Consequently to obtain comparisons, the percentage of correct responses on each question, from each board, was recorded, for each of the subjects.

The results obtained, including the mock score, were as follows:

Table 4.2. Cloze Experiment Data.

CALCULUS

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Further Maths	73	81	83	78	84	80	72	86	81	72
	79	86	91	80	92	80	76	86	81	83
	78	81	87	80	88	80	80	86	81	76
	72	86	83	75	84	80	72	79	77	72
	94	90	91	82	92	87	84	93	85	86
	85	90	91	85	92	93	80	86	85	83
	88	90	91	86	92	93	84	93	85	83
	79	86	87	77	88	87	80	86	85	79
	76	86	87	76	84	80	80	86	81	79
	81	86	87	84	88	93	76	86	81	79
Set 1	73	90	91	82	92	87	76	86	85	83
	63	81	83	74	80	80	72	86	77	76
	76	86	87	79	92	87	76	86	81	79
	64	86	83	76	80	87	72	79	81	72
	57	81	83	77	80	80	68	79	77	72
	80	90	87	81	84	80	76	86	81	83
	55	81	78	74	80	80	68	79	77	79
	62	81	87	75	80	80	72	79	81	79
	88	90	91	85	88	87	80	93	85	79
	75	86	91	80	92	87	76	86	81	83

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 2	60	86	83	80	88	87	80	86	81	79
	64	86	87	81	88	87	80	79	81	76
	57	86	87	73	84	80	68	86	81	72
	42	81	83	74	84	80	68	86	77	72
	69	90	91	83	92	87	76	86	85	83
	62	86	87	82	88	87	72	86	73	76
	41	86	83	73	88	87	68	79	73	76
	50	86	87	74	88	87	72	79	77	76
	74	90	91	84	92	87	80	93	85	79
	45	76	83	72	84	80	72	79	77	79
Set 3	61	90	83	80	88	80	76	79	77	76
	52	90	83	74	84	80	76	86	85	76
	41	81	83	77	84	80	72	79	77	79
	50	90	83	78	84	80	72	86	85	76
	35	81	78	74	80	80	72	79	73	72
	32	76	83	71	80	80	68	79	73	69
	40	76	83	74	84	80	72	79	77	72
	53	76	83	72	88	80	80	79	73	76
	51	90	83	78	88	87	76	79	81	72
	59	86	87	83	84	87	72	86	81	79

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 4	40	86	83	77	84	87	76	86	81	76
	38	86	83	76	80	80	72	79	73	76
	39	81	83	78	80	87	76	79	81	76
	37	81	78	76	84	87	68	79	81	69
	40	86	87	79	88	87	72	86	77	76
	34	86	78	78	84	73	76	79	73	76
	29	86	78	70	80	73	68	79	69	69
	32	86	78	73	80	80	64	79	73	69
	47	86	87	81	84	80	72	79	77	79
	31	81	78	78	84	87	68	79	77	69
Mixed	58	86	87	79	88	87	76	86	81	79
Ability	46	86	87	80	88	87	76	86	81	79
	37	81	83	72	88	80	72	79	73	72
	61	86	91	80	92	80	76	79	77	79
	57	86	83	75	84	87	72	86	77	76
	67	90	91	83	92	93	80	93	85	79
	44	81	87	79	88	80	76	79	73	79
	37	76	83	78	80	80	72	79	77	72
	64	81	87	79	92	87	72	79	81	76
	39	81	83	82	84	80	72	79	81	76
Overall Ave.		85	85	78	86	84	74	83	79	77

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Further Maths	73	76	85	85	88	82	80	89	79	82
	79	80	89	85	88	82	80	94	79	82
	78	76	85	83	88	82	80	89	79	82
	72	80	85	89	81	82	73	83	79	73
	94	88	96	93	94	91	87	94	86	91
	85	84	89	93	94	82	80	94	79	82
	88	84	96	89	94	82	87	94	86	91
	79	84	89	85	88	82	80	94	79	82
	76	84	85	85	81	82	80	89	79	82
	81	80	89	93	94	82	87	89	79	82
Set 1	73	84	93	89	88	82	87	94	79	91
	63	76	85	87	88	82	80	83	71	82
	76	84	89	85	88	82	80	94	79	82
	64	76	89	85	81	82	80	89	79	82
	59	76	85	85	81	73	80	89	71	73
	80	84	89	91	88	82	80	94	79	82
	55	72	81	80	81	73	73	83	71	73
	62	80	89	83	81	82	80	83	79	82
	88	84	93	91	94	82	87	94	79	82
	75	80	93	83	88	82	80	89	79	82

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 2	60	80	89	89	88	82	73	89	79	82
	64	76	85	89	88	82	73	89	71	82
	57	76	81	83	81	73	73	83	79	82
	42	76	85	85	81	73	73	83	71	73
	69	80	89	89	88	82	80	89	79	82
	62	76	85	89	88	82	80	89	71	82
	41	76	81	78	81	73	73	83	71	82
	50	76	85	83	81	82	73	83	79	82
	74	80	93	87	94	82	80	94	79	82
	45	72	85	78	81	73	73	83	71	73
Set 3	61	76	85	91	94	82	80	89	79	82
	52	76	89	91	88	73	80	89	71	82
	41	76	81	85	88	73	73	83	71	82
	50	76	81	85	88	73	73	89	79	82
	35	76	81	80	81	82	73	83	71	73
	32	68	81	78	81	73	73	83	71	73
	40	76	81	83	88	73	73	83	79	73
	53	80	89	83	88	73	73	89	79	82
	51	76	89	83	88	73	80	89	79	73
	59	80	81	89	94	82	80	89	79	82

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 4	40	76	89	85	81	73	80	89	79	82
	38	76	85	83	81	82	80	83	71	73
	39	72	85	85	81	82	80	83	71	73
	37	76	85	83	81	73	80	83	71	73
	40	80	89	85	81	73	80	78	79	73
	34	72	81	78	81	73	73	83	71	73
	29	72	81	76	75	73	67	78	71	73
	32	72	78	80	81	73	67	78	71	73
	47	80	89	83	88	82	73	89	71	82
	31	72	78	76	81	73	73	78	71	73
Mixed Ability	58	80	85	85	88	82	87	89	71	82
	46	76	85	85	81	82	73	89	71	82
	37	76	81	80	88	73	73	83	79	73
	61	76	85	89	88	82	80	83	79	82
	57	76	85	83	88	82	80	83	79	82
	67	80	89	89	88	82	87	94	86	82
	44	76	85	83	81	73	73	89	79	73
	37	72	81	80	75	82	73	89	71	73
	64	76	89	87	88	82	80	89	79	82
	39	72	85	83	81	73	73	89	71	82
Overall Ave.		78	86	85	86	79	78	87	76	80

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Further Maths	73	81	71	85	81	60	76	68	62	81
	79	88	71	92	87	66	76	76	72	84
	78	85	71	85	90	66	71	76	69	81
	72	77	67	85	84	60	62	74	67	78
	94	92	81	92	97	74	76	82	79	91
	85	77	71	85	94	74	76	76	79	84
	88	92	81	92	97	74	76	79	77	91
	79	92	76	92	87	71	76	76	74	81
	76	81	71	85	94	66	68	68	69	81
	81	92	76	92	97	74	76	79	72	84
Set 1	73	85	76	92	90	74	71	74	74	88
	63	85	71	85	90	66	68	74	69	81
	76	77	76	85	90	66	74	74	69	84
	64	85	67	85	84	63	68	71	69	81
	59	77	62	85	81	60	65	71	67	75
	80	88	76	92	84	74	71	74	69	88
	55	81	62	85	84	60	62	65	67	75
	62	85	71	77	84	63	68	71	67	81
	88	92	76	92	94	74	74	76	77	91
	75	85	71	85	81	66	71	74	69	84

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 2	60	85	71	85	84	66	62	74	77	81
	64	85	76	92	84	66	62	74	72	81
	57	77	71	85	87	66	68	71	67	81
	42	77	62	77	81	60	62	68	67	75
	69	85	76	92	90	71	71	79	79	84
	62	81	62	77	84	71	71	71	72	78
	41	81	67	77	81	63	62	65	64	75
	50	81	67	85	87	63	68	68	62	81
	74	88	71	92	94	74	71	76	77	88
	45	77	67	77	81	63	59	68	62	72
Set 3	61	85	71	92	90	71	71	79	69	84
	52	85	71	85	90	69	68	71	69	81
	41	77	67	77	87	60	65	65	64	78
	50	81	67	85	87	66	68	74	67	84
	35	77	67	85	81	60	59	65	64	81
	32	73	62	77	81	60	59	65	62	75
	40	77	67	85	87	60	68	68	62	81
	53	81	67	92	87	74	68	74	74	81
	51	85	76	92	94	77	71	74	69	81
	59	81	76	85	90	71	74	79	69	84

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 4	40	85	62	85	84	57	68	65	69	88
	38	73	67	77	84	66	59	76	64	75
	39	73	71	85	84	66	68	74	62	72
	37	77	71	77	87	57	59	68	62	72
	40	88	71	92	81	74	74	76	69	84
	34	77	67	92	87	63	65	76	67	84
	29	77	67	77	77	57	59	62	62	72
	32	73	62	85	77	60	59	62	62	72
	47	85	62	77	84	69	71	76	74	84
	31	77	62	77	84	57	62	71	67	72
Mixed Ability	58	77	76	92	90	66	68	76	74	84
	46	77	71	85	90	66	68	76	69	81
	37	77	67	77	81	63	62	65	64	75
	61	85	71	92	90	66	68	79	72	84
	57	85	76	85	84	66	68	65	67	78
	67	88	81	92	94	69	74	76	79	84
	44	77	76	77	81	60	65	74	69	75
	37	73	67	85	81	63	65	65	64	75
	64	81	71	92	90	66	68	79	74	81
	39	73	62	85	84	60	62	65	62	81
Overall Ave.		82	70	85	87	66	68	72	69	81

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Further Maths	73	92	70	70	71	67	64	67	68	64
	79	95	75	78	76	67	71	73	80	73
	78	95	75	74	78	67	71	73	80	68
	72	89	70	74	73	61	71	70	80	68
	94	97	80	83	83	72	79	77	84	77
	85	97	70	83	78	72	79	77	84	77
	88	97	75	83	83	72	79	77	84	73
	79	86	70	74	78	61	71	73	76	68
	76	86	65	70	68	67	71	73	72	64
	81	92	75	74	78	72	79	77	80	73
Set 1	73	95	75	78	76	72	79	80	80	68
	63	86	70	74	71	61	71	63	72	64
	76	92	70	78	68	67	71	70	72	68
	64	86	65	70	66	61	64	63	72	64
	59	84	65	70	66	56	64	63	72	59
	80	92	75	78	73	67	71	73	76	68
	55	89	65	70	66	56	64	70	68	64
	62	89	70	74	71	67	71	70	72	64
	88	97	80	83	80	72	79	77	76	73
	75	92	70	78	76	72	79	80	72	64

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 2	60	92	70	78	73	67	79	77	80	64
	64	89	80	78	73	67	79	77	80	68
	57	92	75	74	68	61	71	63	76	73
	42	81	70	70	66	61	64	67	72	64
	69	95	80	83	80	67	79	77	80	68
	62	95	65	70	71	56	71	70	80	59
	41	86	70	70	68	67	79	63	72	64
	50	86	70	78	71	56	71	67	64	59
	74	95	80	83	76	67	79	73	80	73
	45	84	70	70	71	67	79	67	72	59
Set 3	61	95	80	78	76	67	79	73	76	73
	52	89	80	65	68	67	71	73	80	73
	41	92	65	74	68	61	71	70	72	64
	50	89	70	78	73	67	71	77	76	73
	35	92	65	65	66	61	64	67	68	64
	32	81	65	70	66	61	64	60	72	55
	40	81	70	70	73	61	71	70	76	68
	53	95	75	74	80	67	71	70	76	73
	51	89	70	70	68	61	71	70	76	68
	59	89	70	83	80	72	71	77	80	73

SET	Trial Exam	AEB	CAM	JMB	LON	OXF	O&C	MEI	SMP	WEL
Set 4	40	86	70	74	76	61	71	73	76	64
	38	86	70	74	73	61	71	73	68	64
	39	86	70	70	73	61	71	67	72	68
	37	89	70	70	71	61	71	67	68	68
	40	92	70	78	78	61	79	73	80	64
	34	84	70	70	71	61	64	63	68	64
	29	81	65	65	63	56	64	60	64	55
	32	81	70	70	68	61	71	63	64	59
	47	92	80	74	76	72	79	70	72	68
	31	81	70	70	66	61	71	63	64	59
Mixed Ability	58	92	75	74	78	61	71	73	76	68
	46	89	75	78	73	67	71	67	76	73
	37	86	70	70	71	61	64	63	72	64
	61	92	75	78	78	67	79	70	80	73
	57	89	70	74	76	61	71	67	80	73
	67	97	75	83	76	67	79	77	80	73
	44	89	65	78	66	56	71	63	68	73
	37	84	75	74	66	56	64	67	72	68
	64	89	75	83	73	67	79	73	80	73
	39	89	65	78	71	61	64	63	64	68
Overall Ave.		90	72	75	73	64	72	70	75	67

Having collected the data, two questions should be addressed : (i) What type of data do we have ? (ii) What do we want to determine from the data ? The answers to these questions will tell us what type of statistical model we will require to analyse the data.

We wish to decide if several independent samples have come from the same population. Sample values almost always differ somewhat, and the problem is to determine whether the observed sample differences signify differences among populations, or whether they are merely the chance variations that are to be expected among random samples from the same population.

Circumstances sometimes require that we design an experiment so that more than two samples, or conditions, can be studied simultaneously. In this case, it will be necessary to use a statistical test which will indicate whether there is an overall difference among the samples, or conditions, before one picks out any pair of samples, in order to test the significance of the difference between them.

If we wished to use a two-sample statistical test to test for differences among, for example, 5 groups, we would need 10 statistical tests (the number of ways of choosing 2 from 10). We are then giving ourself ten chances, rather than one chance, of rejecting the null hypothesis. If each test is at the 5% level, then the probability of one or more significant differences is $1 - 0.95^5 = 0.40$. This is the actual probability of rejecting the null hypothesis when it is true (a Type I Error). Hence we could easiliy get a fallacious result by testing samples two at a time.

The parametric technique for testing whether several samples have come from identical populations is the analysis of variance, or F test. The advantage of using variances, rather than standard deviations, is that if the samples are independent, then the variances are additive.

If we make the following assumptions then we may use a parametric test : (i) The level of measurement is at least interval. (ii) The sample data is drawn from a normally distributed population. (iii) The variances of samples are not significantly different. For the data we have collected, we shall make these assumptions.

In our case we have three factors (Board, Topic and Student). It could be that any single factor may have no overall effect, but that any factor may have significant effects when individual levels of the other factor(s) are taken into account. This is an interaction effect. Our general strategy will be to calculate point estimates of the variance parameters for the main effects of Board, Topic and Student, and also for the interaction effects of any pair, or of all three. We shall then aim, if possible, to find confidence limits for these parameters. If the value zero is not included in the confidence interval, then we shall conclude that the effect is significant. This is a more rigorous and mathematically satisfying approach than simply performing tests of significance using a prepared package. We shall now move on to the detail of our statistical model.

In order to proceed we shall use a multi-factor analysis of variance (ANOVA). In this statistical model, the 'total variation' is divided into 'explained variation' and 'unexplained variation'. The 'explained variation' comprises 'between groups variation' for each factor, plus 'interaction variation' which covers every possible combinatuon of factors. The 'error' is the 'unexplained', within-groups variation.

Identification of the model

We have a fully crossed 3-way analysis of variance without replications. In what follows, we shall use subscripts *s* for student, *b* for board, *t* for topic and *r* for replication. Hence x_{sbtr} denotes the score of student *s*, on board *b*, on topic *t*, on replication *r*. In our case, *b* and *t* are fixed indices and a '$\bullet$' will indicate an average being taken, whilst *s* and *r* will be random indices and a '$*$' will be used to take a mean. A '$*$' will appear in definitions of effects, but will be replaced by a '$\bullet$' when an estimate is taken.

Distinct letters are used for all effects, Greek letters being used to denote fixed effects and Roman letters being used to denote random or mixed effects.

Hence our model is:

$$x_{sbtr} = \mu + a_s + \beta_b + \gamma_t + d_{sb} + f_{st} + \lambda_{bt} + z_{sbt} + e_{sbtr}$$

where $\mu = x_{*\bullet\bullet*}$

$$
\begin{aligned}
a_s &= x_{s\bullet\bullet*} - x_{*\bullet\bullet*} \\
\beta_b &= x_{*b\bullet*} - x_{*\bullet\bullet*} \\
\gamma_t &= x_{*\bullet t*} - x_{*\bullet\bullet*} \\
d_{sb} &= x_{sb\bullet*} - x_{s\bullet\bullet*} - x_{*b\bullet*} + x_{*\bullet\bullet*} \\
f_{st} &= x_{s\bullet t*} - x_{s\bullet\bullet*} - x_{*\bullet t*} + x_{*\bullet\bullet*} \\
\lambda_{bt} &= x_{*bt*} - x_{*b\bullet*} - x_{*\bullet t*} + x_{*\bullet\bullet*} \\
z_{sbt} &= x_{sbt*} - x_{sb\bullet*} - x_{s\bullet t*} - x_{*bt*} + x_{s\bullet\bullet*} + x_{*b\bullet*} + x_{*\bullet t*} - x_{*\bullet\bullet*} \\
e_{sbtr} &= x_{sbtr} - x_{sbt*}.
\end{aligned}
$$

With these definitions, the expansion of x_{sbtr} is a tautology and the following constraints are automatically satisfied:

$$
\begin{aligned}
a_* &= 0 \\
\beta_\bullet &= 0 \\
\gamma_\bullet &= 0 \\
d_{*b} &= d_{s\bullet} = 0 \\
f_{*t} &= f_{s\bullet} = 0 \\
\lambda_{\bullet t} &= \lambda_{b\bullet} = 0 \\
z_{*bt} &= z_{s\bullet t} = z_{sb\bullet} = 0 \\
e_{sbt*} &= 0.
\end{aligned}
$$

Because we do not have repeated replication data, we are forced to make the modelling assumption $z_{sbt} = 0$. This is equivalent to:

$$x_{sbt*} = x_{sb\bullet *} + x_{s\bullet t*} + x_{*bt*} - x_{s\bullet\bullet *} - x_{*b\bullet *} - x_{*\bullet t*} + x_{*\bullet\bullet *}.$$

For our model we now have

$$e_{sbtr} = x_{sbtr} - x_{sb\bullet *} - x_{s\bullet t*} - x_{*bt*} + x_{s\bullet\bullet *} + x_{*b\bullet *} + x_{*\bullet t*} - x_{*\bullet\bullet *}.$$

We can now replace $*$ by $\bullet$ to obtain effect estimators:

$$\begin{aligned}
\hat{\mu} &= x_{\bullet\bullet\bullet\bullet} \\
\hat{\alpha}_s &= x_{s\bullet\bullet\bullet} - x_{\bullet\bullet\bullet\bullet} \\
\hat{\beta}_b &= x_{\bullet b\bullet\bullet} - x_{\bullet\bullet\bullet\bullet} \\
\hat{\gamma}_t &= x_{\bullet\bullet t} - x_{\bullet\bullet\bullet\bullet} \\
\hat{d}_{sb} &= x_{sb\bullet\bullet} - x_{s\bullet\bullet\bullet} - x_{\bullet b\bullet\bullet} + x_{\bullet\bullet\bullet\bullet} \\
\hat{f}_{st} &= x_{s\bullet t\bullet} - x_{s\bullet\bullet\bullet} - x_{\bullet\bullet t\bullet} + x_{\bullet\bullet\bullet\bullet} \\
\hat{\lambda}_{bt} &= x_{\bullet bt\bullet} - x_{\bullet b\bullet\bullet} - x_{\bullet\bullet t\bullet} + x_{\bullet\bullet\bullet\bullet} \\
\hat{e}_{sbtr} &= x_{sbtr} - x_{sb\bullet\bullet} - x_{s\bullet t\bullet} - x_{\bullet bt\bullet} + x_{s\bullet\bullet\bullet} + x_{\bullet b\bullet\bullet} + x_{\bullet\bullet t\bullet} - x_{\bullet\bullet\bullet\bullet}.
\end{aligned}$$

Hence we have the tautology:

$$x_{sbtr} = \hat{\mu} + \hat{\alpha}_s + \hat{\beta}_b + \hat{\gamma}_t + \hat{d}_{sb} + \hat{f}_{st} + \hat{\lambda}_{bt} + \hat{e}_{sbtr}.$$

When doing arithmetic with the sample values, since we only have one replication, we shall drop the r index and the final '$\bullet$' on all the x's.

So we shall write

$$\begin{aligned}
\hat{\mu} &= x_{\bullet\bullet\bullet} \\
\hat{\alpha}_s &= x_{s\bullet\bullet} - x_{\bullet\bullet\bullet} \\
\hat{\beta}_b &= x_{\bullet b\bullet} - x_{\bullet\bullet\bullet} \\
\hat{\gamma}_t &= x_{\bullet\bullet t} - x_{\bullet\bullet\bullet} \\
\hat{d}_{sb} &= x_{sb\bullet} - x_{s\bullet\bullet} - x_{\bullet b\bullet} + x_{\bullet\bullet\bullet} \\
\hat{f}_{st} &= x_{s\bullet t} - x_{s\bullet\bullet} - x_{\bullet\bullet t} + x_{\bullet\bullet\bullet} \\
\hat{\lambda}_{bt} &= x_{\bullet bt} - x_{\bullet b\bullet} - x_{\bullet\bullet t} + x_{\bullet\bullet\bullet} \\
\hat{e}_{sbt} &= x_{sbt} - x_{sb\bullet} - x_{s\bullet t} - x_{\bullet bt} + x_{s\bullet\bullet} + x_{\bullet b\bullet} + x_{\bullet\bullet t} - x_{\bullet\bullet\bullet}.
\end{aligned}$$

The estimator tautology becomes

$$x_{sbt} = \hat{\mu} + \hat{\alpha}_s + \hat{\beta}_b + \hat{\gamma}_t + \hat{d}_{sb} + \hat{f}_{st} + \hat{\lambda}_{bt} + \hat{e}_{sbt}$$

and $\hat{\alpha}_{\bullet} = 0$

$$\hat{\beta}_{\bullet} = 0$$
$$\hat{\gamma}_{\bullet} = 0$$
$$\hat{d}_{\bullet b} = \hat{d}_{s\bullet} = 0$$
$$\hat{f}_{\bullet t} = \hat{f}_{s\bullet} = 0$$
$$\hat{\lambda}_{\bullet t} = \hat{\lambda}_{b\bullet} = 0$$
$$\hat{e}_{\bullet bt} = \hat{e}_{s\bullet t} = \hat{e}_{sb\bullet} = 0$$

are the estimator conditions.

If we denote the total sum of squares, SS_{tot} by

$$SS_{tot} = \sum_{sbt} (x_{sbt} - x_{\bullet\bullet\bullet})^2$$

we can use the estimator tautology and then the estimator conditions to obtain:

$$SS_{tot} = SS_S + SS_B + SS_T + SS_{SB} + SS_{ST} + SS_{BT} + SS_E$$

where $SS_S = N_B N_T \sum_s \hat{\alpha}_s^2$ and N_B = Number of Boards.

$SS_B = N_S N_T \sum_b \hat{\beta}_b^2$ $\quad N_S$ = Number of Students.

$SS_T = N_S N_B \sum_t \hat{\gamma}_t^2$ $\quad N_T$ = Number of Topics.

$$SS_{SB} = N_T \sum_{sb} \hat{d}_{sb}^2$$
$$SS_{ST} = N_B \sum_{st} \hat{f}_{st}^2$$
$$SS_{BT} = N_S \sum_{bt} \hat{\lambda}_{bt}^2$$
$$SS_E = \sum_{sbt} \hat{e}_{sbt}^2$$

For each sum of squares, SS, above, we define a mean square, MS, as follows:

$$MS_S = \frac{SS_S}{N_S - 1}, \quad MS_B = \frac{SS_B}{N_B - 1}, \quad MS_T = \frac{SS_T}{N_T - 1}$$

$$MS_{SB} = \frac{SS_{SB}}{(N_S - 1)(N_B - 1)}, \quad MS_{ST} = \frac{SS_{ST}}{(N_S - 1)(N_B - 1)}, \quad MS_{BT} = \frac{SS_{BT}}{(N_B - 1)(N_T - 1)}$$

$$MS_E = \frac{SS_E}{(N_S - 1)(N_B - 1)(N_T - 1)}.$$

We obtain expected values for the mean squares in terms of the variance parameters of the model, in our mixed model, from the formulae:

$$E(MS_S) = N_B N_T \sigma_S^2 + \sigma_E^2$$

$$E(MS_B) = N_S N_T \langle \sigma_B^2 \rangle + N_T \sigma_{SB}^2 + \sigma_E^2$$

$$E(MS_T) = N_S N_B \langle \sigma_T^2 \rangle + N_B \sigma_{ST}^2 + \sigma_E^2$$

$$E(MS_{SB}) = N_T \sigma_{SB}^2 + \sigma_E^2$$

$$E(MS_{ST}) = N_B \sigma_{ST}^2 + \sigma_E^2$$

$$E(MS_{BT}) = N_S \langle \sigma_{BT}^2 \rangle + \sigma_E^2$$

$$E(MS_E) = \sigma_E^2.$$

Where $\langle \sigma^2 \rangle$ denotes the fixed effect "finite population" variance parameters and σ^2 represents true variance parameters.

We obtain point estimates $\hat{\sigma}^2$ for all the variance parameters from the above equations, by putting each expected mean square equal to its respective mean square and making $\hat{\sigma}^2$ the subject of the equation. Thus we obtain:

$$\hat{\sigma}_E^2 = MS_E$$

$$\langle \hat{\sigma}_{BT}^2 \rangle = \frac{MS_{BT} - MS_E}{N_S}$$

$$\hat{\sigma}_{ST}^2 = \frac{MS_{ST} - MS_E}{N_B}$$

$$\hat{\sigma}_{SB}^2 = \frac{MS_{SB} - MS_E}{N_T}$$

$$\langle \hat{\sigma}_T^2 \rangle = \frac{MS_T - MS_{ST}}{N_S N_B}$$

$$\langle \hat{\sigma}_B^2 \rangle = \frac{MS_B - MS_{SB}}{N_S N_T}$$

$$\hat{\sigma}_S^2 = \frac{MS_S - MS_E}{N_B N_T}$$

where $\sigma_S^2 = E\left(a_s^2\right)$

$$\sigma_{SB}^2 = E\left(d_{sb}^2\right)$$

$$\sigma_{ST}^2 = E\left(f_{st}^2\right)$$

$$\sigma_E^2 = E\left(e_{sbt}^2\right)$$

and $\left\langle\sigma_B^2\right\rangle = \frac{1}{N_B - 1}\sum_b \beta_b^2$

$$\left\langle\sigma_T^2\right\rangle = \frac{1}{N_T - 1}\sum_t \gamma_t^2$$

$$\left\langle\sigma_{BT}^2\right\rangle = \frac{1}{(N_B - 1)(N_T - 1)}\sum_{bt} \lambda_{bt}^2.$$

The numerical values for all the point estimates of the variance parameters were obtained by

(i) Working out the estimated effect values from the dot formulae.

(ii) Working out the Sums of Squares from their definitions.

(iii) Working out the Mean Squares from their definitions.

(iv) Working out the point estimates for the variance parameters from the above formulae.

		Degrees of Freedom	Sum of Squares	Mean Square	Point Estimate
Between Boards	B	8	29 253	3656.61	15.20
Topics	T	3	28 819	9606.22	17.76
Students	S	59	28 615	485.00	13.22
Interactions	BT	24	43 761	1823.38	30.24
	TS	177	3 018	17.05	0.87
	SB	472	4 252	9.01	-0.05
Residual	E	1416	13 030	9.20	9.20
Total		2159	150 748		

We are now in a position to proceed to tests of significance. We shall assume normality, homoscedasticity and independence of residuals to get the following equations, which are the basis for the tests of significance. We shall make corrections for Kurtosis where necessary.

$$\frac{MS_{BT}}{MS_E} \sim F_{v_{BT},v_E}\left[\frac{N_S v_{BT}\langle\sigma_{BT}^2\rangle}{\sigma_E^2}\right]$$

$$\frac{MS_{ST}}{MS_E} \sim \left(\frac{N_B\sigma_{ST}^2+\sigma_E^2}{\sigma_E^2}\right)F_{v_{ST},v_E}$$

$$\frac{MS_{SB}}{MS_E} \sim \left(\frac{N_T\sigma_{SB}^2+\sigma_E^2}{\sigma_E^2}\right)F_{v_{SB},v_E}$$

$$\frac{MS_T}{MS_{ST}} \sim F_{v_T,v_{ST}}\left[\frac{N_S N_B v_T\langle\sigma_T^2\rangle}{N_B\sigma_{ST}^2+\sigma_E^2}\right]$$

$$\frac{MS_B}{MS_{SB}} \sim F_{v_B,v_{SB}}\left[\frac{N_S N_T v_B\langle\sigma_B^2\rangle}{N_T\sigma_{SB}^2+\sigma_E^2}\right]$$

$$\frac{MS_S}{MS_E} \sim \left(\frac{N_B N_T\sigma_S^2+\sigma_E^2}{\sigma_E^2}\right)F_{v_S,v_E}$$

where $v_B = N_B - 1$ $\quad v_{BT} = (N_B - 1)(N_T - 1)$

$v_S = N_S - 1$ $\quad v_{TS} = (N_T - 1)(N_S - 1)$

$v_T = N_T - 1$ $\quad v_{SB} = (N_S - 1)(N_B - 1)$

$v_E = N_E - 1$

and $F_{m,n}[\rho^2]$ denotes the non-central F distribution with non-centrality parameter ρ^2.

Before proceeding to the F tests, a check was made on the Skewness and Kurtosis.

We denote Skewness by γ_1 and Kurtosis by γ_2.

Asymptotically under a normal distribution

$$\hat{\gamma}_1 \sim N\left(0, \frac{6}{n}\right) \quad \text{and} \quad \hat{\gamma}_2 \sim N\left(0, \frac{24}{n}\right)$$

where n is the number of degrees of freedom.

The following results were obtained:

Effect	df	$\hat{\gamma}_1$	z_1	$\hat{\gamma}_2$	z_2
S	60	0.1953	0.62	−0.5156	−0.82
ST	240	−0.1128	−0.71	0.1746	0.55
SB	540	−0.1342	−1.27	−0.2010	−0.95
E	2160	0.09851	1.87	0.2912	2.76*
* = Reject H_0					

where z_1 and z_2 are the standard scores corresponding to $\hat{\gamma}_1$ and $\hat{\gamma}_2$ respectively.

We see that only the Kurtosis in the case of the error was significantly non-zero. In this case, the degrees of freedom are large and $F_{n,N}$ is virtually $\frac{\chi_n^2}{n}$ when N is large. So the Kurtosis will affect the F tests very little.

The F tests are appropriate here, since, under the null hypothesis, which is being tested, either the expressions in the square brackets (the non-centrality parameters) are zero, or the expressions in the curved brackets are unity. Thus, in all cases, under the null hypothesis, we have a Central F Distribution. We use the upper critical region because, under the alternative hypothesis, the non-centrality parameter is positive, or the expression in the curved brackets is greater than unity and, in both cases, this effectively shifts the distribution to the right.

Summary of F tests

Effect	F value	$z = \frac{1}{2}\log F$	Critical Value	Decision
Fixed Effects				
T	563.3	3.17	0.8453	Sig. 0.1%
B	405.9	3.00	0.5917	Sig. 0.1%
BT	198.2	2.64	0.3786	Sig. 0.1%
Random Effects				
S	52.71	1.98	0.3786	Sig. 0.1%
ST	1.853	0.308	0.2913	Sig. 1%
SB	0.9790	−0.011	0.0000	Not Sig.

To absorb the Kurtosis, when significantly non-zero, into our calculation of confidence intervals for variance parameters, we use the result:

$$\text{var}\left(s^2\right) = \sigma^4\left[\frac{2}{n-1} + \frac{\gamma_2}{n}\right]$$

$$= \frac{2\sigma^4}{n-1}\left[1 + \left(\frac{n-1}{n}\right)\frac{\gamma_2}{2}\right].$$

Now $\frac{s^2}{\sigma^2} \sim \frac{\chi^2_{n-1}}{n-1}$ under normality conditions.

Under non-normal conditions, we choose a distribution with the correct mean and variance and of the same general form, as an approximation. We can do this by simply adjusting the number of degrees of freedom for χ^2.

That is we assume $\frac{s^2}{\sigma^2} \sim \frac{\chi^2_{n^*}}{n^*}$ where $n^* = \frac{n-1}{\left(1 + \frac{\hat{\gamma}_2}{2}\right)}$ and $\hat{\gamma}_2$ is the obvious estimator for γ_2.

Confidence Intervals for Significant Variance Parameters

(a) Residual Variance

$n = df = 1416 \quad n^* = 1235$

We use the approximation $\sqrt{2\chi^2_n} \sim N\left(\sqrt{2n-1}, 1\right)$ which is valid for $n > 100$.

Now $\sqrt{2n^* - 1} = \sqrt{2 \times 1235 - 1} = 49.69$.

Hence 95% confidence interval for $\sqrt{2\chi^2_{1235}}$ is (49.69 - 1.96, 49.69 + 1.96)

$\Rightarrow$ 95% confidence interval for χ^2_{1235} is (1139, 1334)

$\Rightarrow$ 95% confidence interval for σ^2_E is (8.519, 9.978).

(b) Random and Mixed Effects

To establish confidence intervals for the significantly non-zero variance parameters, for the random and mixed effects, we have used the method cited in Ting et al. (1990).

Assuming MS_1 and MS_2 are independently distributed according to

$$MS_1 \sim \left(\phi + \sigma^2\right)\frac{\chi^2_{n_1}}{n_1}$$

$$MS_2 \sim \sigma^2 \frac{\chi^2_{n_2}}{n_2}$$

where ϕ and σ^2 are parameters, the problem is to get a confidence interval on ϕ.

Let the two-sided confidence interval of confidence coefficient $1-2\alpha$ be denoted by

$$L \le \phi \le U \quad \text{or} \quad \phi \in (L, U)$$

where L and U are the lower and upper bounds respectively.

Let $\hat{\phi}$ be the unbiased point estimate of ϕ, so that

$$\hat{\phi} = MS_1 - MS_2$$

writing $L = \hat{\phi} - \sqrt{v_L}$

$$U = \hat{\phi} + \sqrt{v_U}$$

then v_L and v_U are given by:

$$V_L = G_1^2(MS_1)^2 + H_2^2(MS_2)^2 + G_{12}(MS_1)(MS_2)$$

$$V_U = H_1^2(MS_1)^2 + G_2^2(MS_2)^2 + H_{12}(MS_1)(MS_2)$$

where $G_1 = 1 - \dfrac{1}{F^{\alpha}_{n_1,\infty}}$

$$H_2 = \frac{1}{F^{1-\alpha}_{n_2,\infty}} - 1$$

$$G_{12} = \frac{\left(F^{\alpha}_{n_1,n_2} - 1\right)^2 - G_1^2\left(F^{\alpha}_{n_1,n_2}\right)^2 - H_2^2}{F^{\alpha}_{n_1,n_2}}$$

$$H_1 = \frac{1}{F^{1-\alpha}_{n_1,\infty}} - 1$$

$$G_2 = 1 - \frac{1}{F^{\alpha}_{n_2,\infty}}$$

$$H_{12} = \frac{\left(1 - F^{1-\alpha}_{n_1,n_2}\right)^2 - H_1^2\left(F^{1-\alpha}_{n_1,n_2}\right)^2 - G_2^2}{F^{1-\alpha}_{n_1,n_2}}$$

where $F_{n_1,n_2} = \dfrac{\chi^2_{n_1} / n_1}{\chi^2_{n_2} / n_2}$

and $P\left(F_{n_1,n_2} > F^{\alpha}_{n_1,n_2}\right) = \alpha$

and $F_{n_1,\infty} = \dfrac{\chi^2_{n_1}}{n_1}$.

We obtained:

Effect	G_1	G_2	G_{12}	H_1	H_2	H_{12}
S	-0.503	-0.0783	-0.02968	-0.2775	-0.0695	0.01142
ST	-0.25	-0.0783	-0.00467	-0.1776	-0.0695	0.002783

Effect	V_L	V_U	$\hat{\phi}$	L	U
S	59 382	18 165	475.7983	232.114	610.577
ST	17.8515	10.1283	7.8513	3.6262	11.034

95% confidence intervals for variance parameters:

S	6.448 to 16.96
ST	0.4029 to 1.226

(c) Fixed Effects

Assuming homoscedasticity, normality and $\hat{\sigma}_{sbt} = 0$ (since we only have one replication), the distributions of the fixed effect estimators are as follows:

$$\hat{\mu} \sim N\left(\mu, \left[\frac{\sigma_S^2}{N_S} + \frac{\sigma_E^2}{N_S N_B N_T}\right]\right)$$

$$\hat{\beta}_b \sim N\left(\beta_b, \frac{\nu_B}{N_B}\left[\frac{\sigma_{SB}^2}{N_S} + \frac{\sigma_E^2}{N_S N_T}\right]\right)$$

$$\hat{\gamma}_t \sim N\left(\gamma_t, \frac{\nu_T}{N_T}\left[\frac{\sigma_{ST}^2}{N_S} + \frac{\sigma_E^2}{N_S N_B}\right]\right)$$

$$\hat{\lambda}_{bt} \sim N\left(\lambda_{bt}, \frac{\nu_B \nu_T}{N_B N_T}\left[\frac{\sigma_E^2}{N_S}\right]\right).$$

Since $\dfrac{N(0,1)}{\sqrt{\dfrac{\chi_\nu^2}{\nu}}}$ is Student's t_ν distribution, provided numerator and denominator are independent,

$$\frac{\hat{\mu} - \mu}{\sqrt{\dfrac{\hat{\sigma}_S^2}{N_S} + \dfrac{\hat{\sigma}_E^2}{N_S N_B N_T}}} \sim t_{\nu_S}$$

$$\frac{\hat{\beta}_b - \beta_b}{\sqrt{\dfrac{\nu_B}{N_B}\left(\dfrac{\hat{\sigma}_{SB}^2}{N_S} + \dfrac{\hat{\sigma}_E^2}{N_S N_T}\right)}} \sim t_{\nu_S \nu_B}$$

$$\frac{\hat{\gamma}_t - \gamma_t}{\sqrt{\dfrac{\nu_T}{N_T}\left(\dfrac{\hat{\sigma}_{ST}^2}{N_S} + \dfrac{\hat{\sigma}_E^2}{N_S N_B}\right)}} \sim t_{\nu_S \nu_T}$$

$$\frac{\hat{\lambda}_{bt} - \lambda_{bt}}{\sqrt{\dfrac{\nu_B \nu_T}{N_B N_T}\left(\dfrac{\hat{\sigma}_E^2}{N_S}\right)}} \sim t_{\nu_E = \nu_S \nu_B \nu_T}.$$

We need to examine a single confidence statement which gives ranges for all the effect values under a single confidence level. This can be done by reference to Bonferroni's inequality:

$$P\left(\bigcap_i A_i\right) \geq 1 - \sum_i P\left(A_i^c\right)$$

where A^c denotes the complement of A and the A_i are not necessarily independent.

We need this because, for example, the β_b values for different boards are not independent.

If we have n separate statements we can choose each α_i to be $\frac{\alpha}{n}$ to get a single statement

$$P\left(X_i \leq x_i\left(\frac{\alpha}{n}\right), \quad \text{for all } i = 1 \text{ to } n\right) \geq 1 - \alpha.$$

This is a conservative confidence statement. For example if we choose $\alpha = 0.05$ (the 95% confidence level), the probability could be 0.99, or even 0.999, rather than 0.95. Hence the confidence interval obtained may be a lot wider than it need be.

The results were as follows:

Effect	df	z	S.D.	Error
B	472	2.773	0.1827	0.5066
T	177	2.498	0.1539	0.3844
BT	1416	3.2	0.3198	1.023

We list below the values for $\hat{\beta}_b$, $\hat{\gamma}_t$, $\hat{\lambda}_{bt}$ calculated from

$$\hat{\beta}_b = x_{\bullet b \bullet} - x_{\bullet\bullet\bullet}$$
$$\hat{\gamma}_t = x_{\bullet\bullet t} - x_{\bullet\bullet\bullet}$$
$$\hat{\lambda}_{bt} = x_{\bullet bt} - x_{\bullet b \bullet} - x_{\bullet\bullet t} + x_{\bullet\bullet\bullet}$$

together with their 95% confidence intervals.

Board	Point Estimate	95% Confidence Interval
AEB	5.6	(5.1, 6.1)
CAMB	0.5	(0.0, 1.0)
JMB	3.1	(2.6, 3.6)
LON	5.0	(4.5, 5.5)
OXF	-4.6	(-5.1, -4.1)
O & C	-4.8	(-5.3, -4.3)
MEI	0.5	(0.0, 1.0)
SMP	-3.0	(-3.5, -2.5)
WJEC	-1.7	(-2.2, -1.2)

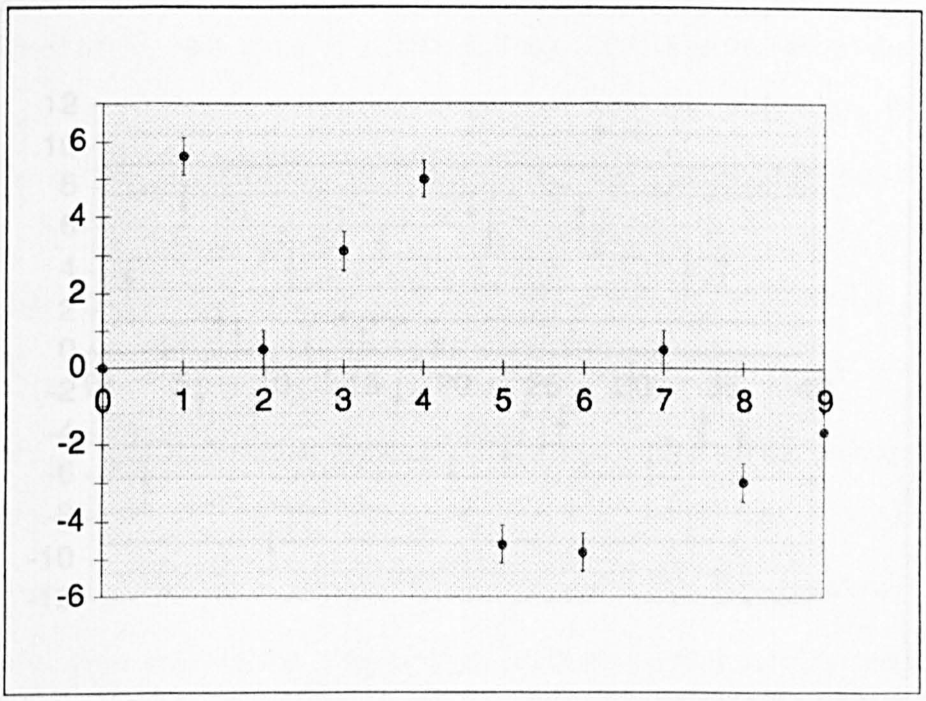
Topic	Point Estimate	95% Confidence Interval
Calculus	3.4	(3.0, 3.8)
Vectors	3.8	(3.4, 4.2)
Projectiles	-2.2	(-2.6, -1.8)
Probability	-4.7	(-5.1, -4.3)

Interaction	Point Estimate	95% Confidence Interval
AEB/CALC	-2.0	(-3.0, -1.0)
CAMB/CALC	3.5	(2.5, 4.5)
JMB/CALC	-6.3	(-7.3, -5.3)
LON/CALC	-0.2	(-1.2, 0.8)
OXF/CALC	7.1	(6.1, 8.1)
O & C/CALC	-2.2	(-3.2, -1.2)
MEI/CALC	1.5	(0.5, 2.5)
SMP/CALC	1.0	(0.0, 2.0)
WJEC/CALC	-2.8	(-3.8, -1.8)
AEB/VEC	-9.6	(-10.6, -8.6)
CAMB/VEC	4.0	(3.0, 5.0)
JMB/VEC	0.3	(-0.7, 1.3)
LON/VEC	-0.9	(-1.9, 0.1)
OXF/VEC	1.8	(0.8, 2.8)
O & C/VEC	1.0	(0.0, 2.0)
MEI/VEC	5.2	(4.2, 6.2)
SMP/VEC	-2.3	(-3.3, -1.3)
WJEC/VEC	-0.3	(-1.3, 0.7)
AEB/PROJ	0.5	(-0.5, 1.5)
CAMB/PROJ	-5.9	(-6.9, -4.9)
JMB/PROJ	6.9	(5.9, 7.9)
LON/PROJ	6.0	(5.0, 7.0)
OXF/PROJ	-5.0	(-6.0, -4.0)
O & C/PROJ	-3.0	(-4.0, -2.0)
MEI/PROJ	-3.8	(-4.8, -2.8)
SMP/PROJ	-3.5	(4.5, -2.5)
WJEC/PROJ	6.9	(5.9, 7.9)
AEB/PROB	10.9	(9.9, 11.9)
CAMB/PROB	-1.9	(-2.9, -0.9)
JMB/PROB	-1.4	(-2.4, -0.4)
LON/PROB	-5.4	(-6.4, -4.4)
OXF/PROB	-4.2	(-5.2, -3.2)
O & C/PROB	3.9	(2.9, 4.9)
MEI/PROB	-3.3	(-4.3, -2.3)
SMP/PROB	4.5	(3.5, 5.5)
WJEC/PROB	-4.0	(-5.0, -3.0)

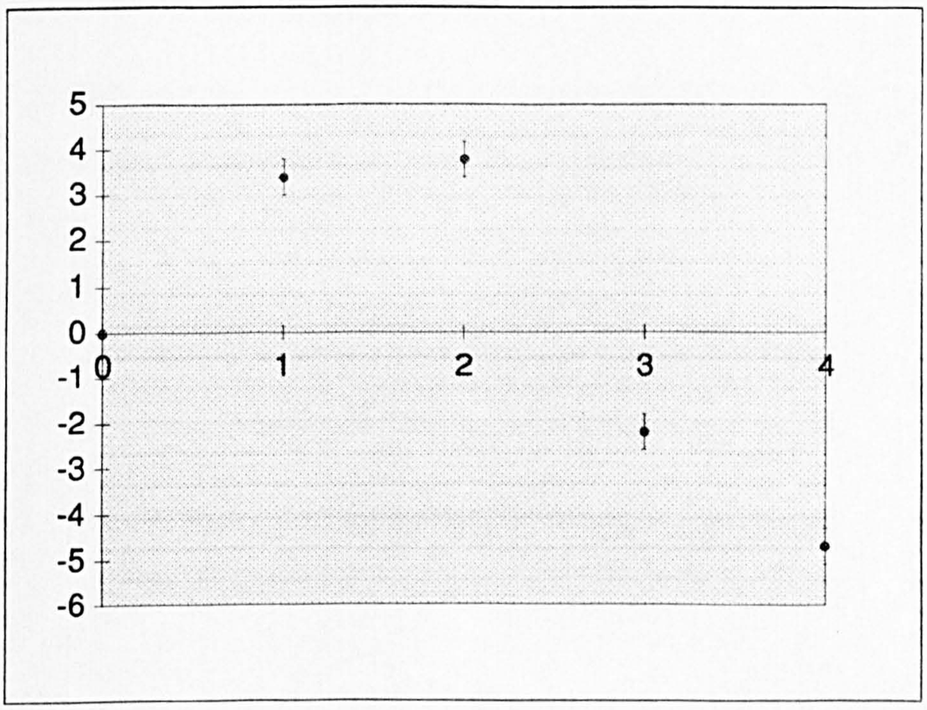
The following charts show the point estimates and the 95% confidence intervals associated with them. We notice, in the case of $\hat{\beta}_b$ values, two cases just touch zero, while in the case of $\hat{\lambda}_{bt}$ values, five intervals cross zero and two touch zero. Overall very few of the confidence intervals cross zero.

Figure 3: Fixed Effects Point Estimates of Variance Parameters.

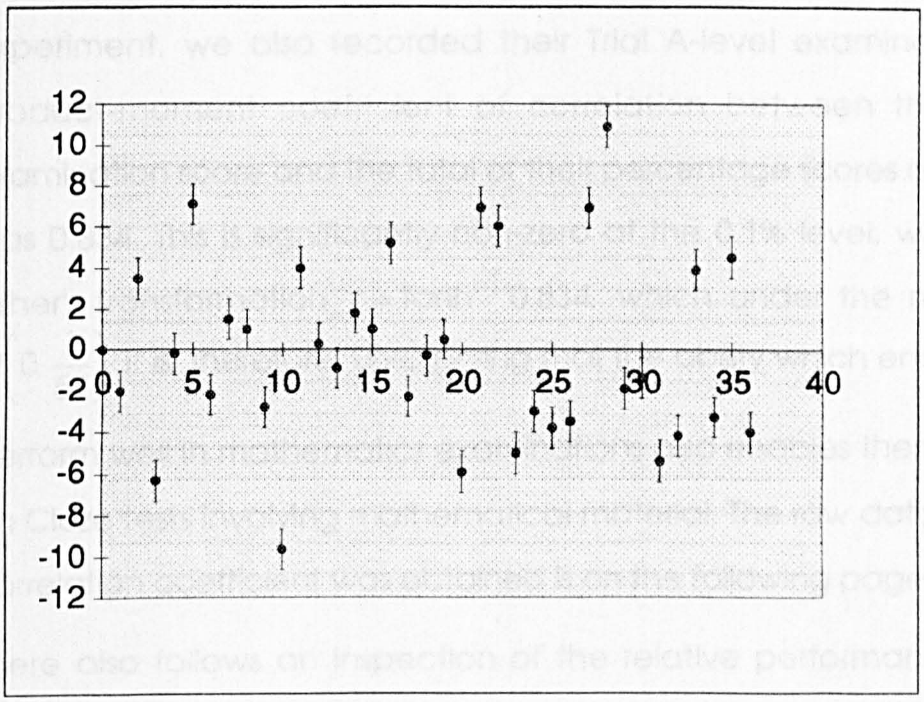
$\hat{\beta}_b$ *values* :



$\hat{\gamma}_t$ *values* :



$\hat{\lambda}_{bt}$ *values* :



Gagatsis (1984) comments on the high correlation between Cloze test scores and I.Q.. It is well documented that performance in mathematics and I.Q. are also highly correlated. For those students who took the Cloze tests in our experiment, we also recorded their Trial A-level examination score. The product-moment coefficient of correlation between their Trial A-level examination score and the total of their percentage scores on the Cloze tests was 0.834. This is significantly non-zero at the 0.1% level, with n = 60, using Fisher's transformation $z = \tanh^{-1} 0.834$, which under the null hypothesis is $N\left(0, \frac{1}{57}\right)$. It is, therefore, unsurprising that the ability which enables students to perform well in mathematics examinations also enables them to score highly on Cloze tests involving mathematical material. The raw data from which the correlation coefficient was obtained is on the following page.

There also follows an inspection of the relative performances on the nine boards in each of the four topic areas, using Kendall's coefficient of concordance, W.

This value of W expresses the degree of agreement among the four sets of rankings of the nine boards.

W lies between 0 and 1 and has a linear relationship with the average value of Spearman's coefficient of rank correlation, if we take all possible pairs of the four sets of rankings.

Bivariate Data from Cloze Experiment

No.	Trial Exam (x)	Total Cloze (y)	No.	Trial Exam (x)	Total Cloze (y)
1	73	2761	31	61	2896
2	79	2914	32	52	2828
3	78	2858	33	41	2701
4	72	2743	34	50	2813
5	94	3106	35	35	2640
6	85	2995	36	32	2568
7	88	3082	37	40	2701
8	79	2900	38	53	2822
9	76	2805	39	51	2826
10	81	2977	40	59	2905
11	73	2986	41	40	2784
12	63	2764	42	38	2700
13	76	2867	43	39	2726
14	64	2743	44	37	2673
15	57	2652	45	40	2840
16	80	2906	46	34	2681
17	55	2636	47	29	2521
18	62	2768	48	32	2574
19	88	3027	49	47	2827
20	75	2887	50	31	2610
21	60	2886	51	58	2869
22	64	2863	52	46	2826
23	57	2754	53	37	2658
24	42	2649	54	61	2883
25	69	2967	55	57	2799
26	62	2783	56	67	3007
27	41	2685	57	44	2717
28	50	2734	58	37	2657
29	74	2989	59	64	2880
30	45	2656	60	39	2684

Mean % on Cloze Tests

	Boards								
	i	ii	iii	iv	v	vi	vii	viii	ix
Calculus	84.7	85.1	77.9	85.9	83.6	74.1	83.1	79.1	76.6
Vectors	77.5	86.0	84.9	85.6	78.7	77.7	87.2	76.2	79.5
Projectiles	81.6	70.1	85.4	86.5	65.9	67.6	72.2	69.0	80.7
Probability	89.5	71.6	74.7	72.5	64.2	72.1	70.2	74.5	67.2

Ranks

	Boards								
	i	ii	iii	iv	v	vi	vii	viii	ix
Calculus	3	2	7	1	4	9	5	6	8
Vectors	8	2	4	3	6	7	1	9	5
Projectiles	3	6	2	1	9	8	5	7	4
Probability	1	6	2	4	9	5	7	3	8
Total Rank	15	16	15	9	28	29	18	25	25

Mean total rank $= 180 \div 9 = 20$

$$S = (15-20)^2 + (16-20)^2 + \dots\dots\dots + (25-20)^2$$
$$S = 386.$$

Kendall's coefficient of concordance W

$$W = \frac{386}{\frac{1}{12} \times 4^2 \times \left(9^3 - 9\right)} \text{ (4 rows, 9 columns)}$$
$$W = 0.402.$$

We may test the significance of W using chi-squared.

Degrees of freedom = 9 - 1 = 8.

$$\chi^2 = \frac{386}{\frac{1}{12} \times 4 \times 9 \times 10} = 12.86 \text{ (4 rows, 9 columns)}$$

Critical value = 15.51 (5 %).

This value, therefore, just fails to reach significance, possibly due to the small sample size.

It is interesting to note that the board with the lowest total ranking was London, while the highest total rankings, in descending order, were Oxford, Oxford and Cambridge, SMP and the Welsh Board. This ordering is in agreement with findings in this study and elsewhere.

5. A-level Mathematics Papers.

5.1 The Concept of 'Demand' in Mathematics Papers.

The Standing Research Advisory Committee (SRAC) (1990) found that defining 'demand' in mathematics was an aim that was "not easily susceptible to the usual forms of empirical research". In a study into 'demand' in the various versions of A-level mathematics, they relied heavily on the ability of professionals in the field to probe their own subject and draw on their own experiences of teaching the subject. They came to the conclusion that 'demand' was best assessed by studying questions on selected topics rather than comparing syllabuses as such. This is a view shared by the author, and later in the study we will be studying questions which have been set in the various versions of A-level mathematics.

Previous studies had experienced similar difficulties:
Bardell (1977) - an Inter-Board Cross Moderation Study into 1976 Advanced Level Pure Mathematics - had felt that criteria for comparing examinations probably existed, but that their complexity was such that it was impossible to state them explicitly.

Johnson and Cohen (1983) had found three types of 'demand' :

(i) 'Academic Demand' whereby topics or subjects have their own intrinsic level of difficulty. For example, deducing the theorems relating to the existence and properties of complex numbers from the underlying axioms might be regarded as more difficult than using the theorems in routine calculations involving complex numbers.

(ii) 'Contextual Demand'. The techniques mentioned above might well be required in the solution of an applied mathematics problem. However, there is a demand created by the problem itself over and above the mere execution of the techniques. The fact that the techniques are embedded within some other material - the problem - gives an additional level of 'demand'. Furthermore, questions may vary in their amount of structure. Routine tasks may be set in unusual contexts. There may be complicated rubrics and mark allocation for questions may vary. Consequently there are a variety of contextual factors which may increase or decrease the 'demand' of a question.

(iii) 'Personal Demands'. As we have seen earlier, there are differences in performance in A-level mathematics due to gender, e.g. girls tend to choose statistics in preference to mechanics. So the nature of questions creates demands of a personal nature due to gender. Other personal factors will also raise or lower the demand made on the candidate. The level of preparation prior to the examination is clearly crucial, as is the candidate's background and motivation. What may be a difficult topic to one person will not be so difficult for another.

SRAC felt that A-level mathematics should be a demanding examination. There should be something to challenge the most able candidates. At the same time the tasks should not be so demanding that they were more appropriate for undergraduates, or that they offered little or nothing for the less able candidate.

One way in which the versions of A-level mathematics differ is in the balance of long and short questions. Hersee (1984) points out that with the grade E (pass) grade boundary set between 30% and 40%, the candidates who just pass are unlikely to get a single question completely correct if the paper consists entirely of long questions. In his experience teachers, and pupils, were happier if papers began with a number of routine short questions, which the average to weaker candidate could get correct. SMP examination papers, he says, are criticised as being too hard for the average candidate. (They are composed of long questions and the questions are set in context, hence the candidate must read and understand the context.)

When considering the choice between A-level examinations in mathematics, a number of questions arise. e.g. Do average candidates get better grades on more routine examination questions? Is it reasonable to use the same title 'Mathematics" for two examinations which make very different demands on candidates? Does A-level try to assess too much in two written papers? Should project work or coursework be included? How should questions of different lengths and styles be used in examination papers? To what extent should longer questions be structured? Should there be any choice of question? Should there be some 'hurdle' to cross before a grade can be awarded?

In these matters SRAC says there is a lack of empirical data. They do point out a number of ways in which a start can be made on this work. It is the intention in this study to look at some of these suggestions and develop them further:

5.2 Syllabus Analysis.

Each syllabus consists of a list of topics, some with explanatory notes. The contents of the Pure Mathematics section is largely determined by the agreed inter-board common core. In the past various comparisons have been made. James (1968), Neill (1976) and Holland (1979) drew up matrices indicating which topics were included in which board's syllabus. SRAC, however, report that no similar exercise has been carried out recently. The study of a mere list of contents may be misleading. If a topic is not specifically mentioned in the list, it does not mean that it will not be required in the examination. A topic may appear in two or more boards' examinations, but may be examined in quite different ways and to greater or lesser depths.

From the syllabus analysis it will be possible to determine whether the board is using a Pure Mathematics and Applied Mathematics approach or a Mathematics and Further Mathematics approach. It will be possible to see how many papers are set and how long is allowed for each paper and possibly the style of each paper; e.g. the number of long and short questions and the number of questions to be answered. Some indication will be given of the mathematical content, which will be pure mathematics plus various amounts and combinations of applied mathematics (mechanics, statistics, numerical methods etc.). From this one can see if the content is broad or narrow and if there are modelling requirements. SRAC found SMP, Oxford and Cambridge and MEI to be particularly broad syllabuses and MEI to contain a good deal of modelling which was of a demanding nature.

The printed syllabus also gives details of the formula sheet that will be available and these show considerable disparity in the amount of help that candidates receive.

Individual features of the syllabus alone do not give a clear picture of how the various versions relate to each other in terms of 'demand'.

5.3 Question Analysis.

To get a better picture of how demanding the various versions are it is necessary to examine, in detail, the questions which are set in the examinations. Griffiths and McLone (1979) provided a framework for analysing questions in mathematics degree examinations at Southampton University. This framework was added to by SRAC (Op. cit.). On each category a question is scored from 0 to 3. (0 = Not demanding , 3 = very demanding). The categories are as follows:

a) Procedure.

How far does the question define in detail the procedure which the candidates should adopt?

b) Objectives.

How far does the question explicitly identify the conclusions which the candidates should reach?

c) Jargon.

How specialised or technical is the vocabulary which is used?

d) Routine processes.

How far does the question demand the reproduction of routine processes?

e) Mathematical content.

How far does the question include mathematical content other than routine processes?

f) Abstraction.

To what extent is the question abstract or theoretical?

g) Mathematical manipulation

To what extent is the manipulation of symbols and calculations required to obtain a solution?

h) Logical manipulation.

To what extent do powers of reasoning have to be deployed?

i) Sustained thinking.

How far are prolonged concentration and marshalling of ideas required?

j) Open solution.

How far is the nature of the solution clearly determined in the question?

k) Real world context.

To what extent does the question relate to the real world?

l) Formula sheet help.

To what extent does the formula sheet help in answering the question?

m) Symbolic / visual.

To what extent does the question require visual or graphical ability?

5.4 Paper Analysis.

There are a number of ways of considering this:

a) Relation between syllabus and papers.

Is there a specific syllabus for each paper? If so this makes the candidate's task easier when he or she comes to revise for the examination. Do the papers cover the syllabus? If they only partially do so then candidates may not be able to demonstrate what they have learned if no question is set on that material. Do candidates have to cover mechanics and statistics or just one of them in order to answer the applied part of the examination?

b) Types of question paper.

Things to consider here are the use of multiple choice questions, the balance of long and short questions and the use, or otherwise, of essay type questions (usually confined to statistics questions). The latter being open-ended could be more demanding.

c) Question choice.

This would have profound influence on how easy or difficult a paper is. In fact in some versions of applied mathematics two candidates taking the same paper could do entirely different sets of questions. In other cases there might be no choice (a situation which is becoming increasingly popular).

d) Mark allocation.

Should all the questions carry equal numbers of marks (clearly not possible when the paper consists of a mixture of long and short questions)? Candidates might have to do different amounts of work to earn the same number of marks on the same paper. The same piece of work on Board X may not carry the same proportion of marks on Board Y. e.g. Find the centre of mass of a solid hemisphere might be worth say 5% of one 3 hour paper for one board and 7% of one 3 hour paper on another board.

e) Rubrics and format.

If the rubrics are complicated the demand on candidates can increase. Use of diagrams may lessen demand, by breaking up pages of unbroken print or increasing understanding of the question more readily, or both.

f) Language.

Some use of language is obviously essential, but if the paper is testing linguistic ability rather than mathematical ability then 'demand' will be unnecessarily high on the candidates.

When questions are set in context, most commonly in applied mathematics, particularly in statistics, then the use of language increases. Putting questions into context may be helpful to candidates, but if the context is unfamiliar it could be a hindrance. SMP especially do this to make questions interesting, but it is criticised by teachers, according to SRAC, for making the questions difficult.

Later in this study the question of the language used in A-level mathematics papers is looked at in some detail.

To start, let us consider the different structures of examinations offered by the various examination boards.

AEB.

Eight papers in all are set (each of 3 hours duration). They are numbered: 1,2,3,4,5,6,9,10.

Paper 1:		The Common Core + extra pure mathematics.
Paper 2:		Further pure mathematics
Paper 3:		Mechanics
Paper 4:		Further mechanics
Paper 5:	)	Both papers contain a mixture of pure mathematics,
Paper 6:	)	mechanics, numerical methods and probability & statistics
Paper 9:		Probability and Statistics
Paper 10:		Further probability & statistics.

For single subject A-level, various combinations are possible:

Papers 1&2: Pure Mathematics

Papers 1&3: Mathematics, Pure and Applied

Papers 1&9: Mathematics, Pure and Statistics

Papers 5&6: Mathematics

Papers 3&4 constitute Mathematics, Applied - which would normally be taken with Papers 1&2 by double subject candidates.

Papers 5&6 have a more 'modern' approach than other combinations, offering pure mathematics, mechanics, statistics and numerical methods. The difference between the 1 and 3 combination and 5 and 6 combination is considerably less marked than it would have been 25 years ago. The existence of both combinations would appear to be an unnecessary duplication.

The style of the papers is:

Paper 1: Section A. Eight short questions totalling 57 marks.
Section B. A choice of 3 questions from 5, each worth 16 marks.
Total mark for the paper 105 marks.

Paper 2: A choice of 7 questions from 10, each worth 15 marks.
Total mark for the paper 105 marks.

Paper 3: Section A. Six short questions totalling 45 marks.

Section B. A choice of 4 questions from 6, each worth 15 marks.

Total mark for the paper 105 marks.

Paper 4: A choice of 7 questions from 10, each worth 15 marks.

Total mark for the paper 105 marks.

Papers 5&6: Section A. Four short questions totalling 30 marks.

Section B. A choice of 5 questions from 12. Each worth 14 marks.

Total mark for the paper 100 marks.

At least two questions must be on the basic pure mathematics syllabus and the remainder from at most two of the three options. Viz. mechanics, numerical methods and statistics.

Papers 9&10: A choice of 6 questions from 10, each worth 17 marks.

Total mark for the paper 102 marks.

The papers are on yellow A5 paper. Diagrams were exceedingly rare. There were only two in the eight papers. Both diagrams were in mechanics questions.

Cambridge.

For single subject mathematics two papers were set, each of 3 hours duration.

Paper 1: The common core + extra pure mathematics.

Paper 2: Three sections comprising mechanics, probability & statistics and further pure mathematics.

The structure of each paper was as follows:

Paper 1: Section A. Eleven short questions totalling 50 marks.

Section B. A choice of 4 questions from 7, each worth 12 marks.

Total mark for the paper 98 marks.

Paper 2: Section A. Five particle dynamics questions.

Section B. Five probability and statistics questions.

Section C. Five pure mathematics questions.

Candidates choose 7 questions with at most 4 coming from one section. Each question is worth 14 marks.

Total mark for the paper is 98 marks.

The paper is white A4. There was one diagram on each paper.

JMB

Five papers are set in all. There are four versions of A-level Mathematics. All candidates take the common Paper 1 and any one of the other four papers as their Paper 2. The various versions are entitled:

(a) Pure and Applied Mathematics.
(b) Pure Mathematics.
(c) Pure Mathematics with Mechanics.
(d) Pure Mathematics with Statistics.

Paper 1: The common core + extra pure mathematics. Sixteen questions totalling 116 marks. The marks per question range from 3 to 16.

Paper 2 (a): Mechanics and statistics. Thirteen questlons totalling 114 marks. The marks per question range from 5 to 16.

Paper 2 (b): Pure Mathematics. Thirteen questions totalling 118 marks. The marks per question range from 4 to 15.

Paper 2 (c): Mechanics. Eleven questions totalling 112 marks. The marks per question range from 5 to 16.

Paper 2 (d): Statistics. Twelve questions totalling 115 marks. The marks per question range from 5 to 14.

In all these papers, candidates may attempt all questions. The maximum score possible on any paper is 100 marks. There are many diagrams on the papers. Some questions are common to the various versions of paper 2. The papers are printed on white A5 paper.

London

Three versions of single subject A-level are available. All candidates take Paper 1, a two and a half hour paper, and Paper 2, a one and a quarter hour multiple choice paper. These cover the common core and some extra pure mathematics topics. Paper 3, a two and a half hour paper, is mechanics or statistics or further pure mathematics, giving three possible subjects:

(a) Mathematics
(b) Pure Mathematics with Statistics
(c) Pure Mathematics.

The structure of the various papers is as follows:

Paper 1: Fifteen questions are set, totalling 100 marks.
The marks per question range from 4 to 14.
Candidates attempt all questions.

Paper 2: Thirty questions are set. For the first 20, candidates select one correct response from 5 possible responses for each question.

The final 10 questions are Multiple Completion questions. Three statements, 1,2 and 3 are made for each question.

Candidates answer:

A if 1,2 and 3 are correct

B if only 1 and 2 are correct

C if only 2 and 3 are correct

D if only 1 is correct

E if only 3 is correct.

Space is provided for rough calculations. Answers are recorded on a separate sheet, which is later marked by computer.

Paper 3: In each version six questions are attempted from eight in the mechanics and statistics papers and from nine in the pure mathematics paper. Each question is worth 17 marks and the total mark is 102.

Papers 1 and 3 are set on blue A5 paper. Paper 2 is on blue A4 paper. There was only one diagram in any of the papers, on the mechanics paper.

Oxford

Five papers in all are set. There are four versions of single subject A-level. Paper 1 is common to all the versions. This is followed by either Paper 2 (mechanics), or Paper 3 (statistics), or Paper 4 (applied mathematics), or Pure Mathematics Paper 2. Each paper is 3 hours long.

Paper 1: Section A is eight short questions (10 marks each).

Section B has a choice of 4 questions from 10 (20 marks each).

Papers 2,3: Section A is six short questions (12 marks each).

Section B has a choice of 4 questions from 6 (22 marks each).

Paper 4: Section A is six short questions (12 marks each), chosen from ten questions (3 mechanics, 3 statistics and 4 decision mathematics).

Section B has a choice of 4 questions from 12 (22 marks each), chosen from 4 mechanics, 4 statistics and 4 decision mathematics questions.

The total mark for each of these papers is 160 marks. The questions are written on white A4 paper and there are numerous diagrams in the mechanics sections.

Oxford and Cambridge

Two papers of 3 hours duration each are set. The structure of each paper is identical. Both include pure mathematics and applied mathematics.

Each paper consists of Part 1 (short questions worth 14 marks each) and Part 2 (longer questions worth 25 marks each). Part 1 has six pure mathematics (Section A) and six applied mathematics (Section B) questions.

Candidates attempt six of the twelve shorter questions, with no more than four being selected from either section. Part 2 has four pure mathematics (Section A) and four applied mathematics (Section B) questions. Candidates attempt four of the eight longer questions, with no more than three coming from either section.

The rubric for these papers is arguably the most complicated of any A-level mathematics paper. Candidates who are well acquainted with the style of past papers probably have a clear idea of what is expected.

The papers are written on white A5 paper and the mechanics questions contain a number of diagrams.

MEI

Two papers of 3 hours duration each are set.

Paper 1: This contains the common core + extra pure mathematics.

Section A consists of eight compulsory questions totalling 52 marks. Each question is worth between 6 and 8 marks.
Section B has a choice of 3 questions from 6, each worth 16 marks.

Paper 2: This contains applied mathematics.

Section A is three compulsory questions (two on probability and one on differential equations). This comprises 30% of the mark for this paper. Five other questions are selected, freely, from Sections B and C. Section B contains seven statistics questions. Section C contains seven mechanics questions. Each of these questions is worth 14% of the mark for the paper.

Each paper is , therefore, marked out of 100. The questions are written on white A5 paper, with two diagrams in the applied mathematics paper.

SMP.

Two papers of 3 hours duration each are set. Both papers contain a mixture of pure mathematics and applied mathematics (mechanics and statistics).

Paper 1: Section A is 12 questions (8 pure, 2 statistics and 2 vectors) worth 5 marks each.
Section B is 8 questions (5 pure, 2 mechanics and 1 statistics) worth 10 marks each. All questions are to be attempted.

Paper 2: Section A is 4 questions (3 pure and 1 statistics) worth 10 marks each.
Section B has a choice of 4 questions from 7 (4 pure, 2 mechanics and 1 statistics) worth 25 marks each.

The total mark for each paper is 140 marks. The papers are written on white A5 paper with many illustrative diagrams.

Welsh Board.

Three versions of Mathematics are possible. Viz. Pure Mathematics with Mechanics, Pure Mathematics with Statistics and Pure Mathematics. All three versions consist of two 3 hour papers. All candidates take Paper 1, which is common to all three versions. Paper 2 is then a mechanics paper, or a statistics paper, or another pure mathematics paper.

Paper A1: The common core + extra pure mathematics.
Section A consists of 8 short questions worth between 3 and 7 marks each. All questions are to be attempted. The total mark for Section A is 40 marks.
Section B has a choice of 4 longer questions from 7. Each question is worth 15 marks.

Papers A2: These are mechanics, statistics and pure mathematics respectively.

Papers A3 and A4:	They have the same structure as paper A1, except that the mechanics paper has only 7 shorter questions in Section A, still totalling 40 marks, however.

The total mark for each paper is 100 marks. The questions are written on white A4 paper. There was one diagram on Paper A1, one diagram on Paper A2 and one diagram on Paper A4.

Thus we find that there is considerable diversity in the papers which are set. We must, therefore, ask ourselves to what extent these various examinations are the same 'beast'. It is the contention of this study that there are significant differences. In the last chapter we examined the issue of 'readability' and in this chapter we see evidence of stylistic differences which, at the least, give rise to some cause for concern. Surely it is not beyond the bounds of possibility that some sort of monitoring should take place, to ensure a degree of uniformity. At the present it seems that each board acts independently and has considerable scope to proceed in a fashion which it deems fit.

6. Question Comparison.

6.1 The A-level Question Matrix.

The fundamental instrument chosen to carry out the comparison was the matrix of Howson and McLone, described in the previous section. A number of amendments were made, so as to include headings which could readily be identified, and also make the scoring more logical. Thus the scale in d) became routine - not routine, scoring 0 - 3 and not the other way round, as in the original. Heading e) was changed from 'mathematical content' to 'complex mathematical content' and rated low - high rather than yes - no. Heading f) became 'level of abstraction' and scored low - high, rather than 'abstraction' and yes - no. 'Mathematical manipulation' was rated low - high rather than 'not required - required'. 'Logical manipulation' was re-named 'powers of reasoning' and scored low - high as opposed to 'required - not required'. The headings 'real world content' and 'symbolic / visual' in the original were omitted, for reasons of definition. It would be difficult to score these on the 0 - 3 scale which we were using. The resulting matrix thus had 11 headings each of which was to be scored 0 (easy) to 3 (difficult). The matrix is shown overleaf.

Table 6.1 A-level Question Matrix.

Board	**Year**			
Topic				
Analysis Headings	0	1	2	3
a) Procedure Defined - not defined				
b) Objectives Defined - not defined				
c) Jargon Not technical - technical				
d) Routine Processes Routine - not routine				
e) Complex mathematical content Low - high				
f) Level of abstraction Low - high				
g) Mathematical manipulation Low - high				
h) Powers of reasoning Low - high				
i) Sustained thinking Not required - required				
j) Open solution Closed - open				
k) Formula sheet help Yes - no				
	Total Score			

6.2 The Question Comparison Experiment.

The author was to use this matrix on all the questions of all the boards being considered, for the 1989 papers. The restraints of time and finance meant that it would not be possible to employ other judges. In order to demonstrate that the judgements being used were reasonable, the assistance of a fellow teacher was employed. Questions on four topics (calculus, vectors, projectiles and probability) from each of the nine boards were selected. These gave a spread of pure, mechanics and statistics questions, roughly in proportion to their appearance on the A-level papers. The fellow teacher and the author used the matrix for each of the 36 questions, scoring each question on each of the eleven headings. The results are compared below.

Since we are concerned with estimating a relative comparison of the fellow teacher's and the author's judgements, we have decided to use Spearman's Rank Correlation Coefficient. In other words, we wish to know the extent to which, if the fellow teacher rated something as difficult, then the author did so as well, and vice versa. Each question was given a score between 0 and 3 under each of the eleven headings, so the maximum possible score would have been 33. In practice the scores for questions fell well short of this, since 3's were rarely awarded. The fellow teacher's score for each question is denoted by x and the author's by y. The scores (out of 33) are shown below:

Table 6.2 Author's and Fellow Teacher's Scores.

Board	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WJEC
Vectors									
x	6	5	7	6	5	6	9	6	5
y	9	8	7	4	4	11	12	5	5
Calculus									
x	5	9	8	8	4	16	14	6	7
y	6	10	11	7	7	14	16	11	11
Projectiles									
x	4	0	2	3	8	8	4	2	7
y	9	3	3	7	12	16	11	8	9
Probability									
x	6	3	3	1	5	4	3	2	6
y	6	5	3	3	10	7	4	3	8

Which yield the following values for R_s:

Vectors : 0.53 Calculus : 0.70 Projectiles : 0.92 Probability : 0.85

(5% Critical value for n = 9 is 0.70 .)

Similarly, for each topic (vectors, calculus, projectiles and probability), we have nine scores (for each board) under each of the eleven headings, giving a maximum score of 27. The scores (out of 27) are shown below:

Table 6.3 Ranks for Author's and Fellow Reacher's Scores.

	Vectors		Calculus		Projectiles		Probability	
	x	y	x	y	x	y	x	y
a)	9	6	11	7	1	7	3	2
b)	0	0	0	0	0	3	1	2
c)	7	10	1	7	5	10	2	7
d)	5	7	10	10	8	11	1	5
e)	6	8	12	15	4	9	2	6
f)	8	11	5	6	3	9	5	0
g)	7	7	13	20	9	12	5	7
h)	10	12	13	14	6	10	8	13
i)	3	3	6	8	2	7	5	4
j)	0	1	2	2	0	0	1	3
k)	0	0	4	4	0	0	0	0

Which yield the following values for R_s:

Vectors : 0.83 Calculus : 0.88 Projectiles : O.99 Probability : 0.45

(5% critical value for $n = 11$ is 0.62 .)

The results indicate a good degree of positive correlation between the fellow teacher's judgements and the author's judgements.

Having established a reasonable degree of correlation between the fellow teacher's judgements and the author's judgements, the author then proceeded to judge each of the questions, for each paper, set by the nine boards in 1989 A-level Mathematics examinations.

The same considerations apply to this data as to the data collected in Chapter 4. In this case, however, the scores under the various headings were integers from 0 to 3 inclusive. The level of measurement here is an ordinal category scale . Hence our first assumption for a parametric test, viz. at least an interval level of measurement, is not fulfilled. It is also doubtful that we have a Normal distribution, which was our second assumption for the use of a parametric test. In order to analyse this data we shall require a non-parametric procedure.

6.3 General Principles for a Friedman Two-way Analysis of Variance.

When we have k matched samples, with at least an ordinal scale of measurement, we can use the Friedman two-way analysis of variance by ranks to decide if the k samples are from the same population. Sample values are subject to fluctuation. The question is whether this fluctuation is due to genuine differences in the population, or whether it is due to the operation of chance factors.

The data is cast in a two-way table having N rows and k columns. The rows represent the matched sets of "subjects" and the columns represent the various "conditions". The Friedman test assesses whether, or not, the different columns of ranks come from the same population. In our case we can use either the headings from our matrix as the "conditions" for each of the nine examination boards, or the nine examination boards as the "conditions" for each of the eleven headings. ($k = 9$ or 11 and $N = 11$ or 9 respectively).

6.4 The Friedman Analysis.

One of the problems encountered was that each board's papers included varying numbers of pure mathematics, mechanics and statistics questions. Hence in the following tables we have taken averages.

For example, AEB had 13 pure mathematics questions. For our heading a) - Procedure, each question was given a score on the 0-3 scale. The total of these thirteen scores was 17, giving an average score of 17/13 = 1.31 . In the same way, all other entries in the tables were calculated.

The table below shows the average score under each heading of the matrix, for each of the boards, on pure mathematics questions.

Table 6.4 Heading-Board Pure Mathematics Scores.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	1.31	1.11	1.19	0.73	1.28	1.50	1.29	0.71	0.87
b)	0.31	0.28	0.13	0.00	0.28	0.20	0.00	0.19	0.07
c)	0.54	0.22	0.25	0.13	0.39	0.45	0.29	0.14	0.20
d)	0.46	0.72	0.63	0.33	1.06	1.25	1.00	0.95	0.47
e)	0.77	1.33	1.06	0.93	1.56	1.60	1.71	1.19	1.20
f)	1.23	1.28	1.25	1.00	1.61	1.60	1.57	1.43	1.33
g)	1.08	1.39	1.25	0.93	1.83	1.75	1.71	1.29	1.27
h)	1.00	1.28	1.38	1.07	1.39	1.80	1.57	1.24	1.00
i)	0.54	0.17	0.19	0.00	0.50	0.65	0.14	0.38	0.33
j)	0.85	0.89	0.81	1.00	1.22	1.20	1.07	1.24	1.07
k)	1.23	1.50	1.75	1.27	1.67	1.80	1.71	1.62	1.40

These values in each row were then ranked, in order to apply the Friedman two-way analysis of variance test. This non-parametric test was used to avoid assumptions of normality implicit in the analysis of variance.

Table 6.5 Heading-Board Ranks for Pure Mathematics.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	8	4	5	2	6	9	7	1	3
b)	9	7.5	4	1.5	7.5	6	1.5	5	3
c)	9	4	5	1	7	8	6	2	3
d)	2	5	4	1	8	9	7	6	3
e)	1	6	3	2	7	8	9	4	5
f)	2	4	3	1	9	8	7	6	5
g)	2	6	3	1	9	8	7	5	4
h)	1.5	5	6	3	7	9	8	4	1.5
i)	8	3	4	1	7	9	2	6	5
j)	2	3	1	4	8	7	6	9	5
k)	2	4	8	1	6	9	7	5	3
Total	46.5	51.5	46	18.5	81.5	90	67.5	53	40.5

If the null hypothesis (that all the columns came from the same population) is in fact true, then the distribution of ranks in each column would be random, and hence we would expect the ranks 1 to 9, in our case, to appear in all columns with approximately equal frequency. This would indicate that for any group it is a matter of chance under which board the highest score occurs and under which board the lowest occurs, which would be the case if the boards did not differ significantly.

Thus, if the scores were independent of the boards, the set of ranks in each column would represent a random sample from the discrete uniform distribution (1,9) and the rank totals (shown in the final row of the above table) for the various columns would be approximately equal. If, on the other hand, the scores were dependent on the boards (i.e. the null hypothesis is false), then the rank totals would vary from one column to another.

The purpose of the Friedman test is to determine whether the rank totals differ significantly.

The Friedman test statistic is:

$$M = \frac{12}{Nk(k+1)} \sum_{j=1}^{k} (R_j)^2 - 3N(k+1)$$

where

N = number of rows

k = number of columns

R_j = *sum of ranks in the jth column*

$M \sim \chi^2_{k-1}$ *as* $N \to \infty$, *approximately.*

In our case with N = 11 and k = 9, the approximation is good.

For the above data:

$$M = \frac{12}{11\times 9\times 10}\left(48.5^2 + \;....\right) - 3\times 11\times 10 = 46.0$$

Referring to the tables for chi-squared, we see that this result is highly significant, indicating that we reject the null hypothesis that there is no significant difference betwee the boards on the pure mathematics questions. Hence we conclude that there is some difference, not attributable to chance, between the boards on the pure mathematics questions.

The procedure, as above, was then applied to mechanics and statistics questions on the same set of papers. We have made this division of question type because it is highly likely that different chief examiners set the questions on these sections of the paper. (The author knows this to be the case for at least three of the boards from his own experience as an examiner and teacher.)

Table 6.6 Heading-Board Mechanics Scores.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	1.92	2.00	1.91	1.63	1.92	2.00	2.00	2.00	1.86
b)	0.00	0.00	0.00	0.00	0.17	0.20	0.29	0.00	0.00
c)	1.17	1.20	0.82	0.25	0.67	0.60	0.57	1.40	0.57
d)	1.00	1.00	1.00	0.00	0.33	0.80	0.71	0.40	0.43
e)	1.67	1.60	1.36	1.38	1.25	1.50	2.00	1.60	1.36
f)	1.33	1.20	1.00	1.38	1.67	1.80	2.00	0.80	1.43
g)	1.67	2.00	1.64	1.63	1.92	2.10	2.00	1.60	1.50
h)	1.67	1.60	1.64	1.63	1.67	1.60	1.57	1.60	1.43
i)	1.25	1.20	1.00	0.88	0.50	0.80	0.57	0.40	0.64
j)	0.00	0.00	0.00	0.00	0.17	0.10	0.29	0.00	0.07
k)	1.58	2.00	1.91	2.00	2.00	1.80	1.86	2.00	1.93

Table 6.7. Heading-Board Ranks for Mechanics.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	4.5	7.5	3	1	4.5	7.5	7.5	7.5	2
b)	3.5	3.5	3.5	3.5	7	8	9	3.5	3.5
c)	7	8	6	1	5	4	2.5	9	2.5
d)	8	8	8	1	2	6	5	3	4
e)	8	6.5	3	4	1	5	9	6.5	2
f)	4	3	2	5	7	8	9	1	6
g)	5	7.5	4	3	6	9	7.5	2	1
h)	8.5	4	7	6	8.5	4	2	4	1
i)	9	8	7	6	2	5	3	1	4
j)	3	3	3	3	8	7	9	3	6
k)	1	7.5	4	7.5	7.5	2	3	7.5	5
Total	61	66.5	50.5	41	58.5	65.5	66.5	48	37

$$M = \frac{12}{11\times9\times10}\left(61^2 + \ldots\ldots\right) - 3\times11\times10 = 11.6$$

Table 6.8. Heading-Board Statistics Scores.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	1.50	1.80	1.83	2.00	1.67	1.80	1.78	2.00	2.00
b)	0.70	0.60	0.25	0.88	1.00	0.20	0.56	0.20	0.00
c)	0.60	0.80	0.67	1.00	0.67	0.20	0.67	0.60	0.33
d)	0.20	0.40	0.17	0.13	0.33	0.30	1.11	0.00	0.80
e)	1.60	2.20	1.50	0.75	0.83	1.20	1.22	1.40	1.53
f)	0.80	1.40	0.75	0.25	0.17	0.80	0.67	0.80	1.20
g)	2.00	1.80	1.25	1.50	1.08	1.30	1.11	1.20	1.60
h)	1.10	1.20	1.25	1.25	1.08	1.60	1.56	1.00	1.13
i)	0.00	0.20	0.25	0.13	0.25	0.30	0.56	0.20	0.40
j)	0.70	0.60	0.25	0.88	1.17	0.20	0.56	0.20	0.00
k)	0.80	1.20	1.50	1.00	1.58	1.70	2.00	1.60	1.67

Table 6.9. Heading-Board Ranks for Statistics.

	AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
a)	1	4.5	6	8	2	4.5	3	8	8
b)	7	6	4	8	9	2.5	5	2.5	1
c)	3.5	8	6	9	6	1	6	3.5	2
d)	4	7	3	2	6	5	9	1	8
e)	8	9	6	1	2	3	4	5	7
f)	6	9	4	2	1	6	3	6	8
g)	9	8	4	6	1	5	2	3	7
h)	3	5	6.5	6.5	2	9	8	1	4
i)	1	3.5	5.5	2	5.5	7	9	3.5	8
j)	7	6	4	8	9	2.5	5	2.5	1
k)	1	3	4	2	5	8	9	6	7
Total	50.5	69	53	54.5	48.5	53.5	63	42	61

$$M = \frac{12}{11\times9\times10}\left(50.5^2 + \ldots \right) - 3\times11\times10 = 6.47$$

For neither the mechanics questions, nor the statistics questions was the result significant at the 5% level, the critical value being approximately 15.5, thus indicating that there was no significant difference between the boards in these questions on areas of applied mathematics. The result on the pure mathematics questions, however, we recall, was highly significant.

The results obtained may initially appear somewhat surprising, when one recalls that the pure mathematics questions are set on the 'common core' topics. Hence, across all the boards, the material for the pure mathematics questions is the same. The differences, therefore, must lie in the way in which these same topics are treated. For example, some questions will be set as straightforward tests of basic principles, or facts, while others may be a complicated puzzle, in which knowing the facts will merely be the start and these facts then have to be re-arranged and manipulated to solve the problem. In applied mathematics the questions may be much more a test of knowing a technique, with restraint exercised on the degree of manipulative algebra or puzzle-solving. Thus candidates are asked to calculate positions and velocities of particles, tensions in strings, probabilities from standard distributions, lines of regression and correlation coefficients. Indeed some syllabuses, referring to statics, specifically state that problems which are 'complicated geometrical puzzles' will not be set. The more testing type of applied mathematics problem may, therefore, be reserved for Further Mathematics or S-level, or not set at all.

We may thus be witnessing a fairly common approach to applied mathematics, perhaps not on common material (the common core does not include applied mathematics), and a varying approach to pure mathematics, where some boards are making more demands than others.

The Friedman test was also applied to our question headings:

Table 6.10. Board-Heading Pure Mathematics Scores.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	1.31	0.31	0.54	0.46	0.77	1.23	1.08	1.00	0.54	0.85	1.23
CAMB	1.11	0.28	0.22	0.72	1.33	1.28	1.39	1.28	0.17	0.89	1.50
JMB	1.19	0.13	0.25	0.63	1.06	1.25	1.25	1.38	0.19	0.81	1.75
LON	0.73	0.00	0.13	0.33	0.93	1.00	0.93	1.07	0.00	1.00	1.27
OXF	1.28	0.28	0.39	1.06	1.56	1.61	1.83	1.39	0.50	1.22	1.67
O&C	1.50	0.20	0.45	1.25	1.60	1.60	1.75	1.80	0.65	1.20	1.80
MEI	1.29	0.00	0.29	1.00	1.71	1.57	1.71	1.57	0.14	1.07	1.71
SMP	0.71	0.19	0.14	0.95	1.19	1.43	1.29	1.24	0.38	1.24	1.62
WJEC	0.87	0.07	0.20	0.47	1.20	1.33	1.27	1.00	0.33	1.07	1.40

Table 6.11. Board-Heading Ranks for Pure Mathematics.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	11	1	3.5	2	5	9.5	8	7	3.5	6	9.5
CAMB	6	3	2	4	9	7.5	10	7.5	1	5	11
JMB	7	1	3	4	6	8.5	8.5	10	2	5	11
LON	5	1.5	3	4	6.5	8.5	6.5	10	1.5	8.5	11
OXF	6	1	2	4	8	9	11	7	3	5	10
O&C	6	1	2	5	7.5	7.5	9	10.5	3	4	10.5
MEI	6	1	3	4	10	7.5	10	7.5	2	5	10
SMP	4	2	1	5	6	10	9	7.5	3	7.5	11
WJEC	5	1	2	4	8	10	9	6	3	7	11
Total	56	12.5	21.5	36	66	78	81	73	22	53	95

$$M = \frac{12}{9\times 11\times 12}\left(56^{2} + \ldots\right) - 3\times 9\times 12 = 76.99$$

Table 6.12. Board-Heading Mechanics Scores.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	1.92	0.00	1.17	1.00	1.67	1.33	1.67	1.67	1.25	0.00	1.58
CAMB	2.00	0.00	1.20	1.00	1.60	1.20	2.00	1.60	1.20	0.00	2.00
JMB	1.91	0.00	0.82	1.00	1.36	1.00	1.64	1.64	1.00	0.00	1.91
LON	1.63	0.00	0.25	0.00	1.38	1.38	1.63	1.63	0.88	0.00	2.00
OXF	1.92	0.17	0.67	0.33	1.25	1.67	1.92	1.67	0.50	0.17	2.00
O&C	2.00	0.20	0.60	0.80	1.50	1.80	2.10	1.60	0.80	0.10	1.80
MEI	2.00	0.29	0.57	0.71	2.00	2.00	2.00	1.57	0.57	0.29	1.86
SMP	2.00	0.00	1.40	0.40	1.60	0.80	1.60	1.60	0.40	0.00	2.00
WJEC	1.86	0.00	0.57	0.43	1.36	1.43	1.50	1.43	0.64	0.07	1.93

Table 6.13. Board-Heading Ranks for Mechanics.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	11	1.5	4	3	9	6	9	9	5	1.5	7
CAMB	10.5	1.5	5.5	3	8.5	5.5	5.5	8.5	5.5	1.5	10.5
JMB	10.5	1.5	3	5	7	5	8.5	8.5	5	1.5	10.5
LON	9	2	4	2	6.5	6.5	9	9	5	2	11
OXF	9.5	1.5	5	3	6	7.5	9.5	7.5	4	1.5	11
O&C	10	2	3	4.5	6	8.5	11	7	4.5	1	8.5
MEI	9.5	1.5	3.5	5	9.5	9.5	9.5	6	3.5	1.5	7
SMP	10.5	1.5	6	3.5	8	5	8	8	3.5	1.5	10.5
WJEC	10	1	4	5	6	7.5	9	7.5	5	2	11
Total	90.5	14	38	32	66.5	61	69	71	41	14	87

$$M = \frac{12}{9\times11\times12}\left(90.5^{2} + \ldots\ldots\right) - 3\times9\times12 = 62.3$$

Table 6.14. Board-Heading Statistics Scores.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	1.50	0.70	0.60	0.20	1.60	0.80	2.00	1.10	0.00	0.70	0.80
CAMB	1.80	0.60	0.80	0.40	2.20	1.40	1.80	1.20	0.20	0.60	1.20
JMB	1.83	0.25	0.67	0.17	1.50	0.75	1.25	1.25	0.25	0.25	1.50
LON	2.00	0.88	1.00	0.13	0.75	0.25	1.50	1.25	0.13	0.88	1.00
OXF	1.67	1.00	0.67	0.33	0.83	0.17	1.08	1.08	0.25	1.17	1.58
O&C	1.80	0.20	0.30	0.30	1.20	0.80	1.30	1.60	0.30	0.20	1.70
MEI	1.78	0.56	0.57	1.11	1.22	0.67	1.11	1.56	0.56	0.56	2.00
SMP	2.00	0.20	0.60	0.00	1.40	0.80	1.20	1.00	0.20	0.20	1.60
WJEC	2.00	0.00	0.33	0.80	1.53	1.20	1.60	1.13	0.40	0.00	1.67

Table 6.15. Board-Heading Ranks for Statistics.

	a)	b)	c)	d)	e)	f)	g)	h)	i)	j)	k)
AEB	9	4.5	3	2	10	6.5	11	8	1	4.5	6.5
CAMB	9.5	3.5	5	2	11	8	9.5	6.5	1	3.5	6.5
JMB	11	3	5	1	9.5	6	7.5	7.5	3	3	9.5
LON	11	5.5	7.5	1.5	4	3	10	9	1.5	5.5	7.5
OXF	11	6	4	3	5	1	7.5	7.5	2	9	10
O&C	11	1.5	4	4	7	6	8	9	4	1.5	10
MEI	10	2	4.5	4.5	8	4.5	6.5	9	2	2	11
SMP	11	3	5	1	9	6	8	7	3	3	10
WJEC	11	1.5	3	5	8	7	9	6	4	1.5	10
Total	94.5	30.5	41	26	71.5	48	67	69.5	21.5	33.5	81

$$M = \frac{12}{9\times11\times12}\left(94.5^2 + \ldots\right) - 3\times9\times12 = 50.7$$

We see that in each case, for pure mathematics, mechanics and statistics, that the test statistic is highly significant, indicating a difference between the headings, not arising from the operation of chance factors.

Examination of the sum of the ranks in each column will give some clue as to an explanation of this.

We see that headings (ii) (objectives) and (iv) (routine processes) always contribute low total ranks. This is much as one would expect. Candidates need to know what to do in the examination, so the objectives are defined in the question in most cases. Furthermore, as this is Mathematics, as opposed to Further Mathematics, for the most part the level of question is fairly routine. One must bear in mind that A-level Mathematics does not just cater for the specialist mathematician, but for the engineer, scientist, economist, geographer etc..

Headings (iii) (jargon), (ix) (sustained thinking) and (x) (open solution) also contribute mainly low values for the total rank. The papers undergo several stages of revision and the removal of jargon is, therefore, likely to have occurred in the process. One can also argue that the subject should not depend on learning jargon for its own sake, which can be accomplished by rote learning. The questions are frequently structured and split into a number of sub-parts. The opportunity for sustained thinking in these circumstances is thus rather limited. We have mentioned above that the objectives are clearly defined in most cases, it is therefore unlikely that there will be many opportunities for an open solution. Indeed, giving the candidates scope for open solutions would cause great difficulty for the consistency of marking. One of the positive virtues in A-level Mathematics is that the marking can be of a consistent standard. It should, in most cases, be clear if the candidate has the correct answer, and if he/she has not, then there should be sufficient working present to apportion credit for reasonable methods, which lacked accuracy. This is not to say that much useful mathematics could not come from offering the chance for an open solution. In fact great benefit could be derived from developing skills in this area. However, it is open to question as to whether an examination with a time limit is the vehicle for assessing this. The scope for open solutions in the papers examined was frequently limited to a comment at the end of a statistics question, or a generalisation from what had gone before.

Headings (xi) (formula sheet) and (viii) (powers of reasoning) always contribute high values for the total rank. All the boards provide a formula sheet to assist the candidates, but one would not expect the candidates to substantially complete questions by referring to information in the formula sheet. Hence, for the majority of questions, there was little help from the formula sheet, resulting in a high total rank. The nature of the subject should explain why heading (viii) scores highly. The subject itself is fundamentally concerned with marshalling facts and deducing results from them, it is scarcely surprising that most questions, therefore, require a fair amount of ability, on the part of the candidate, to reason effectively.

Headings (i) (procedure) and (vii) (mathematical manipulation) also score mainly high total ranks. Although the objectives are defined, in many questions the candidate is not told exactly how to achieve the objective. It is not unreasonable to expect the candidates, at this level, to be able to work things out for themselves, which appears to be the case in many questions. One of the characteristics of A-level questions is the high level of manipulation involved. Whereas O-level contained considerable algebraic manipulative skills, its successor GCSE contains less, because of the inclusion of other material. Consequently, sixth-formers starting A-level mathematics find it difficult to adjust to the level of manipulation required in their work. High levels of manipulative algebraic skill persist through the course and are evident in the examintion questions at the end of the course. Some would argue that the manipulation at A-level is excessive and there are other skills that need to be developed. We shall see that the demand for manipulative ability is prevalent in the requirements of higher education - engineers and physicists, especially, frequently refer to manipulative ability if asked what they require in A-level Mathematics. The questions in A-level Mathematics reflect this demand from users and so the high total rank score we see in this category comes as small surprise.

This leaves headings (v) (complex mathematical content) and (vi) (level of abstraction) falling in the central regions. The examination boards, themselves, would probably feel fairly contented with this state of affairs, as it would appear that the questions are being pitched at a reasonable level, neither too hard nor too easy, in these headings. The examination boards do have the S-level and Further Mathematics papers where more complex content can be put and where the level of abstraction can be increased, as candidates for S-level and/or Further Mathematics would either be specialist mathematicians or highly competent at the subject.

The variation in scores across the headings, for the most part, thus falls into a recognisable pattern, which is not due to chance. The interesting finding to emerge from this analysis is that the pure mathematics questions show variation across the examination boards and in the work which follows we shall look more closely at possible differences between pure mathematics questions and applied mathematics questions.

6.5 The Kruskal-Wallis Analysis.

If we wish to determine if independent samples are from the same population, then the Kruskal-Wallis technique is available for our use. Compared with the most powerful parametric test, the F test, under conditions where the assumptions associated with the statistical model of the F test are met, this non-parametric test has asymptotic efficiency of 95.5%. The Kruskal-Wallis technique tests the null hypothesis that the samples come from the same population, or from identical populations, against location shift alternatives. The test assumes that the variable under study has an underlying continuous distribution.

From our raw data, we are using the total score for each question across the 11 headings. For the pure mathematics questions these are shown on the following page. The scores were then ranked and the ranks totalled for each board. In the Kruskal-Wallis test, all the scores are ranked, whereas in the Friedman test, each row was ranked separately.

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1)$$

where

N = total number of questions

R_j = *sum of ranks in the jth sample*

n_j = *number of cases in the jth sample*

H is distributed approximately as chi-squared with k-1 degrees of freedom, where k is the number of samples.

In our data we have many tied ranks. A correction for ties can be made. In this case the value of H from the above formula is divided by

$$1 - \sum \frac{T}{N^3 - N}$$

where $T = t^3 - t$

and t = number of tied observations in a tied group of scores.

Table 6.16. Pure Mathematics Scores by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
7	6	6	5	7	8	6	6	3
7	7	7	5	9	10	11	6	6
8	7	7	7	11	10	11	7	6
8	8	8	7	11	10	11	8	7
8	9	9	7	11	11	12	8	8
8	9	10	7	12	12	12	8	8
9	10	11	7	13	13	13	9	10
10	10	11	8	13	13	13	9	10
12	11	11	9	13	15	15	10	11
13	11	11	9	13	15	15	11	11
13	11	12	9	14	16	15	11	12
15	12	13	10	15	16	15	11	14
16	13	14	11	15	17	16	12	15
	13	14	11	16	17	18	12	15
	14	15	14	18	17		13	16
	16	15		18	18		13	
	16			19	18		14	
	18			20	19		16	
					20		17	
					21		17	
							19	
(13)	(18)	(16)	(15)	(18)	(20)	(14)	(21)	(15)

Figures in parentheses indicate the number of questions for each board.

Table 6.17. Pure Mathematics Ranks by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
17.5	7	7	2.5	17.5	31	7	7	1
17.5	17.5	17.5	2.5	42.5	53	69	7	7
31	17.5	17.5	17.5	69	53	69	17.5	7
31	31	31	17.5	69	53	69	31	17.5
31	42.5	42.5	17.5	69	69	84.5	31	31
31	42.5	53	17.5	84.5	84.5	84.5	31	31
42.5	53	69	17.5	97	97	97	42.5	53
53	53	69	31	97	97	97	42.5	53
84.5	69	69	42.5	97	118	118	53	69
97	69	69	42.5	97	118	118	69	69
97	69	84.5	42.5	108	129	118	69	84.5
118	84.5	97	53	118	129	118	69	108
129	97	108	69	118	136	129	84.5	118
	97	108	69	129	136	141.5	84.5	118
	108	118	108	141.5	136		97	129
	129	118		141.5	141.5		97	
	129			146	141.5		108	
	141.5			148.5	146		129	
					148.5		136	
					150		136	
							146	
Rank Sums:								
780	1257	1078	550	1790	2167	1319.5	1487.5	896

$$H = \frac{12}{150 \times 151}\left(\frac{780^2}{13} + \ldots\ldots\right) - 3 \times 151 = 36.23$$

In our case we find:

Score	Frequency (t)	T
5	2	6
6	7	336
7	14	2730
8	13	2184
9	10	990
10	11	1320
11	21	9240
12	10	990
13	15	3360
14	7	336
15	13	2184
16	9	720
17	5	120
18	6	210
19	3	24
20	2	6
	Total	24756

$$\textit{Correction divisor for tied ranks} = 1 - \frac{24756}{(150^3 - 150)} = 0.9926$$

H = 36.23/0.9926 = 36.50

Referring to chi-squared tables with 8 df, we see that this result is significant at the 0.1% level. Hence we can reject the hypothesis that the samples come from the same population; i.e., we are saying for pure mathematics questions there is a significant difference between the boards.

The procedure was repeated for mechanics and statistics questions.

Table 6.18. Mechanics Scores by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
7	12	6	9	10	9	11	9	6
12	13	12	10	10	10	12	10	8
12	15	12	11	11	11	13	12	8
13	16	13	12	11	13	14	15	9
13	18	13	12	12	14	17	18	9
14		14	13	12	16	17		9
14		14	14	13	16	20		11
15		15	15	15	16			12
16		15		15	18			15
16		16		16	20			15
19		16		16				15
20				18				18
								18
								18
(12)	(5)	(11)	(8)	(12)	(10)	(7)	(5)	(14)

Figures in parentheses indicate the number of questions for each board.

Table 6.19. Mechanics Ranks by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
3	28.5	1.5	8.5	14	8.5	19.5	8.5	1.5
28.5	39	28.5	14	14	14	28.5	14	4.5
28.5	56	28.5	19.5	19.5	19.5	39	28.5	4.5
39	66.5	39	28.5	19.5	39	47	56	8.5
39	77	39	28.5	28.5	47	72.5	77	8.5
47		47	39	28.5	66.5	72.5		8.5
47		47	47	39	66.5	83		19.5
56		56	56	56	66.5			28.5
66.5		56		56	77			56
66.5		66.5		66.5	83			56
81		66.5		66.5				56
83				77				77
								77
								77
Rank Sums:								
585	267	475.5	241	485	487.5	362	184	483

$$H = \frac{12}{84 \times 85}\left(\frac{585^2}{12} + \ldots\right) - 3 \times 85 = 7.38$$

In our case we find:

Score	Frequency (t)	T
6	2	6
8	2	6
9	6	210
10	5	120
11	6	210
12	12	1716
13	9	720
14	7	336
15	11	1320
16	10	990
17	2	6
18	7	336
20	3	24
	Total	6000

$$\textit{Correction divisor for tied ranks} = 1 - \frac{6000}{84^3 - 84} = 0.9899$$

Corrected for ties H = 7.38/0.9899 = 7.46 is not significant.

Critical value for chi-squared at the 5% level is 17.5 . (8 df)

Table 6.20. Statistics Scores by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
7	12	7	8	6	7	7	9	7
9	12	7	8	8	7	7	9	7
10	13	8	9	9	7	8	10	8
10	13	8	10	9	8	8	10	8
10	16	8	10	9	8	15	13	8
12		9	12	9	11	15		8
12		11	12	9	14	16		9
13		11	17	10	14	17		9
13		11		12	15	22		13
14		15		15	16			15
		16		15				15
		17		19				16
								17
								17
								18
(10)	(5)	(12)	(8)	(12)	(10)	(9)	(5)	(15)

Figures in parentheses indicate the number of questions for each board.

Table 6.21. Statistics Ranks by Board.

AEB	CAMB	JMB	LON	OXF	O&C	MEI	SMP	WELSH
6.5	53	6.5	18.5	1	6.5	6.5	31.5	6.5
31.5	53	6.5	18.5	18.5	6.5	6.5	31.5	6.5
41.5	59.5	18.5	31.5	31.5	6.5	18.5	41.5	18.5
41.5	59.5	18.5	41.5	31.5	18.5	18.5	41.5	18.5
41.5	76	18.5	41.5	31.5	18.5	69.5	59.5	18.5
53		31.5	53	31.5	47.5	69.5		18.5
53		47.5	53	31.5	64	76		31.5
59.5		47.5	81	41.5	64	81		31.5
59.5		47.5		53	69.5	86		59.5
64		69.5		69.5	76			69.5
		76		69.5				69.5
		81		85				76
								81
								81
								84
Rank Sums :								
451.5	301	469	338.5	495.5	377.5	432	205.5	670.5

$$H = \frac{12}{86\times 87}\left(\frac{451.5^2}{10} + \ldots\right) - 3.87 = 3.67$$

In our case we find:

Score	Frequency (t)	T
7	10	990
8	14	2730
9	12	1716
10	8	504
11	4	60
12	7	336
13	6	210
14	3	24
15	8	504
16	5	120
17	5	120
	Total	7314

$$\textit{Correction divisor for tied ranks} = 1 - \frac{7314}{86^3 - 86} = 0.9885$$

Corrected for ties H = 3.67/0.9885 = 3.71 is not significant.

Critical value for chi-squared at the 5% level is 17.5 . (8 df)

The results obtained here confirm the view expressed following the Friedman test; viz. the pure mathematics questions vary in standard of difficulty from board to board. This does not appear to be the case for mechanics and statistics questions. If we look at the total sum of ranks in the Kruskal-Wallis test, for the pure mathematics questions, we see that the highest values, and therefore the most influential in producing the significant result are (in order): Oxford and Cambridge, Oxford, SMP and MEI. This is a very similar grouping to that obtained in the cluster analysis study by Croasdale (1991), to which we have referred elsewhere in this study.

7. A-level Mathematics Teachers' Questionnaire.

The following is an acount of the questionnaire sent out to sixth form teachers of mathematics. Initially the questionnaire was sent to teachers in maintained and independent schools in Bedfordshire. It was hoped to gain opinions from users of A-level Mathematics with a variety of examination boards. In the event, the majority of the teachers, from Bedfordshire, who replied to the questionnaire, were users of the A-level syllabus of the University of Cambridge Local Examinations Syndicate. In order to gather views from teachers who used a a wider cross-section of A-level syllabuses, the questionnaire was circulated to teachers who attended the University of York's Sixth Form Mathematics Teachers Conference in 1992.

In Bedfordshire, copies of the questionnaire were sent to 22 schools. A copy of the questionnaire and the covering letter can be found in Appendix A. A stamped addressed envelope was included to facilitate a reply, which resulted in 15 returns (68%). The schools circulated included a sixth form college, a further education college, maintained comprehensive schools and independent schools. With the exception of the independent schools, the establishments were mixed. At the York conference, there were 48 participants from a wide range of schools and colleges. Each participant was given a copy of the questionnaire on arrival, and, in all, over the course of the the conference (2 days), 34 replies were received (71%).

7.1 Factual Information received.

The following distribution of A-level boards was obtained:

Bedfordshire:

Cambridge 12 JMB 1 London 1 SMP 1

York Conference:

Cambridge 11 JMB 10 London 8 SMP 1 AEB 2 MEI 2

The respondents used the following types of Paper. (Some respondents used more than one version.)

	Beds.	York
(a) Pure Mathematics and Mechanics	3	16
(b) Pure Mathematics and Statistics	3	21
(c) Pure Mathematics and a mixed paper of Mechanics and Statistics.	11	16
(d) Other - Statistics	1	
Pure Mathematics		1

Option (c) was the most popular. This is the style of paper used by the Cambridge Board, which is the largest Board, anyway, and accounted for the majority of our respondents. There are obviously strong geographical factors linking schools in Bedfordshire to the Cambridge Board. Teachers can easily reach the Board's offices to collect copies of past question papers, syllabuses etc.. The independent schools in Bedford, perhaps surprisingly, use the Cambridge Board rather than the Oxford and Cambridge Board, which traditionally dominates the independent school market. Option (c) is also popular on the JMB Board, which is strong in the North of England. The total number of students using each version listed above is approximately as follows, based on the returns from teachers who replied to the questionnaire.

	Beds.	York
(a) Pure Mathematics and Mechanics	62	245
(b) Pure Mathematics and Statistics	112	628
(c) Pure Mathematics and a mixed paper of Mechanics and Statistics	263	952
(d) Other - Statistics	10	
Pure Mathematics		13

The replies certainly indicate that Statistics is a widely taught application of mathematics. Whereas a generation ago, mathematics would almost, without exception, be pure mathematics and mechanics, today the majority view is to teach some mechanics and some statistics in the 'applied' part of the course. It is interesting to note that, in the cases where only one application is taught, Statistics is more popular than mechanics. This would appear to have important implications for the training of engineers, one would expect the potential engineer to be taught mechanics rather than statistics.

The London Board, which has maintained its second paper as either mechanics or statistics until now, has introduced a modular scheme. Under this scheme the three papers have become four modules. There are two pure mathematics modules P1 and P2, two mechanics modules M1 and M2 and two statistics modules S1 and S2. Students can take modules as they progress through their two-year A-level course and obtain points. Modules can be re-taken. When sufficient points have been gathered the students can 'cash' their points for a grade. Four modules (which must include P1 and P2) have to be taken for A-level Mathematics. One suspects that, whereas Pure and Mechanics has been a more popular combination than Pure and Statistics in the past, under the new system many students will take P1,P2,M1 and S1, if the replies to our questionnaire are a guide. This will lead to a further erosion of Mechanics. One also suspects that this may be a rather less challenging course academically, as students will be learning basic work in both statistics and mechanics, and not doing more advanced work in either.

7.2 Opinions on 'Difficulty'.

The respondents were asked their opinions of the 20+ syllabuses for A-level Mathematics that exist currently. In particular did they they consider that:

(a) All are of an equivalent standard of difficulty

(b) Some are harder than the majority

(c) Some are easier than the majority.

Allowing respondents to tick more than one option (e.g. some are harder and some are easier) the following results were obtained:

	Beds.	York
(a)	0	4
(b)	9	16
(c)	3	15
Don't Know	3	5

It is interesting to note that only 4 of the 49 respondents (8%) thought that all of the A-level Mathematics syllabuses were of equivalent standards of difficulty. This would seem to give rise to some concern about variation in levels of difficulty among the various options, which are all supposed to be the same. Certainly, as we shall see in our interviews with university admissions tutors, the commonly held view, at university level, is that they are all of a similar standard of difficulty.

Those respondents, who indicated they thought some boards were either easier or harder than the majority, were asked to indicate which boards they thought fell into these categories, with the following results:

Easier Category:

AEB (12) London (8) Modular MEI (1)

Harder Category:

Cambridge (3) JMB (3) Oxford and Cambridge (5)

MEI (6) Oxford (8) SMP (8).

The number in parentheses is the number of teachers citing this board.

There is an interesting pattern here, which is not too dissimilar to that which emerged in the cluster analysis of Croasdale. AEB and London appear frequently at the easier end and Oxford, Oxford and Cambridge, with its associated projects SMP and MEI, at the harder end. It should be noted, of course, that many of our respondents used Cambridge and JMB, which fell between the extremes. One can argue that they were popular because they were not extreme. Furthermore, one can argue that the users of a particular board feel happy with the version that they are using and are unlikely to criticise it. Nevertheless there was not much instance of Cambridge Board users mentioning JMB and vice-versa.

7.3 Opinions on 'Differences between Boards'.

Respondents were asked to suggest factors which might contribute to differences between the boards.

A number of people thought that the move towards modular assessment was making the syllabus easier; we have already mentioned one way in which this can occur earlier. It is possible that by spreading the load that the syllabus is easier to deal with. By this we mean that examination papers can be taken throughout the course and not just at the end. It also means that papers are shorter and more numerous, rather than two long papers at the end of the course.

Respondents frequently mentioned the predictability of questions, or the paper as a whole. Certainly if the paper, or individual questions, are much the same in style and content from year to year, then candidates will have plenty of opportunity to get acquainted with the type of question that they are likely to encounter. Past papers are available and candidates will work through them quite extensively prior to taking their examination. If the questions vary considerably in style and content then it is likely that practice on past papers, although valuable, is probably not going to instill confidence in the candidates in quite the same way, or to the same extent. Papers of an unpredictable nature will, therefore, seem harder or more forbidding. It was felt by respondents that the papers of some boards were much the same year in and year out, and a clear and predictable pattern, or stereotype, was established. Other boards, it was felt, tried to set the questions in unfamiliar ways, so that more emphasis was placed upon applying knowledge, rather than regurgitating rehearsed routines. Boards which did this were increasing the difficulty of the candidates' task.

Another factor, which emerged, was that in the applied paper or applied questions of some boards, there was a need to use pure mathematics content more extensively. For example, whereas some questions on 'expectation' might require knowledge of the process and how to apply it in a simple numerical example, other questions might involve detailed knowledge of the summation of geometrical progressions or binomial series. The candidate who knew the probability theory, therefore, might still have trouble completing the question if his/her algebra, in particular, was not strong enough. It was felt that there was inconsistency in the extent to which boards required manipulative skills, particularly in applied mathematics questions. The same would be true in mechanics questions, which often involve writing down a number of equations, based on various principles, and then evaluating the unknowns. This latter stage is often the crucial one and the one that will discriminate between the candidates. Again some boards, it was felt, would make more demands in this area than others.

While considering the 'style' of questions, a number of respondents pointed out that some boards were good at using structured questions. This helped to lead the candidates through the question. Often the result obtained in one part of the question could then be used, with advantage, in the next part of the question. If intermediate answers were printed on the question paper this would also be helpful, as the candidate who lost his or her way on part of the question could start the next part using the printed result. There were boards, however, that did not adopt this approach to any great extent. This meant that the questions were inevitably harder. Often the questions to avoid, if possible, were the ones consisting of only a couple of sentences and requiring a single result. The ease of a question, some thought, was often inversely proportional to its length.

Some teachers felt that the papers which required both mechanics and statistics to be answered were more of a challenge than papers where only mechanics or only statistics had to be tackled. One reason for this was that the 'mixture' paper often allowed the candidate more choice of question. Consequently, in the examination, more time was spent reading through the questions, in order to select the ones the candidate wished to answer. Furthermore, the style of thinking required to answer some statistics questions is rather different to that required for mechanics questions. This would particularly be the case if the statistics question was descriptive or open-ended. Moving from one style of thinking to another, in the examination, could present difficulties for some candidates.

Doubts were expressed as to whether Pure Mathematics with Statistics was as demanding a combination of papers as Pure Mathematics and Mechanics. Teachers invariably advised their weak A-level candidates to do Pure Mathematics with Statistics. Statistics questions, many teachers felt, were either easy or hard. Weak candidates could cope with the easy questions. Mechanics questions were more uniform and of a moderate degree of difficulty. Some considerable investment of time had to be made before mechanics questions could be tackled confidently. Several weeks might have to be spent in the course, learning how to draw diagrams, understanding in which direction forces acted, how to resolve forces and take moments etc., before problems could be tackled. The rewards in Statistics were much more immediate and probably more accessible to the weak candidate.

We shall see, in our interviews with university admissions tutors, that staff in higher education are somewhat cautious in allowing students to study mathematics at degree level if their background is Pure Mathematics with Statistics and they have no knowledge of mechanics.

Some respondents felt that some examination boards had a different philosophy behind their examination to other boards. At the Further Mathematics level they certainly have different interpretations of the word 'Further'. Some boards view 'Further' as extra topics at the same level of difficulty, while other boards view 'Further' as harder questions as well as extra topics. At the single mathematics level, some teachers felt that there were boards who made A-level accessible only to those who were able enough to consider pursuing the subject beyond A-level, whether in study of the subject in its own right, or as a supporting subject, perhaps for some form of science or engineering course. Other boards, it was felt, made the A-level accessible to pupils who were not going to pursue mathematics beyond A-level. When an E grade can be obtained with a raw score of between 30 and 35%, one feels that the paper is not really accessible to those candidates obtaining an E grade at the end of two year's work. This is the case on some boards, (see Standards in Advanced Level Mathematics - Report of Study 3, page 30, (1990)) . At the other end of the spectrum, one finds that some boards set questions which a lot of good candidates will succeed in doing completely, whereas other boards set questions which only the outstanding candidate will be able to complete. A professor of Applied Mathematics, looking at two A-level papers from different boards, commented to the author that they were both examining the same work, but in one paper the questions were a straightforward test of the material, whereas in the other the setter was 'dressing the questions up' to add an extra dimension of difficulty; in his view rather needlessly. He said that he would rather be able to see students being successful in what they were doing. The teachers questionned certainly felt that boards had different attitudes towards A-level.

Some teachers felt that this difference in attitude arose because the clientele for boards vary. Oxford and Cambridge is traditionally the board which attracts entries from Public Schools' candidates. The AEB is used by many candidates from Tertiary colleges. London and Cambridge Boards, as well as having a large 'Home' entry from many backgrounds, have a substantial number of 'Overseas' candidates. It was thought that Oxford and Cambridge, for example, would be gearing their A-level papers towards the prospective Oxbridge/Redbrick University entrant, wheras AEB papers, for example, would be aimed more at the student who was likely to pursue a vocational course or go directly into employment after A-level. The latter was described as a more routine approach, perhaps dull in some people's estimation, but likely to produce easier papers for the candidate. 'Tradition', it would seem, is a very potent cause for much which occurs in the educational world. It certainly seemed to be a factor which weighed heavily in the minds of the respondents to the questionnaire.

7.4 Opinions on 'Grading'.

The teachers were asked if they thought that the grades awarded by different boards were comparable. Opinion was fairly evenly divided, 20 thinking 'yes' and 26 'no', with 3 failing to express an opinion. Ideally one would hope that the vast majority would think that the grades were comparable. The results, while not being conclusive, certainly suggest that there is some doubt in the minds of the respondents to this questionnaire.

When asked if they had considered changing boards at A-level, 24 said that they had and 24 said that they had not considered doing so, with 1 undecided. This again, while not being decisive, shows some measure of dissatisfaction with the product produced by some examination boards. The teachers, who said that they had considered changing boards, were asked which board they might adopt. The replies included Modular MEI, London and SMP, both of whom are , or are becoming modular, as well as AEB. It is interesting to note the attractiveness of the modular approach, which is a comparatively new innovation in A-level Mathematics. Perhaps the 'newness' is part of the attraction, but one would hope that there was some intrinsic benefit in the modular approach which was also proving to be attractive to the teachers.

When asked if they thought grades in Mathematics were:

(a) Much harder to gain than the same grades in other subjects

(b) Harder to gain than the same grades in other subjects

(c) About the same level of difficulty as grades in other subjects

(d) Easier to gain than the same grades in other subjects

(e) Much easier to gain than the same grades in other subjects,

the replies were:

(a) 18 (b) 23 (c) 5 (d) 0 (e) 0 Not Sure 3.

Clearly one must allow for a 'well they would say that, wouldn't they' effect, but having done so, there does seem to be a fairly strongly held view, among the mathematics teachers questionned, that grades are relatively harder to come by in Mathematics than the comparable grade in other subjects.

One teacher at the York conference spoke at some length about this. He said that, when a reasonably able student (in relation to his school), at the end of the Fifth Form, is considering what to do for A-level, and realises that he is likely (looking at the results of students who have preceded him) to get a D and two E's in Chemistry, Physics and Mathematics and two C's and a D in English, Economics and Sociology, then that student hasn't really got much of a choice, unless he is very determined to go into some form of engineering. This, he added, has severe implications for the supply of scientists and technologists, which the country needs to produce wealth-creating industry.

We have already seen in the replies to an earlier question that the modular approach to A-level mathematics was proving popular. It came as little surprise, therefore, that when the teachers were asked about modular courses, 10 were strongly in favour of them, 19 in favour and 19 undecided, with only 1 teacher at all opposed to this approach. Certainly in the minds of our respondents, this seemed the way forward and it will be interesting to see in the next few years if other boards adopt this style of course.

7.5 Opinions on 'Content' and 'Style' of Papers.

The teachers were asked if they thought that the syllabus, which they used, had too much material in it, too little in it, or whether the amount of material was about right. No one thought there was too little, which is hardly surprising. Twenty eight thought it was about right and 21 thought there was too much material in the course, 12 of whom were using the Cambridge Board, 6 JMB, 2 London and 1 SMP. On the Cambridge Board syllabus, Paper 2 has three sections; mechanics, statistics and further pure mathematics. The instructions say that candidates should only have to cover the work for two sections. This gives them a choice of 7 questions from 10. Teachers who used this syllabus said that they were tempted to give a bit more choice to their pupils, hence they often cover substantial amounts in all three sections, so the choice may be 7 from 12. This may account, in some measure, for the larger number of teachers, using the Cambridge Board, who said that the syllabus was too extensive.

When asked about the style of question in the papers, used by their school or college, the majority, 32 out of 49, thought questions were about the right standard, with 12 thinking the style of question was too hard and 5 thinking it was too easy. Probably the questions are familiar - as we have mentioned, past papers are available for practice, so these results, which we have obtained, are, possibly, not surprising.

The teachers were asked to consider the hypothetical case of a student who has achieved a grade C in GCSE Mathematics at the end of the Fifth Form Year (Year 11 in National Curriculum terms).Twenty two respondents (slightly less than half) thought such a student had a reasonable chance of achieving a C grade, or better, at A-level. Eighteen thought that a D or E grade at A-level was the likely outcome and eight teachers thought such a student would have only a slight chance of passing A-level and would advise caution before embarking on A-level Mathematics. (The author's own experience of such pupils is that around 35-40% of them secure a pass at A-level.) It is not uncommon to find, in the Sixth Form prospectus of a school, advice to students along the lines that, a B grade at GCSE is required to study Mathematics at A-level and preferably an A grade. It is not uncommon to hear teachers talking of transitional problems between GCSE Mathematics (where the amount of algebra is less than was present in O-level) and A-level Mathematics. Hirst (1993), the admissions tutor at Southampton University, draws attention to this and thinks that the situation may deteriorate with the arrival of the National Curriculum. He points out that A-level Mathematics has to be accessible to students with level 7 of the National Curriculum. (This approximates to grade C at GCSE Mathematics.) In the National Curriculum, trigonometry appears at level 8. Hirst's view is, therefore, that when new A-level syllabuses appear in 1994, there will be greatly reduced content in trigonometry. Calculus, he hopes, will not be too much affected.

Whereas at GCSE Higher Level, the C grade student will have been exposed to the full syllabus, even though he/she may not have mastered it, under the National Curriculum, the Level 7 student may not have covered any of the material for levels 8,9 or 10. This would be equivalent to making A-level accessible to the student with a a C grade from the Intermediate Level of the GCSE. Some schools and colleges allow such students to proceed to A-level, but the students have much foundation work to make good, and one suspects that most schools or colleges would require students to have attempted the Higher Level of GCSE before embarking on A-level, which has seen no reduction of syllabus since the days when it was preceded by O-level.

7.6 Opinions on Coursework.

The advent of GCSE brought with it coursework. Some A-level syllabuses, SMP 16-19, MEI modular, Oxford and Cambridge and Oxford have introduced coursework tasks into A-level work, which can contribute towards the the final assessment. When asked about their feelings towards coursework, the following results were obtained from the respondents to the questionnaire:

Strongly in favour 4 , In favour 22 ,
Undecided 15,
Opposed 6, Strongly Opposed 2.

Teachers have become more accustomed to coursework, while preparing for GCSE. The response above seems to reflect this.

The author was present when the Moderator and Mathematics Subject Officer from one A-level board were discussing coursework at A-level. They felt that it might be possible to devise meaningful tasks in Pure Mathematics and Statistics, but felt it would prove difficult to produce work in Mechanics that was not Physics practical. They also felt uneasy about how such work was going to be reliably assessed at A-level. It seemed rather pointless to go to extreme lengths to get the marking of written papers precise if one was going to allow considerable latitude in the award of up to 20% of the total for coursework, which would presumably be assessed 'in situ' and externally moderated. It was suggested that there might be a written paper to assess the same skills as coursework assessed, thus obviating the need for a coursework component.

It will be interesting to see how the position of coursework at A-level develops in the next few years. The results of our questionnaire seem to suggest mild support for the idea, certainly not strong opposition. The students, clearly, get used to working in certain ways for GCSE/National Curriculum and it does seem to be appropriate for the A-level course to build on these foundations and not be a source of discontinuity. The problems of implementation, however, seem to be a difficult hurdle to cross.

7.7 Miscellaneous Comments.

At the end of the questionnaire, teachers were invited to add any further comments they had about A-level Mathematics.

Several people referred again to the fact that the A-level Mathematics course is a "hard slog", to quote one teacher,"for both staff and pupils", he added. The material was conceptually difficult and there was a lot of it. One respondent thought the material was one and a half A-levels and should be allotted 12 periods per week

It was pointed out that great emphasis was placed upon algorithms and techniques during the entire course, consequently it was hard to 'lighten up the course' and introduce more variety. Some teachers felt that different approaches were required. The introduction of more practical applications, especially for mechanics, were mentioned. As we have pointed out above, this could prove difficult to implement, because of its overlap with Physics. Certainly there seemed to be a need for different approaches to be tried. The Oxford Board's ongoing coursework tasks were favourably mentioned in this context.

There was a feeling that the courses on offer were well-suited to the strong student, but offered little scope for the hard-working but less talented student. When the students find the work difficult then they become discouraged and do not feel inclined to pursue the subject further, was the view that was expressed.

Some of the comments suggested that there should be a more basic, or foundation, course, which concentrated on learning how to differentiate, integrate and solve equations of various types. The SMP already have a limited grade A-level, which received favourable comments, in that it provided something which students required. There was a problem with this course, however, because teachers felt that it was held in low esteem by Higher Education, parents and even the students themselves.

The place of Mathematics at A-level, compared to other subjects, was mentioned quite often. There was a feeling among a number of respondents that higher grades in Mathematics were harder to obtain than in other subjects, which we have already discussed above. There was also the issue of Further Mathematics. Did this still have a place in the Sixth Form? Many schools found difficulty in staffing it, or finding a slot for it on the timetable, or getting sufficient students to make a viable teaching group. Why should Mathematics, alone among A-level subjects, with the possible exception of Music, have two A-levels? Was this not introducing degree level work into school? There was a feeling that Further Mathematics could be taught successfully in large schools, or schools with able staff, who had the time to devote to the subject. There was a suspicion of "elitism" in many of the comments on this particular issue.

A number of teachers wondered where university departments recruit their students from, because a very disappointing number of sixth form students at their schools or colleges went on to study the subject. This has important implications for the supply of mathematicians for industry and also for teacher supply in the future.

Several respondents drew attention to the nature of the language used in A-level Mathematics. They pointed out that there were a large number of students for whom English was a second language and that such students could easily be put off questions by difficult language, or unfamiliar phrases. One of the aspects we have considered in this study is that of language, in particular, is there a consistent level of usage on the part of the various examination boards?

There was an interesting and varied array of comments at the end of the questionnaire and some of the issues raised, like language, mentioned above, are considered elsewhere in the study. The teachers contributing to the questionnaire took considerable time and trouble over their responses and looked at A-level Mathematics in a number of different ways. There were useful ideas for changes that could be made to improve the product, and attention was drawn to a number of problems that have arisen due to recent changes in the educational system, such as GCSE. Warning bells were also sounded about some of the difficulties that are likely to ensue from the implementation of the National Curriculum.

8. University Interviews and First Year Students' Results.

In this chapter we shall examine the views of university mathematics lecturers concerning the content and variety of A-level Mathematics. These views are based on the opinions expressed in a number of interviews conducted by the author at a range of university mathematics departments. Five universities were able to contribute some data, which will be analysed later in the chapter, relating students' A-level results and board with performance in first year university examinations. Some of the departments visited did not have the relevant data, whilst some were unable to provide it as a matter of policy. The five departments, which did contribute data, covered a wide geographical area (North, Midlands, West and South). They each had a well-established and sizeable first year Mathematics class.

8.1 Views of Oxford and Cambridge Tutors.

The author was also able to speak to Fellows at some Oxbridge colleges. At Oxford and Cambridge the Mathematics students, almost without exception, had A grades at A-level, which meant that there was little, or no, detectable variation in A-level performance. Selection at Oxford is based largely on the results of the Fourth Term Entrance Examination, while at Cambridge most students have to gain A grades in Mathematics and Further Mathematics A-levels and achieve a reasonable standard on Sixth Term Entrance Procedure (STEP) papers in Further Mathematics. Recently, Cambridge has introduced a scheme whereby students who have only taken A-level Mathematics (because their school does not offer Further Mathematics) can be admitted to the Mathematical Tripos. Such students have to demonstrate considerable potential in an interview. Currently about

12 out of 250 mathematics students are admitted in this way. The author's impression, from talking to Fellows, was that they much preferred the students to have taken Further Mathematics at school and that the extra provision was for exceptional students, who would clearly benefit from taking the Mathematics course at Cambridge, which was regarded as the most demanding course of its type, certainly in this country, if not the world. The entry criterion here was that the student had to satisfy the selectors that they had the ability to cope. If they could demonstrate this then the A-level background was not regarded as too important. This was clearly different from the system that operated elsewhere.

To some extent, the relaxation of entry requirements at Cambridge was a response to Oxford's Fourth Term Examination. This examination was a sort of pre-emptive strike. Oxford thought that they were getting the first bite at the cherry. Now Cambridge was realising that some of the ablest students were indeed being acquired by Oxford. The Oxford examination was attractive to pupils because they would know the results before they took A-level. The examination was set with a view to attracting pupils from the State System to apply to read Mathematics at Oxford, because it didn't demand that the student should be taking Further Mathematics, necessarily, or have deliberate coaching for the examination (as is frequently the case for pupils from independent schools). At both Oxford and Cambridge, as well as at other universities, staff told the author that the well-groomed independent school pupil might well not do as well as one might suppose. The student who had learnt to work things out for his or herself at A-level was acquiring the skills that would be most valuable at university.

8.2 General Information from University Departments.

At each university the initial contact was made with the admissions tutor. They were often keen to promote their department to a potential supplier of students, as the author was viewed. They usually provided details of their course and copies of the material which they sent out to students before starting the mathematics course. This invariably involved a breakdown of the first year course; e.g.:

1/2 unit	Mathematical Methods and Computing
1/2 unit	Applied Statistics and Data Analysis
1/2 unit	Introduction to Statistical Methods
1/2 unit	Probability with Statistical Applications

Plus one half unit course chosen from:

Biology
Computing
Economics
Statistical Project
History and Philosophy of Science.

Units would be briefly described; e.g. :

"Algebra - The first year work in this subject falls naturally into two parts. First, linear algebra, which develops the ideas of vectors and includes such topics as matrices, linear equations and determinants. The second part deals mainly with the study of groups - a concept of remarkable power and beauty, which has applications to all branches of mathematics, and, in particular, to the study of symmetry and elementary particles in physics".

Potential students, who were only taking one A-level in Mathematics, would seldom know anything of matrices beyond GCSE level (determinant and inverse of 2 X 2). Certainly, AEB, JMB, Cambridge and London Boards, which, as we shall see, are the popular boards, do not include matrices on the A-level Mathematics syllabus. Knowledge of linear equations would be limited to a system of three linear equations with a unique point solution. Only students who were taking Further Mathematics would know of groups.

Prospective students would also be provided with some suggested reading, e.g. books from the Routledge, Keegan Paul Library of Mathematics series, on such topics as Sets and Groups, Linear Equations, Principles of Dynamics. Other more general texts, including titles such as "How to solve it" - G. Polya, "Prelude to Mathematics" - W. Sawyer, "Numbers, their History and Meaning" - G. Flegg, were also recommended.

Most of the universities ran conferences for sixth-formers, giving an introduction to university work. These were perhaps one day events, but in other cases three or four day events at the end of the Lower Sixth Form year, clearly aimed at recruiting the participants in the following autumn.

8.3 A-level Mathematics as Preparation for Higher Education.

After considering this introductory material, we came to the first of our questions posed in Chapter 3. Does the A-level course in Mathematics prepare students for Higher Education? One feature, which nearly all of the lecturers, who were consulted, agreed upon, was that students had to learn to work at a faster pace. In the A-level course, material can be revised and re-tested, so that the weaker student is able to cope. At university, the terms are shorter and there is not the time to go over work again. In order to survive, students have to learn to work faster than they did at A-level. Unfortunately, the pace will be too fast for some students. One Applied Mathematics lecturer complained: "while I am writing on the right hand side of the board, some students are still looking at the left hand side of the board".

In order to address the problem of varying backgrounds, some universities were introducing parallel courses or topics, commensurate with students' abilities and backgrounds. Students, by choosing the appropriate courses, would be able to make progress. At one of the universities, for example, there was an A and a B course in Mechanics. The A course was more challenging and examined the abstract theory more deeply, although the content of the courses was broadly similar. In the university examination A questions carried more marks than B questions.

Many of the lecturers, especially those in Pure Mathematics, felt that students had "narrow horizons" and arrived with misconceptions (based on their A-level experience) about university mathematics. All too often students felt that they would be doing more difficult questions of the same type as they had done at A-level.

At A-level, students would be used to having ideas explained by the use of concrete examples. One Algebra lecturer said : " I am able to begin the abstract algebra course with some concrete examples, not necessarily geometrical ones, but to use too many would be a waste of the available time".

As to what content should be included in an A-level syllabus, there was general satisfaction with the 'Common Core' idea. This guaranteed that certain topics, at least, would be known to all students. Surprisingly, a good many lecturers subscribed to the view: " provided students have learnt to think mathematically in the Sixth Form, it doesn't matter too much what content has been included in the syllabus".

There was a widespread regret at the lack of geometry in schoolwork. The pure mathematicians regarded it as an extremely important vehicle for the teaching of proof. The applied mathematicians felt that many geometrical ideas were directly relevant to mechanics. Geometry was an important unifying factor which related different areas of mathematics to each other. There was a tendency for sixth formers to see various topics, which they were studying, in isolation; e.g. to the sixth former, trigonometry was seen as a separate topic to complex numbers or mechanics, whereas knowledge of trigonometry could be extremely useful in solving problems related to De Moivre's Theorem or projectile motion, for example, or in other branches of the subject.

The lack of understanding of mathematical proof was of special concern to the pure mathematics lecturers, who were spoken to during the interviews. There was fairly general agreement that the decline in the teaching of Euclidean Geometry contributed to the difficulties. This did not mean that if Euclidean Geometry was immediately reinstated that many of the problems would be quickly resolved. Those spoken to felt that understanding the concepts of mathematical proof is a complex subject and is a difficult thing for students to master, regardless of what they have done at the school level. There are definite stages in mathematical development, it was thought, when these ideas make more sense. A good number of the lecturers spoken to felt that it is at the university level where many students appreciate these ideas for the first time. The lecturers regretted, however, that in schools, the pupils did not meet the development of an ordered set of theorems, which were established by logical deductions from already established results.

Some lecturers noted that students came with no knowledge of complex numbers at all, since they are not included in the 'common core'. This was particularly tiresome when some of the class had a very good knowledge in this area, but still had to endure going over the basic material again, albeit rather quickly. It was felt that, as this was a topic that had many applications in physics and engineering, more effort should be made to teach the basics in school.

The author detected, in his conversations, that university mathematics staff still see themselves very strongly allied with physics and engineering. This contrasts with his experience in school, where many A-level mathematics students are not taking physics and are looking for mathematics to help with economics or geography, or take mathematics with two arts subjects. At the end of the Sixth Form, some of these people may well find mathematics is their strongest subject and want to pursue it into Higher Education. If this is the case, then these students will probably take more interest in statistical and numerical applications of the subject, rather than the more traditional view of applications, which still has much support in Higher Education.

Some of the admissions tutors were well aware of these trends at A-level, as they pointed out how much easier it was for sixth formers to get places in physics and engineering departments, than law, psychology and other arts departments at their university.

There was a good deal of criticism of the teaching of certain topics in the Sixth Form. Group theory and linear algebra were the areas most frequently mentioned in this context. It was felt that much of the teaching of groups, for example, did not get off the ground. To be of any use, the teaching of groups should be accompanied by the teaching of rings and fields, so that students get some idea of the 'completeness' and 'structure' of the subject. This could only be done properly at university level. As far as Linear Algebra was concerned, students came with many muddled ideas and did not appreciate the logical development of the subject. One lecturer pointed out the circular arguments that existed in the earlier edition of the SMP linear algebra book, as an example of the sort of poor thinking that goes on.

One analysis lecturer pointed out that when many of his students were asked to prove a result, they immediately thought of mathematical induction as the panacea for all such problems. This, he thought, was because undue emphasis was placed on it at school. Many of the students, he said, believed, quite wrongly, that 'adding the next term' was the inductive step. This was because they had only ever seen mathematical induction used in the context of summation of series. He thought that it would be beneficial to all concerned (staff and students) if sixth formers met proof by induction in a wider context. Proof, he added, should not just stop at this stage, but should include, for instance, proof by contradiction and the use of counter examples.

Pure mathematics lecturers noted, with satisfaction, the investigational type of work that was being done in GCSE. This was encouraging pupils to make conjectures about the things that they were observing, often related to number theory, which was pleasing. All too often, however, these investigations at GCSE level would grind to a halt because pupils did not have the capacity to prove their conjectures. At A-level more progress could be made in this direction, as sixth formers had more tools at their disposal. There was approval for the new syllabuses being published, which allowed students to pursue work in this area at A-level.

An interesting view was expressed by an Algebra lecturer (in fact a Greek). He said that, for cultural heritage reasons, geometry played an important part in Greek schooling. It was wrong to assume that geometry was only for the very able. Many of his contemporaries, with very limited mathematical capabilities, derived much pleasure from geometrical constructions and practical applications of geometry. It was a more worthwhile and rewarding topic for children to study than the meaningless matrix calculations they perform, or the transformation geometry, which can only really be justified, in his view, for those going on to explore the more interesting areas of pure mathematics.

Several lecturers spoke of the need to develop a 'good style' when it came to expressing mathematical ideas. Here again the loss of geometry was mourned. It had introduced students, in the past, to the idea of an axiom, also the idea of rigorous proof and how to set it out. The lack of being able to set out a piece of mathematics was a frequent criticism.

Many of the lecturers complained that students came up to university with an over-reliance on the 'formula booklet', which they had used at A-level. This meant that standard integrals and trigonometric formulae, for example, did not flow freely. The SMP formula booklet came in for considerable criticism for being too comprehensive. This meant that when students came to write out the solution to a problem, they required a fact sheet to refer to, which slowed the whole process down.

Regarding problem-solving, there was a general view that many of the problems, which students encountered at A-level, required 'standard' methods of solution. This did not encourage students to think. It was important that students should be able to develop strategies for solving problems. One lecturer said:

> "When confronted with a problem, it is important to have a systematic approach to dealing with it. Many students think haphazardly, or even randomly!"

In attempting to answer the question as to whether A-level prepares a student for Higher Education, the responses, made by lecturers, often focussed on the methods of study developed in the A-level course, rather than the details of what was included, or not included, in a particular syllabus.

"The whole question of having the correct mathematical attitude is of fundamental importance", was the view expressed by one lecturer. He added: " I get too many people asking me, 'if this topic or that topic is in or out of a syllabus, would it make the situation better or worse?'. Such considerations are quite irrelevant, until you have the correct foundations on which to build your syllabus. I see my task as 'getting students to think mathematically' ".

One analysis lecturer said :

"If you are going to comprehend the material, then you must expect to re-read your lecture notes, possibly several times, in order to gain complete mastery. Students, on the whole, when they come, do not expect this to be so."

Most lecturers, who were spoken to, felt that A-level students had not developed good habits in the use of libraries and textbooks. These attitudes still prevailed when the students arrived at university. In part they thought that this was excusable, since notations varied from book to book and were likely to confuse students. Purchasing books was prohibitively expensive, so they only required a student to purchase a book if it was to be used extensively during the course. Many lecturers said that they aimed to make their lecture notes comprehensive, so that students would have little need to refer to library or textbooks, in order to survive.

8.4 Present Preparation v. Past Preparation.

The second question posed was whether students were as well prepared today as they were some years ago.

Many of the responses to this question concentrated on the issue of students taking one or two A-levels in Mathematics. (Whether in the form Mathematics and Further Mathematics or Pure Mathematics and Applied Mathematics.) As we shall see, from the data collected and analysed later in the chapter, where two A-levels are taken, the Mathematics and Further Mathematics option is much more popular.

A generation ago (say thirty years) it would have been rare for a mathematics undergraduate not to have taken two A-levels in mathematics prior to starting on the university course. Today, although most lecturers said that they would prefer students to have taken two A-levels in mathematics, reluctantly, they said, they had to accept people who had only done one. The main advantage of doing two was that students gained a greater maturity and dexterity in the use of mathematical tools. The content of the second A-level was not so important, as much of the work would be covered in the first few lectures, albeit quickly. This work was covered in the early lectures because one could not assume that everyone had done the second A-level. Some lecturers said that it was not necessary to do the second A-level, because one could not assume that it had been done by everyone, so they would prefer everyone to start at a common level. The advocates of the second A-level were keen to stress that more than one other A-level could be taken with the two mathematical ones. They felt that students who were bright enough to cope with an honours mathematics course were able to take four A-levels in their stride. This would then counteract the claim that most double mathematicians were following too narrow a course.

Surprisingly, the majority of lecturers were not fully aware that there are two distinct interpretations of the word 'Further', when referring to Further

Mathematics A-level. In the various Further Mathematics examinations, the Boards appear to operate two different philosophies. Firstly, one can regard 'Further Mathematics' as extra work to the single subject 'Mathematics', but not harder work. Where Boards operate in this manner, their A-level questions examinations in Further Mathematics consist of straightforward questions on the extensions of the syllabus, which they (the Boards) attempt to make relatively no harder than A-level questions on the Mathematics syllabus. Alternatively, the second interpretation of the word 'Further' is that the mathematics involved should not only require additional syllabus coverage, but also that the questions set should require greater dexterity in applying various techniques, particularly in such areas as calculus, algebra and vectors, than was necessary in Mathematics A-level.

The lecturers, who were aware of the differences between A-levels on different examination boards, were usually involved in the admissions procedure, or had made it their business to find out. One of the universities visited had a deliberate policy of not admitting students, with just a single mathematics A-level on London or AEB, to an honours mathematics course. At the other end of the scale, some who knew of MEI, which was not many, were impressed by the knowledge of students who had done that course. They expressed the view that a student with two C grades on MEI might well turn out a better mathematician than someone with A's or B's on another Board.

Over the years, there had clearly been some erosion of the numbers taking double mathematics A-level, but as we shall see, in our data, the numbers are still holding up fairly well. In our sample, the students taking Further Mathematics are usually doing so on what the lecturers and

Croasdale (1991) regard as the more straightforward syllabuses; viz. AEB, Cambridge, JMB and London (the order being alphabetical). In this respect, then, perhaps students were less prepared than they had been in past days.

Lecturers also noted that manipulative algebraic skills were declining, a feature that Sixth Form teachers referred to in our questionnaire, which they attributed to changes in the GCSE syllabus. Lecturers also pointed out the lack of recall, when it comes to trigonometric identities, standard integrals and other similar features. This we have already commented upon earlier and can possibly be attributed to the provision of Formula Booklets at A-level, which were not available a generation ago.

Students today, however, have covered a wider range of topics in their Sixth Form course. They are likely to have some knowledge of differential equations, vectors, linear algebra, groups, numerical methods, computing, probability and statistics. However, knowledge of mechanics, especially statics, will be far less than previously. This is discussed further, under our fourth question.

8.5 Preparation in relation to Examination Board used.

The third question was whether lecturers had detected any differences in preparation which could be attributed to the Board the student had used at A-level. As mentioned above, some lecturers were aware of differences between the Boards, but beyond that they were certainly not aware which students had used which Boards. Indeed, with a first year class of between 60 and 85 (in our sample of universities) it would be impractical for them to have this information at their fingertips. Even the examples classes, or tutorial groups, typically had anything from 12 to 20 students per group, so personal knowledge was difficult. In this respect, the tutorial system at Oxford and Cambridge was far superior, in that the tutor with a group of no more than 3 students, had personal knowledge of the students' background, strengths and weaknesses.

As lecturers' knowledge of the differences between Boards was somewhat limited, they were shown papers from various Boards and asked for their reactions. Some thought that the more searching papers, e.g. those set by the Oxford and Cambridge Board, were good training for an honours mathematics course. Others thought that the most commonly used Board (London in the case of southern universities, JMB in the case of northern universities) was ideal, as it provided a good base for the university course.

One professor, comparing an Oxford and Cambridge paper with a London paper said :

"The Oxford and Cambridge paper is really dealing with elementary principles and dressing them up, in order to show how clever the examiner is. The London paper is testing knowledge of certain basic techniques. I would prefer that people coming to me know about scalar products, how to evaluate them and how to use them, rather than be able to reproduce 'party tricks' which they have rehearsed."

A statistics lecturer, commenting on an Oxford and Cambridge paper, said:

" If I gave these questions to my first year students in their examination at the end of the year, I would have to give them some hints on how to begin."

The admissions tutor at one college of London University was surprised to learn that the London Board Modular A-level was already underway. He said that the University had a body which monitored the various syllabuses and was supposed to keep him informed of such changes. As far as he knew, they had not given him information about the move to a modular syllabus, by the Board associated with their own University. He thought that this was a significant change, because prospective students may already have taken one or two modules in the summer prior to applying to university. This would put them in a far stronger position, when applying for a place in the autumn term of their Upper Sixth year. The double mathematics student might have taken four modules (the equivalent of A-level Mathematics) before applying. Results could be declared if they had done well, but if they had not done so well, then points could be stored and improved upon in later attempts. In this case, the University would have no knowledge of the weak performance, as the student would not have exchanged his/her points for a grade. He thought that the concept of multiple attempts, until the student had the points he required for a certain grade, was shifting the goal posts somewhat. This made it difficult to compare performance on the modular examination with the conventional A-level examination.

For students using a modular A-level syllabus, there was a danger that once modules had been completed, successfully, they would be be mentally 'ticked off' and fall into disuse. One admissions tutor spoke of an interview that had taken place with a potential student who was following the SMP 16-19 scheme. (This is not strictly a modular course, but built up in units.) The student was asked to express a quadratic function as a perfect square plus a constant, in order to determine the minimum point of the function. When he expressed no knowledge of completing the square, it was suggested that he solved the problem by differentiation. To this suggestion he replied that they had done the unit involving differentiation a term or two ago and he had not revised it recently.

The Mathematics Faculty Officer at Cambridge also commented upon the exchange of points for grades by London Board students. In August, when results were known, he said he received the points total for prospective students, but did not know precisely what this meant in terms of grades, or even subjects, without further enquiries, since the modules can be combined in various ways to give A-level subjects. At a busy time of the year, this added an extra level of administration, which, in his view, made life more complicated than it need be.

The phrase 'A-level Mathematics' encompassed four distinct subjects: (a) Pure Mathematics, (b) Pure Mathematics with Mechanics, (c) Pure Mathematics with Statistics and (d) Mathematics - a mixture of Pure Mathematics, Mechanics and Statistics. This led to our fourth question, concerning the presence, or absence of mechanics, or statistics, or both, in A-level syllabuses.

A significant number of lecturers, of all types, expressed concern and dissatisfaction over A-level subjects entitled 'Mathematics with Statistics'. Mechanics lecturers, naturally, thought that all students should have some experience in mechanics. Pure mathematicians found that it was not uncommon for a good grade at A-level to be achieved largely by an excellent performance in statistics, which masked deficiencies in pure mathematics, these very soon became apparent at university. Statistics lecturers felt that the statistics in A-level dwelt too much on what they termed 'cook-book statistics'. This meant that students either had to learn, or were presented with 'a formula', without any mathematical justification. They were then drilled to perform calculations, when presented with data of a certain type. They were then expected to draw very standard conclusions, without any great deal of analysis of the underlying problem.

A statistics lecturer commented:

"In most A-level questions I look at, all the information in the question has to be used. It has a precise place in the calculation, which the students are expected to perform. The presentation of the information is stereotyped, so that it produces a sort of Pavlovian response on the part of the student. Part of the skill in Statistics is sorting out, from the immense amount of data, which information can be useful and which is redundant, then deciding which test, or tests, is/are appropriate. A-level work does not seem to foster these skills. It might be much richer and interesting if this type of task was included."

There were a few statistics lecturers who would much prefer that their students had not done any statistics or probability before coming to university. They said it took a significant amount of time to undo some misconceptions, which students had acquired. In some cases these misconceptions were irreversible. These misconceptions often related to probability, where the university lecturers saw the notions of probability very closely allied to the development of set theory in pure mathematics. This view is diametrically opposed to current thinking and practice at secondary school level, where much 'Data Handling' is being encountered. (National Curriculum - Attainment Target 5.) Recent GCSE papers include many questions on statistics and probability.

Mechanics lecturers, in contrast, thought that some mechanics teaching in school was very useful groundwork. The wide variation in experience of first year students caused them (the lecturers) considerable problems. Some students, they said, had done nearly all the work that would be covered in the first year. This meant that the course had to be presented in such a way that those students, who had covered the work, would not become bored. Lecturers said that they were able to introduce topics in ways which might be unfamiliar to students, or provide slightly more searching problems than the students had hitherto encountered. In some universities there were quite good practical laboratories, with mechanical demonstrations, that would certainly not be available in schools. Some departments had sophisticated computer simulations, which again would not be available in most schools, on such a large scale. At the other extreme, special facilities had to be provided for the students with no mechanics experience at all. This could be in the form of a separate course, as we have mentioned earlier, or by giving extra reading and examples to assist such students in their catching up process. One lecturer described the parallel courses for experienced students and novices at his university. The novices' course, he explained, was rather like stepping stones. In this course the key concepts were picked out. At the end of the first year, students who had covered the 'stepping stones' route, would be in a position to tackle the same second year courses as the other students, who had spent more time elaborating on the key concepts.

Mechanics lecturers were keen that students should gain a greater facility with three dimensional work. Many students, they said, had very limited 'spatial awareness'. Whether this was an irreversible state, or whether it had just been subdued by their previous experience, they were, on the whole, not sure. The majority of A-level work is concerned with two dimensions. A-level students seem to be reluctant to use vectors. The lecturers pointed out that it was as easy to deal with three dimensions as two dimensions, if the students were familiar with vector methods. As one lecturer put it:

"I think that gaining proficiency in handling vectors is a difficult business and it is necessary to go over the groundwork two or three times before mastery is achieved. Once one is proficient, then three dimensional work is just as easy as two dimensional material."

8.6 Views on Project Work.

Many lecturers in applied mathematics thought that it was good for students to have been introduced to the ideas of practical or project work in mathematics, whilst at school. This comes as an adjunct to the question posed, but the views expressed are certainly pertinent. We shall, therefore, consider them at this stage.

The fact that some A-level courses now include coursework, or project work, were especially welcomed by university staff. The introduction of coursework in GCSE, partly as an outcome of Cockcroft (1982) was cultivating useful skills. The discussions ranged over some of the advantages and difficulties of organising project work. At some of the universities it occurred almost exclusively in the third year, except in computing and statistics courses, where it was very much an ongoing thing. The proponents of this method of teaching and learning thought that it encouraged independence of thought and developed such skills as: selecting material and organising time. The final presentation, usually a written account, gave the student a sense of satisfaction and fulfillment. Other features of projects were that they encourage students to formulate problems, model the problem, report on it, then criticise and evaluate the findings. These skills would be of considerable value to the students in their future careers. Some lecturers said that the results obtained in a traditional lecture course and examination scheme were quite different from the results obtained from project work. There was some fairness in this, as students had an opportunity to demonstrate skills in ways not assessed by examinations.

At the first year level, lecturers were in favour of short assignments, which last a few days. These were especially useful to develop and test a student's grasp of the course. The assignment, in first year work, should relate to the lecture course. In such assignments, it was possible to let students know how they were getting on, as the work progressed. From traditional examinations there was little feedback. The assignment could well take the form of an essay of four or five pages, perhaps on the history of some aspect of mathematics. Statistical surveys were also considered appropriate. Such work gave an opportunity for more realistic problems to be tackled, perhaps involving some lengthy computation. It might involve harder or longer problems than could be tackled on an example sheet.

Lecturers felt it was important that marks were awarded for the project/assignment, to encourage students to take it seriously. This, however, was a long job for the supervisor. The traditional lecture course followed by examination was much less time consuming. Assignments were, for the most part, only possible with small groups, not large ones.

One particular use of project work was to teach modelling skills. One of the universities had a compulsory modelling course in its first year applied mathematics. Burkhardt et al. (1978) list the aims of mathematical modelling as:

(i) To develop the use of mathematical and other skills for handling real situations.
(ii) To establish confidence and a sense of professional identity as an applier of mathematics.
(iii) To reinforce and renew mathematical skills and concepts by concrete illustration and practice.

The same authors list the objectives of modelling as:

(i) To develop the use of mathematical and other skills for handling real situations.
(ii) To develop manipulative skills.
(iii) To translate information between different representations.
(iv) To develop communication skills - including written reports and the ability to work in groups.
(v) To interpret mathematical models.
(vi) To develop a feeling for data and experimental design for testing analytical models.

The lecturers, who were spoken to, felt that modelling was often demonstrated, but not undertaken, because again there were problems involving the size of groups and the time that lecturers were able to devote to teaching. To be effective, one could only have four or five students in a group, so that they all participated. One solution would be to involve postgraduate, or even third year students, in the supervisory programme. This could be of benefit to such supervisors themselves. Many lecturers felt that they agreed with the aims and objectives expressed above, but that the implementation of such ideas was prohibitively difficult. The same limiting factors, they suggested, explained why, to date, so little was done in schools, despite the advantages that could be gained from it.

8.7 Entry Requirements.

As discussion with admissions tutors, in particular, drew to a close, attention focussed on what kind of offer they made to students, who were applying to read Mathematics, at their department. Of particular interest was whether the A-level course, which the student was following, had any direct bearing on the offer that was made. As we have mentioned earlier, one department insisted that applicants who were taking AEB or London Board A-levels, should be taking Further Mathematics, or its equivalent. (Pure Mathematics and Applied Mathematics as separate A-levels, or the appropriate modules, if the modular course was being followed.) Generally speaking, the Board, which the applicant was using, was not taken into account, although some preference for the local Board, was again apparent; e.g. London in the South and JMB in the North. While most universities liked applicants to have a second Mathematics A-level, it was not insisted upon. Some departments made offers such as B in Mathematics, B in Physics or Further Mathematics and C in the third A-level. Rather more important, than the Board which applicants were using, in the mind of the selectors, was the ability to attract the right standard of student. They did not want to let prospective students go to universities which they regarded as their competitors. For example, University X said that it was heavily over-subscribed when their standard offer was A,B,C and they were considering making it A,A,B. However, if they did this, they would lose students to University Y, which had a similar standing and reputation, but was maintaining its standard offer of A,B,C or B,B,B.

Some admissions tutors were aware that the Oxford and Cambridge Board syllabus, for example, was more extensive and the questions more searching, than the syllabus and questions of some other Boards. Generally speaking, at any department, only about one or two students each year had taken Oxford and Cambridge A-levels, so data was scarce. However, some tutors did suggest that a candidate with an A and a D, for instance, in Mathematics and Further Mathematics on this Board, might well turn out to be stronger than a student with an A and a B from some other Board. The author pointed out that that this might be the case, whatever the Board, for what was the difference in marks between a top D and a low B? In all probability it was around 10%. This might largely depend on how a candidate fared on two or three discriminatory questions. Performance on these questions could depend on any number of factors, other than the candidate's inherent ability, such as what had been revised most recently, how a teacher, or a textbook, approached a particular topic, whether a similar question had been met on past papers and so forth.

8.8 A-level and First Year University Results in Mathematics.

At the interviews, the opportunity was available for tutors to inspect past papers from the various Boards. Many did so. A number of tutors made comments about the papers, which we have incorporated in our discussion, earlier in the chapter. The most interesting thing to emerge was the widespread lack of appreciation of the differences between the different sets of papers, accompanied by the widely stated view that the difference between the papers was really not too important.

In order to try to assess, quantitatively, if there were any observable differences between the performance of students who had taken one A-level syllabus and students who had taken another syllabus, during the first year at university, admissions tutors were asked if they could supply data about their first year students. A number of tutors agreed to do this. This information, it was agreed, would not identify either the university or the student. For each first year student, information was supplied, showing:

(i) A-level Board.

(ii) Performance in Mathematics and Further Mathematics A-level.

(iii) First year university mathematics result.

This information can be seen in Appendix C.

The data, which we collected from the universities, has a score in the First Year University Mathematics Examination. This we can regard as a dependent variable, as it is influenced by what has preceded it, namely an A-level course, with a certain board, followed by one year's teaching at a particular university. In all we have scores from five universities and A-level performances from five boards (in one case several boards have been grouped together, because of small numbers). We can almost certainly regard universities as being independent of boards, and it is not unreasonable to think of boards as being independent of each other, likewise universities, save for the external examiner system which operates, although at the First Year level this is likely to be minimal. This situation gives rise to multiple linear regression, whereby the dependent variable (first year university mathematics score) is expressed as a linear combination of the independent variables (A-level performance corresponding to universities and boards).

A linear combination is of the form:

$$y_c = a + b_1x_1 + b_2x_2 + \ldots\ldots\ldots\ldots + b_kx_k$$

where y_c denotes the estimated value of the dependent variable, the a term is the intercept term, the x's are the values of the independent variables and the b's are the regression coefficients.

The full details of our model are set out in the following pages.

The calculations required to calculate the regression coefficients and their standard errors will have to be performed by matrix methods on a computer. before doing so, we will consider certain key concepts which underpin the process.

There are three aspects which we need to consider before proceeding to our regression analysis:

(i) Interpretation of individual regression coefficients.

The intercept term represents an overall mean, the student's basic ability, or estimated value if all the independent variables are zero. Any particular regression coefficient represents the change in the estimated university mathematics score, as a result of a unit change in the corresponding independent variable, while keeping all other independent variables constant. So it represents the average increase (or decrease) in the university result corresponding to a unit change in the A-level performance on Board X, for example. hence we can separate out the effect of each independent variable on the university score, free of any distorting influence from the other independent variables.

(ii) Statistical significance of individual regression coefficients.

We will wish to know whether, or not, any regression coefficient is significantly different from zero. If it is, then this will indicate that the corresponding independent variable has an influence on the estimated university score. To achieve this we will calculate, using the t-distribution, a confidence interval for the regression coefficient concerned. If zero is not contained in this confidence interval, it will indicate a significant result, i.e. the independent variable concerned has some effect on the university mathematics score.

(iii) Statistical significance of overall explanatory power.

Using the t-statistic, we can test the individual regression coefficients. However, we would find it useful to know if the regression coefficients for boards, or for universities, taken as groups, were, statistically, non-zero. We might even like to know if the whole model was statistically significant. This can be accomplished by use of the F statistic, which is the ratio of the explained variance to the unexplained variance.

We shall make the usual assumptions concerning independence, normality and homoscedasticity (viz. that the error term has constant variance) and now proceed to the details of our model before performing the relevant calculations indicated above.

A Specific Linear Model

The A-level scores received may incorporate a "board effect" which adds or subtracts something to get the A-level score from some basic "student A-level mathematical ability". Similarly, university exam scores may incorporate a "university effect", which adds or subtracts something to get the university exam score from some basic "student university exam ability". We assume that both of these basic abilities are independent of board and university respectively.

We shall try to express the relation between these two student abilities as a regression. This will enable us to make corrections for both the board and the university.

Therefore let us think of regressing (Student University Exam Score - A university Effect (y)) on (Student A-level Score - A Board Effect (x)). Let us use s for a student label, u for a university label and b for a board label. Let $b(s)$ be the board label for student s and $u(s)$ be the university label for student s. If we denote the university effect by α_u and the board effect by β_b', we wish to regress

$$y_s - \alpha_{u(s)} \text{ on } x_s - \beta_{b(s)}'.$$

The regression equation would look like

$$y_s - \alpha_{u(s)} = \text{constant} + \gamma(x_s - \beta_{b(s)}') + e_s.$$

If we write $\gamma\beta_{b(s)}' = \beta_{b(s)}$ and absorb the constant into, say, the $\alpha_{u(s)}$ term, we have

$$y_s = \alpha_{u(s)} - \beta_{b(s)} + \gamma x_s + e_s.$$

The model equation is thus

$$y_s = \sum_{t=1}^{1+n_u+n_b} X_{st}\theta_t + e_s, \quad s = 1\ldots\ldots n_s.$$

The General Linear Model

Essentially we have an example of the General Linear Model. Suppose we have n independent random variables $y_1, y_2 \ldots\ldots y_n$ and p unknown parameters $\theta_1, \theta_2 \ldots\ldots \theta_p$ $(p < n)$ from which the means $\mu_1, \mu_2 \ldots\ldots \mu_n$ of $y_1, y_2 \ldots\ldots y_n$ are to be constructed by known linear combinations, then the General Linear Model is that

$$y_i = \sum_{j=1}^{p} X_{ij}\theta_j + e_i = \mu_i + e_i.$$

The model can be written, in matrix notation, in the form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e} = \boldsymbol{\mu} + \mathbf{e}$$

where **y** is a vector of responses

$\boldsymbol{\theta}$ is a vector of parameters

X is a matrix whose elements are zeros or ones or values of 'independent variables'; and

e is a vector of random error terms.

The corresponding elements of **X** are chosen to exclude or include the appropriate parameters for each observation; they are called dummy variables, or indicator variables (where 0's and 1's are used). **X** is called the design matrix.

Models of the type $\mathbf{y} = \mathbf{X}\boldsymbol{\theta} + \mathbf{e}$ are called linear models because the signal part of the model, $\mathbf{X}\boldsymbol{\theta}$, is a linear combination of the parameters, and the 'noise' part, **e**, is also additive. If there are p parameters in the model and n observations, then **y** and **e** are $n \times 1$ random vectors, $\boldsymbol{\theta}$ is a $p \times 1$ vector of parameters (to be estimated) and **X** is an $n \times p$ matrix of known constants.

Construction of the Design matrix

The data was received from five universities. The A-level Boards used by the students fell into four main groups, with approximately equal numbers in each group; viz . AEB, Cambridge, JMB and London. The other Boards combined gave a fifth group of approximately the same size as the other four. This fifth group comprised Oxford, Oxford and Cambridge, MEI, SMP and WJEC, which according to Croasdale's analysis were the harder Boards. (This might well explain, to some extent, why they were less popular.) In all we had data on 351 students. A-level performance in Mathematics was awarded points: A = 5, B = 4, C = 3, D = 2, E = 1. Hence a student with A in Mathematics and B in Further Mathematics scored 9, while a student with just a B in Mathematics scored 4 etc.

The first column of the design matrix **X** was the student's A-level score. The second column was a column of 1's representing an overall mean value. The next four columns were for identifying the university of the student. A 1 was placed in the first column if the student attended University 1 and 0's in the next four columns. A 1 was placed in the second column if the student attended University 2 and 0's in the other three columns etc. If the student attended University 5, then -1 was placed in each of the four columns. The final four columns were for identifying the A-level Board of the student. A -1 was placed in the seventh column and 0's in the next three columns if the student had used AEB. A -1 was placed in the eighth column and 0's in columns seven, nine and ten if the student had taken Cambridge etc. If the student had taken one of the Boards from our combined group, then a 1 was placed in each of the last four columns. The fifth parameter for both University and Board could thus be determined from the other four parameters for Universities and Boards, thus avoiding making **X** singular. The matrix **X** appears in Appendix D.

Define $\mathbf{S} = (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T(\mathbf{y} - \mathbf{X}\boldsymbol{\theta})$.

Differentiate **S** with respect to $\boldsymbol{\theta}_j$ to get a minimum.

$$\frac{\partial S}{\partial \theta_j} = 2\sum_i \left(-X_{ij}\right)(y_i - (X\theta)_i)$$

$$\frac{\partial S}{\partial \theta_j} = 0 \text{ for all } j$$

$$\Rightarrow \mathbf{X}^T(\mathbf{y} - \mathbf{X}\boldsymbol{\theta}) = 0$$

$$\mathbf{X}^T\mathbf{y} - \mathbf{X}^T\mathbf{X}\boldsymbol{\theta} = 0$$

$$\boldsymbol{\theta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

Thus the least squares estimator $\hat{\boldsymbol{\theta}}$ for $\boldsymbol{\theta}$ is

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}.$$

This corresponds to an estimator $\hat{\boldsymbol{\mu}}$ for $\boldsymbol{\mu}$ given by

$$\hat{\boldsymbol{\mu}} = \mathbf{X}\hat{\boldsymbol{\theta}} \quad \text{or} \quad \hat{\mu} = \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}.$$

(We shall assume that $\mathbf{X}^T\mathbf{X}$ is non-singular.)

$\hat{\boldsymbol{\theta}}$ is an unbiased estimator for $\boldsymbol{\theta}$ since

$$E\left(\hat{\boldsymbol{\theta}}\right) = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T E(\mathbf{y})$$

$$= \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{X}\boldsymbol{\theta}$$

$$= \boldsymbol{\theta}.$$

We can also work out the variance-covariance matrix $\mathbf{V}$ of the vector estimator $\hat{\boldsymbol{\theta}}$.

$$\mathbf{V}(\hat{\boldsymbol{\theta}}) = E\left[\left(\hat{\boldsymbol{\theta}} - E\left(\hat{\boldsymbol{\theta}}\right)\right)\left(\hat{\boldsymbol{\theta}} - E\left(\hat{\boldsymbol{\theta}}\right)\right)^T\right]$$

$$= E\left[\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)^T\right]$$

Now $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} - \boldsymbol{\theta}$

$$= \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T(\mathbf{X}\boldsymbol{\theta} + \mathbf{e}) - \boldsymbol{\theta}$$

$$= \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{e}.$$

Hence $\mathbf{V}(\hat{\boldsymbol{\theta}}) = E\left[\left\{\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{e}\right\}\left\{\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{e}\right\}^T\right]$

$$= E\left[\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{e}\mathbf{e}^T\mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\right]$$

Now $E\left(\mathbf{e}\mathbf{e}^T\right) = \sigma^2 \mathbf{I}_n$ where $\mathbf{I}_n$ is the $(n \times n)$ unit matrix.

$$\text{Hence } \mathbf{V}(\hat{\boldsymbol{\theta}}) = \sigma^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1} \mathbf{X}^T \mathbf{I}_n \mathbf{X} \left(\mathbf{X}^T\mathbf{X}\right)^{-1}$$
$$= \sigma^2 \left(\mathbf{X}^T\mathbf{X}\right)^{-1}.$$

In order to estimate σ^2:

Consider the set of estimated residuals $\hat{e}_i$ defined by

$$\hat{\mathbf{e}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\theta}}$$
$$= \mathbf{y} - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$
$$\hat{\mathbf{e}} = \left(\mathbf{I}_n - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\right)\mathbf{y}$$

we have already shown that

$$\hat{\boldsymbol{\mu}} = \left(\mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\right)\mathbf{y}.$$

Let $\mathbf{I}_n - \mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T = \mathbf{M}$

$$\mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T = \mathbf{N}$$

$$\mathbf{M} + \mathbf{N} = \mathbf{I}_n.$$

Both $\mathbf{M}$ and $\mathbf{N}$ are symmetric and idempotent.

i.e. $\mathbf{M}^T = \mathbf{M},\ \mathbf{N}^T = \mathbf{N},\ \mathbf{M}^2 = \mathbf{M},\ \mathbf{N}^2 = \mathbf{N}.$

Also $\mathbf{MX} = 0$

$$\hat{\boldsymbol{\mu}} = \mathbf{Ny}, \quad \hat{\mathbf{e}} = \mathbf{My}.$$

Now $\mathbf{M}^T\mathbf{N} = \mathbf{MN} = \mathbf{M}(\mathbf{I}_n - \mathbf{M}) = \mathbf{M} - \mathbf{M}^2 = \mathbf{M} - \mathbf{M} = 0$

$\therefore \mathbf{M}^T\mathbf{N} = 0$

$\therefore \hat{\mathbf{e}}^T\hat{\boldsymbol{\mu}} = \mathbf{y}^T\mathbf{M}^T\mathbf{N}\mathbf{y} = 0$

$\therefore \hat{\mathbf{e}}$ and $\hat{\boldsymbol{\mu}}$ are orthogonal vectors.

Now $\hat{\mathbf{e}} = \mathbf{My} = \mathbf{M}(\mathbf{X}\boldsymbol{\theta} + \mathbf{e}) = (\mathbf{MX})\boldsymbol{\theta} + \mathbf{Me} = \mathbf{Me}.$

Hence $\hat{\mathbf{e}}^T\hat{\mathbf{e}} = \hat{\mathbf{e}}^T\mathbf{M}^T\mathbf{Me} = \mathbf{e}^T\mathbf{M}^2\mathbf{e} = \mathbf{e}^T\mathbf{Me}$

$$
\begin{aligned}
\therefore E\left(\hat{\mathbf{e}}^T\mathbf{e}\right) &= E\left(\mathbf{e}^T\mathbf{M}\mathbf{e}\right) \\
&= E\left[Tr\left(\mathbf{e}^T(\mathbf{M}\mathbf{e})\right)\right] \\
&= E\left[Tr\left((\mathbf{M}\mathbf{e})\mathbf{e}^T\right)\right] \\
&= E\left[Tr\left(\mathbf{M}\left(\mathbf{e}\mathbf{e}^T\right)\right)\right] \\
&= Tr\left[\mathbf{M}E\left(\mathbf{e}\mathbf{e}^T\right)\right] \\
&= Tr\left[\mathbf{M}\sigma^2\mathbf{I}_n\right] \\
&= \sigma^2 Tr\,\mathbf{M} \\
&= \sigma^2\left[Tr\,\mathbf{I}_n - Tr\left(\mathbf{X}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\right)\right] \\
&= \sigma^2\left[Tr\,\mathbf{I}_n - Tr\left(\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{X}\right)\right] \\
&= \sigma^2\left[Tr\,\mathbf{I}_n - Tr\,\mathbf{I}_p\right] \\
&= \sigma^2(n-p)
\end{aligned}
$$

$$
\therefore \hat{\sigma}^2 = \frac{1}{n-p}\left(\mathbf{y} - \overline{\mathbf{X}}\hat{\boldsymbol{\theta}}\right)^T\left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\theta}}\right)
$$

is an unbiased estimator of σ^2.

Thus we have an unbiased estimate of the variance-covariance matrix $\mathbf{V}\left(\hat{\boldsymbol{\theta}}\right)$.

This is

$$
\hat{\mathbf{V}}\left(\hat{\boldsymbol{\theta}}\right) = \hat{\sigma}^2\left(\mathbf{X}^T\mathbf{X}\right)^{-1}.
$$

Assuming $e_i \sim N\left(0, \sigma^2\right)$

$$
\hat{\sigma}^2 \sim \sigma^2\frac{\chi^2_{n-p}}{n-p} \text{ independently of } \hat{\boldsymbol{\theta}}.
$$

while the different $\hat{\theta}_j$ are not generally independent, we see that individually

$$
\frac{\hat{\theta}_j - \theta_j}{\hat{\sigma}\left[\left(\mathbf{X}^T\mathbf{X}\right)^{-1}_{jj}\right]^{1/2}} \sim t_{n-p}.
$$

So we can obtain a confidence interval for θ_j.

The following values were computed:

$\hat{\theta}$	$\left(\mathbf{X}^T\mathbf{X}\right)^{-1}_{jj}$
2.358	0.001
37.885	0.047
-0.712	0.012
-1.682	0.012
1.168	0.015
-0.642	0.015
2.705	0.012
-0.103	0.013
-2.527	0.014
1.908	0.011

$$\left(\mathbf{y}-\mathbf{X}\hat{\boldsymbol{\theta}}\right)^T\left(\mathbf{y}-\mathbf{X}\hat{\boldsymbol{\theta}}\right) = 44\,930$$

$$\hat{\sigma}^2 = \frac{44\,930}{(351-10)} = 131.759$$

$$\hat{\sigma} = 11.479.$$

The following 95% confidence intervals for parameters were obtained:

$$\theta_1 = (1.730,\ 2.986)$$
$$\theta_2 = (32.990,\ 42.780)$$
$$\left.\begin{array}{l}\theta_3 = (-3.171,\ 1.747)\\ \theta_4 = (-4.143,\ 0.779)\\ \theta_5 = (-1.590,\ 3.926)\\ \theta_6 = (-3.404,\ 2.120)\end{array}\right\}\ \text{Universities}$$
$$\left.\begin{array}{l}\theta_7 = (0.268,\ 5.142)\\ \theta_8 = (-2.686,\ 2.480)\\ \theta_9 = (-5.237,\ 0.183)\\ \theta_{10} = (-0.410,\ 4.226)\end{array}\right\}\ \text{Boards}$$

To calculate the parameters for the fifth university and the fifth board, θ_{11} and θ_{12} respectively, we use

$$\hat{\theta}_{11} = -\left(\hat{\theta}_3 + \hat{\theta}_4 + \hat{\theta}_5 + \hat{\theta}_6\right)$$

and

$$\hat{\theta}_{12} = -\left(\hat{\theta}_7 + \hat{\theta}_8 + \hat{\theta}_9 + \hat{\theta}_{10}\right)$$

whence

$$\theta_{11} = -(-0.712 - 1.682 + 1.168 - 0.642)$$
$$= -1.868$$

and

$$\hat{\theta}_{12} = -(2.705 - 0.103 - 2.527 + 1.908)$$
$$= -1.983.$$

To calculate the corresponding variances we use

$$\sigma^2(\theta_{11}) = \sum_{j=3}^{6} \sigma^2(\theta_i) + \sum_{\substack{i \neq j \\ i=3 \\ j=3}}^{6} \text{cov}(\theta_i, \theta_j).$$

Hence 95% confidence limits are:

$$\hat{\theta}_{11} = 1.868 \pm \left(1.97 \times 11.478 \times \sqrt{0.012375}\right)$$
$$= (-0.647,\ 4.383)$$
$$\theta_{12} = -1.983 \pm \left(1.97 \times 11.478 \times \sqrt{0.0133672}\right)$$
$$= (-4.597,\ 0.631).$$

Tests of significance

If a p-dimensional vector $\mathbf{z}$ is multivariate normal with a positive definite covariance matrix $\mathbf{V}$

i.e. $\mathbf{z} \sim N_p(\boldsymbol{\mu}, \mathbf{V})$

then

$$(\mathbf{z} - \boldsymbol{\mu})^T \mathbf{V}^{-1} (\mathbf{z} - \boldsymbol{\mu}) \sim \chi_p^2.$$

In our case

$$\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right)^T \left(\mathbf{X}^T\mathbf{X}\right)\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\right) \sim \sigma^2 \chi_p^2$$

or $\hat{\boldsymbol{\theta}}^T \left(\mathbf{X}^T\mathbf{X}\right) \hat{\boldsymbol{\theta}} \sim \sigma^2 \chi_p^2 \left[\dfrac{\boldsymbol{\theta}^T \left(\mathbf{X}^T\mathbf{X}\right) \boldsymbol{\theta}}{\sigma^2}\right]$

(the non-central χ^2)

hence $\dfrac{\hat{\boldsymbol{\theta}}^T \left(\mathbf{X}^T\mathbf{X}\right) \hat{\boldsymbol{\theta}}}{p\hat{\sigma}^2} \sim F_{p,n-p} \left[\dfrac{\boldsymbol{\theta}^T \left(\mathbf{X}^T\mathbf{X}\right) \boldsymbol{\theta}}{\sigma^2}\right]$

(the non-central F).

So $\dfrac{\hat{\boldsymbol{\theta}}^T(\mathbf{X}^T\mathbf{X})\hat{\boldsymbol{\theta}}}{p\hat{\sigma}^2} \sim F_{p,n-p}$

under the null hypothesis $\boldsymbol{\theta} = \mathbf{0}$.

$\hat{\boldsymbol{\theta}}$	$(\mathbf{X}^T\mathbf{X})\hat{\boldsymbol{\theta}}$
2.357 5931	152 408.5978
37.884 6935	19 467.52393
-0.712 0762	360.400 8414
-1.681 9974	144.107 5735
1.167 7969	-943.930 5849
-0.642 3862	-619.293 5196
2.705 4838	-360.112 7755
-0.103 4927	60.825 80791
-2.526 6001	-94.706 2447
1.907 5747	-2080.806 627

$\hat{\boldsymbol{\theta}}^T(\mathbf{X}^T\mathbf{X})\hat{\boldsymbol{\theta}} = 1\,090\,924.537$

$$F_{10,341} = \frac{1090\,924.537}{10 \times 131.758\,9723} = 827.97$$

Highly significant.

Hence we can reject the null hypothesis $\boldsymbol{\theta} = \mathbf{0}$.

If we wish to test that $(p - k)$ independent linear combinations of the θ_j are zero.

We can express this as $\mathbf{A}\boldsymbol{\theta} = \mathbf{0}$

where the $(p - k) \times p$ matrix $\mathbf{A}$ has rank $(p - k)$.

Now if $\mathbf{z}$ has covariance matrix $\mathbf{V}$, $\mathbf{Az}$ has covariance matrix $\mathbf{AVA}^T$.

Hence $\mathbf{A}\hat{\boldsymbol{\theta}}$ has covariance matrix $\sigma^2\mathbf{A}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{A}^T$.

So $(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T \mathbf{A}^T\left[\mathbf{A}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{A}^T\right]^{-1}\mathbf{A}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \sim \sigma^2\chi^2_{p-k}$

$$\text{or } \hat{\boldsymbol{\theta}}^T \mathbf{A}^T \left[\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T\right]^{-1} \mathbf{A}\hat{\boldsymbol{\theta}} \sim \sigma^2 \chi^2_{p-k}\left[\frac{\boldsymbol{\theta}^T \mathbf{A}^T \left[\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T\right]^{-1}}{\sigma^2} \mathbf{A}\boldsymbol{\theta}\right]$$

(the non-central χ^2).

$$\text{Hence } \frac{\hat{\boldsymbol{\theta}}^T \mathbf{A}^T \left[\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T\right]^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}}{(p-k)\hat{\sigma}^2} \sim F_{p-k,n-p}\left[\frac{\boldsymbol{\theta}^T \mathbf{A}^T \left[\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T\right]^{-1} \mathbf{A}\boldsymbol{\theta}}{\sigma^2}\right]$$

(the non-central F)

$$\text{whence } \frac{\hat{\boldsymbol{\theta}}^T \mathbf{A}^T \left[\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T\right]^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}}{(p-k)\sigma^2} \sim F_{p-k,n-p}$$

under the null hypothesis $\mathbf{A}\boldsymbol{\theta} = \mathbf{0}$.

So taking independent linear combinations of the university parameters θ_3, θ_4, θ_5, θ_6 we use

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{A}^T = \mathbf{M} = \frac{1}{1000}\begin{pmatrix} 12 & -2 & -4 & -3 \\ -2 & 12 & -2 & -5 \\ -4 & -2 & 15 & -5 \\ -3 & -5 & -5 & 15 \end{pmatrix}$$

$$\mathbf{M}^{-1} = \frac{1}{13\,936}\begin{pmatrix} 1865 & 875 & 940 & 978 \\ 875 & 1905 & 852 & 1094 \\ 940 & 852 & 1632 & 1016 \\ 978 & 1094 & 1016 & 1828 \end{pmatrix}$$

$$\frac{\hat{\boldsymbol{\theta}}^T \mathbf{A}^T \mathbf{M}^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}}{(p-k)\hat{\sigma}^2} = \frac{590.93}{4 \times 131.76} = 1.12.$$

Critical value for $F_{4,351} \doteq 2.37$ (5%).

Hence we retain the null hypothesis $\mathbf{A}\boldsymbol{\theta} = \mathbf{0}$.

So taking independent linear combinations of the Board parameters θ_7, θ_8, θ_9, θ_{10} we use

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B}\left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{B}^T = \mathbf{N} = \frac{1}{1000}\begin{pmatrix} 12 & -3 & -3 & -2 \\ -3 & 13 & -3 & -3 \\ -3 & -3 & 14 & -4 \\ -2 & -3 & -4 & 11 \end{pmatrix}$$

$$\mathbf{N}^{-1} = \frac{1}{12\,634}\begin{pmatrix} 1497 & 657 & 659 & 691 \\ 657 & 1453 & 669 & 759 \\ 659 & 669 & 1421 & 819 \\ 691 & 759 & 819 & 1779 \end{pmatrix}$$

$$\frac{\hat{\boldsymbol{\theta}}^T\mathbf{B}^T\mathbf{N}^{-1}\mathbf{B}\hat{\boldsymbol{\theta}}}{(p-k)\hat{\sigma}^2} = \frac{1300.33}{4 \times 131.76} = 2.47.$$

Critical value for $F_{4,351} \doteq 2.37$ (5%).

Hence we reject the null hypothesis $\mathbf{B}\boldsymbol{\theta} = \mathbf{0}$.

The implications of these results will be discussed in the concluding chapters.

9. Sixth Form Student Interviews.

9.1 Reactions to Trigonometry Questions.

The preparatory work prior to the interviews was taken from the same set of 1989 A-level papers as had been used in the 'Readability' experiment and the 'Question Comparison' exercise. It was decided to make up a set of examples on trigonometry. Some of the questions were relatively short, so, if possible, we selected two short questions from a particular Board's papers, or one longer question. The students, who were the same group that took part in the 'Readability' experiment, were given the trigonometry examples sheet a week before the interviewing began. They were not aware which questions came from which Board. To them it was an exercise on trigonometry, much the same as they might find in their A-level textbook. The week gap enabled them to have time to work through the questions; yet it was not so long before the interview that they would have forgotten about the content of the examples sheet.

Trigonometry was selected since the students had spent a good deal of time on this work in the Lower Sixth, so it should have been familiar. It is a part of the A-level syllabus that forms the Common Core, so it is compulsory. Whereas with Mechanics or Statistics questions, some students may avoid them deliberately. Furthermore, lack of knowledge of trigonometry is often cited as one of the shortcomings in students arriving in higher education (see section of this work on interviews with university staff). Thus we had an opportunity to see how these students coped with these questions on trigonometry.

Although the students were unaware of the source of the questions, in the account which follows, we shall look at each board's questions separately. A copy of the questions is to be found in appendix E.

AEB

A good number of the students could do this question completely correctly. Even though the formula for sin 3θ is given in the formula booklet, many derived it using sin (A+B). Weaker students in the middle part of the question cancelled sinθ and did not appreciate that it might have been zero, so they only found the quadratic in cosθ. The general solution also caused some difficulty to the less competent students. The comments of interest made about this question were:

(i) "The instructions are fairly clear - you know where you are going."

(ii) "You need to remember things like sin 2θ and $\sin^2\theta + \cos^2\theta = 1$ otherwise you have a spot of bother."

(iii) "I get put off when I see 3θ. I can just about handle questions with 2θ. If there was a choice, I wouldn't do this question in the exam." (Weak student).

(iv) "Why do they always ask for general solutions?"

The wording of the question seemed satisfactory. The question did not seem to be too long or particularly difficult, judging by the number who got it correct. The comment (i) above indicates that the question was well structured.

Cambridge

Some students ran into difficulty with the first part of the question, because they tried to start with the left hand side and get the right hand side (or vice versa). The more successful students started by trying to find something for cos 3θ and found that by rearranging an intermediate line, before they got to the version in the formula booklet, that they had the required result. Even if they didn't do the first part, it was possible to recover and substitute the right hand side of the identity for the left hand side of the equation. Answers in radians and general solutions caused predictable difficulties.

Part (b) of the question was completely standard and the vast majority could do it perfectly. Comments received included :

(i) "Part (b) was much easier than part (a), why didn't they put them the other way round ?"

(ii) "I have some trouble when they ask for the answer in radians."

(iii) "The first part of (a) is one of those things you either see or you don't. I didn't. You can still get most of the rest though."

(iv) "I thought there was a difference between identities and equations. Shouldn't the examiner have used three lines for an identity."
(Budding pure mathematician.)

JMB

Question (a)

This question could be done by nearly everyone. It was comforting to realise that by the Upper Sixth, the term 'perimeter' was now well understood. The resulting equation with $l = 7$ was again completely standard. Since no method was specified here, some of the Further Maths set were able to use their t formulae to advantage.

The only significant comment on this question was from a member of the Further Maths set, who was pleased that the examiner was not trying to dictate which way the question should be solved.

The lack of structure here did not seem to be a handicap on such a well known piece of work. The question could hardly be termed long or difficult.

Question (b)

Making a sketch of the situation was not particularly difficult. Everyone realised the need for a diagram of sorts. Only the very weakest failed to see that it was the ambiguous case of the Sine Rule, hence they could only find ϕ and not θ . For those who recognised the ambiguous case, it was a straightforward question. Comments received, mainly from weak students were :

(i) "I couldn't do this question because I wasn't sure which rule to use and the question didn't say."

(ii) "I couldn't remember how to get the second value from the Sine Rule."

(iii) "I wasn't sure whether to write the answer as 1 Km , 1.00 Km or 1.000 Km.

(iv) "The question smacks of Jingoism and our colonial past."
(Pacifist student.)

The examiner, in all probability, obviously felt it was not unreasonable that A-level students should know whether to use the Sine Rule or the Cosine Rule, hence the lack of direction in the question. This clearly prevented some students from starting. Perhaps it would have been kinder to say "use the sine rule", because they would still have to work out that the ambiguous case was required. At least some progress could be made in this case. The question was a sensible length and clearly worded, with a (deliberate) lack of structure.

London

Question (a)

The question was very straightforward, since the approximations sin x = x and cos x = 1 - x^2 are given in the formula booklet, so the students only had to substitute and equate coefficients.

Very little comment was made about this question, except for one or two students who asked why anyone should bother to set something so straightforward.

Question (b)

The first part of the question was another example of the completely standard procedure. Nearly everyone could cope with the least value part as well. The only comments received related to how the final answer should be expressed. Was it best to leave it in surd form or as a decimal value? The equation to be solved at the end was an example of the kind the students were very familiar with.

The students were agreed that they knew what to do in both of these questions, since they were well structured and clearly worded. Neither of the questions was thought to be long.

Oxford

The formula cos 2θ was well known. Proving the identity was troublesome for some students, especially those who thought that 4 should be split up as $\theta + 3\theta$, rather than $2\theta + 2\theta$. Errors with brackets and signs caused others to go wrong. Comments included:

(i) "It wasn't too bad a question, as you only needed the double angle formula, which I knew."

(ii) "It was possible to do the rest of the question, even though I couldn't get the identity right."

(iii) "The general solution was nice and symmetrical here - I could manage this one!"

(iv) "Why radians again?"

(v) "Quite a fair test for people who know the work."

Again well structured, but with two integrals to tackle, using the trigonometric identity, it was quite a long question, though not thought to be excessively difficult by the students who managed to get the identity.

Oxford and Cambridge

There were many pitfalls in this question and only the brightest could cope with it at all well. Inequality signs frequently ended up the wrong way round. In part (i), finding the intersection of (a) and (b) to get (c) proved difficult. In part (ii), even though the factor formulae are given in the formula booklet, using them in reverse is not liked by the students, with resulting inaccuracies. Using part (ii) with part (i) to get (iii) was not straightforward. The following remarks were made:

(i) "I get terribly confused when there are inequalities."

(ii) "When you went through it, I could see that the question really wasn"t that difficult. However, I couldn't sort something like that out, particularly in an exam."

(iii) "If you go wrong in this type of question that's it!"

(iv) "I was pleased to get most of this correct. It's a question for the clever people."

The structuring in this question is more loose than in some other questions. The student is pointed in the right direction, but still has quite a lot to work out for himself. For the able student the question is not too long, but for the average student, who writes rather more than is necessary, it is likely to take all of the 25 minutes allowed, possibly more, if he is to make a reasonable attempt at the complete solution. The language used in this question is quite clear and unambiguous.

MEI

The start of the question was down a familiar avenue, but the question turned into a less familiar route as it progressed. Functions and their inverses were a stumbling-block for the less able or less well prepared student. The students made the following comments on this question :

(i) "It was a bit mean not telling you whether α was positive or negative."
(ii) "Why did they have to spoil a nice trig question with stuff about functions?"
(iii) "I can't remember how you tell whether something has an inverse or not."
(iv) "A more interesting question than usual!"

The question was clear and most made a reasonable attempt at it. The fact that an 'explanation' was required was found difficult by the overseas students, who are often not happy if words have to be used, even though they do not have to use many in the explanation here. Since the word 'explain' was used they thought a verbal description was required, rather than a clear sketch with a minimum of words. No one complained about the length of the question.

SMP

Almost everyone realised that 3x had to be written as 2x + x and the tan (A+B) formula was required. The weaker students found difficulty in dealing with the resulting 'double-decker' fraction. The final value was often correctly stated, but without evidence available for the marker to see how it made the left hand side of the identity undefined and how it made the right hand side also undefined.

There was generally little comment about the question except from one or two students who said that they did not understand the term 'undefined'. One of the Further Maths set, prompted by this question, recalled how the binomial coefficients occur in the formulae for tan nx and thought that 'or otherwise' could be inserted, so that complex numbers and De Moivre's theorem could be used as an alternative method. The question was clear and not too long.

WJEC

Part (a) of the question was unfamiliar to the students. The brighter ones were able to deal with the inverse function by applying some common sense. Some students could successfully apply the formula, even though they could not deduce it.

Part (b) was generally done rather better, although the weakest found getting the formula involving cosecants troublesome. When it came to differentiating, even though the question stated what was constant and what was variable, mistakes were still made, including lack of a negative sign, despite being told that there was a decrease! The following comments about this question were noted :

(i) "Part (b) should have come before part (a)."

(ii) "I don't know much about inverse tangents, so part (a) was a non-starter."

(iii) "They would ask you to differentiate cosec something, wouldn't they."

(iv) "Part (b) would have been more interesting if it was given an astronomical setting. It might have been seen to be relevant."

Neither part seemed to be excessively long and progress could be made with later parts, if mistakes were made, by using the printed results.

The students reported that they found working through these questions a useful exercise. There was a general view that trigonometry was an area of the syllabus which required considerable practice on their part. Many of them said that although there was a formula sheet provided, considerable judgement was required in order to know which formula was appropriate for a particular question. There was a good deal to learn, which was not included in the formula sheet, hence they were quite well motivated to tackle these questions.

9.2 Reactions to the idea of 'Board Difference'.

Having explained briefly the purpose of the study; viz. to assess differences in such things as content, style, structure, wording in the questions set by different examination Boards on a particular topic, it was clear that the students had little awareness that these things might vary. This acted as a spur for them, as they visibly began to think more deeply about the matter. A few commented that they had noted, especially in their statistics textbook, that A-level questions from different Boards were used in the last exercise of each chapter. They said that, generally speaking, they tried the Cambridge Board questions, as this was the Board they were taking. They frequently ignored the questions set by other Boards.

Regarding the trigonometry questions on the examples sheet that they had worked through, they said that some questions were quite straightforward; those involving $a \cos x + b \sin x$ usually came into this category. The students regarded this as a sort of 'banker', as it was almost certain to appear in their final paper at some stage, so they usually made sure that they could cope with questions involving this technique. Other questions, notably those involving inverse trigonometric functions and 'factor formulae', they felt were pretty tough.

Generally speaking, the factor formulae were not well known, seldom remembered and few students could apply them in relevant questions. Most said that because they were on the formula sheet they were not bothering to learn them, so that when an expression such as $2 \sin\frac{1}{2}\theta \sin\frac{3}{2}\theta$ cropped up, they were at a loss, even though the question had said it could be expressed as the difference of two cosines. The critical point of adding the angles for X and subtracting for Y, giving $X = 2\theta$ and $Y = \theta$ was not appreciated by the majority. Some of those who got this far still made a sign error, thus obtaining $\cos 2\theta - \cos\theta$ instead of $\cos\theta - \cos 2\theta$.

It was, therefore, clear to the students that some questions were more difficult than others, in terms of the content. Without prompting this was the only difference noted by the students. With some prompting, most students expressed a preference for questions where there was structuring in the question, so that they knew they were on course. They also said that it was helpful if intermediate answers were provided. If they could not get the intermediate answer, it meant that they might still get somewhere with the rest of the question. A number of more astute students made the observation that short questions, where not much guidance is given, can often be the hardest questions.

Several students pointed out that there were some questions which looked as if they had come straight out of the textbook. There were other questions, however, which would never appear in a textbook. These were written in a style peculiar to examinations. The former were, on the whole, quite straightforward. The latter required careful reading, sometimes several times, before the reader could understand what was required. It was pointed out that it was one thing to be able to do questions from the textbook and quite another to tackle these 'examination style' questions. The straightforward type of questions would not be too different from the questions in the summer examination at the end of the Lower Sixth Form year. The examination at the end of the Lower Sixth was testing whether basic material was learnt. The more demanding A-level questions were also testing whether the knowledge learnt could be applied. This made them considerably more challenging. In the students' view this was far more daunting.

As we have said, earlier, the issues raised by the existence of various Examination Boards were not initially of great importance to the students. Having raised the matter, the students had a number of concerns. It now occurred to them, for instance, that the paper they were to sit from the

Cambridge Board, in due course, might well be substantially different in content, style, difficulty etc., from that done by students sitting with another Board. The fact that it might be more difficult was not, in itself, too much of a worry. The students could appreciate that whereas they might require 40% to pass, on easier papers 45% might be required, or on harder papers only 35% might be required. What mattered to them was whether they would end up with the same grade if they were to do the examinations of another Board. If this was not the case, was the school doing the correct Board? Also of concern was whether universities regarded some Boards as more respectable than others. On this point, the author was able to reassure them, from his work described earlier, that, by and large, the Examination Board being used was not of importance to admissions tutors.

On the first of these two points, currently there is really no way of knowing, for sure, if a student will get the same grade on different Boards' papers, since it is not possible to 'double enter', because A-level Mathematics papers are taken simultaneously across the Examination Boards. One therefore has to rely, almost exclusively on cross-moderation studies, as mentioned in our literature review, or assurances from the Boards that comparability is maintained.

Some of the students expressed the opinion that it was important to do fairly challenging work at A-level, as a good preparation for university. (This was mainly the future engineers, physicists and mathematicians, who would require sound mathematics in future.) The majority regarded A-level as the summit of their mathematical career and were chiefly concerned with getting the best possible grade.

The author was pleased that, with some prompting, the students were able to think seriously about some of the difficulties that lie below the surface when students are taking A-level Mathematics. The students were able to voice some concerns that they might only have realised at a much later stage. Whether these concerns are well-founded, and, if so, whether they can be addressed, are matters we hope to be able to shed light on, as we reach our conclusions of this study.

10. Modular Mathematics

During the period of time in which this study has taken place, a significant development in A-level Mathematics syllabuses has occurred. A number of the examination boards have developed Modular A-level syllabuses. Both ULEAC (University of London Examinations and Assessment Council) and MEI (Mathematics for Education and Industry) have produced Modular schemes, which are already working. The MEI scheme is called "Structured Sixth Form Mathematics". SMP 16-19 is a unit based scheme, which, although strictly not a modular scheme, has many similarities to the modular approach. Both the Oxford Board and the Cambridge Board have launched new Modular syllabuses for candidates who will complete their courses in 1996.

These schemes are in their infancy but offer an alternative approach to A-level mathematics, which is noticeably different from what has gone before. It is , therefore, necessary to devote some attention to them in our discussion. In this chapter we shall look at the SMP 16-19 course, as well as the two truly modular courses, which have been operating for some time, viz. MEI and London. We must limit our discussion to these, as schools and colleges have some experience of their operation, whereas, at the time of writing, no schools or colleges have experience of the Cambridge or Oxford schemes. Because of the restrictions of time, manpower and finance, it will only be possible to visit one school or college, which is operating each of the syllabuses, in the Bedford area. We shall look at each version in turn.

10.1 SMP 16-19

This unit-based scheme has evolved from the SMP Mathematics and Further Mathematics A-levels, which SMP have operated since the mid 1960's. According to the literature describing the scheme, it has been developed at a time when Mathematics is regarded as a difficult A-level subject. The literature points to a high drop-out rate of pupils studying A-level Mathematics, prior to this scheme. It also speaks of the way that A-level Mathematics compares badly with other major A-level subjects, although it does not enunciate in which ways it compares badly.

Certainly it is a well documented fact that the pass rate in Mathematics has been much lower than the pass rate in A-level English, for example. In recent years there has been a decline in entries for A-level Mathematics, of the order of 20-25%. This has been associated with a decline in entries for science subjects, particularly Physics and to a lesser extent Chemistry, with a corresponding rise in entries for arts-based subjects. The closer ties with Europe may well have encouraged some further interest in the study of Modern Languages. Social Sciences, Geography and Economics are also studied more widely. A-level Mathematics can clearly complement these subjects, but frequently students do not study Mathematics with them. A-level Mathematics can, therefore, fare badly against other subjects in terms of pass rate, choice of A-level subject and possibly its intrinsic difficulty for certain sections of the sixth form population.

This difficulty, in part, has been enhanced by GCSE courses, which precede an A-level Mathematics course, according to the SMP advertising literature. The style of GCSE work, including investigational material, is distinctly different from the O-level course, which went before it. Despite this change, there has been little change in the A-level course in acknowledgement of this. A-level Mathematics has still been targeted at the pupil who has achieved an A or B grade at the 16+ level. In GCSE courses, there has been less emphasis on manipulative algebra and trigonometrical skills. A-level courses have tended to assume that these skills were still present, but pupils starting A-level Mathematics, after taking GCSE courses, find the transition problematical. This came out strongly in our Teachers' Questionnaire (Chapter 7). The SMP 16-19 scheme has been written so as to continue in the style of GCSE work and thus smooth the transition. The main principles behind the course are, SMP says, as follows:

"Students are actively involved in developing mathematical ideas."

"Premature abstraction and over-reliance on algorithms are avoided."

"Appropriate use is made of technological advances. For example, it is assumed that students have access to a graphics calculator, or computer with a graph plotting facility."

It is hoped that by improving the presentation of material, the pupils will find the work more relevant and understandable. The SMP 16-19 course is intended for students of all ability ranges. The scheme itself builds on the SMP 11-16 course, which has been running successfully for some years. It is stressed, however, that the 16-19 course can follow on from any GCSE course and should be accessible to a student gaining a grade C in such a course.

The author visited a sixth form college, near to Bedford, which was using the SMP 16-19 scheme. It was one of two sixth form colleges in the town. The particular one visited took pupils from a good many schools. Their intake was from a wide ability range, whereas the other sixth form college in the town was quite selective in its intake. Students arriving at the college, which was visited, would have anything from a grade C at GCSE Intermediate Level to an A grade in GCSE and an A grade in Additional Mathematics; (still available, but not for much longer, on some boards). There was a tendency for pupils to avoid early entry to GCSE, with the arrival of the National Curriculum. The presence of Level 10 on the National Curriculum meant that there was some more advanced work, which would be covered in the fifth form (year 11). Pupils were, therefore, delaying entry, studying these more advanced topics and hoping to gain the starred A grade, rather than take GCSE at the end of year 10 and take Additional Mathematics in year 11. At their previous schools they had used a variety of GCSE courses. Many had taken MEG (Midland) or London, although others had used SEG (Southern), SMP, or in some cases NEAB (Northern) GCSE courses. This was not so much a problem nowadays, as the syllabus for the National curriculum was common to all groups. There were, however, slight differences in emphasis, attributable to the various schemes.

As stated earlier, some pupils had taken only Intermediate (Central Tier) Level. These pupils required an introduction to A-level which included algebraic manipulation and trigonometry, in particular, in order to bridge the gap between GCSE and A-level. The sixth form college felt that the SMP course was able to do this very well, as it did not assume too much prior knowledge.

For A-level Mathematics, in the SMP scheme, the students had to take ten units in all. There are eight 'core' units. Any student , taking SMP A-level Mathematics, has to take these eight units.

The diagram below illustrates the scheme :

2. Newton's Laws of Motion

1. Foundations 3. Introductory Calculus

4. Functions 5. Mathematical Methods 6. Calculus Methods

7. Problem Solving

8. Living with Uncertainty

The students then had to choose two further units. There was a wide choice available, in theory. For example, if one chose the Probability unit to follow on from 'Living with Uncertainty', then there were nine possible choices for the tenth unit; viz: The Normal Distribution, Data Collection, Modelling with Force and Motion, Numerical Methods, Differential Equations, Complex Numbers, Matrices, Information and Coding and Mathematical Structure. In all there are fifty combinations for these last two units. With such a choice, representing 20% of the examination, one must ask if the consistency of the final award is affected.

In practice, the college could not offer anywhere near this total number of choices. If they did, they would end up with very small classes. It would also cause insuperable timetabling difficulties. Essentially the students were offered the chance to take two statistics units, two mechanics units, one probability and one mechanics unit, or to join the Further Mathematics class for two of the extra Pure Mathematics units; e.g. the two Differential Equations units, or the Complex Numbers and the Matrices units. This still gave quite a wide choice and required careful planning to ensure that the class sizes were neither too large nor too small and that the resulting options could be timetabled.

At the Further Mathematics level a further ten units have to be covered. The Pure Mathematics units (Mathematical Structure, Matrices, Complex Numbers, Differential Equations) and the Numerical Methods unit are compulsory. The entire scheme has 23 units, so the double subject candidate elects to omit three applied units. Again, in practice, there was only one Further Mathematics class, the college decided which three units were not to be done, so that each of their pupils did the same course. This choice could vary from year to year, depending on the particular interests of the students and the available teaching staff.

The documentation accompanying the scheme is good. There is a textbook for each of the units. The price of each textbook is between four and five pounds. If a student is purchasing a set, then it is approximately £45, compared to the cost of three textbooks for around £30 for a traditional A-level course. The college had bought class sets, which it hired out to students, otherwise the cost would be prohibitive. Each unit has a teacher's guide, again costing around £45 for a complete set of ten books.

The college said that the setting up of the scheme had been a considerable expense. There are useful aids available, in addition, such as the Mechanics Video and software for the statistical units, as well as programs, produced by David Tall, for (i) Real Functions and Graphs and (ii) Numerical Solutions of Equations. These were of exceptional quality, since David Tall has an international reputation for this sort of material. The author was able to see these programs in use on the college network of BBC+ computers. The students said that they found them helpful in their work, as they aided understanding.

The textbooks are well laid out. There are tasksheets at the end of chapters. This is a self-contained piece of work which is used to investigate a concept, prior to to tackling exercises. There are extension tasksheets for the higher attaining students, which investigate topics in greater depth. There are also supplementary tasksheets for the weaker students, enabling them to gain further practice and consolidate basic skills.

Throughout the text there are 'thinking points' and 'discussion points', the former can be sorted out by the student, without assistance from the teacher, the latter are intended for class discussion, with guidance and help from the teacher.

The number of textbooks required by the students seemed to pose something of a problem, especially when it came to revision. The students said that they would prefer to have all the material in two or three books, which they could carry around with them easily. It was not easy to carry ten books around, even though they were small compared to the usual size of an A-level Mathematics textbook. This meant that they might not have the relevant book, when required. The staff did not think that this was an insurmountable problem, as spare copies were usually available in the department. Difficulties over revision were further eased by the existence of a single volume revision text. This covered the units which were examined in the terminal examination. The revision textbook was clear and well presented, giving an exposition of the basic results, together with worked examples. There were further exercises for the student to attempt and outline solutions were given at the end. This was far better than just giving answers, as the students were able to to see how to arrive at the answers. This revision book would be of considerable value to students taking the examinations for other boards, as a high proportion of the material is that from the 'common core'.

The examination procedure consists of two parts. Part One is a terminal examination, consisting of one three hour paper, containing around fifteen questions, all of which are to be attempted. This paper examines the work of five units, viz. Foundations, Introductory Calculus, Functions, Mathematical Methods and Calculus Methods. The number of marks per question range from about four to fourteen. Most questions are broken down into smaller pieces worth two or three marks. The paper is testing the pure mathematics and methods which are covered by the course. The other five units are assessed by centre-based end-of-unit tests. Each of these tests is of one hour's duration. These tests, as their title indicates, are taken once a unit has been completed. There is, therefore, an element of continuous assessment built into the course. The tests often have three or four sections, one or two of which are related to a datasheet, particularly in the statistics units. There is a need to be selective and decide what information is redundant. (This point was made by a statistics lecturer earlier in the study, who was critical of traditional examination questions, where every piece of information had a precise place in the calculations and nothing was redundant.)

There is clearly a danger with these end-of-unit tests that the subject becomes compartmentalised. When a student has completed a unit and done the test, then the material is shelved and rarely considered again. This point was again mentioned by one of the university lecturers, who was interviewed previously. This particular lecturer was speaking specifically about the SMP 16-19 scheme. On the other hand, the end-of-unit test ensures that students keep working through the course. If they know that there is to be a test towards the end of each term, which will count towards their final assessment, then it will be in their own interests to give the unit considerable attention.

The college, which was visited, was certainly in favour of these regular tests, largely for the reason just stated. They recalled the days before the SMP 16-19 scheme was underway, when it was not uncommon for some students, often bright ones as well as lazy ones, to do little work through the lower sixth, do poorly in the 'mock' examination and then have an intense period of revision, or even learning material for the first time, towards the end of the upper sixth year.

Overall, the college, which had been using the SMP 16-19 scheme for about six years, was very pleased with it. They felt that the grades the students were getting at the end of the course were a decided improvement upon those which had been obtained previously, when they were using the Cambridge or AEB A-levels. In past days, many students had found that they had got little more than a D or E grade after two years hard grind. The new system was still hard work, for the majority, but there was a distinct improvement in the number of C grades and a drop in the number of E grades. The overall pass rate was slightly higher. The college estimated that the improvement had been of the order of half to one grade per student.

In recent years there has been revision of the way grades are awarded in Mathematics, so that there are roughly equal numbers in the grades B,C,D,E, whereas before the numbers in each grade would vary considerably, with C's often being noticeably rare. (This had considerable implications for university entrance, where often a C, or better, was required.) It is interesting to note, that the college felt that the students were getting a better reward. This reward, they felt, was more in line with the grades that could be obtained in other subjects, whereas, previously, the grade in Mathematics was often the weakest.

The college was also pleased that the scheme afforded the opportunity for some students, by taking fifteen units, to get AS Further Mathematics, as well as A-level Mathematics. This was useful preparation, particularly for engineering or mathematics at university. Some students were easily able to cope with the basic six units and required a challenge beyond them, without having to do the entire Further Mathematics course.

The author came away with the impression that the SMP 16-19 scheme was well suited to the students at this college. It was able to fulfill their mathematical needs after completing a GCSE course, at whatever level they had taken GCSE Mathematics. The support materials: textbooks, software and videos were expertly produced and of high quality. There was a question mark against the expense of these materials. The Parents' Association had given some help with the purchase of software and computing facilities. Prudent management, involving the hire scheme, had helped to alleviate the cost of the books, so that it was at a tolerable level. The mixture of a terminal examination on the 'core' material, together with centre-based end-of-unit tests seemed to be a sensible balance. This was of undeniable benefit to those students who found examinations particularly stressful. Having gone with a somewhat ambivalent attitude towards SMP, the observer returned with a positive one, which possibly may be a tribute to the effectiveness of the SMP 16-19 scheme, or possibly the effectiveness of the mathematics department at this particular sixth form college.

10.2 MEI - Structured Sixth Form Mathematics Scheme.

The Mathematics in Education and Industry (MEI) Schools project has been running since the early 1960's. Until recently there have been separate A-levels in Mathematics and Further Mathematics, awarded on the basis of terminal examinations. It was felt that the syllabuses for those examinations represented a compromise between the needs of those intending to read the subject at university and those needing particular techniques to support other subjects or their chosen careers. This had resulted in the content of these syllabuses being set at a level which was inaccessible to many, perhaps the majority, of the sixth formers taking them. (It is interesting to note that elsewhere in the study, the traditional MEI examinations come out consistently in the 'hard' category.) The scheme which we are now considering provides the opportunity for schools to tailor what they offer, more directly, to the needs of their students.

Formal examinations can adequately assess certain areas of mathematics. Other areas benefit by alternative forms of assessment. Thus, by having a coursework element built into the scheme, assessment could involve the use of computing facilities in numerical analysis or statistics.

Topics such as problem-solving and modelling are difficult to assess by examination, since the time required is often greater than that allowed in an examination. As a general principle in this scheme, components are assessed in a 'manner appropriate to the content'.

The scheme consists of 22 components. These are each reckoned to require 45 hours contact time. There are six components in each of Pure Mathematics, Mechanics and Statistics, together with components in Decision and Discrete Mathematics, Commercial and Industrial Statistics, Numerical Analysis and Modelling in Mechanics. The last four are all internally assessed. The first three Statistics components, the first four Mechanics components and the third Pure Mathematics component (Numerical Methods) all require coursework tasks, in addition to the end-of-component examination. All other components are assessed entirely by the end-of-component examination, which consists of a one hour paper.

The first three components in each of Pure Mathematics, Mechanics and Statistics must be done in order, but the last three can be done in any order, although it is recommended that they are done in the order four, five and six.

The award of A-level Mathematics requires the completion of six components. This must include the first three Pure Mathematics components (the common core), plus two from Statistics 1,2 , Mechanics 1,2 , Decision and Discrete Mathematics, together with one further component. This final component can be chosen freely, subject to the rules of dependency, outlined above. Further Mathematics A-level can be gained by completing these six components plus a further six components. If nine components, in all, are completed, then the student gains A-level Mathematics and AS Further Mathematics.

Each component is first marked according to the assessment criteria for the component. A raw mark is thus obtained. This is converted into a component grade, indicated by a lower case letter, corresponding to the capital letters of the grade in the final overall award. The a/b , b/c and e/n boundaries are determined by a qualified analysis of candidates' work, the other boundaries are determined by application of the agreed formula common to all boards.

The raw score is converted to a uniform score, so that each component has equal weight. The uniform scale has a ceiling of 70 marks. Each component grade is assigned a fixed range of uniform scores as follows:

u	n	e	d	c	b	a
0-	10-	20-	30-	40-	50-	60-

For example, if the d category has raw scores between 28 and 32, inclusive, and a candidate has a raw score of 31, then his uniform score is :

$$\frac{(31 - 28)}{(33 - 28)} \times 10 + 30$$

giving him a uniform score of 36. The bonus of 6 marks, within the grade, can be offset against deficiencies in the other components. At the end of the course, the uniform scores for six components are aggregated and a final overall grade awarded as follows:

U	N	E	D	C	B	A
0-	60-	120-	180-	240-	300-	360-

A candidate has to achieve a minimum uniform score of 5 in each of the components.

Candidates may resit a component examination to get a better component grade. Once they are satisfied with the six component grades, they can trade them for a final grade, according to the above table.

In the components which are assessed by examination and coursework, the coursework assesses the particular skills which, by their nature, are unsuitable for assessment within a timed examination. The coursework is assessed over a number of domains according to fixed criteria. The marks for this are added to the marks for the examination and grade boundaries are determined on the total mark. The coursework is assessed by the teacher responsible for that component.

Teachers are allowed to give guidance and instruction to the class as a whole, so that they know what is required. The teacher can also explain the basis for assessment. Teachers can answer reasonable questions and discuss candidate's work with them, up until the point where they begin their final write-up. Candidates may discuss the work amongst themselves. In some cases, where experiments are involved, they may work in groups of two or more. The final write-up, however, must be the candidate's own, unaided, work. If verbal communication is one of the assessment domains, this could take the form of a talk to the class, or an interview with the teacher.

It is permitted to base coursework on work in other subjects, but in this case, a separate write-up for Mathematics must be presented.

The internally assessed components are assessed over five domains: Principles, Techniques, Argument, Strategy and Formulation. In each of these categories, a mark between 0 and 5 is to be awarded, according to a checklist of criteria. This gives a maximum score of 25 for each task in the component. The component consists of three, four or five such tasks. Grades are awarded as follows:

	Total Possible Marks		
	75	100	125
a	64	85	107
b	53	70	88
c	42	55	69
d	30	40	50
e	19	25	32
n	8	10	13
u	0	0	0

(These represent the minimum marks for the stated grade.)

A sample of 15-20% of the scripts should be moderated within the school or college. All coursework has to be available, on request, for moderation by the Board.

The author visited a medium-sized comprehensive school, close to Bedford, which had been operating the above scheme for a period of four years, so two groups had sat the final examination. Previously they had used the Cambridge Board A-level Mathematics course. Their opinions of the scheme, to date, had been quite favourable. The results they had achieved thus far had been broadly in line with their expectations.

In each year of the sixth form there were approximately forty to fifty pupils taking A-level Mathematics. In each year about half a dozen would be aiming for Further Mathematics A-level as well. The school hoped that the students had taken Higher Level (Further Tier) GCSE and had achieved at least a B grade on it, prior to starting the A-level course. The majority had done so. However, there were some students who only had a C grade, some of whom had only achieved a C grade on the Intermediate Level (Central Tier) papers. The head of department explained that there were inevitably those staying on in the sixth form who only had a collection of C grades at GCSE. If, for example, they were taking a scientific or technological subject, then it was sensible for them to try to study some more mathematics. Such candidates clearly faced immense difficulties in achieving an E grade in A-level Mathematics. The modular scheme was helpful for such students, as they could progress at a rate suited to them. It was possible for them to be monitored more closely, as the course progressed, rather than finding that they had failed after a two year struggle. Some of them could do quite worthwhile pieces of coursework, to supplement their rather weak examination performance, which was a further reason for using the modular scheme.

At the other end of the spectrum, the Further Mathematics class, which had left the school the previous term, had been able to extend their knowledge considerably. The class had contained two bright students, who had gone on to study Mathematics at Oxford. They were now reporting that the work covered in the sixth Pure Mathematics component of the MEI Modular Scheme, was well beyond what most of their contemporaries had studied at school. These two had, in fact, studied all five options of the sixth component, although only three are required for the examination. The five components are: Limits, Partial Differentiation, Vectors and Matrices, Differential Geometry and Abstract Algebra. This meant that they were familiar with the Introductory ideas for much of their first year work at university. The teacher who had been taking them explained that there was considerable difficulty in finding textbooks for the sixth Pure Mathematics component. The books available either did not go far enough, or were university texts which went into too much detail. She had resorted, in the end, to using her own university notes and had taken selected examples from more advanced textbooks. The process had been difficult and much of the extra teaching for these two students had taken place at lunch-time, or after school, as they had gone well beyond what the rest of the Further Mathematics class had achieved.

There was a general problem with textbooks for this course. Unlike the SMP 16-19 course, there were none specifically available, when the school began using the MEI scheme. Now, at the start of the fifth year, there were books available covering Pure Mathematics 1 and 2. It was expected that further books would appear, which covered other components. It was thought unlikely, by the teachers, that books would be produced for the more advanced components, as the number of sales would be too small to make it a viable commercial proposition. The books, which had been produced, had good exposition of the material and were well written and interesting. They did, however, lack examples on which pupils could practice skills. Further supplementary material was required, in addition to these books. The school overcame this, to some extent, by the use of past examination papers.

Generally speaking, the school entered all their pupils for the same components. Of the two cohorts who had completed the course, one had taken three Pure Mathematics components (compulsory), two Statistics and one Mechanics components, while the other had taken two Mechanics and one Statistics component in the optional part of the course. The latter group had a predominance of physicists, who were keen to do extra mechanics.

The school had been encouraged by the students' efforts with the coursework involved in Pure Mathematics 3 and the Applied components, to the extent that they were considering attempting the Numerical Analysis component, which was entirely internally assessed. This they proposed to do instead of either Statistics 2 or Mechanics 2. They were considering allowing a choice for the sixth component between Statistics 2, Mechanics 2 or Numerical Analysis, but this could lead to rather unbalanced teaching groups, so they were unsure of the wisdom of this course of action.

The assessment of coursework, to date, had not been either so time-consuming, or as daunting, as some of the staff had anticipated. Within the department, there was a majority for the coursework, which had persuaded the school to adopt this particular modular course. There were some sceptical members of the department, who had come to terms with it. They were impressed by the clear instructions given for awarding marks under the various headings. The results, they maintained, were fairly accurate reflections of the pupils' ability. To illustrate this, they gave the observer two examples of work which had been completed on the Newton-Raphson Method. They asked what grade should be awarded for these pieces of work. One was very apparently 'a' grade standard, as calculations were accurate, the work orderly and well-presented, aims, results and conclusions were clearly visible, even after only a few minutes inspection. The other piece of work was of a moderate standard, there was evidence of some achievement, but there were gaps and explanations were somewhat vague and incomplete. The observer rated this as 'd' grade. The staff then revealed their scores of 23/25 and 12/25 , which were commensurate with the observer's grading, after only a few minutes perusal of each piece of work. The staff said that one could get a good idea of the level of a piece of work very quickly. The marking procedure mostly confirmed one's initial impression. Not all calculations had to be laboriously checked, it was sufficient to take a sample of calculations and sample further if there was some doubt in one's mind.

The author was able to sit in on a number of classes. One was an upper sixth mechanics class. It was towards the end of their first term in the upper sixth. They had taken the first Mechanics component in the previous summer, at the end of their lower sixth year. They had obtained mixed results, so now they were revising for a second attempt. They said that they were somewhat unsure what to expect in their first attempt at the component. They felt much more confident now, as they had started work on the second component. Now they were returning to work on the first component, it seemed much more straightforward. They were pleased to have the opportunity to improve their grade. They thought it would be a good idea if they could give universities some idea of how they were progressing with their A-level course. Under the old system, decisions were made on the basis of GCSE results and what the school wrote about them, neither of which, some admitted, would be all that flattering. If they could say "I have a 'b' grade on Pure 1 and a 'c' grade on Mechanics 1", for example, it would be useful. The students also thought it would be possible for them to indicate to the university if they thought that they could improve on these grades. The students added that they thought that they would be able to cope better by spreading the examination load more widely. If they could get good results on two of the components, at least, before the end of the course, they would be left with, at the most, two examination sessions, each of two hours duration. This, in their view, was a distinctly better prospect than two three hour papers at the end of the course.

The students liked the coursework element in their work. They thought it improved their understanding of the topics. They also felt they could gain some credit, under less pressure than they experienced in examinations.

The school thought that the 45 hour duration for each component was about right. Most pupils, they expected, would take at least two components by the end of the lower sixth. The Further Mathemtics set might even take five or six. So far they had not experienced too much difficulty getting through the course.

The problems of textbooks were really their only source of concern. We have already mentioned the lack of books, specifically written for each component. In addition to this, the teachers felt uneasy about buying, for example, a standard A-level Statistics textbook, costing about ten pounds, if the students ended up only taking the Statistics 1 component. In this case, the students would only be using about one third of the textbook, if that. Shorter textbooks would be more suitable for a GCSE Statistics course. The teachers thought, in general, that these shorter books would be unsuited to an A-level course, as they lacked depth and attention to detail.

The author asked if the school had considered entering some of their brightest fifth formers for Pure 1 at the end of the fifth form year. The school thought that the new National Curriculum offered plenty of scope for keeping the bright fifth-former occupied. They knew that some schools, who take the MEI scheme, had entered fifth formers for Pure 1, having taken GCSE early. They had heard that the results had not been encouraging. Although the content of Pure 1 was broadly similar to what one would have found in Additional Mathematics, the level of question was of A-level standard. This had proved too difficult for fifth formers. The school had thus been deterred from going down this avenue, preferring to give their students greater exposure to A-level teaching and ideas, before entering them for public examinations.

The overall impression of the MEI scheme was that it was essentially the old A-level syllabus broken down into more manageable chunks. This enabled the student, and the school, to select and mix components, in a way which suited their needs. The added dimension of coursework appeared to have more general approval than the author might have supposed. The outcome of this coursework also seemed to be of a high standard. The scheme seemed to be able to stretch the most able, which was in line with the traditional view of MEI, which university lecturers had commented on previously. There was a worrying lack of the sophisticated supporting materials that SMP had. This could well prove to be a hindrance to the widespread adoption of the MEI scheme. The author did see a list of the schools which were participating in the scheme. It was certainly more extensive than the MEI had attracted before the advent of the modular scheme. The list showed a greater number of schools of all types. Previously the clientele for MEI examinations had been largely drawn from selective independent schools. It was, therefore, pleasing to see the MEI modular scheme operating efficiently in a comprehensive school. The facility to take examinations as components were completed, with the attendant possibility of retaking to improve grades, appeared to be attractive to the students, and, therefore, a worthwhile exercise. This must inevitably increase the amount of administration, setting of papers, marking of the same and grading for the Examination Board. Hence, one wonders, in view of this considerable increase, whether the same standards of rigour, accuracy and checking of results are applied to the modular scheme, as were applied to the old A-level. The school visited seemed to be generally well satisfied with the results that had been achieved thus far. One note of caution was expressed, however, by a member of staff, not in the Mathematics Department, who said that the considerable number of modular examinations, which were currently being taken, were disruptive to school life. He had recently been experiencing difficulty in raising sports teams, because many of the team were either taking modular examinations, or revising for them. The modular exams were not just in one subject either. Hence a pupil taking Mathematics, Physics and Chemistry, might well be sitting many papers throughout his time in the sixth-form, as each of these subjects were using a modular system.

10.3 London Modular Mathematics

The London Modular scheme, in contrast to the SMP 16-19 unit scheme and the MEI Structured Sixth Form Mathematics Scheme, had less modules. There are thirteen in all. These are four pure mathematics modules

(P1,P2,P3,P4), four mechanics modules (M1,M2,M3,M4), four statistics modules (T1,T2,T3,T4) and one decision mathematics module (D1). For the award of Mathematics A-level, four modules must be taken and these must include P1 and P2 (the common core). There are five possible combinations :

P1,P2,M1,M2	P1,P2,T1,T2	P1,P2,M1,T1
P1,P2,M1,D1	P1,P2,T1,D1	

A statistics project is included in the assessment of T2. This is 20% of the total mark for that module.

The modules are assessed by one and a half hour written papers, which are set twice a year. Candidates are awarded a number of points between 0 and 12 on the basis of these examinations. The points map onto grades as follows :

0	1	2	3	4	5	6	7	8	9	10	11	12
U	N	N	E	E	D	D	C	C	B	B	A	A

A total score of 44+ is thus required for an A grade, a total score of 36+ for a B grade etc., over the four modules.

AS- level Further mathematics can be obtained from:

P3,M3	P3,T1	P3,T2	P3,T3
T3,T4	P3,D1	P3,P4	M3,M4

A-level Further Mathematics can be obtained from:

P3,P4,M3,M4	P3,P4,M1,M2	P3,P4,M2,T2
P3,P4,T1,T2	P3,P4,M3,T1	P3,P4,T3,T4
P3,P4,T1,D1		

The school visited was an independent school in London, with a mixed sixth form. Previously it had been using the London A-level Syllabus B, which was the present modular scheme's predecessor. The school said that there had, therefore, been little problem with continuity, as the contents of P1,P2 , M1,M2 and T1,T2 were much the same as the former two and a half hour papers in Pure Mathematics, Mechanics and Statistics respectively. The multiple choice papers had disappeared, which they were rather pleased about. Furthermore, the availability of P1,P2,M1,T1 meant that students no longer had to choose between mechanics and statistics, but could do something of each. (We shall comment further on this particular combination later.) This enabled the school to arrange the sixth form sets more easily, as all students were taking the same course now. Previously, problems could, and did, occur when, in addition to the Further Mathematics set, they had two and a half sets wanting to take mechanics and one and a half sets wanting to take statistics.

In 1996, when the first statistics module is to be re-titled T1, having previously been S1, a project ceases to be part of the assessment of that module. Some of the teachers thought that this was against the spirit of statistics, but recognised that it would reduce the load on the students, also on themselves. From 1996, only the Further Mathematics set, who were taking the second statistics module T2, would be required to produce a project.

There was fairly general agreement among the teachers that the new examinations constituted a lighter load. This was welcomed to the extent that the staff thought that previously there had been a mis-match between GCSE and A-level. Students had experienced considerable difficulty in coming to terms with the old A-level course, after a GCSE course, where there had been comparatively little emphasis on manipulative algebra and geometry was minimal. The content of M1 and T1 is of quite a basic nature. This now represents 50% of the A-level course. Staff wondered how university lecturers would react to students, with A-level Mathematics, arriving at university, not having covered Hooke's Law, Circular Motion, Newton's Experimental Law and the Work-Energy Principle in greater detail (i.e. including elastic potential energy), all of which are included in M2, which now only the Further mathematics set covers. True they might encounter them in A-level Physics, but the staff still felt that the mis-match was now being shunted from the GCSE/A-level interface to the School/University interface. It also raises the important question as to whether, or not, P1,P2,M1,T1 is as difficult an option as either P1,P2,M1,M2 or P1,P2,T1,T2, all of which may be done for the award of A-level Mathematics.

The school was used to using the Bostock and Chandler textbooks, which were written with the London Pure Mathematics and Mechanics specifically in mind. These they had continued to use, although the purchase of Mechanics 1 for the M1 module was rather excessive. They used 'Advanced Level Statistics' by Crawshaw and Chambers for statistics, again this went much further than was required for the T1 module. A new set of textbooks, one for each module, was being produced by Chief Examiners and others from the London Board. These clearly covered the necessary work well and had plenty of examples of the right standard. The school thought that they would almost certainly adopt these books in the future.

The school had previously entered some fifth formers for P1 at the end of the fifth form year. These pupils had taken GCSE at the end of the fourth form. Prior to this, such pupils would have taken Additional Mathematics in the fifth form. Taking P1 at the fifth form level had not proved successful, in terms of points awarded. The school said that the results were disappointing. The students, they said, had benefited by being exposed to the material at that stage. They had gone on to be highly successful at the end of their upper sixth year. The problem seemed to be in applying specific skills to A-level style questions, at the stage when they did not have the requisite amount of mathematical maturity. They did not see, in many cases, what technique was required for a particular problem. Had the problem been phrased more directly, or in similar language to their textbook, they would have been able to show what they knew. The school had decided, therefore, not to repeat this entry policy.

Students at the school, particularly if they intended to pursue mathematics, physics or engineering courses were encouraged to do P3 and M2 for AS Further Mathematics. These pupils were mainly in the top set of those sets which were aiming for single-subject A-level Mathematics. Previously the option of AS Further Mathematics had not been available to such students. This was a definite advantage of the modular scheme, because there were always a number of pupils who were not stretched by the single-subject course. These pupils were either not able enough to take Further Mathematics A-level, or did not want the extra load of what was nearly always a fourth A-level (staff thought that the latter was nearly always the case). We have already referred to the items in M2 which the engineers and physicists would find useful. The third pure mathematics module contains: Inequalities, Finite and Infinite Series, Further trigonometry, Hyperbolic Functions, Further Integration, Vectors (including the scalar product), Simple Complex Numbers (excluding De Moivre's Theorem), Differential equations (up to second order linear) and Numerical Methods (including the Newton-Raphson process). This opportunity to do some extra work, without taking a complete A-level, was welcomed by both staff and pupils.

In addition to the AS course, there was one set in each of the upper and lower sixth years which was working towards the double-subject. The Further Mathematics set had a project to do as part of T2. The project had to be relevant to the work of the T1/T2 syllabus. The project was expected to take about 20 hours. It was to include data collection, which could be derived from experiment, observation, questionnaire or simulation. Consideration had to be given to making the data, which was collected, free from bias. Pupils were allowed to work in groups, but the reports had to be their own unaided work.

They could use the data from other subjects such as biology or geography. The report had to include:

(i) Title, (ii) Summary (approximately 100 - 200 words), (iii) Introduction (iv) Description of data collection procedure, (v) Analysis of data - using techniques covered in modules T1 and T2, (vi) Interpretation of results and (vii) Conclusion. The students were encouraged not to regard an inconclusive result as a shortcoming in the work, unless the strategy they were attemting to adopt was flawed. They were also encouraged to make use of computers in their work. Teachers were allowed to give help, but where specific intervention was made, credit was not to be awarded, in order to be fair to students who had not received such assistance.

The assessment of the statistics project in T2 was less rigidly defined than in the MEI scheme. Grade descriptions were given by the Board for grades A,C and E. The teacher, who was assessing the project, had to decide on a grade for the piece. Having decided on the grade, they then had to decide, essentially, whether, within that grade, it was good, moderate or poor. The project could then be awarded a mark between 0 and 25 as follows:

U	N	E	D	C	B	A
0-6	7-9	10-12	13-15	16-18	19-21	22-25

The Mathematics Department felt fairly confident about running this scheme, as it was not radically different from what had gone before. The scheme is less innovative than either the SMP 16-19 or the MEI scheme. There is little emphasis placed on the modelling aspect, and little teacher assessment (only the statistics project for one set). The former textbooks were adequate for the scheme and there was the prospect of at least one good new set of textbooks being available, which covered all the modules. It represented a safe alternative among the modular schemes available. It was, perhaps, a little light in terms of syllabus coverage. Combined wave forms in trigonometry, for example, is not covered until P3, so would not be covered by pupils taking single subject A-level Mathematics. The scheme could, however, be enhanced by adding an AS-level in Further Mathematics for the brighter student. The scheme was less unwieldy than the other schemes, having fewer modules. It was less disruptive to the normal school routine, therefore, a) because there were less modules and b) because the modules were only set twice a year, in January and June, rather than at the end of each term, or when modules had been covered. The administrative arrangements associated with the scheme were, therefore, less extensive, which possibly allowed more time for teaching the material in the course. With more modules, more time would be spent in revision for each module and practice papers prior to sitting the module examination.

The general entry policy was to take P1 and T1 at the end of the lower sixth, students could then repeat these, if necessary, in the following January, or even at the end of the upper sixth.

11. Reflections on Observations and Results.

In the foregoing material, we have looked at the various forms of A-level Mathematics in a variety of ways. These ways have, in some cases, been treated quite independently. In this chapter, we shall try to bring the material together. We shall endeavour to see how it relates to the hypotheses posed in Chapter Three. We shall also try to see if there are any unifying factors, or common themes, which can be discerned.

At the time when our study began, it was the case that a student's performance in A-level Mathematics was entirely assessed by the performance in written examinations. As the study draws to a close, this is still largely the case. However, we have seen, in the last chapter, that modular syllabuses are gaining ground. In some cases a student's performance is assessed by work done during the course, or by a project. However, the written examination paper, if not the only form of assessment, remains, by far, the major component of the assessment. The manner in which these papers are produced and presented to the students is, thus, of crucial importance.

11.1 'Readability'

The assessment of the 'readability' of these papers must, of necessity, be one of the key factors in comparing the examination papers. The vehicle which we chose to investigate this was the Cloze Procedure, with its origins in the Gestalt School of Psychology. The emphasis that the Gestalt School places on seeing a thing as a 'whole', coupled with the principle that 'the whole is more than the sum of its parts', was seen as important in analysing text involving mathematical symbols, diagrams and charts etc.. For this reason the actual text from the papers was used in the experiment conducted in Chapter Four. (See Appendix B .) Various Gestalt concepts underpin the Cloze Procedure. Viz:

(i) Continuity - the ability to see something such as a collection of dots, arranged in a straight line or a curve, as a separate unit.

(ii) Similarity - the ability to categorise features with things in common.

(iii) Closure - the ability to insert missing material, e.g. a circle with a small arc missing will be perceived as a circle.

The Cloze Procedure, whereby deletions were inserted in the actual text of questions seemed a better way of dealing with mathematical text than attempting to modify, or produce, 'readability formulae' for mathematical text.

From the experiment conducted in Chapter Four, it was found that the 'readability' of the papers, as measured by our Cloze Procedure, was not of a uniform standard across the various examination boards. We can see this from the analysis of variance (ANOVA) carried out on the results. The only effect found to be non-significant was the student - board interaction effect. This may, possibly, be due to the fact that students reacted to questions, from whichever board, in much the same manner.

The point estimates of variance parameters for the student - topic interaction effect and for the student effect were 0.87 and 13.22, with associated 95% confidence intervals of 0.40 to 1.22 and 6.45 to 16.96 respectively (correct to two decimal places). The student - topic value, being low, again possibly indicates that the students were treating each question, on whatever topic, in much the same way. Because of the individual differences between students, one would expect the student effect to be of a fair size, as was indeed the case.

The point estimates of the "finite population" variance parameters for the board effect and the topic effect were of a similar magnitude to that for the student effect, 15.20 and 17.76 respectively. For these fixed effects, there was no established procedure for calculating confidence intervals for the "finite population" variance parameters. Conservative Simultaneous Confidence Intervals for the effect values themselves were produced for the fixed effects.

The charts, which we have drawn, for estimates of the board effect and the topic effect have a number of interesting features. Few of the 95% confidence intervals stradle zero, although two of the board values are borderline cases. The boards falling in the positive region are AEB, Cambridge, JMB, London and MEI, whilst those falling in the negative region are Oxford, Oxford and Cambridge, SMP and Welsh. This grouping is quite close to that found by Croasdale (Op. Cit.) in his cluster analysis. The first group may, largely, be regarded as the less demanding papers and the latter the more demanding papers, indicated by the positive and negative values respectively. The topics falling in the positive region are the two pure mathematics topics, calculus and vectors. the applied topics projectiles and probability yielded negative values. This would seem to indicate that the pure mathematics questions were generally easier than the applied questions. Two factors may account for this. Firstly, the pure mathematics questions are taken from the 'common core', hence they have a strictly limited set of techniques, which will, inevitably, become familiar to the students. The students can thus respond to such questions in a more automatic manner when responding to the Cloze procedure tasks on pure mathematics questions. The applied questions, in contrast, will show more variation, from board to board, as the syllabus for applied mathematics does not have an agreed body of material. Secondly, the applied mathematics questions are, frequently, considerably more wordy, with, perhaps, lengthy explanations of a physical situation, in the case of mechanics questions, or a description of events, for which probabilities have to be calculated, in statistics questions. The pure mathematics questions, on the other hand, may be much shorter, with, say, an integral to evaluate, or an equation to solve.

A noteworthy feature of the analysis was the high value for the point estimate of the variance parameter for the board - topic interaction effect. This was 30.24, which was considerably higher than any of the other point estimates of variance parameters. This would seem to indicate that boards treat topics in their own individual manner. One of the subjective impressions one gets, when looking at the papers, is that there is a certain 'style' to the papers. This is beyond the obvious 'look' of the papers. Each paper has its own characteristic layout, e.g. size (A4 or A5) and colour of paper, print size and font etc.. The MEI and SMP papers had a deliberate policy of a modelling approach. This meant that their questions tended to be 'wordy', because they had to set up the modelling situation. Oxford and Cambridge had a propensity to introduce more complicated notation than, say, AEB or London, which gave their questions a certain 'aura', although, in fact, they might be examining the same material. This was precisely the point made in the interview with a professor of applied mathematics, who was comparing questions on an Oxford and Cambridge paper with questions on a London paper. He said that, although they were testing the same material, the London question was much more straightforward in character, whereas the Oxford and Cambridge question was 'dressed up', possibly, he thought, to show how impressive the examiner was. In fact the professor preferred the London question, so, if his suspicion was the case, it had the opposite effect to the intended one.

The results indicate a certain degree of variability in the 'readability' of questions, as measured by our Cloze Procedure, which can be attributed to a board effect and, more significantly, to a board - topic interaction effect. This would support our first hypothesis that the 'readability' of the language used in setting papers in A-level Mathematics, by the various GCE examination boards is not of a uniform standard.

An examination of the data, which we collected, reveals a high product-moment correlation coefficient (0.83) between performance in a traditional Mock Examination and performance in our Cloze tests. This is not unsurprising, as able students are likely to score highly in both areas. We note here the correlation reported by Rankin (1970) between Cloze Procedure performance on scientific material and IQ. Performance in mathematics is well known to be positively correlated with IQ. However, there are similarities in the patterns achieved by the weakest set (Set 4) and the most able set (Further Maths Set). These similarities suggest that the effects of 'readability' are apparent at different levels of ability. In our ANOVA calculations, we recall that the student - board interaction effect point estimate of variance (-0.05) was not significant, while the student - topic interaction effect point estimate (0.87), although significant, was considerably lower than other significant point estimates of variance parameters. This would support our second hypothesis, namely that the differences in 'readability' are experienced by students of all levels of ability in A-level Mathematics.

11.2 Paper Analysis.

Turning now to the papers themselves, in Chapter Five, we examined the structure of the papers. In all there were twenty-two possible versions of A-level Mathematics, offered by the nine boards which we have included in our study. This is taking A-level Mathematics to mean what university entrance requirements would regard as single-subject Mathematics. The mere existence of so many versions of Mathematics causes the notion of a uniform standard to be held in some doubt. AEB, JMB and Oxford each had four versions, while London and the Welsh board had three. This was because, in most cases, there was a first paper which covered the 'common core', plus some extra pure mathematics and a second paper, which was mechanics, or statistics, or further pure mathematics, or a mixture. The only exception was where AEB had a syllabus which they still retained from the days of 'modern' and 'traditional' syllabuses. Historically this had been the 'modern' syllabus. At the time of the study the questions on the 'modern' syllabus were little different from the questions on the other papers, in terms of content. This version of examination, however, provided students with the facility to take two papers, both of which contained pure mathematics, mechanics and statistics questions.

On syllabuses where the first paper is pure mathematics and the second paper one of pure, mechanics, statistics or a mixture, it is difficult to see how the second paper can be held to be equal. Certainly the content is different, if one is all mechanics and another all statistics. How does one 'equate', for example, the ability to solve a projectile problem, with the ability to calculate probabilities for a binomial distribution? The examination boards make some attempt to achieve this by including, if possible, some of the questions from the mechanics or statistics paper on the paper which has a mixture of mechanics and statistics questions. This facility, however, is only available on the JMB and Oxford examinations. In addition, this assumes that the candidature for each type of paper is broadly the same. From our discussions with sixth form teachers, we know this not to be the case. The teachers frequently mentioned that the weakest students would take an alternative where statistics figured prominently, because they regarded statistics as an essentially easier option. In our interviews with university admissions tutors, the ability of students who had taken Pure Mathematics with Statistics was frequently questioned. The university tutors said that they had observed students with this background struggling in their first year courses. They much preferred that students had done mechanics. In fact some insisted on an A grade for students having taken statistics, whereas they might admit a student with a B grade if he/she had taken mechanics.

Each of the remaining boards: Cambridge Oxford and Cambridge, MEI and SMP each had one version of their examination. The Cambridge and MEI examinations had a first paper which was entirely pure mathematics and a second paper which contained a mixture of applied topics. Cambridge also include some further pure mathematics topics. The Oxford and Cambridge and SMP examinations consisted of two papers, each containing pure mathematics, mechanics and statistics.

Thus, among the twenty two versions there was considerable variation in the structure of the examination papers. With such a wide variety it is difficult to see how they can all be examining the same body of knowledge. Even supposing it is the same body of knowledge, the method of examining it is by no means constant. We know that in some cases the second paper may be either statistics or mechanics, but if we compare the mechanics offered on the various boards, Cambridge, for example, only examines particle mechanics and does not examine rigid bodies in any way. Other boards will set questions where the centre of mass of rigid bodies is to be determined. Likewise, in statistics, some boards will include correlation coefficients and others will not, some will include hypothesis testing and others will not, or not so extensively. The applied mathematics content of these papers covers a considerable range of material. It is, therefore, no surprise that the lecturers, which we interviewed, expressed the view that there was very little applied mathematics that they could be sure everyone in their class had covered.

Built into many of these papers is quite considerable question choice, which we have indicated in Chapter Five. This can alter the character of the examination quite considerably. Suppose we place ourselves in the position of an admissions tutor, who is considering a candidate, who has taken an A-level in Mathematics which he knows to have included an applied mathematics content. He has no way of telling how much applied mathematics the candidate has actually tackled, by virtue of the candidates question selection. On the SMP examination, the applied content, attempted by the candidate, may amount to as little as 18% of the total mark. In the Oxford and Cambridge examination the corresponding figure could be as little as 29% or as much as 71%. (This constitutes the maximum amount of applied mathematics on any of the syllabuses.) This latter case would seem to contradict the SEAC (now SCAA - Schools Curriculum and Assessment Authority) ruling that at least 40% of syllabus should be the pure mathematics of the 'common core'. The reality is that at least 40% of the questions set by Oxford and Cambridge cover pure mathematics. In fact 50% of the questions are pure mathematics, however, what the candidate does in the examination, because of question selection, is quite another matter. One should bear in mind, also, that candidates are likely to have practiced on many past papers. They will, therefore, know the structure of their papers and concentrate on their favoured sections of the syllabus accordingly. The other sections of the syllabus they may largely ignore. The admissions tutor, therefore, confronted with only the final grade, has considerable difficulty in knowing what the student may, or may not, have covered.

Cockcroft (1982) found it difficult to recommend whether a student should study mechanics, statistics or a mixture of both. He was, however, explicit (paragraph 568) that 'applied mathematics' should be studied. The problem was that both mechanics and statistics take some while to introduce, before one can make significant progress in either. He, therefore, felt that there may not be sufficient time to study both, without making excessive demands on the student.

It would seem that the indeterminate state of A-level Mathematics described above is in no one's best interest. It would seem appropriate that between 40% and 60% of an A-level Mathematics course should be applied mathematics, of which between 20% and 30% should be mechanics and between 20% and 30% should be statistics. This would give rise to structures such as :

	Pure 60%	Mechanics 20%	Statistics 20%
or	Pure 40%	Mechanics 30%	Statistics 30% .

The pure mathematics in the above patterns comprises at least 40% of the whole, not 29% as may be the case in the Oxford and Cambridge scheme.

A pattern between these limits can readily be achieved in the modular schemes which are emerging. In the four module scheme of London, P1,P2, M1,T1 gives Pure 50%, Mechanics 25% and Statistics 25% . In the six module pattern of MEI and Cambridge (which is just beginning at the time of writing), P1,P2,P3,M1,M2,S1 or P1,P2,P3,M1,S1,S2 gives a similar distribution, with a weighting to either mechanics or statistics, while

P1,P2,P3,P4,M1,S1 is just outside the limits proposed above.

It would seem to be sensible to search for agreement, if possible, on what should constitute M1 and S1 in the six module scheme and make this a subset of M1 and S1 in the four module scheme. In this way, universities would have some clearer idea of what had been studied at A-level. If, further, papers had little, or no, choice of question, then one would know that this material had been studied. Perhaps, too, there would be more uniformity in the forms of A-level Mathematics than is visible in the papers which we have been considering.

While we have A-level Mathematics being set by a number of examining boards, there will always be scope for them to set questions in different forms and mark the papers according to different criteria. In the Standing Research Advisory Committee's (SRAC) cross-moderation study, referred to previously, we see that in 1986 A-level Mathematics examinations, the A/B borderline was at 83% on the London Board papers, but at 67% on the Oxford and Cambridge papers. For the same year's papers, the E/N borderline was at 45% on the London papers and 37% on the Oxford and Cambridge papers. This would indicate that the Oxford and Cambridge papers are pitched at a rather higher level - presumably with the aim of testing the most able students. The London papers would offer more scope for the weakest to show what they could do, whereas the most able were getting the majority of what they did completely correct. So, in order to achieve comparability, this factor has to be taken into account.

SRAC asked the boards what they did to achieve comparability between their A-level Mathematics examinations and those of other boards. Most boards said that they used inter-board statistics selectively. For example, they might focus on other boards whose entry they thought was similar to their own, or the type of centres. Some boards said that they used Chief Examiners with experience of other boards. Boards also thought that script scrutiny and inter-board studies were useful. SRAC felt that the boards take the same steps to ensure comparability in Mathematics as they do in other subjects, but questioned whether, or not, this was adequate. Although, they added, that they could not see what else could be done.

The aspects of papers we have considered earlier would seem to suggest that the situation in Mathematics is considerably more complex than in other subjects. Some progress in achieving comparability could result by:

(i) restricting the number of versions of A-level Mathematics,

(ii) seeking some degree of uniformity in the applied mathematics content,

(iii) boards attempting to set papers in such a way that A/B grade boundaries fall in the 75-80% range and E/N grade boundaries in the 40-45% range, for example.

11.3 Question Analysis.

In Chapter Six we considered questions from the various boards' papers, using our modified form of McLone's matrix. Some comments were made on the interpretation of the statistics in Chapter Six itself. We will not reiterate them here. It will suffice to summarise the main points emerging. There was a significant difference between the boards on the pure mathematics questions, which are set largely on the 'common core' material. There was no significant difference on the mechanics and statistics questions. On the pure mathematics questions, London was the least difficult, judging by the smallest total sum of ranks. Oxford and Cambridge, Oxford and MEI (in descending order of difficulty) were the hardest. This agrees with the findings elsewhere in this study and in other studies, which we have referred to.

As regards the headings under which each question was considered, the headings fell into three identifiable groups:

Low Demand : objectives (ii), routine processes (iv), jargon (iii) sustained thinking (ix) and open solution (x) .

Medium Demand : complex mathematical content (v) and level of abstraction (vi) .

High Demand : formula sheet help (xi), powers of reasoning (viii), procedure (i) and mathematical manipulation (vii).

The Low Demand group contains those headings which contribute to the question being relatively accessible and the High Demand group contains those features which make it less so.

The Kruskal-Wallis analysis on the data revealed significant differences between the boards on pure mathematics questions, but not on applied mathematics questions, supporting the findings of the Friedman analysis. The most difficult board again being Oxford and Cambridge, followed by (in order) Oxford, SMP and MEI. London was the easiest, with AEB the next most easy. There is a consistent pattern to these results, which supports our fourth hypothesis in the case of pure mathematics, but not in the case of mechanics and statistics.

11.4 Teachers' Questionnaire.

We next considered the opinions of A-level teachers from schools in Bedfordshire and the Sixth Form Teachers' Conference held at York University. We found that there was a strong regional bias operating in the selection of examination board. Initially the intention was to use just schools in Bedfordshire, but it soon became apparent that the Cambridge Board, which was the local board, was far too commonly used by Bedfordshire schools to give useful comparisons. Hence a wider and more representative sample was obtained by speaking to teachers at a national conference.

Few (4 out of 49) of the teachers, whom we consulted, felt that Mathematics syllabuses, which were available, were of equal difficulty. Opinion was fairly evenly divided as to whether this was because some boards were too hard or some boards were too easy. The main point is that hardly any thought them all to be the same, which is what they are supposed to be. In the easier category we found mention of AEB, London and Modular

MEI, while in the harder category we found Cambridge, JMB, Oxford and Cambridge, MEI (old syllabus), Oxford and SMP being mentioned. We noted that the pattern is similar to that cited by Croasdale (Op. Cit.) and our findings in Chapter Six. The shift of MEI from the harder end to the easier end, by becoming modular was of interest. There was a general feeling that the advent of modular syllabuses would make Mathematics more accessible at A-level.

As to what made the examinations harder or easier, the content was seldom mentioned, except the occasional reference to statistics being an easier option than mechanics. There was a general feeling that there was too much material in any particular A-level course. Things such as the degree of stereotyped questions from one year to another, structure, or lack of structure, in questions and the amount of algebraic manipulation seemed to be the areas mentioned most frequently.

On the subject of grading, about half of our respondents felt that a student would receive a different grade had he, or she, been taking a different board's examination. Most of those who thought that the grades from board to board were not comparable had changed, or were considering changing, boards. This represented a fair amount of dissatisfaction among those people responding to our questionnaire. It is interesting to recall that the Sixth Form College, which was visited in the Modular Mathematics survey in Chapter Ten, felt that the grades were, on average, half a grade to a grade better, per student, on the SMP 16-19 scheme, compared to the A-level syllabuses which they had used previously. What was more noticeable was that respondents, almost unanimously, felt that grades in Mathematics compared unfavourably with most other A-level subjects.

The sample of 49 respondents included teachers using most of the syllabuses, which we have been considering in this study. Most of the respondents had been in teaching for a number of years and many had used syllabuses other than the one that they were currently using, perhaps at a previous school, or because their school had changed syllabus. We have attempted to gain empirical evidence of some of the anecdotal material one comes across at examiners meeetings, meetings of teachers held by examining boards and visits to other schools, in the course of one's teaching. The views expressed in the questionnaire conform, largely, to the views that one commonly hears and support the Fifth hypothesis, which we have made in Chapter Three. It is clearly not a large sample. If we had the time, money and resources it would be interesting to see if the views expressed were held by a much larger sample.

11.5 University Views and Data.

We then turned to look at A-level mathematics from the standpoint of university lecturers in mathematics. In each of the departments, which were visited, one lecturer was the admissions tutor. Invariably, he/she had a better grasp of A-level matters than most of the other lecturers in their department. Some of the admissions tutors had made considerable efforts to acquaint themselves with the various A-level syllabuses and could talk knowledgeably about them. There were other admissions tutors whose knowledge was incomplete, or rather sketchy.

Most undergraduates had done A-levels on one of AEB, Cambridge, JMB or London Board. There was often a strong regional bias. Thus, students in the North had often taken JMB A-level, whereas students in the South had taken AEB or London. The Cambridge Board students were evenly scattered. Tutors were often keen to foster this regional bias, as it meant that a substantial group in the first year had a similar background.

Some admissions tutors felt that AEB and London A-levels were somewhat thin in material, especially if the A-level was Pure Mathematics with Statistics. One tutor went so far as to say that he would expect a prospective student, who was taking AEB or London, to be doing a second A-level in mathematics, if he was hoping to study for an honours degree in mathematics. One or two tutors mentioned that the occasional student, whom they had taken, with a B or C in Mathematics on the Oxford and Cambridge Board, or on MEI, had turned out to be at least as able as students with a higher grade on some other board.

The majority of departments had a standard offer for their course. If a student achieved this, whatever board's examination they had taken, then they were guaranteed a place. In fact the board they had taken was often not considered, despite the fact that it might be easier to get an A or B on some boards than others. There was quite a widely held view that the grades at A-level were much the same on each board and that any differences were so slight that they need not be considered. One might suppose that this view was adopted because the question had not been addressed, nor was likely to be addressed, because there was not the time nor the manpower available, or because other things took priority, or whatever. In fact, departments which had taken the time, or trouble, had found appreciable differences.

In many of the universities, the university lecturers only assumed that the 'common core' had been covered. Hence other areas were briefly summarised at the start of the university course. Consequently, it was felt, as the common core was included in all the syllabuses, it did not really matter which course had been followed previously. This was especially true in university mechanics courses, as students, in some cases, had done no mechanics at all. Indeed, Crighton (1995), Head of Applied Mathematics and Theoretical Physics at Cambridge University, reports a decline of advanced skills in mechanics. He, further, deplores the disappearance of mathematical proof from schoolwork, the lack of ability to apply skills to new problems and the 'inappropriate concentration on numerical solutions which, allowing no checks for dimension etc., did not provide a feel for problems, hence forming an anti-scientific approach'.

The view expressed above was typical of many university mathematics staff. We noted in Chapter Eight that university mathematics staff remain closely allied to physics and engineering. This is somewhat at variance with the views of sixth form students, who, possibly, see mathematics as a separate subject, with far more diverse applications.

In attempting to answer the question as to whether some A-level syllabuses prepared students for university more thoroughly than others, we found a general dissatisfaction with A-level courses, by university staff. This was not board related. Instead it focussed on such issues as we have indicated above : lack of mathematical proof, poor calculus skills, sometimes non-existent mechanics, inability to apply skills to new problems. Almost, without exception, the universities had not considered a board effect.

In order to investigate this possibility, several departments were able to supply data concerning A-level performance and board, coupled with performance in first year university examinations in mathematics. It is interesting to note that at one university, the data concerning A-level board was kept centrally on the university registry's computer and was not readily available to the department. Some university departments, for reasons of confidentiality could not supply data. However, data was available from five departments and we have analysed this using a Specific Linear Model in Chapter Eight.

Overall there was no significant university effect. This would indicate a consistent standard across universities. This is possibly attributable to the system of external examiners monitoring university courses. It could also be due to the fact that there is a fair degree of mobility in the university system, hence, some staff may have experience of other university departments, having worked in them, or staff attend conferences, or symposia, at other universities. The lack of a university effect would suggest that students would be likely to fare similarly, if they were to have gone to another university department. It can be argued, of course, that mathematics, which has the reputation of being a difficult university subject, is equally difficult, wherever it is taken.

In order to acquire the data, the participating university departments were assured that the results would be treated confidentially. Hence we cannot comment on the sign and size of the university parameters, which we have calculated, in relation to the particular university. Suffice it to say, that the order was, broadly, in line with the author's expectations, as an experienced sixth form teacher, who has taught students, who have gone to most of the departments involved. Although, again, we must stress that there was not a significant overall university effect.

On the other hand, there was a significant overall board effect. This meant, possibly, that some A-level Mathematics syllabuses produced better preparation for university mathematics courses than other syllabuses. Because students at university had predominantly used AEB, Cambridge, JMB or London A-levels, all the other boards had to be amalgamated as a single group, since they occurred far less frequently. The resulting board parameters were:

AEB	2.705
London	1.908
Cambridge	- 0.103
Others	- 1.983
JMB	- 2.527

The suggestion here is that high grades on the AEB and London boards are typically equivalent to lower grades on the remaining boards. We see that in the case of the AEB, the parameter is significantly non-zero, since its 95% confidence limits were 2.707 + 2.437

These findings are consistent with others in this study, and elsewhere, which suggest that, perhaps, the syllabuses and papers in the case of AEB and London lack depth, are more predictable and graded more generously than their competitors.

The data we have gathered supports the sixth and seventh hypotheses, which we have made in Chapter Three. We must be cautious, since it was necessary to amalgamate several boards, when using our Specific Linear Model analysis, because of the small numbers involved. Clearly a much larger sample would be helpful in determining the full extent of the differences between boards. It is comforting, however, that our findings are broadly in line with the expectations of the author and other people, who we have interviewed at various stages in the study. The limited amount of research done in this area is also, largely, consistent with these findings.

11.6 Sixth Form Students' Views.

In addition to the views of university staff, we also considered the opinions of sixth-form students. Initially they looked at some trigonometry questions, taken from the papers which we have used elsewhere in this study. The comments made by the students about the various questions are in Chapter Nine.

The students made comments about each of the questions. There was a general degree of satisfaction about the questions, which the students, in principle, knew how to tackle. From their comments, it is also clear that a structured question aids their ability to tackle questions confidently, consequently this type of question finds approval among the students. In such questions, many, inevitably made mistakes, by not being accurate enough with arithmetic, or algebra, but were essentially using a correct method of approach. This, alone, produced a degree of satisfaction among the students. There was an apparent dislike of questions on which they could not make a start. The type of features causing this were: lack of structure, the introduction of inequalities, inverse functions or unfamiliar notation. Indeed the concept of 'function' - one of the two key concepts of pure mathematics according to Hardy - produced a number of difficulties in these questions. When a question involved several areas of the syllabus this, again, caused difficulties for the majority, only the brightest students were able to cope well in this case. Many of the students displayed an inability to handle converses. Questions could be handled competently when things proceeded in, what the student perceived as the 'normal' direction. If the question required them to reason in the opposite direction, then this caused difficulties. The students at the age of 17-18 have a tenuous grasp of the process which Piaget refers to as 'reversibility'. Perhaps some examiners over-estimate the capacity of sixth-form students to handle such questions competently.

Looking at other questions from these papers, which we have used elsewhere, also from experience gained by teaching sixth-formers for a number of years, it seems that some boards use a higher proportion of structured and straightforward questions than other boards. The students were not aware which questions were produced by which board. Nevertheless, they were able to see, and comment about, differences in the questions. It did not occur to them that some of the differences might be attributable to a 'board factor'. It is interesting to recall that some of the students referred to questions at the end of chapters in their textbooks, where the board is indicated beside the question; after receiving some prompting on the matter of 'board' by the author. The fact that they then revealed that they used questions from the Cambridge Board (the one whose A-level they would be taking), to the exclusion of the rest, was of considerable interest. The students, therefore, did not have any great experience of questions from other boards, from their own study. Consequently, it was unlikely that they would attribute any differences in the questions to a 'board factor'.

Having been alerted to the possibility of a 'board factor', they were able to make some interesting observations of their own, which we have reported in Chapter Nine. The overriding concern, in the students' minds, was one of 'fairness'. They thought it important that questions set by one board should be broadly similar in content and degree of difficulty to those set by another board. They pointed out, particularly, that difficult questions did not give them a chance to show what they did know. More importantly, the students wanted to be sure that the grades awarded would be the same, whatever board they happened to be taking. In the light of the questions they had been doing for this study, they were unsure that the system, at the time of doing these questions, was fair. Clearly the students had little experience of differences between boards and they were only really aware of them after some prompting. Nevertheless, when they began to think about the issue, they produced genuine causes for concern, based on their limited knowledge.

11.7 Modular Mathematics.

The final strand of this study concerned Modular A-level Mathematics courses. This concept was a radical departure from the types of A-level course which had preceded it. From the comments of teachers in Chapter Seven, it was proving, within a short space of time, to be an attractive type of course for both students and staff. Because of its rapid advancement, during the lifetime of this work, it was felt necessary to include some discussion of this form of syllabus and examination. Almost certainly, modular schemes are going to play a prominent part in the development of A-level Mathematics over the next few years, and for some time to come.

The MEI Structured Sixth Form Mathematics Scheme was the first truly modular scheme to appear. Whereas, in the past, the MEI syllabus and examinations had a reputation for being extensive and demanding, the new scheme had been well received and was proving to be accessible to the majority of A-level students, not just the elite few. This was attested both by comments received during our sixth-form teachers' questionnaire and by observation at the comprehensive school near Bedford, which was using the scheme (see Chapter Ten). The introduction of the modular scheme had enabled MEI to move from its rather marginalised position to a mainstream position among the plethora of A-level Mathematics syllabuses.

To a large extent, modular schemes had been launched as a response to the noticeable gap between GCSE courses in Mathematics and the existing A-level Mathematics syllabuses. The limited evidence from our observations suggests that the schemes are enabling students, who have only a B or C grade in GCSE Mathematics, to cope with an A-level course, more readily than they might, otherwise, have done.

The modular schemes have a number of other significant advantages, among these are:

(i) students are able to pace their work more easily,
(ii) there is feedback about progress,
(iii) if satisfactory progress is not being made, units may be retaken,
(iv) credits are 'cashed' for a grade when the student has the required grade,
(v) there is a wider range of choice for students and schools, e.g. mechanics and statistics can both be part of the course,
(vi) able students may do sufficient units to gain AS Further Mathematics, (thus being able to do work beyond A-level Mathematics, without having to take the full double-subject course).

Along with the benefits of the modular system come a number of disadvantages, these include:

(i) There is a considerable degree of choice in the selection of modules beyond the compulsory 'core' pure mathematics. This means that within the one examination there are, in fact, several quite distinct examinations. It is quite difficult to ensure that these different versions are comparable and graded in an equivalent manner.

(ii) Dilution, almost inevitably, occurs in the applied mathematics area, when modules are taken in, for example, mechanics, statistics and discrete mathematics. This means that the student acquires some knowledge in each area, without knowing any area in great detail.

(iii) Related to this degree of choice, within the modular structure, universities will not have a clear idea of the course that has been followed by the students when they arrive at university. This makes planning the first year course, at university, problematic, as little can be assumed to be known by all students.

(iv) Whether one is adopting four or six modules for an A-level scheme, external examinations are going to be taken two or three times during each year of the sixth-form. From the comments of the schools and colleges, which we visited, this is proving somewhat disruptive to the normal routines of the institution. This is particularly the case if the student is following a similar modular course in his/her other A-level subjects. This especially affects extra-curricular activities such as rehearsals for plays and concerts, or training for sports teams, since students will want to be revising for the next modular examination, rather than remain behind after school for these various activities. Furthermore, the actual examinations may occur during the teaching time for other subjects.

(v) The examination papers, being between an hour and an hour and a half, in length, have become fragmented. There are, typically, a number of short questions, with only one or two longer questions. In the three hour papers of previous syllabuses, candidates would have had to cope with a number of longer questions, which would test a topic in some detail. This is no longer the case, for most topics in modular schemes.

(vi) Inherent in a modular scheme, is the very real possibility that areas of the subject become 'compartmentalised'. Thus, once a module has been covered, it is mentally 'ticked off' and to a large extent put to the back of a student's mind. Clearly some ideas, such as integration or differentiation and solving quadratic equations, occur right through the course, in almost any module. These are more the exception, rather than the rule. From the schemes which we have observed, many of the modules are, largely, free-standing and the idea of dependency, although clearly written into the syllabus, is fairly tenuous. Hence a student studying Pure Mathematics Module 3, in the second year, may not be able to tell one a great deal about Statistics Module 1, which was done the previous year. This point came out from discussions with admissions tutors, based on their experience of interviews with prospective undergraduates.

Our final hypothesis in Chapter Three was that "Modular Mathematics syllabuses are a more appropriate way of approaching the study of A-level Mathematics than their competitors and previous A-level Mathematics syllabuses". Whether this is the case, or not, is largely indeterminate. There are clear advantages and, equally, clear disadvantages. Modular Mathematics syllabuses are still in their infancy. Perhaps the passage of time will show if the advantages outweigh the disadvantages, or vice versa. The advantages address some of the concerns of the moment, with little doubt. However, the dangers of acting reactively, by the wholesale embracing of modular syllabuses, are only too apparent. The sensible course appears to be to allow modular and non-modular courses to co-exist for some time to come, in the hope that a clearer assessment can be made.

In this chapter, we have commented on our results and observations, in relation to the hypotheses, which we made in Chapter Three. In the conclusion, which follows, we shall attempt to summarise these findings, indicating those we feel are especially important. We shall also indicate those findings which we feel are inconclusive, but could be established by further research.

12.Concluding Remarks

Throughout our study, using a variety of techniques, we have found differences between the various versions of A-level Mathematics, which were available when this work began. In certain areas we have been able to use quantitative methods. In some cases the differences have proved to be statistically significant. Given the diversity in A-level Mathematics syllabuses, perhaps it is almost inevitable that there will be variation. If we were in a position where there was only one version of A-level Mathematics, students would perform differently on it, if they were to take examinations on two separate occasions. An examiner might interpret work differently at two separate times and different examiners would almost certainly differ in their estimation of an individual's work. The reality is that we have a number of versions of A-level Mathematics, set and marked by different examiners, which are taken by a large number of students each year. Quinlan (1995) gives the figure as nearly 52 000 in 1994. The potential for variation is, therefore, greatly increased.

The overall task has been to assess whether observable differences cause serious problems, which could be alleviated by either reducing the number of versions of A-level Mathematics available, or by taking steps to ensure that the versions which are available are of a more uniform standard. Since A-level performance is important in both the selection procedure and the preparation for both higher/further education and/or employment, it is vital that A-level Mathematics should be as efficient as possible.

In our work we have consistently detected a hierarchy among the examination boards. London, and often AEB, have come out at the 'easier' end, whereas Oxford and Cambridge, Oxford and SMP have come at the 'harder' end. This was noticeable in our work on 'readability', particularly in the applied mathematics area. (We have remarked earlier on the high board-topic interaction effect which was observed.) It was also noticeable in our question comparison exercise. On this occasion, the effects were most noticeable in the pure mathematics area. In addition, a board effect was detected in our analysis of first year university performance. Here the AEB was the only significantly non-zero result, on the 'easy' side, as before, with London tending to the 'easy' end as well. These results are consistent with the few studies that have been carried out previously. The most recent cross-moderation exercise carried put by the GCE Examination Boards, Quinlan (Op.Cit.), shows London to be among the most leniently marked at various grade boundaries. Furthermore it shows that, in the opinion of experienced Chief Examiners from all of the participating boards, London questions "required the least interpretation and greatest level of routine processing", while AEB questions "required the least use of manipulative skills".

We have gathered a considerable amount of qualitative material, and there was a noticeable opinion among the teachers, who responded to our questionnaire, that there was not a state of 'equality' among the various versions of A-level Mathematics. In the post-Thatcher era, where 'market forces','competitiveness' and 'league tables' are much in evidence, teachers are very keen to find the most beneficial product for their students. There is now, possibly more than ever before, a tendency to 'shop around', whereas, in the past, the examination board was more likely to be chosen for traditional or geographical reasons.

It is interesting to recall, at this point, the shift in position of the MEI. At the start of the study, the syllabus was extensive and the papers, which included a high modelling content, were generally regarded as searching. (This was often mentioned by university staff with knowledge of MEI.) The new, modular course, is considerably more straightforward and is being adopted by a wider range of institutions than ever before. Previously, MEI had been largely confined to selective independent schools. In order to survive, some might argue, it has widened its appeal, by becoming modular and ensuring that content and examinations were appropriate to the majority of A-level students.

The examination boards themselves are aware of the competition. The author has come across a case, in his experience, where it was suggested to a chief examiner, by an official of Board X, that the B/C borderline should be placed a little lower, otherwise next year Board Y's entry might improve, because of fewer grade B's being awarded by Board X. Perhaps a mark this year might not have much impact on the perceived standards of A-level Mathematics, however, if this were to continue for a number of years, then B grades, for example, could be gained with a considerably reduced overall mark in, say, ten years time. We find, at the time of writing, that the Government is to launch an enquiry into standards at A-level over the last twenty years.

Furthermore, at the time of writing, we find that Cambridge, Oxford and Oxford and Cambridge Boards are amalgamating. We thus find that two of the boards at the 'difficult' end of our hierarchy (Oxford and Oxford and Cambridge) will be vanishing. This is clearly due to factors other than mathematical ones, since all subjects are involved. Perhaps, however, their more difficult stance has extended to other subjects, causing numbers of entries, overall, to drop, making the Board unviable. Hence it is possible that if boards operate too far from the 'norm', they will 'go to the wall'.

If teachers and boards are aware of the differences between the various syllabuses, from our observations, students are generally not aware of these differences and their importance. When the possibility was pointed out to them, then it was perceived as an important issue. It would seem that more should be done to make the students aware of the choice available and the consequences of those choices.

In contrast, university staff, in general, take a rather more detached view. Apart from a minority, who have investigated the differences between A-level boards, the differences are thought to be, largely, unimportant. Instead, university staff, mostly, are more concerned with the deficiencies of A-level Mathematics, in general, usually centred on the lack of teaching of 'proof' in schools and the lack of students to be able to 'think mathematically'.

We have been examining the position of A-level Mathematics as it is. What of the future? We have mentioned above the reduction in boards by the amalgamation of the three, geographically, Midland boards. Perhaps the creation of a Southern Board, by the amalgamation of London and AEB would be beneficial. The continued provision of diverse courses does not seem a sensible means of promoting harmony. The Cambridge Board, for example, is offering both a Modular and a Linear course. It would seem prudent for boards to be offering a single version of A-level Mathematics. Allowing for the fact that there will be a number of boards operating, in the foreseeable future, SCAA should be able to ensure a choice of modular and non-modular courses - although the weight of current opinion seems heavily in favour of modular, or unit-based courses. Ten years ago, progress seemed to be being made in the reduction of the number of courses, but the advent of modular courses has created a renewed expansion, which must not be allowed to get out of control.

For many years we have had an agreed 'common core' in pure mathematics. Perhaps it is time to make progress in establishing a body knowledge, in the applied mathematics, which is incorporated in A-level Mathematics. From our observations, it seems that higher education cannot rely on their new students having been taught anything in particular, in this area of applied mathematics, which seems highly undesirable.

There are, at present, facilities for inter-board scrutiny. However, there seems to be considerable scope for boards to operate in their own individual ways. For example, some will offer straightforward papers with A grade borderlines in excess of 80% and E grade borderlines above 40%, while other boards will have more searching papers, with A grade borderlines below 70% and E grade borderlines around 30%. There could, and hopefully should, be more done to ensure papers of a similar standard, with A grade, say, at 75% and E grade at 40%, with an allowance of of, say, 2 or 3% either side. This will, of necessiity, involve setting papers where such features as readability and question content are considered. Perhaps the establishment of some kind of 'Question Bank' could be achieved, given the greatly improved computing facilities becoming available.

A-level Mathematics, as mentioned earlier, each year, involves some 50 - 60 000 students. It is important that something which engages such a large number of people, should be as well organised as possible, with high standards of reliability. During the course of this study we have noticed a number of deficiencies and we have made a number of suggestions that could possibly bring some improvement. It is to be hoped that further research will take place in this important area, which, all too often, appears to have had remarkably little attention focussed on it.

Bibliography

Backhouse J.K. et al (1980) — Choice of Mathematics for study at school and university. Oxford Univ. Dept. of Educational Studies

Bardell G.S. (1977) — Report of the Inter - Board Cross-Moderation Study in 1976 Advanced Level Pure Mathematics. Cardiff, WJEC

Bennett N. (1973) — Research Design. Open Univ.

Bormuth J.R. (1966) — Readability: A new approach. Reading Research Quarterly, 1966, 1, pp 79-132

Burkhardt H. et al (1978) — Teaching Methods for Undergraduate Mathematics. proceedings of the University Mathematics teaching Conference 1977. Shell Centre for Mathematical Education, Nottingham.

Chatwin P. (1980) — Some Thoughts on Examining Mathematics at Advanced Level. A-level Mathematics - The Report of the Stoke Rochford Conference. pp 38-46 SMP.

Cockcroft W.H. (1982) Mathematics Counts. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools.

Craggs J.W. (1976) School Examinations and the Well Being of Mathematics. IMA Bulletin Vol 12, 11/12, pp 322-324.

Crank J. (1976) University Mathematics: The First Year Problem and a Remedy. IMA Bulletin Vol 12, 11/12, pp 325-327.

Crighton D. (1995) Action for Mathematics. IMA Bulletin Vol 31, 7/8, p 99.

Croasdale R. (1991) A Cluster Analysis of Mathematics Examination Papers. Int. J. Math. Educ, Sci, Technol. Vol 22, 1, pp 1-8

Dale E. and Chall J.S. (1948) A Formula for Predicting Readability. Educational Research Bulletin Vol 27, pp 11-20 and 37-54.

De Landssheere G. (1973) Le Test de Closure. Mesure de lisibilite et de la Comprehension. Education 2000 Paris, Nathan.

Farr J.N. and Jenkins J.J. (1949) Tables for Use with the Flesch Readsbility Formulae. J. Applied Psychology Vol 33 pp 275-378

Flesch R.F. (1948) A New Readability Yardstick. J. Applied Psychology Vol 32 pp 221-33

Fry E.B. (1969) The Readability Graph Validated at Primary Level. Reading Teacher Vol 22.

Gagatsis A. (1984) Understanding of Mathematical Text: Cloze Tests,Readability Formulae. Scientia Paedagogica Experimentalis Vol 27, 2, pp 251-265.

Government Statistical Service (1994) Statistics of Education. Public Examinations. GCSE and GCE in England.

Griffiths H.B. and McLone R.R. (1979) Qualities Cultivated in Mathematics Degree Examinations. Univ. of Southampton

Gunning R. (1952) The Technique of Clear Writing. New York, McGraw-Hill.

Harrison C. (1980) Readability in the Classroom. CUP.

Hart C.R. (1989) Examinations Post 16: The Role of the School Examinations and Assessment Council. IMA Bulletin Vol 25, 12, pp 290-1.

Hater M.A. (1969) The Cloze Procedure as a Measure of the Reading Comprehensibility and Difficulty of Mathematical English. Purdue Univ.

Hersee J. (1984) A-level Mathematics Problems. Math. Gazette Vol 68, 444, pp 81-87.

Hirst K. (1993) The National Curriculum and A-level Reform. IMA Bulletin Vol 29, 1/2 pp 9-15.

HMI (1982) Mathematics in the Sixth Form. HMI Series: Matters for Discussion No14 HMSO.

Holland B.F. (1979) Advanced Level Mathematics. Common Core. AEB.

IMA (1987) A-level Examinations. IMA Bulletin Vol 23, 10/11 pp 168-9

IMA (1990) Response to the SEAC's Consultative Document on A and AS levels. IMA Bulletin Vol 26, 1/2 pp 20-3.

James W.L. (1968) The Interdependence of Sixth Form Mathematics and the Mathematical Courses of Universities and Colleges of Education. Univ. of Newcastle.

Jennings T.V. (1985) Some Problems at the School/University Interface in Mathematics. M.Phil. Thesis Univ. of Reading.

Johnson S. and Cohen L. (1983) Investigating Grade Comparability Through Cross-Moderation. London, Schools Council.

Jones R.J. McC. (1976) The Usability of Mathematics Textbooks as Found in Third Year Junior Classes. M.Ed. Thesis Univ. of Sheffield.

Kahn R.L. and Cannell C.F. (1957) The Dynamics of Interviewing: Theory, Technique and Cases. New York, Wiley.

Kane R.B. (1967) The Readability of Mathematical English. J. Research in Science Teaching. Vol 5 pp 296-8.

Klare G.R. (1963) The Measurement of Readability. Iowa State Univ. Press.

Klare G.R. (1977) Practical Aspects of Readability. Textual Communication Group. IET Open Univ.

MacGintie W.H. (1966) Proceedings of Symposium on verbal learning research and the technology of written instruction. Columbia. Columbia Univ. (unpublished manuscript).

Malvern D.D. (1979) The Sequencing of Sixth Form Mathematics. An Interim Report. IMA Bulletin Vol 15, 10, pp 228-232

Mathematical Association (1982) Mathematics at A-level. A Discussion Paper on the Applied content. Leicester, Mathematical Association

McLone R.R. (1990) A Study of the Demands made by the Two Approaches to Double Mathematics. Standing Research Advisory Committee of the GCE Examining Boards. Study 1.

Moser C.A. and Kalton G. (1979) Survey Methods in Social Inveatigation. 2nd. Edition. London, Heinemann.

Neill H. (1976) A Survey of A-level Mathematics to Assist Planners of Tertiary Courses which have a Substantial Mathematical Content. 2nd. Edition London, The Royal Society.

Oppenheim A.N. (1966) Questionnaire Design and Attitude Measurement. London, Heinemann.

Payne S.L.B. (1951) The Art of Asking Questions. Studies in Public Opinion No. 3. Princeton, Princeton Univ. Press.

Pitt H.R. (1976) A Survey of A-level Mathematics Syllabuses to assist Planners of Tertiary Courses which have a Substantial Mathematical Content. IMA Bulletin Vol 12, 10, pp 308-315

Pitt H.R. (1978) A Note on the SCUE Syllabus Study: 'A University View on a Possible Minimal Core for A-level. IMA Bulletin Vol 14, 1, pp 26-30

Quadling D. (1980) Mathematics Present, Past and Future. A-level Mathematics - The Report of the Stoke Rochford Conference, SMP.

Quinlan M. (1995) A Comparability Study in Advanced Level Mathematics. London, ULEAC.

Rankin E.F. (1970) The Cloze Procedure, its Validity and Utility. In: Farr (Ed.) Measurement and Evaluation of Reading. New York, Harcourt, Brace and World. pp 237-253.

Rollett A.P. (1963) Honours Courses in Mathematics in the Universities of Great Britain. Incorporated Association of Headmasters and the Association of Headmistresses.

Scott J.F. (1975) Comparability of Grade Standards in Mathematics at GCE A-level, Schools Council Bulletin No. 30. Schools Council Publications. HMSO.

Shuard H. and Rothery A. (1984) Children Reading Mathematics. London, John Murray

Siegel S. (1956) Non-Parametric Statistics for the Behavioral Sciences. New York, McGraw-Hil

SRAC (1990) Standing Research Advisory Committee of the GCE Examining Boards: Standards in Advanced Level Mathematics. Studies 1-5

Stoke Rochford Conference (1980) A-level Mathematics. SMP.

Taylor W.L. (1953) Cloze Procedure: A New Tool for Measuring Readability. Journalism Quarterly No 30.

Taylor W.L. (1957) Cloze Readability Scores as Indices of Individual Differences in Comprehension and Aptitude. J. Applied Psychology No. 41 pp 19-26.

Ting et al. (1990) Confidence Intervals on linear combinations of variance components that are unrestricted in sign.J. Stat., Comput., Simul., No 35, pp 135 - 143.

Travers R.M.W. (1969) An Introduction to Educational Research. New York, Macmillan.

Weaver W. and Kingston A. (1963) A Factor Analysis of the Cloze Procedure and Other Measures of Reading and Language Ability. J. Communication Vol. 13 pp 252-261

Wilmore T.J. (1987) AS-level Mathematics. IMA Bulletin Vol 24, 3/4, pp 52-3.

Appendix A: The A-level Teachers' Questionnaire.

10, Amberley Gdns.,
Bedford
MK40 3BT

23.4.91.

Dear Colleague,

Currently I am a part-time research student at the Open University. I am also an Assistant Chief Examiner in A-level Mathematics for one of the GCE Examination Boards.

I would be extremely grateful if you could take a few minutes to fill in and return the enclosed questionnaire. I enclose an SAE for that purpose.

All replies are anonymous. If you would like to see the results at a future date I would be happy to let you have a copy.

Thank you for your co-operation.

Yours Sincerely,

T.V. JENNINGS

A-level Mathematics Questionnaire.

1) What Examination Board does your school/college regularly use for A-level Mathematics ?
(e.g. Cambridge, JMB, London etc.)

2) Please indicate which type of papers are taken, by placing a tick beside one of the following: (You may tick more than one.)

(a) Pure Mathematics and Mechanics
(b) Pure Mathematics and Statistics
(c) Pure Mathematics and a mixed paper of Mechanics & Statistics
(d) Other (please specify)

3) Approximately how many candidates each year take the subject in each of the four categories above.

(a) (b) (c) (d)

4) Nationally there are over 20 syllabuses for A-level Mathematics. Do you consider that :

(a) All are of equivalent standards of difficulty.
(b) Some are harder than the majority
(c) Some are easier than the majority

(You may tick more than one.)

If you have ticked (b) and/or (c) please state which syllabuses fall into each category:

(b)

(c)

5) What factors do you think contribute to any differences?

contd.)

6) Do you think that the grades awarded by different Boards are comparable? (i.e. If you were able to double enter a candidate, is it likely that he/she would get the same grade?)

(a) Yes (b) No

7) Have you considered changing boards at A-level ?

(a) Yes (b) No

If "Yes" , which Board would you be likely to adopt ?

8) Do you think that the grades awarded in Mathematics are:

(a) Much harder to gain than the same grades in other subjects

(b) Harder to gain than the same grades in other subjects

(c) About the same level of difficulty as grades in other subjects

(d) Easier to gain than the same grades in other subjects

(e) Much easier to gain than the same grades in other subjects ?

(Please tick one)

9) Some Boards are bringing out Modular Courses in A-level Mathematics. Are you

(a) Strongly in favour of this idea

(b) In favour of this idea

(c) Undecided about the idea

(d) Not in favour of this idea

(e) Strongly opposed to this idea ?

(Please tick one)

contd.)

10) For the syllabus which you use, do you find that the amount of material in the syllabus is

(a) too much

(b) about right (please tick one)

(c) too little ?

11) For the syllabus which you use, do you consider that the style of questions used in the examination paper is

(a) too hard

(b) about right (please tick one)

(c) too easy ?

12) Ignoring career factors and individual differences, as far as possible, for a pupil who has gained a B grade in Mathematics GCSE at the end of the Fifth Form year, do you consider that

(a) A C grade (or better) in Mathematics A-level is a readily attainable target.

(b) A low pass grade is the most likely outcome, however much hard work is done during the course.

(c) Considerable caution is advisable before embarking on A-level Mathematics, as the prospects for success are slight.

(Please tick one)

13) What are you feelings about Coursework at A-level ?

(a) Strongly in favour

(b) In favour

(c) Undecided (Please tick one)

(d) Opposed

(e) Strongly opposed

contd.)

14) If you have any further comments that you would like to make about A-level Mathematics please do so below.

Thank you for your help.

APPENDICES PAGES 323-377 HAVE NOT BEEN SCANNED ON INSTRUCTION FROM THE UNIVERSITY