Supply Chain Coordination under Trade Credit and Quantity Discount with Sales Effort Effects

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The purpose of this paper is to investigate the effectiveness of trade credit and quantity discount in supply chain coordination when the sales effort effect on market demand is considered. In this paper, we consider a two-echelon supply chain consisting of a single retailer ordering a single product from a single manufacturer. Market demand is stochastic and influenced by retailer sales effort. As it is found that the trade-credit-only model cannot achieve the perfect coordination of the supply chain, we have developed a hybrid, quantitative, analytical model for supply chain coordination by coherently integrating incentives of trade credit and quantity discount with sales effort effects. The results demonstrate that, providing that the discount rate satisfies certain conditions, and if the income distribution ratio is restricted to a certain range; the proposed hybrid model combining trade credit and quantity discount will be able to effectively coordinate the supply chain by motivating retailers to exert their sales effort and increase product order quantity. Furthermore, the hybrid quantitative analytical model can provide great flexibility in coordinating the supply chain to achieve an optimal situation through the adjustment of relevant parameters to resolve conflict of interests from different supply chain members. Numerical examples are provided to demonstrate the effectiveness of the hybrid model.

1. Introduction

Trade credit is a form of delayed payments for the transfer of goods and services that upstream suppliers allow downstream retailers to settle the payments at the end of the sales period. Trade credit has been referred to be one of the important sources of short-term financing for the firms and could play an important role to firms' growth potential, competitive advantage, and survival [1-3]. In addition, it is also an effective incentive and coordination contract [4-8]. Trade credit is widely used in enterprises of various industries in China, the United States and Europe [1-3]. Suppliers provide trade credit for downstream retailers, allowing them to postpone payments. For retailers, trade credit can reduce their capital occupation and encourage them to increase order quantities. At the same time, suppliers receive more product wholesale income. So trade credit has been regarded as a way to coordinate the supply chain.

Along with the fast advancement of technologies, more intensive competition from markets and more diversifying customer needs, the lifecycle of many products has been greatly shortened in the industries, especially in fashion, electronics and software [9].The sales season for such products is very limited. For short-life-cycle products, if trade credit is reached between a supplier and a retailer, the retailer will be encouraged to increase the order quantity, and then they should exert much sales effort to sell quickly in a shorter period. Otherwise, it is easy to cause inventory risk. In many settings, retailer sales effort is important in influencing market demand. Retailers can influence demand by hiring more sales personnel, improving their sales skills and increasing advertising and so on [10-12]. However, there are relevant costs attached to the retailer's sales effort. By offering trade credit to the retailer, the supplier needs to pay a certain amount of capital cost, but they can enjoy benefits from higher market demand influenced by retailer sales effort. Hence, in the face of such conflicts of interests, how to optimize

the decisions-making and coordinate the supply chain is a very important issue for both the supplier and the retailer.

To understand the role of trade credit in coordinating supply chain while taking into account the effects of retailer sales effort on market demand, we develop a quantitative analysis model based on trade credit. We show that when retailer sales effort influences market demand, trade credit cannot achieve supply chain coordination.

In many cases, the composite mode based on two or more different contracts is often designed to coordinate the supply chain [13-14, 8]. In this paper, we propose a hybrid analytic model under a composite contract with trade credit and quantity discount. We find that when retailer sales effort influences demand, the composite contract can fully achieve supply chain coordination.

The remainder of the paper is organised as follows. Section 2 provides a review of existing article about trade credit. Section 3 discusses the development and analysis of analytic models under a single trade credit considering sales effort effects on demand. The hybrid analytical model under a composite contract with trade credit and quantity discount is proposed in Section 4, followed by an application example in Section 5. Finally, Section 6 draws conclusions.

2. Literature Review

Haley and Higgins[15] firstly studied trade credit in the operation management. Since then, the buyer's inventory policy under given trade credit terms has been concerned for a long time. Goyal [16] considered the EOQ model under trade credit. Huang [17] extended Goyal [16] model to two-part terms of trade credit. Teng et al. [18] and Taleizadeh et al. [19] further supplemented the study of Huang [17]. Later, Mahata [20], Soni and Joshi [21]，Chen et al. [22]，and Wu et al. [23] studied the EOQ or EPQ model for deteriorating items under the two-part trade credit.

Instead of setting trade credit as a given parameter, several articles took trade credit as a decision variable from the vendor's or the supply chain's point of view. Kim et al. [24] and Abad and Jaggi [25] developed a model to find the optimal credit period for the vendor by taking price-sensitive demand into consideration. Zhou et al. [26] determined the supplier's credit policy considering inventory-dependent demand and limited displayed-shelf space. Wang et al. [27] developed the EOQ model to study the supplier's optimal credit period and cycle time for deteriorating items. Tsao [28] built the EPQ model under maintenance, variable setup costs, and trade credits. Pramanik et al. [29] built the EOQ model based on three level partial trade credit.

However, these above studies did not consider the issue of supply chain coordination. Taking trade credit as a mechanism to coordinate a supply chain, Jaber and Osman [30] may be the first study. They studied how to set order quantity and trade credit to minimize the cost of the whole supply chain under constant demand. Luo [7] and Yang et al. [31] proved that trade credit is a new supply chain incentive mechanism which is not equal to the quantity discount. Luo and Zhang [32] further explored the role of trade credit in coordinating supply chains in the case of asymmetric information on the buyer's capital cost. Sarmah et al. [33] investigated the issue of supply chain coordination with trade credit in the context of multi-heterogeneous buyers.

Chaharsooghi and Heydari [34] considered the credit period as a mechanism to develop coordination in a multi-period setting. Arkan and RezaHejazi [35] extended Chaharsooghi and Heydari [34] model by assuming that lead time and ordering cost are controllable.

Zhong and Zhou [36] developed a performance improving model through trade credit by assuming that a retailer had limited storage space and faced an inventory-dependent end demand. Das et al. [37] developed an integrated production inventory model under trade credit for a constant deteriorating item. Ouyang et al. [38] proposed an integrated inventory model under trade credit with capacity constraint and an order-size dependent payment period. Sarkar et al. [6] studied the multi-level trade credit and single-setup-multiple-delivery policy for the coordination of a global sustainable supply chain.

Most previous studies on the role of trade credit in coordinating supply chain failed to consider the newsvendor model which is an important mathematical model for uncertain demand. Lee and Rhee [8] examined relatively early the issue of supply chain coordination based on trade credit in a Newsvendor framework. Zhang et al. [39] proposed a modified quantity discount based on both order quantity and advance payment while considering the manufacturer's risk aversion. Yang et al. [40] proposed a two-period continuous newsvendor model under a two-part trade credit contract and examined the influence of the revenue sharing rates on the optimal early payment time from a coordination perspective. Tsao et al. [41] presented newsvendor models to maximize total profits while taking into account uncertain demand, trade credits, carbon

emissions, and the risk of default simultaneously. Cao and Yu [42] investigated the financing and coordination of an emission-dependent supply chain by trade credit. Heydari et al. [4] and Tsao [5] introduced two-level trade credits into supply chain coordination for uncertain demand. Xiao et al. [14] examined whether revenue-sharing, buyback, and trade credit contracts can coordinate a constrained supply chain.

In many settings, retailer sales effort is very important in influencing the market demand. But literature has not adequately addressed retailer's sales effort effect on market demand while exploring the supply chain coordination through trade credit. In this paper, we construct newsvendor models to explore the issue of supply chain coordination under trade credit when the market demand is influenced by retailer sales effort. We also derive the optimal sales effort policy and order quantity policy. And we further present newsvendor models to maximize total profits while considering trade credit, order quantity discount, and market demand with sales effort effects simultaneously, and examine the effect of quantity discount rates on the retailer's and supplier's policies from a coordination perspective.

3. A Trade-Credit-only Model with Sales Effort Effects

We consider a supply chain with a risk-neutral retailer and a risk-neutral supplier. The retailer makes a single purchase of a product from the manufacturer. The manufacturer dominates and first sets trade credit contract. Then, the retailer determines order quantity and sales effort according to contract terms and market situation, and pays off all payment at the end of sales period. To focus on the essential model feature, we assume that the manufacturer's production capacity is not limited and can provide all goods for the retailer to meet the demand.

The retailer faces the random market demand. We assume the demand which is not influenced by sales effort be δ with a cumulative distribution function $F(\delta)$ and a probability density function $f(\delta)$. Let the market demand with sales effort effects be δ' in which demand is stochastic and multiplicative function of retailer sales effort. Specifically, let δ' be given by $e\delta$, where e is the level of retailer's sales effort [11-12]. The cost to the retailer of exerting e units of sales effort is $C(e)$ which is increasing, convex, and differentiable in e, where $e \geq 1$ and $C(1) = 0$. Thus, the marginal effectiveness of sales effort is constant, and the marginal cost of sales effort is increasing.

Notations used are presented as follows.

q: the order quantity of the retailer

p_m: the unit product wholesale price provided by manufacturer

c_m: the manufacturer's unit product production cost

p_r: the retailer's unit product sales price

c_r: the retailer's unit product sales cost

v: salvage value of unit product unsold at the end of the sales period

t: trade credit period

i_m: the manufacturer's unit capital cost rate

i_r: the retailer's return on investment

e: the retailer's sales effort level

θ: the quantity discount ratio

II_l^m: manufacturer's expected profit

II_l^r: retailer's expected profit

II_l^s: system expected profit

$l=1$，2，3, respectively, denotes the 3 different cases

To avoid unrealistic and trivial cases, we assume that $p_m > c_m > v$, $p_r > p_m + c_r$, and $i_r > i_m$. In addition, the supplier allows the retailer to delay payment, so term period t meets $t > 0$.

In the case of general stochastic market demand, comparing the product market demand δ and the order quantity q, we define the retailer's expected product sales $S(q) = E(min(q,\delta)) = q - \int_0^q F(\delta)\,d\delta$, and the expected product residual amount $T(q) = E(q - \delta)^+ = q - S(q)$. In the case of the market demand affected by the sales effort level, comparing the product market demand δ' and the order quantity q, we define the retailer's expected product sales:

$$S(q,e) = E(min(q,e\delta)) = q - e\int_0^{q/e} F(\delta)\,d\delta$$

And the expected product residual amount is $T(q,e) = E(q - e\delta)^+ = q - S(q,e)$

To help fully expose the benefits of the developed trade credit models, this section will use two scenarios: decentralised and centralised coordination decisions. Literature has extensively discussed how to develop trade credit models without consideration for sales effort level [4-5,8,39-42]. This paper will not repeat the details. However, for comparison purpose the expected profit functions are built for the retailer, the manufacturer and the supply chain system under trade credit contract in the case of the general stochastic demand, as follows:

$$\prod_1^r(q) = (1 + i_r t)\left(p_r S(q) + vT(q)\right) - c_r q - p_m q$$
$$= (1 + i_r t)(p_r - v)(q - \int_0^q F(\delta)d\delta) + [(1 + i_r t)v$$
$$-c_r - p_m]q$$
$$\prod_1^m(q) = p_m q - (1 + i_m t)c_m q$$
$$\prod_1^s(q) = (1 + i_r t)(p_r - v)(q - \int_0^q F(\delta)d\delta) +$$

$$\left((1+i_rt)v - c_r - (1+i_mt)c_m\right)q$$

Thus, for the market demand without sales effort effects, the retailer's optimal order quantity q_1^r and the supply chain system's optimal order quantity q_1^s are as follows, respectively.

$$q_1^r = F^{-1}\left(1 - \frac{c_r + p_m - (1+i_rt)v}{(1+i_rt)(p_r - v)}\right)$$

$$q_1^s = F^{-1}\left(1 - \frac{c_r + (1+i_mt)c_m - (1+i_rt)v}{(1+i_rt)(p_r - v)}\right)$$

3.1 Decentralized Decision Scenario

In the case of the market demand affected by sales effort, the expected profit functions of retailers and manufacturers under decentralized decision model can be developed as follows.

$$\prod_2^r(q,e) = (1+i_rt)(p_r - v)\left(q - e\int_0^{q/e} F(\delta)\,d\delta\right) +$$

$$\left((1+i_rt)v - c_r - p_m\right)q - C(e) \tag{1}$$

$$\prod_2^m(q,e) = p_m q - (1+i_mt)c_m q \tag{2}$$

Proposition 1 Under decentralized decision scenario with sales effort effect, we obtain the retailer's optimal order quantity as

$$q_2^r = eF^{-1}\left(1 - \frac{c_r + p_m - (1+i_rt)v}{(1+i_rt)(p_r - v)}\right) \tag{3}$$

And the optimal effort level e_2^r of the retailer meets

$$\frac{dC(e_2^r)}{de_2^r} = (1+i_rt)(p_r - v)R\left(q/e_2^r\right) \tag{4}$$

Here $R(q) = \int_0^q \delta\,d F(\delta)$

Only when equation (3) satisfies equation (4), the retailer is able to achieve the greatest profit. Thus, the optimal order quantity of the retailer is $q_2^r = e_2^r q_1^r$, and then the optimal sales effort level e_2^r satisfies:

$$\frac{dC(e_2^r)}{de_2^r} = (1+i_rt)(p_r - v)R(q_1^r) \tag{5}$$

Proof: See "Appendix A."

Theorem 1 In the trade credit contract, by considering sales effort level, the optimal order quantity of the retailer in the decentralized decision mode is not lower than that of the general stochastic market demand, that is, $q_2^r \geq q_1^r$.

Proof: comparing $q_1^r = F^{-1}\left(1 - \frac{c_r + p_m - (1+i_rt)v}{(1+i_rt)(p_r - v)}\right)$ with $q_2^r = eF^{-1}\left(1 - \frac{c_r + p_m - (1+i_rt)v}{(1+i_rt)(p_r - v)}\right)$, it is found $q_2^r = q_1^r$ when $e = 1$ in q_2^r. Thus, it is not difficult to find $q_2^r \geq q_1^r$ when $e \in [1, \infty)$. At the same time, it can be found that with the increase of the sales effort, the retailer's order quantity will be also increased.

Theorem 1 shows that improving the sales effort level can increase the retailer's order quantity $\Delta q = q_2^r - q_1^r$ because the high level of sales service quality promotes the market demand for retailer's products.

Theorem 2 In the decentralized decision model, the retailer's expected profits $\prod_2^r(q,e)$ and the manufacturer's expected profits $\prod_2^m(q,e)$ will increase as the sales effort level e increases. Compared with that when sales effort remains constant $(e = 1)$, the retailer will make itself and the manufacturer respectively obtain higher profits if it raises the level of sales effort to the optimal level, that is, $\prod_2^r(q_2^r, e_2^r) > \prod_2^r(q_1^r, e = 1)$, $\prod_2^m(q_2^r, e_2^r) > \prod_2^m(q_1^r, e = 1)$.

Proof: (1) Let

$$\prod_2^r(q,e) = e(1+i_rt)(p_r - v)R(q_1^r) - C(e), \quad \text{where}$$

$q = eq_1^r$. The first order partial derivative of e for equation $\prod_2^r(q,e)$ is

$$\frac{\partial \prod_2^r(q,e)}{\partial e} = (1+i_rt)(p_r - v)R(q_1^r) - \frac{dC(e)}{de}.$$

And it is known $\frac{dC(e_2^r)}{de_2^r} = (1+i_rt)(p_r - v)R(q_1^r)$ from equation (5). Then, $\frac{\partial \prod_2^r(q,e)}{\partial e} = \frac{dC(e_2^r)}{de_2^r} - \frac{dC(e)}{de}$.

$C'(e)$ is monotonically increasing on $e \in [1, e_2^r]$ because $\frac{d^2C(e)}{de^2} > 0$, it can be obtained $\frac{dC(e)}{de} \leq \frac{dC(e_2^r)}{de_2^r}$, that is $\frac{\partial \prod_2^r(q,e)}{\partial e} \geq 0$, further it can be obtained that $\prod_2^r(q,e)$ is an increasing function on $e \in e_2^r$. With the increase of the retailer's sales effort, the expected return of the retailer is also increasing. But the retailer only allows sales effort to be promoted to the best effort level e_2^r.

It is obtained $\prod_2^r(q,e) = \prod_2^r(q_1^r, e = 1)$ when the sales effort level remains constant $(e=1)$. Therefore, it is easy to know as follows.

$\prod_2^r(q = e_2^r q_1^r, e_2^r) > \prod_2^r (q = q_1^r, e = 1)$, that is, $\prod_2^r(q_2^r, e_2^r) > \prod_2^r(q_1^r, e = 1)$.

(2) Let $\prod_2^m(q,e) = (p_m - (1+i_mt)c_m)eq_1^r$, and it is obtained $\partial \prod_2^m(q,e)/\partial e = (p_m - (1+i_mt)c_m)q_1^r > 0$.

Obviously, $\prod_2^m(q,e)$ is an increasing function on $e \in [1, \infty)$, so the manufacturer wants the retailer to maximize the level of sales effort because the manufacturer does not have to bear the effort cost.

Thus, it can be obtained $\prod_2^m(q = e_2^r q_1^r, e_2^r) > \prod_2^m(q = q_1^r, e = 1)$, that is, $\prod_2^m(q_2^r, e_2^r) > \prod_2^m(q_1^r, e = 1)$.

3.2 Centralized Decision Scenario

This subsection analyses the coordination effect of the trade credit contract considering the level of sales effort in the centralized decision mode. In this case, the overall expected profit function of the system is constructed:

$$\prod_2^s(q,e) = (1+i_rt)(p_r - v)\left(q - e\int_0^{q/e} F(\delta)\,d\delta\right) +$$

$$\left((1+i_rt)v - c_r - (1+i_mt)c_m\right)q - C(e) \tag{6}$$

Proposition 2 Under centralized decision scenario with sales effort effect, we obtain the optimal order

quantity as

$$q_2^s = eF^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right) \tag{7}$$

And the optimal sales effort level meets

$$\frac{dC(e_2^s)}{de_2^s} = (1 + i_r t)(p_r - v)R(q_1^s) \tag{8}$$

Where $R(q_1^s) = \int_0^{q_1^s} \delta d\, F(\delta)$

Proof: See "Appendix B."

Theorem 3 In the trade credit contract, when sales effort level is considered, the optimal order quantity of the system under the centralized decision mode is not lower than that under the general stochastic market demand, that is, $q_2^s \geq q_1^s$.

Proof: Comparing $q_1^s = F^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)$ with $q_2^s = eF^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)$, it is found $q_2^s = q_1^s$ when $e = 1$ in q_2^s. Therefore, it is not difficult to find $q_2^s \geq q_1^s$ for $e \in [1, \infty)$.

Theorem 3 shows that the customer demand for the retailer's products will be expanded, furthermore, the retailer will be encouraged to increase its order quantity when sales effort level increases. In this way, the system's demand for funds is satisfied by improving the reasonable allocation of the flow of funds in the supply chain system, thereby increasing the system's order quantity.

Theorem 4 In the centralized decision model, the system's expected profit $\prod_2^s(q, e)$ will increase as the sales effort level e increases. Compared with that when the sales effort remains constant $(e = 1)$, system's overall profit would become higher if the retailer raises the level of sales effort to the optimal level e_2^s, that is, $\prod_2^s(q_2^s, e_2^s) > \prod_2^s(q_1^s, e = 1)$.

Proof: See "Appendix C."

3.3 The Effects of Trade Credit Terms on Decisions and Profits

Theorem 5 In the trade credit contract, when sales effort is considered, the retailer's optimal effort level e_2^r and optimal order quantity q_2^r as well as the supply chain system's optimal effort level e_2^s and optimal order quantity q_2^s will increase as the trade credit period t increases.

Proof: See "Appendix D."

Theorem 6 In the trade credit contract that considers the sales effort level, when the retailer determines the optimal effort level e_2^r, there exists always a trade credit period $\hat{t}$ such that the retailer's optimal expected profit is N^r. $\prod_2^r(q_2^r, e_2^r)$ is not lower than N^r for $t \in$ $[\hat{t}, +\infty)$ and increases with the increase of the trade credit period t. When the supply chain system determines the optimal effort level e_2^s, there is always a trade credit term $\tilde{t}$ so that the optimal expected return of the supply chain system is N^s. $\prod_2^s(q_2^s, e_2^s)$ is not lower than N^s for $t \in [\tilde{t}, +\infty)$ and increases as the trade credit period t increases.

Proof: See "Appendix E."

Theorem 6 shows that the trade credit could encourage the retailer to improve the level of optimal efforts and further to stimulate the retailer to increase the quantity of orders. In addition, the benefits of the retailer and the overall supply chain can increase as the trade credit period is longer. These theorems show that the trade credit provided by the manufacturer helps to positively affect the supply chain while considering the retail sales effort level.

3.4 Comparing Centralized Decision Scenario and Decentralized Decision Scenario

Theorem 7 In the trade credit contract, with the market demand influenced by sales effort, the optimal sales effort level, optimal order quantity of the retailer and the system's optimal profit under the decentralized decision mode are lower than those under the centralized decision mode, that is, $e_2^r < e_2^s$, $q_2^r < q_2^s$, $\prod_2^s(q_2^r, e_2^r) < \prod_2^s(q_2^s, e_2^s)$.

Proof: See "Appendix F."

Theorem 7 shows that both the manufacturer and the retailer are rational, and their goal is to maximize their own interests, so the game will lead to the sales effort level and the order quantity to become distorted, that is, $e_2^r < e_2^s$ and $q_2^r < q_2^s$. Then, the system's profits also appear to be hurt, i.e., $\prod_2^s(q_2^r, e_2^r) < \prod_2^s(q_2^s, e_2^s)$.

For the manufacturer, it could only make the decision on the wholesale price of the product and the trade credit term. The number of products offered depends on the retailer's order quantity.

It can be obtained $\frac{\partial \prod_2^m(q, e)}{\partial q} = p_m - (1 + i_m t)c_m > 0$ from equation (2), and then the expected profit of the manufacturer increases with the increase in the retailer's order quantity. From the retailer's point of view, if the order quantity increases from q_2^r to q_2^s, the retailer cannot accept that because q_2^s exceeds its optimal order quantity. However, this case will result in damage to the overall interests of the system, so trade credit contracts that consider sales efforts cannot achieve perfect coordination between the manufacturer and the retailer.

Of course, to persuade the retailer to increase the order quantity, the manufacturer can provide the retailers with

quantitative discount contracts to make up for the loss of the retailer due to increasing the order.

4. A Hybrid Quantitative Model under Trade Credit and Quantity Discount with Sales Effort Effects

In a quantitative discount mechanism, the manufacturer encourages the retailer to increase the amount of the order by reducing the wholesale price per unit of the product. In this paper, a quantity discount mechanism with a ratio θ is introduced. The expected profit function of the retailer and the manufacturer under the portfolio coordination contract respectively are as follows:

$$\prod_3^r(q,e) = (1 + i_r t)(p_r - v)(q - e \int_0^{q/e} F(\delta) \, d\delta) +$$
$$((1 + i_r t)v - c_r - \theta p_m)q - C(e) \qquad (9)$$
$$\prod_3^m(q,e) = \theta p_m q - (1 + i_m t)c_m \qquad (10)$$

In the combined contract, it is a trade credit contract that takes into account the level of sales effort when $\theta = 1$. In this case, both sides can benefit as the sales efforts improve, but the retailer will lack the intrinsic incentive and motivation to improve sales efforts because it has to bear the full cost of effort. However, the manufacturer who does not have to make any effort but to enjoy the benefits of increasing the level of sales effort expects the retailer to provide a higher level of sales effort. Therefore, there is a certain conflict of interests between the two sides. When $\theta = 0$, the manufacturer's product wholesale price is 0, and its profit is negative, which is unrealistic. Thus, a reasonable discount ratio needs to satisfy $0 < \theta < 1$.

Proposition 3 Under decentralized decision scenario, we consider incentive contract integrating trade credit and quantity discount when sales effort influences the market demand. We obtain the optimal order quantity as

$$q_3^r = eF^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right) \qquad (11)$$

And the optimal sales effort level meets

$$\frac{dC(e_3^r)}{de_3^r} = (1 + i_r t)(p_r - v)R(q_0) \qquad (12)$$

Where $q_0 = F^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)$,

$R(q_0) = \int_0^{q_0} \delta d\, F(\delta)$.

Proof: See "Appendix G."

From Proposition 3, the retailer's optimal expected profit is $\prod_3^r(q_3^r, e_3^r) = e_3^r(1 + i_r t)(p_r - v)R(q_0) - C(e_3^r)$, and the optimal expected profit of the manufacturer is

$$\prod_3^m(q_3^r, e_3^{rc}) = (\theta p_m - (1 + i_m t)c_m)e_3^r q_0$$

Theorem 8 Considering the level of sales effort,

comparing the hybrid model of the trade credit and quantity discount with the single trade credit contract, the optimal sales effort level e_3^r of the retailer in the former case is higher than the optimal sales effort level e_2^r in the latter case, that is, $e_3^r > e_2^r$. The optimal order quantity q_3^r of the retailer in the former case is higher than the optimal order quantity q_2^r in the latter case, that is, $q_3^r > q_2^r$. In addition, the optimal effort level e_3^r and the optimal order quantity q_3^r of the retailer increase with the decrease in θ under the hybrid coordination contract.

Proof: See "Appendix H."

Theorem 8 shows that with the introduction of a quantity discount mechanism in a trade credit contract that considers the level of sales effort, the retailer is encouraged to increase the order quantity by reducing the wholesale price of products. At the same time, under the hybrid coordination contract, with the decrease in θ and the decrease in the wholesale price of the manufacturer's products, the retailer is encouraged to improve the optimal effort level and increase its order quantity.

To achieve the perfect coordination of the supply chain system under the hybrid coordination contract, the retailer's order quantity needs to increase to q_2^s.

Theorem 9 When the quantity discount ratio θ offered by the manufacturer satisfies the following conditions: $\bar{\theta} = \frac{(1 + i_m t)c_m}{p_m}$ (13)

Quantity discount contract based on the trade credit can make the supply chain reach the optimal state. In the meantime, $\bar{\theta}$ increases with the increase in the trade credit period t.

Proof: See "Appendix K."

Theorem 9 shows that the quantity discount ratio $\bar{\theta}$ causing the supply chain to achieve the coordination increases with the increase of the trade credit period provided by the manufacturer. It can be further analysed that when the trade credit period t increases, the manufacturer's commitment to the capital opportunity cost will increase, so it will improve the discount ratio $\bar{\theta}$ to make up for its own losses. Similarly, when the trade credit period t is reduced, the manufacturer can compensate for the reduction in the retailer's earnings by reducing the quantity discount ratio $\bar{\theta}$.

According to Theorem 9, the manufacturer's expected profit is zero when the supply chain achieves the perfect coordination, which is unacceptable to the manufacturer. To achieve coordination, it is necessary to reallocate the total revenue $\prod_3^s(q_2^s, e_2^s)$ of the supply chain system, which assigns φ proportion of $\prod_3^s(q_2^s, e_2^s)$ to the manufacturer and $1 - \varphi$ ratio to the retailer. After the

reallocation, the manufacturer's expected revenue $\prod_4^r(q_2^s, e_2^s)$ should be not lower than the expected revenue $\prod_2^r(q_2^r, e_2^r)$ without the introduction of the quantity discount contract, and the retailer's expected revenue $\prod_4^m(q_2^s, e_2^s)$ should be not lower than the expected revenue $\prod_2^m(q_2^r, e_2^r)$ without the introduction of quantity discount contract.

Theorem 10 When the income distribution ratio φ meets the following condition: $\varphi_1 \leq \varphi \leq \varphi_2$. Here

$$\varphi_1 = \frac{(p_m - (1+i_m t)c_m)q_2^r}{e_2^s(1+i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}$$

$$\varphi_2 = \frac{(1+i_r t)(p_r - v)(e_2^s R(q_1^s) - e_2^r R(q_1^r)) + C(e_2^r) - C(e_2^s)}{e_2^s(1+i_r t)(p_r - v)R(q_1^s) - C(e_2^s)} \quad (14)$$

At this time, the manufacturer is willing to provide the quantity discount ratio which meets the equation (13), and the expressions $\prod_4^r(q_2^s, e_2^s) \geq \prod_2^r(q_2^r, e_2^r)$ and $\prod_4^m(q_2^s, e_2^s) \geq \prod_2^m(q_2^r, e_2^r)$ have been established.

Proof: See "Appendix L."

Theorem 10 shows that in the case of the market demand affected by the retailer's sales effort level, when the quantity discount ratio θ and the profit distribution ratio φ satisfy equation (13) and equation (14), the portfolio coordination contract of the trade credit considering the sales effort level and the quantity discount can realize the coordination of the supply chain, that is, the system stays to achieve the optimal state. By adjusting the distribution of the system income through the parameter φ, the flexibility of the system is increased and the effective cooperation between the two is promoted.

5. An Illustrating Example

We now employ numerical examples to further analyse the trade credit model and the hybrid model integrating trade credit and quantity discount when the sales effort influences the market demand. In the supply chain configuration, we suppose that $p_m = 60, p_r = 188,\ c_r = 40, v = 20, i_m = 10\%, i_r = 12\%$, assume the sales effort cost follows $C(e) = 500(e-1)^2$ and the market demand δ is subject to normal distribution, i.e., $\delta \sim N(100, 50^2)$.

First, we analyse the impact of the retailer's sales effort level on the order quantity and the expected profit. The results are shown in Figure 1 and Figure 2.
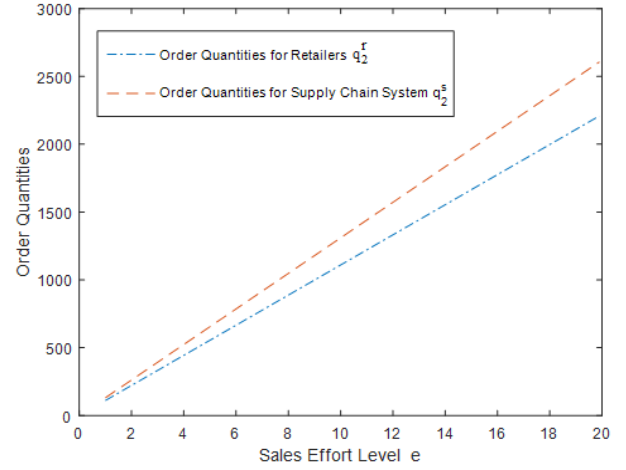

FIGURE 1 Chang trend of order quantity of retailer and supply chain with sales effort level (t=1)
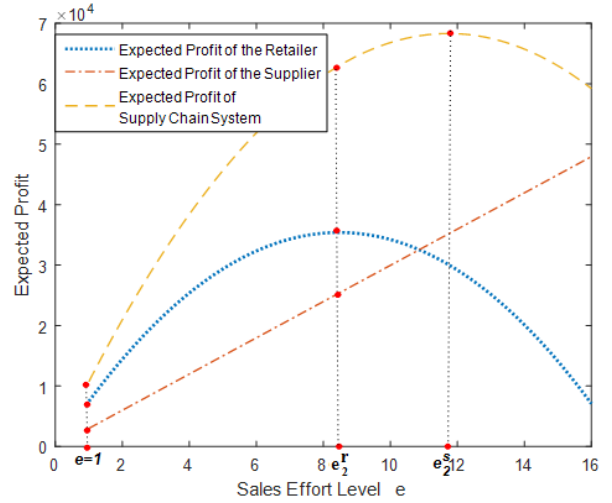

FIGURE 2 Chang trend of expected profit of retailer and manufacturer and supply chain with sales effort level (t=1)

As seen in Figure 1, in the case that the credit period is determined, the increase in the level of sales effort can prompt the retailer and supply chain system to respectively improve the order quantity. This is because the level of sales effort has an impact on the market demand, which indirectly increases the number of products ordered by increasing the market demand, thus increasing the order of the supply chain system.

As found in Figure 2, the expected profit of the retailer tends to rise first and then fall with the increase of the sales effort level. When $e = e_2^r$, the benefits and the effort costs coming with increasing the order quantity reach equilibrium, so the retailer can receive its maximum expected profit. The manufacturer's expected profit increases with the increase of the sales effort level, so it is beneficial for the manufacturer when the retailer invests more effort. The expected profit of the supply chain system tends to rise first and then fall with the increase of the sales effort level. When $e = e_2^s$, the system can receive maximum expected profit. However, in the real world, the retailer does not incur a higher sales effort level than e_2^c, so the retailer and manufacturer's

expected profit will increase with the increase of the sales effort level on $e \in [1, e_2^r]$. Similarly, the supply chain system does not incur a higher sales effort level than e_2^s, so the expected profit of supply chain system will increase with the increase of sales effort level on $e \in [1, e_2^s]$.

Then, the analysis is done about the impact of trade credit period on the decision and the optimal expectation profits of the retailer, the manufacturer and the supply chain system. First, according to the equation $t < \frac{p_m - c_m}{c_m i_m}$ in Section 3.1, we determine the range of credit period t in $0 < t < 10$ is determined based on the above data set. Figures 3, 4 and 5 are obtained by using MATLAB software to the developed model.
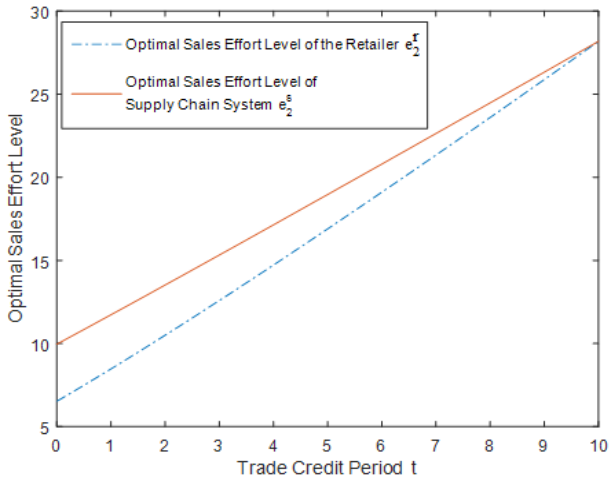

FIGURE 3 Chang trend of optimal sales effort level of retailer and supply chain with trade credit period
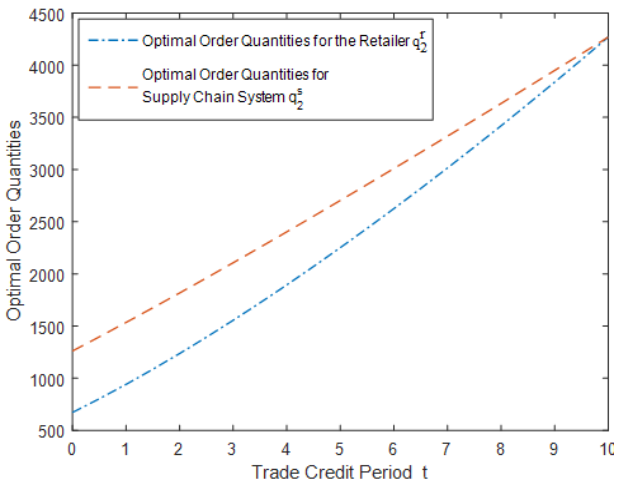

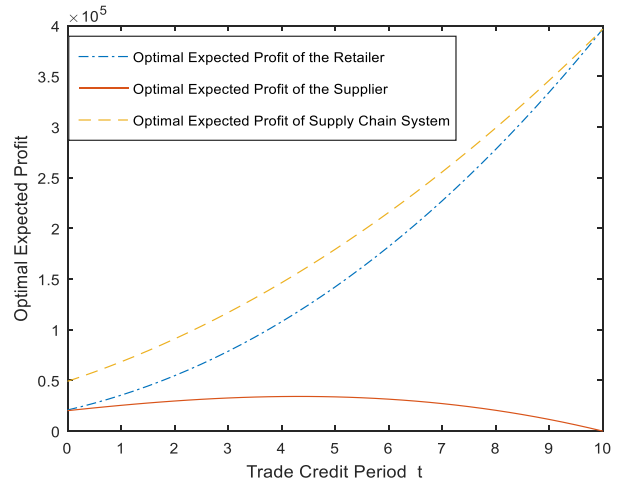FIGURE 4 Chang trend of optimal order quantity of retailer and supply chain with trade credit period

FIGURE 5 Chang trend of optimal expected profits of retailer and manufacturer and supply chain with trade credit period

As seen in Figure 3 and Figure 4, the trade credit period provided by the manufacturer has a positive impact on the optimal sales effort level and optimal order quantity for the retailer and the supply chain system. As found in Figure 5, the optimal expected profits of both the retailer and the supply chain system increase with the increase in the trade credit period, but the optimal expected profit of the manufacturer decreases with the increase in the trade credit period. This is because if the manufacturer allows the retailer to delay the payment of goods, the retailer could use this payment to invest and gain investment income, and the manufacturer would increase capital opportunity cost.
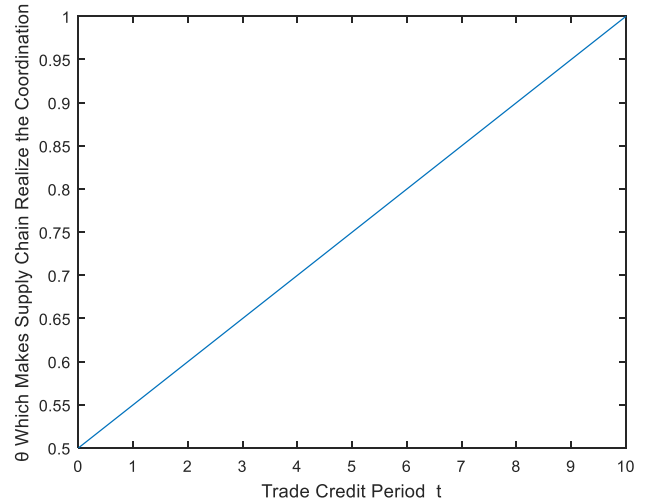

FIGURE 6 Chang trend of quantity discount ratio with trade credit period when the supply chain reaches coordination

As is seen from Figure 6, the longer is the trade credit period t provided by the manufacturer, the greater is the quantity discount ratio $\bar{\theta}$, which means that the manufacturer can adjust the t and $\bar{\theta}$ parameters to balance its own decisions. When the trade credit period t increases, the manufacturer takes on greater capital opportunity costs, but it can increase the discount ratio $\bar{\theta}$ to make up for its own loss, and when the trade credit

period t decreases, the manufacturer can compensate for the reduction in its profit by reducing the quantity discount radio $\bar{\theta}$.

6. Conclusions

Considering trade credit as a tool for supply chain coordination, when market demand is influenced by retailer sales effort, we construct the quantitative models in a newsvendor framework respectively based on the single trade credit contract and the composite contract combining trade credit and quantity discount. And we calculate and analyze the models by Stackelberg game theory and optimization theory. Some interesting insights are obtained as following.

First, when retailer sales effort influences market demand, a single trade credit cannot perfectly coordinate the supply chain, that is, the optimal sales effort level and order quantity and system's profit under the decentralized decision scenario are lower than those under the centralized decision scenario.

Second, based on a single trade credit, when retailer sales effort influences market demand, the optimal order quantity as well as the profits of the manufacturer and retailer and supply chain are higher than those when retailer sales effort does not influence market demand. Whether under decentralized decision scenario or under centralized decision scenario, when sales effort influences the market demand, the optimal effort level and optimal order quantity increase as trade credit period increases, respectively.

Third, considering trade credit and order quantity discount and sales effort effects on the demand, we show that when model parameters meet certain conditions, the composite contract combining trade credit and quantity discount can realize the perfect coordination of supply chain and a "win-win" outcome. Furthermore, the composite contract can provide great flexibility in coordinating the supply chain through adjusting relevant parameters.

In future studies, we can extend the models to the three-tier supply chain (manufacturer-retailer-consumer supply chain). And we can also continue the study from the perspective of information asymmetry. For example, when the sales effort is private information of the retailer, we explore the supply chain coordination under trade credit.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgements

Appendix A. Proof of Proposition 1

We take the partial derivatives of equation (1) and obtain:

$$\frac{\partial^2 \prod_2^r(q,e)}{\partial q^2} = -\frac{1}{e}(1+i_r t)(p_r - v)f\left(\frac{q}{e}\right);$$

$$\frac{\partial^2 \prod_2^r(q,e)}{\partial e^2} = -\frac{q^2}{e^3}(1+i_r t)(p_r - v)f\left(\frac{q}{e}\right) - \frac{d^2 C(e)}{de^2};$$

$$\frac{\partial^2 \prod_2^r(q,e)}{\partial q \partial e} = \frac{q}{e^2}(1+i_r t)(p_r - v)f\left(\frac{q}{e}\right);$$

$$\frac{\partial^2 \prod_2^r(q,e)}{\partial e \partial q} = \frac{q}{e^2}(1+i_r t)(p_r - v)f\left(\frac{q}{e}\right);$$

In addition, with the assumptions $p_r > v$, $\frac{d^2 C(e)}{de^2} > 0$, the Hessian matrix of the binary function $\prod_2^r(q,e)$ can be determined.

$$|H| = \begin{vmatrix} \frac{\partial^2 \prod_2^r(q,e)}{\partial q^2} & \frac{\partial^2 \prod_2^r(q,e)}{\partial q \partial e} \\ \frac{\partial^2 \prod_2^r(q,e)}{\partial e \partial q} & \frac{\partial^2 \prod_2^r(q,e)}{\partial e^2} \end{vmatrix}$$

$$= \frac{\partial^2 \prod_2^r(q,e)}{\partial q^2} \times \frac{\partial^2 \prod_2^r(q,e)}{\partial e^2} - \frac{\partial^2 \prod_2^r(q,e)}{\partial q \partial e} \times \frac{\partial^2 \prod_2^r(q,e)}{\partial e \partial q}$$

$$= \frac{1}{e}(1+i_r t)(p_r - v)f\left(\frac{q}{e}\right)\frac{d^2 C(e)}{de^2} > 0 \quad, \quad \text{and}$$

$$\frac{\partial^2 \prod_2^r(q,e)}{\partial q^2} < 0, \quad \frac{\partial^2 \prod_2^r(q,e)}{\partial e^2} < 0.$$

Therefore, according to optimization theory, $\prod_2^r(q,e)$ has the only optimal solution (q_2^r, e_2^r) on $q \in (0, +\infty) \cup e \in [1, +\infty)$, and respectively satisfies the following first order conditions.

$$\frac{\partial \prod_2^r(q,e)}{\partial q} = (1+i_r t)p_r - c_r - p_m$$

$$-(1+i_r t)(p_r - v)F\left(\frac{q}{e}\right) = 0$$

$$\frac{\partial \prod_2^r(q,e)}{\partial e} = -(1+i_r t)(p_r - v)\int_0^{q/e} F(\delta)\,d\delta +$$

$$\frac{q}{e}(1+i_r t)(p_r - v)F\left(\frac{q}{e}\right) - \frac{dC(e)}{de} = 0$$

At this time, the retailer's optimal order quantity q_2^r can be obtained.

$$q_2^r = eF^{-1}\left(1 - \frac{c_r + p_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right) \tag{A1}$$

Additionally, the optimal effort level e_2^r of the retailer meets the condition.

$$\frac{dC(e_2^r)}{de_2^r} = (1+i_rt)(p_r-v)R\left(\frac{q}{e_2^r}\right) \qquad (A2)$$

Here $R(q) = \int_0^q \delta d\, F(\delta)$

Only when equation (A1) satisfies equation (A2), the retailer is able to achieve the greatest gain. Thus, the optimal order quantity of the retailer is $q_2^r = e_2^r q_1^r$, and the optimal selling effort level e_2^r satisfies:

$$\frac{dC(e_2^r)}{de_2^r} = (1+i_rt)(p_r-v)R(q_1^r)$$

Hence, we conclude the proof.

Appendix B. Proof of Proposition 2

We take the partial derivatives of the equation (6) and obtain:

$$\frac{\partial^2 \prod_2^s(q,e)}{\partial q^2} = -\frac{1}{e}(1+i_rt)(p_r-v)f\left(\frac{q}{e}\right);$$

$$\frac{\partial^2 \prod_2^s(q,e)}{\partial e^2} = -\frac{q^2}{e^3}(1+i_rt)(p_r-v)f\left(\frac{q}{e}\right) - \frac{d^2C(e)}{de^2};$$

$$\frac{\partial^2 \prod_2^s(q,e)}{\partial q\partial e} = \frac{q}{e^2}(1+i_rt)(p_r-v)f\left(\frac{q}{e}\right);$$

$$\frac{\partial^2 \prod_2^s(q,e)}{\partial e\partial q} = \frac{q}{e^2}(1+i_rt)(p_r-v)f\left(\frac{q}{e}\right);$$

In addition, with the assumptions $p_r > v$, $\frac{d^2C(e)}{de^2} > 0$, it can be determined the Hessian matrix $|H|$ of the binary function $\prod_2^s(q,e)$. Here, $|H| > 0$, $\frac{\partial^2 \prod_2^r(q,e)}{\partial q^2} < 0$, $\frac{\partial^2 \prod_2^r(q,e)}{\partial e^2} < 0$.

Therefore, according to optimization theory, $\prod_2^s(q,e)$ has the only optimal solution (q_2^s, e_2^s), and respectively satisfies the following first order conditions.

$$\frac{\partial \prod_2^s(q,e)}{\partial q} = (1+i_rt)p_r - c_r - (1+i_mt)c_m -$$
$$(1+i_rt)(p_r-v)F\left(\frac{q}{e}\right) = 0$$

$$\frac{\partial \prod_2^s(q,e)}{\partial e} = -(1+i_rt)(p_r-v)\int_0^{q/e} F(\delta)\,d\delta +$$
$$\frac{q}{e}(1+i_rt)(p_r-v)F\left(\frac{q}{e}\right)\frac{dC(e)}{de} = 0$$

Then, it is obtained:

$$q_2^s = eF^{-1}\left(1 - \frac{c_r+(1+i_mt)c_m-(1+i_rt)v}{(1+i_rt)(p_r-v)}\right) \qquad (B1)$$

$$\frac{dC(e_2^s)}{de_2^s} = (1+i_rt)(p_r-v)R\left(\frac{q}{e_2^s}\right). \text{ Here, } R(q) =$$
$\int_0^q \delta d\, F(\delta)$

Equation (B1) shows the relationship between the optimal order quantity and the sales effort level, so the optimal order quantity of the system needs to meet
$$q_2^s = e_2^s q_1^s = e_2^s F^{-1}\left(1 - \frac{c_r+(1+i_mt)c_m-(1+i_rt)v}{(1+i_rt)(p_r-v)}\right) \text{ in}$$
order to ensure the system's best benefits, and the optimal sales effort level e_2^s meets

$$\frac{dC(e_2^s)}{de_2^s} = (1+i_rt)(p_r-v)R(q_1^s)$$

Therefore, the system's optimal expected profit is $\prod_2^s(q_2^s, e_2^s) = e_2^s(1+i_rt)(p_r-v)R(q_1^s) - C(e_2^s)$. Hence, we conclude the proof.

Appendix C. Proof of Theorem 4

Let $\prod_2^s(q,e) = e(1+i_rt)(p_r-v)R(q_1^s) - C(e)$, where $q=eq_1^s$. The first order partial derivative of e for equation $\prod_2^s(q,e)$ is

$$\frac{\partial \prod_2^s(q,e)}{\partial e} = (1+i_rt)(p_r-v)R(q_1^s) - \frac{dC(e)}{de},$$

and it is known $\frac{dC(e_2^s)}{de_2^s} = (1+i_rt)(p_r-v)R(q_1^s)$ from equation (8). Then, it is obtained

$$\frac{\partial \prod_2^s(q,e)}{\partial e} = \frac{dC(e_2^s)}{de_2^s} - \frac{dC(e)}{de}$$

Then, $C'(e)$ is monotonically increasing on $e \in [1, e_2^s]$ because of $\frac{d^2C(e)}{de^2} > 0$, it can be obtained $\frac{dC(e)}{de} \le \frac{dC(e_2^s)}{de_2^s}$, that is $\frac{\partial \prod_2^s(q,e)}{\partial e} \ge 0$, further it is obtained that $\prod_2^s(q,e)$ is an increasing function on $e \in [1, e_2^s]$. At this time, the level of sales effort in the supply chain system varies from 1 to e_2^s, and the expected return of the entire system is improved.

$\prod_2^s(q,e) = \prod_2^s(q_1^s, e=1)$ is accurate when the sales effort level remains constant (e=1). Therefore, $\prod_2^s(q = e_2^s q_1^s, e_2^s) > \prod_2^s(q = q_1^s, e = 1)$ proves to be correct, that is, $\prod_2^s(q_2^s, e_2^s) > \prod_2^s(q_1^s, e = 1)$. Hence, we conclude the proof.

Appendix D. Proof of Theorem 5

(1) We take the derivative of t for equation (5) and obtain

$$\frac{d\left(\frac{dC(e_2^r)}{de_2^r}\right)}{de_2^r}\frac{de_2^r}{dt}$$
$$= i_r(p_r-v)R(q_1^r) + (1+i_rt)(p_r-v)\frac{dR(q_1^r)}{dt}.$$
Where

$$\frac{dR(q_1^r)}{dt} = \frac{i_r(c_r+p_m)}{(1+i_rt)^2(p_r-v)}\left(F^{-1}\left(1 - \frac{c_r+p_m-(1+i_rt)v}{(1+i_rt)(p_r-v)}\right)\right)' q_1^r f(q_1^r).$$

Because $F(\delta)$ is a continuously differentiable and reversible strictly increasing function, $\left(F^{-1}\left(1 - \frac{c_r+p_m-(1+i_rt)v}{(1+i_rt)(p_r-v)}\right)\right)' > 0$ is right, thus it can be obtained $\frac{dR(q_1^r)}{dt} > 0$, and for $R(q_1^r) =$

$\int_0^{q_1^r} \delta \, dF(\delta) = \int_0^{q_1^r} \delta f(\delta)d\delta$, it is known that $\delta f(\delta) > 0$ and $q_1^r > 0$, so $R(q_1^r) > 0$ is right. Then, the right-hand side of equation (8) is greater than zero.

In addition, according to the assumption that $C(e)$ in the interval $e \in [1, +\infty)$ is a monotonically increasing convex function, the first part of the left-hand side of the equation (8) is greater than 0, so $\frac{de_2^r}{dt} > 0$ is established.

We take the derivative of t for equation (8) and obtain

$$\frac{d\left(\frac{dC(e_2^s)}{de_2^s}\right)}{de_2^s}\frac{de_2^s}{dt}$$

$$= i_r(p_r - v)R(q_1^s) + (1 + i_r t)(p_r - v)\frac{dR(q_1^s)}{dt}$$

Where

$$\frac{dR(q_1^s)}{dt} = \frac{i_r c_r + c_m(i_r - i_m)}{(1+i_r t)^2(p_r - v)} \times$$

$$\left(F^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)\right)' q_1^s f(q_1^s)$$

In addition, according to $i_r > i_m$, it is obtained $\frac{dR(q_1^s)}{dt} > 0$. Similarly, $\frac{de_2^s}{dt} > 0$ is available, so the retailer's optimal effort level e_2^r and the supply chain system's optimal effort level e_2^s increase with the increase of the trade credit period t.

(2) We take the derivative of t for equation (3) and obtain

$$\frac{dq_2^r}{dt} = \frac{de_2^r}{dt}F^{-1}\left(1 - \frac{c_r + p_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)$$

$$+e_2^r\frac{i_r(c_r + p_m)}{(1+i_r t)^2(p_r - v)}\left(F^{-1}\left(1 - \frac{c_r + p_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)\right)'$$

Because $\frac{de_2^c}{dt} > 0$ has proven to be correct, it is known $\frac{dq_2^c}{dt} > 0$. It is obtained

$$q_2^s = e_2^s F^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)$$

from equation (7).

By taking the derivative of t for equation (7), we obtain

$$\frac{dq_2^s}{dt} = \frac{de_2^s}{dt}F^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right) +$$

$$e_2^s\frac{i_r c_r + c_m(i_r - i_m)}{(1+i_r t)^2(p_r - v)}\left(F^{-1}\left(1 - \frac{c_r + (1+i_m t)c_m - (1+i_r t)v}{(1+i_r t)(p_r - v)}\right)\right)'$$

Similarly, $\frac{dq_2^s}{dt} > 0$ is available, so the retailer's optimal order quantity q_2^r and the supply chain system's optimal order quantity q_2^s increase with the increase of the trade credit period t. Hence, we conclude the proof.

Appendix E. Proof of Theorem 6

(1) With the known condition $\prod_2^r(q_2^r, e_2^r) = e_2^r(1 + i_r t)(p_r - v)R(q_1^r) - C(e_2^r)$, we take the derivative of t for $\prod_2^r(q_2^r, e_2^r)$ and obtain

$$\frac{d\prod_2^r(q_2^r, e_2^r)}{dt} = \left[(1 + i_r t)(p_r - v)R(q_1^r) - \frac{dC(e_2^r)}{de_2^r}\right]\frac{de_2^r}{dt}$$

$$+e_2^r i_r(p_r - v)R(q_1^r) + e_2^r(1 + i_r t)(p_r - v)\frac{dR(q_1^r)}{dt}$$

$$\text{(E1)}$$

It is seen that the first part of the right-hand side of equation (E1) is equal to zero from the equation (5), and because $\frac{dR(q_1^r)}{dt} > 0$, we obtain $\frac{d\prod_2^r(q_2^r, e_2^r)}{dt} > 0$. Therefore, there exists always a minimum trade credit period $\hat{t}$, when the retailer's optimal expected return $\prod_2^r(q_2^r, e_2^r)$ is N^r. Then, it has been proven correct that $\prod_2^r(q_2^r, e_2^r)$ is not lower than N^r for $t \in [\hat{t}, +\infty)$ and increases as the trade credit period t increases.

(2) With the known condition $\prod_2^s(q_2^s, e_2^s) = e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)$, we take the derivative of t for $\prod_2^s(q_2^s, e_2^s)$ and obtain

$$\frac{d\prod_2^s(q_2^s, e_2^s)}{dt} = \left[(1 + i_r t)(p_r - v)R(q_1^s) - \right.$$

$$\left.\frac{dC(e_2^s)}{de_2^s}\right]\frac{de_2^s}{dt} + e_2^s i_r(p_r - v)R(q_1^s) + e_2^s(1 +$$

$$i_r t)(p_r - v)\frac{dR(q_1^s)}{dt}\text{(E2)}$$

It is seen that the first part of the right-hand side of equation (E2) is equal to zero from equation (8), and because $\frac{dR(q_1^s)}{dt} > 0$ has been proven correct, $\frac{d\prod_2^s(q_2^s, e_2^s)}{dt} > 0$ is established. Therefore, there always exists a minimum trade credit period $\tilde{t}$ when the retailer's optimal expected return $\prod_2^s(q_2^s, e_2^s)$ is N^s. Then, it has been proven correct that $\prod_2^s(q_2^s, e_2^s)$ is not lower than N^s for $t \in [\tilde{t}, +\infty)$ and increases as the trade credit period t increases.

Hence, we conclude the proof.

Appendix F. Proof of Theorem 7

(1) Because $\frac{dR(q)}{dq} = qf(q) > 0$ and then $R(q_1^r) < R(q_1^s)$ according to $q_1^r < q_1^s$, $R(q)$ is monotonically increasing on $q \in (0, \infty)$. It is easy to know $\frac{dC(e_2^r)}{de_2^r} < \frac{dC(e_2^s)}{de_2^s}$ by comparing equation (5) with equation (8), and because $C''(e) > 0$, that is, $C'(e)$ is monotonically increasing on $e \in [1, \infty)$, $e_2^r < e_2^s$ is proved to be correct.

(2) $q_2^r < q_2^s$ is correct because of $q_1^r < q_1^s$, $e_2^r <$

e_2^s with known conditions $q_2^r = e_2^r q_1^r$, $q_2^s = e_2^s q_1^s$.

(3) From equation (6), we have

$\frac{\partial \prod_2^s(q,e)}{\partial q} = (1 + i_r t)p_r - c_r - (1 + i_m t)c_m$

$-(1 + i_r t)(p_r - v)F\left(\frac{q}{e}\right)$.

Based on $\frac{\partial \prod_2^s(q,e)}{\partial q} = 0$, We obtain

$(1 + i_r t)p_r - c_r - (1 + i_m t)c_m$
$= (1 + i_r t)(p_r - v)F(q_2^s / e)$.

So

$\frac{\partial \prod_2^s(q,e)}{\partial q} = (1 + i_r t)(p_r - v)(F(q_2^s / e) - F(q/e))$.

$F(\delta)$ is strictly increasing function, and $q_2^s > q_2^r$,
so $\frac{\partial \prod_2^s(q,e)}{\partial q} > 0$ proves correct, that is, $\prod_2^s(q,e)$ is
monotonically increasing on $q \in [q_2^r, q_2^s]$. Therefore,
it can be obtained $\prod_2^s(q_2^r, e_2^s) < \prod_2^s(q_2^s, e_2^s)$ from
$q_2^r < q_2^s$ for any e.

Similarly, it is known as follows.

$\frac{\partial \prod_2^s(q,e)}{\partial e} = (1 + i_r t)(p_r - v)R\left(\frac{q}{e}\right) - \frac{dC(e)}{de}$

As $\frac{dC(e_2^s)}{de_2^s} = (1 + i_r t)(p_r - v)R\left(\frac{q}{e_2^s}\right)$, we obtain

$\frac{\partial \prod_2^s(q,e)}{\partial e} = \frac{R\left(\frac{q}{e}\right)}{R\left(\frac{q}{e_2^s}\right)} \frac{dC(e_2^s)}{de_2^s} - \frac{dC(e)}{de} \cdot \frac{q}{e} \geq \frac{q}{e_2^s}$ is established

when $e \in [e_2^r, e_2^s]$, so $\frac{R\left(\frac{q}{e}\right)}{R\left(\frac{q}{e_2^s}\right)} \geq 1$. In addition,

$\frac{dC(e_2^s)}{de_2^s} \geq \frac{dC(e)}{de}$ is also correct, so it can be obtained
$\frac{\partial \prod_2^s(q,e)}{\partial e} \geq 0$ on $e \in [e_2^r, e_2^s]$, that is, $\prod_2^s(q,e)$ is
monotonically increasing on $e \in [e_2^r, e_2^s]$. Therefore,
we can obtain $\prod_2^s(q_2^r, e_2^r) < \prod_2^s(q_2^r, e_2^s)$ from
$e_2^r < e_2^s$ for any q given. Thus, $\prod_2^s(q_2^r, e_2^r) < \prod_2^s(q_2^s, e_2^s)$ proves correct.

Hence, we conclude the proof.

Appendix G. Proof of Proposition 3

We take the second order partial conduct of q for
equation (9) and obtain

$\frac{\partial^2 \prod_3^r(q,e)}{\partial q^2} = -\frac{1}{e}(1 + i_r t)(p_r - v)f\left(\frac{q}{e}\right)$

It is obvious that $\frac{\partial^2 \prod_3^r(q,e)}{\partial q^2} < 0$ because of
$f\left(\frac{q}{e}\right) > 0$ and $p_r > v$. So $\prod_3^r(q,e)$ has the
only optimal solution q_3^r on $q \in (0, \infty)$, which
meets

$q_3^r = eF^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)$

The retailer's optimal order quantity is q_0 and
meets $q_0 = F^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)$ when $e = 1$.

We take the second order partial conduct of e for
equation (9) and obtain

$\frac{\partial^2 \prod_3^r(q,e)}{\partial e^2} = -\frac{q^2}{e^3}(1 + i_r t)(p_r - v)f\left(\frac{q}{e}\right) - \frac{d^2C(e)}{de^2}$

It is obvious that $\frac{\partial^2 \prod_3^r(q,e)}{\partial e^2} < 0$ because of
$f\left(\frac{q}{e}\right) > 0$ and $\frac{d^2C(e)}{de^2} > 0$. So $\prod_3^r(q,e)$ has the
only optimal solution e_3^r on $e \in [1, \infty)$, which meets
$\frac{dC(e_3^r)}{de_3^r} = (1 + i_r t)(p_r - v)R\left(\frac{q}{e_3^r}\right)$

As a result, the retailer's optimal order quantity is
$q_3^r = e_3^r q_0$, and the optimal sales effort level e_3^r
satisfies

$\frac{dC(e_3^r)}{de_3^r} = (1 + i_r t)(p_r - v)R(q_0)$

Hence, we conclude the proof.

Appendix H. Proof of Theorem 8

Comparing equation (5) and equation (12), since
$0 < \theta < 1$ and $F^{-1}(\delta)$ is a continuous and
monotonically increasing function, it is known $q_0 >
q_1^r$, and $R(q)$ is also a monotonically increasing
function, so $\frac{dC(e_3^r)}{de_3^r} > \frac{dC(e_2^r)}{de_2^r}$. Then, because $C'(e)$ is
a monotonically increasing function, $e_3^r > e_2^r$ has
been proven correct.

It is obvious that $q_3^r > q_2^r$ because $q_3^r = e_3^r q_0$
and $q_2^r = e_2^r q_1^r$ as well as $q_0 > q_1^r$ and $e_3^r > e_2^r$.

We take the first order partial conduct of θ for
equation (12) and obtain

$C''(e_3^r)\frac{\partial e_3^r}{\partial \theta} = (1 + i_r t)(p_r - v)\frac{\partial R(q_0)}{\partial \theta}$

We obtain $\frac{\partial R(q_0)}{\partial \theta} < 0$ from $\frac{\partial R(q_0)}{\partial \theta} = q_0 f(q_0)\frac{\partial q_0}{\partial \theta}$,

where $\frac{\partial q_0}{\partial \theta} = \frac{-p_m}{(1 + i_r t)(p_r - v)}\left(F^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)\right)' < 0$. And $\frac{\partial e_3^r}{\partial \theta} < 0$ because of
$C''(e) > 0$. The following is obtained $\frac{\partial q_3^r}{\partial \theta} = \frac{\partial e_3^r}{\partial \theta}q_0 + e_3^r\frac{\partial q_0}{\partial \theta}$ from the known equation $q_3^r = e_3^r q_0$.
Thus, $\frac{\partial q_3^r}{\partial \theta} < 0$ is obtained from the above conditions
$\frac{\partial e_3^r}{\partial \theta} < 0$, $\frac{\partial q_0}{\partial \theta} < 0$, so the optimal effort level e_3^r and
the optimal order quantity q_3^r of the retailer increase
with the decrease in θ, respectively.

Hence, we conclude the proof.

Appendix K. Proof of Theorem 9

(1) If the supply chain is to be coordinated, it must

be satisfied that the variables (q_3^r, e_3^r) in the decentralised decision mode equal to the variables (q_2^s, e_2^s) in the centralized decision mode. Since $q_3^r = e_3^r q_0$, $q_2^s = e_2^s q_1^s$,

$$\frac{dC(e_3^r)}{de_3^r} = (1 + i_r t)(p_r - v)R(q_0),$$

$$\frac{dC(e_2^s)}{de_2^s} = (1 + i_r t)(p_r - v)R(q_1^s),$$

it can be obtained

$$\begin{cases} \frac{dC(e_3^r)}{de_3^r} = \frac{dC(e_2^s)}{de_2^s} \\ e_3^r q_0 = e_2^s q_1^s \end{cases} \quad (K1)$$

Here, $q_0 = F^{-1}\left(1 - \frac{c_r + \theta p_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)$, $q_1^s = F^{-1}\left(1 - \frac{c_r + (1 + i_m t)c_m - (1 + i_r t)v}{(1 + i_r t)(p_r - v)}\right)$.

It is obtained $\bar{\theta} = \frac{(1 + i_m t)c_m}{p_m}$ by solving equation (K1). At that time, the retailer is encouraged to increase his order quantity to q_2^s and realizes the supply chain's perfect coordination. The retailer's expected profit is

$$\prod_3^r(q_3^r, e_3^r)$$
$$= \prod_3^r(q_2^s, e_2^s) = e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)$$
$$= \prod_2^s(q_2^s, e_2^s) = \prod_3^s(q_2^s, e_2^s)$$

And the manufacturer's expected profit is

$$\prod_3^m(q_3^r, e_3^r) = \prod_3^m(q_2^s, e_2^s) = 0$$

This shows that when the supply chain realizes the coordination, the supply chain system achieves the best income and the retailer enjoys all the benefits, while the manufacturer's earning is 0.

(2) It can be obtained $\frac{d\bar{\theta}}{dt} = \frac{i_m c_m}{p_m} > 0$ by taking the derivative of t for the equation (13).

Hence, we conclude the proof.

Appendix L. Proof of Theorem 10

In order for the manufacturer to be willing to provide a quantity discount to satisfy equation (14), the following conditions must be met:

$$\begin{cases} \prod_4^r(q_2^s, e_2^s) \geq \prod_2^r(q_2^r, e_2^r) \\ \prod_4^m(q_2^s, e_2^s) \geq \prod_2^m(q_2^r, e_2^r) \end{cases} \quad (L1)$$

Where, $\prod_4^r(q_2^s, e_2^s) = (1 - \varphi)\prod_3^s(q_2^s, e_2^s)$, $\prod_4^m(q_2^s, e_2^s) = \varphi\prod_3^s(q_2^s, e_2^s)$.

Therefore, by plugging q_2^s, e_2^s, q_2^r, e_2^r into equation (L1) and calculating, the following are obtained:

$$\varphi \geq \frac{(p_m - (1 + i_m t)c_m)q_2^r}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}, \qquad \varphi \leq$$
$$\frac{(1 + i_r t)(p_r - v)\left(e_2^s R(q_1^s) - e_2^r R(q_1^r)\right) + C(e_2^r) - C(e_2^s)}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}$$

Here, $\varphi_1 = \frac{(p_m - (1 + i_m t)c_m)q_2^r}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}$,

$$\varphi_2 = \frac{(1 + i_r t)(p_r - v)\left(e_2^s R(q_1^s) - e_2^r R(q_1^r)\right) + C(e_2^r) - C(e_2^s)}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}.$$

Then we obtain the following equation:

$$\varphi_2 - \varphi_1 = \frac{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)} -$$
$$\frac{e_2^r(1 + i_r t)(p_r - v)R(q_1^r) - C(e_2^r) + (p_m - (1 + i_m t)c_m)q_2^r}{e_2^s(1 + i_r t)(p_r - v)R(q_1^s) - C(e_2^s)}$$
$$= \frac{\prod_2^s(q_2^s, e_2^s) - \prod_2^s(q_2^r, e_2^r)}{\prod_2^s(q_2^s, e_2^s)}$$

It is obvious that $\varphi_2 - \varphi_1 > 0$ because of $\prod_2^s(q_2^s, e_2^s) > \prod_2^s(q_2^r, e_2^r)$. Then, the condition $\varphi_1 \leq \varphi \leq \varphi_2$ is established.

Hence, we conclude the proof.

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