Formal versus Informal Legislative Bargaining*

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We study how the formality of a bargaining procedure affects its outcome. We compare a formal Baron-Ferejohn bargaining procedure to an informal procedure where players make and accept proposals in continuous time. Both constitute non-cooperative games corresponding to the same bargaining problem: a three-player median voter setting with an external disagreement point. This allows us to study formality in the presence and absence of a core and provides a natural explanation for the effects of preference polarization. Our results show that polarization hurts the median player and that formality matters. The median player is significantly better off under informal bargaining.

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1. Introduction

The outcome of a legislative bargaining process is usually a result of both formal and informal bargaining. When parliament is in session, parliamentary procedures strictly govern what members can do at what time; hence, bargaining is highly formalized. After official sessions have been adjourned, however, bargaining often continues informally in offices, corridors and backrooms, where formal rules barely exist. That bargaining occurs at different levels of formality likely has historical and functional reasons: informal bargaining is arguably faster, whereas formal bargaining provides transparency and legitimacy to the democratic process. The question we address in this paper is whether the formality of bargaining also systematically affects the bargaining outcome. This is important for understanding institutional choice and parliamentary procedures, and therefore for a better understanding of the way (economic) policies are made.

That the bargaining procedure can drastically affect the outcome has been recognized at least since the research boom on spatial voting in the late 1970s. If the procedure favors specific negotiators (e.g., through the order of voting, agenda-setting power, or proposal and voting rights) the outcome may crucially depend on it (e.g., McKelvey 1976, 1979, Romer & Rosenthal, 1978, Schofield 1978, McCarty, 2000). Importantly, the effect of formality is different than that of such variations in formal procedures, on at least two accounts. First, the difference between formal and informal bargaining cannot be captured in terms of changing the agenda or proposal or voting rights. Second, moving from a formal to informal bargaining or vice versa does not *prima facie* favor specific negotiators in any obvious way (whereas reallocating agenda power does, of course). The difference between the two is that informal bargaining provides much more flexibility to the bargaining parties. It does not give more flexibility to some parties than to others, however.

Intuitively, the choice of how much weight to put on formal versus informal procedures may be determined by strategic considerations (Elster, 1998, Stasavage, 2004). For instance, parties with a strong bargaining position may prefer backrooms and wish to reserve formal voting for well negotiated deals. On the other hand, parties with more extreme positions might prefer to avoid backrooms and follow the more formal procedures in order to allow their proposals to have a chance of success. This study intends to help us better understand such preferences.

More specifically, we compare two bargaining procedures, which we believe are representative for formal and informal bargaining in the field. To obtain a clean comparison, in both cases the bargaining procedure is 'fair' in the sense that it does not *prima facie* favor any negotiator. In this important way, our study differs from the legislative bargaining literature of the 1970s discussed above. The main question we address is whether the increased flexibility of the informal compared to the formal procedure affects the legislative outcome. In addition, if it does, does it do so for purely strategic reasons or do psychological effects play a role? To provide an answer to these questions we analyze legislative bargaining both theoretically and in a controlled laboratory experiment.

In the *informal procedure*, players can freely make and accept proposals at any time.[1] Note that this means that multiple proposals may be on the table simultaneously. We did not choose for a completely unstructured face-to-face setting, but instead opted for a computerized setting where players can make and accept proposals in continuous time. This allows us to analyze the procedure as a non-cooperative game and to collect data on the bargaining process. We believe that the procedure is sufficiently unrestricted to be representative for informal bargaining like that which takes place in in parliamentary backrooms. As we will see, the procedure is also not restrictive in the sense that it imposes no strategic constraints on the players. In the *formal procedure*, proposals and voting are regulated by a finite, closed-rule Baron-Ferejohn (1989) alternating offers scheme.[2] Though there are potentially very many fair formal procedures, the

[1] In the 1970s, several experiments used informal bargaining procedures to compare the many cooperative solution concepts that had been proposed. Amongst the first were Fiorina & Plott (1978). The procedures used tend to be rather different from ours, however. More importantly, these studies do not compare their informal procedure to a formal procedure, nor do they model it as a non-cooperative game.

[2] Baron & Ferejohn (1987) compare open and closed amendment rules and find distinct equilibria. Note that both settings constitute formal bargaining procedures.

Baron Ferejohn framework is widely taken to be a suitable model for studying formal legislative bargaining.[3] Our procedure is an elementary Baron-Ferejohn scheme.

We study the effects of formality in the context of a three-player legislative bargaining setting. The game is a straightforward extension of the standard one-dimensional median voter setting (Black, 1948; 1958) and has the following motivation. In the standard setting, the median player's ideal point is the unique (strong) core outcome irrespective of the location of others' ideal points (as long as they are on the same dimension). However, intuitively one may expect that the outcome of a legislative bargaining process or the coalition supporting this outcome is less stable if preferences are far apart –i.e., if polarization is strong–, even if the policy space seems unidimensional. One explanation is that the disagreement point may well lie outside of the line on which all policy proposals are defined. This is an issue we believe has hardly been appreciated in the literature.[4] Such a situation may occur for various reasons.[5] First, a decision often involves a new type of policy or project so that the status quo may not fall in the space under consideration. Second, if the disagreement point consists in the termination of a project or a coalition, then it may involve significant transaction costs (e.g., involving new elections). If so, the disagreement point will be of a qualitatively different nature than the issue under negotiation.

An example serves to illustrate the environments we are thinking of. Imagine a legislature that consists of three factions (doves, moderates and hawks) and is

[3] See, in addition to the work by Baron and Ferejohn, amongst many others, Merlo & Wilson (1995), McCarty (2000), Diermeier, Eraslan & Merlo (2003), Battaglini & Coate (2008) and Banks & Duggan (2000, 2006). These models tend to reach similar conclusions about agenda setting power. The first proposal is often accepted in equilibrium, since players know which proposals would subsequently be accepted or rejected. This gives a great advantage to the player chosen to make the first proposal (Palfrey (2006)). Experiments, however, only partly corroborate these theoretical findings (McKelvey (1991), Diermeier & Morton (2005), Frechette, Morelli & Kagel (2005)). The first proposer does indeed have an advantage, but this is not as large as theoretically predicted, though the advantage increases if communication is allowed (Agranov and Tergiman (2014)). Moreover, the first proposal is sometimes rejected, leading to 'delay'.

[4] The only exception we are aware of is Romer & Rosenthal (1978), who make a similar observation when they compare competitive majority rule to a controlled agenda setting mechanism. They do not consider polarization.

[5] See Eliaz, Ray & Razin (2007) for a theoretical study of bargaining over two alternatives, with varying disagreement payoffs. For our purposes their study is limited, due to restriction to (i) two alternatives and (ii) cases where everyone prefers agreement to disagreement.

deciding on the renewal of a budget for an ongoing war. No single faction holds a majority and any coalition of two does. Doves prefer a reduction of the current war budget, moderates want no change and hawks would like an increase. Preferences are single-peaked with respect to budget revisions. The option to end the war ('retreat') serves as a disagreement point, which cannot simply be represented as a budget revision. (Retreating is qualitatively distinct from a reduction of the budget to zero and, furthermore, spending 5 billion on retreating is quite different from spending this amount on war efforts.) Preferences are such that all parties prefer some revisions to retreating. Polarization is then defined as the distance between the ideal revisions of the factions (relative to the attractiveness of retreating[6]) and captures the extent of divergence of interests. Polarization will most likely drive the stability of the outcome. If polarization is weak, then the factions' preferences lie close together and retreating is a relatively unattractive agreement. Hence, all coalitions will prefer the median ideal to retreating. In this case, we can use Black's Median Voter Theorem (1948, 1958) to predict that the moderates' ideal point will prevail. If the ideal revisions are very far apart, then polarization is strong. In this case retreating is relatively attractive and there are no revisions that any coalition prefers to retreating. With moderate polarization, we get a cyclical pattern. Both doves and hawks prefer retreating to the unaltered budget; however, moderates and doves prefer some negative revisions to retreating; and moderates and hawks prefer the unaltered budget to negative revisions. In this case, it is not clear what the legislature may decide. Intuitively, one may expect the moderates to have the highest bargaining power, as doves and hawks can only coordinate on retreating.

Our model and experimental design take the same basic form as this example. The point of departure is a bargaining *problem* in a median voter setting that is modified to have an exterior disagreement point. Then, we introduce the formal bargaining procedure to obtain the *formal* bargaining game and the informal bargaining procedure to obtain the *informal* bargaining game. We analyze cooperative solutions for the bargaining problem and for both games we derive non-cooperative equilibrium predictions.

[6] In particular, relative to the distance between a player's ideal budget and the budget(s) she finds equally attractive as retreating.

The bargaining outcome will typically depend on specific characteristics of the bargaining problem (*i.c.*, the extent of polarization). As a consequence, the effect of the procedure may also depend on these characteristics. More specifically, the relevance of formality may arguably be dependent on whether or not the bargaining problem has a core.[7] Our setup allows us to obtain distinct outcomes with respect to the core by varying the level of polarization. When polarization is weak, the core consists of the median ideal and with strong polarization the core is the disagreement point. With moderate polarization, the core is empty.

The non-cooperative predictions depend on the procedure.[8] For the formal game, we derive a unique (refined) subgame perfect equilibrium (SPE) that converges (with the number of bargaining rounds) to the core element when this exists. It typically does not converge at all when the core is empty. In the informal game the disagreement point and all points between the players' ideal points can be supported as an SPE-outcome, irrespective of the extent of polarization. In addition, the equilibrium set cannot be refined in any standard way. Our interpretation is that informality offers so much strategic flexibility that strategic considerations alone cannot identify an outcome.

From our experiments, we have two main findings: polarization matters and formality matters. Polarization has a strong impact on the outcome. In accordance with theory, the median player is significantly worse off with moderate than with

[7] When the core is empty, all outcomes can be supported by some agenda-setting institution (McKelvey (1976; 1979), Schofield (1978)). This is important, because the core is empty under many conditions (Gillies (1953), Plott (1967), Riker (1980), Le Breton (1987), Saari (1997)). If the core is non-empty, the (non-cooperative) equilibrium outcome for many procedures tends to lie in it (Perry & Reny (1994), Baron (1996), Banks & Duggan (2000)). There is indeed experimental evidence on the stability of core-outcomes (Fiorina & Plott (1978), McKelvey & Ordeshook (1984), Palfrey (2006)). On the other hand, the outcome is sometimes sensitive to fairness considerations (Isaac & Plott (1978), Eavey & Miller (1984)). Structure may matter even if there is a core, for instance, if some procedures are considered fairer than others (Bolton, Brandts, & Ockenfels, 2005).

[8] Two theoretical breakthroughs have allowed us to overcome challenges in studying the effects of formality. First, for a long time many cooperative solution concepts have been advanced for situations in which the core is empty but none found broad theoretical and empirical support. Miller's (1980) uncovered set as a generalization of the core drew theoretical support (Shepsle & Weingast (1984), Banks (1985), Cox (1987), Feld, Grofman, Hartley, Kilgour & Miller (1987)). However, systematic empirical tests were problematic, as it was impossible to compute the uncovered set in most cases. This problem was largely solved by Bianco, Lynch, Miller & Sened (2006; 2008), who developed an algorithm to find the uncovered set. By doing so, they managed to find solid empirical support using data from many old and new experiments. Second, continuous time bargaining has made the non-cooperative analysis of low-structure settings possible (Simon & Stinchcombe (1989), Perry & Reny (1993)).

weak polarization. However, we find that increased polarization hurts the median player and does so even at weak levels when her most preferred outcome remains the unique core element. Our experimental findings suggest that this is due to negotiations (especially in the first half of the experiment) leading to equal earnings for the coalition partners (reminiscent of Bolton, Chatterjee and McGinn (2003)). Such considerations become less important as negotiators gain more experience. After players have repeatedly played the game (in ever-changing groups), competition between coalitions is strengthened and the position of the median player also becomes stronger.

Our second result is that the formality of the bargaining procedure matters. The median player is significantly better off with the informal than with the formal procedure. One plausible cause seems to be that flexibility in making proposals at any time (even if there are already other proposals on the table) increases her ability to exploit her superior bargaining position, as observed by Drouvelis, Montero & Sefton (2010) in a different setting. This points to the more general idea that parties in a superior bargaining position will prefer institutions that impose less structure on the bargaining procedure. It is important to note, however, that this intuition is not supported by our theoretical analysis of the formal and informal procedures. The theory presented in section 3 is inconclusive with respect to the effects of formality.

The remainder of this paper is organized as follows. Section 2 models the bargaining problem as a cooperative game and derives solutions for it. Section 3 describes and solves the non-cooperative games for the formal and informal bargaining procedures. Our experimental design is presented in section 4 and the experimental results are presented in section 5. Section 6 concludes.

2. The Bargaining Problem and Cooperative Solutions

Formally, the bargaining problem is represented by $\Gamma = \Gamma(N, Z, u_i, W)$ and consists of a finite set N of players, thought of as factions in a legislature; a collection W of subsets of N, thought of as winning coalitions; a set Z of alternatives; and utility functions u_i, one for each player $i \in N$ representing i's preferences over Z. Note that although winning coalitions have been specified, nothing has yet been said about the decision making process itself. Procedures governing this process will be described and formalized in the next section.

In the bargaining problems studied here, three players (N = {1,2,3}) bargain over the set of alternatives represented by $Z \equiv R \cup \delta$ with R denoting the set of real numbers and δ the disagreement point. Each player $i \in N$ has an ideal point $z_i \in Z$. Without loss of generality we normalize by setting $z_1 = -a < 0$, $z_2 = 0$, and $z_3 = b > 0$, with $b \geq a$. Hence, the ideal point of player 2, the *median* player, is $z = 0$. For players 1 and 3, the *wing* players, z_1 is normalized to lie closer to 0 than z_3. We interpret the distance, a, between the closer wing player and the median player as a measure of the *polarization* of players' preferences. We shall distinguish three cases of respectively, weak ($a \leq 1$), moderate ($1 < a < 2$), and strong polarization ($a \geq 2$).

Preferences of all players are single-peaked on R and represented by piecewise linear von Neumann-Morgenstern utility functions $u_i(z) = 1 - |z - z_i|$ We further assume that the utility attributed to the disagreement point is normalized at 0, that is, $u_i(\delta) = 0$, for all $i \in N$. Hence, each player has an open interval of outcomes with strictly positive values; to wit, ($-a - 1$, $-a + 1$) for player 1, (-1, 1) for player 2, and ($b - 1$, $b + 1$) for player 3. Note that the endpoints of these intervals yield utility of 0, while the outcomes outside of these intervals are strictly worse for the respective players than the disagreement point δ. Figure 1 depicts this payoff structure.

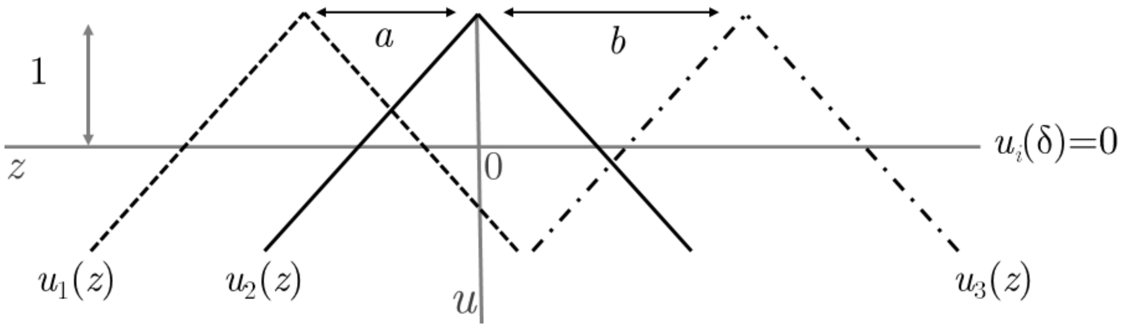

Figure 1
The figure shows the payoff structure of Γ.

As for winning coalitions W, we assume that any majority of two players can implement any $z \in Z$ as the outcome. This can be achieved in various ways, determined by the structure of the bargaining process (see section 3).

Γ can be regarded as a cooperative game and more precisely as a coalitional game without transferable payoff.[9] We start by defining the dominating and covering relations for any given $\Gamma(N, Z, u_i, W)$.

Definition 1 *Let $z, z' \in Z$. We say that*

(i) *z' dominates z, and write $z' > z$, if there is a winning coalition $M \in W$ such that all members in M strictly prefer z' to z;*

(ii) *z' covers z, and write $z'Cz$, if $z' > z$, and $z'' > z' \Rightarrow z'' > z$, for all $z'' \in Z$*

The assumption that the disagreement point δ does not coincide with a point on the line R is important. If δ did lie in R, then we would have a standard median voter setting and the median ideal 0 would dominate all other possible outcomes.

The counter-positive equivalent of Definition 1 reads as follows.

Definition 2 *For an alternative , $z \in Z$ we say that*

(i) *z is undominated if for every z' the set of players who strictly prefer z' over z is not a winning coalition;*

(ii) *z is uncovered if for every z' which dominates z there is a z'' which dominates z' and does not dominate z.*

To obtain a solution we look at the core and, when the core is empty, at the uncovered set, which is a generalization of the core. These are defined by:

[9] Note that individual players cannot achieve any outcome by themselves and hence the payoffs available to singleton coalitions are not independent of the actions of the complementary coalition. Hence, under some definitions it would fall outside of the class of coalitional games with non-transferable utility.

Definition 3

(i) *The core $\mathcal{C}(\Gamma)$ of Γ is the set of all points in Z that are undominated;*

(ii) *The uncovered set $\mathcal{U}(\Gamma)$ of Γ is the set of all points in Z that are uncovered.*

Intuitively, the uncovered set is a 'two step core.' If an outcome z is uncovered, there might be an outcome z' that dominates it, but this outcome z' will itself be dominated by an outcome z'' that does not dominate z. This means for instance, that forward-looking negotiators might be hesitant to move away from a point in the uncovered set. The uncovered set has several appealing theoretical properties. It is never empty, is equal to the core if the latter is nonempty and strict (Miller, 1980), contains all Von Neumann Morgenstern sets (McKelvey, 1986) and it subsumes the Banks set (Banks, 1985).[10] McKelvey argues that the uncovered set could be seen as a "useful generalization of the core when the core does not exist" (1986). More recently, this concept has also attracted significant empirical support (Bianco, Lynch, Miller & Sened 2006; 2008).

Whereas the uncovered set is typically large and difficult to calculate, in our bargaining problem it is small and simple.[11] Table 1 gives the core and uncovered set for our bargaining problems.

TABLE 1

COOPERATIVE SOLUTIONS

Polarization		$\mathcal{C}(\Gamma)$	$\mathcal{U}(\Gamma)$
weak: $a \leq 1$		0	0
moderate: $1<a<2$	$a < b$	$\varnothing$	$\{-a + 1, 0, \delta\}$
	$a = b$		$\{-a + 1, 0, b - 1, \delta\}$
strong: $a \geq 2$		δ	δ

Notes: For Γ=Γ (N, Z, u_i, W), the table gives the elements in the core ($\mathcal{C}(\Gamma)$)and uncovered set ($\mathcal{U}(\Gamma)$) for the levels of polarization distinguished in the first column.

[10] These different papers prove these relations under slightly differing conditions.

[11] In addition, in our game the uncovered set is refined in a nice way by the *von Neumann Morgenstern set* and the *bargaining set*, both of which are unique. More details are given in Appendix A.

When polarization is weak or strong, the core is nonempty and coincides with the uncovered set. It always holds that players 1 and 2 prefer the median ideal to all points right of it, whereas players 2 and 3 prefer the median ideal to all points left of it. In addition, when polarization is weak, players 1 and 2 also prefer the median ideal to the disagreement point. Hence it dominates all points, and, as a consequence, also covers them all. Thus, if polarization is weak, the median ideal is the singleton core and uncovered set. When polarization is strong, no point on the line exists that two players prefer to the median ideal. Hence, the disagreement point dominates all points on the line and constitutes the core and uncovered set. From a behavioral perspective, this case seems less interesting, as no alternative outcome to the disagreement point seems viable.

When polarization is moderate, we get a circular dominance pattern and the core is empty: the median ideal dominates all other points on the line, but is dominated by the disagreement point. The disagreement point itself is in turn dominated by some points on the line (which are dominated by the median ideal etc.). This also means that the median ideal and the disagreement point do not cover each other; they are uncovered. Furthermore, the point closest to the median player that is not dominated by the disagreement point, $-a + 1$, is uncovered as well. (The same holds for $a - 1$ if $a = b$). Consequently, in this case the uncovered set consists of three or four elements: the median ideal, the disagreement point, $-a + 1$ and, if $a = b$, $a - 1$.

For technical details on the dominance relations and the cooperative solutions, see Appendix A.

3. Solutions of Non-Cooperative Games

We now impose procedures on the bargaining problem described in section 2 and analyze the resulting non-cooperative games. One may think of this as the legislature selecting exactly one element of the set of feasible alternatives Z by means of a procedure established (or agreed upon) in advance. Formally, such a procedure can be regarded as an extensive game. We shall present and discuss two such games, exemplary for two important frameworks for legislative bargaining, voting and open bargaining. We do so by introducing a 'formal' and an 'informal' bargaining procedure to $\Gamma = \Gamma(N, Z, u_i, W)$.

3.1. Formal procedure

We begin with a *formal-procedure* framework for the selection of an outcome in Z, represented by a sequential voting game Γ_T^F. The game − similar to that in Baron & Ferejohn (1989) − consists of multiple rounds, with a predetermined maximum of T rounds. For ease of computation, we abstract from discounting between rounds.[12] Each round comprises of three stages.

At stage 1, one player is randomly selected with equal probability across players. At stage 2, the selected player i submits her proposal. This proposal comprises an element of $R \cup \delta$, i.e., either a real number or the disagreement point. At stage 3, players vote independently on this proposal. It becomes the final choice if it is accepted by at least two players. Because the player who submitted it supports her own proposal (by assumption), support by one other player suffices to pass the proposal and end the game. Whenever the proposal is voted down by the two other players, it is off the table and the game proceeds to round $t + 1$, where a player is selected to submit a new proposal, and so on. If the game reaches round T and the final proposal is also rejected, the game ends and the disagreement point δ is implemented.

For any given T and bargaining problem Γ, the game Γ_T^F is an extensive form game of finite length with random moves by nature at the first stage and simultaneous moves by all three players at the third stage of each round.[13] Actions played at any stage are observed before the next stage or round begins. In general, players' best responses will not be unique so one can expect multiple Subgame Perfect

[12] Note that we are modeling negotiations on one topic, not a series of negotiations. In this sense, discounting seems irrelevant. Discounting could lead to small changes in our results, for example because it could make the median player accept offers that are slightly worse for her than in the current analysis. We do not expect that moderate levels of discounting would lead to major changes in our results, however.

[13] Though the player that made a proposal has an action set consisting of one element ('accept') at the 3rd stage.

Nash Equilibria (SPE), possibly with distinct outcomes. In order to select a single best response at each stage, and ultimately to select consistently a single SPE for every given T, we shall adopt a number of tie-breaking rules known from the literature on voting games (Baron & Ferejohn (1989), Baron (1996), Banks & Duggan (2006)):

(*i*) A player accepts a proposal submitted in round t if it provides to her a payoff equal to her expected equilibrium payoff in the subgame beginning at stage 1 of round $t + 1$.

(*ii*) Whenever a player has two best proposals, one that will be accepted and one that will be rejected, she submits the proposal that will be accepted.

(*iii*) Whenever $-c$ and c are both best proposals for player 2 she submits each of them with an equal probability.

(*iv*) Whenever δ and $c \in R$ are both best proposals for a player she submits δ.

The first assumption guarantees that an SPE exists, the remaining assumptions imply that it is unique.[14] Observe that since the number of rounds is finite and the outcome space is continuous, it is intuitive that ties will not occur for a generic parameter set and the SPNE is unique even without the tie-breaking rules (for similar reasons as in Norman 2002). Still, it is convenient to formally guarantee a unique equilibrium for all cases. From now on, we will refer to the equilibrium meeting (i)-(iv) simply as 'the equilibrium' of Γ_T^F. Note that equilibrium strategies in a round depend neither on what happened in previous rounds nor on the total number of rounds (T), but only on the number of rounds left before the game ends.

The equilibrium outcome of Γ_T^F can be characterized by the probability distribution of the equilibrium outcomes μ^T: $Z \to [0,1]$.[15] If all equilibrium proposals are accepted in the first round, μ^T simply allots equal probability to each of the players' equilibrium proposals in the first round. There is, however, the possibility of delay. Though for some values of parameters a, b and T, all three equilibrium proposals are immediately accepted, for other values an equilibrium proposal, in particular

[14] Assumption 4 is particular to our game (as $\delta \notin R$) and is a convenient tie breaking rule for the case $a = 2$. For other parameter values it is not essential which rule one assumes for these ties.

[15] μ^T is a probability mass function. As we show in online Appendix B, μ^T has countable support.

that of player 3, will be rejected.[16] The equilibrium outcome of Γ_T^F may depend in complicated ways on the number of bargaining rounds, T.[17] Hence, our approach is to look at whether the equilibrium outcome converges as T increases. We say that for given values of a and b the equilibrium outcome *converges* if there exists a probability distribution $\mu_{a,b}^*$ on Z such that $\lim_{T\to\infty} \mu^T = \mu_{a,b}^*$. in the sense of weak convergence of probability measures (Billingsley, 1999). If no such limit exists, then we say that the outcome does not converge. The equilibrium outcome converges to some single $z \in Z$ if $\mu_{a,b}^*$ is concentrated at $z \in Z$, i.e. $\mu_{a,b}^*$ = 1. This allows us to summarize the SPE of Γ_T^F in the following proposition:

Proposition 1

(i) *If* $0 \leq a < 1$ *or* $a = b = 1$, *the equilibrium outcome converges to* 0. *For T sufficiently large the first round proposals are accepted without delay.*

(ii) *If* $1 \leq a \leq b < 2$ *and* $b > 1$, *the equilibrium outcome does not converge except for some patches of the values of a and b, and never to a single outcome in Z.*

(iii) *If* $a \geq 2$, *the equilibrium outcome is* δ *for sufficiently large T.*

Proof: The proof is given in online Appendix B. It is a terse and long exercise in backward induction, since the non-convexity of the outcome set precludes the use of standard techniques and results.

Comparing Proposition 1 to Table 1, we conclude that the equilibrium outcome converges to the single element of the core for $a < 1$ and $a \geq 2$, and that it does not converge to a single outcome if the core is empty.[18]

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[16] Delay may only occur when player 3 can make a proposal for the case $b > 2$. In this case, she cannot propose any outcome that would give her and another player a positive pay-off. Moreover, the probability of disagreement increases as the game proceeds (as she will able to enforce disagreement if she is chosen to make the final proposal), which may be more attractive to her than attempting to secure the agreement that is 'best' for her.

[17] Numerically, the equilibrium outcome can be calculated for each value of a, b and T. Simulations show that it always appears to converge to a cycle in T, the length of which depends in erratic ways on a and b. To illustrate, online Appendix B presents simulation results that show how the period of the cycles depends on a and b.

[18] In the non-generic case of $a = 1$ and $b > 1$ the core consists of 0 but the equilibrium outcomes do not converge as T goes to infinity.

Proposition 1 specifies the parameter configurations for which we have robust predictions. Note that legislative bargaining in the field can go through a substantial number of rounds, but that the number is generally finite (due to time constraints, for example). If the outcome converges as the number of rounds increases, this allows for a stable prediction for such cases.[19] If there is a sufficient number of rounds, the prediction will not depend significantly on the exact number, on who gets to propose first or on the precise values of a and b. If the outcome does not converge, however, then the outcome will depend on all these parameters, and typically in a very sensitive and non-linear way. [20] In our experiment we will use $T = 10$. For this case, we can derive a unique prediction, whether the core is empty or not (See Table 3 below). When the core is nonempty, it does not matter much whether T is 9, 10, 11, 20 or 100 or whether a=.4 or a=.41 or whether player 1 or player 3 starts.[21,22] When the core is empty, however, then the outcome depends crucially on all these parameters. Though we have predictions for the specific parameters of our experiment, they will be strongly affected by small changes. This makes the outcome hard to predict in practice. In addition, this arguably makes it more difficult for players to coordinate on the equilibrium. Hence, Proposition 1 helps us understand when specific predictions for finite T are robust (i.e., this is the case for (*i*) and (*iii*)).

3.2. Informal Procedure

Now, we turn to the informal bargaining procedure where all players can make and accept proposals in continuous time, which we denote by Γ_T^I. As noted above, players may at any time propose to disagree (i.e., propose δ). The basic tenet of Γ_T^I is a triple of proposals $\left(p_1^t, p_2^t, p_3^t\right)$ on the table at all times $t \in \left[0, T\right]$ until one of the

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[19] In our experiments, T is "sufficiently large" in the sense of Proposition 1 (i.e. there will be no delay and δ is the equilibrium outcome for $a \geqslant 2$).

[20] See footnote 17.

[21] In addition, in the field (as in our experiment) the outcome set will not be exactly continuous but discrete and very fine-mazed. In this case, the outcome even converges in a finite number of rounds if the core is non-empty.

[22] Parameter values exist (especially those close to values where the core is empty), where more than 10 rounds are required to see converging behavior.

proposals is accepted lest the game ends with the disagreement point δ at time T. It has by now been well established that such a game with continuous time cannot be solved without further assumptions, however (Simon & Stinchcombe 1989, Perry & Reny 1993; 1994). Drawing on Perry and Reny's two-player game (1994), we introduce a reaction and waiting time ρ. Our game is in important ways different from Perry and Reny (1994), nonetheless, as Γ can have an empty core and does not have transferable payoffs.

The rules of the game are as follows. Player i can either be silent (ς), $p_i^t = \varsigma$, have a proposal on the table, $p_i^t = z$, $z \in Z$, or accept the proposal of another player j, $p_i^t = a_j$. For each player i, p_i^t as a function of time is assumed to be piecewise constant and to be right-continuous. We say that a player moves at time t when $p_i^t \neq p_i^{t-} \equiv \lim_{s \uparrow t} p_i^s$. It is natural to only allow only such discrete changes in proposals, since actual negotiations (face-to-face or computerized) consist of discrete actions ('I propose x', 'I accept', 'I withdraw y') in a continuous time.

Players start with no proposal on the table: $p_i^{0-} \equiv \varsigma$. We use a uniform reaction and waiting time, ρ. In particular, if some player moves at time s, no player can move at $t \in (s, s + \rho)$.[23] This models the fact that players cannot react (or act again) immediately after a player has moved and that the time it takes to process information, make a decision and execute it is roughly the same for all players at all times. Essential is that we allow ρ to be arbitrarily small.[24]

Player i accepts j's proposal by setting $p_i^t = a_j$. In order to ensure that a player knows which proposal she accepts, if player i plays $p_i^t = a_j$ she accepts p_i^{t-}. To ensure a unique, well-defined outcome a player i can only accept a proposal at time

[23] We also allow a player to resubmit her old proposal and induce the reaction time ρ. Intuitively, this is the strategic move "I still propose z." Technically, if $p_i^{t-} = z$, then we define the move that resubmits the same proposal as $p_i^t = z^*$. (We set $z^{**} = z$, such that if $p_i^{t-} = z^*$, then resubmitting z is $p_i^t = z$). If accepted, z^* just induces z as outcome.

[24] Note that the waiting time implies that no move (i.e., withdrawal of a proposal, new proposal, or acceptance of a proposal) can take place within the waiting time following any previous move by any player. In our model the waiting time is exactly equal to the reaction time, unlike in Perry and Reny (1993). What is important is that we exclude the possibility of making a proposal and then withdrawing it before it can be accepted. For this purpose, any reaction time smaller than the waiting time would suffice; since there is no time-discounting, this would make the analysis unnecessarily more complex.

t if she is silent herself ($p_i^{t-} = \varsigma$).[25] In addition, one can (naturally) not accept a proposal from a player who is silent. As soon as a proposal has been accepted, the accepted proposal is the outcome of the game. If no proposal has been accepted before or at t = T, then the outcome is the disagreement point δ. After a proposal has been accepted or when $t>T$, no player can move anymore. Formally, we always let the game end at t = T + ρ.[26]

To define strategies and derive equilibria, we need to introduce some further definitions. We do so in Appendix C, where we define strategies, prove Γ_T^I is a well-defined game and derive the set of subgame perfect equilibria. We obtain the following.

Proposition 2 *The set of SPE outcomes contains* $[-a, b] \cup \delta$ *for every continuous game* Γ_T^I with T > ρ.[27]

Many of the SPEs in Proposition 2 may seem unintuitive. For instance, the at first sight unlikely outcome b (which is the ideal point of the wing player furthest from the median player) can be supported by an equilibrium in which players 1 and 2 always propose b. Player 3 will accept b as soon as she can, while players 1 and 2 cannot individually profitably deviate, as the other player will anyhow propose b, which will be readily accepted by player 3.

This large set of SPE cannot be refined in any standard way. A simple set of standard tie-breaking rules, such as those used for Γ_T^F, would be much too weak to have any effect. Stationarity only has a very small bite (b and δ can, for instance, be sustained by stationary strategies for any a and b). A procedure of iterated elimination of weakly dominated strategies, as proposed by Moulin (1979), and used by Baron & Ferejohn (1989), is of little avail in our case, due to the fact that typically

[25] If a player were not required to be silent when accepting a proposal, her proposal could be accepted while she were accepting another. Essentially, we are requiring that a player removes her own proposal before accepting another. Given that ρ can be arbitrarily small, this assumption is not behaviorally restrictive.

[26] This is because at time t, it is not yet known what happens at t itself. Any time after T would do.

[27] More specifically, we derive that the set of SPE outcomes is $[c, b] \cup \delta$, *where* $c = \min\{-a, max\{-b, b-1\}\} \leq -a$.

in many subgames of Γ_T^I multiple actions per player will survive, so that hardly any strategy will eventually be eliminated in the complete game. In addition, a refinement based on trembling-hand perfection (Selten, 1975), if it can be adapted to continuous time and space, will not eliminate these unintuitive equilibrium-outcomes either. For instance, the reason that player 1 does not propose 0 instead of b in the equilibrium discussed above, could be that she is afraid that player 2 might tremble and play $b + \varepsilon$ instead of b. Hence, there are few strategic restrictions on the equilibrium strategies in the informal game.

Nonetheless, there are some points in the outcome set that strike us as more 'likely' than others (for instance points in the uncovered set). Their plausibility might be the result of their *focal* nature due to the constellation of preferences and winning coalitions (which is captured by the cooperative solution concepts of the bargaining problem).

3.3 Overview of Theoretical Results

In Table 2 we summarize the main results obtained for the outcomes of the equilibria of the two strategic games analyzed in this section, Γ_T^F and Γ_T^I, together with the solutions of the cooperative game Γ.

TABLE 2
THEORETICAL RESULTS MAIN CASES

		Polarization		
		Weak: a<1	Moderate: 1<a<2	Strong: a > 2
Cooperative Game Γ (N, X, u_i, W)	Core	0	$\varnothing$	δ
	Uncovered Set	0	{1−a, 0, a−1*, δ}	δ
Formal (convergence), Γ_T^F		0	No convergence**	δ
Informal (for $T \geq \rho$), Γ_T^I			[-a , b]∪ { δ } ***	

Notes: Cells give the solution concepts for the two games as derived above for the three generic cases of polarization (a<1, 1<a<2, a>2). Solution concepts used are described in the previous subsections.

*a−1 is only included if $a = b$; ** There are some exceptions, in which case the outcome may converge but never to a single outcome in Z. ***Outcomes can also lie in the interval [max{b−1, −b}, a] if max{b−1, −b} < −a.

An interesting question to ask at this stage is about the concept of 'formality', i.e., what makes a bargaining setting more or less formal. To start, note that our 'informal', continuous game is characterized by a minimal set of rules, without which one could not define an outcome of the game. From a practical point of view, these rules are not restrictive if the waiting and reaction times are sufficiently small. In contrast, our 'formal' game has a very rigid set of rules governing who can make proposals and at what point in time.

In our view, a Game A can be said to be more formal than a Game B, if A imposes additional rules to B. In this light, it is interesting that with a proper randomization device (such as an external agent to chair the meeting), players in our informal game could agree to an additional set of rules that would result in our formal game. For instance, they could divide the time T in K intervals (of at least 2ρ). In each interval, they (or the external agent) randomly determines which person is allowed to make a proposal (hence, disallowing multiple proposals on the table at the same time, as in our formal game), that person then makes a proposal, after which the others can accept it. This would de facto result in our formal game with K rounds. Because imposing additional rules on our informal game yields the formal game, we conclude that our informal game is indeed less formal.

4. Experimental Procedures and Design

The experiment was run at the Center for Research in Experimental Economics and political Decision making (CREED) of the University of Amsterdam. It was computerized using z-Tree (Fischbacher, 2007). An English translation of the Dutch experimental instructions is provided in online Appendix D. Subjects had to correctly answer a quiz before proceeding to the experiment. In total, 102 subjects were recruited from CREED's subject pool.[28] They earned a €5 show-up fee plus on average €11.65 in 90-120 minutes. In the experiment, payoffs are in 'francs'. The

[28] The subject pool consists of around 2000 individuals who have voluntarily registered. Almost all of these are undergraduate students at the University of Amsterdam, of which approximately 40% major in Economics or Business. When the experiment was announced, all received an invitation to sign up and participation was on a first-come, first-serve basis.

cumulative earnings in francs are exchanged for euros at the end of the session at a rate of 1 euro per 10 francs.

We ran six sessions. Each session consists of 24 periods. In each period subjects are rematched in groups of three. We use matching groups of 6 or 9 subjects.[29] After groups have been formed, subjects are randomly appointed the role of player 'A', 'B', or 'C'. To avoid focality, players do not play the normalized game described above (e.g. B's position is not set equal to 0 and it is not necessarily the case that A's ideal value is closer to B's value than C's is). For analysis, the bargaining problem subjects face can easily be normalized to correspond to the model of section 2.

Each player is appointed an 'ideal value', which is an integer between 0 and 100 (inclusive).[30] Player A's ideal value is always the smallest and player C's the largest. Players know all ideal values. Each group has to choose an integer between 0 and 100 (inclusive). If the group chooses a player's ideal value, this player receives 20 francs. For every unit further from her ideal point, one franc is subtracted from her earnings. Hence, earnings are negative for a player if the group chooses a number that is more than 20 larger or smaller than her ideal value. To avoid negative total earnings at the end of the experiment, each subject starts with a positive balance of 100 francs.[31]

The procedure was varied in a between subjects design, which consisted of a *formal* and an *informal* treatment. We ran three sessions per treatment, and have six matching groups in each. In both treatments proposals are made, consisting of any integer between 0 and 100 (inclusive) or δ (called "end"). If the disagreement point is the outcome, each player receives a payoff of zero.[32]

[29] Subjects are told that they are in a session with 15 or 18 participants and will be rematched in every round.

[30] The restriction to natural numbers is done for practical purposes. It is sufficiently fine-mazed to avoid affecting the equilibria described in the previous section in any relevant way. One difference is that if the outcome converges in T to 0, it converges in finite time and, for our parameter values, in fact it already converges for $T < 10$. In Table 3, we report the equilibria of the high structure game with a discretized line and 10 rounds (which we calculated numerically).

[31] Still, five subjects ended their session with negative earnings. They were sent off with no pay other than the €5 show-up fee. Data which involved these individuals were deleted from the sample due to possible incentive problems. Including these individuals makes little difference, except that statistical results become somewhat less conclusive due to one subject's extreme behavior, He would have lost 14.70 euros and showed erratic behavior after his earnings had become negative.

[32] In both treatments, a round of bilateral messages precedes group negotiations: each player may send a private message (consisting of a number between 0 and 100 or δ) to either or both

In the *formal* treatment, subjects play the game Γ_T^F of section 3.1 with $T = 10$; i.e., negotiations where held for a maximum of 10 rounds per period. We use the strategy method (for proposals) where in every round every player is asked to make a proposal. One proposal is randomly selected and put to the other two group members to vote on. If at least one of the two accepts this proposal, it becomes the group choice and the game ends. If the proposal is rejected by both players, a new round begins, unless 10 rounds have been finished. In the latter case, the outcome is the disagreement point.

In the *informal* treatment, subjects are given two-and-a-half minutes to reach an agreement. At any time, any group member can make a proposal, change a previous own proposal or accept one made by another member. They do so by typing a number (proposal) and clicking on an ok-button, respectively selecting another member's proposal and clicking on an accept-button[33]. As soon as a proposal has been accepted, this becomes the group choice for the period and negotiations are finished. If no proposal is accepted within the time-span, the disagreement point is the outcome. This treatment closely follows our informal model.[34] To help a subject in determining her choices during the negotiations for a group decision, her screen always shows a history of previous rounds,[35] current earnings, a scrollable

other player(s). This is meant to reflect pre-negotiation lobbying. Importantly, because this game is with perfect information, this cheap-talk does not affect the theoretical analysis presented in section 3. Also experimentally, the messages do not influence the outcome: indeed, simple regressions of outcomes on messages show no significant effects, neither on the real-number agreements nor on the decision to disagree. More information is available upon request. Note that this limited message space differs strongly from the unrestricted chat in Agranov and Tergiman (2014). There, these chats increased the amount of resources obtained by the proposer in a Baron-Ferejohn setting.

[33] Note that multiple proposals may be on the table at the same time. A similar procedure was used in a different setting by Montero, Sefton and Zhang (2008).

[34] We do not impose a reaction or waiting time. In the model, these times do not represent procedural restrictions but rather cognitive and physical restrictions, which are allowed to be arbitrarily small. We also do not require that players retract their own proposal before they can accept another. In the model, we need this requirement to guarantee a well-defined outcome. In the experiment, we do not require this, as the probability that a player accepts a proposal at the exact same time her own proposal is accepted is zero. Implementing the additional restriction would not make a big behavioral difference (it would take two mouse clicks instead of one to accept a proposal), but would make the interface unnecessarily more cumbersome. Otherwise, the rules are identical to those in Γ_T^I

[35] The history showed for each previous round what happened in the group the player participated in. In particular, it specified (*i*) the ideal point for each role,(*ii*) the role the player herself had, (*iii*) the outcome and (*iv*) the earnings for all three roles.

help-box with instructions, a history of offers in the current round and a device to calculate payoffs for any hypothetical proposal.

Polarization is varied in a within-subjects design by using 12 sets of ideal values. Each set was used once in the *first half* (first 12 periods) and once in the *second half* (last 12 periods) of a session. The sets were chosen such that for the normalized parameters there were six with $a<1$ and six with $1<a<2$ (*cf*. Table 3, below). We chose not to use parameters with $a \geq 2$ in the experiment because it seems obvious that participants will always agree on the disagreement point of no earnings if there is no outcome where at least two players have positive earnings.

Table 3 gives the (normalized) parameters used, the periods in which they were used and the theoretical predictions for each set. We can conclude a few things from this table about the predictions of the cooperative solutions and the equilibrium of the formal game. First, as long as $a<1$ (weak polarization) the median's payoff does not depend on the level of polarization. Second, when $a>1$ the median's payoff can be expected to decrease with polarization. Third, when the core is empty, there are many instances where the SPE-outcomes of the formal game are not in the uncovered set.

5. Experimental Results

Online Appendix E compares the performance of the various solution concepts. Here, we focus on how formality (and its interaction with polarization) affects the ability of the median player to reach agreements close to her ideal point. All tests used below are two-sided and non-parametric, using each matching group (of six or nine participants) as one independent data point. We use the Wilcoxon signed rank tests for within comparisons and the Mann-Whitney test for between comparisons. Whenever we report statistically significant results for pooled *Formal/Informal* data only, the results are also significant at the 0.05 level for the disaggregated data where *Formal* and *Informal* are tested separately. p-values that are (unrounded) smaller than 0.05 are marked by an asterisk.

TABLE 3

PARAMETERS AND PREDICTIONS

Parameters a	b	Periods	Cooperative[1]	Formal game[2] 1 starts	2 starts	3 starts	Informal game
0.2	1.4	5, 23	0	0	0	0	$[-0.2, 1.4] \cup \{\delta\}$
0.5	1.1	3, 21	0	0	0	0	$[-0.5, 1.1] \cup \{\delta\}$
0.5	1.7	11, 13	0	0	0	0	$[-0.5, 1.7] \cup \{\delta\}$
0.8	0.8	1, 19	0	0	0	0	$[-0.8, 0.8] \cup \{\delta\}$
0.8	1.4	9, 15	0	0	0	0	$[-0.8, 1.4] \cup \{\delta\}$
0.8	2	8, 24	0	0	0	0	$[-0.8, 2.0] \cup \{\delta\}$
1.1	1.1	7, 17	$\{-0.1, 0, 0.1, \delta\}$	−0.3	0	0.3	$[-1.1, 1.1] \cup \{\delta\}$
1.1	1.7	10, 22	$\{-0.1, 0, \delta\}$	−0.4	−0.15	0.4	$[-1.1, 1.7] \cup \{\delta\}$
1.1	2.3	2, 14	$\{-0.1, 0, \delta\}$	−0.45	−0.15	0.45	$[-1.1, 2.3] \cup \{\delta\}$
1.4	1.4	12, 20	$\{-0.4, 0, 0.4, \delta\}$	δ	0	δ	$[-1.4, 1.4] \cup \{\delta\}$
1.4	2	4, 16	$\{-0.4, 0, \delta\}$	−0.55	0.2	δ	$[-1.4, 2.0] \cup \{\delta\}$
1.7	1.7	6, 18	$\{-0.7, 0, 0.7, \delta\}$	δ	$\{-0.5, 0.5\}$	δ	$[-1.7, 1.7] \cup \{\delta\}$

Notes: Cells give the theoretical prediction (*cf.* Table 1) applied to the experimental parameter set. The predictions for the formal procedure are for the game with a discretized outcome set and 10 rounds, as played in the experiment.

[1]For a<1 the prediction is given by the core (=uncovered set); for a>1 it is given by the uncovered set.

[2]The column gives the (refined) SPE conditional on the player chosen to make the first offer. Player 2 is defined as the median position, 1 is the other player closest to the median.

5.1 Earnings

We start with players' earnings from negotiations. Figure 2 shows the payoffs for different levels of polarization (captured by a) and the two treatments. Most relevant are the payoffs of the median player. First consider the effect of polarization. Theory predicts that for weak polarization (a < 1) the median player will be able to secure her maximum payoff (of 1), whereas moderate levels (1<a<2) of polarization would hurt her (*cf.* Table 3).

The experimental results show no obvious change at a = 1. Increasing polarization clearly affects the median player (player 2) negatively, even when a < 1. For example, Player 2 earns approximately 0.9 (close to the maximum of 1) when a=0.2 (for both *Formal* and *Informal*) but only just over 0.79 for a=0.8 in the informal setting. Her earnings are significantly lower for a=0.8 than for a=0.2

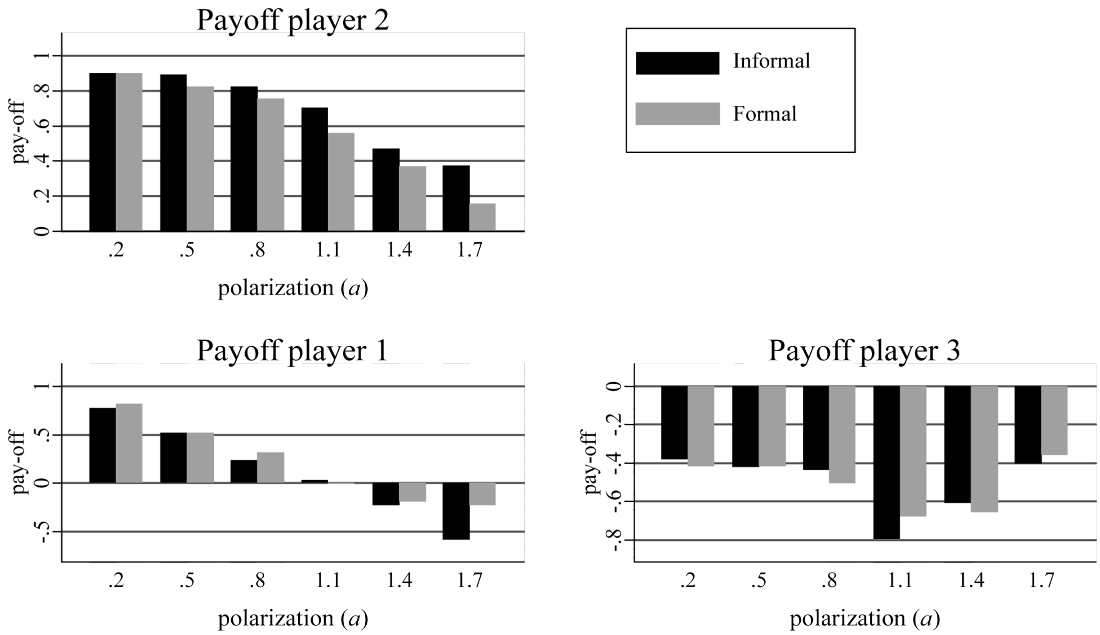

Figure 2
This figure shows payoffs. The bars show the average payoffs of players per period. Player 2 is the median player and player 1 is the other player closest to her.

(p=0.01*). As predicted by theory, the median's payoff is significantly lower for moderate (a>1) than for weak (a<1) polarization (p<0.01*).

Second, formality also has a clear effect. The median player is (significantly) better off in the *Informal* treatment than in the *Formal* treatment (p=0.03*). The difference between treatments seems to increase with the extent of polarization. When polarization is very weak (a=0.2) the procedure does not affect player 2's earnings from negotiations. When it is relatively strong (a=1.7) the median earns more than twice as much in *Informal* than in *Formal*. Next, we further explore what drives these results.

5.2. How Polarization & Formality Affect The Median Player

We start by looking at whether participants manage to reach an agreement before the deadline. Figure 3 shows the number of proposals needed to reach agreement. In both treatments, agreement was reached within the limit (150 seconds or 10 rounds, respectively) in 99% of all cases. Hence, it almost never occurred that the disagreement point was forced upon the negotiators for missing their limit.

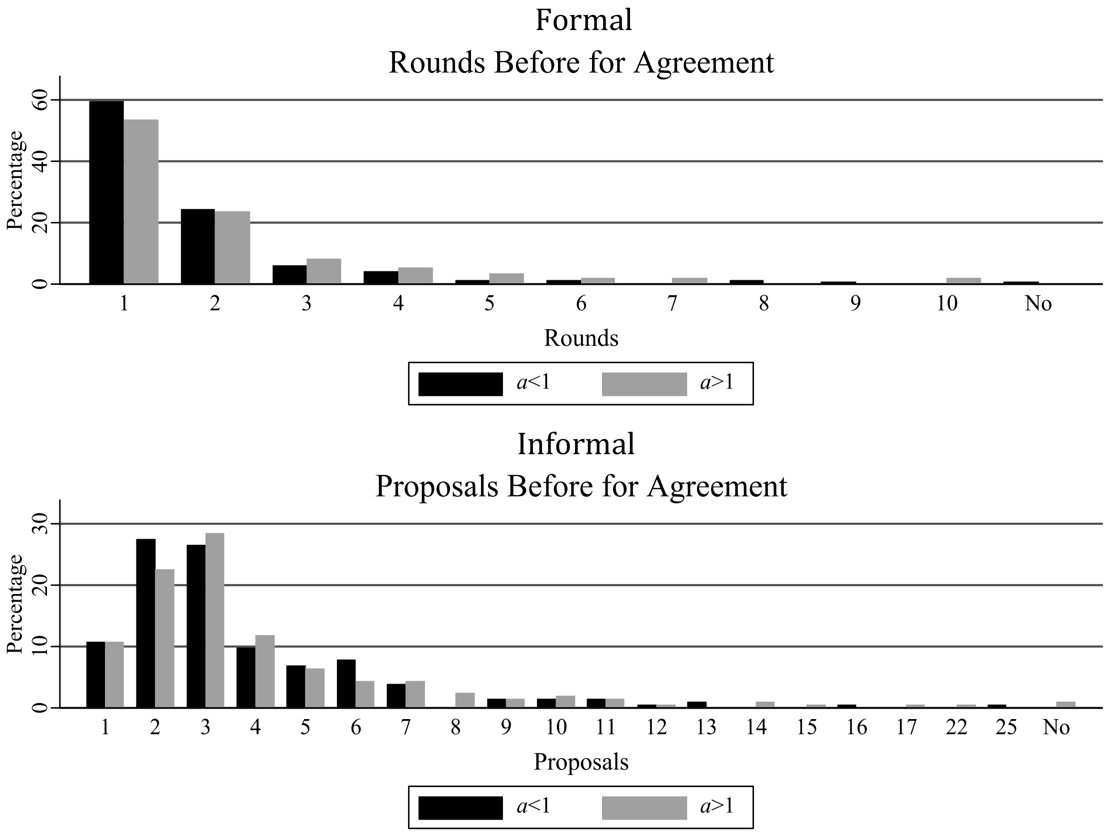

Figure 3

This figure shows the rounds or proposals before agreement. Bars show the fraction of agreements using the number of proposals depicted on the horizontal axis for Formal (top panel) and Informal (bottom panel). In Formal, a proposal in any round could only be made by the player selected to do so and there was a maximum of 10 rounds. In Informal, any player could make a proposal at any time during a period of at most 150 seconds.

Moreover, agreement was generally reached very quickly. In *Formal,* agreement was reached in at most 3 rounds in 88% of the cases and in *Informal* agreement was reached in at most 30 seconds in 82% of the cases. Consequently, binding (time) limits do not appear to be of any influence (for treatment effects). Players make significantly more proposals in *Informal* (4) than in *Formal* (2) (p<0.01*), however.

The outcome of the game can be characterized by three dimensions. First, whether it is a *real number* (as opposed to disagreement). If so, second, its value ('*location*') and, third, its distance to the median position, i.e., its absolute value ('*distance*'). We will look at each of these in turn. Figure 3 shows the percentages of outcomes that were a real number ('frequency'). As long as polarization is weak (a<1), virtually all outcomes are real numbers and polarization is immaterial.

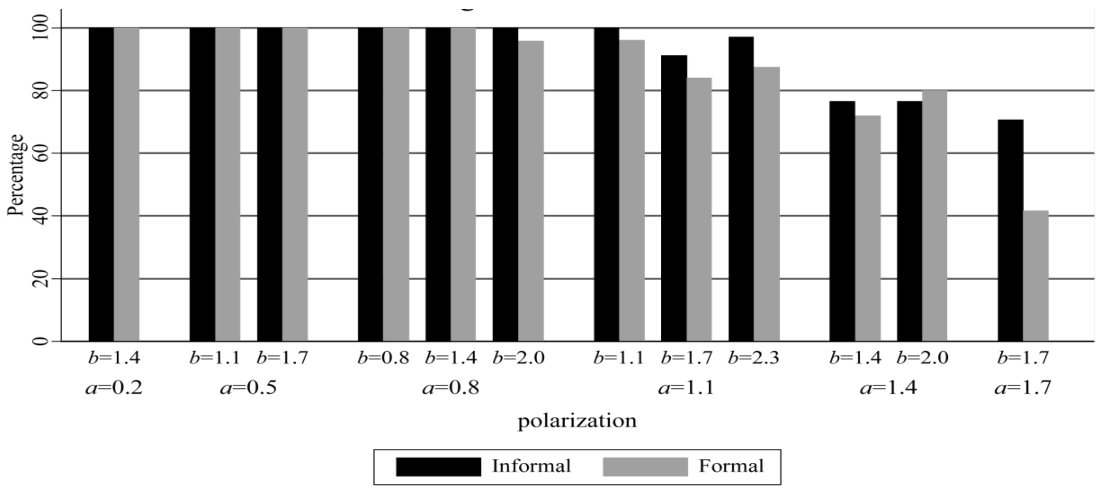

Figure 3
This figure shows the frequency (of outcomes were a real number). The bars show the percentage of outcomes that were a real number.

The frequency (of real number agreements) is, however, clearly and statistically significantly lower for moderate than for weak polarization (p<0.01*). Hence, a decrease in real number agreements may partly explain why moderate polarization is worse for the median player than weak polarization. However, it cannot explain why she cannot obtain her optimal payoff even when polarization is weak. Furthermore, there is no clear treatment effect. Real number outcomes are somewhat less likely in *Formal*, but the effect is small and insignificant (p=0.33 for a<1 and p=0.22 for a>1). Hence, the percentage of real number outcomes cannot explain why the median player is better off in *Informal*.

To consider the combined effects of formality and polarization on the outcome, we provide the results of a logit regression. We regress a binary variable indicating whether or not disagreement was chosen on polarization, a treatment dummy, their interaction (to allow polarization to have a distinct effect in the two treatments) and two controls for period. The results are presented in the second column of Table 4. The results confirm those derived from Figure 4. Polarization has a strong and significant positive effect on the chances of agreeing to disagree but formality does not. Moreover, the interaction term shows that the effect of polarization does not significantly depend on the treatment. Finally, note that there does

TABLE 4

REGRESSIONS

	Dependent Variable	
Independent Variable	Disagreement	Distance
Constant	–8.97***	0.11**
Period	–0.02	–0.01**
Period^2/100	0.00	0.00
Formal	0.74	–0.02
Polarization	4.85***	0.24***
Polarization*Formal	–0.21	0.14**

Notes: Cells give the regression coefficients. Column 2 uses a logit regression where the dependent variable is a dummy with value 1 if the outcome is an agreement to disagree and 0 if the outcome is a real number. The very few (<1%) cases where disagreement was forced upon the players are treated as missing values. Column 3 uses a linear regression where the dependent variable is the absolute distance between the real number agreed upon and the median position of 0. Cases where no real number was agreed upon are treated as missing values. *Period(*^{2}*)* denotes the (squared) period number. *Formal* is a dummy variable equal to 1 (0) in the *(In)Formal* treatment. *Polarization* is the value of variable *a*. In both cases, robust standard errors are clustered at the level of matching groups. ***/** indicates significance at the 1%-/5%-level.

not seem to be much learning going on; there are no significant effects of the period variables.

In search of an explanation of why the median is better off in *Informal*, we take a closer look at the real number outcomes players agreed upon. For completeness sake, we first depict the location of real number agreements in Figure 4 (although the location *per se* is not relevant for player 2's payoff). In this figure, a negative number indicates that an agreement has been reached on an outcome between the ideal points of players 1 and 2 (almost always, this is an agreement between players 1 and 2) while a positive number indicates agreement on a point between the ideal points of players 2 and 3 (almost always agreed upon by these two players). The results show that in almost all cases where $a < b$, the average outcome is negative. In fact, there are only two cases with $a \neq b$ where the average agreement lies between the ideal points of the median player and player 3. In both cases, player 1 still earns more that player 3. This is clear evidence that the average agreement is typically between the ideal points of the median player and the other player closest to the median. We will discuss the coalitions observed in more detail further on.

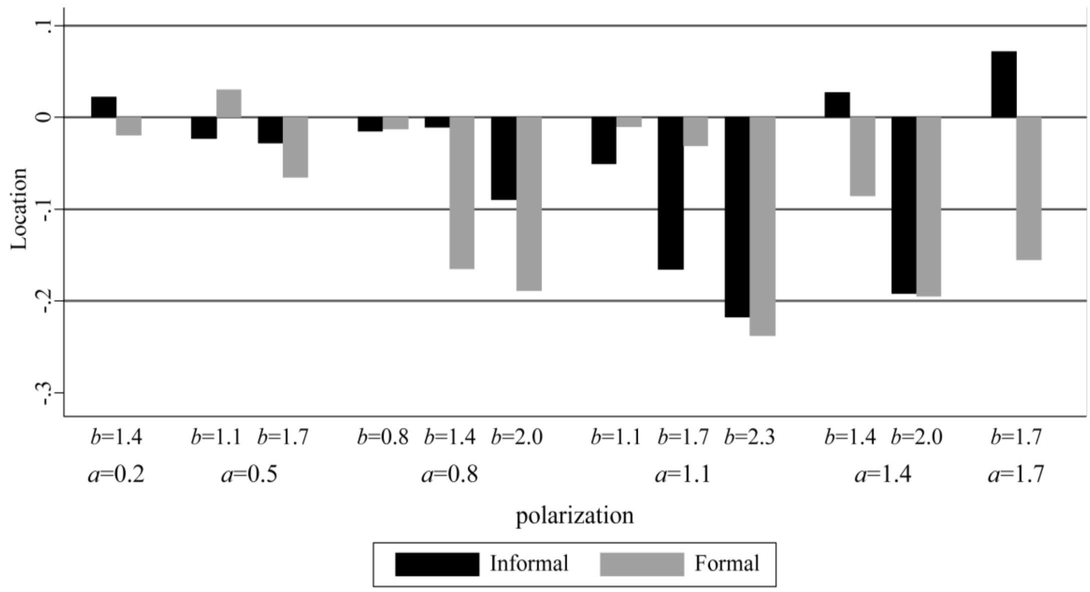

Figure 4

This figure shows the location of real number outcomes. Bars show the average normalized location of agreements, when groups agreed on a real number. Negative (positive) numbers indicate an agreement in between the ideal points of players 1 (3) and 2. The median position is an agreement at 0. Whenever a=b, the non-median players are randomly located as players 1 and 2, so any agreement is equally likely to be normalized to a positive or negative number.

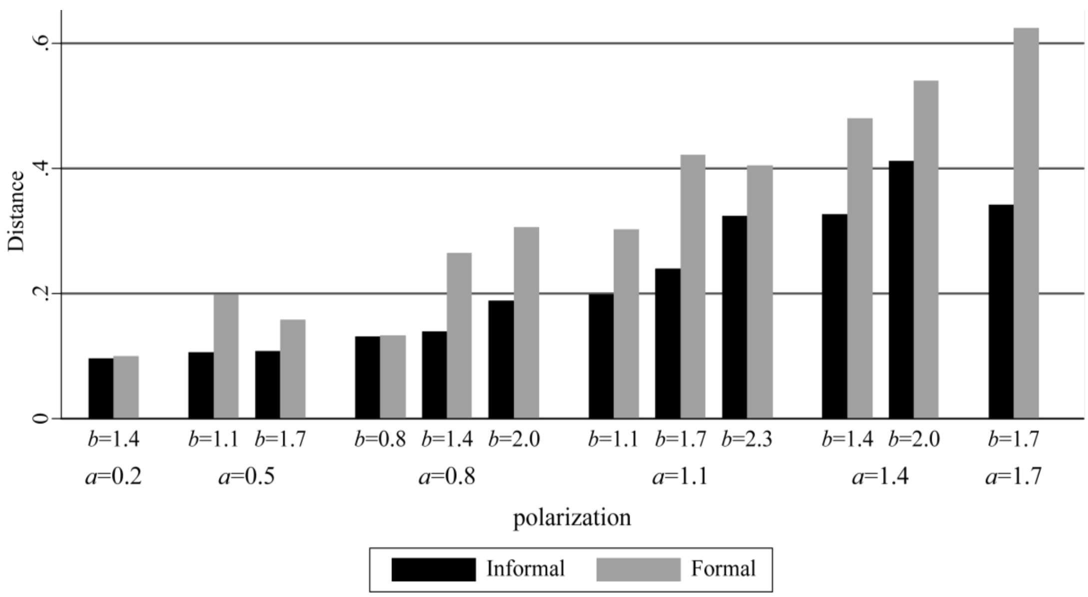

Figure 5

This figure shows the distance of real number outcomes. Bars show the average absolute distance between agreements and the median point, when groups agree on a real number.

Given that an outcome is a real number, the median player's payoff is fully determined by its distance to the median ideal (0). This is shown in Figure 5. This figure clearly shows that the distance increases with polarization. Distance is significantly higher for moderate than for weak polarization (p<0.01*). Distance matters even within weak levels of polarization: it is significantly higher for a=0.8 than for a=0.2 (p=0.01* (pooled), p=0.12(*Informal*), p=0.05* (*Formal*)).

Figure 5 also shows a clear treatment-effect. Distance is significantly lower for *Informal* than for *Formal*. This result can be clarified further by the regression results in column 3 of Table 4. Here, we regress the distance on treatment, polarization, their interaction and period. The results confirm the strong positive effect that polarization has on the distance. As polarization increases, the median finds it harder to achieve an outcome close to her ideal. The interaction term shows that this effect is stronger in *Formal* than in *Informal*. In fact, the insignificant effect of the treatment dummy suggests that it is this differential effect of polarization that is causing the treatment differences in observed distance. That is, in aggregate, the median obtains worse outcomes in *Formal* because polarization has a more negative effect. Finally, there is some learning going on. Over time, the median succeeds in arriving at outcomes closer to her ideal. The lack of a significant effect of the squared period variable suggests that this learning effect is more or less linear.

The effects of polarization and formality are not arrived at by differences in the player making the proposal. Table 5 shows that accepted proposals were made in equal proportions by all three players. It is not the case that some players were more likely than others to make the decisive proposal. The distributions do not differ across the treatments (χ^2=1.60, p=0.45 for all decisions; χ^2=1.3, p=0.52 for real number agreements; χ^2=3.03, p=0.21 for agreements to disagree). In the end, it the outcome (and differences across treatments) cannot be attributed to specific players' final proposals.

Nevertheless, the outcome is closer to the median position when the bargaining is *Informal*. Player 2 somehow exploits her superior bargaining position more than in *Formal*, especially as polarization increases. A possible explanation is that players are freer to make proposals in the *Informal* negotiations, so that they can negotiate better. Contrary to *Formal*, players in *Informal* can also make (counter) proposals when there already proposals on the table. Also, players make significantly more proposals in *Informal* (4.0) than in *Formal* (2.0) (p<0.01*). In addition,

TABLE 5

PROPOSERS

		Accepted proposal			First proposal		
		Player 1	Player 2	Player 3	Player 1	Player 2	Player 3
All	Formal	0.38	0.30	0.32	n.a.	n.a.	n.a.
	Informal	0.37	0.34	0.29	0.41	0.26	0.33
Real number	Formal	0.37	0.33	0.30	n.a.	n.a.	n.a.
	Informal	0.34	0.37	0.28	0.42	0.25	0.33
Disagreement	Formal	0.46	0.05	0.49	n.a.	n.a.	n.a.
	Informal	0.63	0.00	0.37	0.29	0.38	0.32

Notes. In columns 3-5, cells give the fraction of times that an accepted proposal had been made by the player concerned. In columns 6-8, cells give the fraction of times that the first proposal was made by the player concerned. "n.a." = not applicable, because the player to make the proposal was randomly chosen.

we find that in *Formal* players use slightly fewer proposals in the last 12 periods (1.9) than in the first 12 periods (2.1). In contrast, in *Informal,* players use significantly more proposals in the last 12 periods (4.5) than in the first 12 periods (3.5) (p=0.03*). Finally, Table 5 shows that the better negotiation results by median players in *Informal* are not due to the opportunity to make early proposals. In fact, in cases where the final outcome is a real number the median even tends to let others make the first proposal. This leaves the possibility to respond to others' proposals as the main difference between the two treatments.

We conclude that the main driving force underlying the higher profits for the median player in the *Informal* treatment is that the more flexible bargaining procedure allows her to secure real number agreements closer to her preference.

5.3. Intra-coalitional Equality *vs.* Inter-coalitional Competition

One intriguing question that remains is why even weak polarization hurts the median player, while her ideal is the unique core element. To address this question, we consider coalitions and the way in which outcomes distribute payoffs within them. Figure 6 shows the distribution of real number agreements divided by a. Hence, -1 represents an agreement at $-a$ (i.e., player 1's ideal point), 0 represents the median ideal 0 and 1 represents a.

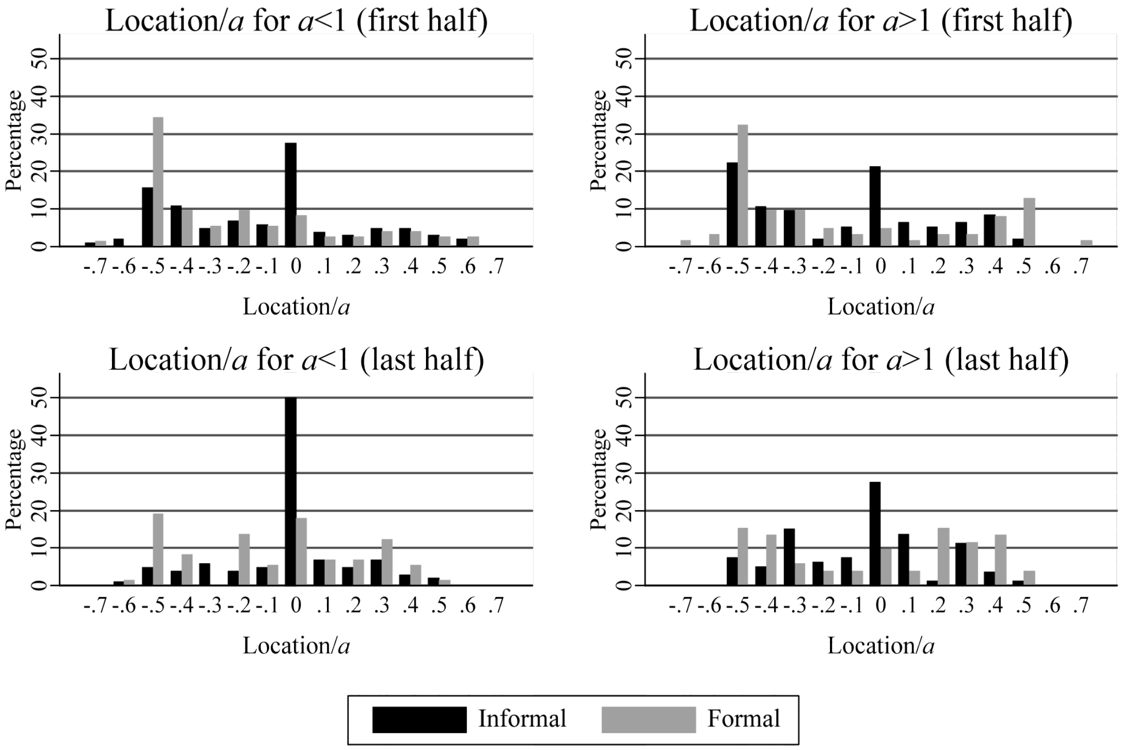

Figure 6
This figure shows the distribution of real number outcomes and learning effects. Bars show the fraction of real number outcomes that are within 0.05 of the outcome depicted on the horizontal axis. The horizontal axis gives the normalized outcome divided by a. The left panels shows the distribution for a<1, the right for a>1. The top panels show the distribution for the first half (first 12 periods), the bottom for the last half (last 12 periods).

Strikingly, almost all real number outcomes lie between $-a/2$ and $a/2$, with $-a/2$ being one of the most frequently chosen outcomes. Note that $-a/2$ equalizes payoffs between players 1 and 2, but is a rather unfair outcome for player 3; in fact worse than the median preference. From a fairness perspective it might seem remarkable that the players in the coalition do not seem to care much about the player outside of the coalition. Nonetheless, this is in line with the findings in the three-person ultimatum games (Güth & Van Damme, 1998, Bolton & Ockenfels 1998), where the third powerless person (who can neither propose nor reject) is given little consideration.

It seems that players 1 and 3 in many cases demand some part of the 'surplus' in a coalition with player 2. However, player 2 does not give more than the equal split to player 1. Furthermore, player 3 does not obtain a better outcome than $a/2$, since player 2 probably feels that she can certainly obtain $-a/2$ in a coalition with player 1. Such considerations yield real number agreements increasing in a, even for weak levels of polarization, as we observe. Note, however, that as *a* increases it becomes more costly to the median player to give her coalition partner an equal share.

There are two possible reasons why the equal shares are observed within coalitions. On the one hand, the median player may simply care about a fair distribution with the partner (though it remains unclear why she does not care about the disadvantages to the third player). Alternatively, the median may simply underestimate the power she has in negotiations within a coalition, because she has the easiest option of switching to the other player to form a coalition on 'the other side'. Changes in outcomes over time allow us to shed some light on these two explanations.

Figure 6 shows that there is a strong learning effect: the distribution of outcomes in the first half (first 12 periods) is very different from that in the last half (last 12 periods). In the first half, intra-coalitional equality considerations seem to play an important role, certainly within coalitions of players 1 and 2. Furthermore, coalitions tend to consist of players 1 and 2, in particular in *Formal* (see Figure 7).

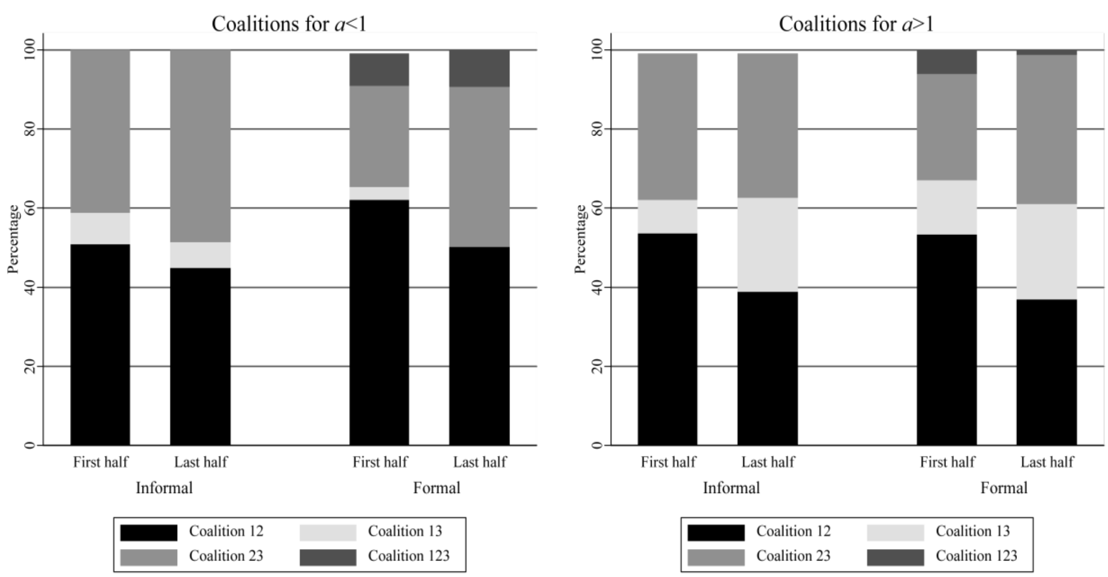

Figure 7
This figure shows coalitions and learning effects. Stacked bars show the distribution of distinct coalitions. A coalition ij is defined as an outcome proposed by i and accepted by j or vice versa. A coalition ijk is an outcome with 2 yes votes (only possible in formal).

In the course of the experiment, inter-coalitional competition becomes more important. In the second half, more coalitions arise of players 2 and 3 than in the first half (p=0.03* (pooled), p=0.46 (*Informal*), p=0.05* (*Formal*)), resulting in a more even spread of positive and negative agreements. Furthermore, for a>1 more coalitions are formed in the second half than in the first half between players 1 and 3 (p=0.03* (pooled), p=0.03* (*Informal*), p=0.21 (*Formal*)).

Seeing the viable 'outside option' of a coalition with player 3 means that the median player can offer less to player 1. Median players appear to realize this remarkably well in the second half of the experiment. Agreements tend to be closer to 0 in the last half (see Figure 6). In particular, the number of balanced 1-2 compromises drops considerably (p<0.01* (pooled), p=0.04* (*Informal*), p=0.05 (*Formal*)) with an accompanying increase in the number of outcomes at the median ideal (p=0.01* (pooled), p=0.03* (*Informal*), p=0.14 (*Formal*)).[36]

This development over time suggests that preferences for fair distributions with the coalition partner are not the dominant force underlying the results in Figure 7 (after all, why would such preferences change over time?). Instead, the differences between the first and second half of the experiment seem to favor the alternative explanation that, over time, the median player is learning to better exploit her central position. For *Informal*, one final piece of evidence provides further support for this learning effect. Here, anyone can make the first offer and this first offer may reflect one's intentions. We compare for each player the mean offer made in the first half to the mean offer made in the second half (in both cases only when making the first offer). For player 1, this 'first shot' was more or less the same in both cases (moving from –0.38 to –0.35). For player 3, the move was larger, but it remained far from the median position (from 0.55 to 0.31). The largest relative change between the first and second half was observed for the median players. In the first half, their opening offer was already aimed towards player 1, with an average of –0.11. In the second half, they opened with an offer very close to their

[36] The absolute distance of the outcome to the median position decreases in both treatments over time. The relative decrease is of a comparable magnitude. In *Informal*, the distance decreases from 0.26 in the first half to 0.17 in the second (a 35% decrease) and in *Formal* this is from 0.38 to 0.26 (32%). For completeness' sake, appendix F provides information about real number agreements, location, and distance, separately for the first and second halves of the experiment.

own ideal point: –0.02. This suggests that the median player learns over time to make more aggressive proposals.

6. Conclusion

In this paper, we have addressed the question whether the outcome of the legislative process is affected by the formality of the bargaining procedure. We compared an informal bargaining procedure where players can freely make and accept proposals to a formal bargaining procedure where agenda-setting and voting is regulated by a Baron-Ferejohn alternating offers scheme. We studied the effect of formality in a legislative bargaining problem that consisted of a three-player median voter setting modified to have an external disagreement point. This allowed us to study formality both when the core exists and when it is empty, and to study whether an external disagreement point can explain why the polarization of a legislature can affect the legislative outcome. We derived cooperative solutions for the bargaining problem, studied the equilibrium properties of the formal and informal bargaining games, and tested the two procedures in the laboratory.

Our first result pertains to polarization. Theoretically, we find that polarization should matter when there is an external disagreement point and in our experiment, we find that this is indeed the case.[37] In particular, polarization hurts the median player. As predicted by theory, in our experiments the median player is significantly worse of at moderate than at weak levels of polarization. In contrast to what theory predicts, however, more polarization hurts the median player even when her ideal is the unique core element. This seems to be the result of intra-coalitional equality considerations. Over time, inter-coalitional competition appears to attenuate such considerations as the median player learns to better exploit her negotiation power and she is hurt less by polarization.

Our second and main result is that formality matters. Theoretically, it is difficult to analyze the effects of formality, as a key characteristic of informal bargaining is

[37] Recall that polarization does not matter in the classic median voter setting (Black, 1948; 1958).

that it imposes very few strategic restrictions on the negotiators. We find that in the informal game, all plausible outcomes are supported as subgame perfect equilibrium points. This is an important motivation to run experiments. The data show a clear treatment effect of formality. The median player in our experiment is significantly better off under an informal bargaining procedure without rules about the timing of proposal and acceptance decisions. Outcomes in the *Informal* treatment are significantly more often the median ideal and significantly less often a compromise between players 1 and 2. It appears that the informal procedure gives the median player more flexibility to exploit her superior bargaining position. This result supports the armchair observation that players in a better bargaining position prefer less regulation of negotiations.

Our results are relevant for the application of game theoretic models to the legislative process. The fact that formality influences the payoffs of certain players and the performance of specific predictions means that 'neutral' simplifying assumptions (i.e., assumptions that do not favor any player *prima facie*) made to obtain tractable results need not be as innocuous as is often assumed. For instance, a highly stylized alternating offers game may not be a suitable model of the legislative process if a significant part of the bargaining is informal.

Finally, understanding the influence of formality is relevant for studying institutional choice and parliamentary procedure. In particular, legislatures have to decide on a bargaining *procedure* –either from scratch or from a set of previously established procedures– before they can decide on the outcome itself. Even if the extent of formality may seem like a neutral parameter, it can significantly influence the bargaining outcome. Consequently, parties may have preferences for a formal or informal bargaining procedure. For instance, parties in the center of a political spectrum may prefer to prolong backroom discussions until agreement has been reached. Our results point to the more general idea that parties in a superior bargaining position will prefer less formal bargaining institutions, as these give them more room to exploit their bargaining position.

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