MNRAS **437,** 675–681 (2014)
Advance Access publication 2013 November 8
doi:10.1093/mnras/stt1910

The most magnetic stars

Dayal T. Wickramasinghe,[1★] Christopher A. Tout[1,2,3] and Lilia Ferrario[1]

[1] *Mathematical Sciences Institute, The Australian National University, ACT 0200, Australia*
[2] *Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA, UK*
[3] *Monash Centre for Astrophysics, School of Mathematical Sciences, Building 28, Monash University, Victoria 3800, Australia*

Accepted 2013 October 7. Received 2013 October 2; in original form 2012 September 12

ABSTRACT

Observations of magnetic A, B and O stars show that the poloidal magnetic flux per unit mass Φ_p/M appears to have an upper bound of approximately $10^{-6.5}\,\mathrm{G\,cm^2\,g^{-1}}$. A similar upper bound to the total flux per unit mass is found for the magnetic white dwarfs even though the highest magnetic field strengths at their surfaces are much larger. For magnetic A and B stars, there also appears to be a well-defined lower bound below which the incidence of magnetism declines rapidly. According to recent hypotheses, both groups of stars may result from merging stars and owe their strong magnetism to fields generated by a dynamo mechanism as they merge. We postulate a simple dynamo that generates magnetic field from differential rotation. We limit the growth of magnetic fields by the requirement that the poloidal field stabilizes the toroidal and vice versa. While magnetic torques dissipate the differential rotation, toroidal field is generated from poloidal by an Ω dynamo. We further suppose that mechanisms that lead to the decay of toroidal field lead to the generation of poloidal. Both poloidal and toroidal fields reach a stable configuration which is independent of the size of small initial seed fields but proportional to the initial differential rotation. We pose the hypothesis that strongly magnetic stars form from the merging of two stellar objects. The highest fields are generated when the merge introduces differential rotation that amounts to critical break-up velocity within the condensed object. Calibration of a simplistic dynamo model with the observed maximum flux per unit mass for main-sequence stars and white dwarfs indicates that about 1.5×10^{-4} of the decaying toroidal flux must appear as poloidal. The highest fields in single white dwarfs are generated when two degenerate cores merge inside a common envelope or when two white dwarfs merge by gravitational-radiation angular momentum loss. Magnetars are the most magnetic neutron stars. Though these are expected to form directly from single stars, their magnetic flux to mass ratio indicates that a similar dynamo, driven by differential rotation acquired at their birth, may also be the source of their strong magnetism.

Key words: magnetic fields – stars: magnetars – stars: magnetic field – white dwarfs.

1 INTRODUCTION

Until recently, observations of magnetism in mid- to early-type stars have focused on the chemically peculiar A and late-B stars. These studies, which are biased towards the stronger field objects, have revealed that there is an upper limit to the dipolar magnetic field emerging from the stellar surface of $B_\mathrm{p} \approx 30\,\mathrm{kG}$ (Elkin et al. 2010) for stars in the mass range $1.2 < M/\mathrm{M}_\odot < 5$. With more sensitive spectropolarimetric studies, Aurière et al. (2010) showed that, at least for stars of spectral type A and late B, there also appears to be a lower limit of $B_\mathrm{p} \approx 300\,\mathrm{G}$ to the effective dipolar field strength, with no stars observed with $30 < B_\mathrm{p}/\mathrm{G} < 300$. Their lower limit is set by sensitivity of their observations. Thus, at least in this mass range, there is evidence that main-sequence stars fall into two groups, magnetic stars and non-magnetic stars. We define the poloidal magnetic flux $\Phi_\mathrm{p} = R^2 B_\mathrm{p}$, where R is the radius of the star. Stars in this group are characterized by

$$10^{-8.5} < \frac{\Phi_\mathrm{p}/M}{\mathrm{G\,cm^2\,g^{-1}}} < 10^{-6.5}, \qquad (1)$$

where M is the total mass of the star. The Magnetism in Massive Stars project (MiMes; Wade et al. 2011) has shown that a similar upper limit to the poloidal magnetic flux per unit mass Φ_p/M also

★ E-mail: dayal@maths.anu.edu.au

applies to massive early-B and O stars (see for example Wade et al. 2012). However, it is not yet clear whether the magnetic field dichotomy seen in the A and late-B stars persists at higher masses.

The origin of the large-scale magnetic fields observed in main-sequence stars with radiative envelopes, namely the intermediate-mass main-sequence A, B and O stars, remains unresolved. Whether the fields are generated by contemporary dynamos in the stellar cores or they are of fossil origin from pre-main-sequence phases of evolution has been debated over the years. A core dynamo is expected to generate small-scale fields but the mechanism by which the field is transported to the surface, where it is seen as a large-scale ordered field, has remained obscure. Numerical magnetohydrodynamical simulations (Braithwaite 2009) of possible stable field structures in stably stratified radiative stars appear to have swung the balance in favour of the fossil field hypothesis. These calculations have confirmed that, while purely poloidal or purely toroidal fields are subject to various instabilities that destroy the fields on short time-scales, stability can be restored in combined poloidal–toroidal field structures under certain conditions. More importantly, these calculations have shown that, regardless of the nature, scale and complexity of the initial field, it relaxes to a stable large-scale poloidal–toroidal structure on an Alfvén crossing time-scale (about 10 yr for Ap stars). The subsequent evolution of the field structure is expected to be slow enough for the field to last for a time-scale similar to the main-sequence lifetime of the star. The presence of a long-lived, nearly dipolar field structure in magnetic Ap and Bp stars is therefore seen to be compatible with the fossil hypothesis. What determines the maximum flux per unit mass, however, remains unexplained but it must be related to the origin of the magnetic flux acquired during the pre-main-sequence history of the star. The detection of magnetic fields in some Herbig Ae and Be stars, with magnetic fluxes similar to those seen in the strong field Ap and Bp stars (for example Wade et al. 2007; Alecian et al. 2008), suggests that the magnetic flux was already present during the pre-main-sequence phase of evolution.

Ferrario et al. (2009) suggested that magnetic main-sequence stars acquire their magnetic fluxes through a dynamo process that occurs during late merging of two protostars on their approach to the main sequence. Because only some stars have merged in such a way this model can explain the magnetic dichotomy. Here, we investigate this hypothesis further and suggest that any ordered primordial poloidal field that is present in the merging stars can be amplified, by differential rotation through a dynamo process, to a field configuration which is mainly toroidal. We present a simple model in which magnetic fields decay whenever they are unstable to short time-scale instabilities. Otherwise toroidal field is generated at the expense of differential rotation and poloidal field is generated as a by-product of the decay of the toroidal field. Once differential rotation has all been removed a long-lived magnetic configuration remains. We propose that such a dynamo, acting after stars merge, provides an explanation for both the magnetic main-sequence stars and the highest field magnetic white dwarfs. The magnetic flux of magnetars suggests that a similar dynamo may play a role in generation of magnetic fields of at least some neutron stars.

2 THE OBSERVED MAXIMUM POLOIDAL FLUX TO MASS RATIO

The incidence of magnetism among A and B stars peaks at about 10 per cent at 3 M$_{\odot}$ and decreases rapidly at lower masses to effectively zero at 1.6 M$_{\odot}$. It appears possible that the incidence increases further at higher masses (Power et al. 2008) although this is yet to be established either way through more complete surveys. We note here that Potter, Chitre & Tout (2012) claim that there is an upper limit to the mass of a star that can maintain magnetic field, for a significant fraction of its main-sequence lifetime, in the presence of hydrodynamic instabilities. The magnetic A, B and O stars have poloidal magnetic fluxes $\Phi_{\rm p} = B_{\rm p}R^2$ in a range that varies over many decades but they all have a similar upper bound. This is illustrated in Fig. 1, where we plot the poloidal magnetic flux per unit mass, $\Phi_{\rm p}/M$, for the most magnetic main-sequence stars (Wade et al. 2012, and references therein).

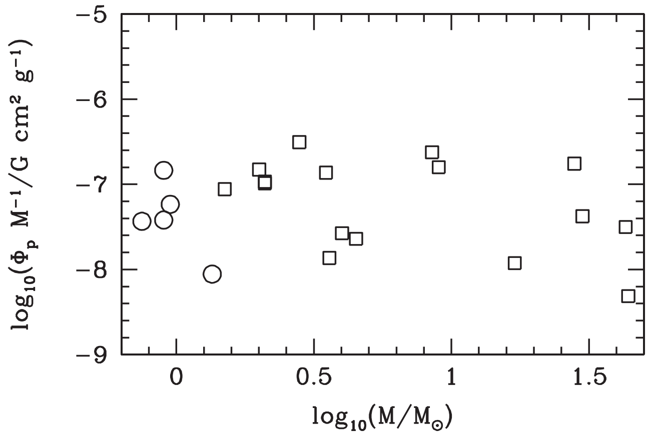

Figure 1. The ratio of magnetic flux to mass $\Phi_{\rm p}/M$ for the most magnetic main-sequence stars (squares) and white dwarfs (circles).

According to our hypothesis, the magnetic A, B and O stars all form when protostellar objects merge late on their approach to the main sequence. Simulations of star formation have shown that the more massive a star is, the more likely it formed by merging of lower mass stars (Railton, Tout & Aarseth, private communication). So a consequence of our hypothesis is that the incidence of magnetism, for as long as the field can be maintained, increases with mass on the main sequence and there is some evidence for this. We have also noted (Ferrario et al. 2009) that such a merging hypothesis provides a natural explanation for the apparent lack of Ap stars in close binary systems with main-sequence companions.

Turning to white dwarfs we find that some 10 per cent have magnetic field strengths in the range 10^6–10^9 G and form the group of high-field magnetic white dwarfs (HFMWDs; Wickramasinghe & Ferrario 2005). We compare, in Fig. 1, the poloidal magnetic flux to mass ratio of the highest field magnetic white dwarfs for which both field and mass are known (Wickramasinghe & Ferrario 2000) with those of the most magnetic main-sequence stars. The similarity in this ratio between the two groups is striking and so suggests a common physical origin for the magnetic fields.

Unlike for the main-sequence A stars, there is not such a clear magnetic dichotomy in the white dwarfs. Among the white dwarfs, there is also a group of low-field magnetic white dwarfs with fields ranging from a few kG to 10^5 G or so. Landstreet et al. (2012) indicate that the discovery rate of white dwarfs with fields in the range 10^3–10^5 G is significantly higher than for the entire high-field range of 10^6–10^9 G. Combined with the apparent dearth of fields in the range 10^5–10^6 G, this suggests that the distribution may be bimodal. Current surveys are not geared to detect fields below 1 kG so the possibility remains that most white dwarfs are magnetic at some low level but this still needs to be confirmed. It seems likely that the magnetic fields in the low-field group must have been generated through dynamo processes in the course of normal single-star evolution to the white dwarf phase.

According to one school of thought, a small proportion of single stars, the magnetic main-sequence stars with strong fossil fields, may retain their fossil flux through to the white dwarf phase to give rise to the HFMWDs (Wickramasinghe & Ferrario 2005). This fossil from the main-sequence hypothesis, however, is not without difficulties. The progenitors of most white dwarfs have complex evolutionary histories with different regions of the star becoming convective and radiative as the star evolves through the red giant and asymptotic giant branches. The survival of fossil magnetic flux thus appears unlikely, although this still needs to be established with detailed calculations. A more serious difficulty with this hypothesis is that the birth rate from this route appears to be too small by a factor of 3 or so to account for the entire class of HFMWDs (Kawka et al. 2007). Alternative explanations for the fields seen in the HFMWDs have therefore been sought.

From the absence of late-type star companions to HFMWDs, while such stars are seen in abundance with other white dwarfs, Tout et al. (2008) established that high magnetic fields in white dwarfs are intimately tied up with stellar duplicity. They then proposed that magnetic fields can be generated through differential rotation during the common envelope evolution which leads to the formation of cataclysmic variables. Such fields could be trapped in the white dwarf and give rise to the magnetic cataclysmic variables. If the cores merge during this phase, a highly magnetic isolated white dwarf can be formed. Though the conditions are ripe for a strong dynamo in the common envelope it is not easy to see how such a field could diffuse into the white dwarf on the time for which the envelope remains in place (Potter & Tout 2010). We might envisage that a weak seed poloidal field, generated when it was last convective and now anchored within the white dwarf, might be wound up by differential rotation in the outer layers of the white dwarf during the common envelope evolution. The ensuing dynamo action can also enhance the poloidal field as long as both poloidal and toroidal fields are sufficient to stabilize one another. The final stable field structure can then freeze into the outer layers of the white dwarf as it cools.

Some of the HFMWDs might have descended from double white dwarf systems that merged, driven by gravitational radiation, after they emerged from a common envelope in a close orbit. Unlike those that merged during the common envelope phase, which we expect to spin down before ejection of their remaining envelope, these HFMWDs can still be spinning more rapidly (Wickramasinghe & Ferrario 2000). Even a small fraction of circumstellar material accreted from a disc by the merged core can spin its outer layers up to break-up speeds (Guerrero, García-Berro & Isern 2004) after which a dynamo can generate the very strong magnetic fields.

3 THE MAGNETIC FLUX TO MASS RATIO AND STABILITY OF FIELD STRUCTURES

For a star of mass M and radius R, the thermal energy can be written as

$$U = \frac{\lambda G M^2}{R}, \tag{2}$$

where G is the gravitational constant and λ is a structural constant of order unity. For a $2\,\mathrm{M}_\odot$ A-star $\lambda \approx 0.5$. The total magnetic energy can be written as

$$E = E_\mathrm{p} + E_\phi = \frac{4}{3}\pi R^3 \left(\frac{B_\mathrm{p}^2 + B_\phi^2}{2\mu_0}\right) = \frac{4}{3}\pi R^3 \frac{B^2}{2\mu_0}, \tag{3}$$

where B_p is the poloidal, B_ϕ the toroidal, B the total magnetic field strength and μ_0 is the permeability of a vacuum. Defining the ratios of magnetic to thermal energy by $\eta = E/U$, $\eta_\mathrm{p} = E_\mathrm{p}/U$ and $\eta_\phi = E_\phi/U$,

$$\eta = \frac{B^2}{2\mu_0}\frac{4}{3}\pi R^3 \frac{R}{\lambda G M^2} = \frac{\Phi^2}{M^2}\frac{2\pi}{3\mu_0 \lambda G}. \tag{4}$$

Thence, scaling to the observed maximum poloidal flux, we find

$$\eta_\mathrm{p} = \frac{10^{-8}}{\lambda}\left(\frac{\Phi_\mathrm{p}/M}{10^{-6.5}\,\mathrm{G\,cm^2\,g^{-1}}}\right)^2, \tag{5}$$

and the observations demonstrate that the maximum η_p is independent of the mass and type of star.

Braithwaite (2009) investigated the evolution of fossil fields in non-rotating stars and found that for stable poloidal– toroidal field structures, the energy in the toroidal and poloidal components of the field must satisfy

$$a\eta^2 < \eta_\mathrm{p} < 0.8\eta, \tag{6}$$

where a is a buoyancy factor and $a \approx 10$ for main-sequence stars. The first inequality is due to the stabilizing effect of a poloidal field on the Taylor instability of the $m = 1$ mode in purely toroidal fields. A lower limit to the poloidal field is ultimately set by the relative importance of magnetic to gravitational–thermal energy through buoyancy effects. The upper limit is due to the stability of poloidal fields which requires they be not significantly larger than the toroidal field. These expressions were obtained analytically from stability studies and were confirmed to be generally consistent with results of numerical studies of non-rotating stars. Braithwaite (2009) argued that the same inequalities are also likely to hold for stable fields in rotating stars.

4 THE DYNAMO MODEL

In a similar spirit to Tout & Pringle (1992), we set up a simple dynamo in which toroidal field is generated from poloidal field by differential rotation $\Delta\Omega$ within the star. Unless the poloidal field is strong enough to stabilize the toroidal, the latter is lost by magnetic instabilities which, in a non-rotating star, operate on an Alfvén crossing time-scale

$$\tau_\mathrm{A} = \frac{R\sqrt{\mu_0 \rho}}{B}, \tag{7}$$

where $\rho = 3M/4\pi R^3$ is the star's mean density. Pitts & Tayler (1985) demonstrated that, in the presence of rotation, the growth rate of magnetic instabilities is reduced by a factor Ω/Ω_B, where Ω is the angular velocity and $\Omega_B = 2\pi/\tau_\mathrm{A}$. The observed spins of magnetic Ap stars are of the order 1–1000 d (Landstreet & Mathys 2000), while their characteristic Alfvén time-scale is of the order of 2 d. Similarly, the observed spin rates of magnetic white dwarfs are of the order of hours to years (Wickramasinghe & Ferrario 2000), while their characteristic Alfvén time-scale is about 1.5 h. Therefore, magnetic field decay is not significantly limited by rotation once the magnetic fields have grown and differential rotation removed. However, we begin our calculations with small fields and differential rotation of the order of the break-up spin of the star. The angular velocity at any point in the star lies between the extremes of ineffectual, $\Omega < \Omega_\mathrm{B}$ and maximal, $\Omega = \Omega_\mathrm{crit}$, where

$$\Omega_\mathrm{crit} = \sqrt{\frac{GM}{R^3}} \tag{8}$$

is the surface break-up spin rate of the star. While we envisage some parts of the star to be rotating slowly, others rotate faster by $\Delta\Omega$. Thus, $0 < \Omega \approx \Delta\Omega < \Omega_{\rm crit}$ in regions where magnetic fields are decaying. We therefore multiply the time-scale for instability growth by $\Omega\tau_{\rm A}/2\pi$, where we first set $\Omega = \Delta\Omega$ and later consider the two extreme cases of $\Omega < \Omega_{\rm B}$ and $\Omega = \Omega_{\rm crit}$, when this is longer than $\tau_{\rm A}$. Our equation for the evolution of toroidal field is then

$$\frac{{\rm d}B_\phi}{{\rm d}t} = \Delta\Omega B_{\rm p} - \frac{B_\phi}{\tau_\phi}, \tag{9}$$

where

$$\tau_\phi = \begin{cases} \infty & \text{if } \eta_{\rm p} > a\eta^2, \\ \max\left(1, \frac{\Omega\tau_{\rm A}}{2\pi}\right)\tau_{\rm A} & \text{otherwise.} \end{cases} \tag{10}$$

Unusually, we regenerate poloidal field from the decaying toroidal field with some efficiency α. Our reasoning is that decay of toroidal field produces buoyant structures that generate a small-scale poloidal field. Reconnection amongst these field elements leaves a large-scale poloidal field in a similar way to that described by Tout & Pringle (1996) for accretion discs. In accord with Braithwaite (2009), our poloidal field decays on the same modified Alfvén time-scale unless there is sufficient total field strength to stabilize it. Thus,

$$\frac{{\rm d}B_{\rm p}}{{\rm d}t} = \alpha\frac{B_\phi}{\tau_\phi} - \frac{B_{\rm p}}{\tau_{\rm p}}, \tag{11}$$

where

$$\tau_{\rm p} = \begin{cases} \infty & \text{if } 0.8\eta > \eta_{\rm p}, \\ \max\left(1, \frac{\Omega\tau_{\rm A}}{2\pi}\right)\tau_{\rm A} & \text{otherwise.} \end{cases} \tag{12}$$

Equations (9) and (11) are non-linear because of the dependence of $\tau_{\rm p}$ and τ_ϕ on B. Differential rotation is gradually reduced by the magnetic torque, so that

$$I\frac{{\rm d}\Delta\Omega}{{\rm d}t} = -\frac{B_{\rm p}B_\phi}{\mu_0}4\pi R^2 R. \tag{13}$$

We now write

$$\zeta = \sqrt{\eta} = B\sqrt{\frac{2\pi R^4}{3\mu_0\lambda G M^2}} = B\sqrt{\frac{1}{2\mu_0\lambda}\frac{R}{GM\rho}}, \tag{14}$$

$$k = \frac{\Delta\Omega}{\Omega_{\rm crit}}, \tag{15}$$

and we define a dynamical time-scale

$$t_{\rm dyn} = \frac{1}{\Omega_{\rm crit}}. \tag{16}$$

We introduce a dimensionless time

$$\tau = \frac{t}{t_{\rm dyn}}, \tag{17}$$

so that

$$\tau_{\rm A} = \frac{t_{\rm dyn}}{\sqrt{2\lambda}\zeta} \tag{18}$$

and

$$\frac{\Delta\Omega\tau_{\rm A}}{2\pi} = \frac{k}{2\pi\sqrt{2\lambda}\zeta}. \tag{19}$$

The moment of inertia of the star is expressed as

$$I = \gamma M R^2, \tag{20}$$

with $\gamma \approx 0.1$ for a $2\,{\rm M}_\odot$ main-sequence star. Our equations now take the dimensionless forms

$$\frac{{\rm d}\zeta_\phi}{{\rm d}\tau} = k\zeta_{\rm p} - \sqrt{2\lambda}\sigma_\phi\zeta\zeta_\phi, \tag{21}$$

where

$$\sigma_\phi = \begin{cases} 0 & \text{if } \zeta_{\rm p}^2 > a\zeta^4, \\ \min\left(1, \frac{2\pi\sqrt{2\lambda}\zeta}{k}\right) & \text{otherwise,} \end{cases} \tag{22}$$

$$\frac{{\rm d}\zeta_{\rm p}}{{\rm d}\tau} = \alpha\sqrt{2\lambda}\sigma_\phi\zeta\zeta_\phi - \sqrt{2\lambda}\sigma_{\rm p}\zeta\zeta_{\rm p}, \tag{23}$$

where

$$\sigma_{\rm p} = \begin{cases} 0 & \text{if } \zeta_{\rm p}^2 < 0.8\zeta^2, \\ \min\left(1, \frac{2\pi\sqrt{2\lambda}\zeta}{k}\right) & \text{otherwise,} \end{cases} \tag{24}$$

and

$$\frac{{\rm d}k}{{\rm d}\tau} = -\frac{6}{\gamma}\zeta_{\rm p}\zeta_\phi. \tag{25}$$

5 RESULTS

We evolve the three coupled differential equations set up in Section 4 by a simple Euler method. Experiments with more precise and stable algorithms revealed no perceptible differences. For a typical A-star of $2\,{\rm M}_\odot$, the structural constants $\lambda = 0.5$ and $\gamma = 0.1$ are chosen according to detailed models made with the Cambridge STARS code (Eggleton 1971; Pols et al. 1995), while $a = 10$ is chosen according to Braithwaite (2009). This leaves a single free parameter α, which describes what fraction of the decaying toroidal field is converted to poloidal, in our simple dynamo model. For a given initial amount of differential rotation, we find that the final field strengths increase monotonically with α because for higher α less of the rotational energy is lost when the toroidal field decays. Thus, we can calibrate α so that the final total poloidal flux per unit mass is equal to that observed (equation 5). Fig. 2 shows the evolution of magnetic fields as the differential rotation decays away from its maximum, $\Delta\Omega = \Omega_{\rm crit}$. We have set $\alpha = 1.52 \times 10^{-4}$ to reproduce the observed maximum in $\eta_{\rm p} \approx 10^{-8}$, so $\zeta_{\rm p} \approx 10^{-4}$ again with $\lambda = 0.5$. For small enough seed fields, toroidal field is initially generated by winding poloidal field owing to the differential rotation. As soon as the toroidal field is large enough that stability criterion (10) is violated the poloidal field begins to grow. While differential rotation is still sufficient the toroidal field grows but it eventually reaches a maximum and decays until it reaches equilibrium with the poloidal field so that $a\eta^2 = \eta_{\rm p}$. Throughout this evolution, $\Delta\Omega$ is destroyed by the magnetic torque so that the final object is rotating as a solid body. At this point, $\tau = 4.88 \times 10^4$. The dynamical time-scale for a $2\,{\rm M}_\odot$ star is about 40 min, so this evolution of the magnetic fields is over in about 3.7 yr which is much less than the corresponding Kelvin–Helmholtz time-scale of 2.3×10^6 yr for a $2\,{\rm M}_\odot$ star. The field configuration is stable according to the Braithwaite (2009) criteria and hence can survive until thermal or nuclear time-scale evolution destroys it.

Table 1 lists the final state for various initial conditions and variation of the regeneration parameter α. The first row is the model of Fig. 2 already discussed. The next two rows illustrate how changing the seed toroidal field has almost no effect. Its growth or decay rapidly wipe out any initial state. Rows four and five show that varying the initial seed poloidal field also has little effect on the final magnetic fields. However, reducing the seed poloidal field does

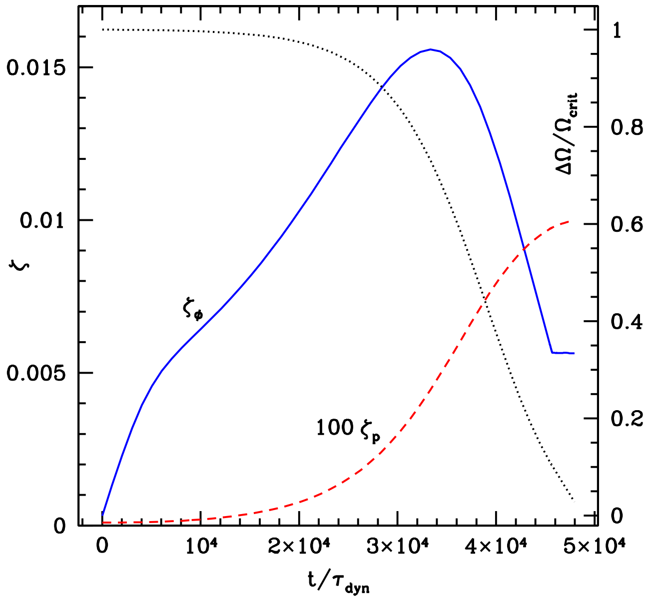

Figure 2. The evolution of magnetic field components following a collision that leaves the greatest differential rotation, equal to its break-up spin, in the merged object. The decay of differential rotation follows the right-hand axis while the indicated fields build up according to the left-hand axis. Toroidal field decays unless $\eta_p > a\eta^2$, and this determines the final ratio of toroidal to poloidal field. The poloidal field is somewhat weaker and so is multiplied by 100 on this figure. Rotational stabilization of magnetic field is included with $\Omega = \Delta\Omega$ in equations (10) and (12).

mean that the differential rotation lasts longer. This is simply because the initial generation of toroidal field depends on the presence of poloidal field. Once the poloidal field has built up the evolution speeds up and proceeds in a similar way to much the same end point. Rows six and seven demonstrate how the parameter α influences the final field strengths. For large α more of the energy extracted from differential rotation goes into magnetic fields. If α were unity no energy would be lost until the stability criterion (12) were violated. We do not expect the generation of poloidal field by decaying toroidal field to ever be sufficient to violate this criterion once the growth of the fields from their seeds is established.

The last two rows of Table 1 show how the final magnetic flux depends on the initial differential rotation. The final poloidal flux is almost directly proportional to the initial k_i. Thus, we hypothesize that, in collisions, the differential rotation left within the merged object ranges up to the maximum which cannot be higher than the break-up spin. The observed lower limit to the total flux per unit mass in main-sequence stars would then result from a lower limit, of about 1 per cent of the break-up spin, to the differential rotation within merged main-sequence stars. It is reasonable to imagine that stars are unlikely to merge without some residual angular momentum because head on collisions are extremely rare. In the case of white dwarfs that merge during common envelope evolution, we might then expect a similar minimum magnetic field of some 10^7 G compared with the maximum of 10^9 G observed for single white dwarfs. Here, the observations suggest a somewhat lower value of about 10^6 G as a lower field limit for the HFMWDs (Külebi et al. 2009).

To investigate the importance of rotational stabilization of magnetic field decay (Pitts & Tayler 1985), we consider the two extreme cases of minimal and maximal rotational stabilization. All observed high-field stars now have $\Omega < \Omega_B$ at their surfaces. Fig. 3 shows the evolution from the same initial conditions and with the same field generation parameter α as the model of Fig. 2 but with $\Omega < \Omega_B$ throughout the evolution. As in that case, the toroidal field initially builds up at the expense of differential rotation but decay sets in earlier leading to the generation of more poloidal field throughout the evolution. At the other extreme, we set $\Omega = \Omega_{crit}$ throughout the evolution. In this case, shown in Fig. 4, for the same initial conditions and α, the early evolution is not much altered because $\Delta\Omega \approx \Omega_{crit}$. Once $\Delta\Omega$ falls toroidal field builds up more at the expense of generating poloidal so that the final poloidal field is smaller. However, we note that the differences are not large and so our model is quite robust to the choice of Ω. The final poloidal field is in the range given by $8.5 \times 10^{-5} < \zeta_p < 1.8 \times 10^{-4}$ from the extreme of maximal rotational stabilization to none at all.

Single HFMWDs can form either when two cores merge during common envelope evolution or when two white dwarfs of total mass below the Chandrasekhar limit emerge from a common envelope with a short enough period and mass ratio close enough to unity that they subsequently merge owing to angular momentum loss by gravitational radiation. Zhang, Wickramasinghe & Ferrario (2009) plot the distribution of magnetic fields of single HFMWDs and magnetic cataclysmic variables in their fig. 5. The single-star fields follow much the same distribution as the combined polar and intermediate polar distribution up to 3×10^8 G or so. Higher fields are found only in the single white dwarfs. In a common envelope, a low-mass main-sequence star merges when it has spiralled in enough to

Table 1. Dynamo models varying initial conditions and the parameter α. Columns are the fraction α of decaying toroidal field that becomes poloidal, the initial ratio of differential rotation to break-up spin, the initial and final ($\Delta\Omega = 0$) toroidal and poloidal fields and the number of dynamical times to reach to reduce $\Delta\Omega$ to zero. The first row shows the model which best fits the observed maximum total flux per unit mass and which is illustrated in Fig. 2.

α	k_i	$\zeta_{\phi,i}$	$\zeta_{p,i}$	$\zeta_{\phi,f}$	$\zeta_{p,f}$	τ_f
1.52×10^{-4}	1	$10^{-3.5}$	10^{-6}	5.36×10^{-3}	10^{-4}	4.88×10^4
1.52×10^{-4}	1	$10^{-4.5}$	10^{-6}	5.35×10^{-3}	10^{-4}	4.91×10^4
1.52×10^{-4}	1	$10^{-2.5}$	10^{-6}	5.35×10^{-3}	10^{-4}	4.60×10^4
1.52×10^{-4}	1	$10^{-3.5}$	10^{-7}	5.31×10^{-3}	9.99×10^{-5}	8.41×10^4
1.52×10^{-4}	1	$10^{-3.5}$	10^{-5}	5.45×10^{-3}	1.03×10^{-4}	2.74×10^4
1.5×10^{-3}	1	$10^{-3.5}$	10^{-6}	1.24×10^{-2}	5.55×10^{-4}	1.08×10^4
1.5×10^{-5}	1	$10^{-3.5}$	10^{-6}	2.31×10^{-3}	1.79×10^{-5}	3.22×10^5
1.15×10^{-4}	0.5	$10^{-3.5}$	10^{-6}	3.94×10^{-3}	4.94×10^{-5}	8.08×10^4
1.15×10^{-4}	0.1	$10^{-3.5}$	10^{-6}	1.67×10^{-3}	1.67×10^{-6}	2.29×10^5

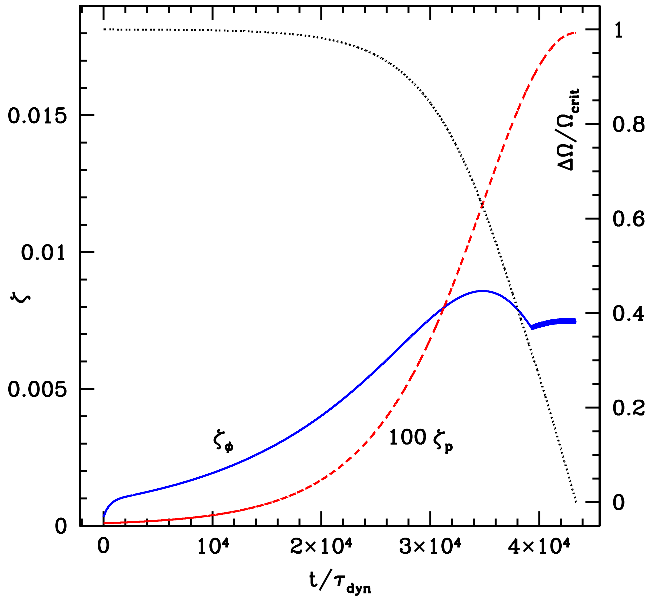

Figure 3. The dynamo when rotational stabilization of field decay is not included, $\Omega < \Omega_B$ throughout. Initial conditions and α are the same as for the model of Fig. 2. In this case, toroidal field decay and consequent poloidal field growth begin much earlier so that the final poloidal field is larger by a factor of 1.8.

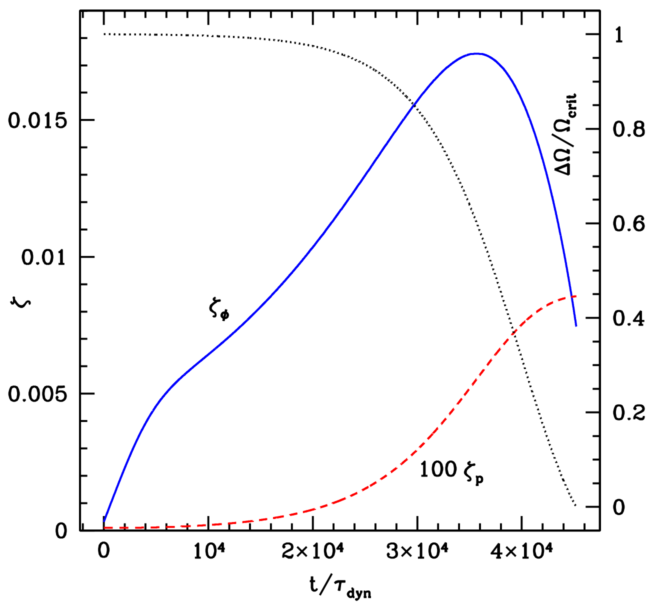

Figure 4. The dynamo when field decay is maximally rotationally stabilized, $\Omega = \Omega_{crit}$ throughout. Initial conditions and α are the same as for the model of Fig. 2. Early on $\Delta\Omega \approx \Omega_{crit}$ so the evolution is very similar. At late stages toroidal field decay is more stabilized so that the final toroidal field is slightly larger and the final poloidal field is smaller by a factor of 0.85.

be tidally disrupted by the degenerate core and it dissolves into the envelope. Tout et al. (2008) hypothesized that the magnetic cataclysmic variables must have emerged from their common envelopes only just detached in order to have acquired their strong fields. For these, we envisage that the hot outer layers of the degenerate core are weakly spun up by the differential rotation between the orbit of the cores and the envelope. A dynamo of the sort discussed here could then build up strong fields from a weak seed field in much less time than would be required for magnetic fields to diffuse into the white dwarf (Potter & Tout 2010). This is much more consistent with the expected time-scale for common envelope evolution. Those that merge account for the bulk of the single HFMWDs that have a similar distribution of fields to the magnetic cataclysmic variables. White dwarfs in the high-field tail have then come from merging degenerate objects. We expect those that merge during common envelope evolution to spin down considerably before the remainder of their envelope is ejected while those that merge subsequently may continue to spin more rapidly as solid bodies. While most magnetic white dwarfs have spin periods far in excess of 1 d a few spin more rapidly. For example, RE J0317–853 has a spin period of only 725 s (Barstow et al. 1995) and a very high magnetic field of 450 MG (Ferrario et al. 1997). We would expect that the highest fields are, on average, associated with higher mass white dwarfs. Future population synthesis studies coupled with more extensive observed data will test this hypothesis further.

Lastly, we comment on the possible relevance of our model to neutron star magnetic fields. Among the neutron stars are the magnetars characterized by their extremely strong magnetic fields of 10^{14}–10^{15} G (Rea & Esposito 2011). In a sense these are the counterparts of the HFMWDs. Merging of two white dwarfs with a total mass that exceeds the Chandrasekhar mass could lead to a collapse to a neutron star under certain circumstances. The white dwarf type fields generated during such a merge could be amplified to magnetar-strength fields by flux conservation during the subsequent collapse (King, Pringle & Wickramasinghe 2001). However, given the large fraction of magnetars among neutron stars, the majority of them are likely to have formed as end products of single-star evolution. Prior to collapse, the cores may be expected to have a range of spin periods depending on the initial angular momenta of stars on the main sequence and angular momentum transport during stellar evolution (Heger, Woosley & Spruit 2005). Models predict that, if angular momentum is conserved during the subsequent core collapse, the resulting neutron stars have spin periods that are generally in accord with the birth spins observed for pulsars. If strong differential rotation is acquired during core collapse, magnetic fields may be generated by a dynamo of the type that we have proposed. We estimate that poloidal fields of $2 \times 10^{14} - 1.2 \times 10^{15}$ G, similar to magnetar fields, could be generated in a neutron star that, at birth, differentially rotates at 5−30 per cent of its break-up speed and if the dynamo efficiency factor α is similar to that which we deduced for main-sequence stars. Such magnetars would also harbour even stronger toroidal fields and we note, significantly, that most models of magnetars require such fields to explain their emission properties (Rea & Esposito 2011).

6 CONCLUSIONS

Observations of magnetic main-sequence stars and white dwarfs show that there appears to be a universal upper limit to the poloidal magnetic flux per unit mass. We have explored recent proposals that the strongly magnetic stars in both these groups arise when stars merge. A simple magnetic dynamo that extracts energy from differential rotation explains the limit if the maximum fields are generated in merged objects that are differentially rotating at break-up. Weaker fields are generated in merged objects with less differential rotation.

In the case of white dwarfs, we envisage that common envelope evolution generates differential rotation in the hot outer parts of the degenerate core. For cataclysmic variables, the amount of differential rotation that can be imparted depends on how close the orbiting cores get before the envelope is lost. A dynamo such as we have described can operate on a rapid time-scale to create stable fields that become frozen in to the white dwarf as it cools. When cores merge in a common envelope, we do not expect a collision. Rather a main-sequence star would dissolve on reaching a depth in the envelope at which it is tidally disrupted by the degenerate core. The fields generated ought to be similar to those in the magnetic cataclysmic variables, which we envisage to have ejected their common envelopes just before they would otherwise have merged. Similarly, a lower mass degenerate core would be tidally shredded and then accreted on to the more massive degenerate core either within the common envelope or subsequently if a double degenerate system merges by gravitational-wave angular momentum loss. In both cases, we expect the differential rotation that can be built up in the outer layers of the hot or heated core to be very large because accretion of less than a tenth of a solar mass of material from the inner edge of an accretion disc can spin a substantial fraction of the white dwarf to break up. So the maximum magnetic field strengths should be found only in the single white dwarfs formed from merged degenerate cores or stars. Those that merge within the common envelope, we expect to rotate very slowly, spun down as angular momentum is lost with the remaining envelope. Those that merge subsequently to the envelope ejection can be expected to have a high fields too but also to be more rapidly rotating.

Merging of two white dwarfs with a total mass that exceeds the Chandrasekhar limit may under certain circumstances lead to an accretion induced collapse to a neutron star with magnetar-type fields. Though most magnetars are likely to have evolved from single stars, a similar dynamo may still play a role to generate fields in these stars if they form highly differentially rotating after core collapse.

ACKNOWLEDGEMENTS

We thank the referee for drawing out attention to the work by Pitts & Tayler (1985). DTW and LF thank the Institute of Astronomy for hospitality while this work was continued. CAT thanks the University of Cambridge, Monash University and the Australian National University for supporting a visit to Australia during which this work was begun, Churchill College for his fellowship and the Bombay Cambridge Society and the Inter University Centre for Astronomy and Astrophysics in Pune for providing an atmosphere in which the writing could be completed.

REFERENCES

Alecian E. et al., 2008, MNRAS, 385, 391
Aurière M. et al., 2010, A&A, 523, 40
Barstow M. A., Jordan S., O'Donoghue D., Burleigh M. R., Napiwotzki R., Harrop-Allin M. K., 1995, MNRAS, 277, 971
Braithwaite J., 2009, MNRAS, 397, 763
Eggleton P. P., 1971, MNRAS, 151, 351
Elkin V. G., Mathys G., Kurtz D. W., Hubrig S., Freyhammer L. M., 2010, MNRAS, 402, 1883
Ferrario L., Vennes S., Wickramasinghe D. T., Bailey J. A., Christian D. J., 1997, MNRAS, 292, 205
Ferrario L., Pringle J. E., Tout C. A., Wickramasinghe D. T., 2009, MNRAS, 400, L71
Guerrero J., García-Berro E., Isern J., 2004, A&A, 413, 257
Heger A., Woosley S. E., Spruit H. C., 2005, ApJ, 626, 350
Kawka A., Vennes S., Schmidt G. D., Wickramasinghe D. T., Koch R., 2007, ApJ, 654, 499
King A. R., Pringle J. E., Wickramasinghe D. T., 2001, MNRAS, 320, L45
Külebi B., Jordan S., Euchner F., Gänsike B. T., Hirsch H., 2009, A&A, 506, 1341
Landstreet J. D., Mathys G., 2000, A&A, 359, 213
Landstreet J. D., Bagnulo S., Valyavin G. G., Fossati L., Jordan S., Monin D., Wade G. A., 2012, A&A, 545, A30
Pitts E., Tayler R. J., 1985, MNRAS, 216, 139
Pols O. R., Tout C. A., Eggleton P. P., Han Z., 1995, MNRAS, 274, 964
Potter A. T., Tout C. A., 2010, MNRAS, 402, 1072
Potter A. T., Chitre S. M., Tout C. A., 2012, MNRAS, 424, 2358
Power J., Wade G. A., Aurière M., Silvester J., Hanes D, 2008, Contrib. Astron. Obs. Skalnaté Pleso, 38, 443
Rea N., Esposito P., 2011, in Rea N., Torres D. F., eds, High-Energy Emission from Pulsars and their Systems. Springer, Berlin, p. 247
Tout C. A., Pringle J. E., 1992, MNRAS, 256, 269
Tout C. A., Pringle J. E., 1996, MNRAS, 281, 219
Tout C. A., Wickramasinghe D. T., Liebert J., Ferrario L., Pringle J. E., 2008, MNRAS, 387, 897
Wade G. A., Bagnulo S., Drouin D., Landstreet J. D., Monin D., 2007, MNRAS, 376, 1145
Wade G. A., Grunhut J. H. the MiMes Collaboration, 2012, in Carciofi A. C., Rivinius T., eds, ASP Conf. Ser. Vol. 464, Circumstellar Dynamics at High Resolution. Astron. Soc. Pac., San Francisco, p. 405
Wade G. A. et al., 2011, in Neiner C., Wade G. A., Meynet G., Peters G. J., eds, Proc. IAU Symp. 272, Active OB Stars: structure, evolution, mass loss and critical limits. Cambridge Univ. Press, Cambridge, p. 118
Wickramasinghe D. T., Ferrario L., 2000, PASP, 112, 873
Wickramasinghe D. T., Ferrario L., 2005, MNRAS, 356, 615
Zhang C. M., Wickramasinghe D. T., Ferrario L., 2009, MNRAS, 397, 2208

This paper has been typeset from a TEX/LATEX file prepared by the author.