PRL 112, 172701 (2014)

2 MAY 2014

Double-Magic Nature of ¹³²Sn and ²⁰⁸Pb through Lifetime and **Cross-Section Measurements**

J. M. Allmond, ^{1,*} A. E. Stuchbery, ² J. R. Beene, ³ A. Galindo-Uribarri, ^{3,4} J. F. Liang, ³ E. Padilla-Rodal, ⁵ D. C. Radford, ³ R. L. Varner, ³ A. Ayres, ⁴ J. C. Batchelder, ⁶ A. Bey, ⁴ C. R. Bingham, ^{3,4} M. E. Howard, ⁷ K. L. Jones, ⁴ B. Manning, ⁷ P. E. Mueller, ³ C. D. Nesaraja, ³ S. D. Pain, ³ W. A. Peters, ⁸ A. Ratkiewicz, ⁷ K. T. Schmitt, ^{3,4,†} D. Shapira, ³ M. S. Smith, N. J. Stone, 4,9 D. W. Stracener, and C.-H. Yu³ ¹JINPA, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA ²Department of Nuclear Physics, Australian National University, Canberra ACT 0200, Australia ³Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA ⁴Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA ⁵Instituto de Ciencias Nucleares, UNAM, AP 70-543, 04510 Mexico, D.F., Mexico ⁶UNIRIB, Oak Ridge Associated Universities, Oak Ridge, Tennessee 37831, USA ⁷Department of Physics and Astronomy, Rutgers University, New Brunswick, New Jersey 08903, USA ⁸Oak Ridge Associated Universities, Oak Ridge, Tennessee 37830, USA ⁹Department of Physics, Oxford University, Oxford OX1 3PU, United Kingdom (Received 4 February 2014; published 30 April 2014)

> Single-neutron states in ¹³³Sn and ²⁰⁹Pb, which are analogous to single-electron states outside of closed atomic shells in alkali metals, were populated by the (9Be, 8Be) one-neutron transfer reaction in inverse kinematics using particle- γ coincidence spectroscopy. In addition, the $s_{1/2}$ single-neutron hole-state candidate in ¹³¹Sn was populated by (⁹Be, ¹⁰Be). Doubly closed-shell ¹³²Sn (radioactive) and ²⁰⁸Pb (stable) beams were used at sub-Coulomb barrier energies of 3 MeV per nucleon. Level energies, γ -ray transitions, absolute cross sections, spectroscopic factors, asymptotic normalization coefficients, and excited-state lifetimes are reported and compared with shell-model expectations. The results include a new transition and precise level energy for the $3p_{1/2}$ candidate in 133 Sn, new absolute cross sections for the $1h_{9/2}$ candidate in 133 Sn and $3s_{1/2}$ candidate in 131 Sn, and new lifetimes for excited states in 133 Sn and 209 Pb. This is the first report on excited-state lifetimes of ¹³³Sn, which allow for a unique test of the nuclear shell model and ¹³²Sn double-shell closure.

> DOI: 10.1103/PhysRevLett.112.172701 PACS numbers: 25.60.Je, 21.10.Jx, 21.10.Tg, 23.20.Lv

Atomic nuclei are finite many-body quantum systems that possess shell structure with closures at Z or N equal to 2, 8, 20, 28, 50, 82, or 126. These closed shells are evident from isotope and isotone abundances and discontinuities in nucleon separation energies [1]. The stable double-magic nuclei, i.e., closed shell in both proton and neutron number, are limited to ⁴He, ¹⁶O, ⁴⁰Ca, ⁴⁸Ca, and ²⁰⁸Pb; radioactive ion beams (RIBs) can provide access to additional nuclei that are potentially double magic such as ⁵⁶Ni, ⁷⁸Ni, ¹⁰⁰Sn, and ¹³²Sn. Experiments on double-magic nuclei test the nuclear shell model [2] and provide input to calculations of properties of neighboring nuclei, many of which are radioactive and experimentally inaccessible. These calculations rely on the inert core [3] to reduce the many-body system to a size that makes the problem tractable. Such calculations for the radioactive ¹³²Sn region have been shown to be vital in controlling simulations for r-process nucleosynthesis [4–8], which is responsible for the origin of nearly half of the elements heavier than Fe (Z = 26).

The double-magic nature of radioactive ¹³²Sn has been elucidated from (1) neutron separation energies [9–12] and (2) radioactive decay [13–19] and Coulomb excitation [20] studies, which have revealed a comparatively large first 2⁺ energy and small electric quadrupole transition strength (similar to ²⁰⁸Pb [21]). Coulomb excitation studies of neighboring even-even nuclei have also supported an inert ¹³²Sn core [20,22–24]. Recently, single-neutron states above the N=82 shell closure in $^{133}\mathrm{Sn}$ were reported in a (d, p) study by Jones et al. [25] using a ¹³²Sn RIB at the Holifield Radioactive Ion Beam Facility (HRIBF). In particular, candidates for the single-neutron $2f_{7/2}$, $3p_{3/2}$, $3p_{1/2}$, and $2f_{5/2}$ states were measured with cross sections that are consistent with shell-model expectations; similar states were observed in 131 Sn [26]. Prior to the (d, p) study of ¹³³Sn [25], candidates for the single-neutron $2f_{7/2}$, $3p_{3/2}$, $1h_{9/2}$, and $2f_{5/2}$ states were reported in a decay study of fission fragments by Hoff et al. [27]; hole-state candidates in ¹³¹Sn were reported in a decay study by Fogelberg *et al.* [28,29].

In this Letter, new information on single-neutron states in ¹³³Sn and ²⁰⁹Pb (cf. Fig. 1), populated by the (⁹Be, ⁸Be) one-neutron transfer reaction in inverse kinematics $(A_{\text{beam}} > A_{\text{target}})$, is reported. Extensive spectroscopic information was obtained by using particle-γ coincidence

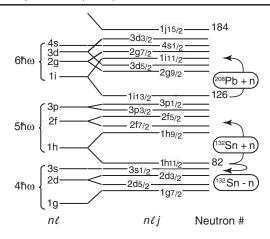


FIG. 1. Single-neutron states expected to be populated in the present one-neutron transfer study of ^{131,133}Sn and ²⁰⁹Pb.

spectroscopy. Evidence is provided for a complete set of single-neutron candidates in $^{133}\mathrm{Sn},\ 2f_{7/2},\ 3p_{3/2},\ 3p_{1/2},\ 1h_{9/2},\ 2f_{5/2},\ \text{and}\ 1i_{13/2}.$ However, the evidence for the $1i_{13/2}$ candidate is inconclusive due to inconsistencies between the spectroscopic results and expectations for an unbound $\ell=6$ neutron. In addition, the $3s_{1/2}$ single-neutron hole-state candidate in $^{131}\mathrm{Sn}$ was populated by ($^9\mathrm{Be},\ ^{10}\mathrm{Be}$). Level energies, γ -ray transitions, absolute cross sections, spectroscopic factors, asymptotic normalization coefficients (ANCs), and excited-state lifetimes are reported and compared with shell-model expectations. This is the first report on excited-state lifetimes of $^{133}\mathrm{Sn}.$

Because 8 Be is unbound ($T_{1/2} = 8.2 \times 10^{-17}$ s [21]), two correlated alphas are detected following the (9 Be, 8 Be) reaction. Scattered targetlike nuclei were measured at forward laboratory angles relative to the beam direction, corresponding to backward angles in the center-of-mass frame, to provide a clean trigger for selecting the γ -ray transitions emitted from the beamlike reaction products. The effectiveness of this technique was recently demonstrated [30–32] in the study of 135 Te and 137 Xe. The selectivity of single-neutron states with heavy-ion induced reactions has been discussed in Refs. [31,33].

Doubly closed-shell 132 Sn and 208 Pb beams were provided by HRIBF at sub-Coulomb energies of 3 MeV per nucleon. A Bragg detector, placed behind a 1.57(8) mg/cm² monoisotopic 9 Be target, measured an energy loss of 140(2) MeV for a 395-MeV beam of 124 Sn and 193(10) MeV for a 624-MeV beam of 208 Pb. The radioactive 132 Sn beam, which was \geq 96% pure [34,35] and had an intensity of 1×10^5 ions/s, was incident on the target for 5 days. The 208 Pb beam was incident on the target for 1 day.

Recoiling target nuclei were detected in the "bare" HyBall (BareBall) CsI(Tl) array [36], using the first four rings at laboratory angles 7° – 14° , 14° – 28° , 28° – 44° , and 44° – 60° relative to the beam direction. Coincident γ rays were detected in the CLARION array of 11 Compton

suppressed, segmented HPGe Clover detectors [37], which was configured with five detectors at 90°, four at 132°, and two at 154°. The Clover detectors were at a distance of 21.75 cm from the target with a total efficiency of 3.00(5)% at 1 MeV. The experimental trigger ($\geq 99\%$ live time) required either a scaled-down particle event or a particle- γ coincidence event.

The particle-gated γ -ray spectra are shown in Fig. 2 for (a) ^{209}Pb from (^{9}Be , $^{8}\text{Be} \rightarrow 2\alpha\gamma$) with a Doppler correction for the 969-keV transition, (b) ^{131}Sn from (^{9}Be , $^{10}\text{Be}\gamma$) with a Doppler correction for the 332-keV transition, and (c) ^{133}Sn from (^{9}Be , $^{8}\text{Be} \rightarrow 2\alpha\gamma$) with a Doppler correction for the 854-keV transition. The recoiling velocity, $\beta = v/c$, and Doppler-corrected energy were determined individually for each transition [32]. The 513- and 2792-keV γ -ray transitions from ^{133}Sn were previously unobserved.

The 513-keV transition in 133 Sn was found to be in coincidence with the previously known 854-keV transition using the (9 Be, 8 Be $\rightarrow 2\alpha\gamma\gamma$) coincidence data (cf. Fig. 3). The 513-keV γ ray originates from a state at 1366.8(4) keV, which corresponds to the $p_{1/2}$ candidate state recently reported at 1363(31) keV in the (d, p) study of 133 Sn [25].

The experimental cross sections were determined from a γ -ray intensity balance, i.e., the difference between the total intensity out of a state and the total intensity feeding that state, which included a kinematic correction to the solid angle. The absolute normalization was obtained from the

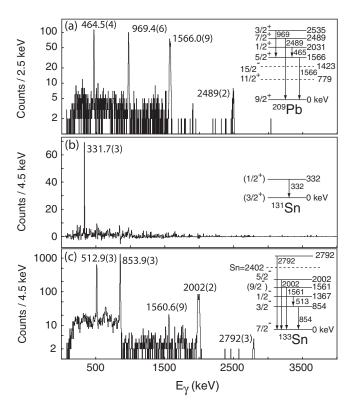


FIG. 2. Gamma-ray spectra of (a) 209 Pb from (9 Be, 8 Be $\rightarrow 2\alpha\gamma$), (b) 131 Sn from (9 Be, 10 Be γ), and (c) 133 Sn from (9 Be, 8 Be $\rightarrow 2\alpha\gamma$).

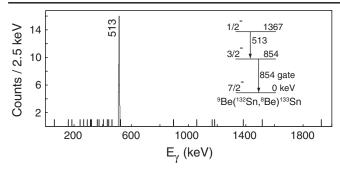


FIG. 3. 854-keV gate on γ - γ coincidence data from ¹³³Sn.

measured elastic scattering yield. The reactions involving ¹³²Sn and ²⁰⁸Pb were below the Coulomb barrier, so the absolute cross-section normalizations were particularly reliable because the transfer-to-elastic ratio could be determined within the same particle detector segment.

The theoretical cross sections were calculated with the distorted wave Born approximation (DWBA) code PTOLEMY [38] using parameter values from previous reaction studies involving $^9{\rm Be}$ [39–43]. The sensitivity of the calculations was explored by varying each parameter over a range of possible values estimated from the previous studies. The bound-state parameter values were $r=1.20\pm0.05$ fm and $a=0.65\pm0.10$ fm for both the real and spinorbit terms, and the spin-orbit potential was $V_{so}=6\pm6$ MeV. The parameter values for the incoming and

outgoing optical potentials were treated identically with $V = 80 \pm 40$ MeV, $r = 1.05 \pm 0.20$ fm, and $a = 0.65 \pm 0.10$ fm for the real component and $W = 17 \pm 17$ MeV, $r_W = 1.21 \pm 0.04$ fm, and $a_W = 0.85 \pm 0.30$ fm for the imaginary component. The absolute cross sections calculated for the (9 Be, 8 Be) reactions weakly depended on the optical-model parameters at the level of a few percent. The absolute cross-section calculations were most sensitive to the bound-state parameters. Uncertainties in the calculated cross sections were estimated by varying each parameter over the range of values assuming a uniform distribution.

Table I summarizes the particle-γ spectroscopic results for the ²⁰⁸Pb and ¹³²Sn beam data. The theoretical cross sections, σ_{thy} , and spectroscopic factors, $S = \sigma/\sigma_{thy}$, assume target spectroscopic factors of $S(^{9}\text{Be}, ^{8}\text{Be}) =$ 0.54(6) (determined from tabulated values in Ref. [40]) and $S(^{9}\text{Be}, ^{10}\text{Be}) = 1.6(2)$ (determined from Refs. [44– 50]). The spectroscopic factors for ²⁰⁹Pb and ¹³³Sn, which represent the fraction (i.e., purity) of the single particle wave function, are reasonably consistent with unity and with previous (d, p) studies [25,51]. The spectroscopic factor for the $\nu 1h_{9/2}$ candidate in ¹³³Sn at 1561 keV, not observed by the (d, p) study [25], shows a reduction in single-particle strength as compared with the other states. This reduction could be due to mixing with a second $9/2^{-}$ state from ¹³²Sn core excitations (e.g., $9/2^-$, $2^+ \otimes 2f_{7/2}$) or to the weak absolute cross section stemming from poor

TABLE I. Summary of particle- γ spectroscopic results for the ²⁰⁸Pb and ¹³²Sn beam data. The cross sections are given for recoiling targetlike nuclei measured at laboratory angles of 7° to 44° with respect to the beam axis. See the text for further details.

E_x (keV)	$J^{\pi_{\mathrm{a}}}$	E_{γ} (keV)	τ (fs)	σ (mb)	σ_{thy} (mb)	Present (⁹ Be, ^{8,10} Be) S	[25,51] (d,p) S	Present (⁹ Be, ^{8,10} Be) C ² (fm ⁻¹)	[25,51] (d, p) $C^2 \text{ (fm}^{-1})$	[53] (¹³ C, ¹² C) C ² (fm ⁻¹)
⁹ Be (²⁰⁸ Pb, ⁸ Be) ²⁰⁹ Pb										
0	$9/2^{+}$				0.0013(4)	-, -, -,	1.21(36)		2.20(17)	2.25(29)
778.9(3) ^b	$11/2^{+}$				0.0005(2)		1.57(47)		0.00187(13)	0.0037(5)
1423(1) ^b	$15/2^{-}$				0.0001(1)		1.19(36)		$2.5(2) \times 10^{-5}$	
1566.0(9)	$5/2^{+}$	1566.0(9)		0.13(4)	0.084(21)	1.5(6)	1.08(32)	14(5)	13.0(7)	
2031(1)	$1/2^{+}$	464.5(4)		0.28(2)	0.22(5)	1.3(3)	1.04(31)	45(8)	48.7(30)	41.7(54)
2489(2)	$7/2^{+}$	2489(2)		0.10(2)	0.062(19)	1.6(6)	1.27(38)	0.026(6)	0.025(2)	
2535(1)	$3/2^{+}$	969.4(6)	87(24)	0.43(3)	0.38(9)	1.1(3)	1.11(33)	2.3(4)	2.93(20)	
⁹ Be (¹³² Sn, ¹⁰ Be) ¹³¹ Sn										
0	$(3/2^+)$				0.15(11)	, -,				
331.7(3)	$(1/2^{+})$	331.7(3)		0.68(8)	0.17(12)	4(3)				
1654.53(8)b	· /	. ,		. ,	0.03(2)	. ,				
	, ,				⁹ Be (¹³² Sı	n, ⁸ Be) ¹³³ Sn				
0	7/2-				3(1)	, 20) 511	0.86(7)		0.64(5)	
853.9(3)	$3/2^{-}$	853.9(3)		12(1)	13(3)	0.9(2)	0.92(7)	6.0(14)	5.6(4)	
1366.8(4)	$1/2^{-}$	512.9(3)	$480(^{+160}_{-100})$	11(1)	12(3)	0.9(2)	1.1(2)	2.5(5)	2.6(6)	
1560.6(9)	$(9/2^{-})$	1560.6(9)	(-100)	0.58(10)	1.1(4)	0.5(2)	, ,	$5.1(15) \times 10^{-6}$. /	
2002(2)	5/2-	2002(2)	$13(^{+10}_{-13})$	8.6(6)	9.6(24)	0.9(2)	1.1(2)	0.0020(4)	0.0009(2)	
2792(3)	·	2792(3)	. 157	0.38(9)	0.18(7) ^c	` '			` '	

 $^{{}^{}a}J^{\pi}$ from ENSDF [21] and spin-flip transitions of present study.

 $^{{}^{}b}E_{x}$ for unobserved excited states from ENSDF [21].

^cCalculated assuming a bound $\nu 1i_{13/2}$ configuration.

momentum matching (i.e., more prone to any multistep processes); another experiment using a higher beam energy would differentiate between these possibilities. In ¹³¹Sn, only the 332-keV transition from the $3s_{1/2}$ neutron-hole candidate was observed in the (9Be, 10Be) reaction, which is consistent with the expectation that it should be the only excited state strongly populated. However, the calculated cross section was extremely sensitive to the bound-state parameters, which resulted in a large uncertainty on the spectroscopic factor. Because these reactions are peripheral and predominately probe the tails of the wave function, ANCs are also reported (model independent), which are insensitive to the bound-state parameters [52] and should be more reliable than the spectroscopic factors (see Refs. [53–56] for further discussion on spectroscopic factor and ANC reliability). A target ANC of $C^{2}(^{9}\text{Be}, ^{8}\text{Be}) =$ $0.24(4) \text{ fm}^{-1}$ is adopted from $0.27(9) \text{ fm}^{-1}$ [57,58], $0.30(28) \text{ fm}^{-1}$ [57–59], and $0.23(5) \text{ fm}^{-1}$ [57–59]. The ANCs for ²⁰⁹Pb are remarkably consistent with previous (d, p) [51] and (¹³C, ¹²C) [53] studies. The ANCs for ¹³³Sn are consistent with the previous (d, p) study [25]. However, a slightly larger ANC is obtained in the present study for the $\nu 2f_{5/2}$ candidate at 2002 keV. In addition, the ANC for the $\nu 1h_{9/2}$ candidate in ¹³³Sn is now reported. No target ANC could be found or determined for (⁹Be, ¹⁰Be) to apply to the ¹³¹Sn case.

Subpicosecond lifetimes of excited states in 209 Pb (2535-keV state) and 133 Sn (1367- and 2002-keV states), given in Table I, were measured by the Doppler shift attenuation method (DSAM) [60], which firmly identified the decays as spin-flip M1 transitions because transitions among other valence orbits or transitions of higher multipolarities must proceed much more slowly. Figure 4 shows the extracted lifetimes for the 1367-keV state (513-keV transition) of 133 Sn for each BareBall ring by comparing the experimental $\beta = v/c$ values to the calculated β versus τ values. The

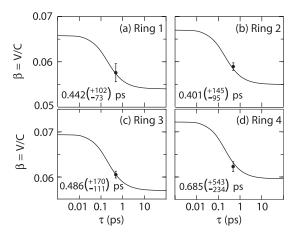


FIG. 4. The lifetime of the $1/2^-$ state at 1367 keV in ¹³³Sn was determined from the 513-keV γ ray using the experimental β values for each BareBall ring and DSAM.

calculated β values were based on the reaction kinematics and stopping powers, measured with the Bragg detector.

M1 transition strengths from the lifetimes reported in Table I are listed in Table II. Unless noted otherwise (i.e., for 207 Pb), these transition rates were evaluated assuming pure M1 transitions, i.e., the E2/M1 mixing ratio $\delta=0$. If $\delta\neq 0$, the M1 transition rate is reduced by the factor $1/(1+\delta^2)$. This reduction factor has a small effect because δ must be small for these strong spin-flip transitions in which the M1 operator simply reverses the spin coupling of a valence nucleon. A comparison of data with theory shows excellent agreement, which suggests relatively pure single-particle wave functions and a robust 132 Sn double-magic core. These M1 transition strengths test the shell model independent of the DWBA bound-state and optical-model parameters.

For a given M1 operator [63], bare or effective,

$$\frac{B(M1; \ell + 1/2 \to \ell - 1/2)}{B(M1; \ell - 1/2 \to \ell + 1/2)} = \frac{\ell}{(\ell + 1)}.$$
 (1)

The new lifetime data allow a parameter-free comparison of the transitions between $3p_{1/2}$ and $3p_{3/2}$ as holes in ^{207}Pb , i.e., $3p_{3/2}^{-1} \to 3p_{1/2}^{-1}$, and particles in ^{133}Sn , i.e., $3p_{1/2} \to 3p_{3/2}$. According to Eq. (1), the B(M1) ratio should be 0.5; experimentally, it is 0.53(16). The agreement is excellent, perhaps better than expected because there should be some differences in the effective M1 operator between the ^{208}Pb and ^{132}Sn regions.

The 2792-keV γ ray [Fig. 2(c)] is a natural $\nu 1i_{13/2}$ candidate for 133 Sn; the energy is consistent with expectations from systematics (cf. Fig. 8 in Ref. [31]) and with the 2694(200)-keV prediction by Urban *et al.* [64]. However, the γ -ray Doppler shift suggests a relatively long lifetime of $\tau > 1$ ps, which is consistent with expectations for an E3 decay but contradicts expectations for an unbound $\ell = 6$ neutron ($\tau \sim 0.1$ ps) that is nearly 400 keV above the neutron separation energy of 2396(4) keV [10–12]. The neutron-decay width should be much larger than the γ -decay width, but a relatively large cross section was determined from the γ -ray intensity. A 2792-keV γ ray from an $(11/2^-)$ state in 133 Sb is reported in the literature [21]. This state could be populated by one-neutron transfer

TABLE II. M1 transition strengths in units of (μ_N^2) .

Nuclide	Transition	$B(M1)^{exp}$	$B(M1)^{thy}$
²⁰⁹ Pb	$3d_{3/2} \rightarrow 3d_{5/2}$	0.72(20)	0.71 ^b
²⁰⁷ Pb	$3p_{3/2}^{-1} \rightarrow 3p_{1/2}^{-1}$	$0.47(6)^{a}$	0.40^{6}
¹³³ Sn	$2f_{5/2}^{5/2} \rightarrow 2f_{7/2}^{1/2}$	$0.55(^{+\infty}_{-14})$	0.52°
¹³³ Sn	$3p_{1/2} \rightarrow 3p_{3/2}$	$0.88(^{+23}_{-22})$	0.67°

^aFrom the Evaluated Nuclear Structure Data File [21], including a mixing ratio of $\delta = +0.091(9)$.

^b*M*1 operator for the ²⁰⁸Pb region from Castel and Towner [61]. ^c*M*1 operator for the ¹³²Sn region from Brown *et al.* [62].

on a 132 Sb beam contaminant. However, there was no evidence for 132 Sb in the beam from either the Bragg detector or Coulomb excitation, which would produce a 162 -keV γ ray. Population of 133 Sb could be achieved by 132 Sn (9 Be, 8 Li), but this channel would not lead to prompt detection of two correlated α particles. In addition, no γ rays were observed from the lower-lying states of 133 Sb [21].

In summary, single-neutron states have been measured in ¹³¹Sn, ¹³³Sn, and ²⁰⁹Pb by a novel particle-γ coincidence technique following sub-Coulomb heavy-ion induced oneneutron transfer in inverse kinematics. This technique yielded an extensive set of spectroscopic information. In particular, the particle- γ technique involving (9 Be, ⁸Be $\rightarrow 2\alpha\gamma$) provided a clean selection of the one-neutron transfer channel, high-precision γ-ray and excitation energies, relatively high-precision absolute cross sections, and excited-state lifetimes in a single experiment. Candidates for all expected single-neutron states in ¹³³Sn were observed, including a new transition and precise excitation energy for the $3p_{1/2}$ state. However, the evidence for the $1i_{13/2}$ candidate is inconclusive. New absolute cross sections have been measured, including the $3s_{1/2}$ neutron-hole candidate in ¹³¹Sn. Overall, the experimental cross sections are consistent with shell-model expectations. Furthermore, three new excited-state lifetimes were measured in ¹³³Sn and ²⁰⁹Pb from spin-flip M1 transitions, which show consistency with shell-model expectations. These lifetime results provide a unique test of the shell model without the uncertainties of optical-model and bound-state parameters. Despite being neutron rich and radioactive, ¹³²Sn is determined to be a robust doublemagic nucleus from both excited-state lifetime and crosssection measurements. The shell model can be applied with relative confidence to calculations of both ground- and excited-state properties of nuclei in the ¹³²Sn region, many of which are beyond current experimental access.

The authors thank B. A. Brown, C. J. Gross, B. P. Kay, R. L. Kozub, M. McCleskey, N. Michel, K. A. Miernik, and W. Nazarewicz for fruitful discussions and the HRIBF operations staff for developing and providing the stable and radioactive beams used in this study. This research was sponsored by the Office of Nuclear Physics, U.S. Department of Energy, by the Australian Research Council under Grant No. DP0773273, by CONACyT (Mexico) under Grant No. CB103366, and by the National Science Foundation. This work was also supported in part by the U.S. DOE under Contracts No. DE-AC05-76OR00033 (UNIRIB), No. DE-FG02-96ER40963 (UTK), and No. DE-FG52-08NA28552 (Rutgers).

- [1] M. G. Mayer, Phys. Rev. 74, 235 (1948).
- [2] M. G. Mayer and J. H. D. Jensen, *Theory of Nuclear Shell Structure* (John Wiley and Sons, London, 1955).
- [3] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C 87, 034309 (2013).
- [4] S. Chiba, H. Koura, T. Hayakawa, T. Maruyama, T. Kawano, and T. Kajino, Phys. Rev. C 77, 015809 (2008).
- [5] R. Surman, J. Beun, G. C. McLaughlin, and W. R. Hix, Phys. Rev. C 79, 045809 (2009).
- [6] J. Beun, J. C. Blackmon, W. R. Hix, G. C. McLaughlin, M. S. Smith, and R. Surman, J. Phys. G 36, 025201 (2009).
- [7] P. Mohr, Phys. Rev. C 86, 068803 (2012).
- [8] S.-S. Zhang, M. S. Smith, G. Arbanas, and R. L. Kozub, Phys. Rev. C 86, 032802(R) (2012).
- [9] B. Fogelberg, K. A. Mezilev, H. Mach, V. I. Isakov, and J. Slivova, Phys. Rev. Lett. 82, 1823 (1999).
- [10] M. Dworschak et al., Phys. Rev. Lett. 100, 072501 (2008).
- [11] J. Hakala et al., Phys. Rev. Lett. 109, 032501 (2012).
- [12] J. Van Schelt et al., Phys. Rev. Lett. 111, 061102 (2013).
- [13] J. W. Grüter, K. Sistemich, P. Armbruster, J. Eidens, and H. Lawin, Phys. Lett. **33B**, 474 (1970).
- [14] A. Kerek, G. B. Holm, L.-E. De Geer, and S. Borg, Phys. Lett. 44B, 252 (1973).
- [15] T. Björnstad et al., Phys. Lett. 91B, 35 (1980).
- [16] K. Kawade, K. Sistemich, G. Battistuzzi, H. Lawin, K. Shizuma, and J. Blomqvist, Z. Phys. A 308, 33 (1982).
- [17] T. Björnstad et al., Nucl. Phys. A453, 463 (1986).
- [18] B. Fogelberg, M. Hellström, D. Jerrestam, H. Mach, J. Blomqvist, A. Kerek, L. O. Norlin, and J. P. Omtvedt, Phys. Rev. Lett. 73, 2413 (1994).
- [19] P. Bhattacharyya et al., Phys. Rev. Lett. 87, 062502 (2001).
- [20] J. R. Beene *et al.*, Nucl. Phys. A746, 471 (2004);
 D. C. Radford *et al.*, Nucl. Phys. A752, 264 (2005).
- [21] Evaluated Nuclear Structure Data File (ENSDF), http://www.nndc.bnl.gov/ensdf/.
- [22] J. M. Allmond et al., Phys. Rev. C 84, 061303(R) (2011).
- [23] J. M. Allmond et al., Phys. Rev. C 87, 054325 (2013).
- [24] A. E. Stuchbery et al., Phys. Rev. C 88, 051304(R) (2013).
- [25] K. L. Jones *et al.*, Nature (London) **465**, 454 (2010); Phys. Rev. C **84**, 034601 (2011).
- [26] R. L. Kozub et al., Phys. Rev. Lett. 109, 172501 (2012).
- [27] P. Hoff et al., Phys. Rev. Lett. 77, 1020 (1996); Hyperfine Interact. 129, 141 (2000).
- B. Fogelberg and J. Blomqvist, Phys. Lett. 137B, 20 (1984);
 B. Fogelberg and J. Blomqvist, Nucl. Phys. A429, 205 (1984).
- [29] B. Fogelberg et al., Phys. Rev. C 70, 034312 (2004).
- [30] D. C. Radford et al., Eur. Phys. J. A 15, 171 (2002).
- [31] J. M. Allmond *et al.*, Phys. Rev. C **86**, 031307(R) (2012).
- [32] J. M. Allmond, AIP Conf. Proc. 1525, 610 (2013).
- [33] P.D. Bond, Comments Nucl. Part. Phys. 11, 231 (1983).
- [34] D. W. Stracener, G. D. Alton, R. L. Auble, J. R. Beene, P. E. Mueller, and J. C. Bilheux, Nucl. Instrum. Methods Phys. Res., Sect. A **521**, 126 (2004).
- [35] J. F. Liang et al., Phys. Rev. C 75, 054607 (2007).
- [36] A. Galindo-Uribarri, AIP Conf. Proc. **1271**, 180 (2010).
- [37] C. J. Gross *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **450**, 12 (2000).
- [38] M. H. Macfarlane and S. C. Pieper, ANL-76-11 Rev. 1 Argonne National Laboratory Report (1978) (unpublished).

^{*}jmallmond@gmail.com

Present address: AMETEK-ORTEC, Oak Ridge, Tennessee 37831, USA.

- [39] D. P. Stahel, G. J. Wozniak, M. S. Zisman, B. D. Jeltema, and J. Cerny, Phys. Rev. C 16, 1456 (1977).
- [40] J. Lang, R. Müller, J. Unternährer, L. Jarczyk, B. Kamys, and A. Strzałkowski, Phys. Rev. C 16, 1448 (1977).
- [41] A. Osman, M. Y. Ismail, and M. M. Osman, Phys. Rev. C 27, 650 (1983).
- [42] S. K. Pandit et al., Phys. Rev. C 84, 031601(R) (2011).
- [43] V. V. Parkar et al., Phys. Rev. C 82, 054601 (2010).
- [44] D. L. Auton, Nucl. Phys. A157, 305 (1970).
- [45] J. P. Schiffer, G. C. Morrison, R. H. Siemssen, and B. Zeidman, Phys. Rev. 164, 1274 (1967).
- [46] S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
- [47] M. L. Roush, F. C. Young, P. D. Forsyth, and W. F. Hornyak, Nucl. Phys. A128, 401 (1969).
- [48] R. E. Anderson, J. J. Kraushaar, M. E. Rickey, and W. R. Zimmerman, Nucl. Phys. A236, 77 (1974).
- [49] S. E. Darden, G. Murillo, and S. Sen, Nucl. Phys. A266, 29 (1976).
- [50] J. Yan, F. E. Cecil, J. A. McNeil, M. A. Hofstee, and P. D. Kunz, Phys. Rev. C 55, 1890 (1997).
- [51] A. Strömich, B. Steinmetz, R. Bangert, B. Gonsior, M. Roth, and P. von Brentano, Phys. Rev. C 16, 2193 (1977).
- [52] A. M. Mukhamedzhanov, C. A. Gagliardi, and R. E. Tribble, Phys. Rev. C **63**, 024612 (2001).

- [53] M. A. Franey, J. S. Lilley, and W. R. Phillips, Nucl. Phys. A324, 193 (1979).
- [54] G. J. Kramer, H. P. Blok, and L. Lapikás, Nucl. Phys. A679, 267 (2001).
- [55] D. Y. Pang, F. M. Nunes, and A. M. Mukhamedzhanov, Phys. Rev. C 75, 024601 (2007).
- [56] B. P. Kay, J. P. Schiffer, and S. J. Freeman, Phys. Rev. Lett. 111, 042502 (2013).
- [57] N. K. Timofeyuk, Phys. Rev. C 81, 064306 (2010);N. K. Timofeyuk, Phys. Rev. C 88, 044315 (2013).
- [58] I. R. Gulamov, A. M. Mukhamedzhanov, and G. K. Nie, Phys. At. Nucl. 58, 1689 (1995).
- [59] P. H. Barker, A. Huber, H. Knoth, U. Matter, A. Gobbi, and P. Marmier, Nucl. Phys. A155, 401 (1970).
- [60] P. J. Nolan and J. F. Sharpey-Schafer, Rep. Prog. Phys. 42, 1 (1979).
- [61] B. Castel and I. S. Towner, *Modern Theories of Nuclear Moments* (Clarendon Press, Oxford, 1990), p. 20.
- [62] B. A. Brown, N. J. Stone, J. R. Stone, I. S. Towner, and M. Hjorth-Jensen, Phys. Rev. C 71, 044317 (2005).
- [63] K. H. Maier, K. Nakai, J. R. Leigh, R. M. Diamond, F. S. Stephens, Nucl. Phys. A183, 289 (1972).
- [64] W. Urban et al., Eur. Phys. J. A 5, 239 (1999).