

# Analytical approximations to numerical solutions of theoretical emission measure distributions

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## ABSTRACT

Emission line fluxes from cool stars are widely used to establish an apparent emission measure distribution,  $Emd_{app}(T_e)$ , between temperatures characteristic of the low transition region and the low corona. The true emission measure distribution,  $Emd_t(T_e)$ , is determined by the energy balance and geometry adopted and, with a numerical model, can be used to predict  $Emd_{app}(T_e)$ , to guide further modelling. The scaling laws that exist between coronal parameters arise from the dimensions of the terms in the energy balance equation. Here, analytical approximations to numerical solutions for  $Emd_t(T_e)$  are presented, which show how the constants in the coronal scaling laws are determined. The apparent emission measure distributions show a minimum value at some  $T_0$  and a maximum at the mean coronal temperature  $T_c$  (although in some stars, emission from active regions can contribute). It is shown that, for the energy balance and geometry adopted, the analytical values of the emission measure and electron pressure at  $T_0$  and  $T_c$  depend on only three parameters: the stellar surface gravity and the values of  $T_0$  and  $T_c$ . The results are tested against full numerical solutions for  $\epsilon$  Eri (K2 V) and are applied to Procyon ( $\alpha$  CMi, F5 IV/V). The analytical approximations can be used to restrict the required range of full numerical solutions, to check the assumed geometry and to show where the adopted energy balance may not be appropriate.

**Key words:** stars: coronae – stars: individual:  $\epsilon$  Eri – stars: individual:  $\alpha$  CMi – stars: late-type.

## 1 INTRODUCTION

The observed fluxes of emission lines from cool stars, including the Sun, are now routinely used to derive apparent emission measures ( $Em_{app}$ ) for given lines and apparent emission measure distributions [ $Emd_{app}(T_e)$ ], by using different lines formed over a range of electron temperatures ( $T_e$ ). The precise definition of the emission measure differs between authors, and that adopted here is given in Section 2.

When the *International Ultraviolet Explorer (IUE)* was operating, the  $Emd_{app}(T_e)$  of cool stars was known only between  $T_e \simeq 10^4$  and  $\simeq 2 \times 10^5$  K (see e.g. Jordan et al. 1987). Early X-ray satellites and the *Extreme Ultraviolet Explorer (EUVE)* together provided information above  $\simeq 8 \times 10^5$  K, and under favourable circumstances, the *EUVE* could detect a few lines formed around  $3 \times 10^5$  K (see e.g. Drake, Laming & Widing 1995). The Goddard High Resolution Spectrograph (GHRS) and the Space Telescope Imaging Spectrograph (STIS) onboard the *Hubble Space Telescope (HST)* have given improved spectral resolution and sensitivity in the ultraviolet regions (e.g. Wood et al. 1996). In particular, in addition to

lines formed in the low to mid transition region (at  $10^4$ – $2 \times 10^5$  K), the spectrum of an M-dwarf flare, obtained with the GHRS, showed the forbidden line of Fe xxI at 1354 Å (Maran et al. 1994). Also, spectra of G/K dwarfs obtained with the STIS showed the forbidden lines of Fe xII at 1242 and 1349 Å (Jordan et al. 2001), formed in their upper transition region or inner corona. The *Far Ultraviolet Spectroscopic Explorer (FUSE)* has observed the resonance lines of O vi (formed around  $3 \times 10^5$  K), a number of lines that are formed below  $10^5$  K and further forbidden lines of Fe xvIII and Fe xix (Redfield et al. 2003). Both *Chandra* and *XMM-Newton* now provide extensive information from lines formed at  $T_e \geq 8 \times 10^5$  K (see e.g. Sanz-Forcada, Favata & Micela 2004). Thus, the  $Emd_{app}(T_e)$  of a range of stars is now reasonably well constrained over all the temperature range of interest, and in some stars the observations of forbidden lines of highly ionized iron with the STIS and *FUSE* provide simultaneous measurements in the lower and upper transition region/corona.

The true emission measure distribution [ $Emd_t(T_e)$ ] is determined by the energy balance in the outer atmosphere and the actual geometry. By making numerical calculations in a chosen geometry, theoretical values of both  $Emd_t(T_e)$  and  $Emd_{app}(T_e)$  can be predicted and the latter can then be compared with that observed. In

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an example of this approach, Sim & Jordan (2003a) used *EUVE* observations of  $\epsilon$  Eri (HD 22049, K2 V) to determine  $Emd_{app}(T_e)$ . The numerical model assumed that in the upper transition region there is an energy balance between the divergence of the conductive flux and the radiation losses and, as a boundary condition, was required to match the observed minimum in  $Emd_{app}(T_e)$ . A spherically symmetric geometry was adopted and a non-thermal pressure term was included in the equation of hydrostatic equilibrium. The observed  $Emd_{app}(T_e)$  (and observed electron densities,  $N_e$ ) could be fitted by the values from the model, provided the area filling factor was  $\simeq 1$  in the inner corona, and  $\simeq 0.2$  in the mid transition region. More recently, Ness & Jordan (2008) have included an analysis of the X-ray spectrum obtained with the Low Energy Transmission Grating Spectrometer (LETGS) onboard *Chandra* to obtain an improved observed  $Emd_{app}(T_e)$  and also the relative element abundances. The atomic data from CHIANTI (v5.2) (Landi et al. 2006) were used to update the values of the electron pressure ( $P_e$ ). Improved area factors, which were slightly smaller, were also found by iterating the first solution with a variable area factor.

Early work on the implied area of emitting material in the solar transition region was carried out by Kopp (1972). Also, Torricelli-Ciamponi, Einaudi & Chiuderi (1982) used the presence of a minimum in  $Emd_{app}(T_e)$  to constrain models for the heating of solar loop structures, in an extension of the approach used by Rosner, Tucker & Vaiana (1978).

Hearn (1975, 1977) derived scaling relations for the energy losses by conduction and radiation from coronae, by assuming minimum energy loss (mel) from a corona. His approach has some similarities to that adopted below and we compare his predictions with our results in Section 2.

The results of a range of numerical models made for  $\epsilon$  Eri by Ness & Jordan (2008) showed that, provided the heating function can be expressed as an energy flux, there exist scaling laws between the calculated coronal emission measures, electron pressures, temperatures and the value of the stellar gravity,  $g(r_c)$ , at the radial distance at which the coronal temperature,  $T_c$ , is reached. This is to be expected, since these scaling laws can be derived from simple dimensional arguments, with the constants being determined by the specific assumptions made. However, in numerical calculations in other than plane-parallel geometry, and in calculations that include the non-thermal pressure term, it is difficult to see exactly what determines the constants.

Here we adopt some simplifications regarding the variation in  $P_e$  and the geometry, to find which chosen parameters control the full solutions to  $Emd_{app}(T_e)$ . It is shown that the analytical solutions given below depend only on the stellar surface gravity ( $g_*$ ) and the boundary temperatures,  $T_c$  and  $T_0$ . The values of  $P_0$ ,  $P_c$ ,  $Em_{\iota}(T_0)$  and  $Em_{\iota}(T_c)$  are all determined by the choice of  $g_*$ ,  $T_0$  and  $T_c$ .

The simple approach adopted is also useful as a starting point for testing the following questions. Does the emission have to come from some fraction of the surface area? Is local deposition of a heating flux (other than from conduction) required below the corona? Is a mean corona, rather than closed magnetic structures (with heights smaller than the isothermal pressure-squared scaleheight), appropriate?

Although  $T_0$  is similar in the main-sequence stars studied ( $\simeq 2 \times 10^5$  K), it is significantly higher in the F-star Procyon (HD 61421,  $\alpha$  CMi, F5 IV/V) and other evolved stars, including single giants and RS CVn binaries (as pointed out for Capella by Dupree et al. 1993). The giant stars and RS CVn binaries are not considered here because we assume a plane-parallel geometry, which becomes inappropriate in lower gravity stars and in the coronae of close binaries. Also,

consideration of their escape velocities suggests that the highest temperature material is likely to be magnetically confined, rather than occurring in a quiescent corona with a large scaleheight.

The simple theoretical model adopted for stars with a quiescent corona is set out in Section 2, together with the analytical relations derived. Comparisons between the simple analytical results and those derived from the full computational models for  $\epsilon$  Eri, by Ness & Jordan (2008), are made in Section 3. Procyon is used as a further example in Section 4. The conclusions are discussed and summarized in Section 5.

## 2 THEORY: STARS WITH A QUIESCENT CORONA

In the Sun, the maximum in the observed  $Emd_{app}(T_e)$  occurs at the average temperature of the inner quiescent corona ( $T_c \simeq 1.6 \times 10^6$  K). Except in observations at or above the limb, the emission lines are formed predominantly over the first pressure-squared isothermal scaleheight, since the lines are collisionally excited, with fluxes  $\propto N_e^2$  (see Section 2.2). As apparent in movies made using *Yohkoh* data, the emission from the inner corona, away from active regions, shows little inhomogeneity and very few variations with time. If active regions are present, the emitting material is observed as a gradual decrease in  $Emd_{app}(T_e)$  at temperatures larger than  $T_c$ . In very early spatially unresolved studies, Neupert (1965) observed the variation with time of lines formed at different values of  $T_e$ , over more than one solar rotation period. These support the above picture.

In other main-sequence stars, we expect a quiescent corona to be present, as well as active regions.  $T_c$  is assumed to correspond to the temperature at which the observed  $Emd_{app}(T_e)$  has its maximum value. At worst, this assumption gives an upper limit to  $T_c$ . Ideally, the contribution of active regions should be studied through rotational modulation of high-temperature X-ray lines, but this is currently difficult, owing to the large amount of observing time required.

### 2.1 The upper transition region

The full numerical calculations were made using a spherically symmetric atmosphere and included a non-thermal pressure term in the equation of hydrostatic equilibrium. The relevant equations have been given in Sim & Jordan (2003a) and/or Griffiths & Jordan (1998). In the analytical results below, we adopt a plane-parallel geometry and a constant emitting area. In Section 3, we compare the analytical results with those from the full spherically symmetric numerical models calculated by Ness & Jordan (2008).

The true emission measure for a given line is defined as

$$Em_{\iota}(0.3) = \int_{\delta h} N_H N_e dh, \quad (1)$$

where  $h$  is the height,  $\delta h$  corresponds to the region over which  $\log T_e$  changes by  $\pm 0.15$  about the optimum temperature of formation for the line and  $N_H$  is the number density of hydrogen.

In plane-parallel geometry, the apparent emission measure for an optically-thin line is given by

$$Em_{app}(0.3) = \frac{1}{2} \int_{\delta h} N_H N_e dh, \quad (2)$$

where the factor of 1/2 allows for the photons emitted in the outward direction.

Equation (1) can also be expressed as

$$Em_t(0.3) = \frac{0.86P_e^2}{\sqrt{2}T_e} \frac{dh}{dT_e}, \quad (3)$$

where the electron pressure is defined as  $P_e = N_e T_e$ , and the factor of 0.86 arises from  $N_H \simeq 0.86N_e$ . Here it has been assumed that  $P_e^2$  and  $dh/dT_e$  are constant over the region in which an individual line is formed. The former is a good assumption, but the latter can initially be less accurate. However, in actual calculations of line fluxes using a theoretical model, any variation in  $dh/dT_e$  over the region of line formation is taken into account. The *starting values* of  $Em_{app}(0.3)$  are used to find the initial  $Emd_{app}(0.3)(T_e)$ , but following the calculation of the line fluxes, including the *full* contribution function for each line,  $Emd_{app}(0.3)(T_e)$  is optimized. Relative element abundances are also adjusted during this process. The details are given in Ness & Jordan (2008). Note that equation (3) gives an expression for the true emission measure for a line formed at a given  $T_e$ . Equation (3) is also used to define the true emission measure distribution,  $Emd_t(0.3)(T_e)$ , since the quantities on the right-hand side are all differentiable functions of  $T_e$ . For simplicity of presentation, the label 0.3 is omitted in the equations below.

The energy flux carried by thermal conduction,  $F_c(T_e)$ , is given by

$$F_c(T_e) = -\kappa T_e^{5/2} \frac{dT_e}{dh}, \quad (4)$$

where  $\kappa T_e^{5/2}$  is the coefficient of thermal conduction and, from Spitzer (1956),  $\kappa$  is taken to be  $1.1 \times 10^{-6} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-7/2}$ . Here, as in our full numerical calculations, we ignore the small variation in  $\kappa$  with  $N_e$  and  $T_e$  (Spitzer 1956). In  $\epsilon$  Eri, this amounts to only 30 per cent between  $\log T_e = 5.3$  and 6.5. There will be a difference between  $F_c(T_e)$  in the analytical and numerical, spherically symmetric, solutions, but given the limited extent of the region considered, this is not expected to be large in dwarf stars [see comments after equation (12)].

Equation (3) allows  $F_c(T_e)$  to be expressed in terms of  $Em_t$ , that is,

$$F_c(T_e) = -\frac{0.86\kappa}{\sqrt{2}} \frac{P_e^2 T_e^{3/2}}{Emd_t(T_e)}. \quad (5)$$

From equation (5), it is clear that one cannot use a boundary condition that sets  $F_c(T_e)$  to exactly zero at some base temperature, since this would lead to an infinite value of  $Emd_t(T_e)$ .

As carried out in earlier work (see Jordan & Brown 1981), equation (5) can be differentiated to give

$$\frac{d \log Emd_t(T_e)}{d \log T_e} = 3/2 + \frac{2d \log P_e}{d \log T_e} - \frac{d \log[-F_c(T_e)]}{d \log T_e}. \quad (6)$$

So far, no assumptions have been made about the energy balance. It is now assumed that the corona is heated by a flux of energy from lower layers and that this energy is not dissipated until high in the corona, far above the first pressure-squared isothermal scale-height,  $H$ , over which the spatially averaged stellar emission lines are mainly formed. (The same situation is relevant to solar lines observed near the Sun centre.) In this case, the heated region can be studied only through solar observations above the limb. In the Sun,  $T_e$  rises slowly up to a height of  $0.70R_\odot$ , far higher than  $H \simeq 0.06R_\odot$ , while the non-thermal velocities (interpreted using  $T_e$ , rather than the unknown ion temperatures  $T_i$ ) continue to increase (Landi, Feldman & Doschek 2006).

Thus, below the heated region, it can be assumed that the divergence of the thermal conductive flux is balanced by the radiation

losses. If any dissipation of the mechanical heating flux ( $F_m$ ) were present in the upper transition region and inner corona, then this would add a term  $-dF_m/dh$  to the right-hand side of equation (7) given below. This would result in a steeper  $Emd_t(T_e)$  (Jordan 2000), since it adds a positive term to the right-hand side of equation (10) given below. An enthalpy flux is not included in either our full numerical solutions or the analytical approximations, because large enough systematic flows are not usually observed in the upper transition region and inner corona. Hence,

$$\frac{dF_c(T_e)}{dh} = -\frac{dF_r(T_e)}{dh}, \quad (7)$$

where the radiation losses are given by

$$\frac{dF_r(T_e)}{dh} = \frac{0.86P_e^2 P_{rad}(T_e)}{T_e^2}, \quad (8)$$

where  $P_{rad}(T_e)$  is the radiative power-loss function.

or, using equation (3) for  $dh/dT_e$ ,

$$\frac{dF_r(T_e)}{dT_e} = \frac{\sqrt{2}Emd_t(T_e)P_{rad}(T_e)}{T_e}. \quad (9)$$

Equation (6) then becomes

$$\frac{d \log Emd_t(T_e)}{d \log T_e} = 3/2 + \frac{2d \log P_e}{d \log T_e} - \frac{2Emd_t(T_e)P_{rad}(T_e)}{0.86\kappa P_e^2 T_e^{3/2}}. \quad (10)$$

Here we approximate  $P_{rad}(T_e)$  by  $\alpha T_e^{-1/2}$ , where  $\alpha$  is a constant (taken as  $2.8 \times 10^{-19} \text{ erg cm}^3 \text{ s}^{-1} \text{ K}^{1/2}$ ), on the grounds that we are considering only collisionally excited lines where  $2 \times 10^5 \leq T_e \leq 10^7 \text{ K}$ . At higher temperatures, continuum processes cause an increase in  $P_{rad}(T_e)$ . Alternatively, numerical values can be used, for example, from CHIANTI (v6) (Dere et al. 2009). The form adopted here is useful in elucidating the physics since it results in the simplest analytical relations.

The variation in the total pressure (including a non-thermal pressure term) with  $T_e$  is included in our numerical solutions using hydrostatic equilibrium. In this paper, the non-thermal pressure is neglected, since in  $\epsilon$  Eri, above  $T_e = 2 \times 10^5 \text{ K}$ , it does not exceed 0.08 of the gas pressure (Sim 2002). Hydrostatic equilibrium should be a good approximation in the upper transition region and inner corona, since flows with velocities approaching the sound speed are not usually observed in these regions.

The third term in equation (10) is given by hydrostatic equilibrium and can be expressed as

$$2 \frac{d \log P_e}{d \log T_e} = -2 \frac{\sqrt{2}\mu m_H Emd_t(T_e)g_* T_e R_*^2}{0.86k_B P_e^2 (R_* + h)^2}, \quad (11)$$

where  $\mu$  is the mean molecular weight, taken to be 0.619, and  $R_*$  and  $g_*$  are the stellar radius and surface gravity, respectively. The variation in this term is small in the mid transition region, but becomes more important as  $T_e$  increases, since  $Emd_t(T_e)T_e/P_e^2$  increases with  $T_e$ .

Although we drop the variation in  $P_e$  with  $T_e$  in general, we do use the integral of  $dP_e/dT_e$  to relate  $P_o$  and  $P_c$ , using

$$P_o^2 = P_c^2 + \frac{2\sqrt{2}\mu m_H g_*}{0.86k_B} \int_{T_o}^{T_c} Emd_t(T_e) \frac{R_*^2}{(R_* + h)^2} dT_e. \quad (12)$$

To find an analytical solution (see Section 2.2), the variation in the gravity with height has to be neglected. The full numerical solutions available for dwarf stars (e.g. Philippides 1996; Sim 2002; Ness & Jordan 2008) show that this variation is not very large in G/K dwarfs; for example, the final numerical model by Ness & Jordan

(2008), using a radial height of  $\leq 3000$  km at  $\log T_e = 5.30$ , gives  $g(r_o)/g_* \geq 0.99$  and  $g(r_c)/g_* \geq 0.90$  at  $\log T_c = 6.53$ .

From equation (10), ignoring the variation in pressure, a *minimum* in  $Emd_t(T_e)$  occurs at some  $T_o$ , when

$$Em_t(T_o) = \sqrt{\frac{3 \times 0.86\kappa}{4\alpha}} P_o T_o. \quad (13)$$

Thus, at a given  $T_o$ ,  $Em_t(T_o) \propto P_o$ .

## 2.2 Global constraints and resulting scalings

At the top of the transition region/base of the corona,  $F_c(T_c)$  is the energy conducted back from the overlying heated corona. At  $T_o$ , the conductive flux is  $F_c(T_o)$ . The easiest way to use the global constraint that the overall net conductive flux is balanced by the total radiation losses is to use the approach of Rosner et al. (1978), but in this work no explicit boundary conditions on the values of  $F_c(T_c)$  and  $F_c(T_o)$  have been imposed.

From equation (7), one can write

$$\int_{T_o}^{T_c} F_c(T_e) dF_c(T_e) = - \int_{T_o}^{T_c} F_c(T_e) \frac{dF_r}{dh} \frac{dh}{dT_e} dT_e. \quad (14)$$

Hence, using equations (4) and (8),

$$\frac{1}{2} [F_c(T_c)^2 - F_c(T_o)^2] = 0.86\alpha\kappa \int_{T_o}^{T_c} P_e^2 dT_e. \quad (15)$$

Then, on the left-hand side of equation (15), equation (3) can be used to express  $dT_e/dh$  in terms of  $Emd_t(T_e)$ . On the right-hand side of equation (15), the pressure term is taken to be constant at  $P_o^2$ . This is close to the mean value of  $P_e^2$  in the full numerical solutions. The pressure variation between  $T_o$  and  $T_c$  is included in the conductive flux terms. The result is

$$\frac{0.86\kappa}{4} \left[ \frac{T_c^3 P_c^4}{Em_t(T_c)^2} - \frac{T_o^3 P_o^4}{Em_t(T_o)^2} \right] = \alpha P_o^2 (T_c - T_o). \quad (16)$$

Substituting for  $Em_t(T_o)$  from equation (13), equation (16) can be rearranged to give

$$Em_t(T_c) = \sqrt{\frac{0.86\kappa}{4\alpha}} \frac{T_c P_c^2}{P_o} \frac{1}{\left(1 - \frac{2T_o}{3T_c}\right)^{1/2}}, \quad (17)$$

where  $T_c$  can be replaced by a general  $T_e$  ( $> T_o$ ), and, similarly,  $P_c$  can be replaced by a general  $P_e$ , to give a general equation for  $Emd_t(T_e)$ .

$Em_t(T_c)$  can also be found by making the approximation that the coronal emission is formed mainly over the first pressure-squared isothermal scaleheight. This is justified by using the equation of hydrostatic equilibrium to show that  $P_e^2$  decreases exponentially with height according to  $P_c^2 \exp[-(h - h_c)/H]$ , where  $h_c$  is the height at the base of the corona and  $H = k_B T_c / 2\mu m_H g_*$ . Solar observations by Gibson et al. (1999) confirm the hydrostatic decrease in  $N_e^2$  under the near-isothermal conditions in the low corona. In this case,

$$Em_t(T_c) = \frac{0.86k_B}{2\mu m_H} \frac{P_c^2}{g_* T_c}. \quad (18)$$

Combining equations (17) and (18) allows  $P_c$  to be eliminated, leading to an explicit expression for  $P_o$ ,

$$P_o = \sqrt{\frac{\kappa}{0.86\alpha} \frac{\mu m_H}{k_B} \frac{g_* T_c^2}{\left(1 - \frac{2T_o}{3T_c}\right)^{1/2}}}. \quad (19)$$

Thus,  $P_o$  scales as  $g_* T_c^2$ , with only a weak dependence on  $T_o/T_c$ , and, provided  $g_*$  is known, can be found from the values of  $T_o$  and  $T_c$  that match the observed behaviour of  $Emd_{app}(T_e)$ .

Once  $P_o$  is known,  $Em_t(T_o)$  is known from equation (13) and can also be expressed in terms of  $g_*$ ,  $T_c$  and  $T_o$ , i.e.,

$$Em_t(T_o) = \sqrt{3} \frac{\kappa \mu m_H}{2\alpha k_B} \frac{g_* T_o T_c^2}{\left(1 - \frac{2T_o}{3T_c}\right)^{1/2}}. \quad (20)$$

The ratio of  $Em_t(T_c)$  to  $Em_t(T_o)$  is given by

$$\frac{Em_t(T_c)}{Em_t(T_o)} = \frac{T_c P_c^2}{T_o P_o^2} \frac{1}{\sqrt{3} \left(1 - \frac{2T_o}{3T_c}\right)^{1/2}}. \quad (21)$$

To separate  $Em_t(T_c)$  and  $P_c^2$  requires equation (12), which can be written as

$$\frac{P_o^2}{P_c^2} = 1 + \frac{\sqrt{2} \int_{T_o}^{T_c} Emd_t(T_e) dT_e}{Em_t(T_c) T_c}. \quad (22)$$

The detailed numerical models show that  $P_o^2/P_c^2$  varies slowly when  $T_c$  is varied, because the last term in equation (22) is almost constant. The general form of  $Emd_t(T_e)$ , when  $T_c$  in equation (17) is replaced by  $T_e$ , can now be applied. This time, guided by the full calculations, we take the variable  $P_e^2$  as  $P_c^2$  and remove it from the integral. The integral of  $T_e/[1 - (2/3)(T_o/T_e)]^{1/2}$  has a standard solution (see e.g. Jeffrey 1995), which reduces to

$$\frac{8T_o^2}{9} \int \frac{1}{(1 - Y^2)^3} dY = \frac{T_o^2}{9} \frac{Y(5 - 3Y^2)}{(1 - Y^2)^2} - \frac{T_o^2}{6} \ln \left( \frac{|Y - 1|}{|Y + 1|} \right), \quad (23)$$

where  $Y^2 = 1 - (2T_o/3T_e)$ . The first term on the right-hand side of equation (23), evaluated at  $T_c$ , dominates and reproduces the full solution to within 0.1 per cent. Thus, when the constants in the general expressions for  $Emd_t(T_e)$  and  $Em_t(T_c)$  are included, the last term in equation (22) becomes

$$\frac{\sqrt{2} \int_{T_o}^{T_c} Emd_t(T_e) dT_e}{Em_t(T_c) T_c} = \frac{1}{\sqrt{2}} \left(1 - \frac{2T_o}{3T_c}\right) \left(1 + \frac{T_o}{T_c}\right). \quad (24)$$

Hence,  $P_o^2/P_c^2$  can be found from equations (22) and (24), and  $P_c^2$  can then be found, using  $P_o$  from equation (19).

Using equation (18),  $Em_t(T_c)$  can then be expressed as

$$Em_t(T_c) = \frac{\mu m_H \kappa}{2k_B \alpha} \frac{g_* T_c^3}{\left(1 - \frac{2T_o}{3T_c}\right) \left[1 + \frac{1}{\sqrt{2}} \left(1 - \frac{2T_o}{3T_c}\right) \left(1 + \frac{T_o}{T_c}\right)\right]}. \quad (25)$$

This gives the scaling with  $g_* T_c^3$ , expected from dimensional arguments, and also the absolute value, which arises from the energy balance adopted. The terms in  $T_o/T_c$  vary by only 1 per cent over the range of  $T_c$  in the models discussed below. Thus, for a given choice of  $\kappa$  and  $\alpha$ , only  $g_*$ ,  $T_c$  and  $T_o$  are required to evaluate  $Em_t(T_c)$ .

We now make comparisons with results from Hearn's (1975, 1977) formulation for mel coronae. Hearn applied  $dF_c = -dF_r$  to a corona, but took a partial differentiation with respect to the coronal temperature, at constant coronal pressure. He assumed that  $T_o$  and  $F_c(T_o)$  can be neglected.

Making the same assumptions, by dropping the second term in equation (15) and using equation (18), it can be seen that

$$P_o(\text{mel}) = \sqrt{\frac{\kappa}{0.86\alpha} \frac{\mu m_H}{k_B} g_* T_c^2}. \quad (26)$$

However, in our approach, if  $F_c(T_o)$  tends to zero, then  $Em_t(T_o)$  tends to infinity. Thus, equation (16), together with equation (18), can be used to define a critical (maximum) value of  $P_o$ . This pressure cannot be reached when there is a minimum in  $Emd_t(T_e)$  at  $T_o$ .

$P_o(\text{crit})$  is given by

$$P_o(\text{crit}) = \sqrt{\frac{\kappa}{0.86\alpha}} \frac{\mu m_H}{k_B} \frac{g_* T_c^2}{\left(1 - \frac{T_o}{T_c}\right)^{1/2}}. \quad (27)$$

This approaches the value of  $P_o(\text{mel})$  when  $T_o$  is much smaller than  $T_c$ . If observations of density/pressure sensitive lines show clearly that  $P_o$  exceeds  $P_o(\text{crit})$ , then models including a fractional emitting area, or heating, additional to that provided by thermal conduction, must be considered.

Although the differences between our predicted values of  $P_o$  and  $P_o(\text{mel})$  are not large, the advantage of our solutions is that they allow for the difference between  $P_o$  and  $P_c$  in hydrostatic equilibrium and predict the values of  $Em_t(T_o)$ , as well as  $Em_t(T_c)$ . Hearn (1975, 1977) assumed a constant pressure with a value defined at the ‘base of the corona’. Applying the pressure from equation (26) to find  $Em_t(T_c)$  from equation (18) gives larger values than found from our analytical solutions.

### 3 RESULTS AND COMPARISONS WITH FULL MODELS

In optimizing our full solution for  $Em_d(T_c)$  and  $Em_{d,\text{app}}(T_c)$  of  $\epsilon$  Eri (Ness & Jordan 2008), we ran five models with different values of  $T_c$ . The results for the full models are those for which a minimum in  $Em_{d,\text{app}}(T_c)$  is just possible at the chosen value of  $T_o = 2 \times 10^5$  K. Such solutions are found by gradually increasing the value of  $Em_{\text{app}}(T_c)$ . As stressed earlier, the full numerical models are in hydrostatic equilibrium, including a non-thermal pressure term based on observed linewidths, and adopt a spherically symmetric atmosphere. Although we do not expect exact agreement between the analytical and the full models, it is of interest to examine the size of the differences between them, since the analytical models can be useful in the process of homing in on the optimum solution for a given star.

Table 1 gives the values of the parameters discussed in the previous section, with those from the above full numerical models given in the lines labelled ‘n’ and those from the analytical predictions in the lines labelled ‘a’. The order of the parameters listed reflects the order in which the calculations can be made, that is,  $P_o$  from equation (19),  $Em_t(T_o)$  from equation (13),  $(P_o/P_c)^2$  from equations (22) and (24),  $P_c$  from equations (19), (22) and (24), and  $Em_t(T_c)$  from using  $P_c^2$  in equation (17) or (18).

The full numerical models give the same scaling laws for  $P_o$  and  $P_c$  as found from the analytical approach, but with multiplication

**Table 1.** Comparison of parameters from the numerical models for  $\epsilon$  Eri (lines labelled ‘n’) and from the analytical equations (lines labelled ‘a’). The quantities involved, including their units, are  $\log [T_e(\text{K})]$ ,  $\log [P_e(\text{cm}^{-3} \text{K})]$  and  $\log [Em_t(T_c)(\text{cm}^{-5})]$ ;  $\log T_o$  is fixed at 5.30.

$\log T_c$		6.50	6.53	6.55	6.60	6.65
$\log P_o$	(n)	15.91	15.97	16.01	16.10	16.20
$\log P_o$	(a)	15.86	15.92	15.96	16.06	16.16
$\log Em_t(T_o)$	(n)	27.50	27.56	27.60	27.69	27.79
$\log Em_t(T_o)$	(a)	27.37	27.43	27.47	27.56	27.66
$2\log (P_o/P_c)$	(n)	0.254	0.254	0.253	0.252	0.251
$2\log (P_o/P_c)$	(a)	0.236	0.235	0.235	0.235	0.235
$\log P_c$	(n)	15.79	15.84	15.88	15.98	16.07
$\log P_c$	(a)	15.75	15.81	15.85	15.95	16.04
$\log Em_t(T_c)$	(n)	28.22	28.31	28.36	28.51	28.66
$\log Em_t(T_c)$	(a)	28.10	28.19	28.25	28.40	28.55

**Table 2.** Values of  $\log [Em_t(T_c)/Em_t(T_o)]$  and  $Em_t(T_c)/P_c T_c$  calculated from the full numerical models for  $\epsilon$  Eri (lines labelled ‘n’) and the analytical solutions (lines labelled ‘a’). Units as in Table 1.

$\log T_c$		6.50	6.53	6.55	6.60	6.65
$\log [Em_t(T_c)/Em_t(T_o)]$	(n)	0.72	0.74	0.76	0.82	0.87
$\log [Em_t(T_c)/Em_t(T_o)]$	(a)	0.73	0.76	0.78	0.83	0.88
$\log [Em_t(T_c)/P_c T_c]$	(n)	5.93	5.93	5.93	5.93	5.94
$\log [Em_t(T_c)/P_c T_c]$	(a)	5.85	5.85	5.85	5.85	5.85

factors of about 1.1. Similarly, the full numerical emission measures  $[Em_t(T_o)$  and  $Em_t(T_c)]$  follow the same scaling laws as found from the analytical approach, but are systematically larger by mean factors of about 1.3. The differences arise from the approximations to the electron pressure used in the analytical equations, including the neglect of the non-thermal pressure term, and to the different geometries adopted. The variation in  $g$  with the radial distance  $r$  is also included in the numerical solutions.

Table 2 gives the combinations of parameters that appear in equation (21) and in the fourth term of equation (10).

It can be seen that the ratio of the emission measures at  $T_c$  and  $T_o$  given by equation (21) agrees better with the results from the full models than do the absolute values. The numerical models show that the ratio  $Em_t(T_c)/P_c T_c$  is almost constant, as expected from the scaling laws,  $Em_t(T_c) \propto g(r_c)T_c^3$  and  $P_c \propto g(r_c)T_c^2$ .

In the numerical solutions for  $\epsilon$  Eri, the origins of the values of the constants of proportionality in the empirical scaling laws are hard to pin down. The analytical expressions give similar scalings, but now the actual value of the constant of proportionality can be clearly tracked back to the global energy balance equation assumed.

### 4 EXAMPLE APPLICATION

Here we apply the analytical expressions to Procyon (HD 61421, F5 IV–V) to illustrate what can be learnt before detailed modelling is carried out. Earlier work on Procyon has shown that it is difficult to reconcile different measurements of  $P_e$  without invoking limited areas of emission or the presence of active region loops (Schmitt et al. 1985, 1996; Jordan et al. 1986). Because of the lower gravity and larger linewidths in Procyon, compared with those of cool dwarf stars, the simple methods used in Section 2 are expected to be less accurate than for  $\epsilon$  Eri. For a quiescent corona, the nature of the heating flux is not relevant in our energy balance model, so either a magnetohydrodynamic (MHD) wave flux or an acoustic wave flux, as suggested by Mullan & Cheng (1994), is possible.

Emission line fluxes are available for Procyon from a number of the instruments mentioned in Section 1. Jordan et al. (1986) analysed spectra obtained with the *IUE* and the *Einstein Observatory* and derived emission measures, but not a mean emission measure distribution. Early observations with *Copernicus* were used to constrain the line emission measure around  $3 \times 10^5$  K. They found that the ratio of the coronal emission measure to that of the lower transition region was significantly smaller than in main-sequence dwarf stars and that the mean coronal temperature was around  $1.5 \times 10^6$  K. This was in broad agreement with earlier work by Schmitt et al. (1985), who used data from the *Einstein Observatory*. In particular, even with the uncertainty in the line fluxes from *Copernicus*, it appeared that  $T_o \geq 3 \times 10^5$  K, rather than  $2 \times 10^5$  K in the main-sequence stars. Drake et al. (1995) included data obtained with the *EUVE*, plus some adjusted data from *Copernicus*, to produce the emission measure distribution above  $T_c \simeq 1.6 \times 10^5$  K, while Sanz-Forcada,

Brickhouse & Dupree (2003) combined the observations from the *IUE* and *EUVE* to improve the overall  $Em_{\text{app}}(T_c)$ . Although there were differences in detail, owing to the abundances and atomic data adopted, the  $Em_{\text{app}}(T_c)$  found by Sanz-Forcada et al. (2003) showed a similar form to that indicated in Jordan et al. (1986), but with  $T_o \simeq 4 \times 10^5$  K. Sanz-Forcada et al. (2004) improved the higher temperature part of the  $Em_{\text{app}}(T_c)$  using spectra obtained with the LETGS onboard the *Chandra* satellite. They found  $T_o \simeq 5 \times 10^5$  K and  $T_c \simeq 2 \times 10^6$  K. Raassen et al. (2002) also analysed spectra from the LETGS and the *XMM-Newton* satellite, but made a global three-temperature fit using the *SPEX* computer package, rather than an individual line based approach. Wood et al. (1996) analysed the linewidths and redshifts measured from spectra obtained with the GHRS instrument onboard the *HST*. The linewidths were found to be larger than in the main-sequence stars, such as  $\epsilon$  Eri (Sim & Jordan 2003b). In summary, in Procyon,  $\log [T_o(\text{K})]$  lies between 5.5 and 5.7 and  $\log [T_c(\text{K})]$  lies between 6.2 and 6.3. The equation of hydrostatic equilibrium should strictly include the effects of any non-thermal pressure associated with the larger linewidths observed.

We adopt the following stellar properties: a distance of 3.53 pc (Girard et al. 2000); an angular diameter of 5.51 mas (Mozurkewich et al. 1991) and hence  $R_* = 2.09 R_\odot$ ; a mass of  $M_* = 1.5 M_\odot$  (Girard et al. 2000) and hence  $\log [g_*(\text{cm s}^{-2})] = 3.98$ .

The results of applying the analytical expressions given in Section 2 are summarized in Table 3, for  $\log [T_o(\text{K})] = 5.5$  and 5.7, and  $\log [T_c(\text{K})] = 6.2, 6.25$  and 6.3.

As yet there are no completely satisfactory numerical models of the outer atmosphere of Procyon. Philippides (1996) used observations from the *Röntgen SATellit* (*ROSAT*) to find  $Em_{\text{app}}(T_c)$  and  $T_c$ , and made models in a spherically symmetric geometry, including a non-thermal pressure term. She noted that the latter term causes  $P_e$  to increase with  $T_e$  within the transition region, before decreasing again by  $T_c$ . The later observations with the *EUVE* (Drake et al. 1995; Schmitt et al. 1996), and with both the LETGS and the *EUVE* (Sanz-Forcada et al. 2004), showed that, as expected, the earlier one-temperature and two-temperature fits to *ROSAT* spectra overestimated the coronal emission measure. Sim (1998, M Phys project report, unpublished, 2002) made a model of the chromosphere and lower transition region using line fluxes and widths measured by

**Table 3.** Predicted analytical values of  $P_o$ ,  $P_c$ ,  $Em_i(T_o)$  and  $Em_i(T_c)$  for Procyon, together with combined parameters. Results using  $\log T_o = 5.5$  are given in the upper part of the table, and those using  $\log T_o = 5.7$  in the lower part. Units as in Table 1.

$\log T_c$	6.20	6.25	6.30
$\log T_o = 5.5$			
$\log P_o$	14.62	14.71	14.81
$\log P_c$	14.50	14.59	14.69
$\log Em_i(T_o)$	26.32	26.41	26.51
$\log Em_i(T_c)$	26.57	26.71	26.86
$\log [Em_i(T_c)/P_c T_c]$	5.87	5.87	5.87
$\log T_o = 5.7$			
$\log P_o$	14.64	14.73	14.82
$\log P_c$	14.52	14.61	14.70
$\log Em_i(T_o)$	26.54	26.63	26.73
$\log Em_i(T_c)$	26.61	26.75	26.89
$\log [Em_i(T_c)/P_c T_c]$	5.89	5.89	5.89

**Table 4.** Values of  $P_o$ ,  $P_c$ ,  $Em_{\text{app}}(T_o)$  and  $Em_{\text{app}}(T_c)$  from numerical models and observations. Units as in Table 1.

$\log T_c$	6.30
$\log T_o = 5.5$	
$\log P_o$	14.81 <sup>a</sup>
$\log P_c$	14.73 <sup>a</sup>
$\log Em_{\text{app}}(T_o)$	26.27 <sup>b</sup>
$\log Em_{\text{app}}(T_c)$	26.84 <sup>c</sup>
	27.16 <sup>d</sup>

<sup>a</sup>From the numerical model by Sim (2002).

<sup>b</sup>From observations fitted by Sim (1998).

<sup>c</sup>Starting value adopted by Sim (1998).

<sup>d</sup>From observations fitted by Sanz-Forcada et al. (2004), scaled as described in the text.

Wood et al. (1996), which supersede the fluxes from *Copernicus* by Jordan et al. (1986), and included the radiative transfer in lines formed up to  $\simeq 2 \times 10^4$  K. He adopted a plane-parallel atmosphere up to  $3 \times 10^5$  K and a spherically symmetric atmosphere at higher temperatures, including the non-thermal pressure term throughout the atmosphere. However, he interpolated  $Em_{\text{app}}(T_c)$  between  $\simeq 3 \times 10^5$  K and a coronal temperature of  $2 \times 10^6$  K, rather than making an energy balance model.

Here we use the work by Sanz-Forcada et al. (2004) to give information on  $Em_{\text{app}}(T_c)$ , but adopt an iron abundance of  $\log N_{\text{Fe}} = 7.51$ , rather than their value of 7.32 (on the scale where  $\log N_{\text{H}} = 12.0$ ). Their absolute scale has been changed to match our definition of  $Em_i(0.3)$  given by equation (1). Their maximum value of the apparent volume emission measure at  $\log [T_c(\text{K})] = 6.30$  has been divided by  $4\pi R_*^2$  to find  $Em_{\text{app}}(T_c)$ . Because of this conversion, in a spherically symmetric geometry, the apparent and true emission measures are related by

$$Em_{\text{app}}(T_c) = f(r)G(r)Em_i(T_c), \quad (28)$$

where  $f(r) = (r/R_*)^2$  and  $G(r) = 0.5(1 + \sqrt{1 - [1/f(r)]})$  is the fraction of emitted photons not intercepted by the star. Thus, in an extended atmosphere, the product  $f(r)G(r)$  can be greater than 1, so that  $Em_{\text{app}}(T_c)$  can be larger than  $Em_i(T_c)$ .

Table 4 gives the observed values of  $Em_{\text{app}}(T_o)$  and  $Em_{\text{app}}(T_c)$  from Sim (1998) and Sanz-Forcada et al. (2004), respectively. Values of  $\log P_o$  and  $\log P_c$  from Sim's (2002) numerical models are also given.

We can now compare the predicted values of  $Em_i(T_o)$  and  $Em_i(T_c)$ , given in Table 3 for  $\log [T_c(\text{K})] = 6.30$ , with the values of  $Em_{\text{app}}(T_o)$  and  $Em_{\text{app}}(T_c)$  given in Table 4. The value of  $\log Em_{\text{app}}(T_o)$ , when  $\log [T_o(\text{K})] = 5.50$ , found by Sim (1998, M Phys project report, unpublished) is smaller than the value of  $\log Em_i(T_o)$ . In a plane-parallel atmosphere, these are expected to differ by about a factor of 2, which is indeed the case. Bearing in mind the comparisons between the analytical and numerical results for  $\epsilon$  Eri, this suggests that the emission at this height is *not* from a highly restricted area of the atmosphere. The value of  $Em_i(T_c)$  is similar to, or less than the value of  $Em_{\text{app}}(T_c)$ . Using the pressure-squared isothermal scaleheight ( $H$ ) in the corona, plus  $R_*$ , to estimate  $r$  at  $T_c$  leads to  $f(r)G(r) \simeq 0.85$ . Given the uncertainties in, and differences between, the atomic data used by the above authors, the disagreement between the predicted and observed values is not large and is in the direction found from the numerical and analytical results for  $\epsilon$  Eri. However,

Procyon is a case where full numerical models are required to make more detailed comparisons between observed and predicted results.

It is very difficult to establish values of  $N_e$  in Procyon. Schmitt et al. (1996) made a careful analysis of density-sensitive lines of Fe x to Fe xiv observed with the *EUVE* and concluded that the average value of  $N_e$  at coronal temperatures lies between  $10^9$  and  $10^{10}$  cm $^{-3}$ , with a value of  $3 \times 10^9$  cm $^{-3}$  being adopted. However, for some ions, the results depended on weak lines, and a range of  $N_e$  was found from different pairs of lines within a given ion. As Schmitt et al. (1996) point out, the uncertainties in  $N_e$  mask any systematic variation in  $N_e$  or  $P_e$  with the stage of ionization. At around  $\log [T_e(\text{K})] = 6.20\text{--}6.30$ , their smallest value of  $\log [P_e(\text{cm}^{-3} \text{K})] = 15.2$  is significantly larger than those given in Table 3.

Ness et al. (2001) measured the ratio of the fluxes in the forbidden line ( $1s^2 \ ^1S_0\text{--}1s2s \ ^3S_1$ ) and intersystem (plus quadrupole) line ( $1s^2 \ ^1S_0\text{--}1s2p \ ^3P_{1,2}$ ) in the He I-like ions C v, N vi and O vii, using spectra obtained with the LETGS. We have found revised flux ratios by also including further spectra available from more recent observations with the LETGS. We have also updated the values of (and limits on)  $N_e$  by using CHIANTI (v6) (Dere et al. 2009). In the Sun, only the flux ratio in C v is affected by the photospheric/chromospheric radiation field at the wavelength of the  $^3S_1\text{--}^3P$  transitions (Gabriel & Jordan 1969). In Procyon, because of the higher photospheric temperature, Ness et al. (2001) found that this photoexcitation is also significant in N vi. As a result,  $N_e$  is not constrained by the observed flux ratio in C v. In N vi, the combined uncertainties in the radiation field and the measured flux ratio are too large to yield a definitive value of  $\log [N_e(\text{cm}^{-3})]$ , which can lie between  $\leq 8.0$  and 10.0. O vii provides the best diagnostic, since the effects of the radiation field are small and make little difference to the value of  $\log [N_e(\text{cm}^{-3})] = 9.13$  obtained (when these effects are included). However, the uncertainty of about  $\pm 10$  per cent in the measured flux ratio of 3.60 includes the value of 3.75 at  $\log [N_e(\text{cm}^{-3})] = 8.0$ . The observed ratio yields  $\log [P_e(\text{cm}^{-3} \text{K})] = 15.43$ , with an upper limit of 15.99 and a lower limit of  $\leq 14.3$ .

Liang, Zhao & Shi (2006) investigated density-sensitive X-ray lines of Si x, using LETGS fluxes from Raassen et al. (2002). From the strongest lines at 50.524 and 50.691 Å, they found  $\log [N_e(\text{cm}^{-3})] \simeq 8.41\text{--}8.45$  at  $T_e = 1.26 \times 10^6$  K, and that calculations by other authors gave only slightly higher values. The resulting values of  $\log [P_e(\text{cm}^{-3} \text{K})]$  were in the range 14.5–14.7. These are similar to the value found from Sim’s (2002) numerical model and to the value predicted by the analytical scaling laws. The Si x lines are expected to be formed at the same  $T_e$  as those of N vi. However, in Si x, the high density limit for the flux ratio occurs at a lower value of  $N_e$  than in N vi. If the emitting regions contained both a quiescent corona and active regions (of smaller area), then the higher density regions could contribute relatively less to the electron density measured from Si x. Thus, one cannot rule out the possibility that N vi could detect a higher value of  $N_e$ .

Jordan et al. (1986) used simple line-opacity arguments to deduce that at  $T_e = 2 \times 10^5$  K,  $\log [P_e(\text{cm}^{-3} \text{K})] \leq 14.4$ . This is a factor of 2 lower than the value of 14.76 in the model by Sim (2002). Wood et al. (1996) made a more sophisticated estimate of line-opacity effects and deduced that the profile of the line of Si iii at 1206 Å implied a pressure of  $\log [P_e(\text{cm}^{-3} \text{K})] \leq 14.8$  at  $\log [T_e(\text{K})] = 4.70$ , in reasonable agreement with the value of 14.68 in Sim’s (2002) model. However, the intersystem lines of O iv analysed by Wood et al. (1996) led to  $\log [P_e(\text{cm}^{-3} \text{K})] \simeq 15.0$  (or  $\leq 15.5$ , when possible errors were considered). However, as they pointed out, the

O iv flux ratios were not far from those expected in the low density limit. Analyses of spectra observed recently by Ayres<sup>1</sup> with the STIS should give at least improved limits on values of  $P_e$ . The values of  $\log P_o$  from the methods used here are given in Table 3 and agree well with Sim’s (2002) model value of 14.81.

Overall, the higher coronal values of  $P_e$  found by Schmitt et al. (1996), and from our present analysis of O vii, could be reconciled with the values from the analytical predictions and the numerical models if higher pressure active region loop structures were present, as well as a quiescent corona (see also Schmitt et al. 1985; Jordan et al. 1986). Such active regions could also contribute to  $E_{md,app}$  near its apparent maximum value. However, it seems very unlikely that the quiescent atmosphere has pressures that significantly exceed those given in Tables 3 and 4. Whether or not the upper part of the quiescent atmosphere is heated by acoustic or MHD waves is still an open question, but in the time-averaged acoustic heating model by Mullan & Cheng (1994), the maximum coronal temperature is only  $6.5 \times 10^5$  K. If this temperature is adopted for  $T_c$ , the predicted value of  $E_{m_i}(T_o)$  is about a factor of 5 smaller than the observed value given in Table 4, so any acoustic heating must either lead to a hotter corona or be limited to the region below  $T_o$ .

By applying the analytical solutions, we have found that the value of  $E_{m_i}(T_o)$  is acceptable without invoking a limited area of emission and that current acoustic heating models are not entirely satisfactory. We have also predicted the values of  $P_e$  in the quiescent transition region and corona. While the former is acceptable, higher pressures cannot be ruled out above about  $T_e \geq 10^6$  K. Procyon is clearly a star for which full numerical models are required. These will be carried out following the methods used by Ness & Jordan (2008) in studies of  $\epsilon$  Eri, and will include recent atomic data for the emission lines used.

## 5 DISCUSSION AND CONCLUSIONS

Full numerical solutions giving the  $E_{md_i}(T_e)$  and  $E_{md,app}(T_e)$  of  $\epsilon$  Eri were carried out by Ness & Jordan (2008). The solutions for different values of  $T_c$  and fixed  $T_o$  and  $g(r_c)$  showed scaling laws between these parameters and  $P_o$ ,  $P_c$ ,  $E_{m_i}(T_o)$  and  $E_{m_i}(T_c)$ . These solutions were calculated in spherical symmetry and in hydrostatic equilibrium, including a non-thermal pressure term. The link between the scaling laws and the assumed energy balance was not obvious. Here, analytical solutions for a plane-parallel atmosphere, with some approximations for the variation in  $P_e$ , are presented, which give scaling laws that can be linked directly to the energy balance adopted. These analytical solutions reproduce well the values of  $P_o$  and  $P_c$  and give values of  $E_{m_i}(T_o)$  and  $E_{m_i}(T_c)$  that are smaller than in the full solutions by less than about a factor of 1.3. The analytical solutions are therefore useful in finding the best initial conditions in full numerical solutions. Only the values of  $g_*$ ,  $T_o$  and  $T_c$  are required, the latter two being simple to measure.

When a ‘critical solution’ is found by setting the conductive flux at  $T_o$  to zero, and when  $T_o$  is very much less than  $T_c$ , our value of  $P_o(\text{crit})$  is the same as that given by Hearn’s (1975, 1977) mel hypothesis. In our approach, the use of the condition that  $E_{md_i}(T_c)$  passes through a minimum at some observed  $T_o$  has the advantage that  $P_c$  and  $E_{m_i}(T_o)$  can be found, as well as  $P_o$  and  $E_{m_i}(T_c)$ .

We have applied our results to Procyon, for which the approximations made should be less accurate than for  $\epsilon$  Eri. It is difficult to measure the electron density in Procyon, but our results rule out

<sup>1</sup> archive.stsci.edu

suggestions that in the spatially averaged atmosphere the values significantly exceed about  $7 \times 10^9 \text{ cm}^{-3}$  in the transition region above  $10^5 \text{ K}$ , or about  $3 \times 10^8 \text{ cm}^{-3}$  in the quiescent corona. Improvements to the density-sensitive line ratios in the *EUVE* wavelength range will have to await the flight of new instruments.

Some earlier problems in reconciling coronal and transition region pressures have been reduced by using the full  $E_{\text{md,app}}(T_e)$ , rather than single  $T_c$  fits to earlier measurements of X-ray fluxes. If the higher electron densities found from the *EUVE* and possibly from the LETGS could be confirmed, these would show that active region material is indeed present (Schmitt et al. 1985; Jordan et al. 1986).

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