Proceedings of Meetings on Acoustics

Volume 19, 2013 http://acousticalsociety.org/

ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Signal Processing in Acoustics
Session 4pSP: Sampling Methods for Bayesian Analysis and Inversions in Acoustic Applications

4pSP3. Probabilistic two dimensional joint water-column and seabed inversion

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This paper develops a probabilistic two-dimensional (2D) inversion for geoacoustic seabed and water-column parameters in a strongly range-dependent environment. Range-dependent environments in shelf and shelf-break regions are of increasing importance to the acoustical-oceanography community, and recent advances in nonlinear inverse theory and sampling methods are applied here for efficient probabilistic inversion in 2D. The 2D seabed and water column are parameterized by highly efficient, self-adapting irregular grids which match the local resolving power of the data and provide parsimonious solutions requiring few parameters to capture complex environments. The self-adapting parameterization in the water-column and seabed is achieved by implementing the irregular grid as a trans-dimensional hierarchical Bayesian model which is sampled with the Metropolis-Hastings-Green algorithm. To improve sampling, population Monte Carlo is applied with a large number of interacting parallel Markov chains employing a load balancing algorithm on a computer cluster. The inversion is applied to simulated data for a vertical line array and several source locations to several kilometers range. Complex pressure fields are computed using a parabolic equation model and results are considered in terms of 2D ensemble parameter estimates and marginal uncertainty distributions. [Supported by NSERC]

Published by the Acoustical Society of America through the American Institute of Physics

© 2013 Acoustical Society of America [DOI: 10.1121/1.4800601]

Received 19 Jan 2013; published 2 Jun 2013

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INTRODUCTION

Matched-field inversion (MFI) has received significant attention for range-independent geoacoustic parameter estimation (e.g., [1, 2, 3]) as well as probabilistic inference for uncertainty estimation (e.g., [4, 5, 6]). Range-dependent inversions have been investigated to a lesser extent and are typically limited to relatively simple environments and/or make strong simplifying assumptions [e.g., noise-free simulations allow identifying features of range dependence prior to inversion, sediment layering is parallel to the seabed and/or range dependence consists of a small number of one-dimensional (1D) segments, see [7] and references therein].

Geoacoustic inversion with uncertainty estimation is challenging due to several factors: The inverse problem is known to be significantly nonlinear [8] and requires computationally intensive sampling techniques to quantify uncertainty. In addition, the information content of full-field data requires careful choice of parameterizations (model selection): Simple (subjective) parameterizations have been shown to lead to significant residual-error dependence [9] and may indicate an under-parametrized model [6]. Difficulties associated with parameterization are greatly exacerbated in range-dependent environments.

A substantial generalization of model selection is achieved by trans-dimensional (trans-D) models that relax the requirement of specifying a single model to a group of reasonable models [10]. For example, instead of assuming a certain number of sediment layers in a geoacoustic inversion, the number of layers is specified in terms of lower and upper bounds and treated as an unknown in the inversion. Efficient parameterizations are of paramount importance for probabilistic 2D inversion, where the curse of dimensionality can preclude the use of regular-grid parameterization. Irregular grids have been applied in tomography [11], and irregular grids based on Voronoi cells [12] have been used for interpolation applications [13]. The latter approach can be used as a trans-D hierarchical model for efficient parameterization in inverse problems [14] as it allows nodal density to vary in space and be sparse where little parameter variability and/or low data information content exist and dense where variability and/or information is high. Hence, the parameters consist of the number of nodes, node positions, and environmental properties at each node.

This paper applies a trans-D nonlinear acoustic inversion for 2D seabed and water-column variability [15]. The environment is parametrized in terms of an irregular grid of Voronoi nodes where the number of nodes and nodal locations are assumed unknown. In the seabed, an unknown sound velocity, density, and attenuation are associated with each node; in the water-column, only sound velocity at each node is unknown. These parameters define the 2D environment used for acoustic forward computations. During the inversion, the nodes self-adapt to local structure/data resolution based on Bayesian parsimony. Through this intrinsic parsimony, structure is controlled in such a way as to be consistent with prior and data information, and no subjective choices are required. Sampling from trans-D posterior distributions is achieved by the Metropolis-Hastings-Green (MHG) algorithm [16, 10]. Here, trans-D interacting Markov chains [17, 18, 6] are applied to improve dimension jumps and fixed-dimension chain mixing, which is instrumental in extending trans-D sampling to nonlinear 2D inversion. The algorithm is applied to simulated full-field acoustic data for several frequencies and source ranges, as recorded at a vertical line array (VLA). Acoustic-field predictions are computed using a range-dependent wide-angle Padé parabolic equation (PE) model [19].

BAYESIAN TRANS-DIMENSIONAL MODELS

Let $\mathbf{d}$ be N observed data and $\mathscr{I}_k$ denote a group of models specifying particular choices of physical theory, model parameterization, and error statistics. Let $\mathbf{m}_k$ be M_k parameters

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representing a realization of model $\mathscr{I}_k$. Bayes' rule can be written for a Bayesian hierarchical model [16]

$$P(k,\mathbf{m}_k|\mathbf{d}) = \frac{P(k)P(\mathbf{d}|k,\mathbf{m}_k)P(\mathbf{m}_k|k)}{\sum_{k'\in\mathscr{K}}\int P(k')P(\mathbf{d}|k',\mathbf{m}'_{k'})P(\mathbf{m}'_{k'}|k')d\mathbf{m}'_{k'}}, \tag{1}$$

where $P(k)$ is the prior over k. The state space is trans-D and given by the union of all fixed-dimensional spaces in $\mathscr{K}$. A Markov chain that converges to the posterior $P(k,\mathbf{m}_k|\mathbf{d})$ can be formulated using the MHG algorithm. Note that $P(k,\mathbf{m}_k|\mathbf{d})$ intrinsically addresses model selection and typical inferences about expectations do not require the computation of normalizing constants. This is a substantial advantage over model selection by way of computation of normalizing constants/evidence [20].

Here, the seabed and water column are parametrized with independent 2D partition models with a variable number of nodes [21, 14] and with the bathymetry assumed to be known. The partition model extends over a range given by the largest source-array separation and a maximum depth chosen to be sufficiently large for the PE model. Including creation and deletion of nodes, the Voronoi cells provide a natural re-partitioning of the space. Complex acoustic pressure fields measured at a VLA of H hydrophones and F frequencies ($N = FH$) are given by $\mathbf{d}=\{\mathbf{d}_f, f=1,F\}$, and errors are assumed here to be independent. The likelihood function used in this work is given by [22]

$$\log_e L(k,\mathbf{m}_k) \propto -H\sum_{f=1}^{F}\log_e\left|\mathbf{d}_f - \mathbf{d}_f(k,\mathbf{m}_k)\right|^2 . \tag{2}$$

Equation (2) is a hierarchical model in terms of the standard deviation of the residual error which is implicitly sampled. Data for several source ranges are included by summing over the $\log_e L$ terms for each range.

TWO-DIMENSIONAL PARAMETERIZATION

The 2D environment is parametrized here as a partition model in terms of Voronoi cells [12]. Voronoi cells are non-overlapping nearest-neighbor regions that are fully described by a node position (in range and depth) and a norm that defines the distance between the node position and any point in space. Here, we choose the Euclidean distance $d^{(i)}$ for the ith node for normalized coordinates in range r_n and depth z_n (normalized by maximum range and depth, respectively)

$$d^{(i)} = \sqrt{(r_n^{(i)} - r_n)^2 + (z_n^{(i)} - z_n)^2}, \tag{3}$$

where $r_n^{(i)}$ and $z_n^{(i)}$ are the range and depth for the ith node, respectively. An example of partitioning a unit area into Voronoi cells is shown in Fig. 1. The seabed or water-column parameters associated with a node are considered constant within the cell area. A vector of Voronoi nodes can represent a 2D environment efficiently with a small number of parameters. The trans-D hierarchical formulation of the model ensures parsimony, constraining the number of nodes to be consistent with data and prior information. The computation of predicted pressure fields is carried out using the Padé-expansion PE model PECan [19] which requires a regular grid in range and depth. Hence, for each prediction, a nearest-neighbor interpolation is performed from the irregular Voronoi nodes onto the regular PE grid.

INVERSION RESULTS

The simulation environment extends from 0- to 4-km range and 0- to 300-m depth (Fig. 2). Discretization for the PE algorithm involves a grid of 401 and 1001 grid points in range and depth, respectively. Sound velocity varies as a function of range and depth in the water column

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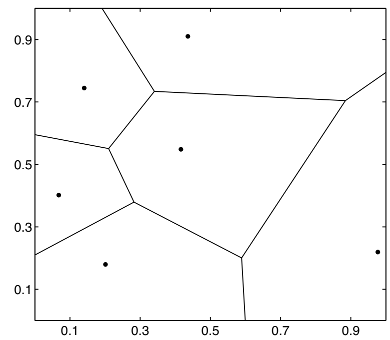

FIGURE 1: Example of a unit area partitioned by 6 Voronoi nodes using the Euclidean norm. Node positions are indicated as dots and nearest-neighbor cell boundaries with lines.

and seabed. Density and attenuation are constant and assumed known in the water, but vary with range and depth and are unknown in the seabed. However, since this inversion is predominatly sensitive to velocities, results are only shown for velocities. The water column is strongly stratified at shallow depths and includes a low-velocity range-dependent intrusion from 0- to 2-km range. The bathymetry changes from 200-m to 100-m depth over the total range with a flat section near the center. The seabed has a 10-m thick low-velocity (1550 m/s) layer that forms the water-sediment interface. Below this is a wedge (1600 m/s) extending from 2 to 4 km and decreasing in thickness from 70 to 10 m before cutting off entirely. Otherwise, the seabed consists of a basement of 1700 m/s (Fig. 2).

Data are simulated for a source at 50-m depth, 5 ranges (1, 2, 3, 3.5, 4 km), and 3 frequencies (50, 100, and 200 Hz). The frequency range was chosen to be consistent with the scale of the environmental variation. The data were simulated for a 32-element VLA at 0-km range with 4-m sensor spacing from 10–134-m depth. Complex Gaussian noise with a standard deviation of 15% of the maximum data magnitude at each frequency was added. The inversion was initiated at a model drawn randomly from the prior and constrained to a low dimension (3 nodes each in water and seabed).

Figure 3 shows the marginal distributions for the number of nodes in sediment and water column. In addition, the sampling of k is shown for a randomly selected chain. The uncertainty for k in the sediment is much larger than in the water but both peak at the same value of 7. The chain jumps between different numbers of nodes more frequently for the seabed than for the water column.

Posterior velocity estimates are considered in terms of ensemble-mean models and standard deviations. Figure 2 shows a detailed picture of the structure in the water column which is well recovered. However, gradients in velocity make for a much more challenging problem. The ensemble mean for velocity in the seabed (Fig. 2) shows that the sediment structure is accurately recovered by the inversion. In addition, the k-marginal distributions (Fig. 3) show that a parsimonious parameterization requiring only few Voronoi nodes is sufficient to obtain the results in Fig. 2. All interfaces are recovered as relatively sharp transitions and are less smeared out when the velocity contrast is high, such as from 0- to 1.7-km range near the water-sediment interface. Figure 2 also shows posterior standard-deviation estimates for the 2D water-column and geoacoustic parameters. Uncertainties for velocity are generally low in the water and larger in the seabed. Uncertainty also increases rapidly with increasing depth below the seafloor. These uncertainty estimates would likely decrease for inversions with increased data bandwidth, decreased range, and increased number of sources.

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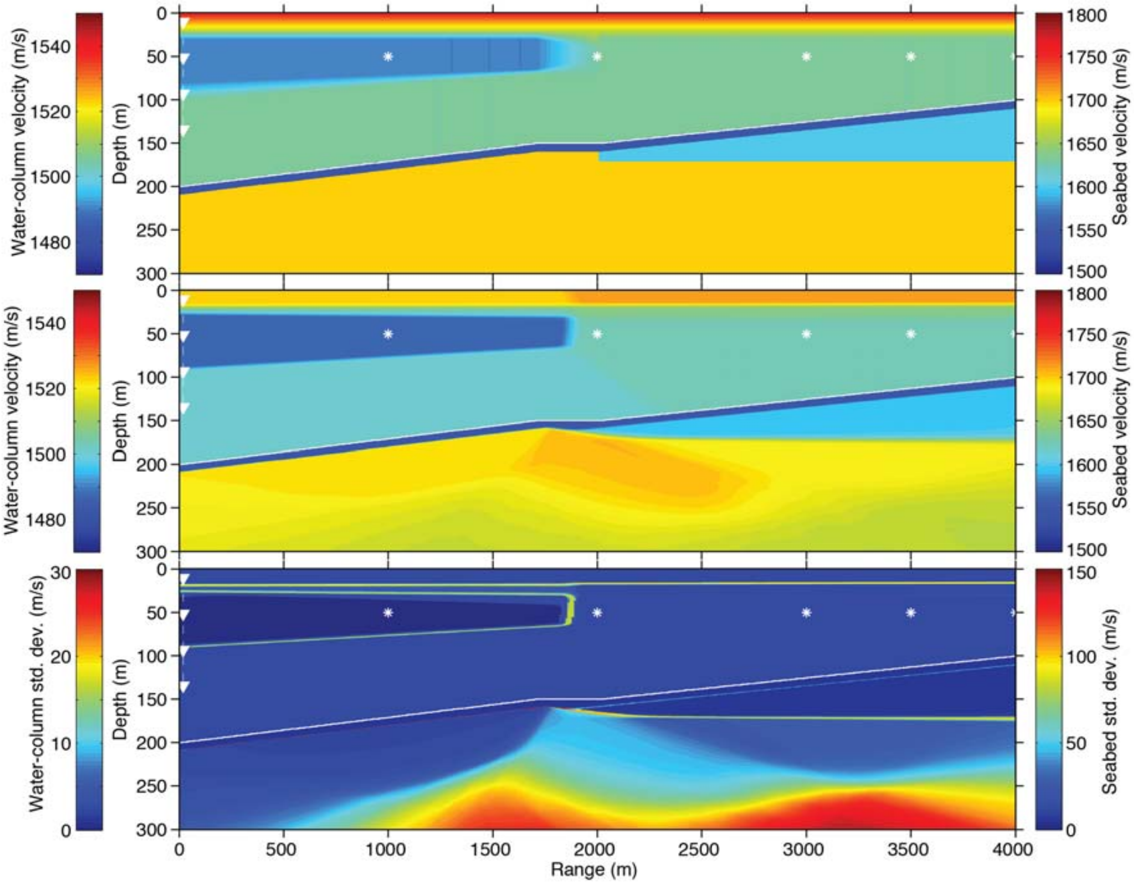

FIGURE 2: Nonlinear 2D inversion results for water column and seabed (top–true model, middle–ensemble mean, bottom–standard deviation). The inversion self adapts to structure, specifying locations of interfaces/layering is not required. Bathymetry is indicated by a white line and source (∗) and receiver (▼) locations are shown. Water-column and seabed structure are shown at different color scales for better visibility.

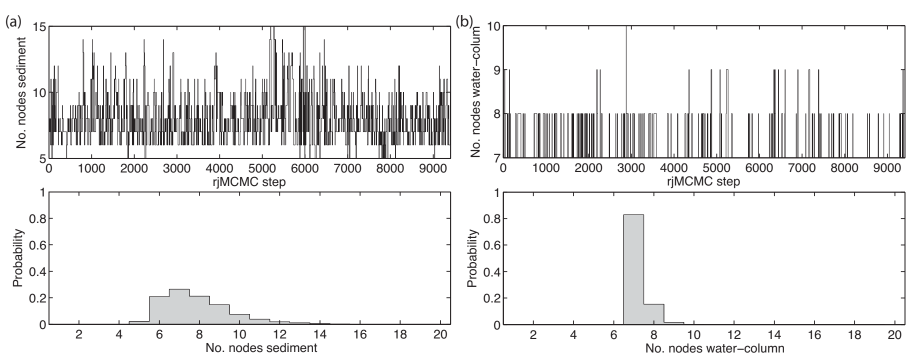

FIGURE 3: Posterior sampling of the number of nodes in (a) seabed and (b) water column. Chain plots are given for a randomly chosen single chain.

SUMMARY

This paper developed and applied a probabilistic and fully-nonlinear 2D inversion for a geophysical/acoustical problem. Parameter and uncertainty estimation in 2D poses substantial challenges for all aspects of inversion. Efficient parameterizations are required, and choosing an appropriate parameterization for 2D problems *a priori* is much more difficult than in 1D.

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Therefore, quantitative model selection is essential to a successful algorithm. Here, model selection was addressed intrinsically by formulating the inversion for a trans-D hierarchical model which relaxes model specification from a single model to a reasonable group of models. In particular, a 2D partition model was chosen to represent the environment in terms of nearest-neighbor regions (Voronoi cells) which can represent complex environments with few parameters. The partition model for the 2D ocean system was split into water-column and seabed partition models separated by known bathymetry. Including unknown bathymetry is possible in principle but is non-trivial in terms of sampling from the posterior distribution efficiently.

The application to simulated data gave promising results with velocity structure well resolved in the water column and seabed. The ability to jointly recover water-column and seabed parameters in a general 2D inversion illustrated the potential of the method for experiments in challenging environments with significant 2D water-column structure which may not be straightforward to measure during an experiment.

Acknowledgments

The authors gratefully acknowledge the support of the Natural Sciences and Engineering Research Council of Canada. The computational work was carried out on a parallel high-performance computing cluster operated by the authors at the University of Victoria funded by the Natural Sciences and Engineering Research Council of Canada and the Office of Naval Research. The authors thank Dr. Gary Brooke, Dr. David Thomson, Dr. Dag Tollefsen, and Dr. Malcolm Sambridge for valuable discussions.

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