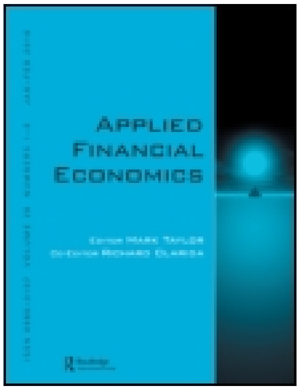

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Strategic risk aversion

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This article demonstrates that exaggerated risk aversion may comprise a rational form of strategic behaviour in the face of asymmetric information. Unlike some other forms of strategic behaviour analysed previously, this behaviour confers a benefit in the form of higher *ex post* consumption (not merely higher expected consumption or expected utility) and whether or not markets are perfectly competitive.

Keywords: strategic behaviour; risk aversion; contingent claims; asymmetric information

JEL Classification: G14; D82

I. Introduction

More than 20 years after Mehra and Prescott's (1985) pioneering study, the high risk aversion seemingly implied by historical equity premia remains a puzzle (Cochrane, 2005, p. 481). While many attempts have been made to rationalize the equity premium without implausibly high risk aversion, some have suggested that extreme risk aversion may be a genuine phenomenon (Fama, 1991; Kandel and Stambaugh, 1991). However, such explanations may predict implausible behaviour with respect to large changes in consumption, so the postulated behaviour may constitute a type of mistake or irrationality (Siegel and Thaler, 1997). Without impugning alternate models, this article demonstrates how a high degree of risk aversion may in fact be fully rational as a form of strategic behaviour in the face of asymmetric information. The contribution of the analysis lies not only in identifying an additional factor that may help explain the observed equity premium, but also in extending and refining the known potential for strategic misrepresentation in financial markets previously identified by Kyle (1989) and others.

The model exhibits both similarities and differences with respect to previous analysis of investor behaviour. As in the framework of Kyle (1989) and others, the traders here observe private signals and report demand functions to a market maker or auctioneer; previous studies have interpreted such reported functions as limit orders (ibid.) or as a limit order book (Holden and Subrahmanyam, 1996; Biais *et al.*, 2000).[1] Unlike in Kyle (1989) or Jackson (1991), traders here are not restricted to report linear demand schedules, a generalization that permits strategic risk aversion to be addressed. Also, the focus of analysis here is very different, aiming to characterize the allocational effect of a particular strategic distortion

[1] Black (1995) documents the observed use of sequences of scaled limit orders placed far in advance (before final prices are known) on financial exchanges. Other studies in which traders report demand functions or schedules to a market maker include Jackson (1991), Holden and Subrahmanyam (1996) and Biais *et al*. (2000). Massoud and Bernhardt (1999) likewise interpret the auctioneer as a market maker, while Barlevy and Veronesi (2000) similarly analyse a market in which a trader's strategy is a demand schedule, though without a precise description of the institutional mechanism by which those demand schedules are aggregated to generate an equilibrium price and allocation vector.

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rather than to quantify the aggregation of private information in equilibrium prices. Because of this, I allow private information to be exogenous, unlike Jackson (1991) or Holden and Subrahmanyam (1996).

The main result is that traders in our model can improve their *ex post* consumption levels (not merely expected utility) by reporting preferences that overstate their risk aversion. This result, moreover, arises *whether or not* traders can actually influence the equilibrium market prices. Thus, in contrast to models by Kyle (1989), Jackson (1991) and others, the distortion here does not depend on imperfect competition in the rational expectations equilibrium. This generalization demonstrates that the efficacy of strategic risk aversion is exceptionally robust to variations in market structure.

This main result contrasts with an earlier literature that identified conditions under which agents could benefit from *understating* their risk aversion (Kurz, 1977; Crawford and Varian, 1979; Sobel, 1981). Those studies relied on a particular bargaining process (Nash, 1950; Raiffa, 1953; Kalai and Smorodinsky, 1975) in contrast to the quasi-Walrasian equilibrium in the model here. Similarly, one could interpret the linear demand schedules strategically utilized in Kyle (1989) as benefiting from an understatement of the true risk aversion corresponding to the traders' assumed exponential utility functions, also in contrast to the result obtained below.

The analysis proceeds as follows. Section II introduces the general framework, which subsequent sections further develop and apply in both perfectly and imperfectly competitive settings. Section III formally derives the main result under perfect competition, while section IV presents a graphical interpretation of that result. Section V demonstrates by numerical example that the main result does not require perfect competition but can also arise even when individual agents have substantial unilateral influence on the equilibrium market prices. Section VI concludes this article.

II. The Framework

The model is the simplest that is capable of exhibiting the main result. Initially, uncertainty is portrayed using the standard information partition model of a state space, in which an agent's information is a particular element of the partition (Li, 2009). Agents receive state-contingent endowments and trade in contingent claims before the true state is revealed. A contingent claim is defined in the usual way as a contract that provides a specified payoff conditional on a particular state of the world occurring, representing uncertain returns.[2] Contingent claims have often been interpreted as financial derivatives (as in Heath *et al*., 1991; Muroi, 2005; Phillips and Yu, 2009) but have also been applied to other financial choices such as optimal cash holdings (Anderson and Carverhill, 2007) and to other topics as diverse as measures of systemic risk in banking (Lehar, 2005).

As noted above, traders in this model report their demand functions to a market maker after observing an exogenous private signal, and we wish to analyse the resulting allocations. The calculations below are presented directly in terms of the associated utility functions, for three reasons. First, this approach makes it clear that the demand functions correspond to a specific utility function, unlike in some previous models such as Kyle (1989, where the linear demand functions introduced on page 323 do not in general correspond to the exponential utility functions assumed on page 320, as noted by Jackson, 1991, p. 5).[3] Second, this approach permits an explicit characterization of the form of misrepresentation that increases a trader's consumption. Third, given that our central research question requires an analysis of utility functions, the exposition can be streamlined by working directly with those functions rather than introducing the associated demand functions as an extra step. Standard theory establishes that every continuous utility function has an associated demand function, so this simplification is without loss of generality.[4]

I assume state-independent utility functions, state-dependent private endowments, private signals and fixed aggregate endowments. The benefit derived from exaggerated risk aversion is in *ex post* consumption, not merely *ex ante* expected utility – a stronger benefit than previously found for other types of strategic behaviour.

As in Kyle (1989), Jackson (1991), and other studies, equilibrium allocations are determined on the basis of the submitted demand functions without

[2] In its simplest form, a contingent claim may provide a unit payoff in a single state (LeRoy and Werner, 2001, p. 18). More generally, a contingent claim such as an option may provide a range of payoffs that vary across states.
[3] Kyle's assumption of normally distributed random variables is needed to reconcile linear demand functions with exponential utility. The analysis here does not impose the assumption of normal distributions.
[4] Other studies have analysed competition in schedules in the theory of industrial organization, such as Bernheim and Whinston (1986) under complete information, Klemperer and Meyer (1989) for supply functions under uncertainty (but not asymmetric information), and McAfee (1993).

allowing subsequent recontracting. Thus, traders learn nothing beyond their private signal until final allocations have been established. Even so, as in all such models, it is possible to identify a strategy that benefits from the (*ex ante* unknown) aggregate information. Intuitively, the strategy of exaggerated risk aversion works because risk averse behaviour mitigates the downside risk of trading against a collectively better informed market.

III. Perfect Competition

To demonstrate that imperfect competition is not essential to the efficacy of strategic misrepresentation in our framework (unlike in Kyle, 1989; Jackson, 1991; and other previous models), we begin by assuming price-taking traders with general utility functions and distributions of outcomes (probabilities of states). These generalizations are important because previous analysis has demonstrated that some widely accepted properties of financial markets are highly sensitive to the assumed form of utility functions and stochastic distributions (Barlevy and Veronesi, 2000).

I compare equilibrium consumption under a general, concave, differentiable utility function $U(x)$ versus a strictly concave transformation thereof, representing strategic exaggeration of the reported degree of risk aversion. I show that a given trader consumes more in the *ex ante* unknown true state by reporting the transformed (i.e. more risk averse) function, holding constant the reported demand functions of other traders. The sequence of events is as follows. Each trader observes an exogenous private signal, then reports a preference function to the market maker, who determines Walrasian equilibrium allocations and prices conditional on the reported preferences. Finally, consumption occurs.

The trader's exogenous private information yields prior probabilities π_i for each state i. The number of states is arbitrary, but it suffices to compare the *ex ante* unknown true state with any other state, as follows. The aggregate information of all other traders is superior to that of the given trader in the sense that, as in any revealing equilibrium, the equilibrium prices will be found *ex post* to satisfy the relation

$$P_j/P_i > \pi_j/\pi_i \tag{1}$$

where j is the *ex ante* unknown true state, i any other state and P_i the equilibrium price of a contingent claim in state i. To impose perfect competition, I assume that equilibrium prices are independent of any trader's reported preferences; i.e. any trader is small relative to the market.

First-order conditions for asset allocation to maximize the reported utility of $U(x)$ are, for any two states i and j,

$$P_j/P_i = (\pi_j/\pi_i)U'(x_j)/U'(x_i) \tag{2}$$

for given π_i, π_j and equilibrium values of P_i, P_j. This is the condition that the market maker will apply for every trader (using each trader's own reported utility function) and every pair of states. Letting j denote the *ex ante* unknown true state (without loss of generality), we obtain $U'(x_j)/U'(x_i) > 1$ from Equations 1 and 2 or, by concavity of U, $x_j < x_k$. Thus, whichever state is finally revealed to occur, actual consumption in the true state will be less than the quantity that would have been consumed contingent on any other state.

Now let g be a positive, increasing, strictly concave, differentiable function. When demand is reported according to $g(U(\cdot))$ instead of $U(\cdot)$, denote the associated equilibrium consumption quantity by y instead of x. Then the first-order condition becomes

$$P_j/P_i = (\pi_j/\pi_i)[g'(U(y_j))U'(y_j)]/[g'(U(y_i)U'(y_i)] \tag{3}$$

Again let j denote the *ex ante* unknown true state. As shown in the Appendix, Equations 2 and 3 together imply that $y_j > x_j$. That is, *ex post* consumption in the true state is higher when equilibrium is computed according to the more concave utility function. This result means that a trader as modelled has an incentive to exaggerate his risk aversion to the market maker.

Because this incentive applies to any initial utility function, no matter how risk averse, the result implies an unbounded degree of risk aversion as optimal for each trader. In the context of a discrete state space and given the nature of the assumed signals, this would imply lexicographic preferences across states (rectilinear indifference contours) as the limiting case, though I do not provide here an explicit analysis of such equilibrium utility functions.

IV. A Graphical Illustration

Because the formal proof of the competitive case may not be completely transparent, this section presents a graphical version of the competitive result, depicted in Fig. 1. The information structure, sequence of actions and other conditions are as in the

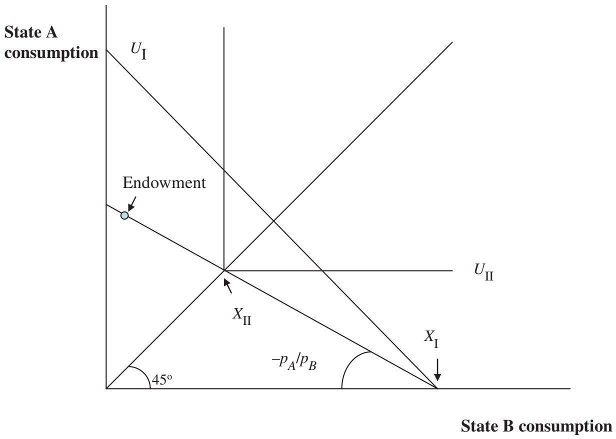

Fig. 1. One trader's equilibrium consumption in the true state and a false state

previous section. In this framework, Fig. 1 depicts the situation for an individual trader where the true state (*ex ante* unknown to the trader) is A and an alternative possible state is B. As in the previous section, other states may exist, but it suffices to focus attention on these two states, the true state and one false state. I compare the trader's *ex post* consumption under two alternative state-independent utility functions, one risk neutral and the other risk averse.

The shape of the trader's iso-expected utility curves is fully determined by his subjective probabilities of the two states and his degree of risk aversion. Figure 1 depicts both a risk neutral indifference curve (U_I) and a risk averse indifference curve – in this instance, the extreme case of a fixed-coefficient or maximin indifference curve (U_{II}). The latter indifference curve has a vertex on the 45° ray through the origin, by state-independence.

As in the previous section, the exact nature of the trader's information remains unspecified, but I assume that it assigns at least as high a probability to state A as to state B. If the former probability is higher than the latter, the risk neutral indifference curve has a slope flatter than -1. Very poor information might leave the trader with equal priors, giving U_I a slope of -1. The result can be shown in either case; Fig. 1 depicts U_I as having a slope of roughly -1 for simplicity.

A crucial condition is that the combined information of all other traders is superior to that of an individual trader in the following sense. Given that state A is going to occur, the budget constraint must pass through the trader's endowment point with a slope that is not only flatter than -1, but also flatter than U_I (the opposite would be true if state B were going to occur). Although the slope of the budget line here is the price ratio, relative prices in the two states are related to the relative probabilities assigned to the states by the aggregate market. Thus, 'superior information' translates into the condition that the market is able to assign a higher *ex ante* probability to the state that is revealed *ex post* to occur than can an individual trader. As in the previous section, assume this condition. In the imperfectly competitive case presented in Section II, this condition was derived from other considerations.

The consequence of these conditions is that, once the true state is realized, the trader will consume the point x_I if he has reported risk neutral preferences, and x_{II} instead if he has reported maximin preferences. However, given that state A occurs, only the y-component of the consumption point is relevant *ex post*. Thus we see that *ex post* consumption is higher at x_{II} than at x_I. Again, the analysis is symmetric in the event that state B occurs, and leads to the same conclusion: *ex post* consumption is higher in the true state for risk averse preferences than for risk neutral preferences.

Several comments about this scenario are in order. First, if prices were announced before trading was implemented, or if recontracting were allowed, then each trader could identify the true state as the one having the highest price. The market would not clear in this case because everyone would want to buy and no one would be willing to sell contingent claims in that state. Thus, lack of prior announcement of prices and prohibition of recontracting are necessary in this example not only for the result shown, but also for markets to clear. This property is common to rational expectations equilibria in general.

Second, although only the two extremes of risk neutrality and maximin preferences are shown in the diagram, intermediate degrees of risk aversion will give intermediate consumption levels in the true state. The relevant indifference curves will cross the 45° ray at a slope equal to the ratio of the trader's subjective probabilities of the states, and be tangent to the budget line at some point between the 45 degree ray through the origin and the x axis. This result is especially easy to see in case of equal priors so that the indifference curve is symmetric about the 45° ray through the origin.

Third, as noted above, the example generalizes to any number of states; B can represent any state other than the true one. Finally, the fixed budget line assumes that the trader's reported preferences have no effect on equilibrium prices (i.e. the trader is 'small' relative to the market). As in the previous section, this condition demonstrates that market power is not necessary to generate the main result. However, to demonstrate that perfect competition is also not necessary for the main result, the next section

Table 1. Description of the market

		Utility function			Endowment if true state is		
		Case I	Case II	Information structure	*a*	*b*	*c*
Trader	1	$U = x$	$U = \ln(x)$	[*a*] [*bc*]	5	1	3
	2	$U = \ln(x)$	$U = \ln(x)$	[*b*] [*ac*]	3	5	1
	3	$U = \ln(x)$	$U = \ln(x)$	[*c*] [*ab*]	1	3	5

Note: Each state occurs with probability 1/3. Consumption is constrained to be nonnegative.

Table 2. Equilibrium consumption in each state

		Case I			Case II		
State		*a*	*b*	*c*	*a*	*b*	*c*
Trader	1	5.79	0	0	5.79	1.25	1.96
	2	1.96	6.31	2.18	1.96	5.79	1.25
	3	1.25	2.69	6.82	1.25	1.96	5.79

will derive a similar outcome under imperfect competition.

V. Imperfect Competition

To address the imperfectly competitive case, I impose additional structure for clarity and tractability. There are three states of the world and three traders. Each trader observes an exogenous, *ex ante* signal consisting of an element of a partition of the state space, as in Bond (2003), Gunderson (2006) and others. This partition, or information structure, varies from trader to trader and is coarser than the state space.[5] After observing their signals, each trader reports a preference function to the market maker, who then calculates an allocation vector and price vector across the traders and states to maximize the reported utility functions subject to market clearing. Finally, consumption occurs.

To show the key effect, I focus without loss of generality on the reported preferences of the first trader. I compare two cases. In both cases, the second and third traders have a utility function $U_i = \ln(x_i)$; the actual utility function is immaterial, but the crucial assumption is to hold constant their preferences while allowing the reported preferences of the first trader to vary. The first trader has a utility function $U_1 = x_1$ (case I, risk neutral) but may instead report a demand function corresponding to $U_1 = \ln(x_1)$ (case II, risk averse). Table 1 summarizes the endowments, information structures and utility functions of these traders.

In such a market, allocations and prices are determined by standard calculations, as in Shubik (1977) and Wilson (1978), after imposing a restriction of nonnegative consumption. The consumption quantities and prices for each state constitute a Walrasian equilibrium in that they satisfy the conditions to maximize each trader's reported expected utility, conditional on his private information and endowments, as well as satisfying each trader's budget constraint and the market clearing constraint. In our model, the equilibrium allocations and prices are determined and administered by the market maker.

Table 2 shows the equilibrium consumption quantities for each trader in each realized state. It is evident that, should state *b* or *c* occur, trader 1 consumes more when he reports a demand function corresponding to $U_1 = \ln(x_1)$ than when he reports his true demand function. If the true state is *a*, his consumption is unaffected by this misrepresentation because his signal in that case is as precise (i.e. is as fine a

[5] In particular, the signal will be that element of the partition containing the true state. For example, suppose that the possible states are [*a b c*] and the trader's information structure is the partition [*a*] [*bc*]. Then, if state *c* is actually going to occur, the agent will 'observe' the element [*bc*] *ex ante*. He then assigns an equal probability to states *b* and *c*, and zero probability to state *a*. This type of information structure is further explained in Huang and Litzenberger (1988) and Osborne and Rubinstein (1994). Li (2009) has shown how this framework can be extended to incorporate the possibility that agents are unaware *ex ante* of certain potential states or outcomes.

Table 3. Equilibrium prices in each state

		Case I			Case II		
State	True	*a*	*b*	*c*	*a*	*b*	*c*
Contingent State	*a*	1	0.30	0.24	1	0.14	0.22
	b	0.22	1	0.53	0.22	1	0.14
	c	0.14	0.42	1	0.14	0.22	1

partition of the state space) as the aggregate information of the other traders for that state. Thus, in state *a* alone, he is at no informational disadvantage with respect to the rest of the market. Similarly, it can be shown that trader 2 benefits from reporting a demand function corresponding to $U_2 = \ln(x_2)$ rather than a linear demand function if the true state is *a* or *c*, while trader 3 benefits from reporting a demand function corresponding to $U_3 = \ln(x_3)$ instead of a linear demand function if the true state is *a* or *b*.

Table 3 presents the equilibrium price vectors in each state and for each case. Two features are apparent. First, the price is always highest in the true state. This result stems from the combination of a constant aggregate endowment across states and the ability of aggregate market information to identify the true state. Thus, even though aggregate supply is independent of the state, aggregate demand is highest in the true state, given the signals received by the various traders.

Second, the equilibrium price vectors are different in the two cases. Thus, the reported preferences of trader 1 have a nonneglible effect on prices, as might be expected when there are only three traders. Thus, this analysis exhibits imperfect competition, which has previously been found to encourage other forms of strategic misrepresentation as in Kyle (1989), Jackson (1991) and others.

This version of the model is similar to that of Green and Lin (2000), who establish a contrasting result in a different context. In their analysis, there are likewise three investors (bank depositors), each investor's consumption depends on the reported preferences of all investors, and the bank plays the role of 'auctioneer' or market maker. However, in contrast to our result, truth-telling is a dominant strategy in their model.

VI. Concluding Remarks

The extremely risk averse behaviour apparently exhibited by investors in aggregate has been recognized as puzzling for more than two decades. While various explanations have been proposed, none has yet proven universally convincing. This article identifies a new mechanism that may contribute to the observed behaviour, perhaps in addition to other mechanisms previously suggested.

In particular, an incentive is demonstrated for traders to misrepresent their degree of risk aversion when reporting preferences to a market maker in contingent claims. For a given trader, a higher degree of reported risk aversion yields higher *ex post* consumption (a stronger result than increased expected utility), taking the reported preferences of other traders as given. This result is seen in both imperfect and perfect competition, for general distributions of states, and for general utility functions.

Besides large empirical risk premia, other possible consequences of strategically exaggerated risk aversion may also arise and could be usefully explored in future research. The welfare effects of such strategic misrepresentation constitute another potentially important open question. On the one hand, the resulting pattern of allocations may conform more nearly to a 'full insurance' economy, benefiting all risk averse consumers in an expected utility sense. On the other hand, such risk reduction is typically achieved only at a cost, and the misrepresentation of risk aversion may lead to a socially wasteful level of spending to reduce risk. If, on balance, exaggerated risk aversion was found to reduce aggregate welfare, such a finding would warrant additional research into the design of market mechanisms to mitigate the incentive for such strategic behaviour.

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Appendix: Proof that $y_j > x_j$ in the Competitive Case

Condition (1) implies that $g'(U(y_j))U'(y_j) > g'(U(y_i))U'(y_i)$ or, by concavity of g and U, $y_j < y_i$. But $y_j < y_i$ implies that $U(y_j) < U(y_i)$ and so we obtain

$$g'(U(y_j)) > g'(U(y_i)) \tag{4}$$

by concavity of g. Now $U'(x_j)/U'(x_i) = (P_j/P_i)(\pi_i/\pi_j)$ while $U'(y_j)/U'(y_i) = (P_j/P_i)(\pi_i/\pi_j) \cdot [g'(U(y_i))/g'(U(y_j))]$. Then (4) implies that

$$U'(x_j)/U'(x_i) > U'(y_j)/U'(y_i) \tag{5}$$

Given this inequality (5), four cases arise.

(i) Suppose $y_i \geq x_i$ and $y_j \leq x_j$ for some $i \neq j$. Then $U'(y_i) \leq U'(x_i)$ and $U'(y_j) \geq U'(x_j)$. But this result contradicts Equation 5, so this case cannot occur in equilibrium.

(ii) Suppose $y_i \leq$ (respectively, $<$) x_i while $y_j <$ (respectively, $\leq$) x_j for all $i \neq j$, in such a way as to satisfy Equation 5. But then the budget constraint $p \cdot y = p \cdot x$ fails, assuming that the price vector is unaffected by the trader's reported utility function. That is, either x (the consumption vector of contingent claims given that U was reported) is not feasible, or else y (the consumption vector given that $g(U)$ was reported) is suboptimal by monotonicity of U. Therefore, this case cannot occur in equilibrium.

(iii) Suppose $y_i \geq (>) x_i$ while $y_j > (\geq)\ x_j$ for all $i \neq j$. Then the budget constraint is similarly violated. Either y is not feasible or x is suboptimal. Thus, this case cannot occur.

(iv) Suppose $y_i < x_i$ while $y_j > x_j$ for all $i \neq j$. This is the only remaining case, and the only one that raises no contradictions. This case therefore must occur in equilibrium.