Reducing Blur in X-ray Micro-CT Due to a Non-point Source

H. Li*[1], A. Kingston[1], G. Myers[1] and T. Varslot[2]

[1]Research School of Physics and Engineering, Department of Applied Mathematics, Australian National University. [*email: lhy110@physics.anu.edu.au]

[2]Head of Software and Systems services, Numerical Rocks, Trondheim, Norway.

Keywords: deconvolution, iterative algorithms, micro tomography, source deblur

ABSTRACT

In micro-CT, X-rays are typically emitted from a "source spot" several microns wide. This paper investigates two algorithms for correcting the resulting penumbral source-spot blurring. We test Richardson-Lucy (R-L) and Conjugate Gradient (CG) methods (regularised and un-regularised), on simulated projection data. The CG method enforces self-consistency in the sinogram, whilst the R-L method independently deconvolves each radiograph. Both methods deliver a better reconstruction than the standard filtered back-projection (FBP), at the cost of increased computation time. Reconstructions from simulated data indicate that the CG method preserves fine details better than the R-L method.

1. INTRODUCTION

X-ray computed tomography (CT) imaging is typically modelled using the Radon transform. For simplicity, conventional CT reconstruction algorithms assume X-rays emanate from an infinitesimal point (Natterer 2001). This assumption is valid when the source spot is relatively small compared to the voxel size of the volume being reconstructed, but is increasingly violated as high-resolution micro-CT moves towards voxels approximately the same size as the source spot. In these cases the non-negligible size of the source (with respect to the voxel size) will lead to penumbral blurring in the radiographs, resulting in blurring artefacts in conventional CT reconstructions. In this paper we model the source as a sum of incoherent (i.e. non-interfering) point sources. To better isolate the effects of source-spot blurring, we assume the linear attenuation coefficient of the sample to be approximately independent of X-ray energy (i.e. we assume the incident beam is sufficiently filtered such that beam-hardening is negligible). This simulated projection data is used to test the R-L and CG methods.

For large source-sample distances the imaging system is approximately linear shift invariant, and the inverse problem reduces to 2D deconvolution of the radiographs, followed by standard CT reconstruction (Paganin, 2006). We explored deconvolution algorithms, such as: Fourier Deconvolution, Landweber's algorithm (Landweber 1951), Super resolution (Hunt 1992), etc. The R-L method was chosen as it is expected to be comparatively stable in the presence of high-frequency noise (Lucy 1974).

The radiographs in a CT data set will contain some redundant data at low spatial frequencies, and must be self-consistent (i.e., lie on the range space of the forward problem). In the absence of beam hardening and refraction, the de-blurred radiographs will lie on the range space of the Radon transform. Inconsistent radiographs will lead to artefacts in the 3D reconstruction: unlike R-L deconvolution, CG reconstruction enforces self-consistency in the CT data set, and is valid for source-sample distances small enough that blur can no longer be modelled as a 2D convolution of the radiographs.

2. MODELLING THE EFFECT OF THE NON-POINT SOURCE

2.1. Source Blurring and Noise function

As stated above, blurring caused by non-point source is modelled by the convolution of the simulated intensity with a source-spot kernel. For simulated data, the source-spot kernel is a normal distribution with standard deviation of 2 pixels, truncated to be 9 pixels wide. Some Poisson noise is added to the radiograph to model noise:

Noisy radiograph = -ln{Possion[10*2^15*exp(-Original radiograph)]/(10*2^15)}.

For real data, the source spot-kernel may be measured by analysing radiographs of a known phantom. In our case we used a block of steel. A radiograph of the vertical edge of the block was taken, the line-spread function was measured from the block edge, and the point-spread function of the source was then calculated.

2.2. Richardson-Lucy (R-L)

The R-L method (Lucy 1974) is a maximum-likelihood statistical deconvolution algorithm. It maximises the likelihood of producing the observed sinogram, given a point-spread function (PSF) and assuming Poisson noise. Let the operator B represent convolution with the PSF, and B* be its adjoint. Let g_b be source-blurred radiograph, and g_r be the deblurred radiograph. Using the TV-minimisation filter (Molina 1994) for regularisation, the i^{th} iteration of the regularised R-L algorithm is calculated as follows:

$$g_r^{i+1} = C(g_r^i B\{g_b / [B^*(g_r^i)]\}),$$

where C is the TV-minimisation filter with parameter 0 for the R-L method, and 0.0018 for the RL TV-minimisation regularised method. In this work 16 iterations are used. This regularisation step is similar to a prior that favours smooth radiographs. A volume reconstruction is obtained from the R-L deblurred sinogram using the standard FBP algorithm.

2.3. Conjugate Gradient (CG)

As discussed in the introduction, a CT data set must be self-consistent to avoid artefacts in the 3D reconstruction. The CG method presented here iterates between sinogram and volume space, ensuring that the deblurred sinogram is self-consistent. The Conjugate Gradient method requires the adjoint, of the forward imaging operator $A=\sum(c_jA_j)$, where A_j models projection from a point-source with index j, and relative intensity c_j. Note that this is a convolution over several incoherent sources, not a convolution of the radiographs. The adjoint operator A* can be shown to be: $A^*=\sum(c_jA_j^t)$. The Conjugate Gradient method is an algebraic solver with quadratic convergence (Katsaggelos 1991) that minimises $||g_r-Af_r||_{L2}$. For the regularised iterative method:

$$f_r^{i+1}=C(f_r^i+\alpha^i p^i),$$

where C is the TV-minimisation filter with parameter 0 for the CG method, and 0.0002 for the CG TV-minimisation regularised method. Here p^i is the step direction and α^i is the magnitude, both of which are determined by the conjugate gradient method (Fletcher 1964). In this work 64 iterations are used.

3. SIMULATION RESULTS

We simulated CT imaging of a phantom image (see figure 1, left), adding noise to the radiographs as per section 2.1. Reconstruction was then performed with FBP, R-L deconvolution followed by FBP, and CG reconstruction (regularised and un-regularised). Results are shown in Figures 1 and 2. For quantitative comparison of noise levels, we will use the square area with coordinates (X,Y)=([31,50],[35,54]), which is near-constant in the original image. The standard deviation of pixel values in this area is calculated, and normalised with respect to the standard, uncorrected FBP reconstruction (see Table 1). Lower values indicate a smoother reconstruction. The sharpness measure (i.e. the Laplacian) is used to quantify contrast in the various images (see Table 1). Ideally, we desire a low-noise, high-contrast reconstruction. Relative signal to noise ratio (RSNR) is just contrast divided by noise. The difference between each reconstruction and the original in the L2 norm (L2FO) is also calculated, although this tells us little about whether edges in the reconstruction are correctly positioned.

Table 1 demonstrates that all deconvolution methods improved upon FBP, as measured by RSNR and L2FO. CG produced a better image than RL according to both measures. Regularisation degrades the fine detail in the Richardson-Lucy reconstruction (see the reduced contrast in Table 1), but does not appear to do so for the Conjugate Gradient method (very little reduction in contrast, see Table 1).

Since both regularised reconstructions have a similar amount of noise, we can conclude that the CG method preserves more fine structure in the images than the R-L method. We expect this gap in performance to widen as source-sample distance is reduced (violating the assumptions made by the R-L reconstruction). Encouragingly, we note that both regularised reconstructions had similar noise levels to the blurry FBP reconstruction, whilst exhibiting significant increases in contrast.

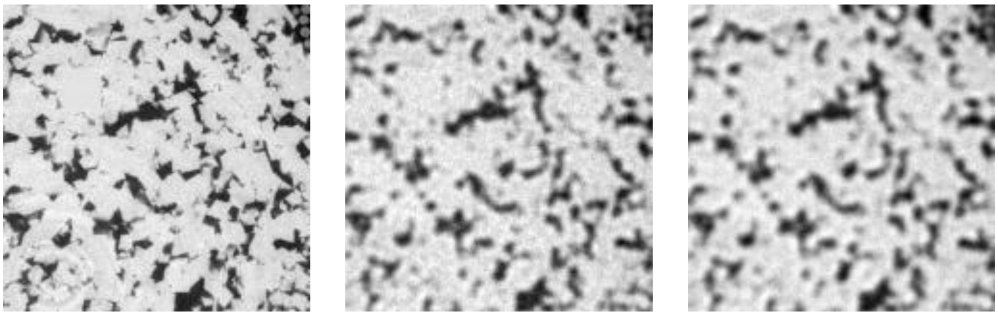

Figure 1: *Original image, Richardson-Lucy, Conjugate Gradient*

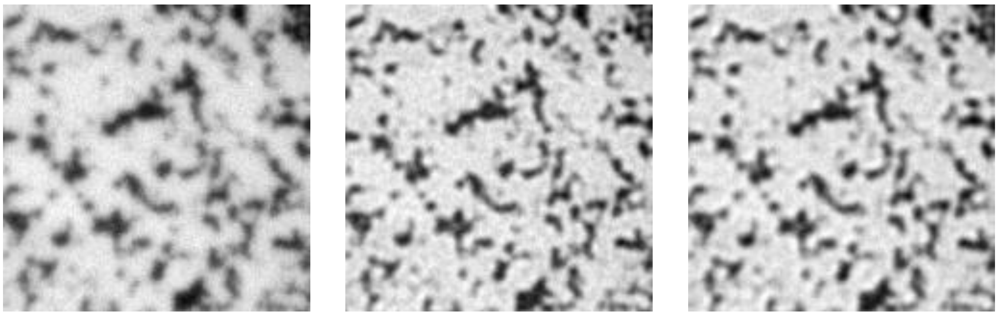

Figure 2: *Filtered Back-projection, Richardson-Lucy and Conjugate Gradient with regularisation*

Reconstruction	Noise relative to FBP (Std. Dev.)	Sharpness relative to FBP (Contrast)	Relative Signal to Noise Ratio	L2 norm distance from original
Original	0.138	1.975	14.3	0
FBP	1.000	1.000	1.00	0.0543
RL	1.509	1.633	1.08	0.0435
RL TV-Reg	1.179	1.380	1.16	0.0435
CG	1.251	1.484	1.18	0.0389
CG TV-Reg	1.154	1.409	1.22	0.0393

Table 1: *Quantitative analysis of various method of reconstructions*

In terms of computational time, Richardson-Lucy takes about twice the computational time of the standard filtered back-projection, while Conjugate Gradient takes about 2.0*[number of Iterations] times the computational time of the standard filtered back-projection. The overwhelming majority of the computation time in the CG algorithm is consumed by projection and backprojection operations, which may be accelerated by a factor of ~40 using a GPGPU (Myers et al. 2011).

4. CONCLUSIONS

Both the Richardson-Lucy and Conjugate Gradient methods are able to partially correct for source-spot blurring, for large source-sample distances, at the cost of increased computing time. Preliminary results from simulated data indicate that the regularised Conjugate Gradient method preserves fine detail better than the R-L method, and we expect the results from the R-L method to degrade as source-sample distance is decreased.

5. REFERENCES AND ACKNOWLEDGEMENTS

Fletcher, R., and Reeves, C. M. (1964). Function minimization by conjugate gradients. The Computer Journal.

Hunt, B R. and Sementilli, P. (1992). Description of a Poisson Imagery Super Resolution Algorithm. Astronomical Data Analysis Software and Systems I, A.S.P. Conference Series.

Katsaggelos, A K., Biemond, J., Schafer, R W. and Mersereau, R M. (1991). A Regularized Iterative Image Restoration Algorithm. IEEE Transactions on Signal Processing. Vol 39, 914-929.

Lucy, L B. (1974). An iterative technique for the rectification of the observed distriutions. The Astronomical Journal. Vol 79, 745-754.

Landweber, L. (1951). An Iteration Formula for Fredholm Integral Equations of the First Kind. American Journal of Mathematics.

Molina, R., Mateos, J., and Abad, J. (1994). Prior Models and the Richardson-Lucy Restoration Method. The Restoration of HST Images and Spectra II, 118-122.

Myers, G., Kingston, A., Varslot, T., Turner, L., Sheppard, A. (2011). Dynamic X-ray micro-tomography for real time imaging of drainage and imbibition processes at the pore scale. Presented at the 25th International Symposium of Core Analysts, Austin, TX, USA, 18-21 September 2011.

Natterer, F. (2001). The Mathematics of Computerized Tomography. Society for Industrial and Applied Mathematics.

Paganin, D. M. (2006). Coherent X-Ray Optics. Oxford Science Publications.