

## Robust Optimal Attitude Control of Multirotors

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### Abstract

In this paper, a robust and optimal control method is proposed for the attitude control problem of multirotors. The designed controller consists of an optimal controller and a robust compensator. The optimal controller is designed based on the linear quadratic regulation control method for the desired tracking of the nominal linear system, whereas the robust compensator is introduced to restrain the influence of multiple uncertainties. Tracking errors are proven to be bounded with specified boundaries ultimately. Simulation and experimental results on the hexarotor demonstrate the effectiveness of the proposed robust optimal control approach.

### 1 Introduction

In the automatic control field, multirotors have received much attention in respect that they have some advantages over the regular helicopters. First, fixed-pitch rotors are used instead and their lift thrusts can be altered without swashplate. Second, the payload and the maneuverability of the rotorcrafts can be increased by multiple rotors. Third, for a given size, each rotor of the multirotors is smaller than the equivalent main rotor of a regular helicopter due to the usage of multiple rotors and thus the safety of the operations can also be enhanced. Consequently, in the automatic control circle, intensive efforts, have been devoted to the control problem of multirotors. In this paper, we investigate the robust control problem for hexarotors (see, Fig. 1). Compared to the quadrotors, hexarotors can carry more payload and are more highly maneuverable.

Previous studies on multirotors mainly focused on achieving automatic flight for quadrotors. The PID control methods (see, [Mahony et al., 2012], [Hoffmann et al., 2011], and [Altug et al., 2005]) and the nonlinear control methods (see, [Castillo et al., 2004], [Tayebi and McGilvray, 2006], [Das et al., 2009], [Bertrand et al., 2011], [Aguilar-Ibanez et al., 2012], and [Guerrero-Castellanos et al., 2011] to mention a few)



Fig. 1. The hexarotor in the air.

were discussed for the quadrotors to achieve attitude and position control. These works involve designing flight controllers based on accurate model, whereas the effects of uncertainties were not further discussed.

Many efforts have been done on achieving the robust flight control of the quadrotors. In [Zhang et al., 2011], under the effects of a class of time-varying disturbances, the attitude control was achieved for a quadrotor. In [Alexis et al., 2011], a novel switching model predictive controller was designed based on an affine model of the quadrotor to achieve the attitude control. By combining command-filtered backstepping based control approach and position control method, the attitude angles were stabilized and the trajectory tracking control was achieved in [Zuo, 2010]. However, the uncertainties, considered in [Zhang et al., 2011], [Alexis et al., 2011], and [Zuo, 2010], were limited to being time-invariant in their simulation and experimental tests. Furthermore, further restraining the effects of various uncertainties including nonlinear dynamics, parametric uncertainties, unmodeled dynamics, and external disturbances simultaneously, still remains open.

The robust controller design of a hexarotor is similar to that of a quadrotor. However, there also exist

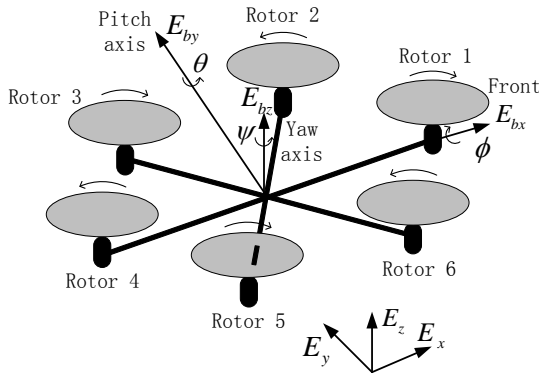


Fig. 2. The schematic of the hexarotor.

differences between them due to different aerodynamic forces and torques generated by the rotors. As the hexarotor possesses two more rotors, the resulted torque of the hexarotor is different from that of the quadrotor. Many works focused on achieving the automatic flight control of the quadrotors, whereas the discussions on the robust hexarotor control problem is rare.

In this paper, the attitude system is divided into three subsystems: that is, the pitch, roll, and yaw subsystems. For each subsystem, a linear nominal model is obtained by the linearizing method, whereas coupling, nonlinear dynamics, parametric uncertainties, and external disturbances are regarded as uncertainties, which are named as the named equivalent disturbances. A robust and decoupled control method is proposed based on the linear quadratic regulation (LQR) control scheme and the robust compensation technique. For each subsystem, the designed controller consists of a nominal optimal controller and a robust compensator. The nominal optimal controller is designed to achieve the desired tracking of the nominal linear system, whereas the robust compensator is designed for restraining the influence of the various uncertainties mentioned above.

Compared to the previous studies on the control problem of the multirotors, the influence of uncertainties can be restrained in this article. The tracking errors of the hexarotor are proven to be bounded with specified boundaries in a finite time. Moreover, the robust compensation enables the decoupled controller design for each axis, making the system architecture more modular. For each angle, this decoupled control method results in a linear time-invariant controller, which is comparatively easy to implement in practical applications. Furthermore, the parameters of the optimal controllers can be determined easily according to the specified requirements of the tracking performance by selecting the values of weighting matrices in cost functions.

The following parts of this paper are organized as follows. In Section 2, the hexarotor mathematical model is briefly described. In Section 3, the optimal controller and the robust compensator are designed. In Section 4, robust tracking properties of the whole closed-loop system are proved. In Section 5, simulation results are given. In Section 6, experimental results of the hexarotor in hovering conditions are provided and we conclude the paper with future direction Section 7.

## 2 Model of Hexarotor

The schematic of the multirotor is depicted in Fig. 2.

Define  $\alpha = \{E_x, E_y, E_z\}$  the earth-fixed inertial frame and  $\beta = \{E_{bx}, E_{by}, E_{bz}\}$  the frame attached to the body with origin in the mass center. Let  $\eta = [\theta \ \phi \ \psi]^T$  denote the three Euler angles: the pitch angle  $\theta$ , the roll angle  $\phi$ , and the yaw angle  $\psi$ , which define the rotation from  $\alpha$  to  $\beta$ . The mathematical model of angular dynamics three angle motions of hexarotor can be written as

$$\begin{aligned} J_\theta \ddot{\theta} &= C_\theta(\eta, \dot{\eta})\dot{\eta} + a_{\theta 1} \tau_\theta, \\ J_\phi \ddot{\phi} &= C_\phi(\eta, \dot{\eta})\dot{\eta} + a_{\phi 1} \tau_\phi, \\ J_\psi \ddot{\psi} &= C_\psi(\eta, \dot{\eta})\dot{\eta} + a_{\psi 1} \tau_\psi, \end{aligned} \quad (1)$$

where  $J_i$  ( $i = \theta, \phi, \psi$ ) are the moments of inertias,  $C_i(\eta, \dot{\eta})$  ( $i = \theta, \phi, \psi$ ) are the Coriolis terms, and  $\tau = [\tau_\theta \ \tau_\phi \ \tau_\psi]^T$  are the external body-fixed frame torques.

The desired references of the pitch, roll, and yaw angles are denoted by  $r_\theta$ ,  $r_\phi$ , and  $r_\psi$  respectively. The control inputs for  $\theta$ ,  $\phi$ , and  $\psi$  are  $u_i$  ( $i = \theta, \phi, \psi$ ), which are proportional to  $\tau_i$  ( $i = \theta, \phi, \psi$ ) and satisfy  $u_i = \tau_i / a_{i2}$ , where  $a_{i2}$  is a positive constant. A power distribution board is applied to distribute the control inputs to the six rotors, therefore the three rotational motions of the vehicle are being controlled by  $u_i$  ( $i = \theta, \phi, \psi$ ) directly.

Let us define an angular error vector with the error and its derivative and integral terms as

$$e_i = [e_{i1} \ e_{i2} \ e_{i3}]^T \quad (i = \theta, \phi, \psi),$$

where  $e_{i1} = i - r_i$ ,  $e_{i2} = \dot{e}_{i1}$ , and  $e_{i3} = \int e_{i1}$ . This paper will investigate the robust control of attitude so that the attitude tracking errors are bounded with specified boundaries ultimately.

Let  $a_i = a_{i1} a_{i2} / J_i$  ( $i = \theta, \phi, \psi$ ). The vehicle parameters  $a_i$  ( $i = \theta, \phi, \psi$ ) can be split up into the nominal parts (denoted by  $N$ ) and the uncertain parts (denoted by  $\Delta$ ) as

$$a_i = a_i^N + \Delta a_i, \quad i = \theta, \phi, \psi.$$

Define

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ a_i^N \\ 0 \end{bmatrix}.$$

From the hexarotor model (1), we can obtain that

$$\dot{e}_i = A_i e_i + B_i (u_i + q_i), \quad i = \theta, \phi, \psi, \quad (2)$$

where  $q_i$  ( $i = \theta, \phi, \psi$ ) are the equivalent disturbances. They have the following forms

$$q_i = (C_i(\eta, \dot{\eta})\dot{\eta} / J_i + (a_i - a_i^N)u_i + w_i - \ddot{r}_i) / a_i^N, \quad (3)$$

where  $w_i$  ( $i = \theta, \phi, \psi$ ) are external disturbances.

From (2), we can see that the model can be divided into three subsystems: the pitch subsystem ( $i = \theta$ ), the roll subsystem ( $i = \phi$ ), and the yaw subsystem ( $i = \psi$ ). The coupling between each angle is considered as equivalent disturbances.

*Assumption 1:* The uncertain parameters are bounded. The nominal parameters  $a_i^N$  ( $i = \theta, \phi, \psi$ ) are positive and satisfy that  $|a_i^N - a_i| < a_i^N$ .

*Assumption 2:* The external disturbances are bounded.

*Assumption 3:* The reference signals and the derivatives  $r_i^{(k)}$  ( $i = \theta, \phi, \psi; k = 0, 1, 2$ ) are bounded.

### 3 Robust Optimal Controller Design

The designed controller includes two parts: a nominal optimal controller and a robust compensator and thereby the control inputs  $u_i$  ( $i = \theta, \phi, \psi$ ) have the forms as

$$u_i = u_i^{NO} + u_i^{RC}, \quad i = \theta, \phi, \psi, \quad (4)$$

where  $u_i^{NO}$  is the nominal optimal control input and  $u_i^{RC}$  is the robust compensating input.

An optimal LQR controller is designed for the nominal part of the error model (2), which is rewritten below

$$\dot{e}_i = A_i e_i + B_i u_i, \quad i = \theta, \phi, \psi. \quad (5)$$

The cost functions  $J_i$  ( $i = \theta, \phi, \psi$ ) to be minimized are

$$J_i = \frac{1}{2} \int_0^{\infty} [e_i^T(t) Q_i e_i(t) + \pi_i (u_i^{NO}(t))^2] dt,$$

where  $Q_i$  ( $i = \theta, \phi, \psi$ ) are  $3 \times 3$  weighting matrices which are symmetric and positive-definite, and constants  $\pi_i$  ( $i = \theta, \phi, \psi$ ) are positive. by solving the following Riccati equations, one can obtain the positive-definite matrices  $P_i$  ( $i = \theta, \phi, \psi$ )

$$A_i^T P_i + P_i A_i - \pi_i^{-1} P_i B_i B_i^T P_i + Q_i = 0, \quad i = \theta, \phi, \psi.$$

Then, one can obtain the state-feedback gains as

$$K_i = \pi_i^{-1} B_i^T P_i, \quad i = \theta, \phi, \psi.$$

The optimal control inputs can be obtained as

$$u_i^{NO} = -K_i e_i, \quad i = \theta, \phi, \psi. \quad (6)$$

Actually, for each subsystem considered here, the designed nominal linear time-invariant feedback controller only depends on its own state feedback. Therefore, a decoupled nominal control is achieved for each nominal subsystem.

Moreover, the robust compensator is introduced to produce a compensating signal to restrain the influence of uncertainties. They are designed based on the robust filters as

$$F_i(s) = \frac{f_{li} f_{si}}{(s + f_{li})(s + f_{si})}, \quad i = \theta, \phi, \psi,$$

where  $s$  is the Laplace operator and  $f_{li}$  and  $f_{si}$  are positive constants. The robust filters have the following property (see, [Zhong, 2002] and [Liu et al., 2013]): if the robust compensator parameters  $f_{li}$  and  $f_{si}$  ( $i = \theta, \phi, \psi$ ) are sufficiently large and  $f_{li}$  is much larger than  $f_{si}$ , the gains of the filters would approximate to 1.

The robust compensating inputs can be designed as follows

$$u_i^{RC}(s) = -F_i(s) q_i(s), \quad i = \theta, \phi, \psi. \quad (7)$$

They can be realized as follows by introducing two new

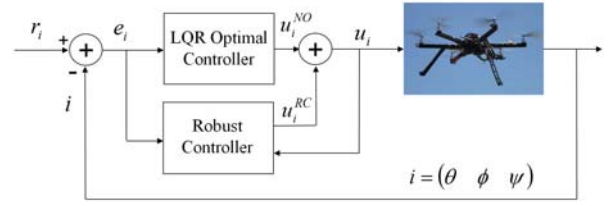


Fig. 3. The block diagram of the closed-loop control system with three subsystems: the pitch subsystem ( $i = \theta$ ), the roll subsystem ( $i = \phi$ ), and the yaw subsystem ( $i = \psi$ ).

states

$$\dot{z}_{1i} = -f_{si} z_{1i} - f_{si}^2 e_{i1} + a_i^N u_i, \quad (8)$$

$$\dot{z}_{2i} = -f_{li} z_{2i} + (f_{li} + f_{si}) e_{i1} + z_{1i},$$

$$u_i^{RC} = -f_{li} f_{si} (e_{i1} - z_{2i}) / a_i^N, \quad i = \theta, \phi, \psi.$$

In this paper, second order robust filters are applied instead of first order robust filters. Therefore, the robust compensating inputs  $u_i^{RC}$  ( $i = \theta, \phi, \psi$ ) can be designed without the angular velocities, which might contain much noise in practical applications. Actually, if the three angular velocities are reliable and obtainable, first order filters, which depend on the angular velocities, can be used instead.

It should be noted that the designed robust compensators are also linear time-invariant and decoupled. Although there exist coupling between the three Euler angles, the robust compensator of each channel only involves its own state of each subsystem.

The configuration of the robust optimal closed-loop control system can be depicted as shown in Fig. 3.

### 4 Robust Properties Analysis

In this section, the attitude tracking errors of the three Euler angles are proved to converge into a given neighborhood of the origin ultimately for a given initial condition.

Define

$$A_{Hi} = A_i - B_i K_i, \quad i = \theta, \phi, \psi.$$

Let  $q = [q_\theta \quad q_\phi \quad q_\psi]^T$ ,  $e = [e]_{9 \times 1} = [e_\theta^T \quad e_\phi^T \quad e_\psi^T]^T$ , and

$u^{RC} = [u_\theta^{RC} \quad u_\phi^{RC} \quad u_\psi^{RC}]^T$ . Then, we can obtain by using (5)-(7) that

$$\dot{e} = A_H e + B(u^{RC} + q), \quad (9)$$

where

$$A_H = \text{diag}(A_{H\theta}, A_{H\phi}, A_{H\psi}), \quad B = \text{diag}(B_\theta, B_\phi, B_\psi).$$

The robust properties of the closed-loop control system can be summarised as follows

*Theorem 1:* If the assumptions described in the second section hold, for the given positive constant  $\varepsilon$  and the given initial state, we can obtain positive robust compensator constants  $f_{li}$  and  $f_{si}$  ( $i = \theta, \phi, \psi$ ), which have sufficiently large values and satisfy that  $f_{li}$  is sufficiently larger than  $f_{si}$ , and a positive constant  $T^*$ , such that the attitude tracking error is bounded and satisfies that  $|e(t)| \leq \varepsilon, \forall t \geq T^*$ .

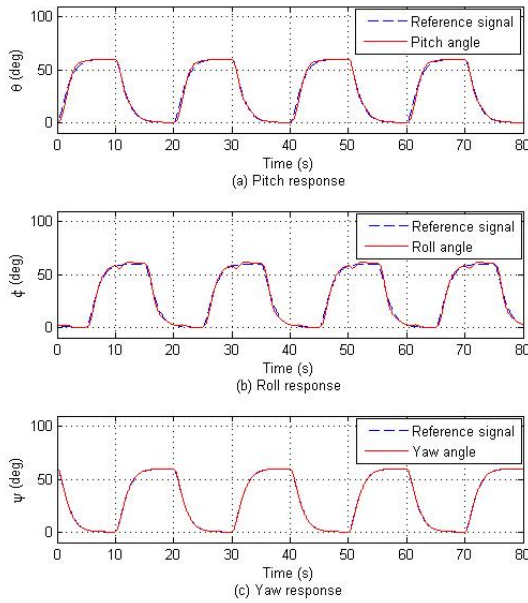


Fig. 4. Responses with the LQR control method for the nominal model.

Theorem 1 can be proven based on the small gain theory. It should be pointed out that the robust compensator parameters  $f_{li}$  and  $f_{si}$  ( $i = \theta, \phi, \psi$ ) can be tuned on-line monotonously, that is, set them with some initial values satisfying that  $f_{li} \geq \eta_i f_{si}$  ( $i = \theta, \phi, \psi$ ), where  $\eta_i$  is a large positive constant and is selected according to practical situations. Then, run the closed-loop control system for some specified missions. If the tracking performance is not satisfactory, we can set  $f_{li}$  and  $f_{si}$  ( $i = \theta, \phi, \psi$ ) to larger values satisfying  $f_{li} \geq \eta_i f_{si}$  ( $i = \theta, \phi, \psi$ ), until the desired tracking performance of the closed-loop system can be achieved.

## 5 Simulation Results

The nominal parameters used in this section are  $a_\theta^N = 2.9630$ ,  $a_\phi^N = 1.8286$ , and  $a_\psi^N = 0.3626$ .

First, we simulate the LQR controller with a nominal model without considering uncertainties and disturbances. Let  $Q_i = \text{diag}(100, 10, 1)$  and  $\pi_i = 1$  ( $i = \theta, \phi, \psi$ ). Simulation results with the LQR control method are presented in Fig. 4. We can see that the nominal controller parameters are selected appropriately and thus the closed-loop system by the nominal optimal controller achieves good tracking performance.

Then, we consider the effects of the parametric uncertainties and external disturbances. The parameter perturbations are assumed up to 80% of the nominal values. The external disturbances are given as follows

$$\begin{aligned} w_\theta(t) &= 2 \sin 10t, \\ w_\phi(t) &= 10 \cos 10t, \\ w_\psi(t) &= 20 \sin 10t. \end{aligned}$$

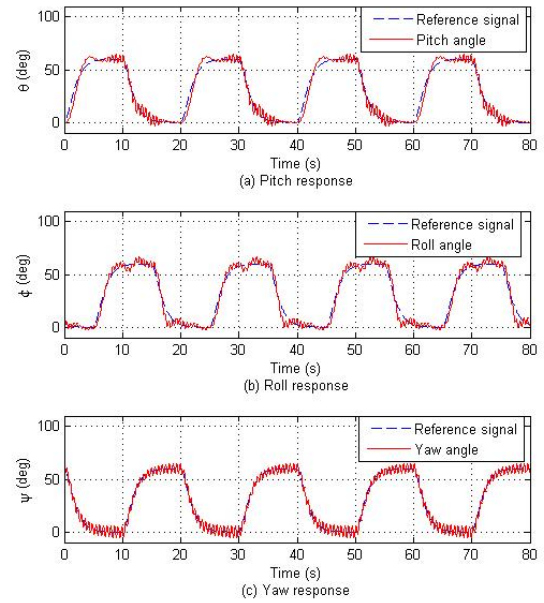


Fig. 5. Responses with the LQR control method under the effects of disturbances.

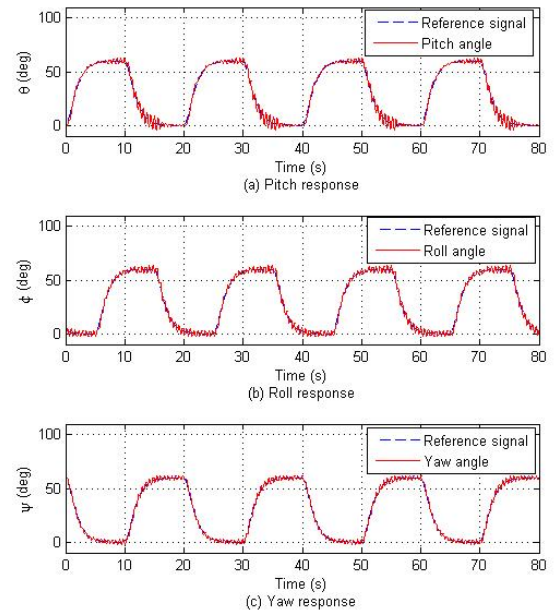


Fig. 6. Responses with the robust control method under the effects of disturbances.

The responses with the LQR control approach and with the design robust optimal control method are presented in Fig. 5 and Fig. 6. The tracking errors are compared in Fig. 7. We can see that the proposed robust control method achieves slightly better tracking performance.

Lastly we, set the parameters of the robust compensators to be larger values as:  $f_{li} = 1000$  and  $f_{si} = 200$  ( $i = \theta, \phi, \psi$ ). Corresponding responses and



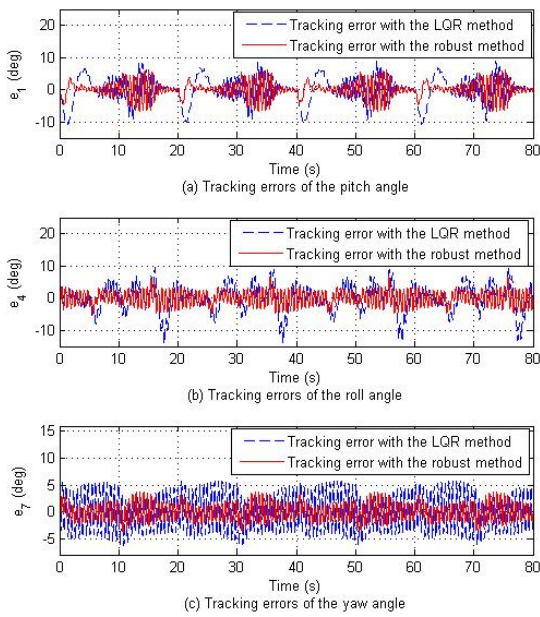


Fig. 7. Comparisons of tracking errors under the effects of uncertainties.

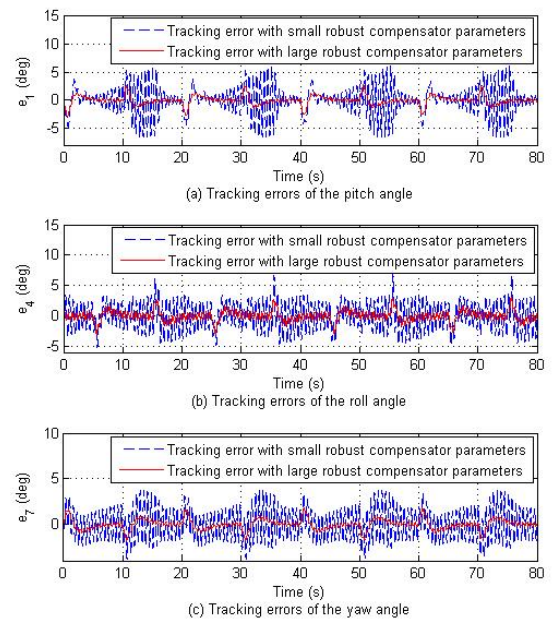


Fig. 9. Comparisons of tracking errors under the effects of uncertainties.

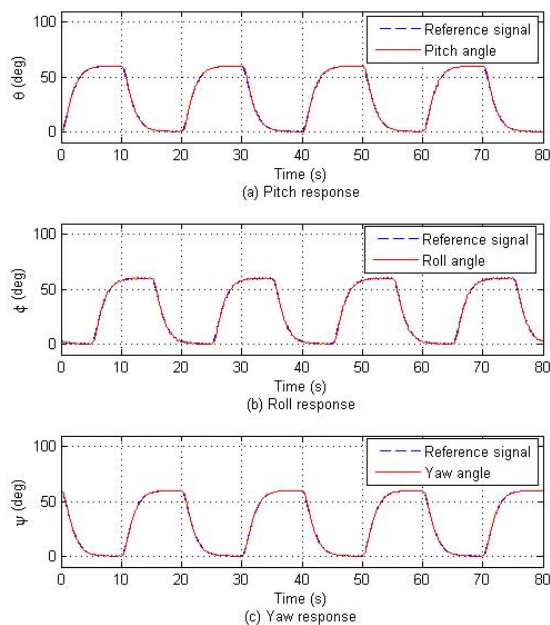


Fig. 8. Responses with larger robust compensator parameters under the effects of uncertainties.

tracking errors are depicted in Fig. 8 and Fig. 9. Compared to the first and second test, we can see that the tracking performance is greatly improved, since the tracking errors are much small. Actually, the tracking performance of the closed-loop control system can be improved if  $f_{li}$  and  $f_{si}$  are selected with larger values.

## 6 Real-time Experimental Results

The hexarotor used in this section is developed based on Arducopter UAVs [Arduino, 2005]. An onboard avionic electronic system is used here to obtain the attitude and position information and implement the control algorithms. It consists of an inertial measurement unit module, a global positioning system module, a magnetometer, a sonar sensor and a barometer. The updating rate of attitude loop is 50 Hz. The robust controller parameters are selected as:  $Q_j = \text{diag}(800, 1, 0.0031)$  ( $j = \theta, \phi$ ),  $Q_\psi = \text{diag}(800, 1, 20)$ ,  $\pi_i = 1000$ ,  $f_{li} = 5$ , and  $f_{si} = 1$  ( $i = \theta, \phi, \psi$ ) for better tracking performance.

For the hovering conditions, experimental results are presented in Fig. 10. We can see that all of three tracking errors of the attitude angles are less than 0.1 deg, which means that the closed-loop control system achieves good tracking performance. The control inputs are depicted in Fig. 11.

## 7 Conclusions

A robust optimal controller was proposed to achieve the attitude control of a multirotor. It includes an nominal optimal controller for the nominal system to obtain desired tracking performance and a robust compensator to restrain the influence of various uncertainties. Simulation and experimental results on the hexarotor demonstrated the effectiveness of the designed robust controller.

Currently, we have only tested our control method on the attitude control of the hexarotor. Currently we are applying the method for trajectory control under severe outdoor wind disturbances.

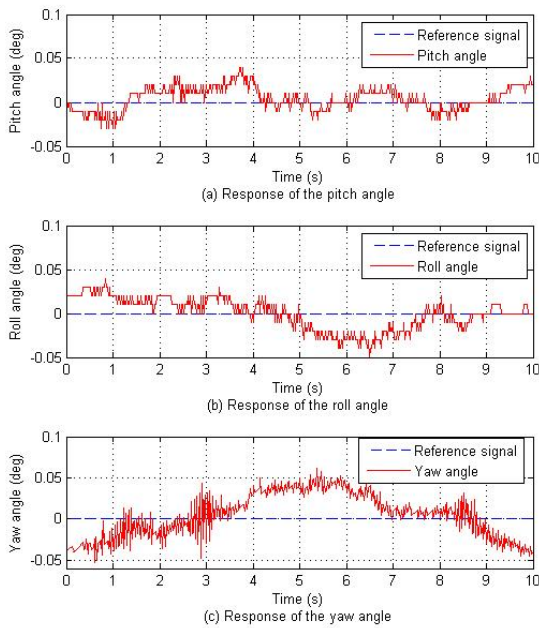


Fig. 10. Experimental responses in the hovering experiment.

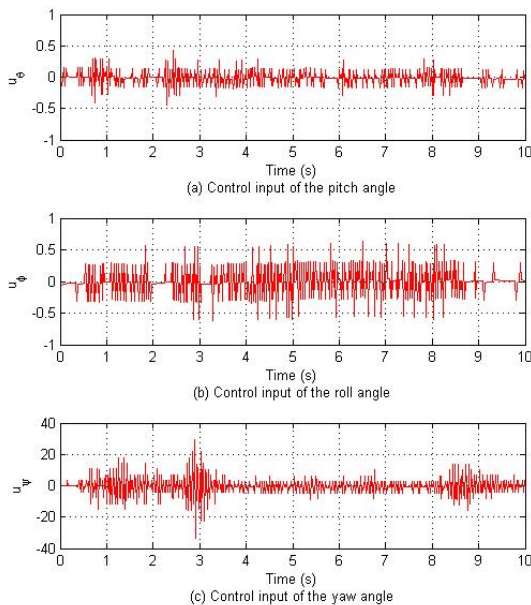


Fig. 11. Control inputs in the hovering experiment.

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