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SPECTOR3D: a resistive magnetohydrodynamic stability code for stellarators

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Abstract. SPECTOR3D is a magnetohydrodynamic (MHD) code designed to study the resistive stability and mode spectrum of stellarators. It finds the spectrum of the linearized, compressible, resistive MHD equations. Some typical results are given for a tokamak model with toroidal field ripple and a 17-period Heliotron.

1. Introduction

We describe a code, SPECTOR3D, for the study of the stability and spectrum of compressible resistive magnetohydrodynamic (MHD) perturbations in stellarators. The code has been checked for ideal cases against the standard benchmark Large Helical Device (LHD) equilibria of Nakamura et al. [1]. This code is designed to find the spectral properties (particularly for the unstable modes) of stellarators. A version of Variational Moments Equilibrium Code (VMEC) and a Boozer coordinate mapper based on a module in the TERPSICHORE code provide the equilibrium data to the stability code SPECTOR3D, which uses the Jacobi–Davidson technique [2, 3] to find a range of spectral properties.

2. Code formulation

The equations solved for the compressible case are the linearized MHD equations with appropriate normalization: the factor $\eta = S^{-1}$ is the inverse magnetic Reynolds number (Lundquist number). In the following the capital letters refer to the equilibrium magnetic field $\mathbf{B}$ and pressure P and the lower case letters to the perturbed magnetic field $\mathbf{b}$, velocity $\mathbf{v}$ and pressure p. The normalized density profile, $\rho^* = \rho(r)/\langle\rho\rangle$, can account for variations in plasma density across the radius. The ratio of specific heats, γ, is usually set to $\frac{5}{3}$, but it can be chosen arbitrarily. In particular, if γ is taken to be very large, the incompressible limit can be approximated. Thus, we have

$$\rho^* \frac{\partial \mathbf{v}}{\partial t} = (\nabla \times \mathbf{B}) \times \mathbf{b} + (\nabla \times \mathbf{b}) \times \mathbf{B} - \nabla p,$$

$$\frac{\partial p}{\partial t} = -\mathbf{v} \cdot \nabla P - \gamma P \nabla \cdot \mathbf{v},$$

with $\nabla \cdot \mathbf{b} = 0$. To satisfy this last condition the magnetic field is written in terms of the vector potential $\mathbf{b} = \nabla \times \mathbf{a}$. Ampere's law (integrated once) thus becomes

$$\frac{\partial \mathbf{a}}{\partial t} = \mathbf{v} \times \mathbf{B} - \eta(\nabla \times \nabla \times \mathbf{a}).$$

Boundary conditions are given by the imposition of a conducting wall at the plasma edge, and analyticity at the origin.

The code VMEC is used to find numerical stellarator equilibria, and an external program is used to convert this equilibrium into Boozer coordinates. The equilibrium is then interpolated on to the stability grid using cubic splines of appropriately scaled equilibrium quantities, to account for any singularities at the origin. The magnetic vector potential is expressed in term of its covariant components and the velocity in terms of its contravariant components; seven scalar variables $(v_1, v_2, v_3, p, a_1, a_2, a_3)$ represent the physical quantities $(\mathbf{v}, p, \mathbf{a})$, which are discretized radially using finite elements, and poloidally and toroidally using Fourier components. A divergence-free and sufficiently smooth finite element scheme is ensured by representing v_1, a_2 and a_3 by cubic spline functions, and v_2, v_3, p and a_1 by quadratic Hermite polynomials. The algebraic complexities were assisted by using Mathematica to output the matrix elements in a form which could be directly inserted into a Fortran code. Integral forms of the linearized resistive MHD equations are expanded using the finite element representations and the resulting coupled equations are reduced to matrix form: the overall formulation which is similar to the two-dimensional resistive code CASTOR [4]. The code has been tested using some analytical one-dimensional results [5], two-dimensional Solov'ev equilibria [6] and LHD model equilibria [1, 6, 7].

3. Some results

3.1. Toroidal ripple

We have investigated resistive ballooning modes in a model tokamak system with toroidal field ripple as shown in Fig. 1. The rotational transform profile is $\iota \approx (0.80 - 0.75\psi + 0.22\psi^2)$, where ψ is the normalized toroidal flux. In this case the modes are toroidally localized in the region of bad curvature. Perturbation components with toroidal mode numbers $n = 1, -4, 6, \ldots$ are considered, and a range of m is chosen so that the field-line-resonant components have $n = 1$. The most unstable mode is then at relatively small wavenumber.

3.2. A 17-period Heliotron

We consider the stability of a two parameter family of 17-field-period Heliotrons. The shape of the plasma boundary is specified in VMEC coordinates: the position of the plasma boundary is parameterized by angles θ, ζ and expressed in cylindrical coordinates R, Z, ζ with $R = \sum R_{M,N} \cos(m\theta - n\zeta)$ and $Z = \sum Z_{M,N} \sin(m\theta - n\zeta)$. The rotational transform arises in the model Heliotron due to the $m = 1/n = 17$ components which produce the helical twist: we choose $R_{1,17} = \epsilon$ and $Z_{1,17} = -1.05\epsilon$. The vacuum rotational transform is dependent on ϵ, with $\iota \approx \iota_\epsilon(0.286 + 0.714\psi)$, where ψ is the normalized toroidal flux. We then introduce a strongly peaked toroidal plasma current into these configurations, with $J = J_0(1 - \psi)^4$, with the pressure gradient profile still constrained to be zero. For a range of ι_ϵ and J_0 we consider perturbations in the $n = \cdots -16, 1, 18 \ldots$ toroidal mode family, with

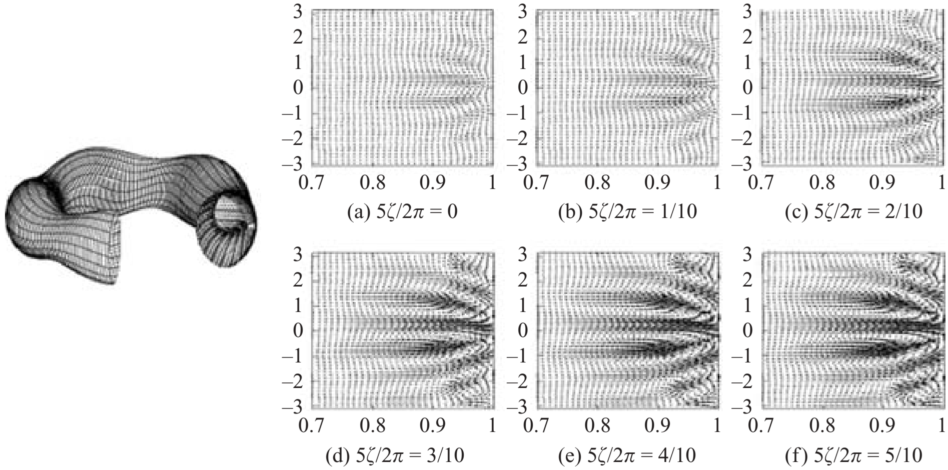

Figure 1. A tokamak-like equilibrium with a strong toroidal field modulation with toroidal mode number $n = 5$ and the mode structure, plotted against flux coordinate (horizontal) and poloidal Boozer angle (θ), for various toroidal Boozer angles (ζ). The maximum displacement is at the edge of the section with largest minor radius ($\zeta = \pi/5$). We could expect the dominant poloidal mode m to be approximately n/ι, which starts from about 7 for $n = 6$.

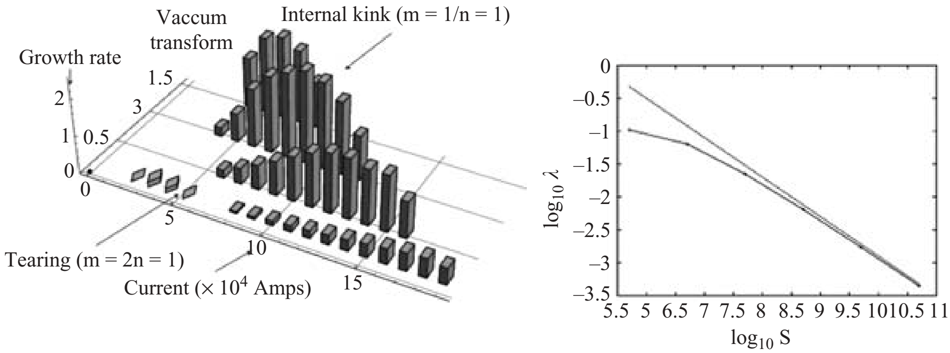

Figure 2. Kink mode ($m = 1$, $n = 1$ dominant; bars) and resistive tearing modes ($m = 2$, $n = 1$ dominant; diamonds), showing relative growth rates for varying vacuum rotational transform ι_ϵ (measured at the plasma boundary) and plasma current generated transform for a 17-period Heliotron and the growth rate of an $m = 2/n = 1$ tearing mode as a function of S. The straight line shows the predicted asymptotic dependence $\lambda \propto S^{-3/5}$.

poloidal mode numbers chosen so that only the components with $n = 1$ are field-line resonant. (Note that a mode family consists of groups of n values differing by integral multiples of the number of field periods [8]). Figure 2 shows the tearing and internal kink modes resolved by the code.

4. Conclusion

The code SPECTOR3D has been developed to calculate the spectrum and, in particular, the most unstable mode for resistive MHD modes in stellarators. The two applications described in the paper are the first steps in a numerical survey of the nature of global resistive MHD stability in stellarators.

In the first application, a Heliotron test case, current-driven tearing modes are found only in configurations with very small vacuum transforms. It is possible that stellarators in general will not be found to be strongly unstable to global tearing modes. The second investigation, of the toroidally modulated Tokamak, demonstrated that toroidal localization of global resistive MHD modes can occur. Toroidal localization is expected from local stability analysis, but it is still interesting to have resolved this effect in a global code.

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