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Generalization of the Langmuir–Blodgett laws for a nonzero potential gradient

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The Langmuir–Blodgett laws for cylindrical and spherical diodes and the Child–Langmuir law for planar diodes repose on the assumption that the electric field at the emission surface is zero. In the case of ion beam extraction from a plasma, the Langmuir–Blodgett relations are the typical tools of study, however, their use under the above assumption can lead to significant error in the beam distribution functions. This is because the potential gradient at the sheath/beam interface is nonzero and attains, in most practical ion beam extractors, some hundreds of kilovolts per meter. In this paper generalizations to the standard analysis of the spherical and cylindrical diodes to incorporate this difference in boundary condition are presented and the results are compared to the familiar Langmuir–Blodgett relation. © 2005 American Institute of Physics. [DOI: 10.1063/1.1850480]

I. INTRODUCTION

The extraction of ion beams from plasmas has been employed for several decades in a varied gamut of fields from surface treatment of semiconductors^{1–4} to space propulsion.^{5,6} Typical extraction systems employ an assembly of three biased electrodes to extract either negative or positive particles from a plasma. Commonly, extraction in such systems is treated using the familiar Langmuir–Blodgett relations for cylindrical⁷ and spherical⁸ diodes by assuming that the plasma/beam interface, or meniscus, and the 0 V equipotential surface where it is in contact with the beam, are concentric spheres in the case of circular apertures and concentric cylinders in the case of slit beams (cf. Fig. 1). Linear design of ion extraction systems from plasmas such as those proposed by Coupland⁹ and Green¹⁰ in the 1970s and later extended by Coste¹¹ in the 1990s repose on these basic equations. Indeed, even the analytical approach of Pierce¹² in the 1940s and subsequent developments by Radley,¹³ Daykin,¹⁴ and others in the 1950s are based on the Child–Langmuir and Langmuir–Blodgett assumptions of space-charge limited flow between concentric and planar surfaces.

An alternative approach that was developed around the same time as the work of Coupland and Green for the design of high perveance electrodes models the plasma/sheath/beam system as a whole and solves the nonlinear Poisson–Vlasov equations with numerical techniques.^{15,16} The method has the advantage that key figures of merit such as beam brightness can be expressed directly from the model but comes at the expense of high computational complexity.

The most recent work in the literature on the extraction of ions from plasmas still seems to be based on the Pierce method and therefore relies on the Child–Langmuir and

Langmuir–Blodgett laws.^{17,18} Two important assumptions underpinning these relations are that both the electric field and the initial velocity of the particles at the emission surface are zero. Although these may hold for thermionic emission from a hot filament (as was Child, Langmuir, and Blodgett’s initial intention), the assumption that the electric field at the emission surface is zero, central to the Langmuir–Blodgett derivations, is incorrect in the case of particle extraction from a plasma because the field across the meniscus is commonly several hundred kilovolts per meter. In this work, we consider the meniscus to be at the end of the plasma sheath where the electron and ion fluxes are equal, i.e., where the potential is equal to the floating potential. This separates the mathematical problem into two distinct parts. The potential distribution in the sheath, which is determined by the plasma (Bohm criterion), and the potential distribution in the beam. These two distributions are then “stitched” at the meniscus boundary to form a smooth and continuous distribution. This is where the approach proposed in this paper differs from the standard Langmuir–Blodgett methodology in that the plasma is not simply considered as a charge neutral body from which particles are extracted. Rather, the plasma *imposes* a flux and an electric field at its boundary with the beam. Moreover, it has been shown experimentally¹⁹ and numerically²⁰ that the electric field is continuous everywhere inside the plasma and sheath, thus excluding the possibility of a surface charge being built up at any arbitrary boundary defined inside the plasma or sheath. Extraction from a plasma can therefore not be transformed into the extraction of particles from a virtual cathode or anode. Moreover, as it will be shown in this paper, the electric field at the meniscus must be taken into account as it has a strong effect on the beam distribution

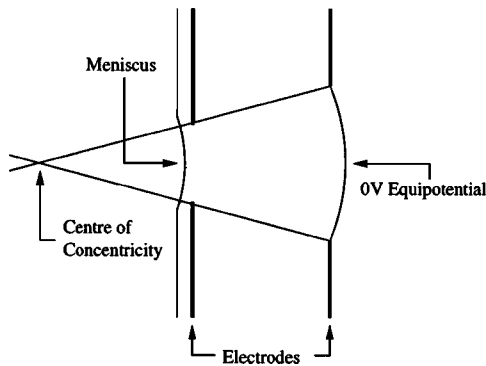


FIG. 1. Concentric meniscus and 0 V equipotential.

functions. However, as with the original Child–Langmuir and Langmuir–Blodgett laws the assumption of zero ion velocity upon entry to the extraction gap is maintained because the strong accelerating fields used in ion extractors increase the mean velocity of the particles to several orders of magnitude above their exit velocity from the plasma sheath.

In light of these observations, the authors of this paper were not able to find any generalizations of the planar, cylindrical, or spherical cases applicable to plasmas in the literature, and, as many papers on the extraction of ions from plasmas employ the Langmuir–Blodgett relations either directly^{10,9} or indirectly by using elements of the Pierce electrode design method,^{18,21,17} it is of some interest to develop an expression for the flow of particles in diodes based on the boundary conditions implied by extraction from a plasma.

It is noted, however, that theory has already been developed within the field emitter community, with particular contributions from Barbour,²² Anderson,²³ and van Veen,²⁴ that consider current flow from planar, cylindrical, and spherical emission tips in the presence of strong electric fields at the emission surface. Though this body of work describes a situation akin to that which will be studied in this paper there are two major differences which make it difficult to use in conjunction with plasmas. First, the Fowler–Nordheim relation, which gives the current density as a function of the applied electric field at the emission surface, cannot be integrated into a theory for the extraction of particles from plasmas. Second, unlike the emission surface of a field emitter which does not change geometry as a function of extraction voltage, the shape of the plasma meniscus is sensitive to the plasma density, extraction potential, and the electric field structure in the extractor.²⁵ This implies that for given plasma conditions the strict geometry assumptions of the standard diode models can only be guaranteed by specific electrode structures. An analytical approach for the design of such electrodes was developed by Radley¹³ and relied on the series formulation of the Langmuir–Blodgett laws but assumed a zero electric field at the emission surface. Though the expressions presented by Barbour, Anderson, and van Veen treat the nonzero electric field case they are not in a formulation suitable for use in Radley’s methodology. This is a further motivation to develop a series solution for the extraction of particles from plasmas generalized to take into account the nonzero electric field at the meniscus.

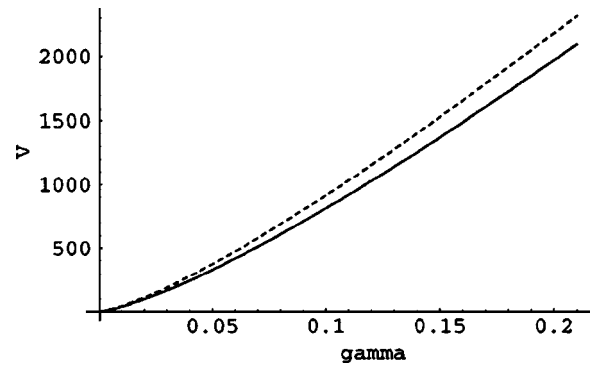


FIG. 2. Generalized distribution (dashed line) vs Langmuir–Blodgett (solid line). Density $n=10^{14} \text{ cm}^{-3}$ and normalized gradient $dV/d\gamma = 130.01 \text{ V/unit}$.

A. Statement of the problem

The Bohm sheath criterion stipulates the minimum ion velocity for entry into the sheath to maintain a stable sheath at a plasma boundary. In conjunction with some distribution relation for electrons, this defines a potential structure within the sheath. In particular, the electric field at the plasma boundary is nonzero and is typically several hundred kilovolts per meter. For continuity of the electric field across the plasma/beam interface the potential gradient must be equal on both sides of this surface. This is not assured by the original assumptions of Langmuir and Blodgett who were modeling particle flow from thermionic cathodes. In that case the source of particles was assumed to be undepletable and to have no intrinsic electric field so that for equilibrium the boundary condition in the extractor was for the electric field to be zero at the entry to the acceleration gap. In the case of plasmas, the flux of ions is fixed and an intrinsic electric field does exist in the sheath which separates the beam from the bulk plasma. This strongly implies that particle beam extraction from plasmas is not accurately described by the Langmuir–Blodgett relation as it is usually stated. Figure 2 shows that taking the meniscus field into account for a plasma density of 10^{14} cm^{-3} alters the potential distribution by over 10% at an extraction potential of 2000 V and that this error increases monotonically with extraction potential. As will be shown in Sec. V, this discrepancy between the basic Langmuir–Blodgett relation and the generalized form is also affected by the plasma density.

Furthermore, in space-charge limited systems it is commonly thought that a virtual anode/cathode is formed in response to an increase in extraction voltage, however, in the case of ion extraction from a plasma there is no physical reason for one to appear at the meniscus and it has never been observed in particle-in-cell or ray-tracing simulations undertaken by Sutherland *et al.*²⁵ Instead, the meniscus adjusts its position and shape so as to enable the electric field in the extractor to equal that in the sheath. It is this variation in the shape of the meniscus that results in the various beam shapes that are observed.²⁶

In this paper, we shall concentrate on spherical and cylindrical symmetry but the reasoning in both cases can be extended to planar diodes, where the extraction field is so as

to be equal to the sheath voltage for a flat sheath and the electrodes are designed to accommodate a parallel beam. In addition, focus will be on positive ion extraction.

II. SPHERICAL SYMMETRY

A. The Langmuir–Blodgett relation

In the case of ion beam extraction from a circular aperture, the charge and potential distributions in the beam are assumed to be analogous to those in a complete spherical diode. Following Langmuir and Blodgett,⁸ we state Poisson's equation between two concentric spheres:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dV}{dr} \right) = - \frac{\rho}{\epsilon_0}, \quad (1)$$

where V is the potential at a point distant r from the common center and ρ is the ion charge density. The current flowing in the diode can be written in terms of the particle velocity v :

$$I = 4\pi r^2 \rho v, \quad (2)$$

where the velocity can be written in terms of the voltage V by using the kinetic energy relation

$$\frac{1}{2} M v^2 = -eV. \quad (3)$$

Combining Eqs. (1)–(3) yields

$$r^2 \frac{d^2 V}{dr^2} + 2r \frac{dV}{dr} = A \tilde{V}^{-1/2}, \quad (4)$$

where $\tilde{V} = -V$ and

$$A = \frac{I}{4\pi\epsilon_0} \sqrt{\frac{M}{2e}}. \quad (5)$$

Equation (4) can probably not be integrated directly but a solution can be found in terms of a series. The form of the solution as a function of the ratio $R = r/r_s$ presented by Langmuir and Blodgett is

$$V(R) = \left(\frac{9}{4}A\right)^{2/3} f^{4/3}(R), \quad (6)$$

where f is the analytic function to be found and r_s is the radius of curvature of the emitting surface. We use the same form here. The $(\frac{9}{4}A)^{2/3}$ term serves to normalize for the constant term A related to the current and hence the plasma density and meniscus curvature, and the $f^{4/3}(R)$ term to remove the square root from Eq. (4) and hence to simplify subsequent derivations. One further transformation is performed by setting

$$\gamma = \ln(R) \quad (7)$$

so that a solution to Eq. (6) can be expressed in terms of a MacLauren series

$$f = \sum_{n=0}^{\infty} a_n \gamma^n. \quad (8)$$

By substituting Eq. (6) into Eq. (4) and imposing Eq. (7) we find

TABLE I. Expansion terms for the coefficients of the MacLauren series [see Eq. (11)]. Spherical case. Note: the α_n are very close to the original Langmuir–Blodgett series coefficients.

n	α_n	β_n	γ_n
1	1.003 5	4.049	-10.92
2	-0.308 4	-8.008	25.11
3	0.083 38	7.791	-25.85
4	-0.018 25	-3.96	13.47
5	0.002 870	1.000 4	-3.448
6	-0.000 222 7	-0.099 04	0.3441

$$3ff' + f'^2 + 3ff'' - 1 = 0, \quad (9)$$

where $f' = df/d\gamma$ and $f'' = d^2f/d\gamma^2$.

B. Correcting for nonzero gradient

Taking the first derivative of Eq. (6), we find that

$$V'(\gamma) = \lambda \sqrt{p}, \quad (10)$$

where $\lambda = \frac{4}{3}(\frac{9}{4}A)^{2/3}$ and $p = f'(\gamma)^2 f(\gamma)^{2/3}$, for $\gamma=0$. In the classic Langmuir–Blodgett derivation $f(\gamma)=0$ in Eq. (9) leads to $f'(\gamma)=0$ for $\gamma=1$ (assuming potential increases as a function of position in the extractor), so that $p=0$ and hence $V'(\gamma=0)=0$. In other words, the Langmuir–Blodgett derivation requires a zero initial gradient in potential. Numerically, however, it is possible to have f approach zero without requiring $V'(0)$ to annul by setting $f'(0)$ such that Eq. (10) holds for the desired value of $V'(0)$. For a given value of $V'(0)$ and A , there is a limit to how small $f(0)$ can be set, but in most practical cases it is several orders of magnitude less than unity.

We find that the solution depends only on p , rather than the individual values of $f(0)$ and $f'(0)$, for the range of p which is of interest. The series coefficients a_n are expressed as a quadratic

$$a_n = \alpha_n + \beta_n p + \gamma_n p^2, \quad (11)$$

where α_n , β_n , and γ_n are the expansion coefficients found by a least squares method. These terms are presented in Table I.

III. CYLINDRICAL SYMMETRY

A similar derivation can be made for the case of cylindrical symmetry. Poisson's equation becomes

$$r \frac{d^2 V}{dr^2} + \frac{dV}{dr} = B \tilde{V}^{-1/2}, \quad (12)$$

where

$$B = \frac{I}{2\pi l \epsilon_0} \sqrt{\frac{M}{2e}} \quad (13)$$

and l is the length of the extraction slit. The solution takes the form

TABLE II. Expansion terms for the coefficients of the MacLauren series [see Eq. (18)]. Cylindrical case. Note: the δ_n are very close to the original Langmuir–Blodgett series coefficients.

n	δ_n	ε_n	ζ_n
1	1.003 4	3.989	-10.69
2	-0.408 6	-8.223	25.43
3	0.100 5	7.974	-26.22
4	-0.018 66	-4.038	13.65
5	0.002 658	1.0184	-3.491
6	-0.000 201 1	-0.1007	0.3482

$$V = \left(\frac{9}{2}Br\right)^{2/3} g^{4/3}, \quad (14)$$

where g is the analytic function to be found over the desired range of r . Again substituting Eq. (14) into Eq. (12) and setting Eq. (7), we find

$$3gg'' + g'^2 + 4gg' + g^2 - 1 = 0, \quad (15)$$

which is the same result as was previously presented by Langmuir and Blodgett.⁷ We take the first derivative of the potential and write it in terms of the parameter p ,

$$V'(R) = \mu(g^{4/3} + 2\sqrt{p}), \quad (16)$$

where $\mu = \frac{2}{3}\left(\frac{9}{2}Be\gamma\right)^{2/3}$ and $R=1$, and note that in the limit the term in g disappears, so that $V'(R) \approx 2\mu\sqrt{p}$. This has the same form as the spherical case, so that by writing

$$g = \sum_{n=0}^{\infty} b_n \gamma^n \quad (17)$$

and plotting the series coefficients in terms of the parameter p , we obtain

$$b_n = \delta_n + \varepsilon_n p + \zeta_n p^2, \quad (18)$$

where δ_n , ε_n , and ζ_n are coefficients found in the same fashion as for the spherical case. Results are presented in Table II.

IV. PRACTICAL VALUES FOR V

A. The plasma sheath

There are several ways to model the sheath. Lieberman²⁰ presents a method in terms of the Bohm sheath criterion and the Boltzmann equation in one dimension, which is easily extended to spherical and cylindrical symmetry. This model is intuitively pleasing since it incorporates both the nonzero ion velocity v_B at the entry to the sheath, required by the Bohm criterion, and also the presence of electrons. Another popular method is the Child sheath, which is extended to spherical and cylindrical symmetry by the use of the standard Langmuir–Blodgett corrections. In this case, the presheath/sheath boundary and the meniscus are considered to be concentric spheres for extraction from a circular aperture and concentric cylinders for extraction from a slit. Though solving the Boltzmann sheath is possible in terms of a series and has been done by the authors, it requires a somewhat more drawn out analysis than is necessary to demonstrate the the-

sis of this work, and so for simplicity, the Child sheath method will be employed with Langmuir–Blodgett corrections. As such, the presheath can be ignored and we assume that the velocity of ions and the potential at the bulk plasma/sheath edge are zero. However, it should be noted that the Child sheath yields smaller gradients than the Boltzmann sheath.

B. Sheath potential at the meniscus

To solve Eq. (4), we first note that in the case of extraction from a plasma I must equal the ambipolar flux for ions:

$$I^+ = 0.6en_s v_B \mathcal{A}, \quad (19)$$

where n_s is the plasma density at the sheath edge and $\mathcal{A} = 4\pi r_s^2$ is the area over which current is extracted, r_s being the radius of curvature of the sheath. It follows from Eqs. (5) and (19), that the solution to Eq. (4) is strongly related to both n_s and r_s .

In addition, we require three boundary conditions. At the entry to the sheath we set $V(1)=0$ and $dV(1)/dR=0$. To determine the sheath potential at the meniscus we equate ion flux, assumed constant throughout the sheath,

$$\Gamma_i = \frac{n_s v_B}{R^2}, \quad (20)$$

to the electron flux at the meniscus,

$$\Gamma_e = \frac{n_s(\bar{v}_e)e^{eV_m/kT_e}}{4R^2}, \quad (21)$$

where $\bar{v}_e = (8eT_e/\pi m)^{1/2}$ is the mean electron velocity and V_m is the potential of the meniscus with respect to the plasma-sheath edge. Thus upon substitution of the Bohm velocity,

$$n_s \left(\frac{eT_e}{M}\right)^{1/2} = \frac{1}{4}n_s \left(\frac{8eT_e}{\pi m}\right)^{1/2} e^{eV_m/kT_e} \quad (22)$$

which becomes

$$V_m = -T_e \ln\left(\frac{M}{2\pi m}\right)^{1/2}. \quad (23)$$

This can be expressed in a more convenient form by substituting the mass of the extracted ion species. Krypton is a typical gas used in ion beam extraction from plasmas and has a mass of $M=84$ a.u. Therefore in this case, Eq. (23) can be rewritten as $V_m = -5.05T_e$, which, assuming $T_e=3$ eV, is ≈ -15 V.

This now leads to suitable boundary conditions for Eq. (4),

$$V = 0 \text{ V}, \quad r = r_s,$$

$$V = -15 \text{ V}, \quad r = r_m,$$

$$dV/dR = 0, \quad r = r_s, \quad (24)$$

where r_m is the radius of curvature of the meniscus.

C. Potential gradient at the meniscus

Since both the voltage and its first derivative are zero at r_s , f is independent of the parameter p and we have the familiar Langmuir–Blodgett relation

$$\alpha(\gamma) = \gamma - 0.3\gamma^2 + 0.075\gamma^3 - 0.0143182\gamma^4 + 0.0021609\gamma^5 - 0.00026791\gamma^6. \quad (25)$$

This series expansion in conjunction with Eq. (6) and the boundary conditions now determine both the sheath width and the potential gradient at the meniscus edge. The sheath width is taken as the value of $r_m - r_s$ for which Eq. (6) is equal to Eq. (23). The potential gradient at the meniscus edge is then equal to the first derivative of Eq. (6) taken at this value of r_m .

Assuming a constant electron temperature and gas type, Eqs. (6), (7), and (25) show that the potential gradient at the meniscus edge is dependent on the bulk plasma density and the radius of curvature of the meniscus.

V. DISCUSSION

The extent to which the adjusted solution presented here deviates from that of Langmuir–Blodgett depends on the magnitude of the potential gradient at the sheath edge. From Eq. (10) we find that

$$p = \alpha(\gamma_m)^{1/3} \frac{d\alpha}{d\gamma}(\gamma_m), \quad (26)$$

where γ_m is the value of γ at the meniscus edge. We note that $d\alpha/d\gamma$ does not vary much with γ so that the dominant term is $\alpha^{1/3}$. Consequently, p decreases with increasing density as the sheath becomes ever thinner, with the result that the first and second order terms in the quadratic expansions of the MacLauren series coefficients disappear in the limit where p tends to zero. Though it is noted that the constant terms (cf. Tables I and II) are not exactly equal to the Langmuir–Blodgett coefficients, due to the fact that a_n and b_n are fitted for a given range of p , they are sufficiently close that convergence between the generalized and the Langmuir–Blodgett series can easily be recognized. Nevertheless, the densities required for p to approach zero are unrealistically

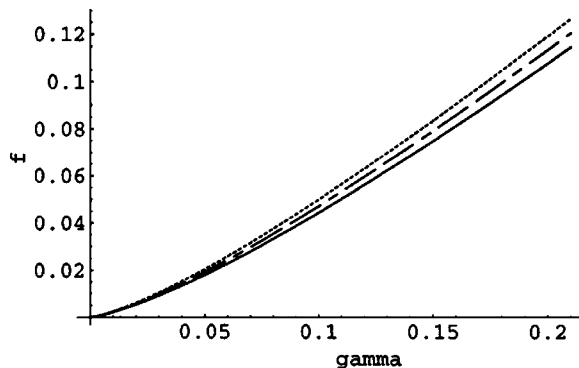


FIG. 3. The function f vs γ for various values of the parameter p compared to the Langmuir–Blodgett distribution. Solid line: Langmuir–Blodgett distribution. Dashed line: $n = 10^{14} \text{ cm}^{-3}$ ($p = 0.013\ 293$). Dotted line: $n = 10^{13} \text{ cm}^{-3}$ ($p = 0.028\ 525$).

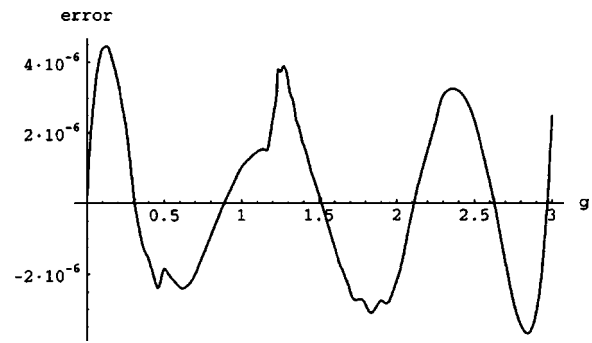


FIG. 4. Error of the first six terms of the generalized series solution relative to the exact solution as a function of γ (here $g = \gamma$). Spherical case.

high for practical ion extractors. A density of 10^{14} cm^{-3} is on the limit of practicality and only engenders a p of 1.5376×10^{-3} . Therefore in most practical circumstances, the generalized series coefficients are disparate to those in the Langmuir–Blodgett case, and increasingly so, as the plasma density is lowered (cf. Fig. 3).

The range of γ over which the generalized series representation is accurate depends on the number of terms in the series. However, as can be seen in Fig. 4, the six term generalized series solution has an error of less than 10^{-5} for $\gamma < 3$. By way of comparison a four-term and a six-term Langmuir–Blodgett series are plotted against the exact solution, for an initial gradient of zero, in Fig. 5. There is a clear deviation from the exact solution at $\gamma = 1.5$ and $\gamma = 2.5$, respectively. As a reference to the scale magnitude of γ , we consider a 5° diverging beam extracted through a 1 mm circular aperture. In this case the meniscus curvature $r_m = 5.737 \text{ mm}$, so that a γ of 3 represents a beam length of 115 mm. The interelectrode spacing in extraction gaps is typically only a few millimeters so that in most practical cases γ is less than unity.

Using a quadratic representation for the series coefficients a_n and b_n , the accuracy of the series solutions can be maintained to within 10^{-5} for values of $p < 0.1$. This means that almost exact distributions can be calculated for densities down to $2 \times 10^{11} \text{ cm}^{-3}$ and to within 10^{-3} for densities down to 10^9 cm^{-3} .

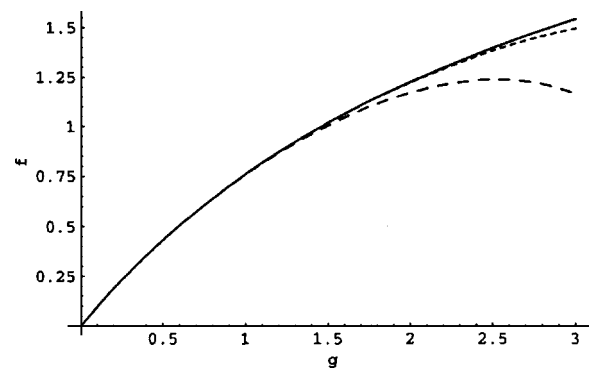


FIG. 5. Range of the four-term (long dash) and six-term (short dash) series solutions as a function of γ (here $g = \gamma$). The solid line is the exact solution.

VI. AN EXAMPLE

As an example, we further consider the case of a 5° diverging beam as extracted through a 1 mm circular aperture. The plasma source is assumed to be a krypton plasma of density 10^{14} cm^{-3} at the sheath edge. As previously mentioned, the problem can be considered analogous to that of a complete spherical diode of curvature $r_m = 5.737 \text{ mm}$. The current density flowing across the meniscus yields $A = 1.0932 \times 10^7$ according to Eq. (5) and assuming that the collection area is given by $\mathcal{A} = \pi r_m^2$. The ratio of the sheath to the presheath is given by first solving Eq. (6) for the boundary conditions set in Eq. (24)

$$\alpha(\gamma_m) = \frac{15^{3/4}}{\sqrt[9]{4A}} = 1.5368 \times 10^{-3}, \quad (27)$$

and then solving Eq. (25) to find $\gamma_m = 1.5376 \times 10^{-3}$. From Eq. (7) we get $R = 1.00154$ so that the sheath width is given by

$$r_m - \frac{r_m}{1.00154} = 8.8 \text{ } \mu\text{m}. \quad (28)$$

The gradient at the sheath edge in terms of γ_m is

$$\frac{dV}{d\gamma}(\gamma_m = 1.537610^{-3}) = 0.26396A^{2/3} = 13001 \text{ V/unit}, \quad (29)$$

which is equivalent to $2.27 \times 10^6 \text{ V m}^{-1}$.

According to Eq. (10), $p = 1.3293 \times 10^{-2}$ which we note is well below the established limit of $p_{\text{limit}} = 0.1$. In combining Table I and Eq. (11) we find the series expansion of f ,

$$1.05539\gamma - 0.410413\gamma^2 + 0.182378\gamma^3 - 0.06851\gamma^4 + 0.015559\gamma^5 - 0.00147843\gamma^6, \quad (30)$$

and hence an expression for the voltage in terms of γ through relation (6). The result and the basic Langmuir–Blodgett distribution are shown in Fig. 2.

VII. CONCLUSION

The alteration of the potential structure inside the beam when taking the meniscus field into account also has an effect on the current-voltage relationship for the diode. Though for a given extractor geometry (or interelectrode gap spacing for a diode) the current approaches the space-charge limit and is proportional to $V^{3/2}$ according to

$$I = \frac{16\pi\epsilon_0}{9} \sqrt{\frac{2e}{M}} \frac{V^{3/2}}{f^2} \quad (31)$$

for spherical symmetry and

$$I = \frac{4\pi l\epsilon_0}{9} \sqrt{\frac{2e}{M}} \frac{V^{3/2}}{\tilde{r}g^2} \quad (32)$$

for cylindrical symmetry (where \tilde{r} is the extraction gap length and V is the extraction potential), this only holds for a unique combination of density, extraction potential, and extraction gap spacing. The flow is not limited in the sense that the charge density in the extraction gap impedes the flow of

particles from the plasma (as this is fixed by the plasma's natural ambipolar flux), but the potential and charge distributions in the extraction gap do approach those that would be obtained if the emission surface had a fixed geometry and an undepletable source of particles. However, a change in any of these three quantities causes a change in the shape of the plasma meniscus²⁵ which, in order to maintain the strict symmetry constraints of the theory, would require a recalculation of the extractor geometry (or interelectrode spacing for a diode). In this sense Eqs. (31) and (32) do not follow a simple $V^{3/2}$ power law as the current is a function of both the extraction potential V and the geometric functions f and g (for spherical and cylindrical symmetries, respectively) which in turn are functions of the electric field at the plasma meniscus and hence the plasma density. This is in stark contrast to the way that the Langmuir–Blodgett laws are typically applied to plasmas where it is assumed that an increase in the extraction potential simply results in an increased current flow in the extraction gap (without changing any other parameters). Ultimately, it is the plasma which dictates the amount of current that can flow in the interelectrode gap and the extractor which must be built around the extracted beam.

Finally, experimental¹⁹ and numerical²⁰ studies of plasmas have shown that the potential distribution in the plasma bulk, presheath, and sheath is smooth and continuous and thus precludes the possibility of charge built up on any arbitrary boundary within the system. This implies that the electric field is continuous and smooth from the plasma bulk to the ground reference of the beam and that no virtual cathode or anode can arise. In particular, there can be no charge buildup at the meniscus (as defined in this paper) because by definition the electron and ion fluxes are equal there. The conservation of flux implies that there are no choke points in the system.

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