# Three-Core Weakly-Guiding Mode-Selective Fibre Couplers

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## Abstract Summary

The coupling behaviour of two-core mode-selective couplers (MSC) depends on the spatial-orientation of the asymmetric higher-order modes. This restricts their use for mode de-multiplexing in few-mode fibre networks. The use of three-core MSC's is presented as a solution.

Keywords- few-mode fibres; mode-selective couplers; modesplitting; spatial-mode-division-multiplexing.

## I. INTRODUCTION

The ever-increasing demand for high-bandwidth optical fibre communications is destined to challenge the limits of singlemode fibre. This has led to a renewed interest in the use of fewmode fibre to exploit the extra degrees of freedom provided by several propagating modes. Few-mode fibre avoids the severe modal crosstalk experienced in highly-multimode fibre, thus allowing for each mode to be considered an independent data channel. The independent excitation and detection of the modes however presents major challenges. Currently there are no simple techniques available for multiplexing and demultiplexing more than two orthogonal spatial modes. The multiplexing/de-multiplexing of two (and possibly more) modes could however in principle be achieved using simple waveguide structures such as mode-selective couplers (MSC) [1–3], asymmetric Y-junctions [4–6] or multimode interference devices (MMI) [7]. The use of asymmetric Y-junctions has recently been delineated [8]. Asymmetric Y-junctions allow for mode-division-multiplexing of elliptical and highlybirefringent fibre. As for multimode interference devices, they are typically limited to merely separating (or combining) the asymmetric from the symmetric planar modes [7].

This paper is concerned with the use of mode-selective couplers. More specifically, a novel multiplexing/demultiplexing technique is presented that uses simple three-core mode-selective fibre couplers (MSC's). Mode-selective fibre couplers involve a higher-order mode in one fibre coupling to the fundamental modes of one or more other fibres, and vice versa. This situation differs from a traditional fibre coupler in which the mode-order remains unchanged. Mode-selective couplers rely on the matching of the modal propagation constants (e.g. using non-identical fibres). However if the higher-order mode is asymmetric (i.e. non-axisymmetric or having non-zero azimuthal number), the coupling behaviour also depends on the spatial-orientation of its electric field. This poses somewhat of an issue if MSC's are to be used in fibre mode de-multiplexers. This is because the spatial-orientations of the asymmetric modes are inherently unstable in any sizeable length of circularly-symmetric fibre. In the case of a two-core MSC the power transfer can range anywhere from 0 to 100% depending on the spatial-orientation [2–3].

however demonstrates that This paper certain configurations of three-core MSC's can have 100% power transfer regardless of the spatial-orientation of the higher-order mode. This implies that three-core fibre MSC's can be used for mode-division-multiplexing of circularly-symmetric fibre. This is an obvious advantage over planar devices (i.e. asymmetric Y-junctions or multimode interference devices) which can only de-multiplex modes of fixed spatial-orientation (e.g. the modes of highly-birefringent or elliptical-core fibre). The main drawback of mode-selective couplers are their inherent wavelength dependence.

#### II. COUPLED-MODE THEORY

This section provides an analysis of three-core mode-selective fibre couplers, highlighting their ability to de-multiplex modes of arbitrary spatial-orientation. The general three-core modeselective fibre coupler used in the analysis is shown in Fig. 1. The fibre cores are assumed to be both circularly-symmetric and weakly-guiding.



Figure 1. Three-core mode-selective fibre coupler (with LP<sub>11</sub> shown in fibre 1)

N. Riesen is the recipient of both ANU, Canberra and CSIRO, Lindfield research scholarships.

A higher-order linearly-polarized mode (LP<sub>*lm*</sub>) in core 1 is assumed to have the same propagation constant  $\beta$ , as the fundamental modes of two identical outer cores 2 and 3. Furthermore, the electric field of the higher-order mode in core 1 is assumed to have a cosine azimuthal dependence. This means that for the LP<sub>11</sub> mode, for instance, the line defined by the angle  $\alpha$  is perpendicular to the zero line of the antisymmetric mode's field. So in other words if  $\alpha = 0$  the lobes of the LP<sub>11</sub> mode, for instance, are bisected by a horizontal line.

Referring to Fig. 1, the angle between cores 2 and 3 relative to core 1 is given by  $\phi$ . The coupling coefficients between cores 1 and 2, and cores 1 and 3 are given by C and  $\overline{C}$ , respectively. The coupling coefficient between cores 2 and 3 is given by D. It is assumed that  $D \ll C \cup \overline{C}$  and to a first order approximation D = 0. The assumption is accurate for sufficiently large values of  $\phi$ . Note that even if  $D \neq 0$ , the coupled-mode equations could still be solved analytically. The assumption of D = 0 is however necessary to ensure a truly  $\alpha$ -independent MSC (i.e. 100 % power decoupled irrespective of the spatial-orientation,  $\alpha$ ). In the case of finite values of D, coupling between the outer cores will occur at length scales of  $\pi/D$ . This length scale should be much larger than the coupler length scale  $\pi/C_R$ .

In this paper it is also assumed that the mode-selective coupler is weakly-coupled (i.e. coupling length per unit length is small) such that,  $|C| \ll |\beta|$  and  $|\overline{C}| \ll |\beta|$ . This means that the distance *d*, between core 1 and either of the outer cores is assumed to be sufficiently large such that there is a small overlap of the modal fields. In this case the fields of the fibre modes can be considered in isolation [2], [9]. The fields of the modes are also assumed to be orthonormal.

Now, the *z*-dependences of each mode of the three fibre cores can be expressed as [9],

$$a_i(z) = b_i(z)e^{j\beta z}$$
  $i = 1, 2, 3.$  (1)

where the amplitudes  $b_1$ ,  $b_2$  and  $b_3$  satisfy the following coupled-mode equations,

$$\frac{db_1}{dz} = jCb_2 + j\overline{C}b_3; \quad \frac{db_2}{dz} = jCb_1; \quad \frac{db_3}{dz} = j\overline{C}b_1. \quad (2)$$

Note that only the forward-mode equations are considered, which are assumed to be decoupled from the backward-mode equations. This is a valid assumption for weakly-coupled MSC's that are also phase-matched (i.e.  $\beta_1 = \beta_2 = \beta_3 = \beta$ ). Assuming core 1 initially contains all the power (i.e.  $b_1(0) = 1$  and  $b_2(0) = b_3(0) = 0$ ), the modal powers along the three cores are solved analytically,

$$P_{1}(z) = |b_{1}(z)|^{2} = \cos^{2}(\Omega z)$$
(3)

$$P_{2}(z) = |b_{2}(z)|^{2} = \frac{C^{2}}{\Omega^{2}} \sin^{2}(\Omega z)$$
(4)

$$P_{3}(z) = |b_{3}(z)|^{2} = \frac{\overline{C}^{2}}{\Omega^{2}} \sin^{2}(\Omega z)$$
(5)

where  $P_1(z)+P_2(z)+P_3(z) = 1$ , and  $\Omega^2 = C^2+\overline{C}^2$ . If  $C_R$  denotes the radial dependence of the coupling coefficients then [2],

$$C = -C_R \cos(l(\alpha - \phi)) \qquad \overline{C} = -C_R \cos(l\alpha) \tag{6}$$

where l is the azimuthal number of the higher-order mode. Note that since the coupler is assumed to be phase-matched, the coupling behaviour is also reciprocal. From (6),

$$\Omega = C_R \sqrt{\cos^2(l(\alpha - \phi)) + \cos^2(l\alpha)}.$$
 (7)

This implies that when the outer cores are at angle  $\phi = \pi/2l + n\pi/l$ , with n = 0, 1, 2, 3, ..., we have that  $\Omega = C_R$ . Therefore the power in each core is given by,

$$P_1(z) = \cos^2(C_R z) \tag{8}$$

$$P_2(z) = \sin^2(l\alpha)\sin^2(C_R z) \tag{9}$$

$$P_3(z) = \cos^2(l\alpha)\sin^2(C_R z).$$
(10)

The power decoupled from core 1 to the outer cores is therefore,

$$P_T = P_2(z) + P_3(z) = \sin^2(C_R z)$$
(11)

where the coupling length is given by  $z_c = \pi/2C_R$ . Therefore all the power of a higher-order mode can be decoupled regardless of the azimuthal orientation of its field  $\alpha$ , when using a threecore MSC with outer cores at angle  $\phi = \pi (n+1/2)/l$ . In the special case where l = 0,  $\Omega = \sqrt{2C_R}$  but  $C_R$  becomes  $C_R/\sqrt{2}$  so  $P_{2,3}(z) = \sin^2(C_R z)/2$ . In the case of l = 0 only a two-core MSC is of course needed for total power transfer, although a threecore structure could be used to reduce the coupling length.

The radial dependence of the coupling coefficient (i.e. for coupling between core 1 and core 2, 3) depends on the fibre and source parameters and was previously derived [2],

$$C_{R} = (-1)^{l} \frac{2\sqrt{2}k\rho_{2}\Delta_{2}u_{1}u_{2}n_{co,2}^{3/2}}{\rho_{1}v_{1}v_{2}^{3}n_{co,1}^{1/2}} \frac{K_{l}(w_{1}d/\rho_{1})}{K_{1}(w_{1})\sqrt{K_{l-1}(w_{2})K_{l+1}(w_{2})}}$$
(12)

where, k is the wave-number,  $\rho_i$  is the core radius,  $\Delta_i$  is the relative index difference,  $u_i$  and  $w_i$  are the core and cladding modal indices,  $n_{co,i}$  is the core index, K is the modified Bessel function of the second kind, d is the spacing between the axes of cores 1 and 2 (or 1 and 3), and  $v_i$  is the normalized frequency. As before, subscripts denote the higher-order mode fibre (i = 1) or one of the two outer cores (i = 2). The coupling coefficient (12) assumes weak-coupling and weak-guidance. Note that in the case where l = 0, C<sub>R</sub> becomes C<sub>R</sub>/ $\sqrt{2}$ .

#### III. NUMERICAL SIMULATIONS

The following numerical Beam Propagation Method (BPM) simulations confirm the predicted  $\alpha$ -independent behaviour of the three-core fibre MSC's. The parameters common to both simulations are:  $n_{co,l,2} = 1.4446$ ,  $\Delta_{l,2} = 6.85 \times 10^{-3}$  and  $\lambda = 2\pi/k = 1550$  nm. The simulations shown in Fig. 2, demonstrate the  $\alpha$ -independent decoupling of an LP<sub>11</sub> mode when using a three-core MSC with two outer cores that have an angular offset  $\phi = \pi/2$  with l = 1 and n = 0. Here,  $\rho_1 = 4.338 \mu$ m,  $\rho_2 = 2 \mu$ m and

the coupler length is  $z_c \approx 3.0$  mm. Note also that the power in the two outer cores could be combined in practice in order to avoid a power penalty.



Figure 2. The decoupling of an LP<sub>11</sub> mode (i.e. see insets) from a two-mode core to two outer cores which have an angular offset  $\phi = \pi/2$ . The total decoupled power is independent of the mode's spatial-orientation  $\alpha$ , as demonstrated for (a)  $\alpha = 0$ , (b)  $\alpha = \pi/8$ , (c)  $\alpha = \pi/4$  and (d)  $\alpha = \pi/2$ .

The simulations of Fig. 3 also demonstrate the  $\alpha$ independent behaviour of three-core MSC's for the case of the LP<sub>21</sub> mode. In this example  $\phi = 3\pi/4$  with l = 2 and n = 1. The parameters for this simulation are  $\rho_1 = 6.48 \mu m$ ,  $\rho_2 = 1.992 \mu m$ and the coupler length is  $z_c \approx 4.1 mm$ .



Figure 3. The decoupling of an LP<sub>21</sub> mode (i.e. see insets) from a four-mode core to two outer cores which have an angular offset  $\phi = 3\pi/4$ . The total decoupled power is again independent of the mode's spatial-orientation  $\alpha$ , as demonstrated for (a)  $\alpha = 0$ , (b)  $\alpha = \pi/16$ , (c)  $\alpha = \pi/8$  and (d)  $\alpha = \pi/4$ .

## IV. MSC MULTIPLEXERS/DE-MULTIPLEXERS FOR FEW-MODE FIBRE NETWORKS

This paper has shown that three-core mode-selective fibre couplers allow for the de-multiplexing of any given asymmetric spatial-mode (i.e. l > 0) regardless of its spatial-orientation. As suggested, two-core mode-selective couplers are nonetheless sufficient for de-multiplexing the symmetric

spatial-modes (i.e. l = 0). Therefore a linear sequence of twoand three-core couplers constituting the *de-multiplexer*, could allow for the separation of all modes at the end of a few-mode fibre. A linear sequence of two-core mode-selective couplers would be sufficient for the *multiplexer*, since the spatialorientation of the modes excited at the start of the fibre is irrelevant.

# V. CONCLUSION

In summary this paper has presented and numerically verified, theory describing the behaviour of three-core mode-selective fibre couplers. The paper has shown that these simple devices could be used for de-multiplexing all the power of any given fibre mode, regardless of its spatial-orientation. A series of inline two- and three-core MSC's could therefore be used as the mode demultiplexer of a few-mode fibre network, whereas a series of inline two-core MSC's could be used as the multiplexer. Difficulties are expected to arise when multiplexing/de-multiplexing modes with closely-spaced propagation constants. This problem can be avoided by limiting the number of modes guided by the fibre, so as to avoid a 'continuum' of propagation constants. The wavelength dependence also limits the level of wavelength division multiplexing (WDM) possible when using mode-selective couplers.

Major practical challenges concerning MSC's include their fabrication as well as minimizing their dependences on external variables [1], [10]. The principles presented in this paper also apply to other waveguide structures, such as buried-channel waveguides and possibly also microstructured optical fibres.

## ACKNOWLEDGMENT

The authors would like to thank John W. Arkwright and Jong H. Chow for useful suggestions and discussion.

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