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of Maximal Elements on Constrained  
Reconfigurable Meshes**

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# Optimal Computation of the Contour of Maximal Elements on Constrained Reconfigurable Meshes

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## Abstract

The Reconfigurable Mesh (RM) attracted criticism for its key assumption that a message can be broadcast in constant time independent of bus length. To account for this limit Beresford-Smith *et al.* have recently proposed  $k$ -constrained RM where buses of length at most  $k$ , a constant, are allowed to be formed. Straightforward simulations of optimal RM algorithms on this constrained RM model are found to be non-optimal. This paper presents two optimal algorithms to compute the contour of maximal elements of a set of planar points. The first algorithm solves this problem of size  $p$  in  $O(\frac{p}{k})$  time on an  $k$ -constrained RM of size  $k \times p$ ,  $k \leq p$ , and the second algorithm solves this problem of size  $n$  in  $O(\frac{q}{k})$  time on an  $k$ -constrained RM of size  $p \times q$ ,  $p \leq q$ , and  $pq = kn$ .

## 1 Introduction

It is well-known that interprocessor communications and simultaneous memory accesses often act as bottlenecks in

present-day parallel machines. Bus systems have been introduced recently to a number of parallel machines to address this problem. Examples include the *Bus Automaton* [16], the *Reconfigurable Mesh (RM)* [12], the *content addressable array processor* [20], and the *Polymorphic torus* [11]. Among them RM draws much attention because of its simplicity. A bus system is called *reconfigurable* if it can be dynamically changed according to either global or local information.

In the most common model of RM, it is assumed that a message can be broadcast in constant time along any bus independent of its length. This assumption attracted criticism and cast a shadow of doubt on the implementation of RM. Although investigations of bus delays in [8, 9, 10] has confirmed that broadcast delay is very small, theoretically it cannot be correctly modelled by a constant independent of bus length. To account for this limit, Beresford-Smith *et al.* [2] has proposed the  $k$ -constrained RM where buses of length at most  $k$ , a constant, are allowed to be formed at any time.

In [2, 5, 6] it has been pointed out that

straightforward simulations of RM algorithms on  $k$ -constrained RM compromise with the  $AT^2$  [17, chapter 2] optimality. To address this issue, optimal algorithms have already been developed for sorting and computing convex hull [2], broadcasting [5], and multiplying sparse matrices [6]. In this paper we have explored one further problem from a similar point of view. We have presented here two optimal algorithms on  $k$ -constrained RM to compute the contour of the maximal elements of a given set of planar points. The first algorithm computes the m-contour (see Section 2.2) of  $p$  planar points on a  $k$ -constrained RM of size  $k \times p$  in  $O(\frac{p^2}{k})$  time and the second algorithm computes the m-contour of  $n$  planar points on a  $k$ -constrained RM of size  $p \times q$  in  $O(\frac{q}{k})$  time, where  $p \leq q$  and  $pq = kn$ .

The paper is organised as follows. In the next section we present the basic issues associated with RM and its various models based on message propagation delay. This section also presents definition of the problem and describes the  $AT^2$  lower bound metric.  $AT^2$  optimal m-contour algorithms on  $k$ -constrained RM are developed in Section 3. Section 4 concludes the paper.

## 2 Preliminaries

For the sake of completeness, here we briefly define the reconfigurable mesh and its various models based on message propagation delay. We also give definition of the problem of computing the contour of the maximal elements of a given set of planar points and describe the  $AT^2$  lower bound metric. Throughout the paper, we use  $\Theta()$  to mean “order exactly,”  $O()$  to mean “order at most,” and  $\Omega()$  to mean

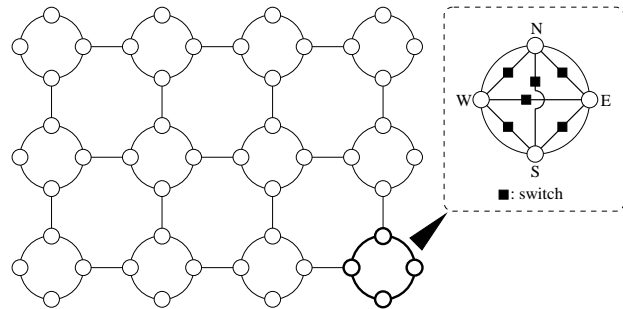


Figure 1: A reconfigurable mesh of size  $3 \times 4$

“order at least.”

### 2.1 Reconfigurable Mesh and Models Based on Propagation Delay

The reconfigurable mesh is primarily a two-dimensional mesh of processors connected by reconfigurable buses. In this parallel architecture, a processor element is placed at the grid points as in the usual mesh connected computers. Processors of the RM of size  $X \times Y$  are denoted by  $PE_{i,j}$ ,  $0 \leq i < X - 1$ ,  $0 \leq j < Y - 1$  where processor  $PE_{0,0}$  resides in the south-western corner. Each processor is connected to at most four neighbouring processors through fixed bus segments connected to four I/O ports **E** & **W** along dimension  $x$  and **N** & **S** along dimension  $y$ . These fixed bus segments are building blocks of larger bus components which are formed through switching, decided entirely on local data, of the internal connectors (see Figure 1) between the I/O ports of each processor. The fifteen possible interconnections of I/O ports through switching are shown in Figure 2. The connection patterns are represented as  $\{p_1, p_2, \dots\}$ , where each of  $p_i$  represents a group of switches connected

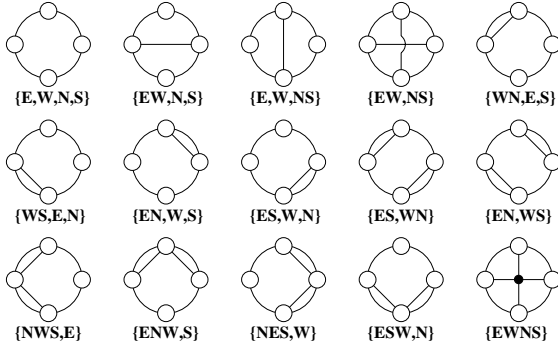


Figure 2: Possible internal connections between the four I/O ports of a processor

together such that  $\cup_{\forall i} p_i = \{\mathbf{N}, \mathbf{E}, \mathbf{W}, \mathbf{S}\}$ . For example,  $\{\mathbf{NS}, \mathbf{E}, \mathbf{W}\}$  represents the connection pattern with ports  $\mathbf{N}$  and  $\mathbf{S}$  connected and ports  $\mathbf{E}$  and  $\mathbf{W}$  unconnected.

A reconfigurable mesh operates in the single instruction multiple data (SIMD) mode. Besides the reconfigurable switches, each processor has a computing unit with a fixed number of local registers. A single time step of an RM is composed of the following four substeps:

**BUS substep.** Every processor switches the internal connectors between I/O ports by local decision.

**WRITE substep.** Along each bus, one or more processors on the bus transmit a message of length bounded by the bandwidth of the fixed bus segments as well as the switches. These processors are called the *speakers*. It is assumed that a collision between several speakers will be detected by all the processors connected to the bus and the transmitted message will be discarded.

**READ substep.** Some or all the processors connected to a bus read the mes-

sage transmitted by a single speaker. These processors are called the *readers*.

**COMPUTE substep.** A constant-time local computation is done by each processor.

Other than the buses and switches the RM of size  $p \times q$  is similar to the standard mesh of size  $p \times q$  and hence it has  $\Theta(pq)$  area in VLSI embedding [17], under the assumption that processors, switches, and links between adjacent switches occupy unit area.

One critical factor in the complexity analysis of reconfigurable algorithms is the time needed to propagate a message over a bus. In the most common *unit-time delay* model [19], it is assumed that in any configuration any message can be transmitted along any bus in constant time, regardless of the bus length. Unfortunately this assumption, based on which a large number of algorithms with constant time complexity are developed, is theoretically false, as the speed of signals carrying information is bounded by the speed of light. This partially explains why the reconfigurable meshes have not gained wide acceptance initially. Recently some VLSI implementations of reconfigurable meshes have demonstrated that the broadcast delay, though not a constant, is nevertheless relatively small in terms of machine cycles. For example, only 16 machine cycles are required to broadcast on a  $10^6$  processor YUPPIE (Yorktown Ultra Parallel Polymorphic Image Engine) [9, 10]. GCN (Gated-Connection Network) [8] has even shorter delays by adopting precharged circuits. Broadcast delay can further be reduced by using optical fibers for reconfigurable bus system and electrically controlled directional coupler switches as pro-

posed in [1]. Although the above observations serve the practical purposes, unit-time delay can never be theoretically sound. To account for this theoretical limit, two different models have been introduced in the literature.

In the *log-time delay* model [13] it is assumed that each broadcast takes  $\Theta(\log s)$  time to reach all the processors connected to a bus, where  $s$  is the maximum number of switches in a minimum switch path between two processors connected on the bus.

Beresford-Smith *et al.* [2] have recently proposed the *k-constrained* model where it is assumed that in any situation any message can propagate at most  $k$  fixed bus segments and thus buses of length at most  $k$  are allowed in any step, where  $k$  is a constant. We use the notation  $\text{RM}_A^k$  to refer to a  $k$ -constrained RM of area  $A$ .

A *linear bus* is a bus which never branches, thereby excluding the connection patterns  $\{\text{NSE}, \mathbf{W}\}$ ,  $\{\text{NSW}, \mathbf{E}\}$ ,  $\{\mathbf{EWN}, \mathbf{S}\}$ ,  $\{\mathbf{EWS}, \mathbf{N}\}$ , and  $\{\text{NSEW}\}$ . Now, any RM algorithm which uses only linear buses can be simulated by the  $k$ -constrained RM by propagating signals  $k$  processors at a time.

**Lemma 1** *Any message can be broadcast over a linear bus of length  $l$  in  $O(\frac{l}{k})$  time on a  $k$ -constrained RM.*  $\square$

Beresford-Smith *et al.* [2] have shown in obvious way that such a simulation loses  $AT^2$  optimality unless the area of the mesh is reduced. To address this issue,  $AT^2$  optimal algorithms have already been developed for sorting and computing convex hull [2], broadcasting [5], and multiplying sparse matrices [6].

Throughout this paper RM is assumed to be unconstrained if not stated otherwise.

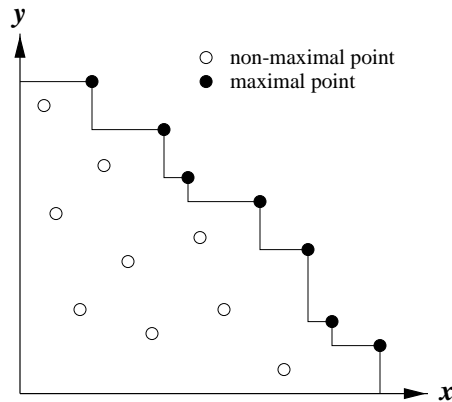


Figure 3: m-contour of a set of planar points

## 2.2 Problem Definition

Let the planar point at coordinate  $(i, j)$  be defined as  $P(i, j)$ . Again, let for any point  $p$ ,  $x(p)$  denote the  $x$ -coordinate and  $y(p)$  denote the  $y$ -coordinate of  $p$ , e.g.,  $x(P(i, j)) = i$  and  $y(P(i, j)) = j$ .

**Definition 1** *A point  $p$  dominates a point  $q$  (denoted by  $q \prec p$ ) if  $x(q) \leq x(p)$  and  $y(q) \leq y(p)$ . (The relation “ $\prec$ ” is naturally called dominance.)*

Let  $S$  be a set of  $N$  planar points. To simplify the exposition of our algorithms, the points in  $S$  are assumed to be distinct.

**Definition 2** *A point  $p \in S$  is maximal if there is no other point  $q \in S$  with  $p \prec q$ .*

We are interested in the contour spanned by the maximal elements of  $S$ , called the *m-contour* of  $S$  which can be obtained by simply sorting the maximal elements in ascending order of their  $x$ -coordinates (Figure 3). Let the m-contour of a set  $S$  be denoted as  $m(S)$ .

We have mentioned two interesting observations on m-contour in our paper [14, 15] which are given below for the sake of completeness.

**Lemma 2** *Every  $m$ -contour is sorted in descending order of the  $y$ -coordinates.*

**Proof.** Suppose the contrary. Then there exists at least one pair of maximal elements  $p$  and  $q$  such that  $y(p) < y(q)$  while  $x(p) \leq x(q)$ , which contradicts with the assumption that point  $p$  is maximal.  $\square$

Let for any set  $S$  of some planar points functions  $\min_x(S)$  and  $\max_x(S)$  denote the *minimum* and *maximum*  $x$ -coordinates in the set respectively. Let two more functions  $\min_y(S)$  and  $\max_y(S)$  be defined similarly w.r.t.  $y$ -coordinate.

**Lemma 3** *Given  $K$  sets  $S_0, S_1, \dots, S_{K-1}$  of planar points such that  $\forall t : 0 \leq t < K - 1, \max_x(S_t) \leq \min_x(S_{t+1})$ , then  $\forall i : 0 \leq i < K - 1, \forall p \in m(S_i) \wedge y(p) > \max_y(m(S_j)), \forall j > i$ , if and only if,  $p \in m(\bigcup_{t=0}^{K-1} S_t)$ .*

**Proof.** The *necessity* part can be proved by arranging a contradiction of Lemma 2. To prove the *sufficiency* part we take a point  $p \in m(S_i), \exists i : 0 \leq i < K - 1 \wedge p \notin m(\bigcup_{t=0}^{K-1} S_t)$ . Then by the definition of maximality we get  $\exists q \in \bigcup_{t=i+1}^{K-1} S_t$  such that  $p \prec q$ , i.e.,  $y(p) \leq y(q)$ .  $\square$

The  $m$ -contour problem is also known as finding the maxima of a set of vectors and has been extensively explored for serial computers in [7, 8, 21]. Computation of maximal elements is important in solving the *Largest Empty Rectangle Problem* [4] where a rectangle  $R$ , and a number of planar points  $S \in R$ , are given and the problem is to compute the largest rectangle  $r \subseteq R$  that contains no point in  $S$  and whose sides are parallel to those of  $R$ . If  $R$  is divided into four quadrants then the maximal elements w.r.t. the northeast(NE), northwest(NW), southwest(SW), and southeast(SE) directions as depicted in Figure 4

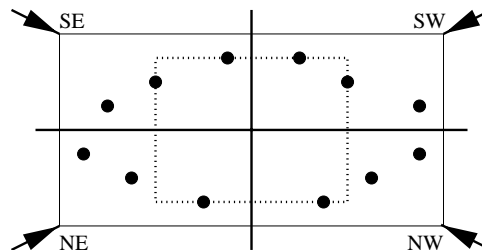


Figure 4: Importance of maximal elements in computing largest empty rectangle

remain the only candidates to be the supporting elements of the empty rectangles lying in all the four quadrants.

It is well known that the time complexity for computing the contour of the maximal elements of  $n$  planar points is  $\Theta(n \log n)$  using a serial computer [8]. This lower boundary can be concluded from the fact that the problem of sorting can be easily transformed into an  $m$ -contour problem of same size. The  $AT^2$  lower bound of  $m$ -contour problem of size  $n$ , as shown in Section 2.3, is  $\Omega(n^2)$ . Dehne [3] gives an  $AT^2$  optimal algorithm for solving  $m$ -contour problem on a mesh of size  $\sqrt{n} \times \sqrt{n}$  in  $O(\sqrt{n})$  time. Manzur *et al.* [14, 15] have presented three constant time  $m$ -contour algorithms on RM of various dimensions. Using the result of optimal simulation of a multidimensional RM by a 2-dimensional RM in [18], it can easily be shown that all the three algorithms in [14, 15] are  $AT^2$  optimal.

## 2.3 $AT^2$ Lower Bounds

Let a problem  $\mathcal{P}$  of size  $n$  have  $I$  information content [17, pages 51-54]. If this problem  $\mathcal{P}$  is realised in a VLSI circuit with aspect ratio  $\alpha$  then, by Ullman [17, page 57],  $AT^2$  lower bound of solving  $\mathcal{P}$  will be  $\Omega(\alpha I^2)$ .

Problem of sorting  $n \log n$ -bit numbers has information content  $I = \Omega(n)$ . As the problem of sorting  $n$  numbers can be easily transformed into a problem of computing the contour of maximal elements of  $n$  planar points, it can be concluded that the m-contour problem of size  $n$  has information content  $I = \Omega(n)$ . Hence, the  $AT^2$  lower bound of computing m-contour of  $n$  planar points is  $\Omega(n^2)$ .

Let m-contour of  $p$  planar points on an  $RM_{kp}^k$  of size  $k \times p$ ,  $k \leq p$ , be done in  $O(T)$  time. This solution will be  $AT^2$  optimal if

$$kpT^2 = \frac{p}{k}p^2 ,$$

i.e.,  $T = \frac{p}{k}$ . Similarly, to compute m-contour of  $n$  planar points,  $AT^2$  optimally, on  $RM_{kn}^k$  of size  $p \times q$ ,  $p \leq q$ , the solution time must be  $O(\frac{q}{k})$ .

### 3 Optimal m-contour Algorithms on $k$ -constrained RMs

We first briefly outline a few published results to be used in our algorithms.

**Lemma 4** *Given  $p$  items in the first row of an  $RM_{kp}^k$  of size  $k \times p$ ,  $k \leq p$ , these items can be sorted in  $O(\frac{p}{k})$  time, which is  $AT^2$  optimal.*

**Proof.** See in [2]. □

**Lemma 5** *Given  $n$  items in the first  $\frac{n}{p}$  columns of an  $RM_{kn}^k$  of size  $p \times q$ ,  $p \leq q$ , these items can be sorted in  $O(\frac{q}{k})$  time, which is  $AT^2$  optimal.*

**Proof.** See in [2]. □

Given a binary sequence,  $b_j$ ,  $0 \leq j < N$ , the *prefix-and computation* is to compute,  $\forall i : 0 \leq i < N$ ,  $b_0 \wedge b_1 \wedge \dots \wedge b_i$ .

Similarly the *prefix-or computation* computes  $b_0 \vee b_1 \vee \dots \vee b_i$ ,  $\forall i : 0 \leq i < N$ . Adapting the technique of bus splitting [13] it is easy to show that:

**Lemma 6** *Given a binary sequence of length  $k$  in the only row of an  $RM_k^k$  of size  $1 \times k$ , both the prefix-and and the prefix-or of the elements in the sequence can be computed in  $O(1)$  time.*

**Proof.** It is easy to check that the constant time algorithm in [13] on an RM of size  $1 \times k$  uses only horizontal buses of length at most  $k$ . Hence, the same algorithm can be used for  $RM_k^k$  of size  $1 \times k$ . □

**Lemma 7** *Computing m-contour of  $k$  planar points in the first row of an  $RM_{k^2}^k$  of size  $k \times k$  can be done in  $O(1)$  time.*

**Proof.** Again it is easy to check that the first algorithm in [14, 15] on an RM of size  $k \times k$  can compute the m-contour of  $k$  planar points in constant time using only linear buses of length  $O(k)$ . Hence, the same algorithm can be used for  $RM_{k^2}^k$  of size  $k \times k$  to compute the same problem in  $O(1)$  time. □

#### 3.1 Optimal Computing of the m-contour of $p$ Planar Points on $RM_{kp}^k$ of Size $k \times p$ , $k \leq p$

Let the  $RM_{kp}^k$  of size  $k \times p$  be divided into  $\frac{p}{k}$  submeshes of size  $k \times k$  each and the given  $p$  planar points in the first row be distributed in such a way that each processor  $PE_{i,jk}$ ,  $0 \leq i < k$  and  $0 \leq j < \frac{p}{k}$ , receives a point. It is obvious that such



a redistribution of elements can be carried out in constant time using a column broadcast followed by a row broadcast with bus splitting [13]. Now, we sort the points w.r.t.  $x$ -coordinate in column-major order by Lemma 4 in  $O(\frac{p}{k})$  time.

Let the points residing in column  $jk$  be denoted by the set  $S_j$ ,  $0 \leq j < \frac{p}{k}$ . Clearly these  $\frac{p}{k}$  sets of planar points follow the condition of Lemma 3, i.e.,  $\forall j : 0 \leq j < \frac{p}{k} - 1, \max_x(S_j) \leq \min_x(S_{j+1})$ . The m-contours  $m(S_j)$ ,  $0 \leq j < \frac{p}{k}$ , are now computed in parallel using a submesh of size  $k \times k$  for each computation. By Lemma 7 this operation takes only  $O(1)$  time. Now, we transfer the  $\max_y(m(S_j))$  values to the first row of the  $RM_{kp}^k$  in the following single RM step using Lemma 2:

1. b: Any processor in column  $j$  containing a point  $\in m(S_j)$  disconnects all port interconnections while the rest of the processors connect port **N** with **S** for all  $0 \leq j < \frac{p}{k}$ .
- w: Any processor in column  $j$  containing a point  $\in m(S_j)$  now writes the  $y$ -coordinate of the point to port **S** for all  $0 \leq j < \frac{p}{k}$ .
- r: Every processor in the first row reads port **S** in.

Here, the substeps are labelled as “b:”, “w:”, “r:”, and “c:” to denote the BUS, WRITE, READ, and COMPUTE substeps respectively. As the buses formed in the above RM step are of length at most  $k$ , this RM step can also be used as a single  $k$ -constrained RM step.

Now, the m-contour of the entire  $p$  points can be computed in the following RM steps using Lemma 3:

1. Iterate the following for  $t = 0$  to  $\lceil \frac{p}{k^2} \rceil - 1$  in step 1:

- 1.1 Copy  $\max_y(m(S_{tk+j}))$  to processors  $PE_{i,j+rk}$ ,  $0 \leq i < k$ ,  $0 \leq r < \frac{p}{k}$ , for all  $0 \leq j < k$ , using a column broadcast then a row broadcast and finally a column broadcast.
- 1.2 Copy the  $y$ -coordinate of the point residing in processor  $PE_{i,kj}$  to the processors  $PE_{i,kj+r}$ ,  $0 \leq r < k$ , for all  $0 \leq j < \frac{p}{k}$ ,  $0 \leq i < k$ , using a row broadcast.
- 1.3 Now in the  $j$ th submesh of size  $k \times k$ , the  $i$ th row contains  $k$   $\max_y$  values paired with the  $y$ -coordinate of a particular point, say  $d$ . Now, apply Lemma 3 to eliminate  $d$  by computing prefix-or over the comparison values.

The bus length in the column broadcasts in Step 1.1 is at most  $k$  while the same in the row broadcast is at most  $p$ . Hence, by Lemma 1 Step 1.1 can be used on  $k$ -constrained RM in  $O(\frac{p}{k})$  time. Step 1.2 uses buses of length at most  $k$  and thus it can be used on  $k$ -constrained RM in constant time. By Lemma 6 Step 1.3 takes  $O(1)$  time. It can then be concluded that the above iteration takes  $O(\lceil \frac{p}{k^2} \rceil \frac{p}{k})$  time which is non-optimal.

Now, the clever idea of *systolic computing* can be applied to achieve optimal time. It can be easily shown that  $\frac{p}{k^2}$  iteration steps can be done systolically at each step of simulation of broadcasting over longer buses in  $k$ -constrained RM. As  $\frac{p}{k^2} \leq \frac{p}{k}$ , it can be concluded that

**Theorem 1** *Given  $p$  planar points in the first row of an  $RM_{kp}^k$  of size  $k \times p$ ,  $k \leq p$ , the m-contour of these points can be com-*

puted in  $O(\frac{p}{k})$  time, which is  $AT^2$  optimal.

□

### 3.2 Optimal Computing of the m-contour of $n$ Planar Points on $RM_{kn}^k$ of Size $p \times q$ , $p \leq q$

Let the  $RM_{kn}^k$  of size  $p \times q$  be divided into  $\frac{q}{k}$  submeshes of size  $p \times k$  each and the given  $n$  planar points in the first  $\frac{n}{p}$  columns be distributed in such a way that each processor  $PE_{i,jk}$ ,  $0 \leq i < p$  and  $0 \leq j < \frac{q}{k}$ , receives a point. It can easily be shown that such a redistribution can be carried out systolically in  $O(\frac{n}{p}) = O(\frac{q}{k})$  time using only row broadcasts. Now, we sort the points w.r.t.  $x$ -coordinate in column-major order by Lemma 5 in  $O(\frac{q}{k})$  time.

Let the points residing in column  $jk$  be denoted by the set  $S_j$ ,  $0 \leq j < \frac{q}{k}$ . Clearly these  $\frac{q}{k}$  sets of planar points follow the condition of Lemma 3, i.e.,  $\forall j : 0 \leq j < \frac{q}{k} - 1$ ,  $max_x(S_j) \leq min_x(S_{j+1})$ . The m-contours  $m(S_j)$ ,  $0 \leq j < \frac{q}{k}$ , are now computed in parallel using a submesh of size  $p \times k$  for each computation. By Theorem 1 this operation takes only  $O(\frac{p}{k})$  time. Now, we transfer the  $max_y(m(S_j))$  values to the first row of the  $RM_{kn}^k$  in the following single RM step using Lemma 2:

1. b: Any processor in column  $j$  containing a point  $\in m(S_j)$  disconnects all port interconnections while the rest of the processors connect port **N** with **S** for all  $0 \leq j < \frac{q}{k}$ .
- w: Any processor in column  $j$  containing a point  $\in m(S_j)$  now writes the  $y$ -coordinate of the point to port **S** for all  $0 \leq j < \frac{q}{k}$ .

r: Every processor in the first row reads port **S** in.

As the buses formed in the above RM step are of length at most  $p$ , this RM step can also be used for  $k$ -constrained RM in  $O(\frac{p}{k})$  steps by Lemma 1.

Now, the m-contour of the entire  $n$  points can be computed in the following RM steps using Lemma 3:

1. Iterate the following for  $t = 0$  to  $\lceil \frac{q}{k^2} \rceil - 1$  in step 1:
  - 1.1 Copy  $max_y(m(S_{tk+j}))$  to processors  $PE_{i,j+rk}$ ,  $0 \leq i < k$ ,  $0 \leq r < \frac{q}{k}$ , for all  $0 \leq j < k$ , using a column broadcast then a row broadcast and finally a column broadcast.
  - 1.2 Copy the  $y$ -coordinate of the point residing in processor  $PE_{i,kj+r}$ ,  $0 \leq r < k$ , for all  $0 \leq j < \frac{q}{k}$ ,  $0 \leq i < p$ , using a row broadcast.
  - 1.3 Now in the  $j$ th submesh of size  $p \times k$ , the  $i$ th row contains  $k$   $max_y$  values paired with the  $y$ -coordinate of a particular point, say  $d$ . Now, apply Lemma 3 to eliminate  $d$  by computing prefix-or over the comparison values.

The bus length in the column broadcasts in Step 1.1 is at most  $p$  while the same in the row broadcast is at most  $q$ . Hence, by Lemma 1 Step 1.1 can be used on  $k$ -constrained RM in  $O(\frac{q}{k})$  time as  $p \leq q$ . Step 1.2 uses buses of length at most  $k$  and thus it can be used on  $k$ -constrained RM in constant time. By Lemma 6 Step 1.3 takes  $O(1)$  time. It can then be

concluded that the above iteration takes  $O\left(\left\lceil \frac{q}{k^2} \right\rceil \frac{q}{k}\right)$  time which is non-optimal.

Again, the clever idea of systolic computing can be applied to achieve optimal time. It can be easily shown that  $\frac{q}{k^2}$  iteration steps can be done systolically at each step of simulation of broadcasting over longer buses in  $k$ -constrained RM. As  $\frac{q}{k^2} \leq \frac{q}{k}$ , the following can be concluded:

**Theorem 2** *Given  $n$  planar points in the first  $\frac{n}{p}$  columns of an  $RM_{kn}^k$  of size  $p \times q$ ,  $p \leq q$ , the  $m$ -contour of these points can be computed in  $O\left(\frac{q}{k}\right)$  time, which is  $AT^2$  optimal.  $\square$*

## 4 Conclusion

In [2, 5, 6] it has been pointed out that straightforward simulations of RM algorithms on  $k$ -constrained RM compromise with the  $AT^2$  optimality. To address this issue, we have developed two optimal algorithms to compute the contour of maximal elements of a set of planar points. The first algorithm solves this problem of size  $p$  in  $O\left(\frac{p}{k}\right)$  time on an  $k$ -constrained RM of size  $k \times p$ ,  $k \leq p$ , and the second algorithm solves this problem of size  $n$  in  $O\left(\frac{q}{k}\right)$  time on an  $k$ -constrained RM of size  $p \times q$ ,  $p \leq q$ , and  $pq = kn$ .

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