# Biologically Inspired Vision and Control for an Autonomous Flying Vehicle 

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## Declaration

This thesis is an account of research undertaken between February 2000 and October 2007 at The Research School of Biological Sciences, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university. The following summarises my own contributions and any contributions made by other people:

Chapter 1: This chapter is all my own work.
Chapter 2: I completed the literature review in this chapter independently.
Chapter 3: The simulation of the Eagle helicopter, including development of all the C-code was implemented by myself. Mr Bilal Ahmed, provided assistance with generating the frequency plots for roll, pitch and vertical motion using the $C I F E R^{\circledR}$ program. I conducted all of the systems identification experiments and was the pilot for all flights.

Chapter 4: I designed the avionics system architecture for the RMAX and PC104 Eagle helicopters. I selected the PC104 hardware and implemented a variant of the Linux operating system which would boot from a compact flash card on the various flight computers. Electronics design for the inertial systems and autopilots was completed at the UNSW@ADFA workshop under my direction. All of the onboard software on the Eagle and RMAX helicopters was written by myself except for the frame grabber driver and software for computing optic flow, which was provided by Dr Javaan Chahl. I wrote the real-time driver software to interface to the Yamaha Attitude Control System on the RMAX, a multi-port serial card, the NovAtel DGPS system, laser rangefinder, ultrasonic sonar, servos and bluetooth modems. I wrote all of the ground control, telemetry and command and control software. I developed the calibration software for all of the sensors including accelerometers, magnetometers and gyroscopes.

Chapter 5: I developed the beacon algorithm, sensor fusion, controller software and associated simulations. The visual tracking code for the beacon experiments was provided by Dr Chahl. I conducted the flight experiments for the beacon tracking hover and the optic flow damped hover, including operating the ground control computer, tuning control gains and piloting the helicopter in-between closed loop flights.

Chapter 6: All work involving forward flight simulations, controller designs and test flights in this chapter is my own.

Chapter 7: All work involving simulations, network designs, code, techniques and flight test presented on neural networks is my own.

Chapter 8: The designs of the EKF algorithms are all my own. All of the code, simulations, interpretation of results and conclusions presented in this chapter are my own.

Chapter 9: The conclusions and recommendations in this chapter are all my own.

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## Abstract

This thesis makes a number of new contributions to control and sensing for unmanned vehicles. I begin by developing a non-linear simulation of a small unmanned helicopter and then proceed to develop new algorithms for control and sensing using the simulation. The work is field-tested in successful flight trials of biologically inspired vision and neural network control for an unstable rotorcraft. The techniques are more robust and more easily implemented on a small flying vehicle than previously attempted methods.

Experiments from biology suggest that the sensing of image motion or optic flow in insects provides a means of determining the range to obstacles and terrain. This biologically inspired approach is applied to control of height in a helicopter, leading to the World's first optic flow based terrain following controller for an unmanned helicopter in forward flight. Another novel optic flow based controller is developed for the control of velocity in hover. Using the measurements of height from other sensors, optic flow is used to provide a measure of the helicopters lateral and longitudinal velocities relative to the ground plane. Feedback of these velocity measurements enables automated hover with a drift of only a few cm per second, which is sufficient to allow a helicopter to land autonomously in gusty conditions with no absolute measurement of position.

New techniques for sensor fusion using Extended Kalman Filtering are developed to estimate attitude and velocity from noisy inertial sensors and optic flow measurements. However, such control and sensor fusion techniques can be computationally intensive, rendering them difficult or impossible to implement on a small unmanned vehicle due to limitations on computing resources. Since neural networks can perform these functions with minimal computing hardware, a new technique of control using neural networks is presented. First a hybrid plant model consisting of exactly known dynamics is combined with a black-box representation of the unknown dynamics. Simulated trajectories are then calculated for the plant using an optimal controller. Finally, a neural network is trained to mimic the optimal controller. Flight test results of control of the heave dynamics of a helicopter confirm the neural network controller's ability to operate in high disturbance conditions and suggest that the neural network outperforms a PD controller. Sensor fusion and control of the lateral and longitudinal dynamics of the helicopter are also shown to be easily achieved using computationally modest neural networks.

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## Abbreviations

| ADC | Analog to Digital Converter |
| :--- | :--- |
| AHRS | Attitude Heading Reference System |
| ANN | Artificial Neural Network |
| ARC | Australian Research Council |
| CAN | Controller Area Network bus protocol |
| CEP | Circular Error Probability |
| CIFER ${ }^{\circledR}$ | Comprehensive Identification from FrEquency Responses software |
| CSIRO | Commonwealth Science and Industrial Research Organisation |
| CMU | Carnegie Mellon University |
| COTS | Commercially Available Off-The-Shelf |
| CPU | Central Processing Unit |
| DGPS | Differential Global Positioning System |
| DSTO | Defence Science and Technology Organisation |
| EKF | Extended Kalman Filter |
| ESDU | Engineering Sciences Data Unit |
| FFT | Fast Fourier Transform |
| FPGA | Field Programmable Gate Array |
| GPS | Global Positioning System |
| GUI | Graphical User Interface |
| HOTO | Hand Over Take Over |
| $I^{2} A$ | Optic Flow Image Interpolation Algorithm |
| I2C | Inter Integrated Circuit Serial Computer Bus |
| $I^{3} A$ | Optic Flow Iterative Image Interpolation Algorithm |
| IMU | Inertial Measurement Unit |
| INS | Inertial Navigation System |
| LM | Levenberg-Marquardt |
| LQG | Linear Quadratic Gaussian |
| LQR | Linear Quadratic Regulator |
| LRF | Laser Rangefinder |
| LSB | Least Significant Byte |
| MAV | Micro Air Vehicle |
| ms | Milliseconds |
| MSB | Most Significant Byte |
| MSE | Mean Square Error |
| PC | Personal Computer |
| PCH | Pseudocontrol Hedging |
| PWM | Pulse Width Modulation |
| RAM | Random Access Memory |
|  |  |


| RC | Radio Controlled |
| :--- | :--- |
| RF | Radio Frequency |
| RDIP | Rotational Dynamics Inversion Processor |
| ROM | Read Only Memory |
| RTK | Real Time Kinematic |
| RTOS | Real Time Operating System |
| SBC | Single Board Computer |
| SD | Standard Deviation |
| TPP | Tip Path Plane |
| UAV | Unmanned Aerial Vehicle |
| UHF | Ultra High Frequency |
| USC | University of Southern California |
| VBI | Video Blanking Interval |
| VLSI | Very Large Scale Integration |
| YACS | Yamaha Attitude Control System |
| $\mu S$ | Microseconds |

## Nomenclature

| $a$ | Blade 2D lift curve slope |
| :--- | :--- |
| $a_{0}$ | Coning angle |
| $a_{1}$ | Longitudinal flapping |
| $a_{z}$ | Vertical acceleration |
| $A$ | Rotor disk area |
| $A$ | Magnitude of magnetometer sine wave during calibration |
| $A_{b}$ | Blade area |
| $\hat{\boldsymbol{A}}_{\boldsymbol{i}}$ | Unit vector in direction of $i^{t h}$ beacon landmark |
| $A_{1}$ | Lateral cyclic |
| $A_{l a t}$ | Lateral cyclic to main rotor pitch ratio |
| $\boldsymbol{b}$ | Earth's magnetic field vector |
| $b_{1}$ | Lateral flapping |
| $b_{i j}$ | Element of rotation matrix $\boldsymbol{B}$ from row $i$ and column $j$ |
| $\boldsymbol{B}$ | $3 \times 3$ rotation matrix |
| $B_{l o n}$ | Longitudinal cyclic to main rotor pitch ratio |
| $B_{1}$ | Longitudinal cyclic |
| $c$ | Blade chord |
| $c$ | Lateral flybar flapping |
| $c_{x}, c_{y}$ | Optic flow calibration constants |
| $C_{D}$ | Drag coefficient, $D / 0.5 \rho V^{2} S$ |
| $C_{D 0}$ | Profile drag coefficient |
| $C_{L}$ | Lift coefficient, $L / 0.5 \rho V^{2} S$ |
| $C_{l o n}$ | Longitudinal cyclic to flybar pitch ratio |
| $C_{P}$ | Power coefficient, $P / \rho A(\Omega R)^{3}$ |
| $C_{P_{0}}$ | Blade profile power coefficient, $P_{0} / \rho A(\Omega R)^{3}$ |
| $C_{T}$ | Thrust coefficient, $T / \rho A(\Omega R)^{2}$ |
| $d$ | Longitudinal flybar flapping |
| $D$ | Drag |
| $D_{l a t}$ | Lateral cyclic to flybar pitch ratio |
| $e$ | Rotor blade hinge osffset |
| $e_{e f f}$ | Effective rotor hinge offset |
| $f$ | Equivalent flat plate area |
| $F_{x}$ | Force acting in x-axis direction |
| $F_{y}$ | Force acting in y-axis direction |
| $F_{z}$ | Force acting in z-axis direction |
| $g$ | Acceleration due to gravity (scalar) |
| $\boldsymbol{g}$ | Gravity vector |
| $H$ | Height above ground |


| H | EKF observation matrix |
| :---: | :---: |
| $i_{s}$ | Main rotor shaft angle |
| $I_{b}$ | Blade mass moment of inertia |
| $I_{x}$ | Second mass moment of inertia about x-axis |
| $I_{y}$ | Second mass moment of inertia about y-axis |
| $I_{z}$ | Second mass moment of inertia about z-axis |
| $I_{x y}$ | Cross-product moment of inertia about xy-axes |
| $I_{y z}$ | Cross-product moment of inertia about yz-axes |
| $I_{x z}$ | Cross-product moment of inertia about xz-axes |
| $k$ | Sensor gain |
| $k_{\text {ind }}$ | Induced power correction factor |
| $k_{\beta}$ | Blade root moment equivalent spring |
| K | EKF gain matrix |
| $\mathcal{K}$ | Slope of gyroscope output per rate of rotation |
| $K_{1}$ | Cyclic due to servo command, $d A_{1} / d \delta_{\text {lat }}$ |
| $K_{d}$ | Derivative feedback gain for a PID controller |
| $K_{I}$ | Integral feedback gain for a PID controller |
| $K_{p}$ | Proportional feedback gain for a PID controller |
| $K_{u}$ | Ultimate proportional feedback gain for marginal stability |
| $L$ | Lift |
| $L$ | Rolling moment |
| $L_{m r}$ | Rolling moment due to main rotor |
| $L_{u}$ | Longitudinal turbulence scale length |
| $L_{v}$ | Lateral turbulence scale length |
| $L_{w}$ | Vertical turbulence scale length |
| $m$ | Helicopter mass |
| $m$ | Unit vector aligned with magnetometer |
| $m_{b}$ | Mass of a rotor blade |
| M | Pitching moment |
| $M_{m r}$ | Pitching moment due to main rotor |
| $N$ | Yawing moment |
| $N$ | Number of blades |
| $P$ | EKF covariance matrix |
| $\boldsymbol{q}$ | Vector of quaternion parameters $\left[q_{0} q_{1} q_{2} q_{3}\right]^{T}$ |
| $q_{0}, q_{1}, q_{2}, q_{3}$ | Quaternion parameters |
| $\boldsymbol{Q}$ | EKF process noise matrix |
| $Q_{x}$ | Longitudinal optic flow |
| $Q_{y}$ | Lateral optic flow |
| $R$ | Rotor blade radius |
| $\boldsymbol{R}$ | EKF measurement noise matrix |
| $\mathcal{R}$ | Range computed from optic flow |
| $\mathcal{R}_{\psi}$ | Rotation matrix due to a rotation about the x -axis |
| $S$ | Planform area used to non-dimensionalise lift and drag |
| T | Main rotor thrust |
| $T_{\text {tr }}$ | Tail rotor thrust |


| $u$ | Longitudinal body-axis velocity |
| :--- | :--- |
| $u_{\text {mean }}$ | Mean longitudinal velocity |
| $U$ | Raw sensor output |
| $v$ | Lateral body-axis velocity |
| $\hat{V}$ | Air velocity relative to TPP |
| $V_{i}$ | Rotor induced velocity |
| $V_{n}$ | Freestream velocity normal to TPP |
| $V_{R}$ | Velocity of air relative to blade |
| $V_{t}$ | Freestream velocity tangential to TPP |
| $w$ | Vertical body-axis velocity |
| $V_{\infty}$ | Freestream velocity |
| $V_{z}$ | Vertical velocity |
| $X$ | X position in Earth centred coordinates |
| $\hat{\boldsymbol{X}}_{b}$ | x body axis unit vector |
| $\hat{\boldsymbol{X}}_{g}$ | x global axis unit vector |
| $\mathcal{W}$ | Vertical relative velocity between the ground and heliopter |
| $Y$ | Y position in Earth centred coordinates |
| $\hat{\boldsymbol{Y}}_{b}$ | y body axis unit vector |
| $\hat{\boldsymbol{Y}}_{g}$ | y global axis unit vector |
| $z_{m r}$ | Vertical distance from main rotor hub to centre of gravity |
| $Z$ | Z position in Earth centred coordinates |
| $Z$ | Vertical position |
| $\hat{\boldsymbol{Z}}_{b}$ | z body axis unit vector |
| $\hat{\boldsymbol{Z}}_{g}$ | z global axis unit vector |
| $\mathcal{Z}$ | Terrain clearance |
| $\alpha$ | Blade section angle of attack |
| $\beta$ | Rotor blade flapping angle |
| $\delta_{p}$ | Offset error of roll rate gyroscope |
| $\delta_{q}$ | Offset error of pitch rate gyroscope |
| $\delta_{r}$ | Offset error of yaw rate gyroscope |
| $\delta_{x}$ | Offset error of x body axis accelerometer |
| $\delta_{y}$ | Offset error of y body axis accelerometer |
| $\delta_{z}$ | Offset error of z body axis accelerometer |
| $\delta_{c o l}$ | Collective pitch servo command |
| $\delta_{l a t}$ | Lateral (roll) servo command |
| $\delta_{l o n}$ | Longitudinal (pitch) servo command |
| $\delta_{p e d}$ | Yaw servo command |
| $\delta_{z}$ | Offset error of z body axis accelerometer |
| $\Delta T$ | Time Step |
| $\gamma$ | Locke number, $\rho a c R^{4} / I_{b}$ |
| $\gamma_{i x}^{2}$ | CIFER® correlation factor |
| $\zeta_{i x}, \zeta_{i y}, \zeta_{i z}$ | Sensor alignment coefficients for the $i^{t h}$ sensor |
| $\theta$ | Plangle |


| $\theta_{\max }$ | Servo rate limit |
| :--- | :--- |
| $\theta_{s}$ | Commanded sinusoidal servo amplitude |
| $\theta_{0}$ | Blade collective pitch angle |
| $\kappa$ | Forward flight profile drag power correction factor |
| $\lambda$ | Rotor inflow ratio |
| $\lambda$ | Attitude vector correction factor |
| $\lambda_{x}, \lambda_{y}, \lambda_{z}$ | Components of Earth's magnetic field along magnetometer axes |
| $\lambda_{i}$ | Rotor induced inflow ratio, $V_{i} / \Omega R$ |
| $\mu$ | Rotor advance ratio |
| $\rho$ | Air density |
| $\sigma$ | Rotor solidity, $A_{b} / A$ |
| $\sigma_{u}$ | Longitudinal turbulence intensity factor |
| $\sigma_{v}$ | Lateral turbulence intensity factor |
| $\sigma_{w}$ | Vertical turbulence intensity factor |
| $\tau_{f}$ | Time constant of main rotor flapping |
| $\tau_{s}$ | Time constant for flybar flapping |
| $\Upsilon$ | Sensor offset |
| $\phi$ | Roll angle |
| $\phi$ | Phase angle of magnetometer sine wave during calibration |
| $\Phi$ | Beacon elevation angle |
| $\Phi$ | EKF fundamental matrix |
| $\psi$ | Blade azimuth angle measured anti-clockwise from above |
| $\psi$ | Yaw angle |
| $\psi$ | Rotation of IMU about an axes during calibration |
| $\Psi$ | Beacon azimuth angle |
| $\Omega$ | Angular velocity of main rotor |
| $\Omega$ | Servo corner frequency due to rate limit |

## Chapter 1

## Introduction

### 1.1 Aim

The aim of this thesis is to investigate and develop an effective control system for a flying vehicle using biologically inspired vision as the primary sensor. The underlying motivation for this work is that it might make autonomy for Unmanned Aerial Vehicles more practical in a real-world environment. This should be achievable using cheaper and smaller vision based sensors to replace or augment sensors based on Global Positioning Systems (GPS) and expensive Inertial Navigation Systems (INS). In addition, vision potentially offers a better solution to the problem of obstacle avoidance and terrain clearance than conventional techniques such as radar or laser rangefinding.

Work in this thesis is confined to the control of helicopters in both hover and forward flight. This is deliberately done as it increases the generality of the techniques used, making them applicable to not just forward flight at high speed, but also to vehicles attempting to operate in a confined region where motion may involve occasionally stopping, flying sideways, vertically, turning on the spot or even flying backwards. This makes the work more challenging owing to the additional difficulties of controlling such a platform. Helicopters are dynamically unstable and require constant external manipulation of the control inputs by a human or a machine pilot to prevent divergence from the desired flight path. Helicopter control is highly non-linear due to the complex nature of the rotor aerodynamics. Helicopter control channels are also highly coupled. For example, a commanded increase in rotor thrust causes an increase in torque which must be compensated by application of increased tail rotor pitch; this in turn requires a lateral tilt to the main rotor disk to prevent sideways motion; this rotor tilt then needs to be offset by an increase in thrust and a rotor longitudinal tilt to compensate for rotor flapping cross-coupling effects.

### 1.2 Motivation

### 1.2.1 Unmanned Aerial Vehicles

Autonomous flying vehicles have many applications for operations in dangerous areas where the risk to a human pilot would be unacceptable. Some examples include:
mine detection and disposal, operations near a volcano and operations in radioactive environments (e.g. dumping coolant on Chernobyl). They also have great potential for applications such as search and rescue, exploration, crop dusting, interplanetary exploration, survey, coastal patrol, pollution monitoring and atmospheric study.

Recent trends in modern warfare are showing an increased reliance on autonomous flying vehicles to provide reconnaissance information and battlefield intelligence in hostile environments. In April 2005, the New York Times [1] reported that the number of UAVs in use in the skies over Iraq had exceeded 700. Such craft remove humans from regions of potential danger and offer the hope of one day eliminating humans from the battlefield.

### 1.2.2 Why vision sensing?

Laser Rangefinders (LRF) and radars both tend to be bulky which precludes them from use on vehicles smaller than about 20 kg . A typical LRF used for robotics is the SICK LMS291 which weighs approximately 4.5 kg [2]. The smallest Synthetic Aperture Radar (SAR) planned for UAVs is likely to be the miniSAR from Sandia Labs at $4-5 \mathrm{~kg}[3]$. By contrast, a self-contained device capable of measuring image motion for terrain following has been produced by the Defence Science and Technology Organisation (DSTO) which weighs less than 5 grams. The miniaturisation of sensors will contribute to the practicality of Micro Air Vehicles (MAVs), classified as aircraft with a wingspan of 15 cm or less. MAVs could be used in a multitude of new roles, benefiting from the low cost of manufacture and the difficulty of detection. For military use, MAVs could be manufactured as a disposable item. The reduced airworthiness regulatory requirements of MAVs makes them much easier to integrate into the human environment since an MAV crash presents practically no risk to life or property. The problem of fully integrating larger UAVs into civilian airspace is still largely an unsolved problem and financial insurance for UAVs is hence difficult to obtain for commercial operations.

The passive nature of a vision sensor provides many advantages over other forms of ranging devices. Firstly, by not producing any electromagnetic emissions, a vision sensor can be used in an operational environment where stealth is important, such as on the battlefield or in a law enforcement situation. Secondly, there are health risks associated with the use of radar and laser which need management. By using a wide field of view, a camera also provides simultaneous ranging information over a large area. Radars and laser rangefinders are essentially point sensors and need to be scanned over the environment to build up a 3D map of the terrain. This adds mechanical complexity or introduces the need for large antenna arrays to be installed.

The most common navigation sensor used on UAVs is GPS. This is an appropriate sensor for medium to high altitude aircraft flying away from terrain. However, close to the ground, where terrain clearance must be maintained, GPS is only useful when very detailed maps of the ground relief and other obstacles is held onboard. In many cases this is not practical. In particular, when operating in urban environments, the obstacles to be avoided may be in motion such as cars and people. Vision
provides an immediate observation of the environment and should be able to provide a means of navigating through it which does not require a priori information about the position of objects.

There is an increasing need for small UAVs to operate in urban cluttered environments where GPS coverage cannot be guaranteed. Most accurate GPS implementations require at least 8 satellites to be observable in the sky. When flying close to buildings, beneath underpasses and for indoor flight, this coverage will not be achieved. One solution may be to augment GPS with visual sensing for parts of the mission where GPS is not providing full accuracy. Alternatively, GPS would be used to navigate a UAV into close proximity to a desired location before visual guidance is activated to achieve high fidelity positioning near obstacles.

### 1.3 Approach

We learn from biology that it must be possible to build systems that are of small size yet able to deal with complex environments. Many of the techniques used by others in the related work section make use of feature matching and tracking approaches which are difficult to achieve in repeated experiments due to changes in environment such as varying lighting conditions. In this thesis, I have attempted to search for ways to make the flight control system simple yet robust. Further I have tried to apply biologically inspired vision to a more complicated and general flight control problem than has been done before. Work by others on optic flow sensing (see Chapter 2 for references) is mainly concerned with fixed wing aircraft, ground robots and blimps which are either inherently stable or have such long time constants in their motion that control stability becomes trivial.

As this thesis has progressed, the availability of new hardware has often superceded work already completed. Initially, an off the shelf Hirobo Eagle helicopter was adapted to autonomous operations by addition of sensors and a telemetry system so that automated control could be executed from a ground-based computer. Later developments in computer hardware and further competitive grant based funding, permitted construction of an onboard processing system for another Eagle helicopter. On these helicopters, all of the sensors were developed in-house including a series of three-axis inertial measurement systems. The small payload capabilities of these helicopters made miniaturisation of systems a primary concern. In 2004, through an ARC Linkage grant, a Yamaha RMAX 90 kg helicopter became available. This helicopter had a much larger payload carrying capacity ( 30 kg ) and came with its own inertial measurement system and convenient RS-232 based interfaces to controls and telemetry. The longer endurance of the RMAX (1 hour compared to 15 minutes for the Eagle) made the experimental side of the project much easier. For the purposes of this thesis, the availability of the RMAX made much of the systems integration work completed on the Eagle redundant. I have however persevered with the Eagle architecture in parallel, to prove that the same concepts can be applied to a much smaller rotorcraft with lower grade sensors.

Although the visual control in the thesis starts with fairly complex algorithms, I
have been able to move to much simpler and less computationally intensive schemes based on optic flow. The first visual guidance system used the more conventional machine vision approach, involving the tracking of features on the ground with detailed mathematics to resolve egomotion from the observed relative position of the features. This method was found to be brittle and time consuming to implement. The schemes based on optic flow were found to be much easier to demonstrate in the field, owing to not having a reliance on feature tracking or complicated mathematical processing.

### 1.4 Contributions

This thesis is primarily concerned with how to take processed visual sensor output and use it in conjunction with the inertial sensors to control the flight path of a helicopter. The thesis comprises the following key achievements:

- Development of a fully non-linear simulation of a small unmanned helicopter and its sensors,
- A system for controlling a helicopter in hover using 3 visual landmarks on the ground,
- Use of optic flow for controlling the longitudinal and lateral drift of a helicopter attempting to hover over the ground without reference to any landmarks,
- The use of optic flow based ranging in forward flight to achieve terrain following,
- The application of artificial neural networks to the control of a helicopter using vision as the primary sensor, and
- An examination of new techniques for sensor fusion of inertial and visual information to produce estimates of attitude and velocity suitable for control of the vehicle,


### 1.5 Layout of the Thesis

The thesis contains nine chapters. In Chapter 2, related work in the areas of visual control of flight vehicles, calculation of optic flow and control techniques is presented. Chapter 3 provides an overview of helicopter dynamics and details the implementation of a helicopter simulation for testing of control schemes and sensor fusion used in the thesis. A detailed system description is provided for the helicopters used in Chapter 4. Chapter 5 describes the experiments completed to control a helicopter in hover using vision as the primary sensor. The control of forward flight is tackled in Chapter 6. Control schemes based on Artificial Neural Networks (ANN), which reduce the mathematical complexity yet deal with the unmodelled helicopter
dynamics are discussed in Chapter 7. In Chapter 8, advanced sensor fusion techniques using Extended Kalman Filters are developed for combining the available sensory inputs to provide useful variables for control. Finally, my conclusions and recommendations for future work are provided in Chapter 9.

## Chapter 2

## Related Work

### 2.1 Introduction

This thesis project draws upon work from a number of diverse fields. I have structured my review of the related literature in terms of biologically inspired vision, visual flight control, optic flow, sensor fusion and helicopter control.

### 2.2 Biologically Inspired Vision

This thesis was originally inspired by work done by Srinivasan et al [4-6] on the mechanisms by which insects use optic flow to aid in navigation. Optic flow is the motion of visual features across the field of view of the observer caused by translation or rotation of the observer. Optic flow has various definitions in a machine vision context, but is commonly described as the apparent motion of brightness patterns in an image sequence [7]. Optic flow can be used to perceive relative range of objects in the environment since close objects exhibit a higher angular motion in the visual field than distant objects, when the observer is in motion.

A number of researchers e.g. [8-10] have suggested that insects in forward flight perceive the range to objects in their field of view using optic flow. In [11], Srinivasan theorises that insects use this process to maintain terrain and obstacle clearance by effectively flying away from places in their field of view where angular motion is high. This idea is at least 50 years old, and was put forward by Kennedy $[12,13]$ as early as 1939. Through a series of experiments, Srinivasan and colleagues gathered evidence for this idea based on the notion that bees use optic flow to centre their flight path through narrow gaps. In the experiment, bees were trained to fly down a tunnel. One of the walls of the tunnel was able to be moved longitudinally in either direction using a conveyor belt arrangement. Averaged over many flights, a very clear trend was that bees flying in the same direction as the moving wall flew closer to the wall while bees flying in the opposite direction to the moving wall flew further away. Further experiments demonstrated that this effect was not influenced by the spatial period, intensity profile or contrast of the patterns placed on the tunnel walls. Together, these experiments demonstrate that bees must judge their distance from objects using the apparent angular speed of the environment. This makes sense since it will provide a means of measuring range which is independent of the visual texture of the environment, provided flight speed or height is known.


Figure 2.1: The bee centring response. In Srinivasan's experiments, bees fly down tunnels that are adorned with vertical strips as depicted in the top image. Over hundreds of flights, the flight path of the bees was recorded and exhibits the mean and standard deviation shown by the shaded regions in $[\mathrm{a}-\mathrm{f}]$. These results show that bees use image motion to determine range from obstacles. Reproduced with permission from Srinivasan and Zhang [14].

Another observation of bee behaviour leads to a proposed scheme for landing a UAV based on optic flow [15]. In experiments by Srinivasan et al [16], the 3D landing trajectories of bees were filmed using a video camera. The bees were seen to hold longitudinal optic flow constant whilst maintaining a constant glide angle of about $28^{\circ}$ to the touchdown point. The glide angle could be maintained by keeping the target touch down point at the same angular position in the retina, or by regulating a constant ratio of descent speed to forward speed. The two conditions of constant optic flow and constant glideslope are all that are needed to fly the insect down to a landing. In addition, as the optic flow is inversely proportional to range, the bee's forward and descent speed will reduce as the landing points get closer, to both reach zero at touchdown, providing a smooth landing. A major advantage of this method is that the flight speed or height of the insect does not need to be measured, eliminating the need for other sensing strategems such as stereo disparity until the insect is about to flare for landing. In a fixed wing aircraft, the ever decreasing flight speed would at some point would cause the aircraft to stall. However, one could imagine this technique used to control the landing approach of a helicopter, which has no limits on slow speed flight.

The fact that insects have such tiny brains, yet can operate so effectively in flight, suggests that their mechanisms for visual control of flight must be efficient. Hence, I have chosen to attempt to use optic flow sensing as one of the key elements in the design of the control system for this thesis project.

Insects do not use vision alone for stabilisation [17-21]. Instead, they use a combination of inertial, visual and other sensing modes. They also undoubtedly have dynamics state models built into their nervous system which allow them to anticipate, in feed-forward, the effect of their muscular actions on their flight path. With this in mind, I have attempted to design a system that integrates basic inertial sensors with visual sensing as the primary sensor.

### 2.3 Application of Visual Control to Robots

Vision has been used to control robots since at least the early 1970s [22]. A number of researchers have applied vision in control loops to ground-based robots using optic flow [5, 23,24] for collision avoidance. Other techniques that have been commonly used in ground-based robots include stereo vision and target tracking.

During the life of this thesis project, visual control of UAVs has been a very active area of research. Consequently, there have been a number of other groups using similar approaches to mine which have published in the literature after I completed the fundamental milestones of successful landmark tracking hover [25] in 2000 and optic flow controlled hover [26] in 2001. I have included the later work in the following brief overview of progress in the field.

### 2.3.1 Biologically Inspired Visual Flight Control

For his PhD dissertation [27], Barrows developed a VLSI optic flow sensor for use in a micro-UAV. The sensor consisted of a single chip sensor head containing photo receptors and analog processing combined with a microcontroller that completed the processing and output servo commands. The optic flow sensor was tested on a small glider and used to avoid the floor and walls of an indoor hallway. The control algorithm employed was 'bang-bang' in that if a certain threshold of optic flow was exceeded, the aircraft elevator (or rudder) would be moved to a preset deflection. In later work [28], optic flow sensors have been used outdoors by Barrows to control the altitude of a small radio-controlled aircraft at altitudes ranging from 2 metres to 10 metres.

In [29] a tethered 100 gram helicopter is described which used optic flow for terrain following over an indoor circular track. The autopilot called OCTAVE (Optical altitude Control sysTem for Autonomous VEhicles) was used to control a single rotor mounted on a whirling arm. This arrangement constrained the rotorcraft to flight in a circular path so that only one dimensional flow in the tangential direction needed to be considered. The pitch of the rotor was adjusted by the operator to set the desired speed of the helicopter. The height of the rotorcraft was controlled by the autopilot by changing the speed, and hence thrust, of the rotor in response to the perceived height over the terrain determined by optic flow. This technique allowed the rotorcraft to maintain a relatively constant height over the terrain which included a shallow ramp. In another experiment, the helicopter was able to land by maintaining constant optic flow whilst the forward speed was reduced, a technique suggested by Srinivasan et al in [4]. For these experiments, the optic flow was measured using a 1D Elementary Motion Detector (EMD) [30] inspired by the physiological structure of a housefly's visual system. The EMD circuit measured the output of 20 photoreceptors arranged in a line and was implemented firstly on a Field Programmable Gate Array (FPGA) [31] and later using a tiny microcontroller to produce a sensor weighing less than one gram. In a related project, Zufferey [32] used optic flow to control a number of robots. In Zufferey's work, the output of a 1D camera was processed by a PIC microcontroller to determine optical flow in
one direction. The resulting optic flow computation was used to control a ground vehicle and a small fixed wing aircraft. A number of behaviours were achieved:

- Steering control. Two 1D cameras, oriented at $45^{\circ}$ from the forward axis, were mounted on the fuselage of the fixed wing. The aircraft was flown inside a large room and when optic flow on either camera exceeded a certain threshold, the aircraft was turned by about $90^{\circ}$ to avoid hitting a wall. The control was achieved by smoothly applying rudder to full deflection and turning the aircraft over a period of one second. Using this technique, the airplane was able to fly four minutes without colliding with a wall.
- Simulated altitude control. A wheeled robot was used to maintain a constant distance from a wall to simulate altitude control. By keeping optic flow constant in a feedback loop, the robot maintained a relatively constant distance from the wall.

Reiser and Dickinson [33] describe object avoidance using optic flow in an insectinspired robotic control test bed comprising a 5 degree of freedom gantry. A vision sensor comprising a 30 frame per second $115^{\circ} \times 95^{\circ}$ field of view camera was placed on the gantry and was free to move within a 92 cm diameter cylindrical arena. A PC attached to the sensor using a frame grabber was able to calculate optic flow using an array of spatio-temporal correlation elements, using the method proposed by Hassenstein and Reichardt [34]. For the experiments, the vertical position and translational speed of the robot was kept constant and pitch and roll attitude was kept fixed. Detectors for image expansion were used to trigger insect-like saccades, such that the robot would rapidly change heading when approaching an obstacle. The results showed that simple image loom detectors were enough to prevent the robot from hitting the walls of the arena and obstacles placed inside the arena.

In $[35,36]$ experiments were conducted on the AVATAR helicopter which suggest that slow flying UAV may be able to avoid obstacles in an urban environment using vision. For this experiment, the AVATAR was fitted with a forward looking stereo camera and two cameras providing optic flow computation on the side. In forward flight the sideways looking optic flow cameras determined range to objects in their field of view. As the helicopter was flown towards obstacles on the side, such as trees, the helicopter tended, in most instances, to turn away from those obstacles. This technique might be extended to the point where a helicopter could safely pick a path through an urban environment, such as flying down the middle of a street autonomously.

Chahl tested a scheme for terrain following on a 1.2 m wingspan delta-wing UAV using optic flow [37]. For flight test, a downward looking camera with a field of view of 100 degrees was used with a control-by-telemetry scheme implemented on a 500 Mhz Pentium III ground computer. For each local optic flow vector calculated, the scheme calculated the corresponding climb angle needed to clear the obstacle represented by that flow vector. The maximum climb angle from the set of all climb angles calculated was used as a reference input for the longitudinal controller. Chahl noted that the rotational effect on the optic flow calculation dominates the
raw results, so that it is critical to subtract off the effect of rotations using a rate gyroscope before calculating the range to obstacles.

### 2.3.2 Non-Biological Examples of Visual Flight Control

One of the earliest successful uses of vision to control a helicopter was undertaken by Amidi for his PhD dissertation [38], submitted in 1996. In Amidi's work, carried out at Carnegie Melon University (CMU), a pair of cameras was used to track natural features on the ground using high-speed template matching. The initial features were selected by picking a window of pixels at the centre of the image. By locking on to ground objects as the helicopter moves, the vision system was able to estimate the helicopter's position and velocity. Stereo vision was used to determine the range to the features. A series of PD controllers were used in a feedback loop to control the position of the helicopter in hover and slow forward flight. For outdoor experiments, the vision system was implemented on a 67 kg Yamaha R- 50 helicopter and was shown to be able to control the helicopter with a position accuracy of $3-10 \mathrm{~cm}$ in hover.

Saripalli et al, at the University of Southern California (USC), have landed their AVATAR autonomous helicopter on a helipad using vision and inertial information [39]. The AVATAR is a small radio controlled helicopter with a PC-104 based avionic architecture. In their work, a pattern comprised of polygons painted on the helipad was detected and tracked so that the helicopter could align itself with the pattern and then use its relative pose to land. The image was first segmented with a fixed intensity threshold and then the first, second and third invariant moments of the thresholded pixels were calculated and compared against a template descriptor of the moments from the known target geometry (see [40] for an explanation of this technique). The advantage of this approach is that the invariant moments are not affected by translation, rotation or scaling and that the target descriptor is stored as a relatively small vector. The helicopter was able to track the pattern even when the helipad was moving, however, the helipad motion was stopped for the actual landing.

In [41], Mejias et al worked with members from the USC group to achieve visual servoing of the AVATAR and a similar small autonomous helicopter designated the COLIBRI from the Universidad Politécnica de Madrid. The visual servoing target was a window from a building. In the case of the COLIBRI, template matching based on the Lucas-Kanade tracker [42] was used to match the image of the window being tracked to a stored reference template of the window. This provided the coordinates of the corners of the window which were used to generate a set of velocity commands to an inner loop controller, to move the helicopter into a position where the target was centred in the field of view. The helicopter was flown to within about 4 m of the window and then the visual servoing loop was activated. The helicopter trajectory successfully converged to a position where the target was centred in the image.

Researchers at the University of California Berekely used a Yamaha R-50 unmanned helicopter to test a vision system for control of landing [43, 44]. In this work, a target comprising a black and white pattern of small squares of different
sizes was placed on the ground. A camera image was first thresholded by using a histogram technique with the intensity cuttoff set by trial and error. Once the landing target was segmented from the background, the corners of the squares were detected and each square was identified based on the centres of gravity. A non-linear optimisation technique was used to calculate the position and attitude of the UAV relative to the target. In their flight tests, the output of the vision algorithm was compared to the output of the onboard GPS/INS system. The results compared to within 5 cm in translation and $5^{\circ}$ in attitude.

Amongst other UAV programs, Georgia Tech operate a Yamaha RMAX UAV dubbed the GTMax. This helicopter has been fitted with a variety of sensors including differential GPS, inertial, magnetometers, an ultrasonic height sensor and a camera. In their experiment, a target on the ground comprising a dark square on a light background was tracked from a camera on the GTMax. Given the position and orientation of the target apriori, an Extended Kalman Filter (EKF) was used to fuse inertial data with data from the visual system, to provide the attitude and position of the helicopter without GPS [45]. Using an EKF output as a reference, the GTMax was able to follow position commands from the ground [46].

Proctor et al [47] constructed a glider with a camera and onboard video transmitter which responds to commands telemetered from a control computer on the ground. The guidance and control system used a high-speed pattern matching technique $[48,49]$ to track the outline of an open window using only a nose mounted camera on the glider, with the aim of getting the glider to fly through the window. A discrete EKF was used to estimate the states of the glider from the observed window geometry. The results of simulation and flight test of the glider provided an indication that, under some conditions, it would be possible to guide an aircraft using vision alone.

### 2.4 Methods for Calculating Optic Flow

In selecting a method for calculating optic flow for my outdoor experiments, attention was directed towards methods that are computationally efficient and fast enough to be implemented in real-time on hardware small enough to be flown onboard the helicopter. The methods had to be deterministic so that the execution time never takes longer than the allocated time for processing within the control loop (20 milliseconds). Finally, the method chosen had to be robust to noise so that artifacts resulting from vibration, radio interference, dust and outdoor lighting effects did not cause the method to fail.

Considerable work has been completed over the last three decades on developing techniques for calculating optic flow robustly. The main approaches are correlation, gradient models, energy methods and phase methods. An explanation of the classical techniques can be found at [50]. A quantitative comparison of some of the bestknown techniques for calculating optic flow is provided by Barron and Beauchemin in [51]. I will provide an overview of the main techniques described in the literature before describing the image-interpolation technique used in this thesis.

### 2.4.1 Correlation

Correlation is probably the most obvious technique and involves matching image regions or features between frames from different times. The objective is to find the shift between two image regions, that correspond to the same features being observed, which maximises some quantitative measure of similarity. The correlation coefficient defined by the integral in Equation (2.1) is one such measure where $f(x, y)$ and $g(x, y)$ are the intensities of the two frames being compared. Several types of correlation measures are used including mean-normalised correlation, variancenormalised correlation and sum of squared differences (SSD).

$$
\begin{equation*}
\iint f(x+\delta x, y+\delta y) g(x, y) d x d y \tag{2.1}
\end{equation*}
$$

Anandan developed a hierarchical framework and algorithm for matching overlapping patches in an image [52]. Groups of pixels are compared with the surrounding pixel blocks to find the best match in terms of SSD to whole integer values of pixel location. A quadratic approximation to the SSD surface centred on this pixel location is then used to do a sub-pixel match. The search is conducted using a coarse grid first which is used to guide the match at finer levels. A smoothness constraint is used at each level of coarseness with a finite-element based minimisation technique to find a smooth displacement field that approximates the displacements found from the matching process.

Singh describes another two stage framework for computing image flow [53]. The first stage comprises a search strategy to find the best match between adjacent patches to find local measurements of optic flow. The second stage combines the velocity measurements for the pixels in a given image neighborhood using a Gaussian weighting function which weights measurements from the centre of the neighborhood more than those further from the centre.

Other correlation techniques are described in the literature including those by Bülthoff et al [54]; Dutta and Weems [55]; Little and Kahan [56]; Burt et al [57]; and Glazer et al [58]. According to Barron [51], matching techniques tend to have poor sub-pixel accuracy compared to other techniques such as gradient based methods. The correlation techniques can be robust but also tend to be computationally demanding [59].

### 2.4.2 Gradient Methods

Gradient methods exploit the spatiotemporal gradients of image intensity to calculate optic flow [60]. Gradient methods begin by assuming that the image intensity is constant, which is a reasonable assumption provided that lighting conditions are uniform and do not change suddenly, that specular reflections are small, shadows are not present and surfaces are not translucent. Note that this assumption is also required by most other optic flow methods [50]. With this assumption in place, the intensity of a given point in the image $f(x, y, t)$ does not change with time, so that we can write:


Figure 2.2: The aperture problem. Only the component of the image velocity which is perpendicular to a straight edge (b) can be determined when viewed through a small aperture. The global velocity ( V ) of a feature can only be found by combining several local velocity measurements or by making use of local curvature or texture if it exists.

$$
\begin{equation*}
\frac{d f}{d t}=0 \tag{2.2}
\end{equation*}
$$

This can be expanded using the chain rule of differentiation to:

$$
\begin{equation*}
\frac{\partial f}{\partial x} \frac{d x}{d t}+\frac{\partial f}{\partial y} \frac{d y}{d t}+\frac{\partial f}{\partial t}=0 \tag{2.3}
\end{equation*}
$$

or

$$
\begin{equation*}
F_{x} u+F_{y} v+F_{t}=0 \tag{2.4}
\end{equation*}
$$

where $F_{x}, F_{y}$ are spatial gradients and $F_{t}$ is a temporal gradient. The terms $u$ and $v$ are the image velocities in the $x$ and $y$ directions respectively. The spatial gradients can be calculated from the image by comparing adjacent pixels and the temporal gradient can be calculated from the difference between consecutive frames arriving in the image sequence. This leaves two unknown variables $u$ and $v$ which cannot be determined using Equation (2.4) alone. The inability to solve for both $u$ and $v$ is known as the aperture problem [61]. The aperture problem can be understood in physical terms by imagining a straight edge translating within a small image window as in Figure 2.2. Only components of motion perpendicular to the edge can be determined without further information. A number of additional constraints have been proposed to solve Equation (2.4) and avoid the aperture problem. These generally involve measuring the local velocity components along the direction of the local intensity gradient and then combining them together to obtain the global velocity [62]. Methods for doing this include simple averaging [63] and neural networks [64].

Lucas and Kanade [42] proposed a solution to the aperture problem by assuming that the unknown flow components are constant within some image window defined by $\psi$. The values of flow are found by minimising the error function $\mathcal{E}_{a}$ defined in Equation (2.5).

$$
\begin{equation*}
\mathcal{E}_{a}=\iint \psi \cdot\left(F_{x} u+F_{y} v+F_{t}\right)^{2} d x d y \tag{2.5}
\end{equation*}
$$

Another popular way to combine the local velocities to get a global velocity
field is with a smoothness constraint which assumes that the flow velocities vary smoothly between adjacent points in the image. For example, Horn and Schunk [65] use a smoothness constraint based on minimising the square of the magnitude of the gradient of the optical flow velocity $\mathcal{E}_{c}^{2}$ where:

$$
\begin{equation*}
\mathcal{E}_{c}^{2}=\left(\frac{\partial u}{\partial x}\right)^{2}+\left(\frac{\partial u}{\partial y}\right)^{2}+\left(\frac{\partial v}{\partial x}\right)^{2}+\left(\frac{\partial v}{\partial y}\right)^{2} \tag{2.6}
\end{equation*}
$$

In Horn and Schunk's method, the smoothness criterion and the sum of the errors $E_{b}$ in the image intensity equation are minimised simultaneously. A weighting factor $\alpha$ is used to describe the relative weighting of these errors as in Equation (2.8). An iterative scheme is then used to find the velocities which minimise the total error $\mathcal{E}$.

$$
\begin{gather*}
\mathcal{E}_{b}=f_{x} u+f_{y} v+f_{t}  \tag{2.7}\\
\mathcal{E}^{2}=\iint\left(\alpha^{2} \mathcal{E}_{c}^{2}+\mathcal{E}_{b}^{2}\right) d x d y \tag{2.8}
\end{gather*}
$$

Nagel [66] also uses a smoothness constraint but uses second order derivatives to orient the smoothness constraint so that it is not imposed across steep intensity gradients. This enables real-world images containing edges and occlusions to be processed. Numerous other smoothness constraints have been proposed. Incorporation of smoothness constraints are computationally intensive [62] and do not always lead to the correct global velocities [67].

Lucas and Kanade's technique is classified as a local method whilst techniques using smoothness constraints are classified as global methods. The major drawback of the local methods are that they do not overcome the aperture problem in parts of the image where the image gradient is small [68], leading to sparse flow fields. Global methods on the other hand provide denser flow fields but are thought to be more sensitive to noise [51, 69].

Srinivasan developed a generalised gradient based algorithm [70] which addresses some of the problems with gradient algorithms, namely deriving the correct global velocity without first getting the local velocities and requiring higher order derivatives. Srinivasan's technique uses six spatiotemporal filters applied to the same image patch. The size of the patch determines how local or global the resulting velocities are. In later work [71], however, Srinivasan develops the Image Interpolation algorithm and shows that it has better robustness and accuracy with a negligible increase in execution time [72].

### 2.4.3 Energy methods

It has been suggested that some image motion properties are more evident in the frequency domain $[73,74]$. There is biological evidence to support the use of such elements in nature [75]. Energy methods work by finding the energy peaks in the spatiotemporal spectrum from a sequence of images. The term energy tends to be used rather than power to be compatible with the spectrum involving coordinates in
both time and space. Energies are calculated by means such as summing and squaring of the outputs from the applied filters. Various combinations of spatiotemporal filters of different orientation sensitivities are used to span the spectrum adequately.

Adelson and Bergen [76] explain that image motion presents itself as orientation in space-time. They propose a concept using a series of spatiotemporal filters to extract the orientation and hence the flow from the image. Heeger [75] points out that the power spectrum of a translating two dimensional texture occupies a tilted plane in the frequency domain. Using Gabor filters, Heeger presents results for an optic flow algorithm based on extracting orientation.

Watson and Ahumada [73] use a combination of delays, temporal and spatial filters to form sensors which are tuned to particular spatial frequencies and directions. Multiple sensors at different orientations and centred at different locations in the image are used to determine a complete flow field. For testing Watson and Ahumada's sensors, 16 consecutive frames were required. This type of approach is therefore computationally intensive [71]. Energy models can also be quite susceptible to variations in the contrast of image components [73].

### 2.4.4 Phase-Based Techniques

Phase based methods use the phase behaviour of band-pass filter outputs to measure optic flow. Phase methods have the advantage of being able to cope with several different velocities occurring on the same spatial patch, which would occur in the case of changing occlusion and transparency. They are also more robust to noise [74] than the amplitude-based energy methods discussed previously, owing to the independence of phase from amplitude variations resulting from changes in lighting conditions.

The use of phase dates back to the work of Hildreth [77]; Buxton and Buxton [78]; and Duncan and Chou [79]. In this early work, binary edge maps were generated using various filters such as the Laplacian of a Gaussian which can be seen as finding phase zero-crossings. Fleet and Jepson have provided a generalised treatment of the use of phase information for optic flow in reference [74] in which the local optic flow is computed from the motion of contours of constant phase. Their method uses the output from a series of linear velocity tuned filters similar to the ones used by Heeger [75].

### 2.4.5 Image Interpolation Technique

I have chosen to use Srinivasan's Image Interpolation Algorithm [71], abbreviated $I^{2} A$, to compute optic flow resulting from the motion of a plane perpendicular to the camera. The technique is non-iterative, does not require identification or tracking of individual features in the image, and does not require the calculation of high order spatial or temporal derivatives. These features make it robust to noise and quick to execute. Of all the techniques reviewed, these properties make $I^{2} A$ most suited to use for flight control.

Srinivasan discusses versions of the algorithm for any combination of translations
parallel to the plane with a rotation about an axis perpendicular to the plane. A simplified version of the algorithm is also discussed which calculates the flow due to pure lateral and longitudinal translation. For small rates of rotation, e.g. less than about 10 degrees between consecutive frames, it is possible to accurately extract translation signals at different parts of the image whilst neglecting the rotation effects. For the work in this thesis, where optic flow is applied to the control of hover and forward flight, the yaw rate of the helicopter is constrained by the tail rotor control loop to be less than 0.1 degree per frame, hence, only the translatory flow in various parts of the image needs to be considered. Furthermore, it is possible to determine rotation from this flow field by looking at the distribution of translation flow components in the image. This makes inclusion of the effects of rotation in the algorithm of limited benefit, allowing us to dispense with the added computation which is significant since the rotated reference images would need to be generated using trigonometric functions. A simplified version of the algorithm is therefore used which calculates the flow due to lateral and longitudinal translation only. A detailed derivation of the $I^{2} A$ algorithm can be found in [71].

The algorithm compares the current image with a set of four reference images which are translated from a previous frame. We define the pixel intensity functions at times $t_{0}$ and $t$ as $f_{0}(x, y)$ and $f(x, y)$ respectively where $x$ and $y$ are the image coordinates measured in pixels. The reference images $f_{1}, f_{2}, f_{3}$ and $f_{4}$ are formed from the first image by shifting them by reference shifts $\Delta x_{r e f}$ and $\Delta y_{\text {ref }}$ pixels along the x and y axes, so that Equation (2.9) applies.

$$
\begin{align*}
& f_{1}(x, y)=f_{0}\left(x+\Delta x_{r e f}, y\right) \\
& f_{2}(x, y)=f_{0}\left(x-\Delta x_{r e f}, y\right) \\
& f_{3}(x, y)=f_{0}\left(x, y+\Delta y_{r e f}\right) \\
& f_{4}(x, y)=f_{0}\left(x, y-\Delta y_{r e f}\right) \tag{2.9}
\end{align*}
$$

The $I^{2} A$ algorithm relies on the assumption that the image at time $t$ can be linearly interpolated from $f_{0}$ and the reference images so that Equation (2.10) applies.

$$
\begin{equation*}
\hat{f}=f_{0}+0.5\left(\frac{\widehat{\Delta x}}{\Delta x_{\text {ref }}}\right)\left(f_{2}-f_{1}\right)+0.5\left(\frac{\hat{\Delta y}}{\Delta y_{\text {ref }}}\right)\left(f_{4}-f_{3}\right) \tag{2.10}
\end{equation*}
$$

Our objective is to calculate the translations $\Delta x$ and $\Delta y$ which give the best match between the actual image $f$ and the interpolated approximation $\hat{f}$. We can express this as a least square problem over an image patch defined by the window function $\Psi$ by minimising the error $E$ where $E$ is defined by Equation (2.12). The two dimensional window function $\Psi$ specifies the size and form of the patch. Gaussian, triangular and boxcar kernels could be used, but the latter requires much less computation time.

$$
\begin{align*}
E & =\iint \Psi \cdot[f-\hat{f}]^{2} d x d y  \tag{2.11}\\
& =\iint \Psi \cdot\left[f-\left\{f_{0}+0.5\left(\frac{\widehat{\Delta x}}{\Delta x_{r e f}}\right)\left(f_{2}-f_{1}\right)+0.5\left(\frac{\hat{\Delta y}}{\Delta y_{r e f}}\right)\left(f_{4}-f_{3}\right)\right\}\right]^{2} d x d y
\end{align*}
$$

The error may be minimised by taking the partial derivatives of $E$ with respect to $\Delta x$ and $\Delta y$ and setting them to zero. After simplification, the two resulting linear equations are at (2.12-2.13) can be solved simultaneously for $\Delta x$ and $\Delta y$.

$$
\begin{array}{r}
\left(\frac{\widehat{\Delta x}}{\Delta x_{r e f}}\right) \iint \Psi \cdot\left(f_{2}-f_{1}\right)^{2} d x d y+\left(\frac{\widehat{\Delta y}}{\Delta y_{r e f}}\right) \iint \Psi \cdot\left(f_{4}-f_{3}\right)\left(f_{2}-f_{1}\right) d x d y \\
=2 \iint \Psi \cdot\left(f-f_{0}\right)\left(f_{2}-f_{1}\right) d x d y \tag{2.12}
\end{array}
$$

$$
\begin{align*}
\left(\frac{\widehat{\Delta x}}{\Delta x_{\text {ref }}}\right) \iint \Psi \cdot\left(f_{2}-f_{1}\right)\left(f_{4}-f_{3}\right) d x d y & +\left(\frac{\widehat{\Delta y}}{\Delta y_{\text {ref }}}\right) \iint \Psi \cdot\left(f_{4}-f_{3}\right)^{2} d x d y \\
& =2 \iint \Psi \cdot\left(f-f_{0}\right)\left(f_{4}-f_{3}\right) d x d y \tag{2.13}
\end{align*}
$$

### 2.5 Helicopter Control

The last decade has seen a plethora of control techniques applied to unmanned helicopters appear in the literature. My aim in reviewing this literature has been to determine the most simple yet practical control system for a helicopter that would allow demonstration of visual control. I have examined both classical, modern and black-box control techniques.

### 2.5.1 Classical Control

The essence of classical control is successive feedback loop closure using linear control elements such as Proportional (P), Integral (I), Derivative (D) and combinations of these (e.g. PI for Proportional and Integral feedback). The gain of these controllers can be tuned by trial and error, with empirical techniques such as those employed by Ziegler and Nichols [80] or, in the presence of an accurate system model, using tools such as root-locus [81] or frequency analysis [82, 83].

In Amidi's work [84] with a Yamaha R-50 helicopter, the helicopter controller was implemented as a series of PD controllers with output saturation. Single PD controllers were used with the collective and tail rotor pitch servos to control height and yaw respectively. The control of horizontal position and velocity was achieved
with a series of three nested loops with two PD controllers in each loop for separate control of the lateral and longitudinal channels. In the outermost loop, position errors were used to control two reference velocities. In the middle loop, the reference velocities were used as the input to the PD controllers for pitch and roll attitude reference values. Finally the reference pitch and attitude values were used as inputs to the PDs controlling lateral and longitudinal cyclic. This system effectively controlled the helicopter in hover to within 15 cm .

Saripalli et al [39] implement a very similar control scheme to Amidi except in their system a PI controller is used instead of a PD controller for control of altitude and a simple proportional controller is used to control the inner loop attitude. Mettler points out in [85] that the derivative feedback for the attitude inner loop does not provide any advantage since the Bell-Hiller stabiliser bar fitted to most unmanned helicopters already provides attitude rate feedback. Similar systems have also been used by Buskey et al [86]; Sanders [87]; and Mettler et al [85].

Small-scale helicopters have faster dynamics than full-size helicopters which makes them harder to control [88]. Most small helicopters designed for human piloted control are therefore fitted with a mechanical stabilizer bar that provides lagged attitude rate feedback to the main rotor, to improve stability. However, the presence of a stabilizer bar complicates automatic control as it decreases the stability margin for the coupled dynamic mode between the rotor and the fuselage. This prevents high gain controllers from being implemented. Mettler et al [85] describe the use of a notch filter in combination with a series of PD controllers to develop an attitude controller for a Yamaha R-50 helicopter. The notch filter was set to the natural frequency of the fuselage-rotor mode allowing the roll angle feedback gain to be increased more than fivefold.

### 2.5.2 Modern Control Methods

The design of controllers using classical methods such as root locus and Bode methods is essentially trial and error [89]. The two concepts defining modern control are that: (a) the design is based directly on the state-variable model and (b) performance specifications are expressed in terms of a mathematically precise performance criterion [90]. All the gains are solved simultaneously and are directly solved from the state space matrix algebra. Modern controllers offer the prospect for improved performance and can, in some cases, adapt to changing conditions. The downside of most modern controller methods is that they require unmeasured states which can be hard to accurately estimate. Whilst many modern control techniques are shown to be closed loop stable and function on real helicopters, improvements in performance over equivalent classical controllers on the same plant are rarely confirmed or quantified in the literature. Furthermore, they may require accurate system models which, in the case of a highly non-linear helicopter, may be difficult to achieve. It is often stated that the use of PID control requires laborious tuning to take place. Whilst this is true to some extent, the systems identification step required to develop a modern controller is no less laborious and opens up many opportunities for numeric error during the controller design stage. I have included a brief overview
of the application of modern control theory to helicopter control for the sake of completeness.

Many of the modern control techniques are linear methods. As the helicopter is non-linear, linearisation of the system model is necessary in order to design a working controller. Koo and Sastry present approaches to this based on differential flatness [91] and approximate feedback linearisation [92]. In these techniques, approximate models are developed which are minimum phase, whereas the actual plant being modelled is in fact non-minimum phase [93]. Unfortunately their work was conducted in simulation models which ignore the rotor and actuator dynamics, so successful performance on a real system is not guaranteed.

Optimal control is a branch of control theory that aims to find control laws to maximise some optimality criterion. A typical optimality criterion involves a linear quadratic form which serves to simultaneously minimise the control energy and the time it takes the system to reach steady-state. Noting that the model has to be linearised, and is therefore only an approximation of the real plant, it can be argued that the condition of optimality can become somewhat invalid. Furthermore, the choice of weighting factors used in the optimality criterion can become somewhat arbitrary. Optimal controllers for helicopters have been proposed by Morris [94]; Mettler et al [95] and Bogdanov et al [96]. Morris et al tested a Linear Quadratic Gaussian (LQG) controller [97, 98] on a small electric helicopter constrained to moving in only pitch, roll and yaw. Even with this simplified system, the system performance was not good, owing to unmodelled dynamics and the lack of angular rate sensors. Mettler et al developed a successful LQG controller for the lateraldirectional dynamics of a small helicopter by ignoring the flapping dynamics and treating the helicopter as a rigid body. Using a notch filter to attenuate the fuselagerotor mode, the controller was shown to successfully control bank angle in flight, however removal of the notch filter resulted in instability.

Bogdanov et al describe State-Dependant Riccati Equation (SDRE) control [99] of an X-cell 8 kg RC helicopter and the Yamaha RMAX helicopter. The approach consists of approximating the non-linear model with a linear set of state update equations at each time step based on the current state. Treating the state equations as linear, the Ricatti equation is then solved and used to calculate optimal control inputs based on a linear quadratic cost function. Bogdanov's results for trajectory control of an RMAX flying a $183 m \times 183 \mathrm{~m}$ square racetrack pattern are good with a position accuracy of 2 m and an altitude accuracy of about 1.2 m . However, the computational power required to solve the Ricatti equation is demanding, requiring 14 ms out of 20 ms at 50 Hz on a 300 MHz Pentium CPU to compute. This level of processing could not be achieved on either of the systems used for this project as it would have required an additional CPU to be added.

One method of dealing with inaccurate systems identification is to apply the principles of robust control. In robust control methods, such as $\mathcal{H}_{2}$ and $\mathcal{H}_{\infty}$ [100], the objective is to ensure sufficient stability remains in all scenarios that could arise from the presence of noise, disturbances and bounded uncertainties in the plant model. A number of researchers have applied robust controllers to small helicopters. Takahashi applied a $\mathcal{H}_{2}$ controller to a control of hover in a full-size helicopter simulator.

Bendotti and Morris discuss a $\mathcal{H}_{\infty}$ controller for a model helicopter constrained to 3 degrees of freedom only and show that it out performs an LQG controller [101]. Hashimoto et al develop and flight test a third order $\mathcal{H}_{\infty}$ velocity controller for a full-scale unmanned helicopter which augments an existing stability augmentation system [102]. Other examples of $\mathcal{H}_{\infty}$ techniques applied to helicopter control are La Civita et al [103] ; Shim et al [93] and Yang et al [104-107]. Recent results from test of a $\mathcal{H}_{\infty}$ loop shaping controller on a Yamaha R-50 helicopter at CMU have shown tracking performance which is claimed to exceed the performance of any other techniques in the published literature [108]. On the basis of this result, one would expect that $\mathcal{H}_{\infty}$ would be a good candidate for future unmanned helicopter control designs. However, this work was only made possible by the presence of a high fidelity simulation model [103], so I have not pursued the approach in this thesis.

A number of schemes for controlling a helicopter using backstepping [109] have appeared in the literature. Backstepping is a recursive technique whereby the complete system is built up in cascaded stages, starting with a simple system that can be stabilised using a known Lyapounov function. As each system is stabilised, an integrator is added to the input and then the same process is repeated to design a Lypanov function and feedback law to stabilise the new system. The process continues until the complete system is stabilised. The advantage of backstepping is that it can be applied to nonlinear system without having to make linearising approximations. It also guarantees stability and can be made to be adaptive [110]. Mahoney and Hamel present simulation results for a robust trajectory tracking controller using backstepping [111] which can enable a priori bounds on the tracking performance to be set. A very similar approach is taken by Frazzoli et al [112] who also develop a backstepping trajectory control in simulation. The disdavantage of backstepping is that, even for simple systems, the algebra can become very complicated and unwieldy. Hence, the currently published applications of backstepping to helicopters tend to make a number of assumptions to simplify the models that are unrealistic [113], such as assuming that the rotor can be manipulated to provided instantaneous control forces and moments. For example the backstepping algorithm by Mahoney et al [114] assumes that the flapping angles are directly measurable and controllable, neither of which is practical. Whilst backstepping does show promise, the complexity is not consistent with my stated aim of keeping the control systems as simple and practical as possible.

### 2.5.3 Black-box Approaches to Control

The complexities and mathematical tedium of developing high-fidelity systems identification models of a plant can be avoided by using a black-box approach where no assumptions are made about the internal structure of the plant. The two dominant approaches for achieving this are fuzzy logic and Artificial Neural Networks (ANN). Of these two, I have only pursued neural control because it has the greatest similarity to the neural foundation of insect flight control, which is the inspiration for this thesis. In addition, most of the work on fuzzy logic for helicopter control has been tested in simulation only, whereas there are some encouraging results using

ANNs that have been demonstrated on real helicopters. In some situations neural networks and fuzzy logic can be shown to be equivalent.

Control based on fuzzy logic uses the concept of simultaneous membership in various fuzzy sets. For example, if temperature feedback was being used in a control system, the controlled variable might have a membership in both hot and warm sets. As the temperature varies towards a state of being hot, the membership in the hot set would increase whilst decreasing in the warm set. This approach allows for smooth changes in parameters rather than the abrupt changes that would result from using a simple threshold with binary logic. The advantage of fuzzy control is that it can be built up from rules of logic that have physical meaning to the designer, unlike neural networks for which the internal mechanisms can be difficult or impossible to conceptualise.

The work on fuzzy logic control for helicopters dates back to the early 1990s and has continued until present day. In 1995, Sugeno et al [115, 116] implemented a set of 57 if-then type control rules for control of hover and slow speed flight on a Yamaha R-50 helicopter. Jiang et al [117] designed a fuzzy logic controller to adaptively cancel the errors in another controller which was based on approximate inversion of a helicopter plant model. The controller was then tested on an AH64 Apache helicopter simulation. Sasaki et al [118] applied a learning fuzzy logic controller to a linearised simulation of an RC helicopter in hover and showed that it provided stable hover control. Amaral et al $[119,120]$ showed in simulation that a neuro-fuzzy inference controller based on a combination of ANN and fuzzy logic controllers could be used to control a Sikorsky S-61 full-size helicopter in hover. Kadmiry and Driankov tested the use of fuzzy logic to schedule the gain for an unmanned helicopter in a non-linear simulation [121,122]. In a non-linear simulation for an X-Cell RC helicopter, Sanchez et al tested a combination of PID controllers for control of attitude and altitude and fuzzy logic controllers for controlling translation movement [123]. Other fuzzy logic controllers for helicopters have been tested in simulation by Cavalcante et al [124] ; Phillips et al [125]; and Hoffmann et al [126].

### 2.5.4 Artificial Neural Networks

A neural network is a computational system comprised of interconnected processing elements called neurons or nodes. Neural networks from the basis of mental function in the brains of animals. Artificial neural networks (ANN) are a biologically inspired paradigm for computing, implemented either in software or hardware. The idea of using an ANN to perform computation was first proposed by McCulloch and Pitts in 1943 [127].

The computational requirements for executing ANN (once trained) are generally small and can be readily implemented for real-time control, making them suited to implementation on a small robot where only modest computing resources may be available. ANN can also be trained to perform a task without having to explicitly model or understand the underlying dynamics. This is good for complicated systems which may be unweildy or analytically non-tractable. ANNs can act as excellent function interpolators, but cannot be relied on to extrapolate into regions where


Figure 2.3: Model of a neuron [129].


Figure 2.4: A multilayer network.
they have not been trained. Care must therefore be exercised in their use.
The nodes in a neural network are connected to other nodes in such a way that the nodes collectively produce some useful output in response to an input stimuli. Owing to the interconnectivity of the nodes, processing in the network is done in parallel, rather than in series as would be done in a conventional digital computer program. For example, a single neuron in a human brain may be connected to up to 10,000 other neurons. This parallelism enables fast processing. Networks may be arranged into layers as in the example shown in Figure 2.4. The network shown consists of an input layer, hidden layer and output layer. The hidden layer is so named because the outputs of its neurons are not seen directly from outside the system. This topology is by no means fixed and any number of layers may be used and connected arbitrarily. For example in the Hopfield Network [128], every node is connected to both inputs and outputs.

In an ANN, the connections to other neurons are usually represented as weights, such that the input to one neuron is the sum of the products of the outputs of the other neurons and the respective weighting terms $W_{i j}$. The effect of each node is to apply an activation function to the sum of the weighted inputs as shown in figure 2.3 . Equations (2.14) and (2.15) define the neuron mathematically.

$$
\begin{align*}
& u_{k}=\sum_{j=1}^{m} w_{k j} x_{j}  \tag{2.14}\\
& y_{k}=\varphi\left(u_{k}+b_{k}\right) \tag{2.15}
\end{align*}
$$

The activation function can be linear or non-linear and combinations of different activation functions can be used in the same network. One of the major breakthroughs in ANNs was the use of a sigmoidal transfer function which allowed the network to approximate any non-linear function, given sufficient nodes. A sigmoidal function is an s-shaped function that has an approximately linear middle region with a graceful non-linear saturation. A common sigmoidal activation function is the logistic function $\varphi_{\text {log }}(\cdot)$ defined in Equation (2.16).

$$
\begin{equation*}
\varphi_{\log }(\nu)=\frac{1}{1+e^{(-a \nu)}} \tag{2.16}
\end{equation*}
$$

In the various training algorithms that have been developed for ANNs, the objective is to adaptively change the weights in the internodal connections so that the performance of the network is optimised. The most common methods for training neural networks are supervised learning, unsupervised learning, and reinforcement learning techniques. In supervised learning the network is provided with example pairs of input data $x$ and output data $y$ from which it can learn a mapping between the two. In unsupervised learning, the ANN is given the input data $x$ but instead of providing the desired outputs $y$, a cost function is provided, which is to be optimised in some way. The cost function can be any function of the inputs and outputs. Reinforcement learning is similar to Unsupervised learning, except that the input data is not provided beforehand, but generated by the interaction of the network with its environment. In reinforcement learning, the network must learn by trying various policies to optimise a reward signal provided to the network as a consequence of its actions (outputs) [130].

After a period of dormancy in the 1970's [129], neural networks underwent a resurgence of interest following the publication of a book edited by McClelland and Rumelhart [131] which outlined a method for training multi-layer ANNs using the back-propogation algorithm. Backpropogation $[132,133]$ is a gradient descent technique designed to minimise the least square error between the ANN outputs and the desired outputs. The training proceeds with a series of forward and backward passes through the network using a set of training data. In the first pass, the outputs of the neurons are propogated forwards using Equations (2.14-2.15), starting from the input side of the network, working through each layer in turn, until the output layer is reached. In the forward pass the network weights are fixed. In the second pass, the output errors are propogated backwards through the network and used to adjust the network weights using an error-correction learning rule. As the cycle is repeated for different training inputs and desired outputs, the weights of the network gradually adapt to improve the match to the desired outputs. Whilst backpropogation is well-known and easy to implement, it can be very slow to converge. Other algorithms have been proposed which have much faster convergence for most problems. The Levenberg-Marquardt (LM) algorithm [134] is renowned as one of the fastest techniques for training ANNs with up to several hundred weights [135]. The LM algorithm was used for the work on ANNs in this thesis.

Certain kinds of ANNs can achieve temporal processing by making use of delayed inputs. This allows ANNs to perform functions such as differentiation, integration, filtering and state space representation of plant. A number of mechanisms for temporal processing can be used, such as placing tapped delays on some of the inputs to provide the network with a time history. The outputs of any layer can also be passed back to the input layer. For example, the outputs of the hidden layer could be passed back to the input layer through a tapped delay to provide a state-space representation of the dynamics being modeled. Networks that have at least one feedback loop in their structure are known as recurrent networks, whereas feed-forward neural networks do not feedback the output of any neuron to a pathway which can affect the input of the same neuron. In this thesis, use is made of recurrent ANNs to perform numerical integration for state-space propogation.

### 2.5.5 Flight Control using Neural Networks

Teaching by Showing control for the attitude control of a small helicopter was demonstrated in simulation by Montogomery and Bekey [136]. Their approach was to augment a fuzzy logic controller with a neural network. The fuzzy logic controller was implemented using a set of fuzzy rules relating angles and rate errors to roll and pitch control inputs. The number and type of membership functions making up the structure of the fuzzy controller were decided upon by a competent human pilot based on their perceptions of how they flew a helicopter. A fuzzy learning algorithm devised by Wang [137] was used to tune the fuzzy rule set as a teacher flew the helicopter. In practise, the teacher could be a human pilot, but in this case a non-linear controller described in reference [138] was used. Once trained satisfactorily, the fuzzy logic controller was activated and the teacher relinquished control. Whenever the control performance fell below some criterion, due to changes in the system for example, the trainer would take over control again and then a neural net would be trained to predict the difference between the fuzzy controller and the trainer. Once trained, the neural net output would then be used to augment the fuzzy controller so as to better mimic the teacher. The combined fuzzy/ANN controller was found to outperform the standalone fuzzy controller and provide stable control of attitude.

Buskey et al trained a simple feed-forward ANN with 10 hidden units to mimic the actions of a human pilot [139]. The network inputs were accelerometer and rate gyroscope outputs. For flight test, only the aileron servo was driven by the ANN whilst the other servos were controlled by the pilot. The helicopter hovered for 15 seconds in this fashion until a wind gust necessitated the pilot taking over control. The authors of this work attempted to use recurrent neural networks at first but abandoned the attempt as no advantage was achieved from doing this with their simple approach.

In 2001, Bagnell and Schneider demonstrated a reinforcement learning policy for an autonomous helicopter [140]. In their scheme, a pair of very simple feed-forward networks were used, comprising 5 weighted connections for each of the pitch and roll control channels. The inputs to the roll channel network were lateral position, lateral velocity and roll angle. Equivalent inputs were used for the pitch channel which was treated as decoupled from the roll axis. A single hidden layer node was used between the position input and the control output. The network functioned in a similar fashion to a linear PD controller but the sigmoidal nature of the hidden unit provided the ability to deal with large changes in the position set point. The network weights were optimised using the amoeba technique outlined in [141] based on various trajectories calculated in simulation. Cost functions which minimised the velocities, position errors and control inputs were used. After proving the stability of the resulting controller in simulation, the controller was tested on a R-50 helicopter belonging to CMU. The helicopter was able to track moving position set points in the presence of strong wind gusts.

In 2004, Ng et al published the results for a simple neural network controller for a Yamaha R-50 helicopter using another reinforcement learning technique [142]. The approach was to first identify a non-linear 12 state model of the helicopter
dynamics in hover. Next the PEGASUS reinforcement learning algorithm of [143] was used to teach the ANN a policy for hovering in place. The inputs to the ANN were the difference between the actual and desired helicopter state. The outputs of the net were the cyclic pitch, collective pitch and tail rotor controls. The ANN was structured so such that the lateral, longitudinal and vertical channels were separated. For trajectory following, feed-forward between the yaw channel and the other channels was also added. A quadratic cost function based on the control input energy, velocities, orientation and the errors in position was used to optimise the ANN weights. This technique was successful in not only hovering the helicopter, but also in allowing it to follow trajectories, including a nose in circle and a rectangle in the vertical plane. In later work [144], autonomous control of inverted hover was successfully demonstrated using the same techniques.

Extensive work has been progressed at Georgia Tech to use neural networks to augment other controllers. The aim of this approach is to reduce the reliance on high-fidelity mathematical models when designing high-bandwidth controllers and to allow the controller to adapt to changing environmental conditions. In 1994, Calise et al published a scheme for adaptively correcting the inversion errors for an attitude controller based on an inverted model of the helicopter attitude dynamics [145]. The inverse controller known as the Rotational Dynamics Inversion Processor (RDIP) takes as inputs the desired angular accelerations $\ddot{\phi}, \ddot{\theta}$ and $\ddot{\psi}$ and outputs the control signals which it predicts should approximately accomplish these accelerations. The desired angular accelerations were the output of PD controllers for roll, pitch and yaw channels which were in turn driven by the output of a second order command filter. Pilot commands were passed through the command filter to provide desired outputs $\theta_{d}, \dot{\theta}_{d}, \ddot{\theta}_{d}$. The outputs used to generate a commanded angular acceleration $U_{c}$ using $U_{c}=K_{p}\left(\theta_{c}-\theta\right)+K_{d}\left(\dot{\theta}_{c}-\dot{\theta}\right)+\ddot{\theta}_{c}$ where $K_{p}$ and $K_{d}$ are the gains of the PD controller. The inversion model was a linear function of collective pitch angle, linear velocities, angular rates and $U_{c}$. As the RDIP was based on a simple linear model of the dynamics it contained errors owing to model uncertainty and the unmodelled non-linearities. The ANN outputs were added to the desired angular accelerations coming from the PD controllers to correct the inversion errors. A training scheme based on the notion of Lyapunov stability and described in reference was used to train the network online so that it could adapt to the inversion errors in flight [146]. When tested in a high fidelity Apache helicopter simulation, the controller was found to track attitude commands much more precisely with the neural augmentation than without it.

The work by Calise et al was extended by Leitner et al in 1998 to include a second controller based on inversion which controlled the translational dynamics [147]. A trajectory outer loop was built around the attitude inner loop and tested in simulation. The outer loop uses an inverse dynamics block to calculate attitude commands for the attitude controller. The inputs to the translational inverse dynamics block were the desired linear accelerations which were calculated from the position/velocity commands in a similar fashion to that used in the attitude command filter. The translational inverse was a very simple analytic expression derived from consideration of the desired tilt of the platform to obtain the desired accelerations. In 1999,

Prasad et al published another application of this technique tested on an R-50 helicopter which implemented a rate control attitude hold (RCAH) function, by adding an integrator to the commanded rate system [148].

In [149], Calise et al simulated R-50 pitch attitude control using the same neural augmentation techniques with the addition of Pseudocontrol Hedging (PCH). PCH was used to adjust the output of the command filter (i.e. the pseudocontrol) so that the ANN did not see the actuator characteristics as model tracking error. Actuators suffer from saturation and add high order dynamics and a pure delay effect. The inverse model neglected the actuators and the neural network could not adequately cope with effects such as saturation, so that the PCH was found to be necessary to allow the ANN to adapt correctly. The 2nd order command filter was also replaced by a 3 rd order command filter as it better matched the dynamics of the underlying system, dominated by the actuators. The new command filter and the PCH gave much better results than previously attained. Other variations of this approach were tested by Hovakimyan et al [150] who added a state space observer to estimate the inversion errors and Corban et al [151] who added tapped delay lines to the network inputs. In [152], Corban et al added an outer loop velocity controller around the attitude loop already described and used a first order velocity filter to convert the velocity commands from the pilot to pseudocontrol accelerations. PCH was used to protect outer loop from adapting to inner loop dynamics. Using the same control structure as Corban et al, Johnson and Kannan have obtained excellent tracking results on the GTMAX unmanned helicopter [153]. These have included precise tracking of racetrack, circular and square patterns in forward flight as well as following a complex 3D trajectory previously flown by a human pilot and logged. The ANN structure used in the neural augmentation work predominantly consisted of two layers with five nodes in the hidden layer. The inputs to the ANN were the full state and pseudocontrol error.

### 2.5.6 Controller Selection

Based on the literature, I have chosen to initially apply a classical helicopter controller based on PID controllers and an attitude inner loop. This approach has been shown to work on other small-scale helicopters and has a number of advantages over other methods for experimentation. One such advantage is that the inner-loop can be used as a stability augmentation system in the event of the outer loops being deactivated or malfunctioning. The controller can be built up in stages with the inner loop being designed and tested first. The inner loop also provides a bound on the reference attitude used so that if the outer loop were to malfunction, the effect on the helicopter can be constrained.

PID controllers are preferred because they are easily implemented and debugged, can be readily tuned by trial and error and analysed without having a detailed plant model. Whilst using the wrong gains can result in instability during tuning, this can be overcome experimentally with a safety pilot. If the pilot is simultaneously able to provide control inputs to augment the controller, gains can be gradually ramped up until instability is reached and then backed off to provide desirable
control performance.
Noting the biological parallels, I also intend to pursue the use of neural networks for sensor fusion and control. There are many examples in the literature that show that ANN can be used for flight control. ANN are computationaly efficient and can easily be implemented with the hardware on the helicopter.

### 2.6 Summary

I have identified that visual control of an unstable rotorcraft is feasible. Based on the biological observations, the use of optic flow would appear to be a very useful sensing technique for control of flight, and I will aim to make use of this approach. Noting its reported robustness and ability to be computed in real-time, I will use the optic flow image interpolation algorithm $\left(I^{2} A\right)$ to calculate optic flow.

# Helicopter Simulation and Control 

### 3.1 Introduction

The testing of new control and sensor algorithms is made practical for this thesis by using a time domain simulation in SIMULINK ${ }^{\circledR}$ of the helicopter dynamics, sensors and controller. This allows the practicality of algorithms to be tested rapidly without putting the helicopter at risk. The aim of the simulation is not to develop controllers that can be copied directly from the simulation to the helicopter without further tuning of control gains. Rather, the aim of the simulation is to determine what style of controller can be used on the helicopter, taking into account the types of sensors proposed. The simulation is also an excellent tool for developing sensor fusion algorithms as in most cases the code can be transferred directly from the simulation as a C file to the helicopter autopilot. For this thesis, a simulation of the Eagle helicopter is used for all of the development work.

A number of other researchers have constructed simulations for small unmanned helicopters. Munzinger [154] created a real-time hardware-in-the-loop (HIL) simulation running on a Single Board Computer for a Yamaha R-50 unmanned helicopter. The output of the simulation could be used to mimic the sensor data passed to the actual onboard control system. Cunha and Silvestre [155] have developed a generic helicopter dynamic simulation model and used this model to demonstrate an LQ state feedback hover controller. Gavrilets et al developed and validated $[156,157]$ a HIL simulation under the QNX ${ }^{\circledR}$ real-time operating system for their X-Cell 60size helicopter. The Gavrilets simulation incorporated servo dynamics and sensor characteristics and provided a good match with experimental data for a wide range of conditions. Kowalchuk et al [158] created a 3 degree of freedom longitudinal dynamics model of their Raptor 50 sized helicopter, created within MATLAB ${ }^{\circledR}$ with a good match to flight data for pitch rate and pitch angle.

Mathematical models for small-scale helicopters have been extensively documented in the literature in recent years [94,102,159-161]. The general approach is to combine a linearised rotor aerodynamic model with the non-linear rigid body equations. Sometimes a non-linear thrust and rotor inflow model is incorporated, usually involving an iterative scheme to simultaneously solve for the induced velocity through the rotor and the thrust. The non-linear fuselage and tailplane aerodynamics forces are only usually added when forward flight is considered. An underlying theme in simulating small helicopters has been the realisation that owing to the
complexity of the helicopter, minimum complexity simulations tends to be most practical, so most researchers have chosen to use the minimum level of detail required to capture the key system dynamics. This approach was championed by Heffley who published a report for NASA in 1988 on a minimum complexity helicopter model [162] that is regularly cited in the field. Heffley maintains that adding higher order effects such as the off-axis moments due to blade flapping does not automatically lead to a better match with flight data.

While hardware-in-the-loop models are very useful for specific platforms, a variety of different platforms and sensor combinations have been used in this thesis, so that the time spent writing interface code to mimic the data output protocols of certain kinds of sensors would be prohibitive. SIMULINK ${ }^{\circledR}$ was chosen as a development environment for convenience as the graphical drag and drop building block approach speeds up the development cycle and allows the simulation to be quickly adapted to other helicopters. Balancing simulation fidelity against practicality, a simulation has been created that is capable of simulating the following effects:

- Exact non-linear rigid body equations of motion;
- Wind gusts and turbulence;
- First order main rotor flapping dynamics;
- Hover, rearwards, sideways and forward flight;
- Dynamic effects of the Bell-Hiller stabiliser bar;
- Fuselage and tailplane aerodynamic forces;
- Approximate servo dynamics; and
- Sensor lags, filtering, offsets and noise.

The simulation does not model ground effect, interaction of rotor downwash with the fuselage and rotor lead-lag mechanisms. Higher order flapping modes are only really of interest when studying vibration and aeroelastic problems [163] so they have also been ignored. Only low rates of descent are allowed on the helicopter so that momentum theory can still be applied. This seems reasonable noting that all of the experiments conducted for the thesis involve flight close to the ground so that high descent rates cannot be achieved.

To simplify debugging, the simulation is divided into a number of blocks and subsystems. On the highest level, the simulation consists of an aerodynamics subsystem, sensor subsystem, sensor fusion block and controller subsystem as shown in Figure 3.1. Diagrams of the internal structure of each subsystem are provided in Appendix B. Most of the computational blocks have been implemented as C code S-functions. In all, the base simulation consists of 19 C -files not including the variety of blocks used later for testing sensor fusion algorithms and artificial neural networks.


Figure 3.1: Top level of eagle simulation


Figure 3.2: Body axes system

### 3.2 Helicopter Dynamics

Having discussed the motivation for the simulation, a brief overview of the underlying theory is now provided.

### 3.2.1 Aircraft Conventions

The body axes system of the helicopter is fixed to the aircraft centre of gravity and rotates as the aircraft's attitude changes as shown in Figure 3.2. This set of axes is particularly useful as the sensors are fixed with respect to the body axes. Another set of axes, known as the inertial axes, are required for navigation and defined with respect to the surface of the earth. The inertial axes are aligned so that the x -axis is horizontal and points North, the y-axis is horizontal and points East and the z-axis is positive down towards the centre of the earth. A precise mapping between the inertial and body axes can be made based on the attitude of the helicopter.

Using standard aircraft nomenclature, the velocity components of the helicopter along the body axes $x, y$ and $z$ are given the designations $u, v$ and $w$ respectively. Likewise, the body axes rotation rates of the helicopter are $p, q$ and $r$. The sense of the rotations are defined in accordance with a right hand axes system.

### 3.2.2 Attitude Representation

In order to properly define the orientation of an aircraft it is not only necessary to define a coordinate system about which to apply rotations, but also the order in which they are applied. Perhaps the easiest way to visualise attitude is using Euler angles. In aviation three Euler angles are used to describe the orientation of an aircraft with respect to an axes system fixed to the earth. These angles use the familiar designations roll, pitch and yaw. They are also commonly attributed the Greek names and symbols: $\phi, \theta$ and $\psi$, respectively. In aviation, the order of the rotations is yaw, pitch, then roll. Hence an attitude of $\left(45^{\circ}, 20^{\circ}, 15^{\circ}\right)$ would mean that you would start with a horizontal line aligned with heading North-East. Inclining this line by an angle of $20^{\circ}$ to horizontal would form the longitudinal axis of the aircraft (ie pitched up at $20^{\circ}$ ). The specified attitude would then be completed by rolling the aircraft to the right $15^{\circ}$ around the inclined line.

Although, physically meaningful, Euler angles suffer from some major drawbacks. At pitch angles of $\pm 90^{\circ}$, the mathematical description of attitude becomes a singularity so that yaw ceases to have any meaning. At angles close to $\pm 90^{\circ}$, significant computational errors may arise when using equations based on Euler angles to simulate or track aircraft attitude. There is also a problem with wraparound such that integrated values of Euler angles may exceed $\pm 90^{\circ}$ in pitch or $\pm 90^{\circ}$ in roll, which may make determination of a unique attitude difficult. Finally, the equations which propagate the attitude of a vehicle in Euler angles are highly non-linear and involve many trigonometric terms. For simulation and control, this makes them computationally expensive and less robust than alternative representations.

Other forms of attitude representation do exist. The most widely used of these, involves four variables instead of three, and gets past the problem of the singularity at high pitch angles. The four variables $q_{i}$ used are called quaternion parameters and are related to the Euler angles as shown below. The quaternions are more difficult to understand conceptually but do not suffer from wraparound or singularities and can be propagated with simple linear matrix algebra. However, it is usual to convert the quaternion attitude back to Euler angles when it is necessary to display the attitude or use it for control.

$$
\begin{align*}
& q_{0}=\cos \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2}+\sin \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \\
& q_{1}=\sin \frac{\phi}{2} \cos \frac{\theta}{2} \cos \frac{\psi}{2}-\cos \frac{\phi}{2} \sin \frac{\theta}{2} \sin \frac{\psi}{2} \\
& q_{2}=\cos \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2}+\sin \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2} \\
& q_{3}=\cos \frac{\phi}{2} \cos \frac{\theta}{2} \sin \frac{\psi}{2}-\sin \frac{\phi}{2} \sin \frac{\theta}{2} \cos \frac{\psi}{2} \tag{3.1}
\end{align*}
$$

### 3.2.3 Rigid Body Dynamics

The helicopter airframe is much more rigid than the rotor system, and can be treated as a rigid body for the purposes of control analysis. Newton's second law of motion

Table 3.1: Eagle helicopter inertia properties

| Parameter | Meaning | Units | Value |
| ---: | :--- | :--- | ---: |
| m | mass | kg | 8.2 |
| $I_{x}$ | Mass Moment about x-axis | $\mathrm{kgm}^{2}$ | 0.30 |
| $I_{y}$ | Mass Moment about y-axis | $\mathrm{kgm}^{2}$ | 0.82 |
| $I_{z}$ | Mass Moment about z-axis | $\mathrm{kgm}^{2}$ | 0.40 |
| $I_{x z}$ | Product of Inertia | $\mathrm{kgm}^{2}$ | -0.01 |

can be used to derive the relationships between the forces and moments acting on the helicopter and the linear and angular accelerations. Assuming that the helicopter is of a conventional mass distribution, it is usual that the $x z$ plane is a plane of symmetry, so that the cross product moments of inertia $I_{y z}=I_{x y}=0$. In this case the equations of motion are those in Equation (3.2). A good derivation of these equations can be found in the flight mechanics text by Nelson [89].

$$
\begin{align*}
F_{x} & =m(\dot{u}+q w-r v) \\
F_{y} & =m(\dot{v}+r u-p w) \\
F_{z} & =m(\dot{w}+p v-q u) \\
L & =I_{x} \dot{p}-I_{x z} \dot{r}+q r\left(I_{z}-I_{y}\right)-I_{x z} p q \\
M & =I_{y} \dot{q}+r p\left(I_{x}-I z\right)+I_{x z}\left(p^{2}-r^{2}\right) \\
N & =-I_{x z} \dot{p}+I_{z} \dot{r}+p q\left(I_{y}-I_{x}\right)+I_{x z} q r \tag{3.2}
\end{align*}
$$

where

$$
\begin{align*}
I_{x} & =\iiint\left(y^{2}+z^{2}\right) d m & I_{x y} & =\iiint x y d m \\
I_{y} & =\iiint\left(x^{2}+z^{2}\right) d m & I_{x z} & =\iiint x z d m  \tag{3.3}\\
I_{z} & =\iiint\left(x^{2}+y^{2}\right) d m & I_{y z} & =\iiint y z d m
\end{align*}
$$

The mass $m$ and mass moments of inertia $I_{x}, I_{y}, I_{z}$ and $I_{x z}$ of the helicopters are given in table 3.1. The mass for each helicopter was found simply by weighing them on a set of digital scales. The moments of inertia were calculated in an Excel ${ }^{\circledR}$ spreadsheet by weighing individual components and treating them as lumped or distributed masses at various distances from a datum position.

For robustness, the attitude of the helicopter is stored as a quaternion and updated using the following equations provided by Stevens and Lewis in [90]. The quaternion attitude update also removes the need to use trigonometric functions which would be required if integrating the Euler angle differential equations.

$$
\left[\begin{array}{c}
\dot{q}_{0}  \tag{3.4}\\
\dot{q}_{1} \\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]=-\frac{1}{2}\left[\begin{array}{cccc}
0 & p & q & r \\
-p & 0 & -r & q \\
-q & r & 0 & -p \\
-r & -q & p & 0
\end{array}\right]\left[\begin{array}{l}
q_{0} \\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]
$$

The final step in updating the rigid body states is to update the position of the helicopter in global coordinates relative to an earth-based axes system. The local velocities $u, v$ and $w$ are first converted to global velocities $\dot{X}, \dot{Y}$ and $\dot{Z}$ by multiplying the local velocities by the rotation matrix $\boldsymbol{B}$ as in Equation (3.5). The rotation matrix can be determined directly from the quaternions using Equation (3.6). These velocities are then integrated to obtain the global position $[X, Y, Z]$

$$
\left[\begin{array}{c}
\dot{X}  \tag{3.5}\\
\dot{Y} \\
\dot{Z}
\end{array}\right]=\boldsymbol{B}\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]
$$

where

$$
\boldsymbol{B}=\left[\begin{array}{ccc}
q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{1} q_{2}+q_{0} q_{3}\right) & 2\left(q_{1} q_{2}+q_{0} q_{3}\right)  \tag{3.6}\\
2\left(q_{1} q_{2}-q_{0} q_{3}\right) & q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{2} q_{3}+q_{0} q_{1}\right) \\
2\left(q_{1} q_{3}+q_{1} q_{2}\right) & 2\left(q_{2} q_{3}-q_{0} q_{1}\right) & q_{0}^{2}-q_{1}^{2}-q_{2}^{2}+q_{3}^{2}
\end{array}\right]
$$

Equations (3.2) have been implemented as a C code SIMULINK ${ }^{\circledR}$ S-function in the simulation. An S-function is a block diagram component in SIMULINK ${ }^{\circledR}$ that provides access to user defined functions written in either C, Ada or MATLAB ${ }^{\circledR}$ mfile format. The SIMULINK ${ }^{\circledR}$ software calls the S-functions when the outputs and state derivatives corresponding to the S-function need to be updated. The states for the dynamics block are position, local velocity components in the helicopter axes system, rotation rates and quaternion attitude. Inputs to the block are the forces and moments acting on the helicopter while the outputs are accelerations, local velocities, position, body angular rates and attitude. A simple trapezoidal integration scheme executing at 2000 updates per second is used to update the states. A graphical mask for the dynamics block allows the user to change the mass, moments of inertia and initial states by double clicking on the block icon.

### 3.3 The Aerodynamics of a Helicopter

Unlike an aeroplane, where the wing is fixed with respect to the fuselage, a rotary wing vehicle has rotor blades that are free to flap. This allows the plane of the rotor disc to tilt in relation to the shaft, depending on the balance of aerodynamic and centrifugal forces acting on each blade. In turn, the amount of tilt of the rotor disc determines the forces and moments acting at the rotor hub. The flapping motion of the blades creates a relationship between the control inputs, helicopter motion and the forces and moments acting at the hub which is much more complex than similar relationships between control deflections and the forces and moments acting upon a fixed wing aircraft.

The controls of the helicopters used in this thesis are the same as a conventional helicopter and consist of 5 control channels: collective pitch, lateral cyclic pitch, longitudinal cyclic pitch, tail rotor pitch and throttle. Observations of the RMAX and Eagle helicopters with an optical tachometer have shown that the RPM does not vary more than 1 percent during flight owing to onboard governors controlling the throttle channel. For this reason the throttle channel has been ignored and a constant RPM assumed. This simplification may not be appropriate during aggressive manoeuvring.

The main rotor forces and moments are controlled by the collective and cyclic pitch channels. The collective pitch control varies the average blade incidence of all of the blades. Increasing the collective pitch control results in an increased angle of attack of each blade and a subsequent increase in main rotor thrust. Decreasing the collective pitch has the opposite effect. The vertical motion of the helicopter is thus controlled by varying the collective pitch.

In order to achieve pitching and rolling moments, the orientation or tilt of the rotor disk is changed by applying pitch that varies cyclically, once per revolution. Increasing the blade pitch on one side of the rotor disk and decreasing it on the opposite side causes the path of the blade tips, known as the Tip Path Plane (TPP) to be tilted. Since the thrust vector acts essentially perpendicular to the TPP [164], this can be used to change the trim of the helicopter. In addition to tilting the thrust vector, moments acting at the rotor hub are generated by the effect of cyclic pitch on the blade aerodynamic forces. Cyclic pitch is applied through a swashplate comprising one fixed and one rotating plate joined by bearings. The servos are connected to the fixed part of the swashplate as shown in Figure 3.6. The pitch change links controlling the main rotor blade pitch are connected to the rotating part of the swashplate so that tilting the swashplate causes the blade pitch to be varied once per revolution in a sinusoidal fashion. Collective pitch can be achieved by raising or lowering the swashplate to increase or decrease the blade pitch collectively. The blade pitch $\theta$ can be described as the first three terms in a Fourier series as per Equation (3.7) where $\psi$ represents the azimuth angle of the rotor blade measured anti-clockwise when viewed from above and starting with $\psi=0^{\circ}$ at the tail. Note that, as a general rule, blades may be twisted along their length, in which case the blade pitch is also a function of radius. However for the Eagle, this is not the case. In accordance with standard conventions [165], I will use the symbol $\theta_{0}$ for collective pitch at the root of the blade and $A_{1}$ and $B_{1}$ for cyclic pitch.

$$
\begin{equation*}
\theta=\theta_{0}+A_{1} \cos \psi+B_{1} \sin \psi \tag{3.7}
\end{equation*}
$$

The variable $A_{1}$ is known as lateral cyclic as it predominantly causes a rolling moment while $B_{1}$ is known as longitudinal cyclic as it results in predominantly a pitching moment. The rotor control inputs are stimulated by three angular servos on the Eagle which receive commands at 50 Hz using Pulse Width Modulation (PWM). Horns fitted to the shaft of each servo move to an angular position which is linearly proportional to the width of the PWM pulse, with neutral servo angle being defined as when the pulse width is 1.5 milliseconds. The angular displacements of the


Figure 3.3: Actuator disk theory of vertical flight
servo horns are transmitted to the rotor blade through a number of mechanical mixing systems including the swashplate. For pitch and roll, I will denote the servo commands in terms of the difference between the trimmed pulse width for hover and the actual pulse width in microseconds as $\left(\delta_{l a t}\right)$ and ( $\delta_{\text {lon }}$ ) respectively. The collective pitch angle ( $\delta_{\text {col }}$ ) represents the raw PWM signal to the collective pitch servo.

### 3.3.1 Momentum Theory

Now that the control inputs to the main rotor are defined, we need to simulate how the forces and moments acting on the rotor hub are manipulated by these controls. To develop this, we must first understand the basic aerodynamics of a rotor. The flow through a rotor is shown in Figure 3.3 for the case of hover. The simplest analysis we can perform is momentum theory in which we treat the rotor disk as an infinitely thin discontinuity across which the pressure changes an amount $\Delta P$ due to the energy imparted by the rotor blades. We further assume that the flow velocity is uniform in the slipstream in a direction parallel to the disk, and that the conditions outside the slipstream are not perturbed by the rotor. This approach is known as actuator disk theory [164]. The velocity $V_{i}$ is the induced velocity through the rotor disk. Applying conservation of energy and conservation of momentum together, we can show that, in hover, the relationship between the induced velocity and thrust $(T)$ is given by Equation (3.8) where A is the rotor disc area and $\rho$ is the density of the air.

$$
\begin{equation*}
T=2 \rho A V_{i}^{2} \tag{3.8}
\end{equation*}
$$

Non-dimensional coefficients are often used in helicopter aerodynamics as these simplify the equations. Table 3.2 summarises the coefficients that I will use in this thesis. Using non-dimensional form, Equation (3.8) can be expressed in nondimensional form as per Equation (3.9).

| Symbol | Description | Formulae | Symbol | Description | Formulae |
| :---: | :--- | :---: | :---: | :--- | :---: |
| $C_{L}$ | Lift Coefficient | $\frac{L}{0.5 V^{2} S}$ | $C_{D}$ | Drag Coefficient | $\frac{L}{0.5 \rho V^{2} S}$ |
| $C_{T}$ | Thrust Coefficient | $\frac{T}{\rho A(\Omega R)^{2}}$ | $\mu$ | Advance Ratio | $\frac{V}{(\Omega R)^{2}}$ |
| $\lambda_{i}$ | Inflow Coefficient | $\frac{V_{i}}{(\Omega R)}$ | $C_{P}$ | Power Coefficient | $\frac{P o w e r}{\rho A(\Omega R)^{3}}$ |
| $\gamma$ | Lock Number | $\frac{\rho a c R^{4}}{I_{b}}$ | $C_{Q}$ | Torque Coefficient | $\frac{T o r q u e}{\rho A(\Omega R)^{2} R}$ |

Table 3.2: Rotorcraft non-dimensional coefficients

$$
\begin{equation*}
C_{T}=\frac{T}{\rho A(\Omega R)^{2}}=2 \lambda_{i}^{2} \tag{3.9}
\end{equation*}
$$

The constant $\gamma$ in Table 3.2 is known as the Lock number which is a nondimensional parameter, representing the ratio of aerodynamic to centrifugal forces. The Lock number becomes important when studying the flapping of the rotor blades and will be used later in the chapter. The constant $I_{b}$ is the blade $2^{\text {nd }}$ mass moment of inertia about its flapping hinge.

### 3.3.2 Blade Element Theory

Momentum theory does not provide us with any guidance on what collective pitch is required to achieve a given thrust, or how cyclic pitch affects the rotor forces and moments. To determine these relationships, we need to use Blade Element Theory, in which the rotor blade is treated as an infinite series of discrete aerofoil sections. Each section of rotor blade of span $d r$ experiences lift and drag forces defined as the forces acting normal and parallel to the local wind vector at each radial location on the rotor blade. Because the flow conditions can change both radially and with blade azimuth, it is necessary to consider the elemental forces $d D$ and $d L$ shown at Figure 3.4 for each radial location and azimuth angle.

As for any aerofoil, the elemental forces can be defined from the fundamental lift and drag Equations (3.12). The subscripts 0 and i on the drag term refer to the profile drag and induced drag respectively.

$$
\begin{align*}
d L & =\frac{1}{2} \rho V_{R}^{2} a \alpha c d r  \tag{3.10}\\
d D_{0} & =\frac{1}{2} \rho V_{R}^{2} C_{d_{0}} c d r  \tag{3.11}\\
d D_{i} & =d L \sin \phi \tag{3.12}
\end{align*}
$$

Referring to Figure 3.4, the angle of attack of the blade is $\alpha=(\theta-\phi)$. The lift curve slope $a$ defines the linear relationship between lift coefficient $C_{L}$ and $\alpha$ for the 2D blade section, such that $C_{L}=a \alpha$. Air density has the symbol $\rho$. The constant


Figure 3.4: Blade element diagram
$C_{D_{0}}$ represents the profile drag of the blade which is the drag of the blade due to viscous effects in the air. For flight at moderate angles of attack, it is reasonable to treat $C_{D_{0}}$ as a constant [163]. Based on indicative values for similar size rotors provided at [166], a value of 0.012 is choosen for $C_{D_{0}}$. The induced drag results from the component of the lift vector which acts perpendicular to the shaft and this adds to the total torque requirement of the rotor.

The total thrust of the rotor can be calculated by integrating the lift and drag components which act in a direction normal to the rotor disc $(d L \cos (\phi)+d D \sin (\phi))$ with a double integral with respect to $r$ and $\psi$. Similarly, the total torque of the rotor can be found by integrating the term $(r(d D \cos (\phi)+D L \sin (\phi)))$. Expressions for pitching and rolling moments, side force and horizontal force can also be derived by integrating the elemental forces. Unfortunately, these expressions tend to be unwieldy and require some assumptions to be made to solve them analytically. One such problem is that the induced flow through the rotor, $V_{i}$ is an unknown which needs to be calculated by some other means. Usually this is done by first calculating an approximate value of the induced velocity from momentum theory and then applying blade element theory using this value of $V_{i}$. In forward flight, the induced velocity varies with azimuth due to the asymmetry of the conditions on the advancing and retreating side of the rotor disk. Predominantly this manifests itself as an upwash at the front of the disk and a downwash at the rear of the disk. For accurate performance work, it is common to incorporate an assumed distribution of induced velocity based on experiment to account for these variations. However for minimum complexity simulations, a uniform induced flow is generally assumed.

In general flight, the helicopter experiences a relative freestream velocity due to its own motion of $V_{\infty}$ given by Equation (3.13) and shown at Figure 3.5. The Figure has much similarity to the momentum theory diagram used in vertical flight. The airstream is deflected through the actuator disc by speed $V_{i}$ at the disc and can be shown to change the downstream flow by $2 V_{i}$. This flow is made up of components $V_{n}$ and $V_{t}$ perpendicular and tangential to the TPP respectively.

$$
\begin{equation*}
V_{\infty}^{2}=u^{2}+v^{2}+w^{2}=V_{n}^{2}+V_{t}^{2} \tag{3.13}
\end{equation*}
$$

Making small angle approximation for the flapping angles is justified since the flapping angles rarely exceed $10^{\circ}$ in normal flight [164]. Hence the expressions for $V_{n}$ and $V_{t}$ are:

$$
\begin{align*}
V_{n} & =\left(a_{1}+i_{s}\right) u-b_{1} v-w  \tag{3.14}\\
V_{t}^{2} & =u^{2}+v^{2} \tag{3.15}
\end{align*}
$$

The most basic inflow assumption that can be made is to assume uniform inflow, which is to say that the inflow $V_{i}$ does not change with radius or azimuth. In this case, we can integrate the elemental forces to achieve a closed form solution for thrust in terms of collective pitch, inflow relative to the TPP $\left(\lambda^{\prime}\right)$ and advance ratio $(\mu)$ as per Equation (3.16) (see [167] for a detailed derivation).

$$
\begin{equation*}
T=\frac{\rho a(\Omega R)^{2} A_{b}}{2}\left[\frac{1}{3} \theta_{0}\left(1+\frac{3}{2} \mu^{2}\right)-\frac{1}{2} \lambda^{\prime}\right] \tag{3.16}
\end{equation*}
$$

where

$$
\begin{equation*}
\lambda^{\prime}=\frac{V_{i}+V_{n}}{\Omega R} \quad \text { and } \quad \mu=\frac{V_{t}}{\Omega R} \tag{3.17}
\end{equation*}
$$

### 3.3.3 Rotor Thrust

In order to calculate the inflow and hence thrust, use is made of Glauert's induced flow model [168] which likens a rotor in forward flight to an elliptically loaded fixed wing of circular planform. This approach has been taken by several others [154,157, 162]. Based on the downwash for the equivalent wing, Glauert suggested that the mean induced velocity $V_{i}$ can be expressed as in Equation (3.18).

$$
\begin{equation*}
V_{i}=\frac{T}{2 \rho A \hat{V}} \quad \text { where } \quad \hat{V}=\sqrt{V_{T}^{2}+\left(V_{n}+V_{i}\right)^{2}} \tag{3.18}
\end{equation*}
$$

Note that no theoretical basis exists for using the assumed flow pattern of Figure 3.5, other than: (a) in hover, Equation (3.18) reduces to $V_{i}=\sqrt{\frac{T}{2 \rho A}}$ which is the momentum equation derived for axial flight; and (b) in fast forward flight $\hat{V} \approx V_{\infty}$ and the formula becomes the same as that predicted for an elliptically loaded wing. However there does appear to be a good match between this model and experiment [167]. After some manipulation [165] of Equation (3.18) we can write:

$$
\begin{equation*}
V_{i}^{2}=\sqrt{\left(\frac{\hat{V}}{2}\right)^{2}+\left(\frac{T}{2 \rho A}\right)^{2}}-\frac{\hat{V}^{2}}{2} \tag{3.19}
\end{equation*}
$$

The combination of momentum theory and blade element theory using Glauert's simple inflow model results in two coupled non-linear Equations (3.16) and (3.19)


Figure 3.5: Glauert's assumed flow model
which must be solved numerically. Heffley [162] uses an iterative scheme based on a converging guess of induced velocity. In certain situations, I have found that this does not always converge, being sensitive to both the initial guess of $V_{i}$ and the flight condition. Leishman and Padfield $[163,169]$ have suggested schemes based on Newton-Raphson techniques, but these schemes are also not guaranteed to converge. For this thesis, a new method has therefore been developed using a simple binary search algorithm to find $V_{i}$, making use of the fact that the thrust varies monotonically with $V_{i}$. Whilst taking roughly double the number of iterations, the technique always converges provided the upper and lower bounds of the search are set correctly.

In the binary search algorithm, the difference in the thrust $\Delta T$ predicted by Equations (3.16) and (3.19) is calculated based on a guess of $V_{i}$. The objective of the algorithm is to find a value of $V_{i}$ which makes $\Delta T$ as close to zero as possible. The algorithm starts by assuming a lower bound $\left(V_{i}=0\right)$ and an upper bound of $V_{i} . \Delta T$ is then calculated for $V_{i}$ halfway between the upper and lower bounds and also for the value of $V_{i}$ at the lower bound. If this $\Delta T$ is of opposite sign to a $\Delta T$ calculated at the lower bound, then the solution must lie between the lower bound and the midpoint guess. Hence for the next iteration, the new upper bound becomes the old midpoint guess. Conversely, if $\Delta T$ is of the same sign as a $\Delta T$ calculated at the lower bound, then the solution must lie between the upper bound and the midpoint guess. In this case, the lower bound is set to the old midpoint guess. This continues until $\Delta T$ falls below a prescribed convergence criterion. Typically $\Delta T$ converged to within a tolerance of $10^{-6} \mathrm{~N}$ after no more than 25 iterations. The algorithm for calculating $V_{i}$ and thrust is embedded in a C code S -function block.

### 3.3.4 Flapping Dynamics

Rotor blade flapping takes place under conditions of dynamic equilibrium, about the hinge, between the aerodynamic lift, the centrifugal force and the blade inertia. Since, in any steady state of the rotor, the flapping motion is periodic, the flapping angle can be expressed in the form of an infinite Fourier series:

$$
\begin{equation*}
\beta=\beta_{0}+a_{1} \cos \psi+b_{1} \sin \psi+a_{2} \cos 2 \psi+b_{2} \sin 2 \psi+\ldots \tag{3.20}
\end{equation*}
$$

For most work, only the constant term and the two first harmonic terms are used. The constant term represents the coning angle. The magnitude of the coefficients dies off by approximately an order of magnitude for each harmonic, so that the $\sin 2 \psi$ and $\cos 2 \psi$ terms are usually about $1 / 10^{\text {th }}$ as significant as the $\sin \psi$ and $\cos \psi$ terms. When only the first order terms are considered, the flapping angle $\beta$ is defined in terms of the coning angle $a_{0}$, longitudinal flapping $a_{1}$ and lateral flapping $b_{1}$ as per Equation (3.21).

$$
\begin{equation*}
\beta=a_{0}+a_{1} \cos \psi+b_{1} \sin \psi \tag{3.21}
\end{equation*}
$$

The flapping is affected by the cyclic pitch and also by the pitching and rolling rates. Gyroscopic effects on the blade hinge moments result in coupling with the rotation rates, which causes the TPP to lag behind the shaft when the aircraft is pitching and rolling. For a typical helicopter, a pure pitch rate will generate a longitudinal flapping effect which is about double the magnitude of the lateral flapping. This is a source of cross-coupling known as rate cross coupling. However, Heffley suggests that the rate cross-coupling be ignored, as it does not necessarily produce a good match to flight data. Prouty [165] derives a rather complex expression for the effect of the rotation rates in terms of the hinge offset, advance ratio and other constants. For low speed and zero hinge offset, this simplifies to Equations (3.223.23).

$$
\begin{align*}
& a_{1}=-\frac{16}{\gamma}\left(\frac{q}{\Omega}\right)+\left(\frac{p}{\Omega}\right)  \tag{3.22}\\
& b_{1}=-\frac{16}{\gamma}\left(\frac{p}{\Omega}\right)-\left(\frac{q}{\Omega}\right) \tag{3.23}
\end{align*}
$$

Tischler and Remple [170] show that the main flapping of the main rotor, which is a second order systems, can be accurately represented as two coupled first order equations. If the cross-coupling terms are initially ignored as suggested by Heffley [162], these equations become those in Equations (3.24-3.25).

$$
\begin{align*}
\dot{a_{1}} & =-q-\frac{1}{\tau_{f}}\left(a_{1}+\frac{d a_{1}}{d B_{1}} B_{1}\right)  \tag{3.24}\\
\dot{b_{1}} & =-p-\frac{1}{\tau_{f}}\left(b_{1}+\frac{d b_{1}}{d A_{1}} A_{1}\right) \tag{3.25}
\end{align*}
$$

For a teetering rotor, consisting of a rotor blade hinged on the axis of the shaft, it can be shown that the natural frequency of the flapping is equal to the angular velocity of the rotor $\Omega$. As the system is therefore in resonance, the phase lag between the excitation (the once per revolution cyclic pitch) and the flapping is exactly $90^{\circ}$. The Eagle main rotor hub consists of a pivot on the shaft axis about
which the blades can flap and an elastomeric element that allows moments, as well as forces, to be transferred from the rotor blade to the hub. The arrangement can be conceptualised as the combination of a moment from a centre spring and a tilted TPP [157] as shown in Figure 3.7. A consequence of the centre spring is that the natural frequency of the rotor blade flapping is no longer in resonance with the rotational excitation, leading to a reduced phase shift between flapping and cyclic pitch and a decreased flapping time constant. Prouty [165] derives the time constant for flapping of a hinged rotor blade to arrive at Equation (3.26) in terms of the hinge offset from the shaft. The time constant is typically between one-quarter and one half of a rotor revolution, depending on the effective hinge offset and spring constant.

$$
\begin{equation*}
\tau_{f}=\frac{16(1-e / R)}{\gamma \Omega(1-e / R)^{4}(1+e / 3 R)} \tag{3.26}
\end{equation*}
$$

Cooke [171] shows that the effect of a spring on the rotor blade hinge is equivalent to the effect of hinge offset and goes on to derive equations for the effect of the spring on the natural frequency and damping ratio. Re-arranging Cooke's equations which deal with both hinge offset and spring constant, provides Equation (3.27) which allows the effective hinge offset $e_{\text {eff }}$ to be calculated.

$$
\begin{equation*}
e_{e f f}=\frac{K_{\beta}}{m_{b} r_{g} R^{2} \Omega^{2}} \tag{3.27}
\end{equation*}
$$

where $K_{\beta}$ is the spring stiffness, $m_{b}$ is the mass of the blade and $r_{g}$ is the radial location of the centre of gravity of the blade. Substitution of the effective hinge offset back into Equation (3.26) provides a means to calculate the time constant. For the case of the Eagle, this results in an effective hinge offset of $e=0.12$ and a flapping time constant of approximately 6 milliseconds.

One other important effect on flapping needs to be considered. A helicopter rotor responds to changes in sideslip through an effect well-known for fixed wing, dihedral effect. The effect of dihedral is for the helicopter to roll away from sideslip. This is a stabilising influence which acts to maintain wings level, since a bank to one side results in the aircraft sideslipping towards the low wing. The resulting sideslip and dihedral effect then causes the aircraft to roll away from the low wing. Consider a hovering helicopter with blades rotating anti-clockwise. If a helicopter were to suddenly experience a gust from the right, the rear of the disk would be advancing into the flow and generate increased lift. Blades at the front of the disk would be retreating from the relative wind and receive a net decrease in lift. Due to phase lag, the TPP would be tilted to the left and the helicopter would get a rolling moment to the left, rolling the helicopter away from the gust. I make use of Heffley's equation for dihedral effect (3.28) which is provided at [162]. Since the thrust coefficient does not change greatly between hover and forward flight, its value is calculated once at the start of the simulation based on the thrust coefficient for steady hover.

$$
\begin{equation*}
\frac{d b_{1}}{d v}=\frac{d a_{1}}{d u}=\frac{2}{\Omega R}\left(\frac{8 C_{T}}{a \sigma}+\sqrt{\frac{C_{T}}{2}}\right) \tag{3.28}
\end{equation*}
$$



Figure 3.6: Flybar arrangement with Bell-Hiller stabilisation system

The main rotor flapping is implemented as a discrete S -function block in SIMULINK ${ }^{\circledR}$. The inputs to the block are the velocities (for dihedral effect), the rotation rates for calculation of gyroscopic effects and the cyclic pitch. The outputs of the block are the longitudinal and lateral flapping angles, $a_{1}$ and $b_{1}$.

### 3.3.5 Flybar Dynamics

A feature of practically all small unmanned helicopters is a control rotor, known as a flybar, which augments the stability of the main rotor. The flybar is a hybrid of stabilising arrangements patented by Bell and Hiller. The primary effect of the flybar is to provide feedback from the helicopter pitch and roll rates to the cyclic pitch of the blades. The time constant of the flybar is much lower than for the main rotor which slows down the dynamics of the helicopter, making it easier for a human pilot to fly. A number of groups have provided an analysis of the flybar with varying degrees of complexity [172]. All of the UAV platforms utilised for this thesis project employ a flybar arrangement consisting of a teetering rotor rotated 90 degrees out of phase with the main rotor. The flybar blades comprise small paddles which are connected to the main rotor pitch change horns through a Bell-Hiller mixing lever as shown in Figure 3.6. The incidence of the paddles is controlled by linkages to the swashplate (not shown in the figure).

Unlike the Hiller stabiliser bar, the flapping angle of the flybar is used to control the main rotor blade pitch. Application of cyclic pitch causes the flybar pitch to change, resulting in flapping of the flybar. The flapping of the flybar is fed through mechanical linkages to adjust the main rotor blade cyclic pitch, resulting in the TPP tilting as desired. This scheme introduces significant lag into the control loop, which would be problematic for the pilot, so that some cyclic pitch is fed through a mechanical mixing system directly into the main rotor to provide some faster control. The flybar is hinged at the shaft so that the time constant Equation (3.26) simplifies to Equation (3.29). For the Eagle, the time constant of the flybar is approximately 0.23 seconds which is significantly slower than that for the main rotor.

$$
\begin{equation*}
\tau_{s}=\frac{16}{\gamma_{s} \Omega} \tag{3.29}
\end{equation*}
$$

The flapping $\left(\beta_{s}\right)$ of the flybar is defined by the longitudinal flapping $c$ and the lateral flapping $d$ such that $\beta_{s}=c \cos \psi+d \sin \psi$. A set of first order Equations (3.30) defines the flapping dynamics. The constants $D_{l a t}$ and $D_{l o n}$ represent the gearing ratio between the servo commands $\delta_{l a t}$ and $\delta_{l o n}$ and the corresponding pitch change of the flybar paddles.

$$
\begin{align*}
\tau_{s} \dot{d} & =-d-\tau_{s} p+D_{l a t} \delta_{l a t} \\
\tau_{s} \dot{c} & =-c-\tau_{s} q+C_{l o n} \delta_{l o n} \tag{3.30}
\end{align*}
$$

The linearised relationship between the cyclic pitch fed to the main rotor blades, the flybar flapping and the servo commands are presented in Equation (3.32). The constants $A_{\text {lat }}, B_{\text {lon }}$ and $K_{s}$ represent the combined gain of the servos and the mechanical mixing ratios of the linkages. The values of the constants $A_{\text {lat }}$ and $B_{l o n}$ were found by measuring the variation in pitch with variation in the control signal sent to the servos. The value of the constant $K_{s}$ was estimated based on the kinematics of the linkage geometry. For the purposes of simulation, the mechanical mixing equation was treated as linear, although some non-linearity does exist. The flybar dynamics are implemented in their own S-function block. Inputs are the rotation rates and cyclic pitch. Outputs are the flapping which is fed to the input of the main rotor flapping block.

$$
\begin{gather*}
A_{1}=A_{l a t} \delta_{l a t}+K_{s} d  \tag{3.31}\\
B_{1}=B_{l o n} \delta_{l o n}-K_{s} c \tag{3.32}
\end{gather*}
$$

### 3.3.6 Main Rotor Control Forces and Moments

The longitudinal and lateral forces and moments acting on the main rotor hub are a result of tilting the TPP. Referring to Figure 3.7, it can be seen that the effect of the centre spring is to create a moment to be transferred to the rotor hub which is linearly proportional to the flapping angle $\beta$. The stiffness of the centre spring acting on the Eagle rotor hub, was measured in a force deflection test and found to be $K_{\beta}=270 \mathrm{Nm} /$ radian. The TPP tilt also causes the thrust vector to act on a line of action which is offset by $Z_{m r} \beta$ from the centre of gravity, creating additional moments. The combined effect is to generate main rotor pitching and rolling moment contributions as per Equation (3.33).

$$
\begin{equation*}
L_{m r}=K_{\beta} b_{1}+T z_{m r} b_{1} \quad \text { and } \quad M_{m r}=K_{\beta} a_{1}+T z_{m r} a_{1} \tag{3.33}
\end{equation*}
$$

The forces acting on the main rotor are commonly approximated as acting perpendicular to the TPP [169]. Making small angle approximations, we can write the forces acting at the rotor hub as per equation (3.34).


Figure 3.7: Centre spring representation of rotor forces and moments

$$
\begin{equation*}
F_{x}^{m r}=-T_{m r} a_{1} \quad F_{y}^{m r}=T_{m r} b_{1} \quad F_{z}^{m r}=-T_{m r} \tag{3.34}
\end{equation*}
$$

The lateral and longitudinal rotor forces and moments are simulated by an Sfunction block which takes thrust and flapping as its inputs.

### 3.3.7 Main Rotor Torque

The yawing torque $N_{m r}$ generated by the main rotor results from the drag of the rotor blades through the air. This torque must be balanced by the tail rotor thrust to stop the helicopter yawing. The main rotor torque can be calculated by dividing the main rotor power $P_{m r}$ by the angular velocity as in Equation (3.35).

$$
\begin{equation*}
N^{m r}=\frac{P^{m r}}{\Omega} \tag{3.35}
\end{equation*}
$$

The power of the main rotor is due to a number of sources: the induced power $\left(P_{\text {ind }}\right)$ which is the power required to create the induced velocity $V_{i}$; the profile power $\left(P_{0}\right)$ which is the power to overcome the profile drag of the blades; parasite power $\left(P_{\text {fus }}\right)$ which is the power required to overcome the drag of the fuselage in forward flight; and climb power $\left(P_{\text {climb }}\right)$ which is the rate of change of gravitational potential energy due to climbing. The power equation may be written compactly in non-dimensional form as in Equation (3.40):

$$
\begin{equation*}
C_{P_{\text {tot }}}=C_{P_{\text {ind }}}+C_{P_{0}}+C_{P_{f u s}}+C_{P_{\text {climb }}} \tag{3.36}
\end{equation*}
$$

where

$$
\begin{align*}
C_{P_{\text {ind }}} & =k_{i n d} C_{T} \lambda_{i}  \tag{3.37}\\
C_{P_{0}} & =\frac{\sigma C_{D_{0}}}{8}\left(1+\kappa \mu^{2}\right)  \tag{3.38}\\
C_{P_{\text {fus }}} & =\left|X^{\text {fus }} u\right|+\left|Y^{\text {fus }} v\right|+\left|Z^{\text {fus }}\left(w-V_{i}\right)\right|  \tag{3.39}\\
C_{P_{\text {climb }}} & =\frac{m g \dot{H}}{\rho A(\Omega R)^{3}} \tag{3.40}
\end{align*}
$$

The constant $k_{\text {ind }}$ is a correction factor to compensate for tip losses and for the inflow not being uniform. The constant $\kappa$ compensates the profile drag power for the effects of the reverse flow region and the spanwise flow over the rotor blade in forward flight. Values of $k_{\text {ind }}=1.2$ and $\kappa=4.7$ have been assumed based on suggestions by Leishman [163].

### 3.3.8 Tail Rotor

The calculation of tail rotor thrust is treated in the same way as the main rotor, except the orientation is changed and flapping effects are not included. Another S-function block was created to calculate the tail rotor thrust $T^{t r}$ and the tail rotor power. The yawing moment from the tail rotor is calculated from the tail rotor thrust using $N^{t r}=T^{t r} d^{t r}$ where $d^{t r}$ is the distance from the centre of the tail rotor to the centre of gravity.

### 3.3.9 Tailplane Forces and Moments

The Eagle helicopter is fitted with a vertical fin on the tailplane which helps to stabilise the helicopter directionally in forward flight. The fin is treated like a wing which creates lift when exposed to an angle of attack. The equations for the force and moment contributions of the stabiliser are provided in Equation (3.44). The drag force of the tailplane is lumped in with the drag force of the fuselage as a whole.

$$
\begin{align*}
V_{R}^{v t} & =\left(u^{2}+w^{2}\right)  \tag{3.41}\\
\alpha^{v t} & =\frac{v+r d^{v t}}{u}  \tag{3.42}\\
F_{y}^{v t} & =\frac{1}{2} \rho V_{R}^{v t} a^{v t} \alpha^{v t}  \tag{3.43}\\
N^{v t} & =F_{y}^{v} d^{v t} \tag{3.44}
\end{align*}
$$

### 3.3.10 Fuselage Forces

The fuselage drag forces are calculated along each body axis using the formulae in Equation (3.47). The values $S_{x}, S_{y}$ and $S_{z}$ are the equivalent flat plate areas of the fuselage in the respective directions.

$$
\begin{align*}
F_{x}^{f u s} & =\frac{\rho}{2} S_{x}^{f u s} u^{2}  \tag{3.45}\\
F_{Y}^{f u s} & =\frac{\rho}{2} S_{y}^{f u s} v^{2}  \tag{3.46}\\
F_{Z}^{f u s} & =\frac{\rho}{2} S_{z}^{f u s}\left(w+V_{i}\right)^{2} \tag{3.47}
\end{align*}
$$

### 3.3.11 Dynamics Subsystem

The helicopter dynamics are combined into a SIMULINK ${ }^{\circledR}$ subsystem containing all of the aerodynamic effects and rigid body dynamics as shown in Figure 3.8.

Figure 3.8: Helicopter dynamics simulation subsystem

The inputs to the subsystem are the collective pitch, tail rotor pitch and cyclic pitch angles produced by the servos. The outputs are the state vector comprised of position, local velocities, quaternion attitude, angular rates and accelerations. The parameters used in the dynamics simulation are summarised in Appendix A.

### 3.4 Servo Dynamics

The Eagle helicopter has been fitted with DS8411 fast-response digital RC servos manufactured by the Japan Radio Control Company (JR). Each servo contains an integral PID controller that positions the servo in response to a Pulse Width Modulated control signal (PWM). The controller receives updates every 20 milliseconds from the PWM stream. Various representations of RC servo dynamics have been used ranging from first order to third order systems [157,173]. Gavrillets et al [157] used a first order representation of the servo dynamics based on an identified bandwidth for a typical RC servo of 6 Hz , defined as being the frequency at which the output lags the input with a 90 degree phase lag.

Brennan [174] conducted a detailed analysis of high-end digital servos and notes that the frequency response of the servos is not in fact linear, being dependent on amplitude. This agrees with observations made by driving a servo with sinewaves ranging in frequency from 0.125 Hz to 6.5 Hz and measuring the servo horn position with a sampled potentiometer output. My tests show that the magnitude response begins to fall below unity at a speed of 1 Hz for maximum deflection and 4 Hz for 5 percent of full travel. At amplitudes below this, the servo Bode plot exhibits a corner frequency of between $3.5 \mathrm{~Hz}-4.0 \mathrm{~Hz}$. By comparing the frequency at which the magnitude begins to drop, $\Omega_{c r}$, and the commanded amplitude $\theta_{s}$, it is possible to calculate the rate limit $\dot{\theta}_{\text {max }}$ using Equation (3.48) which is derived from considering when the derivative of the commanded sine wave exceeds the rate limit. At amplitudes of $60^{\circ}, 48^{\circ}$ and $36^{\circ}$, the indicated rate limits were $314^{\circ} / \mathrm{sec}, 301^{\circ} / \mathrm{sec}$ and $302^{\circ} / \mathrm{sec}$ respectively. This agrees well with the $300^{\circ} / \mathrm{sec}$ maximum angular rate specification for the servo, provided by the manufacturer.

$$
\begin{equation*}
\dot{\theta}_{\max }=\Omega_{c r} \theta_{s} \tag{3.48}
\end{equation*}
$$

Brennan achieved a good match to experimental data in his work by modeling the servo as a second order dynamic with a rate limiter. Increased fidelity can be achieved by including a small deadband of about 1 degree of servo horn travel that is incorporated into COTS servos to reduce their power consumption. I have used a similar approach to Brennan except I have used the simpler first order transfer function $F_{s}(s)$ in Equation (3.49) combined with a rate limiter of $300^{\circ} / \mathrm{sec}$, as the measuring equipment was not accurate enough to determine the damping ratio of the servo dynamics from the resonant peak and hence characterise the second order dynamics properly. Noting that most other researchers use only a first order representation this is not expected to be a significant source of error.

$$
\begin{equation*}
F_{s}(s)=\frac{1}{1+0.05 s} \tag{3.49}
\end{equation*}
$$

### 3.5 Atmospheric Disturbances

The aim of including turbulence into the simulation is to ensure that the sensing and control system can cope with a real-world environment where wind effects can be a significant challenge to station keeping. Only linear velocities are considered as the small size of the helicopter means that gradient effects which might cause local rotational flow are negligible [173]. The velocities corresponding to turbulence are added to the body axis velocities provided by the rigid body dynamics block and these velocities are fed as the freestream velocity input to the aerodynamic blocks.

The dominant techniques for simulating the effects of atmospheric turbulence on flying vehicles are based on the Dryden and von Karman spectral models of atmospheric turbulence. The Dryden spectra has been used for this simulation as it can be most readily generated, simply by passing white noise through appropriate linear filters [175]. The filters used to generate the Dryden spectra are described by the transfer functions at Equations (3.50-3.51), reproduced from [176], where the constant lengths $L_{u}, L_{v}$ and $L_{w}$ refer to the scale of turbulence. The output of the velocities will be the relative wind velocities on all three axes. The parameter $V$ in the transfer functions is the relative speed of the aircraft to the airmass. The equations are aimed at aircraft in forward flight where the longitudinal velocity is dominant, hence the longitudinal filter is different to the lateral and vertical filters which are the same. Whilst, in the case of a hovering helicopter, the wind can be from any direction, for most work, the helicopter is flown with its nose into wind, so that the speed $V$ can be approximated by the mean wind speed. The scale lengths can be found from a number of empirical sources such as ESDU [177] and MIL-F8785C [178]. The latter states that, at heights below 6 m , which is representative of the conditions that the experiments for this thesis were conducted, constant values of $L_{u}=L_{v}=722.5 \mathrm{~m}$ and $L_{w}=3 \mathrm{~m}$ may be used. The turbulence intensity factors $\sigma_{u}, \sigma_{v}$ and $\sigma_{w}$ depend on the mean speed of the relative wind and the altitude $H$. They are described by Equations (3.52-3.53) which are reproduced from [178]. The scale lengths and mean wind speed are entered in a graphical mask of the turbulence block by the user before starting the simulation.

$$
\begin{gather*}
F_{u}(s)=\sigma_{u} \sqrt{\frac{2 V}{\pi L_{u}}}\left(\frac{1}{s+\frac{V}{L_{u}}}\right)  \tag{3.50}\\
F_{v}(s)=\sigma_{v, w} \sqrt{\frac{3 V}{\pi L_{v, w}}}\left(\frac{s+\frac{V}{\sqrt{3} L_{v, w}}}{\left(s+\frac{V}{L_{v, w}}\right)^{2}}\right)  \tag{3.51}\\
\sigma_{w}=0.1 u_{\text {mean }}  \tag{3.52}\\
\frac{\sigma_{u}}{\sigma_{w}}=\frac{\sigma_{v}}{\sigma_{w}}=\frac{1}{(0.177+0.000823 H)^{0.4}}
\end{gather*}
$$

### 3.6 Sensor Models

The sensor block functions to simulate the characteristics of the sensors fitted to the helicopter. The input to the block are the state variables from the helicopter dynamics block comprising position, velocity, acceleration and atttitude. It consists of the following subsytems:

- Inertial Measurement Unit. This block simulates the accelerometers, magnetometers and gyroscopes. A graphical mask allows offsets, gains and random noise to be changed for each sensor. A two pole butterworth filter is implemented for each sensor with the same corner frequencies as the actual sensors. The accelerometers and gyroscopes have a 20 Hz corner frequency with a 400 Hz sample rate whilst the magnetometers have a 10 Hz corner frequency. All of the inertial sensors are sampled at a 400 Hz sample rate.
- Optic Flow. The optic flow sensor subsystem calculates the flow that would be seen by a downwards looking camera. The inputs to the sensor are the local velocities and the height above ground. Gaussian white noise is added to the flows with the same variance as that measured from experiment.
- GPS. The position from the dynamics block is converted into global $X, Y$ and $Z$ coordinates with an adjustable sample rate and added position white noise. For the experiments in this thesis, a 20 Hz sample rate was used to simulate the NovAtel DGPS sensor fitted to the Eagle helicopter.
- Rangefinder. This block simulates a generic rangefinder for determining height above ground such as a laser rangefinder or stereo camera. A graphical mask allows sensor offset, scale error and random noise to be set.
- Beacon Position. This block generates a set of three azimuth and elevation angles to simulated visual landmarks. The block simulates a camera sensor with a 50 Hz frame rate and a single pixel quantisation. Angles are calculated relative to the aircraft body axes.


### 3.7 State Estimator

To control a helicopter in hover, the following states are required: position, velocity, attitude and rotation rates. These states must be deduced from the sensor outputs using a state estimator. An S-function block was implemented to do this. The block is capable of accepting input from a variety of different types of sensors. The sensory inputs are grouped into inertial, GPS, optic flow, height and visual position measurements. The outputs of the block are output with an update rate of 50 Hz to match the controller sample rate.

The estimator executes in a predict and correct cycle. The predict cycle occurs at the sample rate of the incoming inertial data which is 400 Hz . At each time step the state equations are propagated based on the measured accelerations and
rotation rates. The same equations used for the rigid body dynamics block are used in the estimator with the exception of attitude which is propagated as two separate reference vectors.

Position and velocity are corrected when measurements of these state variables are taken. The GPS measurements arrive at 20 Hz , the visual data arrive at the speed of the frame grabber which is 50 Hz and the height data arrive at the speed of the sonar which is 25 Hz . The corrections to each state $\hat{x}$ are made using Equation (3.54) where $k$ is an appropriate weighting factor and $z$ is an observation of the state. The correction term $k$ for each sensor measurement is able to be modified by the user by modifying the variables in the S-function GUI mask. The performance of the estimator is generally not that sensitive to the values of $k$ and these values can be quickly set by trial and error to an appropriate value for the sensor noise being simulated.

$$
\begin{equation*}
\hat{x}^{*}=\hat{x}+k(z-\hat{x}) \tag{3.54}
\end{equation*}
$$

Attitude is stored and propagated inside the estimator as two vectors corresponding to the gravity or downwards vector and a vector corresponding to the direction of the local magnetic field. The gravity and magnetic field vectors are converted into a rotation matrix and quaternion attitude representation as required. The vectors are corrected at 50 Hz using the measured magnetic vector from the magnetometers and the gravitational vector measured by the accelerometers. It is assumed that over a long time period, the accelerometer vector averages to that due to gravitational acceleration. Hence the gravitational correction is based on the simple correction cycle at Equation (3.54) with the observations being the measured accelerations. During periods of rapid acceleration, this assumption causes errors in the pitch and roll angles but I have found this does not upset the control of the helicopter in hover or steady forward flight. For future work, it needs to be borne in mind that, if the helicopter were to enter a banked turn, the attitude from the state estimator would erect to a false horizon after a few seconds.

### 3.8 Simulation Validation

Initial adjustments of the simulation were made to match the trim control settings of the simulation to the helicopter. The trim collective, aileron and elevator PWM settings of the simulation were matched to values observed from flight test. The simulation was validated against actual flight test data using frequency response techniques. Time domain data were not used for validation since the helicopter is dynamically unstable and requires constant attention by the pilot or control system to keep in a stable hover. Due to small differences between the simulation and plant, attempts to feed open loop control inputs recorded from flight test into the simulation (or vice versa) result in rapid divergence from stable flight.

A chirp signal was used to stimulate the control inputs in both the simulation and the actual plant (see Figure 3.9b), to generate a frequency response spectrum. The chirp signal is made up of a sinusoidal signal of constant amplitude with a frequency


Figure 3.9: Test signals used to determine frequency response
that increases linearly with time. In the simulation, the chirp signal was generated continuously at the update rate of the simulation. The chirp was varied in frequency from 0.1 Hz to 10 Hz over 120 seconds. Separate simulation runs were completed for aileron, elevator and collective frequency sweeps. A low-gain control system was implemented to maintain the helicopter in a hover with zero mean velocity. The control inputs from the control system were added to the chirp signal. The control inputs and the simulation state data were sampled at 50 Hz and recorded to file.

For flight test, the chirp signal was generated offline using MATLAB ${ }^{\circledR}$ with unity amplitude and a 50 Hz sample rate to match the 50 Hz PWM update rate. The same chirp used in the simulation was used in the flight test experiments. As the test signal is implemented with a discrete sample rate of 50 Hz , it is not practical to use a chirp to stimulate frequencies greater than about 5 Hz , due to aliasing effects. To achieve higher frequency stimulation for the flight test, a series of doublet waveforms was appended to the chirp signal. A doublet consists of consecutive square wave pulses of opposite sign, as shown in Figure 3.9a. The pulse width of the doublet is matched to the frequency of interest. Doublets tuned to $3 \mathrm{~Hz}, 5 \mathrm{~Hz}, 7 \mathrm{~Hz}$ and 10 Hz and consisting of 4 pulses each were used with a spacing of 3 seconds between them.

The test signal data were incorporated into the helicopter control system so that a switch on the pilot's transmitter could be used to initiate the start of the test signal sequence. The amplitude of the test signal could be scaled to any desired setting by the ground control operator using the command and control telemetry system. The aileron and elevator test signals were given an amplitude of $20 \mu \mathrm{sec}$ each and the collective channel was stimulated with an amplitude of $30 \mu \mathrm{sec}$.

The systems identification flight tests were carried out on one axis at a time. For each axis, the chirp/doublet signal was superimposed on top of the pilot's control inputs. During the tests, the pilot attempted to use as few control inputs as possible whilst keeping the helicopter in a steady hover. Each test signal was executed for at least two complete cycles of the test signal, so that each run was between 3 and 4 minutes. Figure 3.10 shows the collective input after the test signal has been applied and the resulting vertical (heave) acceleration measured by the accelerometers. As the test signal was superimposed on the pilot inputs, there are, at times, large


Figure 3.10: Test signals applied to the helicopter and simulation
excursions from the trim collective due to the pilot's effort at keeping the helicopter at a safe and relatively constant altitude.

A software package called CIFER ${ }^{\circledR}$ (Comprehensive Identification from FrEquency Responses) [179] was used to analyse the data and produce Bode plots [82] representing the frequency response of the helicopter. CIFER ${ }^{\circledR}$ was developed by the US Army/NASA Rotorcraft Division (Ames Research Centre) to perform systems identification using frequency response methods. The program uses Fast Fourier Transforms (FFT) applied to overlapping windows of time domain data to generate smooth spectral estimates. The frequency curve is found by averaging the local response estimates over adjacent windows to provide a smooth curve. For the work in this thesis, multiple FFT windows of $24 \mathrm{sec}, 20 \mathrm{sec}, 15 \mathrm{sec}, 10 \mathrm{sec}$ and 6 sec duration were used and the results averaged.

The CIFER ${ }^{\circledR}$ package outputs a classical Bode plot from sets of input data $X(j \omega)$ and output data $Y(j \omega)$. The Bode plots consist of: (a) the logarithm of magnitude ratio $\left(20 \log _{10}(|Y| /|X|)\right.$ (in Decibels) versus $\log _{10} \omega$; and (b) the phase shift $\phi$ between input excitation and response versus $\log _{10} \omega$ plot. The software also generates a third plot for the correlation factor versus $\log _{10} \omega$. The correlation factor $\gamma^{2}$ is a measure of the reliability of the frequency response curves which can be defined as the fraction of the output spectrum that is linearly attributable to the input spectrum [170]. The correlation factor can range between zero and unity with 1.0 being the ideal case for a system with zero noise and perfect linearity. Reduced $\gamma^{2}$ can arise from system non-linearities, measurement noise, cross-coupling and
secondary inputs arising from unmeasured system disturbances. A general guideline provided by the authors of the software is that provided $\gamma^{2}>0.6$ and not oscillating, the frequency spectrum is reliable [170].

For the purposes of simulation validation, the effects of cross-coupling between control inputs was ignored. Three different control channels were analysed: aileron, elevator and collective. The rudder channel was not treated as it was considered too dangerous to excite the rudder servo at high frequency, owing to warnings from the manufacturer about tail rotor servo failure in such conditions. The simulation and flight test results were plotted on the same Bode plots for each of the three control inputs. The following frequency response were examined: aileron excitation of roll rate; elevator excitation of pitch rate; and collective excitation of vertical acceleration.

The correlation factor for the plots is satisfactory in most parts of the spectrum. At high frequencies, the correlation factor drops off and begins to oscillate due to the aliasing effects of the sample rate. At low frequencies, the correlation factor is seen to deteriorate because of the lack of low-frequency stimulation. For the aileron and elevator channels, stimulation of less than about 0.15 Hz is not very effective due to unavoidable low bandwidth feedback from the pilot or control system which tends to cancel out most of the motion. This feedback cannot be prevented, otherwise, the low bandwidth oscillations in roll and pitch would result in excessive attitudes developing. Other sources of reduced correlation factor would include the presence of wind gusts, neglecting cross-coupling effects, vibration and the non-linearities introduced by the servo linkages.

The lateral and longitudinal response both exhibit a lightly damped second order response which agrees with results for other small-scale helicopters found in the literature $[85,102,180]$. The attenuation of the control inputs at high frequency can be attributed to the servo transfer function and the first order flapping lag for the flybar/main rotor.

The simulation and flight test magnitude responses matched relatively well with some small discrepancies in the resonant peak for the lateral and longitudinal cases. Differences in phase response are most likely due to the variable fixed lag present in the real helicopter due to the 50 Hz servo and telemetry update rates. This fixed lag can vary between 0 ms and 40 ms , depending on the non-deterministic synchronisation between the PC104 logging system and the MPC555 autopilot. The resonant peaks represent the coupling between the fuselage rigid body mode, rotor flapping and the flybar [85]. The resonant frequency in simulation was adjusted to better match the flight test results by changing the lateral and longitudinal moments of inertia by about $10 \%$. The longitudinal case also had a slightly different lowfrequency gain which was corrected by changing the scale of the longitudinal cyclic to flybar pitch $C_{\text {lon }}$. After making these adjustments, the improved results were checked and the simulation was deemed to be a satisfactory representation of the Eagle helicopter. The final results are shown in Figures 3.11-3.12.


Figure 3.11: Lateral frequency response: This chart shows the roll rate response $(p)$ of the helicopter to lateral cyclic pitch input.


Figure 3.12: Longitudinal frequency response: This chart shows the pitch rate response $(q)$ of the helicopter to longitudinal cyclic pitch input.


Figure 3.13: Vertical frequency response: This chart shows the vertical acceleration response ( $a_{z}$ ) of the helicopter to collective pitch input.

### 3.9 Closed Loop Simulation

In chapter 2, I examined the control approaches of other groups working on autonomous helicopter control. A review of previous work suggests that a relatively simple yet practical scheme for controlling a helicopter would be the use of an attitude feedback inner loop combined with a PID based position and velocity outer loop. To prove this scheme would work with the quality of sensors, data rates, control lags that would exist in the real-system, I first tested it in simulation. In later chapters, the simulation will be used to demonstrate the practicality of various vision-based control modes in hover and forward flight. For an initial test of the control paradigm, I used simulated GPS and inertial sensor models to provide state measurements to the state estimator.

The controller consists of a number of PID control loops. Each PID controller is implemented as a discrete S-function with a time step equal to the 50 Hz servo update found on the actual helicopter. Equation (3.55) defines the PID control function based on the error signal $e$ and its derivative. The constants are the gains of the control block. Rather than calculating the error signal and derivative inside the block, the derivative is brought in separately from an external source. Calculating the derivative inside the block leads to unacceptable noise since the error signal may be noisy. Also, the derivatives are already available, being either measured directly, such as angular rates, or available from the state estimator.

$$
\begin{equation*}
y=K_{p} e+K_{d} \dot{e}+K_{I} \int e \tag{3.55}
\end{equation*}
$$

The tail rotor PID is fed the error in heading angle and the yaw rate as its
inputs. The inputs to the collective pitch PID are the error in altitude and the vertical velocity. The lateral and longitudinal control scheme consists of an outer and inner loop as proposed in Chapter 2.

The gains were tuned systematically using trial and error to converge on an acceptable solution. In the first instance, the attitude control loop was tuned independently by turning off the outer loop and stopping the integration of velocity and position in the dynamics block. Once satisfactory stability was demonstrated, the outer loop was re-activated and the position and velocity gains were then tuned. By disconnecting selected force and moment signals it was possible to tune pitch and roll independently at first. Based on the approach of Ziegler and Nichols [80], proportional gains on all loops were steadily increased to an ultimate gain $K_{u}$ for which the helicopter was marginally stable. The gains were then reduced to about $60 \%$ of $K_{u}$ and then small adjustments were made to obtain a good compromise between stability and speed of response.

Figure 3.14 shows the results of simulation of hover using the PID control scheme. The desired reference position for the hover is $X=0 \mathrm{~m}, Y=0 \mathrm{~m}, Z=0 \mathrm{~m}$ and $\psi=0^{\circ}$. Turbulence corresponding to $5 \mathrm{~m} / \mathrm{s}$ mean wind speed is applied to the relative air velocities. Sensor noise equivalent to that recorded from the actual sensors is applied to the inertial and DGPS sensor blocks. The results show that the helicopter is able to hold position and remains stable throughout the test. The helicopter height wanders no more than 20 cm from its datum position due to the effect of the turbulence.

### 3.10 Summary

A closed-loop simulation of the Eagle helicopter has been developed using SIMULINK ${ }^{\circledR}$. The model incorporates realistic sensor models, state estimation and control. In simulation, a position control loop scheme using PID control with an attitude feedback inner loop has been shown to be a practical means of controlling the helicopter. The simulation will be used in the following chapters to demonstrate the feasibility of various sensing and control schemes.


Figure 3.14: Simulation of closed loop hover

## Chapter 4

## System Overview

### 4.1 Introduction

This chapter provides an overview of the platforms and systems used to conduct the various flight experiments. This includes a discussion of the extensive embedded systems development carried out by the author to field the avionics, telemetry and sensors required to perform close loop flight experiments.

### 4.2 Helicopter Platforms

Experiments for this thesis were conducted on two separate helicopter types, both comprising a conventional helicopter design with a single main rotor and a single tail-rotor for anti-torque compensation. The first of these were built from kits manufactured by the Japanese hobby company Hirobo. These kits are all based on variants of the Eagle 60 size radio-controlled helicopter used by radio-control enthusiasts. The competition grade Eagles were selected on the basis of the perceived quality of construction. Being of all carbon fibre construction, the airframe weight is reduced, providing more capacity for avionics to be carried. Also, the Eagles had all metal control linkages and high quality rotor parts, reducing concerns relating to control slop and wear. In all, four Eagle helicopters were constructed and instrumented for visual control research. A gasoline powered Eagle was also built and tested but was abandoned due to low payload capability and high vibration. Three of the helicopters were powered by internal combustion engines running on methanol based fuel. A fourth helicopter was converted to electric power in 2005 using a brushless DC motor. This modification reduces vibration, eliminates fuel spills on to avionics, eliminates problems with exhaust smoke obscuring the camera and means that it is no longer necessary to account for fuel burnoff changing the helicopter weight and balance in flight. I made a decision to transition all experiments to the electric helicopter owing to these advantages. Unfortunately, it has only been in the last two years that advances in battery technology have made an electrical autonomous helicopter practical.

The second platform is an RMAX unmanned helicopter manufactured by Yamaha. The RMAX helicopter used for this thesis is designated as a model L-50 by the manufacturer and is designed for agricultural work. This RMAX has no inherent autonomy other than a Yamaha Attitude Control System (YACS) which provides


Figure 4.1: UNSW@ADFA RMAX in flight
stability augmentation to assist the pilot when flying with a remote control system.

### 4.3 Autopilot Systems

Two schemes are devised for conducting closed loop experiments. In the first scheme, a control by telemetry approach is taken, such that the sensor information is sent to a ground computer using a radio link. The ground computer processes the sensor information, calculates corresponding control inputs and sends these back to the helicopter via another radio link. The second scheme involves a fully embedded control system such that all of the processing is done on the helicopter in real-time. Different systems are used on the Eagle and RMAX but both make use of PC104 computers based on the Pentium III chipset for vision processing.

### 4.3.1 Control by Telemetry

In the control by telemetry scheme, most of the processing for vision, sensor fusion and control is completed on a standard PC on the ground. This provides ample processing power for the vision calculations. This scheme also has the advantage that software can be rapidly modified on the ground control computer without reprogramming the helicopter systems. The disadvantage of the system is the reliance on telemetry links which add lag to the control loop, restrict sensor bandwidth and introduce occasional random losses of data. This is more of a problem with the video data where the quality of the imagery is, at times, degraded.

A flight computer, based on an 8 bit 8051 microcontroller architecture, samples the onboard sensors sequentially and organises the telemetry information for transmission to the ground. The sensor data is sent to the ground digitally using the part of the video signal normally reserved for teletext called the Vertical Blanking Interval (VBI). The flight computer's other functions are to interrogate a GPS unit for position and velocity once per second, perform sensor self-test functions and to


Figure 4.2: Eagle with control by telemetry
determine the magnetometer offsets on start-up.
A UHF video transmitter is fitted to the side of the helicopter and used to transmit video and telemetry data to the ground. Various types of cameras can be bolted onto the helicopter front end. The video and telemetry signal generated by the helicopter is received by a purpose built UHF aerial and down-converter. The resulting video signal is then fed into a BT848 frame grabber fitted to a standard PC PCI slot. For the initial visual landmark based hover test phase, the video signal was also passed to a second PC so that parallel processing could take place. In this case, the first PC completes vision processing whilst the second PC decodes the telemetry stream, propagates the state estimate and communicates with the radio control interface. The two computers communicate using a parallel port cable between them. Later tests have been conducted on a single computer. The single computer was found to have sufficient speed to complete the optic flow computation and run the control system on one CPU at an update rate of 50 Hz .

A Graphical User Interface (GUI) for the control system was written by the author to allow update of control parameters in real time. Again, this made the process of tuning control gains much more efficient as changes could be made even when the helicopter is airborne. The GUI also serves as a monitoring tool, providing graphical display of navigation and state information. A particularly useful component of the GUI is a stripchart display that plots a moving history of selected variables to the screen in real-time. Some parts of the control GUI are shown in Figure 4.3. These include the main control panel, attitude indicator, horizontal situation indicator, control position dials and a map display with position of beacon landmarks shown.

The PC running the control system is interfaced to a microcontroller that in turn is interfaced to a hand held Radio Control (RC) model transmitter. A switch on the transmitter allows control to be passed between the computer control system and the human pilot. This enables the pilot to launch the helicopter manually and then


Figure 4.3: Control by telemetry GUI components
hand over control to the computer at an appropriate moment. This is an invaluable capability as it also allows the pilot to regain control of the helicopter instantly in the event of a control system failure. It also allows the pilot to pass control to the helicopter for a short period even when the control system is unstable, so that the closed loop behaviour can be observed.

### 4.3.2 Eagle Embedded Control

A Motorola MPC555 microcontroller was selected as the main processor for the embedded autopilot on the Eagle. The MPC555 is a 32 -bit device based on a 40 Mhz PowerPC core. It has a large array of peripheral equipment, which makes it suitable for embedded applications that require intensive computation, high integration, and expandability. Various single board computers (SBC) based on the MPC555 are available. A Phycore-MPC555 SBC was chosen due its small credit-card size and the abundance of onboard storage, including 4MB of RAM and 4MB of Flash ROM memory. The final autopilot design consists of three printed circuit boards surrounding the SBC. The additional circuit boards are stacked in line with the SBC and provide interfacing to the radio control system, servos, I2C bus, CAN-Bus, IMU and RS-232 devices such as the RF modem used for telemetry. The boards also contain power supply circuitry including a facility to monitor the battery and bus voltages.

For experimental purposes and safety, a mechanism is necessary to allow the helicopter to be switched between manual and automatic control. Some of the experiments conducted were of a high risk nature and required repeated attempts to achieve successful closed loop behavior, so special care had to be taken to ensure


Figure 4.4: Eagle with onboard embedded control
that pilot control could be regained rapidly in event of excursions from stable flight. A robust scheme for Hand Over Take Over (HOTO) is therefore implemented on the helicopter, allowing switching between these two modes. In manual mode, a human pilot flies the helicopter with a hand held radio control transmitter. In automatic mode, the helicopter controls are set by the autopilot. The $7^{\text {th }}$ receiver channel has been assigned for controlling the HOTO function and the autopilot sets automatic mode when this channels lie within a certain range of values. A switch on the pilot's transmitter is used to set the value of the $7^{\text {th }}$ channel to one of two possible values corresponding to automatic or manual.

The current set of servo actuators consists of five servo channels: collective, throttle, aileron, elevator and tail rotor pitch. To simplify testing, it is desirable to be able to choose which channels the pilot controls and which channels are controlled by the autopilot at any instance. This enables each control loop to be tuned individually, which is much easier to cope with experimentally. To facilitate this, a pass-thru parameter can be changed from the ground, which controls the source of each channel. The pass-thru parameter only applies in automatic mode and is implemented in software. The control channels to be set by the pilot are decoded by the autopilot and passed through to the servos as a Pulse Width Modulated (PWM) signal unchanged. In manual mode, all of the servo channels are driven directly from the receiver through a hardware switch.

In addition to the above HOTO system, a failsafe mechanism is present which de-activates automatic control when a watchdog circuit is not reset periodically by the main control loop. In the event of a software failure stopping the program from running, the main control loop is unable to toggle a designated data output from the MPC555. After a period of 0.1 seconds, the failure to toggle results in control being passed back to the radio control receiver.

Servo channels are controlled using pulse width modulation at an update rate
of 50 Hz . The MPC555 autopilot generates the PWM servo signals for up to eight channels. Currently only five channels are required to control the helicopter. The servo channel PWM outputs are sent in the order aileron, elevator, throttle, rudder, collective with 2.5 milliseconds (ms) spacing between each pulse. Every 20 ms the sequence repeats. In order to minimise control lag and therefore increase controllability, the leading edge of the PWM pulse for each channel is activated immediately after the control value for that channel has been calculated. As the range of PWM pulse widths is $1-2 \mathrm{~ms}$, this infers that the maximum control lag due to the PWM transmission is only 2 ms . Without this immediate update, the PWM lag would vary from $1-22 \mathrm{~ms}$ depending on what part of the PWM cycle the control update was made.

The current control loop executes at 400 Hz . At the start of each control loop iteration, sensor data is read in from the various sampling buffers on the MPC555 microcontroller. The sensor data is then processed to remove errors found in calibration and filter out unwanted noise. The corrected data is then used to update the onboard state estimate. As each control channel is only updated at 50 Hz , only one channel is calculated and updated per control loop. Based on this sequence of events, the maximum delay between a sensor being sampled and a control signal arriving at the servo is approximately 5 ms . Additional effective lags of about $5-10 \mathrm{~ms}$ are also present due to analog pre-filtering of the inertial data.

### 4.3.3 PC104 Implementation

Whilst the current MPC555 microcontroller based autopilot is satisfactory for embedding a basic attitude control system and position controller, its processing power is limited. This prevents onboard high-end processing such as image processing or sophisticated state estimation algorithms. A PC104 based flight computer was therefore developed. In the case of the RMAX helicopter, the MPC555 microcontroller system was not required as all of the interfacing to sensors and servos could be achieved using a single PC104 computer.

A Toronto Micro Electronics Incorporated (TME) PC104 board [181] was selected for the RMAX and Eagle after bench-testing its performance and confirming its ability to boot from an onboard compact flash card. A modified version of the Slackware ${ }^{\circledR}$ Linux operating system [182] was developed by the author for this board. The Kernel for the operating system is patched using RTLinux [183] version 3.1 source code to make it a Real Time Operating System (RTOS). This provides a real-time capability which runs entirely from the onboard compact flash card. The operating system includes X -windows and a full set of development tools, so that a keyboard and monitor can be connected to the PC104 and the system used as a hardware in the loop development system. For flight tests, the RTOS boots from the solid state compact flash disk on startup and automatically loads the flight control software.

A convenient feature of the PC104 architecture is a native USB interface. This allows a data logging feature using COTS memory stick devices. This means that high bandwidth flight test data can be recorded in flight and then transferred to


Figure 4.5: Eagle avionics architecture
a PC workstation immediately after landing for analysis. Memory sticks are now available with 4 GB capacity enabling practically unlimited amounts of flight test data to be recorded at the fastest sensor sampling rates $(1600 \mathrm{~Hz})$.

The integration of the PC104 into the Eagle avionics system is shown in Figure 4.5. An RS-232 communications link between the PC104 system and the autopilot allows data to be passed between the two systems. Attitude, rotation rate and control data will be passed from the autopilot to the PC104. High-level control information and commands are passed from the PC104 to the autopilot. The inner loop control function to control attitude is performed on the autopilot whilst the high-level vision processing functions are performed by the PC104 computer.

A Parvus Incorporated FG104 ${ }^{\mathrm{TM}}$ frame grabber module based on the Coxenant Fusion ${ }^{\mathrm{TM}} 878 \mathrm{~A}$ chipset is interfaced to the Eagle PC104 and the RMAX vision processing PC104. Using the PCI bus (also known as PC104Plus), the frame grabber writes the captured frames directly into the RAM of the host PC104 at 50 frames per second. As the video is interlaced, odd and even frames are written to separate parts of memory. Each captured frame consists of 288 x 384 pixels and for each pixel an 8-bit grey-level intensity is stored.

### 4.3.4 RMAX Systems

Some of the experiments for this research were conducted on the 80 kg Yamaha RMAX helicopter shown in Figure 4.1. This platform has been used for autonomous helicopter research at a number of other institutions including Georgia Tech [184], University of California (Berkeley) [185], Linköping University [186], Carnegie Mellon University [187] and NASA [188]. The Yamaha RMAX platform is predomi-
nantly used for agricultural work in Japan, although a fully autonomous version has been marketed for airborne surveillance. A number of variants have been produced but the underlying systems are similar in each model. The UNSW@ADFA RMAX provides a 30 kg payload with an endurance of approximately one hour. The performance of the RMAX makes it an ideal platform for research. The Yamaha control system is known as the Yamaha Attitude Control System (YACS). The YACS on the UNSW@ADFA RMAX has been configured to output inertial information to the PC-104 flight computer via an RS-232 link which includes the output of three fibre optic rate gyroscopes and three accelerometers. A Microstrain 3DM-GX1 ${ }^{\circledR}$ attitude sensor [189], incorporating a 3 -axis magnetometer, is fitted to the RMAX to provide heading information to the flight computer.

The RMAX came with a stability augmentation system based on an attitude control inner loop. The YACS provides attitude stabilisation using a simple proportional feedback scheme shown in Figure 4.7. The RMAX servos are driven by Pulse Width Modulated (PWM) signals such that the width of the PWM pulse corresponds to the desired servo position. The PC-104 flight computer sends servo commands to the YACS as 2 byte unsigned integer values corresponding to the desired PWM pulse width in microseconds. The YACS applies attitude feedback to these commands as shown in Figure 4.7.

The flight computer is interfaced to a Laser Rangefinder (LRF), a radio modem, a Novatel RT2 DGPS, YACS and a vision processing computer. The flight computer performs sensor fusion and generates the control inputs to drive the collective, cyclic and tail rotor servos. An 8-port PC104 serial card is necessary to connect the flight computer to all of the RMAX systems and the sensors. The image processing computer runs a free version of the RTLinux RTOS. For the flight computer the more heavily tested RTLinuxPro RTOS commercial variant is used. Whilst two PC104 computers are used for mission flexibility, both the control software and image processing software could run on one computer. Both PC104 computers are based on 800 MHz Pentium 3 CPUs with 256MB RAM. Figure 4.6 shows the avionics system architecture for the UNSW@ADFA RMAX.

The PC104 flight computers communicate with each other over a bi-directional RS-232 connection. The vision computer sends 27 byte packets containing optic flow and other visual information, PC104 status data and a checksum. The control computer is able to send commands to the vision computer over the same RS-232 link in order to change the mode that the flight computer operates in, for example to switch to a stereo vision computation.

### 4.4 Telemetry Systems

Bluetooth modem pairs operating at 2.4 GHz were installed on the Eagle and RMAX helicopters. The Eagles use a modem with a range of 100 m and a weight of 18 grams. The RMAX, with its larger payload capacity, has a different brand of modem with an external aerial that has a range of 500 m and a weight of 30 grams. The Bluetooth equipment permit full-duplex communications and baud rates up to 115 Kbaud . A


Figure 4.6: RMAX avionics architecture


Figure 4.7: Yamaha Attitude Control System (YACS)


Figure 4.8: Eagle control GUI. Various parameters can be changed in flight. The page shown in the diagram is used to adjust the controller gains remotely.
communications protocol was developed which intersperses 50 Hz telemetry from the helicopters with bi-directional command and control data between a ground station and the helicopters. The protocol used by the Eagles and the RMAX is the same except some platform specific packets apply to only one helicopter type.

The ground station comprises a personal computer running RTLinux with a serial port connected to the Bluetooth modem. Both desktops and a laptop computer have been used as the ground station for field work. The author has developed a real-time communications stack for the ground station which encodes and decodes telemetry packets between the helicopters and ground station. A GUI shown in figure 4.8 has also been developed to allow the operator to easily interact with the communications stack and send commands. This allows the type of telemetry being sent from the helicopter to be changed by the press of a button. Graphical utilities have been implemented to display all of the variables that can be sent from the helicopters.

Commands are sent to the helicopters in a set format depicted in Table 4.1. A header character and checksum allows the helicopter to decode command packets of variable length. The second character in the command data packet establishes the length of the packet being received. Depending on the type of the command being sent, the amount of command data being transmitted varies. Some commands such as 'start logging' have no option data while some commands such as an instruction to change a control gain may contain several bytes of option data. Every time a command is sent, the command identifier field is incremented to define a unique command. Upon successful receipt of a command packet, the autopilot sends an acknowledgement packet containing the command identifier embedded in the original message. The ground computer waits for a number of seconds after sending a command to see if an acknowledgment has been sent by the autopilot. A record of all commands sent to the helicopter is stored on the ground control computer. If an acknowledgement is not received within a set time (usually about 5 seconds) for

Table 4.1: Telemetry packet structure

| Byte Index | Description |
| :---: | :--- |
| 0 | Header character 0x55 |
| 1 | Command character |
| 2 | Length of packet in bytes (n) |
| 3 | Command identifier MSB |
| 4 | Command identifier LSB |
| $5 . . n$ | Data Options |
| $n+1$ | 8 bit checksum |

any of the commands sent, the command is resent. Fields within the control GUI corresponding to transmitted commands are highlighted in a bright red colour until an acknowledgement has been received. At present, 35 different types of command packets have been implemented to perform tasks such as zeroing of sensors, changing gains for PID controllers, changing flight modes and controlling logging of data.

Telemetry from the helicopters uses a similar scheme for transmitting sensor, state and control data so that it may be displayed on the ground station GUI or logged to a file. The type of telemetry being sent to the ground may be varied by sending a command from the ground station.

### 4.5 Sensors

### 4.5.1 Vision Sensors

The RMAX and Eagle helicopter are fitted with a compatible camera mounting bracket so that the cameras can be swapped between helicopters when required. The mount allows the camera to be pointed to any tilt angle from straight down to pointing forward along the helicopter longitudinal axis. For most of the experiments discussed in this thesis, an analog Sony CCD image sensor is used. The camera outputs a PAL standard composite video signal which is captured by the frame grabber. The camera is housed in a $3 \mathrm{~cm} \times 3 \mathrm{~cm} \times 2.5 \mathrm{~cm}$ enclosure with a lug to attach to the mounting bracket on the nose of each helicopter.

### 4.5.2 Inertial Measurement Unit

Whilst the RMAX comes fitted with gyroscopes and accelerometers, the Eagle helicopters had to be fitted with inertial sensors. When the project commenced, funding was not available for a satisfactory COTS Inertial Measurement Unit (IMU), forcing the construction of an in-house system. Three different IMU designs have been constructed, calibrated and test flown on the Eagle helicopters during the evolution of this project. The first of these comprises a sensor cluster with three accelerometers and three Murata ENV-05D gyroscopes in the same case. A separate box with a three-axis magnetometer system is used in conjunction with this system to provide
all the sensors needed for a complete Attitude Heading Reference System (AHRS). A 16bit Analog to Digital Converter (ADC) is installed in each box and interfaced to a remote microcontroller.

The other two IMU designs comprise three accelerometers, three magnetometers and three rate gyroscopes with ADC circuity and an integrated microcontroller in the same housing. The primary difference between these two IMUs is that the latest device uses smaller Analog Devices ADXRS-150ABG [190] gyroscopes rather than the Murata gyroscopes.

The axes of each set of sensors are arranged in an orthogonal fashion so that the orientation and motion of the helicopter along all three axes can be determined. The sensors rotate with the body axes system of the helicopter, which is fixed to the aircraft centre of gravity and rotates as the aircraft's attitude changes. The faces of the metal IMU case are deliberately machined to be square to within 0.1 degrees. The IMU $x, y$ and $z$ reference axes are defined in a direction normal to the faces of the IMU case. The naming of the IMU $\mathrm{x}, \mathrm{y}$ and z reference axes were selected so that when the IMU is installed in the helicopter, the axes would then be coincident with the helicopter body axes. The sensors used in the Inertial Measurement Units are as follows:
a. Accelerometers. The accelerometers measure the sum of gravitational acceleration and the velocity derivatives of the IMU. If the assumption is made that the motion derivatives average out to zero over a long enough time period, then the mean values of the accelerometers can be assumed to represent the gravity vector. As the axes of the accelerometers are aligned with the helicopter axes, the gravity vector is measured relative to the helicopter. In effect, the accelerometers act like a pendulum hanging from beneath the helicopter. Such a pendulum would tend to hang vertically downwards but would swing from side to side in response to horizontal accelerations. As well as providing a noisy estimate of the vertical direction, the accelerometers provide motion derivatives, which can be integrated to provide velocity.
b. Magnetometers. Each magnetometer provides a voltage output in proportion to the degree of alignment with the local magnetic field. Using three orthogonal magnetometers it is possible to measure the relative orientation of the earth's magnetic vector to the helicopter. The horizontal component of the earth's magnetic field points towards the magnetic North pole. When combined with the gravity vector measured by the accelerometers, the helicopter is able to determine its complete orientation including pitch, roll and yaw.
c. Gyroscopes. Three piezoelectric gyroscopes provide a voltage output proportional to measured rotation rate around the aircraft body axes. By integrating the gyroscopic rates, the attitude of the helicopter can be updated. The Murata gyroscopes used have a range of $-90^{\circ} / \sec$ to $+90^{\circ} / \sec$ and the Analog Devices gyroscopes have a range of $-150^{\circ} / \mathrm{sec}$ to $+150^{\circ} / \mathrm{sec}$.

In designing the IMUs for the MPC555 autopilot, the aim is to achieve the highest sampling rate possible without interrupting the processor continuously. This is achieved by having a separate processor to carry out the low-level sampling, leaving the MPC555 to run the state estimation algorithm without interruption.


Figure 4.9: Eagle IMU interface

A Microchip PIC18F6720 microcontroller [191] was selected on the basis of its low power and high speed to do the low level sampling of sensors. Figure 4.9 is a blockdiagram of the hardware components relating to the attitude state estimator.

The estimation cycle on the MPC555 executes at 400 Hz . The cycle is initiated by a timing pulse, which is sent from the MPC555 to the PIC. On receipt of the timing pulse, the PIC begins a set sequence of sampling and data transmission. The order of sampling and transmission is shown in the timing diagram at Figure 4.10. Each frame of data consists of 4 sets of accelerometer samples, 2 sets of gyroscope rate samples, 1 set of multiplexed accelerometer temperatures and 1 set of magnetometer values. All sensors are sampled at 1600 Hz , however, some averaging takes place onboard the PIC to reduce the bandwidth of the magnetometer, temperature and gyroscope data transferred to the MPC555.

All data from the PIC is sent as a synchronous serial stream transmitted at approximately $1 \mathrm{Mbits} / \mathrm{sec}$. A 64 -byte hardware buffer on the MPC555 receives the data. At the start of each cycle, the data from the previous 2.5 ms frame is read from the buffer and converted to raw sensor values with 16 -bit precision. The large hardware buffer on the MPC555 allows the entire frame of data from the PIC to be received without any software interrupts being required. Hence minimal overhead is associated with inter-processor communication.

To allow calibration, the PIC can be switched to a mode where sensor data is transmitted at 50 Hz using the RS-232 protocol. This protocol is compatible with standard PC serial port hardware and allows the IMU to be connected directly to a PC. Calibration software running in RTLinux on a host PC enables calibration data to be generated for the IMU. This data is in the form of alignment matrices, scale factors, offsets and thermal correction curves. After calibration, the calibration data is hard-coded into the Phycore-MPC555 using on-board non-volatile memory.

### 4.5.3 Vibration Issues

In the initial stages of this project, a major problem was the vibration due to the internal combustion engine and harmonics of the rotor aerodynamic excitation. The first attempt at installing accelerometers on an Eagle helicopter quickly showed


Figure 4.10: AHRS timing loop


Figure 4.11: Eagle avionics vibration isolation system. Four elastomeric isolators protect the PC104, GPS and IMU from vibration. Another four isolators protect the MPC555 autopilot.
that, without mechanical isolation, the accelerations due to vibration exceeded the 5 g dynamic range of the accelerometers used. Furthermore, the Murata gyroscopes used were based on a mechanical vibrating tuning fork arrangement which would fail from vibration fatigue after less than a minute of flight time. Many of the other groups working on autonomous helicopters have experienced similar problems with vibration and have used ad-hoc techniques such as rubber and foam mountings selected by trial and error. Dunabin et al [192], provide a good narrative of their attempts to isolate vibration on an X-cell 60 size helicopter, starting from wrapping components in foam, to designing an elaborate spring-mass damper system. A similar route was followed for the Eagle helicopters, beginning with rubber bushes and ending with an isolation housing consisting of 12 springs and damper pairs. This system was used for 5 years but required the foam dampers to be replaced regularly as they were subject to wear and degradation from the oil sprayed on them by the motor exhaust. In 2004, a much better commercially available set of vibration isolators was found. This design, shown in Figure 4.11, has proven to be very reliable and does not require maintenance. The inertial sensors for the RMAX were pre-fitted with an isolation system so that further isolation design was not required.

Even with mechanical isolation, the vibration of the helicopter imparts significant synchronous noise on the acceleration sensors. This vibration is predominantly at the rotor speed of 25 Hz with other high frequency components due to the engine and tail rotor. The accelerometer raw output can vary from 0 to 65536 corresponding approximately to a sensor analog value of -5 g to +5 g . To deal with vibration, a low pass filter with a 10 Hz cutoff frequency has been implemented digitally. A 255th order Finite Impulse Response digital filter is used to provide satisfactory attenuation of unwanted high frequency components ( 40 dB above 15 Hz ) whilst not exceeding the computational capacity of the MPC555 microcontroller.

An advantage of the filter type chosen is a linear phase response so that all frequencies are lagged by a constant amount, in this case 80 ms . The filtered output


Figure 4.12: Laser scanning system
from the accelerometers is used in conjunction with gyroscopic rates to calculate the vertical orientation of the helicopter. To eliminate the lag associated with the filtering process, the attitude estimate is corrected with filtered accelerometer data and then propagated forward using gyroscope rate date to cancel the 80 ms phase lag created by the filtering.

### 4.5.4 Differential GPS

For the purposes of this experiment, highly accurate carrier phase DGPS measurements were available to provide a monitoring system to record the helicopter motion during closed loop. Novatel OEM4-G2L GPS [193] cards are mounted adjacent to the Eagle and RMAX flight computers. The OEM4 cards are used with differential corrections from a nearby base station fitted with another Novatel OEM4 card. The card operates in a Real-Time Kinematic (RTK) positioning mode, to provide 20 Hz position and velocity with an accuracy of $1-2 \mathrm{~cm}$ Circular Error Probability (CEP).

### 4.5.5 Laser Rangefinder

A Laser Rangefinder (LRF) with a novel rotating mirror was integrated with the RMAX helicopter by the author as shown in Figure 4.12. Owing to the orientation of the axis of the mirror and the angle of the mirror to the axis, the laser traces an elliptical cone shape on the ground below. As the laser traces out a locus of points on the ground, an array of 3D coordinates is assembled that defines the intersection of the laser scan pattern and the ground. Each scan takes place in less than 40 milliseconds and typically comprises 100 points. As the range accuracy of each point on the ground is better than 2 cm in practice, the error in the ground position is small and suitable for guiding the vertical trajectory of the helicopter in close proximity to the ground. A plane fitted through these points using an appropriate technique such as least squares defines the relative distance and orientation of the ground with respect to the helicopter.

An AccuRange 4000 Laser Rangefinder from Acuity [194] is used for this project. This rangefinder uses a modulated beam to measure range using a time-of-flight method. The 20 milliwatt beam is transmitted by a laser diode at a wavelength of


Figure 4.13: Rotating mirror assembly

780 nm . The manufacturer claims a stated range of a few cm up to 16.5 m with an accuracy of 2.5 mm . Although Acuity provide a linecaster system of their own, this system is replaced to obtain the conical scan pattern peculiar to this application. A mirror is machined out of a block of aluminium with a reflecting face cut at $86.5^{\circ}$ to the axis of the motor. This provides a distorted cone with an average included angle of approximately $7.0^{\circ}$ as the mirror spins. The mirror is hand polished and then electroplated with gold to provide a reflectivity to the laser light of $95 \%$. Figure 4.13 shows the mirror assembled on the drive motor.

To obtain a sufficiently fast scan rate, the mirror is spun at least 1500 RPM or 25 cycles per second. As the mirror is not symmetrical about the axis, the imbalance of the mirror needed to be addressed to operate at these speeds. A balancing collar was therefore designed out of stainless steel to offset the static and dynamic imbalance of the mirror. The collar is of constant thickness but the end profile is machined to maintain the centre of gravity of each longitudinal slice of the combined collar/mirror assembly on the axis of the shaft. Once assembled, the balance of the assembly is finely adjusted by adding tiny weights and removing material where required. The entire assembly with LRF and mirror is mounted under the belly of the RMAX using 4 cable mounts for vibration isolation. The mounts were tuned to attenuate vibrations at the main rotor frequency and above.

The mirror is mounted directly onto the shaft of a small DC motor, which is itself mounted at $45^{\circ}$ to the beam of the laser rangefinder. The speed of the motor can be adjusted by changing the input voltage using a multi-position switch. An optical encoder is fitted on the shaft of the motor. The encoder outputs a quadrature pulse train with a precision of 4096 pulses per revolution. An index pulse is triggered once per revolution for synchronisation purposes. The pulse train from the encoder is monitored by an analogue safety interlock system that automatically disrupts power to the laser in case of the mirror speed falling below 1000 rpm . This stops the laser from being concentrated on to a single spot for too long and causing a hazard to observers.

The rangefinder and encoder signals are read into a PC104 form factor High

Speed InterFace (HSIF) manufactured by Acuity. The HSIF uses an ISA bus interface to communicate with the CPU and enables a sample rate of up to 50,000 samples per second. For this application, a sample rate of 2 KHz is adequate to provide the accuracy required without overloading the processing capability of the PC104. The HSIF comes standard with a 2 K hardware buffer. A half-full interrupt on the HSIF triggers a real-time driver program on the PC104 to download the next series of samples and load them into RAM. As each sample comprises 8 bytes including amplitude, range, encoder and intensity fields, the buffer becomes half-full after 128 samples are received. A control thread running on the PC104 executes at 100 Hz and checks for the latest data in RAM each time it is woken up. With a sample rate of 2 KHz , the interrupt is triggered about every 64 ms and takes about 1 ms to execute. The processing takes place after the full data set for each rotation of the mirror is received which occurs about every 40 ms . Combining all of the latencies together results in a maximum latency of approximately $10+64+1+40=124 \mathrm{~ms}$.

For each scanned sample, a distance measurement and encoder output are taken from the laser apparatus. The range measurement is corrected for known errors due to changes in temperature, ambient light and reflectivity. The range measurement is scaled into meters using a look up table based on calibration data. The encoder measurement is converted to an azimuth angle measured from a reference point designated as the nose of the vehicle. Each pair of range and azimuth angle measurements can be converted in to the coordinates of a point in 3D relative to the laser sensor. In other work by the author [195], a system is described for fitting a plane to the points in a complete revolution of the spinning mirror. The orientation and position of the plane can then be used to determine the slope of the ground and its proximity. For the purposes of this thesis, the average range calculated from one revolution of the mirror is used to calculate the helicopter's height above the ground.

A flight test was conducted to check that the height output of the laser sensors was reasonable and compared well with a second measurement of height. The helicopter was placed in vertical flight with climbs and descents set up using pilot collective. The output of the sensor was compared against the altitude output of the Novatel DGPS system after subtracting the elevation of the ground under the helicopter from the DGPS altitude. The results of this comparison, in Figure 4.14, show a very close match between the two systems.

### 4.6 Sensor Calibration

### 4.6.1 Optic Flow Calibration

A special purpose calibration machine was constructed to enable calibration of the optic flow sensing. This is required so that the constants $c_{x}$ and $c_{y}$ can be found in the linear relationships $Q_{x}=c_{x} v_{x}$ and $Q_{y}=c_{x} v_{y}$ which relate the pixel shifts per frame to the optic flow angular velocities in radians $/ \mathrm{sec}$. The two constants can be approximately calculated from the field of view of the camera and the number of pixels in the image. However, due to distortion from the optics and other errors, it


Figure 4.14: Validation of laser rangefinder sensor height measurement


Figure 4.15: Optic flow calibration machine
is prudent to re-calibrate the system with each new camera that is installed. The machine consists of two 42 cm rollers guiding a 30 cm wide belt as shown in Figure 4.15. Photographs of grass were taken with a digital camera and printed on to the belt to provide a realistic textured environment. A DC electric motor is used to drive one of the rollers through a set of gears. The belt is driven at various speeds by changing the voltage to the motor, and the average optic flow measured by the algorithm is recorded. A stopwatch is used to record the time taken for the belt to travel around an integral number of revolutions, and the actual optic flow in ( $\mathrm{rad} / \mathrm{sec}$ ) is then calculated by dividing the belt velocity by the distance of the camera from the belt. Figure 4.16 shows a graph of the known optic flow generated by the belt at a number of different belt speeds versus the estimated flow measured by the sensor after scaling based on the known camera specifications. The calibration constants were determined from a simple linear fit to the data.


Figure 4.16: Calibration of optic flow scale

### 4.6.2 IMU Calibration

One of the major problems facing attitude estimators is the effect of sensor error. Various forms of sensor errors exist including misalignment, offset, gain and thermal drift. With careful calibration, many of the fixed errors can be reduced significantly. Specifically, calibration procedures have been devised for the following errors:
a. Sensor misalignment. The alignment of the axis of each sensor within its package is generally only known to within a few degrees. Consequently, it is necessary to determine the actual alignment of the sensors with respect to the IMU housing. Misalignment of the sensors results in cross-coupling between the various sensor axes. Misalignment is corrected by multiplying the sensor values by a 3 x 3 alignment matrix. This alignment matrix is determined in a set of separate calibration processes devised for each of the accelerometer, gyroscope and magnetometer sensors.
b. Gain. Each sensor outputs a voltage between 0 and 5 V which is proportional to the variable being measured. The sensors have a linear relationship between input and output and the gain of each sensor refers to the slope of the voltage versus input variable. The gain is slightly different for each sensor and is determined during calibration.
c. Offset. Each sensor has a nominal output of about 2.5 v when the measured variable is zero. For accelerometers and gyroscope, this applies to the case where there is no acceleration or no rotation respectively. For the magnetometers, a zero reading occurs when the magnetic field is perpendicular to the sensor. The null position of each sensor is usually slightly offset from 2.5 v due to variations during the manufacturing process. This offset is measured during calibration and stored for each sensor in non-volatile memory on the MPC555 autopilot.
d. Thermal Drift. Variations in temperature cause a shift in the offset of each sensor. This shift can be measured during calibration by cycling the temperature of
the IMU in a thermally controlled environment and measuring the change in sensor reading. As the thermal drift is the only error that changes dynamically, it is the only error that needs to be accounted for post-calibration.

I have developed my own methods and software for calibration, for which a detailed description is provided in Appendix C.

### 4.7 Attitude Determination

Helicopters are dynamically unstable and require constant control inputs to prevent them from diverging from a level-flying attitude. Accurate knowledge of attitude (pitch, roll and yaw) is therefore vital to robust control. The RMAX helicopter is fitted with an in-house sytem for attitude determination. However the Eagle helicopter requires development of an attitude state estimator.

Determination of attitude using inertial sensors is made by measuring the direction of two vectors: the gravity vector and the magnetic vector aligned with the Earth's local magnetic field. The global z-axis (vertical) is aligned with the gravity vector. Knowledge of this vector alone is sufficient to determine pitch and roll angles. Determination of yaw, however, requires at least one other orthogonal measurement vector. The magnetic vector provides this information.

I have developed a simple but robust attitude estimation algorithm for determining attitude for the control by telemetry Eagle and on the MPC555 autopilot. This algorithm does not attempt to estimate sensor errors during operation and uses a fixed gain correction to the attitude estimate from observed attitude vectors at each time step. The simple attitude algorithm has many advantages over a more complex filtering algorithm. These include:
(a) Low computational overhead. The filter can therefore be run at much higher speed than would otherwise be possible and will have much faster dynamic response. This trait is essential for robust real-time control.
(b) Robustness. The filter is highly robust to initial conditions being inaccurate and can withstand errors in attitude of 180 degrees. Acceleration noise levels due to vibration which are several times larger than the acceleration due to gravity, do not prevent the filter from functioning adequately. This is especially important for operations on a small rotary wing vehicle, which will be subject to a high level of vibration.

Attitude is stored in the autopilot as six state variables. The six states comprise the estimate of the three components of the gravity vector and the three components of the magnetic vector. Gyroscopic information is used to update the estimate of the two vectors by rotating their orientation in accordance with measured rotation rates. If the vectors were only updated by the gyroscopic rates, the attitude would gradually diverge from the actual attitude because of sensor errors and numerical rounding. This divergence is prevented by making a small correction to the vector estimates based on measurements of the acceleration and magnetic vectors. Equation (4.1) is used to correct the estimated vectors from the observed vectors.

$$
\begin{equation*}
x_{k+1}=x_{k}+\lambda\left(x_{o b s}-x_{k}\right) \tag{4.1}
\end{equation*}
$$

where $x_{k}$ is the current estimate of the vector, $x_{o b s}$ is the noisy measurement of the vector and $\lambda$ is the correction factor. Experiments have shown that a good choice of value for $\lambda$ is about 0.01 .

Attitude is estimated by comparing the difference between the body and inertial frames of reference. The first step is to express the orientation of each inertial axis as a unit vector in body coordinates. Now, the inertial frame of reference is defined by a set of axes $x, y, z$ where $x$ represents the horizontal North direction, $y$ the horizontal East direction and $z$ represents vertically down. The unit vectors representing the inertial axes in inertial coordinates are given by the trivial expressions in Equation (4.2).

$$
\hat{\boldsymbol{X}}_{\boldsymbol{g}}=\left[\begin{array}{lll}
1 & 0 & 0
\end{array}\right]^{T}, \hat{\boldsymbol{Y}}_{\boldsymbol{g}}=\left[\begin{array}{lll}
0 & 1 & 0
\end{array}\right]^{T}, \hat{\boldsymbol{Z}}_{\boldsymbol{g}}=\left[\begin{array}{lll}
0 & 0 & 1 \tag{4.2}
\end{array}\right]^{T}
$$

The inertial axes unit vectors are transformed to body coordinate unit vectors by multiplying by the rotation matrix $\boldsymbol{B}$ as follows,

$$
\hat{\boldsymbol{X}}_{\boldsymbol{g}}=\left[\begin{array}{lll}
\hat{X}_{b} & \hat{Y}_{b} & \hat{Z}_{b}
\end{array}\right]^{T}=\boldsymbol{B}\left[\begin{array}{lll}
\hat{X}_{g} & \hat{Y}_{g} & \hat{Z}_{g} \tag{4.3}
\end{array}\right]^{T}
$$

where each unit vector forms one column of a 3 x 3 square matrix. Since the inertial unit vectors together form the identity matrix, the rotation matrix $\boldsymbol{B}$ is simply equal to the 3 x 3 square matrix formed from the body coordinates of the inertial axes unit vectors.

Unit vectors representing the inertial axes can be obtained in body coordinates from the IMU state estimate. As the $\hat{\boldsymbol{Z}}_{g}$ unit vector represents vertically down, its value in body coordinates is taken directly from the gravity estimate. The $\hat{\boldsymbol{Y}}_{\boldsymbol{g}}$ axis represents the horizontal East direction. The East direction is perpendicular to the plane of the magnetic and gravity vectors. Hence the $\hat{\boldsymbol{Y}}_{\boldsymbol{g}}$ unit vector in body coordinates is found by taking the vector cross product of the magnetic and gravity vector estimates. Finally, the $\hat{\boldsymbol{X}}_{\boldsymbol{g}}$ axis represents the horizontal North direction. It is perpendicular to the other two axes and is found by taking the vector cross product of the body coordinates of the $\hat{\boldsymbol{Y}}_{\boldsymbol{g}}$ and $\hat{\boldsymbol{Z}}_{\boldsymbol{g}}$ axes.

Once the rotation matrix has been determined, the attitude can be calculated from it. If $\boldsymbol{B}$ represents the complete transformation from inertial to body axes for an attitude $(\phi, \theta, \psi)$, then $\boldsymbol{B}$ is given by Equation (4.4). (Note the use of the shorthand for the sin and $\cos$ functions e.g. $c_{\phi}$ means $\cos (\phi)$.)

$$
\boldsymbol{B}=\left[\begin{array}{ccc}
c_{\theta} c_{\psi} & c_{\theta} s_{\psi} & -s_{\theta}  \tag{4.4}\\
-c_{\phi} s_{\psi}+s_{\phi} s_{\theta} c_{\psi} & c_{\phi} c_{\psi}+s_{\phi} s_{\theta} s_{\psi} & -s_{\phi} c_{\theta} \\
s_{\phi} s_{\psi}+c_{\phi} s_{\theta} c_{\psi} & -s_{\phi} c_{\psi}+c_{\phi} s_{\theta} s_{\psi} & c_{\phi} c_{\theta}
\end{array}\right]
$$

The attitude angles can be readily found from elements in the first row and last column of $\boldsymbol{B}$. Thus, if $b_{i j}$ is an element of $\boldsymbol{B}$ from row $i$ and column $j$,

$$
\begin{equation*}
\phi=\operatorname{atan} 2\left(b_{23}, b_{33}\right), \theta=-\operatorname{asin}\left(b_{13}\right), \psi=\operatorname{atan} 2\left(b_{12}, b_{11}\right) \tag{4.5}
\end{equation*}
$$

The attitude estimator implemented on the control by telemetry computer executes at 200 Hz whilst that on the MPC555 updates at 400 Hz . Trapezoidal integration is used to integrate the gyroscope rotation rates and update the estimated gravity and magnetic vectors. The attitude is converted to Euler angles only when required for control purposes. As the servo update rate is 50 Hz , the conversion to Euler angles is only made at 50 Hz .

### 4.8 Summary

In this chapter, the development of autonomy systems needed to conduct the flight experiments in this thesis have been described. The systems are well-suited to rapid prototyping of algorithms, owing to the ability to change control parameters and software quickly in the field.

## Chapter 5

## Control of Hover

### 5.1 Introduction

Various visual mechanisms for controlling hover on the RMAX and Eagle helicopters have been field tested. Hover is the most difficult phase of flight to control as there is no additional stability provided by the vertical tail and horizontal tail. Hence once control of hover can be achieved, transition to more elaborate flight modes should be possible.

The first means of controlling the helicopter was chosen to prove that the control by telemetry system could be used with a vision sensor to control the helicopter. In the first instance, mimicking biological sensing and control was not a high priority, so a conventional visual servoing method was used where the position of the helicopter was triangulated from known landmarks on the ground. Once the landmark or beacon hover was completed, work then turned to achieving a biologically inspired means of sensing.

### 5.2 Beacon Hover

The so-called beacon algorithm uses the known position of three visual landmarks (beacons) placed on the ground to triangulate position and attitude. As well as application to hover, the method could also provide a means for guiding the helicopter during a landing approach to a prepared site, provided that the separation and size of beacons is sufficient for the camera to resolve. The accuracy of the technique for a conventional camera system is about 2 cm in position and 1 degree in attitude for a helicopter hovering in front of the beacons, when the beacons are spaced about 5 m apart. The technique is therefore comparable to the most accurate implementations of differential carrier phase GPS, without the need for costly base station infrastructure.

A single camera is used to track the elevation and azimuth angles of the beacons with respect to the helicopter local axes. The visual segmentation software produces six observations, consisting of three pairs of azimuth and elevation angles. The conversion of these angles into 6 DOF position and attitude coordinates is nontrivial and has no closed form solution [196].

The method chosen is a numerical procedure based on a search through angle space to determine all possible solutions. For each set of 6 beacon bearings, there


Figure 5.1: Beacon geometry
are between 1 and 4 possible solutions for position and attitude [197]. The correct solution from all possible solutions must be determined using some means to eliminate invalid solutions.

The first step in solving the geometry is to express the problem in a 2D diagram. Lines between the camera and the beacons can be thought of as the edges of a triangular based pyramid. When folded out flat, the pyramid has the form shown in Figure 5.1.

The distances $d_{12}, d_{23}, d_{13}$ are known as they are determined directly by the spacing used to separate the beacons on the ground. The distances $r_{1}, r_{2}$ and $r_{3}$ between the camera and the beacons are not known and must be determined. In order to solve for the helicopter position and attitude, all angles and lengths in the diagram must be calculated. The beacon algorithm starts by guessing geometry for the inner triangle such that $r_{2}$ and $r_{3}$ are defined. Assuming a value for $\alpha_{2}$ achieves this. Using trigonometry it is then possible to come up with two solutions for $r_{1}$ for each of the two triangles. By intelligently searching through all possible values of $\alpha_{1}$ it is possible to find the cases where the values of $r_{1}$ calculated in the left and right outer triangles are equal. Such solutions fully define both the range and bearing to all beacons which permits determination of position and attitude. The steps used in the beacon algorithm are listed below:

Step 1 - Known geometry can be evaluated first. Distances between the beacons are known from their separation along the ground. The included angles $\beta_{1}, \beta_{2}$ and $\beta_{3}$ can be solved analytically. The bearing to each beacon is then converted into a unit vector $\hat{\boldsymbol{A}}_{\boldsymbol{i}}$ centred on the camera location using the coordinate transformation given in Equation (5.1).

$$
\left[\hat{\boldsymbol{A}}_{\boldsymbol{i}}\right]=\left[\begin{array}{lll}
\cos \left(\Psi_{i}\right) \sin \left(\Phi_{i}\right) & \sin \left(\Psi_{i}\right) \sin \left(\Phi_{i}\right) & \cos \left(\Phi_{i}\right) \tag{5.1}
\end{array}\right]^{T}
$$

where $\Psi$ and $\Phi$ are the azimuth and elevation angle of the beacon as seen from the helicopter camera. The included angle between each unit vector pair, denoted by $\hat{\boldsymbol{A}}_{\boldsymbol{i}}$ and $\hat{\boldsymbol{A}}_{\boldsymbol{j}}$, is then found using Equation (5.2).


Figure 5.2: Multiple solutions to the beacon problem

$$
\begin{equation*}
\cos (\beta)=\frac{\left\langle\hat{\boldsymbol{A}}_{\boldsymbol{i}}, \hat{\boldsymbol{A}}_{\boldsymbol{j}}\right\rangle}{\left\|\hat{\boldsymbol{A}}_{\boldsymbol{i}}\right\|\left\|\hat{\boldsymbol{A}}_{\boldsymbol{j}}\right\|} \tag{5.2}
\end{equation*}
$$

Step 2 - The range of possible values of $\alpha_{2}$ is $0^{\circ}$ to $\left(180^{\circ}-\beta_{2}\right)$. The objective of the algorithm is to determine the values of $\alpha_{2}$ which make the difference $\Delta r=$ $r_{1 L}-r_{1 R}$ equal to zero where $r_{1 L}$ and $r_{1 R}$ are the estimates of $r_{1}$ from left and right triangles respectively. A simple binary search through all possible values of $\alpha_{2}$ is not satisfactory as there may be multiple solutions. Figure 5.2 shows how for a given value of $r_{2}$, there are two possible values of $r_{1}$, denoted by $r_{1}^{+}$and $r_{1}^{-}$that will fit the geometrical constraints of the problem. As this happens in both left and right triangles, there can be up to 4 different beacon ranges that satisfy the problem.

There will be values of $\alpha_{2}$ for which outer triangles cannot be constructed, as the base of the triangles are too short to reach the side where $r_{1}$ is measured. A search is conducted to determine all of the intervals of $\alpha_{2}$ in which real solutions for $r_{1 R}$ and $r_{1 L}$ are possible. There can be up to 4 intervals of $\alpha_{2}$ and each of these intervals may contain a solution. A fixed depth binary search is carried out on each of these intervals to places where $\Delta r$ is zero.

Step 3 - Once solutions to the pyramid geometry are available, the coordinates of each beacon are known by their elevation, azimuth and range relative to the helicopter. These spherical coordinates are converted into Cartesian coordinates in the helicopter local coordinate frame. To extract the helicopter position and attitude from this information, the known orientations of the beacons are exploited. Firstly the local coordinates of the centre of the beacons, which is the position datum, is found by taking the average of the local coordinates of each beacon. Each beacon local coordinate is then translated with respect to this datum. After this step, the beacons are now in a coordinate frame such that they are arranged equidistant from the datum and in a plane parallel to the helicopter reference frame. The transformation between the beacon global coordinates and local coordinates is completed using a 3 x 3 rotation matrix $\boldsymbol{B}$ as shown in Equation (5.3).

$$
\left[\begin{array}{lll}
x_{1} & x_{2} & x_{3}  \tag{5.3}\\
y_{1} & y_{2} & y_{3} \\
z_{1} & z_{2} & z_{3}
\end{array}\right]_{\text {local }}=\boldsymbol{B}\left[\begin{array}{lll}
x_{1} & x_{2} & x_{3} \\
y_{1} & y_{2} & y_{3} \\
z_{1} & z_{2} & z_{3}
\end{array}\right]_{\text {global }}
$$

The positions of the beacons in global coordinates relative to the datum are known from the constraints of the problem. The beacon positions in local coordinates are the output of the beacon algorithm. Hence the equation above can be re-arranged to solve for the rotation matrix in terms of the beacon observations and their known positions as shown in Equation (5.4).

$$
\boldsymbol{B}=\left[\begin{array}{lll}
x_{1} & x_{2} & x_{3}  \tag{5.4}\\
y_{1} & y_{2} & y_{3} \\
z_{1} & z_{2} & z_{3}
\end{array}\right]_{\text {local }}^{-1}\left[\begin{array}{lll}
x_{1} & x_{2} & x_{3} \\
y_{1} & y_{2} & y_{3} \\
z_{1} & z_{2} & z_{3}
\end{array}\right]_{\text {global }}
$$

With the rotation matrix defined, the aircraft attitude can be extracted. The most robust method is to compare elements of the rotation matrix with the quaternion representation of $\boldsymbol{B}$ derived from first principles. It can be shown [198] that the magnitude of each quaternion parameter $q_{i}$ can be expressed in terms of the main diagonals of the rotation matrix as in Equation (5.5).

$$
\begin{align*}
& 4 q_{0}^{2}=1+b_{11}+b_{22}+b_{33} \\
& 4 q_{1}^{2}=1+b_{11}-b_{22}-b_{33} \\
& 4 q_{2}^{2}=1-b_{11}+b_{22}-b_{33} \\
& 4 q_{3}^{2}=1-b_{11}-b_{22}+b_{33} \tag{5.5}
\end{align*}
$$

where $b_{i j}$ are the elements of the rotation matrix $\boldsymbol{B}$. The sign of each quaternion parameter can be found from the off-diagonal terms in a similar fashion. As an alternative, Euler angles can also be calculated directly from the rotation matrix, however, Euler angles are an inferior method of storing attitude since they suffer from singularities. With attitude determined, the position of the helicopter is found by rotating the position of the datum point to global coordinates.

Step 4-Corresponding to each unique set of beacon ranges, will be a helicopter position and attitude which would produce the same three pairs of elevation and azimuth angles to target. For example, the set of beacon bearings in table 5.1 gives rise to the two 6DOF solutions in table 5.2. Clearly, only one solution can be correct at any one time. The method used to determine which of the 1-4 beacon algorithm solutions is valid is in two parts. Firstly, a comparison with the inertial attitude is made. If the beacon attitude differs more than $15^{\circ}$ from the inertial attitude, it is discarded. This part is not sufficient by itself, as in some circumstances the beacon attitude can be very close to the actual attitude, but still be the wrong solution. Secondly, if more than one solution to the beacon algorithm exists at this point, the current estimate of position is compared to the solutions. The solution closest to the current position estimate is then assumed to be correct.

| Beacon | Azimuth <br> $\Psi$ | Elevation <br> $\Phi$ |
| :--- | :---: | :---: |
| 1 | $95.81^{\circ}$ | $41.96^{\circ}$ |
| 2 | $94.30^{\circ}$ | $50.51^{\circ}$ |
| 3 | $80.17^{\circ}$ | $46.89^{\circ}$ |

Table 5.1: Example set of bearings for a North facing helicopter hovering level 2 m above and to the left of the beacon datum

| DOF | Solution 1 | Solution 2 |
| :--- | :---: | :---: |
| x | -0.4718 | 0.0000 |
| y | 1.9872 | -2.0000 |
| z | -1.8166 | -2.0000 |
| roll | $95.29^{\circ}$ | $0^{\circ}$ |
| pitch | $9.43^{\circ}$ | $0^{\circ}$ |
| yaw | $5.26^{\circ}$ | $0^{\circ}$ |
| $r_{1}$ | 2.875 | 2.555 |
| $r_{2}$ | 2.425 | 2.988 |
| $r_{3}$ | 2.751 | 2.780 |

Table 5.2: Multiple solutions to the beacon problem: A pair of solutions for the given beacon bearings. Solution 2 is valid but solution 1 is invalid. Note ranges to the beacons $\left(r_{1-3}\right)$ are different for each solution

### 5.2.1 Sensor Fusion

The obvious choice of algorithm for fusing sensor data and predicting the unmeasured state velocities would be an Extended Kalman Filter (EKF). For the practical purposes of the experiment, however, gains had to be tuned in the field, so that closed loop flights of only a few seconds each were achieved initially, before recovery action had to be initiated by the pilot. This short time frame was not enough for an EKF to properly converge. Consequently a fixed gain filter was chosen to correct position and velocity estimates. The helicopter position and velocity is propagated as a six state vector, consisting of three Cartesian position estimates and three velocities in local coordinates. The state vector is updated by prediction and correction cycles:

Predict Cycle - The first step is a prediction based on the measured accelerations, rotation rates and attitude. The state update equations were based on the rigid body equations of motion at (5.6) and (5.7). These were integrated at 200 Hz using a trapezoidal integration scheme. The variables $a_{x}, a_{y}$ and $a_{z}$ are the measured accelerations from the IMU. The variables $p, q$ and $r$ represent the rotation rates sensed by the IMU rate gyroscopes. The variable $u, v$ and $w$ are the velocities of the helicopter in local body axes. The rotation matrix $\boldsymbol{B}$ is determined directly from the attitude of the helicopter using Equation (3.6).

Position:

$$
\left[\begin{array}{l}
\dot{x}  \tag{5.6}\\
\dot{y} \\
\dot{z}
\end{array}\right]=\boldsymbol{B}\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]
$$

Velocity:

$$
\left[\begin{array}{c}
\dot{u}  \tag{5.7}\\
\dot{v} \\
\dot{w}
\end{array}\right]=\boldsymbol{B}\left[\begin{array}{l}
0 \\
0 \\
g
\end{array}\right]+\left[\begin{array}{l}
a_{x} \\
a_{y} \\
a_{z}
\end{array}\right]+\left[\begin{array}{ccc}
0 & r & -q \\
-r & 0 & p \\
-p & q & 0
\end{array}\right]\left[\begin{array}{l}
u \\
v \\
w
\end{array}\right]
$$

Correct Cycle - The second step is a correction based on measurements of position and attitude. As position measurements are only available at a video frame rate of 50 Hz , there are 4 predict cycles for every correct cycle. A fixed gain correction is used to correct position and velocity in accordance with Equations (5.8) and (5.9). The - and + superscripts below represent the pre and post priori estimates of the state variables respectively. The * superscript denotes an observation and the time shift between values with subscript k and $(\mathrm{k}-1)$ is $\Delta t=0.02$ seconds.

Position:

$$
\left[\begin{array}{c}
x_{k}^{+}  \tag{5.8}\\
y_{k}^{+} \\
z_{k}^{+}
\end{array}\right]=\left[\begin{array}{c}
x_{k}^{-} \\
y_{k}^{-} \\
z_{k}^{-}
\end{array}\right]+K_{1}\left[\begin{array}{c}
x_{k}^{*}-x_{k}^{-} \\
y_{k}^{*}-y_{k}^{-} \\
z_{k}^{*}-z_{k}^{-}
\end{array}\right]
$$

Velocity:

$$
\left[\begin{array}{c}
u_{k}^{+}  \tag{5.9}\\
v_{k}^{+} \\
w_{k}^{+}
\end{array}\right]=\left[\begin{array}{c}
u_{k}^{-} \\
v_{k}^{-} \\
w_{k}^{-}
\end{array}\right]+\frac{K_{2}}{\Delta t} \boldsymbol{B}\left[\begin{array}{c}
x_{k}^{*}-x_{k-1}^{*} \\
y_{k}^{*}-y_{k-1}^{*} \\
z_{k}^{*}-z_{k-1}^{*}
\end{array}\right]
$$



Figure 5.3: Beacon algorithm hover controller

### 5.2.2 Control Strategy

Simulation studies have shown that an effective strategy for the control of a model helicopter is to use an inner loop to control attitude and an outer loop to control velocity. Vertical velocity and height estimates were used to control collective pitch. Attitude, gyro rates, horizontal position and velocity estimates were used to control cyclic pitch. Finally yaw information from the attitude filter and yaw gyro rate was used to control the tail rotor pitch. Figure 5.3 is a block diagram depicting the control architecture.

PD controllers were selected in the attitude loop because of their simplicity and robustness. PID controllers were selected for velocity and position for similar reasons but with error integration enabled to prevent position offsets from taking hold. Altitude and yaw are both controlled as independent channels using a PID control scheme.

### 5.2.3 Simulation of Beacon Hover

Before attempting to use the beacon algorithm in a real-flight, a closed loop simulation using the algorithm was performed. An S-function block was created to simulate the process of obtaining elevation and azimuth to landmarks from a camera image. Another S-function block was created to convert the elevations and azimuth into attitude and position. The blocks were inserted into the SIMULINK ${ }^{\circledR}$ subsystem shown in figure 5.4 which was placed in the existing Eagle simulation.

Closed loop hover of the Eagle was simulated for 100 seconds using a desired


Figure 5.4: SIMULINK ${ }^{\circledR}$ model of beacon sensor system
datum position of $\mathrm{X}=5 \mathrm{~m}, \mathrm{Y}=0 \mathrm{~m}$ and $\mathrm{Z}=-2 \mathrm{~m}$ relative to the center of the beacon triangle. The helicopter position was set to an initial position that was 1 m behind, 0.5 to the right and 0.25 m above the datum position. This was to check the effect of an initial error on the stability of the helicopter trajectory. This is important, noting that the helicopter would not be exactly at the desired point on control handover owing to the pilot's inability to judge the exact location of the helicopter. Wind turbulence corresponding to a $20 \mathrm{~km} / \mathrm{hr}$ mean wind speed was incorporated in to the simulation.

The feedback scheme depicted in figure 5.3 was implemented in the controller block of the simulation. Position and velocity estimates were produced by the fusion of inertial and beacon algorithm data as described in section 5.2.1. Although attitude information was produced by the beacon algorithm, the attitude inner loop was instead driven by attitude information provided by the inertial attitude estimator. This approach was taken in the field experiments also, noting the critical nature of the attitude estimate and the devastating effect a large attitude error could have.

The observed beacon location was assumed to be able to be resolved down to 1 pixel resolution in the camera image. Based on a field of view of $120^{\circ}$ and a $384 \times 288$ pixel camera image, one pixel corresponds to approximately 0.005 radians in the horizontal. Being conservative, a quantisation of 0.01 radians was simulated in both azimuth and elevation. Figure 5.5 shows the azimuth and elevation angles recorded from the simulation.

The velocity estimates shown were produced by using Equation (5.9) in a 50 Hz correction cycle. Suitable values for the correction cycle gains $K_{1}$ and $K_{2}$ in Equations (5.8-5.9) were found by trial and error in simulation to be 0.1 and 0.025 respectively. These values provided a good measure of robustness against sensor offsets whilst smoothing out the data.

A comparison between the actual state and that estimated from the beacons is shown in figure 5.6. The position and attitude estimates in the figure are the raw output of the algorithm. There is a good match between the two sets of data, except the effect of quantisation makes the signal appear noisier. Over the 100 second simulation, the error in the beacon algorithm estimate has a standard deviation of $\left[\sigma_{x}, \sigma_{y}, \sigma_{z}, \sigma_{\phi}, \sigma_{\theta}, \sigma_{\phi}\right]=\left[0.023 m, 0.062 m, 0.035 m, 0.42^{\circ}, 0.41^{o}, 0.54^{o}\right]$. As a comparison, NovAtel Incorporated claim a CEP horizontal accuracy of 1 cm [199] for the


Figure 5.5: Azimuth and elevation angles to beacons

DGPS card used in this thesis. Using the conversion table of [200], this corresponds to an RMS error of $\sqrt{\sigma_{x}^{2}+\sigma_{y}^{2}}=2.1 \mathrm{~cm}$ horizontal position. The standard deviation of the errors in the velocity estimates in the $\mathrm{x}, \mathrm{y}$ and z directions were $0.03 \mathrm{~m} / \mathrm{s}$, $0.077 \mathrm{~m} / \mathrm{s}$ and $0.044 \mathrm{~m} / \mathrm{s}$ respectively. This can be compared to a horizontal velocity error of $0.03 \mathrm{~m} / \mathrm{s}$ RMS for the DGPS hardware [199]. The position accuracy from the beacon compares reasonably well with the DGPS accuracy for the simulation scenario, however, it must be noted that the accuracy will improve as the helicopter flies closer to the beacons and degrade further away.

The initial error in position is restored in about 5 seconds. During the simulation, the helicopter wanders less than 30 cm horizontally and maintains its altitude to within 50 cm , despite the turbulence. The simulation demonstrates that the use of the beacon algorithm to triangulate the position of the helicopter should be sufficient to enable stable closed loop hover using inertial sensing and visual reference to landmarks on the ground. Based on these results, an experiment was devised to test the principle on the actual helicopter.

### 5.2.4 Beacon Image Processing

For the actual flight tests, the video images were processed using a video-interrupt driven task under real time Linux. High pixel-count operations were undertaken using Intel's SIMD instruction set known as MMX, which speeds processing by a


Figure 5.6: Position and attitude from simulated beacon algorithm
factor of four over the normal instruction set. The image size was $344 \times 288$ pixels. The camera was an inexpensive security camera rated at 400 TV lines with a comparable wide-angle lens giving a horizontal field of view of $120^{\circ}$. Due to the inexpensive nature of the optics, the image produced by the system was highly distorted and unacceptable for this precision positioning application. In order to overcome these issues, Tsai's camera calibration algorithm [201, 202] was applied. A non-linear optimisation of camera radial distortion parameters is performed based on comparing ideal pixel locations to measured pixel positions. Using this technique, there was no sign of visible distortion in the image. Rather than apply a look-up-table to the entire image on every frame, all coordinates measured from the screen were transformed by the camera calibration parameters prior to passing the data to the beacon solution.

The images were initially pre-processed using a centre surround also known as Mexican hat operator [203], which is essentially a bandpass filter on the image. The effect of this operator was to remove regions of the image that did not contain high contrast. The beacons were made from styrofoam, and were thus very bright. Certain sites where testing was undertaken were very bright in the near infrared wavelengths, as is typical of vegetation in sunlight. Short pass filters were placed over the camera to accentuate the intensity of the beacons. The filters used did not cut wavelengths longer than 700 nm (Kodak Wratten filters), so only undesirable visible wavelengths could be blocked.

After pre-processing to eliminate regions of no contrast, a threshold was applied to segment the beacons from the background. Figure 5.7 shows a typical view of the beacons from the helicopter during a hover trial and a view of the beacons after a threshold has been applied. Due to the contamination with IR, thresholds of as little as 16 out of 256 grey levels were used, rather than the expected 40 or 50 . Since the essential configuration of the beacons was known, it was possible for the tracking system to search for the beacons from the bottom of the screen towards the top, labelling the beacons in sequence. In order to qualify as being a beacon, a certain threshold number of white pixels was required to be within a 60 x 60 pixel window. The size of each beacon was measured based on the first moment of intensity (ie. measuring only white pixels) within each tracking window.

The tracking system was a standard alpha, beta, gamma tracker, with online adjustable tracking parameters [204]. Values of alpha, beta and gamma of 1,0 , and 0.04 were found to be successful, and no modifications were made after early tests. Essentially the tracking stage was not required. In addition to position, the size of each window was also tracked while tracking, so as to reduce the effect of additional 'speckle' (particularly dandelions) around the beacons. The window size was adjusted by assuming that the beacon was elliptical, and thus that the first moment in each major axis would define the shape. The size of the window was adjusted to be 1.2 times the size of the moments of the beacon, on each major axis. Track parameters for the window resizing operation were an alpha of 0.1 and the other parameters all zero. Using this system, track was held for all beacons under almost all situations until a beacon departed the $120^{\circ}$ field of view of the camera.

The beacon tracking system converts the 2D Cartesian screen coordinates deter-


Figure 5.7: Use of visual landmarks to control hover. View of targets from helicopter (top), view of beacons after thresholding (middle), external view of helicopter hovering over targets (bottom).
mined for each beacon into an azimuth angle $\Psi$ and elevation angle $\Phi$. Elevation angle is measured from the aircraft z axis such that a point directly underneath the aircraft would have an elevation of $0^{\circ}$. The azimuth angle of the helicopter is measured by projecting the line between the beacon and camera into the aircraft xy plane. The angle between this projection and a line through the nose of the aircraft is the azimuth angle. Points on the right hand side of the helicopter range in azimuth from $0^{\circ}$ at the nose to $180^{\circ}$ at the tail. Points on the left hand side of the helicopter vary in azimuth from $0^{\circ}$ to $-180^{\circ}$.

The beacons were arranged in an equilateral triangle such that each side of the triangle was 5 m . The base of the triangle was orientated with magnetic East-West so that the heading calculated by the beacon algorithm could be easily compared against the magnetic heading measured by the magnetometers. The position of the helicopter in global coordinates was defined such that the origin was at the centre of the beacons, the x-axis was aligned with North, the y-axis aligned with East and the $z$-axis was vertically straight down.

Position information is fused with information from accelerometers, magnetometers and gyroscopes to determine a complete state estimate suitable for use in control.

### 5.2.5 Beacon Experiment

The helicopter was fitted with a wide angle field of view camera, IMU and a 3 -axis magnetometer. All sensor information and video from the camera were telemetered to the ground. A PC workstation on the ground was used to decode the sensor telemetry and process the video in real-time. The same PC was also interfaced to the pilot's radio control transmitter so that control could be passed between the pilot and PC control software as required. A fixed gain filter operating at video frame rate, was used to estimate the height and velocity of the helicopter. These estimates were then applied in the same closed loop feedback scheme depicted in Figure 5.3. The helicopter was set up to hover closed loop at a desired position of 7 m behind the beacons and 3 above ground. The helicopter hovered stably for approximately 90 seconds under closed loop control of all 6 DOF before the pilot resumed control to land and refuel.

A logging system interfaced to the ground control PC recorded video and sensor data. The position of the helicopter found by applying the beacon algorithm is shown at Figure 5.8 for 15 seconds of flight. Figures 5.9 and 5.10 show the corresponding elevation and azimuth angles of all 3 beacons. The helicopter mean position was approximately 7 m behind, 1 m to the left and 3 m above datum. The video data records show that over the 15 second period, the helicopter moved less than 50 cm from mean in all 3 axes.

Testing the beacon algorithm in flight was a difficult task. The human pilot was required to position the helicopter so that the beacons were centred in the field of view of the camera. However, the pilot could not simultaneously fly the helicopter and examine the video data from the helicopter so that a second operator was required to vocalise steering commands to the pilot. Also, due to the limited field of view of the camera, beacon track is rapidly lost if the helicopter strays from its


Figure 5.8: Camera position determined from beacon algorithm


Figure 5.9: Beacon elevation angle


Figure 5.10: Beacon azimuth angle


Figure 5.11: Panoramic camera: A polynomial mirror is used to map a large part of the visual surroundings (best represented by a sphere) onto the camera plane
assigned station. During the initial stages of tuning control gains, station keeping by the helicopter is poor, resulting in only very short closed loop runs. The brevity of closed loop data hampers decisions on gain optimisation. Field of view problems may be alleviated in future using a panoramic camera [205] similar to the one shown in Figure 5.11 which has a field of view of $360^{\circ}$ in azimuth and more than $160^{\circ}$ in elevation.

### 5.3 Optic Flow Damped Hover

In this section, an optic flow based hover and landing system is presented which provides a means to land in an unknown environment without a reliance on GPS or other navigation aids. The sensor is used to reduce the lateral and longitudinal drift of the helicopter to negligible rates so that a safe landing can take place without an absolute position reference. An operational scenario could be envisaged where an unmanned helicopter could first search for a suitable landing area using visual or other means. During the search, terrain clearance could be maintained with sensing based on visual, laser range finding or radar systems. As aprt of other research, I have developed a novel laser ranging system [195] that determines the slope and distance of the ground in one 40 millisecond scan using a conical scan pattern. This sensor could be used to search for a piece of ground that was level enough to land on, and then, in combination with the optic flow sensor, an automated landing could be executed. The optic flow and inertial sensor compliment each other well. The optic flow provides a noisy measurement of horizontal velocity which does not suffer from offsets. When integrated, the accelerations match the velocity well but due to slight offsets, the integrated velocity diverges quickly if not corrected by a separate measure of velocity. Accelerometers suffer offsets due to thermal effects, misalignment and calibration errors. By combining the integrated inertial data with the vision sensing, the noise of the optic flow measurement can be smoothed whilst simultaneously compensating for offsets in the measured accelerations.

Optic flow can result from translations and rotations. For a downwards looking camera, like the one used here, the effect of positive (nose-up) pitch rate $q$ is the same as the effect of forward velocity. Likewise, the effect of positive roll rate $p$ (right wing down) is the same as lateral translation to the left. In these cases, for a relatively planar surface under the helicopter, the entire image is translated by the motion. The effect of yaw is to produce an image flow field consisting of vectors rotated around the point in the image corresponding to the center of rotation. Without any translation, the average of these vectors cancels out. Also, during hover, yaw rates are kept relatively small by the heading control loop which is very effective. Yaw can therefore be ignored in calculating the lateral and longitudinal drift. Likewise the effect of vertical motion results in image loom, however, due to the collective control loop, the vertical velocity are very small ( $3 \mathrm{~cm} / \mathrm{sec}$ for autoland) and can be neglected. Using an inertial system, the effect of rotations can be removed simply by subtracting the rotations rates measured by onboard rate gyroscopes. The net remaining angular motion of the image is proportional to the horizontal velocity. In terms of the components of motion, we can express the longitudinal $u$ and lateral $v$ velocities in terms of the helicopter height above ground level $H$ and the flow components $Q_{x}$ and $Q_{y}$ as in Equation (5.10). In these equations, the flow is defined to be positive when it corresponds to a positive translation along one of the helicopter body axes.

$$
\begin{equation*}
u=\left(Q_{x}-q\right) H \quad v=\left(Q_{y}+p\right) H \tag{5.10}
\end{equation*}
$$

The Yamaha attitude system (YAS) outputs pitch angle $(\theta)$, roll angle $(\phi)$, roll
rate (p), pitch rate (q) and yaw rate (r). The YAS also outputs the net accelerations $\left[a_{x}, a_{y}, a_{z}\right]$ sensed in the body axes coordinates after the gravitational acceleration vector has been subtracted. The offsets from these sensors are relatively small, less than $0.05 \mathrm{~m} / \mathrm{s}^{2}$ in acceleration and $0.1 \mathrm{deg} / \mathrm{sec}$ in rotation rates. I have previously described an Extended Kalman Filter [26] which enables the accelerometer offsets to be estimated online. This was for a low grade IMU on a very small helicopter which suffered from significant sensor offsets and drift. For the RMAX, this extra complexity is not required and a simpler complimentary filter has been applied. This is a similar approach to that taken by Corke in [206] to fuse optic flow data with inertials on a small helicopter. The fusion of inertial and optic flow information takes part in two parts consisting of a state update phase and then a correction phase:

Predict Cycle - The first step is a prediction based on the measured accelerations, rotation rates and attitude. The state update equations were based on the rigid body motion Equations (5.11). These were integrated at 100 Hz using a trapezoidal integrations scheme. The variables $a_{x}, a_{y}$ and $a_{z}$ are the measured accelerations from the IMU. The variables $\mathrm{p}, \mathrm{q}$ and r represent the rotation rates sensed by the IMU rate gyroscopes. The variable $\mathrm{u}, \mathrm{v}$ and w are the velocities of the helicopter in local body axes.

$$
\left[\begin{array}{c}
\dot{u}  \tag{5.11}\\
\dot{v} \\
\dot{w}
\end{array}\right]=\left[\begin{array}{c}
a_{x} \\
a_{y} \\
a_{z}
\end{array}\right]+\left[\begin{array}{ccc}
0 & r & -q \\
-r & 0 & p \\
-p & q & 0
\end{array}\right]\left[\begin{array}{c}
u \\
v \\
w
\end{array}\right]
$$

Correct Cycle - The second step is a correction based on measurements of the optic flow and height. As optic flow measurements are only available at a video frame rate of 50 Hz , there are 2 predict cycles for every correct cycle. A fixed gain correction is used to correct velocity in accordance with Equation (5.12) . The and + superscripts below represent the pre and post priori estimates of the state variables respectively. The * superscript denotes an observation and the time shift between values with subscript k and $(\mathrm{k}-1)$ is $\mathrm{t}=0.02$ seconds. The vertical velocity is corrected from the GPS vertical velocity output or from the derivative of the laser-range finder height, depending on which sensor is being used at the time.

$$
\left[\begin{array}{c}
u_{k}^{+}  \tag{5.12}\\
v_{k}^{+}
\end{array}\right]=\left[\begin{array}{c}
u_{k}^{-} \\
v_{k}^{-}
\end{array}\right]+\alpha\left[\begin{array}{l}
Q_{x}^{*} H^{*}-u_{k}^{-} \\
Q_{y}^{*} H^{*}-v_{k}^{-}
\end{array}\right]
$$

A value of 0.25 was used for the gain $\alpha$. This was thought to be an adequate compromise between smoothing out noise in the sensor and minimising the effect of accelerometer drift.

The helicopter was flown at a height of approximately 1 m above a grass field with no artificial texture in view of the camera. Once the helicopter was established in a reasonably stable hover by the pilot on the ground, control was switched to the flight computer. GPS altitude was used to control the height of the helicopter by varying the collective pitch using a PI controller. The height of the helicopter was regulated to within 10 cm of the set height. The aileron and elevator inputs were controlled by PI control loops using the lateral and longitudinal velocities


Figure 5.12: Longitudinal velocity $\left(V_{x}\right)$ calculated from optic flow versus $V_{x}$ from DGPS
respectively from the optic flow calculation. Due to the integral feedback in the PI loop, any short term excursions in the position of the helicopter due to wind gusts tend to be compensated. The helicopter was flown for nearly four minutes using the optic flow to control the hover. During the flight, the average wind velocity was $30 \mathrm{~km} / \mathrm{hr}$ with frequent gusts.

Figure 5.12 shows a comparison between the longitudinal velocity measured by the GPS and the longitudinal velocity derived from optic flow and inertial sensing. The optic flow based velocity is clearly a noisier signal than the velocity measured by the GPS. We anticipate vibration to be a significant source of noise. The camera position for these experiments was not ideal as it was mounted to the vision computer for which vibration was severe at times with both rotational and translational components. For future work, I intend to design a vibration isolation system to protect the camera.

Figure 5.13 shows the position of the helicopter over time and Figure 5.14 shows the lateral and longitudinal velocities as measured by the GPS during 120 seconds of flight. In this time, the aircraft drifted 68 cm North and 2.57 m in an Easterly direction as measured by the GPS. This equates to an average drift velocity of $2.2 \mathrm{~cm} / \mathrm{sec}$ which is easily low enough to permit a safe and gentle landing to take place. During strong gusts, there are peaks of velocity up to $20 \mathrm{~cm} / \mathrm{sec}$ which are still within the bounds of landing.

For the experiment, the gains in the PI controllers were tuned manually with only a few iterations and improved performance might well be achieved with further tuning. Also, as shown in Figure 4.7, the YACS system includes a low pass filter with 2.8 Hz corner frequency on the aileron and elevator channels which restricts the bandwidth of the controller. In future work, I aim to develop an in-house controller to replace the YACS which will remove the prefiltering of the aileron and elevator channels.


Figure 5.13: Helicopter position during closed loop hover


Figure 5.14: Helicopter longitudinal $\left(V_{x}\right)$ and lateral $\left(V_{y}\right)$ velocities measured by DGPS during closed loop hover

### 5.4 Summary

Visual sensing provides a cheap, lightweight and passive way of controlling hover or low speed flight in an autonomous flying vehicle. A method using triangulation of postion in three dimensions from visual bearings to landmarks was used succesfully to control a helicopter in hover.

## Chapter 6

## Control of Forward Flight

### 6.1 Introduction

A number of researchers have suggested that insects use optic flow in forward flight to control their terrain clearance $[8-10]$. The aim of this chapter is to show, in simulation and experimentally, that calculation of range from optic flow can also be used to avoid terrain by an inherently unstable rotorcraft. Experiments were completed with both the Eagle helicopter and the RMAX helicopter. The concept was first tested in closed loop on a relatively flat surface using the control by telemetry scheme on an Eagle helicopter. Later, the RMAX helicopter completed terrain following experiments over a more irregular landscape using all onboard processing.

The tests in this chapter make use of a downwards looking camera that measures the optic flow of the ground underneath the helicopter. If speed is known, the terrain clearance $(H)$ can be calculated from Equation (6.1) where V is the horizontal speed of the helicopter and Q is the magnitude of the optic flow vector due to pure translation. The basic assumption in this work is that the ground speed of the vehicle is known approximately. In our case, the speed of the helicopter is known from GPS measurements or from the observed open loop behaviour of the helicopter.

$$
\begin{equation*}
H=V / Q \tag{6.1}
\end{equation*}
$$

### 6.2 Sensor Fusion

Equation (6.1) can be used to measure height above terrain. This estimate tends to be rather noisy owing to the vibration of the platform and the resulting noise in the optic flow measurement. To reduce the effects of vibration, the optic flow and velocity data are both smoothed using a moving average filter comprising 5 samples, before the velocity is divided by the optic flow to get range. Figure 6.1 shows ground speed and optic flow range versus time for the same period of flight for the RMAX helicopter flying over a rough grass paddock. The output of the laser rangefinder (LRF) is provided as a benchmark. The optic flow range is calculated by dividing the longitudinal velocity by the longitudinal optic flow after applying the averaging filter. The plot of standard deviation in figure 6.1 was calculated using a sliding window of 50 samples (i.e. 1 second) width applied to the discrepancy between the optic flow range and the LRF. At $5 \mathrm{~m} / \mathrm{s}$ the standard deviation of the noise is about


Figure 6.1: Effect of velocity on optic flow ranging noise. The top diagram demonstrates how the noise in optic flow ranging increases at low speed. Results for height are benchmarked against the laser rangefinder (LRF).

5 cm . At low speeds, this noise increases dramatically and can be seen to reach approximately 0.6 m standard deviation for a ground speed less than $1 \mathrm{~m} / \mathrm{s}$.

A state prediction and correction cycle is used to estimate the terrain clearance $\mathcal{Z}$ and relative vertical velocity $\mathcal{W}$. First, at each inertial sensor sample time $\Delta T$, the relative vertical velocity estimate is updated using the vertical accelerometer measurement $a_{z}$ as per Equation (6.2). A measurement of the relative speed between the ground and the helicopter is determined by differentiating the range calculated by optic flow, $\mathcal{R}$. At low speeds this derivative is very noisy, so at speeds below $2 \mathrm{~m} / \mathrm{s}$ in the RMAX, the vertical velocity was corrected using GPS vertical velocity instead. The Eagle helicopter was not flown closed loop at low speed, so an alternative means of correcting the vertical velocity was not required. The vertical velocity is calculated using a first order approximation to the derivative of height as shown in Equation (6.5). The terrain clearance estimate is updated using the relative vertical velocity estimate and corrected using the range from optic flow. The prediction and correction cycle is outlined below:

Predicting the relative vertical velocity estimate $\hat{\mathcal{W}}$, where $\phi$ and $\theta$ are the helicopter roll and pitch angles respectively and $g$ is the acceleration due to gravity:

$$
\begin{equation*}
\hat{\mathcal{W}}_{k}^{-}=\hat{\mathcal{W}}_{k-1}^{+}+\left(a_{z}+g \cos \phi \cos \theta\right) \Delta T \tag{6.2}
\end{equation*}
$$

Updating the terrain clearance estimate $\mathcal{Z}$ from the velocity estimate $\hat{\mathcal{W}}$ :

$$
\begin{equation*}
\hat{\mathcal{Z}}_{k}^{-}=\hat{\mathcal{Z}}_{k-1}^{+}+\hat{\mathcal{W}}_{k-1}^{-} \Delta T \tag{6.3}
\end{equation*}
$$

Calculating the optic flow range measurement $\mathcal{R}$ from the longitudinal velocity $u$ and the longitudinal optic flow $Q_{x}$ :

$$
\begin{equation*}
\mathcal{R}_{k}=u /\left(Q_{x}-q\right) \tag{6.4}
\end{equation*}
$$

Conditioning the relative velocity estimate from the range measurements where the constant $\alpha$ is a filtering parameter between 0 and 1 :

$$
\begin{equation*}
\hat{\mathcal{W}}_{k}^{+}=(1-\alpha) \hat{\mathcal{W}}_{k-1}^{-}+\alpha\left(\mathcal{R}_{k}-\mathcal{R}_{k-1}\right) / \Delta T \tag{6.5}
\end{equation*}
$$

Conditioning the terrain clearance estimate from the measured optic flow range where the constant $\beta$ is a filtering parameter between 0 and 1:

$$
\begin{equation*}
\hat{\mathcal{Z}}_{k}^{+}=(1-\beta) \hat{\mathcal{Z}}_{k-1}^{+}+\beta \mathcal{R}_{k} \tag{6.6}
\end{equation*}
$$

The equations outlined above are an approximation of the exact equations of motion of the helicopter in that the pitching and rolling motions of the helicopter are ignored. This eliminates the cross-product terms between angular velocity and linear velocity. For this work, the camera is also assumed to be pointing straight down at the ground, however, the optic flow due to rotations is eliminated by subtracting the rotation rates measured by the gyroscopes from the measured optic flow. Whilst the attitude of the helicopter will change as the helicopter manoeuvres, on average, the vertical body axis of the helicopter will be closely aligned with the normal to the ground plane, and the approximation will only introduce a small error. In an aircraft that is pitching or rolling violently whilst terrain following, it would be neccessary to use the full equations of motion of the helicopter and to stabilise the camera so that it always pointed downwards.

The filter gains $\alpha$ and $\beta$ were choosen as a compromise between correcting drift in the velocity and position estimates, and smoothing out the noise introduced by the optic flow range measurement. Values of 0.02 and 0.2 respectively were found to be suitable in simulation.

The inertial sensors provide no information on the rate of change of terrain, only the absolute height of the helicopter with respect to some fixed datum. Consequently the terrain clearance will tend to lag behind the actual terrain clearance slightly, particularly when abrupt changes in terrain clearance occur. Using filter gains of $\alpha=0.2, \beta=0.02$ and a forward speed of $5 \mathrm{~m} / \mathrm{s}$, the time to adapt to a 1 m step change in terrain height would be approximately 0.2 seconds. Owing to the spike in velocity estimate caused by the step change, the terrain clearance estimate tends to overshoot the actual terrain clearance by about $5 \%$.

### 6.3 Simulation

The Eagle simulation was used to test the ability of optic flow ranging to control the height of the helicopter in forward flight. A simple 'P' classical controller in combination with the existing attitude inner loop was used to maintain a near constant longitudinal velocity of $5 \mathrm{~m} / \mathrm{s}$. Lateral velocity was kept close to zero using
the same control arrangement. Heading of the helicopter was kept constant using a PID controller. The collective pitch control was controlled using a PD controller set to maintain a constant distance from terrain. Terrain clearance and vertical velocity were calculated using the predict and correct cycle described in Equations (6.2)-(6.6).

The Eagle simulation was modified to take terrain data as input from a file. The relative height between the helicopter and the ground was then calculated by subtracting the terrain height from the helicopter height, so that the simulated optic flow could be computed using Equation (6.1). Gaussian noise was added to the calculated flow with statistical properties based on the optic flow measured in hover at approximately 1 m above a level grass field. Comparison between flight results and data from the camera when it is held in a fixed position above the ground confirm that the optic flow on the flying helicopter is much noiser owing to the vibration of the camera mount. The dominant source of the noise is the once per rotor revolution vibration of the helicopter at a frequency of about $25-26 \mathrm{~Hz}$. Higher frequency harmonics from the main rotor and tail rotor are also present. It is neccessary to isolate the noise component of the flow from vibration from the change in the flow due to the actual motion of the helicopter. This was achieved by calculating the standard deviation of the optic flow about mean over short periods and then averaging. Groups of 4 consecutive 50 Hz optic flow samples, spanning 2 complete rotor revolutions, were used to calculate standard deviation. These standard deviations were then averaged over 80 seconds. The resulting standard deviations were $5.2 \mathrm{deg} / \mathrm{sec}$ for $Q_{x}$ and $9.5 \mathrm{deg} / \mathrm{sec}$ for $Q_{y}$. Results for standard deviation to within $20 \%$ of these values were achieved using $2,6,8$ and 10 samples before averaging. It should be noted that the translatory components of vibration will produce less noise at heights above 1 m , owing to the reduced angular effect of the vibration induced displacements at long ranges. It should also be noted that when the camera is in close proximity to the ground, such as when the helicopter is about to take off or just before landing, that the vibrations can lead to significantly increased noise in the optic flow measurements. For the simulation purposes here, the helicopter is always higher than 1 m above the ground, so the use of optic flow noise statistics for a fixed height of 1 m is is conservative.

The first test performed in simulation was to see if the helicopter could track a simple vertical terrain feature consisting of sequence of rising and falling ramps and a step. The desired terrain clearance was set to 3 m , the desired forward speed was set to $5 \mathrm{~m} / \mathrm{s}$ and the initial state of the helicopter at $t=0$ was set to match these values. The simulation was set to run for 100 seconds, producing the results shown in figure 6.2. The helicopter follows the terrain well, with a maximum error in desired height of 0.48 m up until the step in terrain occurs. As the helicopter cannot execute an instantaneous change in height, the error in terrain height reaches a maximum as the step is sensed which is equal to the height of the step plus the previous terrain clearance. Within 3 seconds of the step occuring, the helicopter re-establishes a close terrain following behaviour with a terrain clearance of $3 \pm 0.1 \mathrm{~m}$. An overshoot in height occurs which brings the helicopter to within 1 m of the ground at 2.5 seconds after the step. Some overshoot to a step is expected from a properly tuned


Figure 6.2: Simulated terrain following for ramp and step changes in terrain

PD controller, but the overshoot is clearly increased owing to the spike in velocity created by using the derivative of relative height to correct the vertical velocity estimate. The simulated optic flow data does not take in to account the field of view of the camera, treating the camera as a point sensor which is only affected by the point directly underneath the helicopter. If the real helicopter were to fly over a step change in terrain, such as a cliff, the optic flow would not change instantly as for a short time, the camera image would contain points from both the high and low side. The algorithm used on the helicopter averages the optic flow vectors calculated over the flow field, so that there would be a smooth transition from one optic flow scale to another as the helicopter flew over the cliff. This would reduce the effect of the velocity spike, and hence reduce the helicopter overshoot.

The next test was to see how well the helicopter could respond to features of different spatial frequency. A chirp signal was used to create sinusoidally varying terrain with an amplitude of $\pm 3 \mathrm{~m}$ and a frequency varying between 0.01 Hz and 0.2 Hz over 100 seconds. The results of the test are shown in figure 6.3. The simulation was stopped after 65 seconds, because the helicopter could no longer adjust its trajectory fast enough to keep up with the rate of change of the terrain, resulting in a collision with the terrain. The estimate of the terrain height is seen to closely match the real height above terrain. The velocity estimate from optic flow ranging is attenuated by about $40 \%$ of the original values. This attenuation is desirable as it smoothes out the effect of small duration variations in the terrain which might cause the helicopter to overshoot the terrain.

Finally, terrain data from an actual experiment were used in the simulation. The data collected by the RMAX helicopter's LRF from the terrain following experiment later in the chapter was used as an input to the simulation. The helicopter was set


Figure 6.3: Simulated terrain following for sinusoidal terrain
to fly at the average speed of the RMAX during the test. The results are shown in figure 6.4.

### 6.4 Terrain Following Experiments

### 6.4.1 Experimental Procedure

For the experimental work, a mechanism was required to allow the pilot to take over control of the helicopter in event of a control or sensor failure. As the helicopter quickly flies out of sight of the pilot in forward flight, a technique was used to chase the helicopter at a relatively safe constant distance. The pilot was accomodated in the back of a utility vehicle as shown in figure 6.5. The pilot crouched over the back of the vehicle so that he had a clear view of the helicopter in flight. The driver of the vehicle was instructed to maintain a constant distance from the helicopter.

### 6.4.2 Calculation of Optic Flow

For robustness, the optic flow calculation must be able to cope with flight at various altitudes and speed. In forward flight, it must simultaneously measure small lateral optic flow values whilst measuring large optic flows in the longitudinal direction. When the optic flow shift exceeds the reference shift, the $I^{2} A$ ceases to function, and its performance becomes unpredictable and degraded [71]. Ideally, reference shifts would be choosen to be about double the expected optic flow so that there is a sufficient margin over saturation whilst maintaining optimum measurement precision. Because of these considerations, an adaptive algorithm is required, otherwise


Figure 6.4: Simulated terrain following for real terrain


Figure 6.5: Forward flight experimental procedure (Pilot: Matt Garratt)
the optic flow measurement would be saturated at low altitude, too insensitive at high altitude, and operable only in a very narrow range of height and speed.

To overcome the limited range of flow rates that can be measured using $I^{2} A$, the simple tracking estimator in Equation (6.7) was used to estimate the size of the next image shift $S_{k+1}$ based on previous image shifts $S_{k}$ and the measured optic flow $Q$. The constant $\gamma$ is a parameter between 0 and 1.0 adjusted to give stable tracking.

$$
\begin{equation*}
S_{k+1}=\gamma\left(S_{k}+Q\right)+(1-\gamma) S_{k} \tag{6.7}
\end{equation*}
$$

The expected shift is applied to the image before computation of optic flow, and added to the result after computation, so that all measured shifts are less than 2 pixels. Despite the narrow range of the measurement, the adaptive feature of the modified algorithm allows rates in excess of 10 pixels per frame to be measured (over $700^{\circ} / s$ with the camera used in the experiments). The adaptive version of the $I^{2} A$ algorithm is known as the Iterative Image Interpolation Algorithm, abbreviated $I^{3} A$.

### 6.4.3 Terrain Following using Control by Telemetry

This test involved a control by telemetry scheme. All of the optic flow processing, state estimation and control was executed on a real-time Linux PC in the chase vehicle. A PC mounted in the chase vehicle was interfaced to the pilot's transmitter so that control could be switched between the pilot and the computer using a switch on the transmitter.

The camera was oriented at $45^{\circ}$ to the vertical axis of the helicopter to provide some anticipation of terrain. For this test, DGPS information was not available. As closed loop control of forward speed was not possible, the helicopter was instead flown at a constant pitch attitude using a proportional feedback loop. The a priori observed forward flight speed at this attitude was then used with Equation (6.1) to provide a measurement of the height above ground using optic flow. Note that this technique is only really applicable in low wind conditions as the open loop behaviour of the helicopter can only be used to estimate air speed and not the ground speed which is needed for accurate optic flow ranging. The height estimate was combined with accelerometer information to provide an estimate of height and vertical velocity.

A PD controller was used to control collective pitch to maintain a datum height of 2 m above terrain. The pilot retained control of roll cyclic in order to keep the helicopter on the desired ground track. Heading was controlled using a PID controller.

The tests were performed on a flat, dry lake bed with the chase vehicle positioned behind the helicopter. Before control was handed over to the computer system, the helicopter was first established in forward flight under manual control. Once in steady flight and telemetry confirmed, control was handed over to the computer. After some initial tuning of gains, the helicopter was flown closed loop in a straight line for over 2 km with the optic flow ranging controlling height. The motion of the helicopter was recorded by an external video camera but no provision was made for recording the inertial data embedded in the telemetry stream. During the closed


Figure 6.6: Helicopter height measured by LRF versus the height estimated from optic flow.
loop phase, the helicopter was observed to speed up and slow down, possibly due to the presence of wind. In response to the fluctuations in speed, the controlled height of the helicopter was observed to change, as would be expected from the error in the constant speed assumption.

This experiment demonstrated the feasibility of using optic flow height control for a rotorcraft in forward flight. Further testing of forward flight was put on hold until the deficiencies in the experiment could be rectified. Specifically, these included obtaining an accurate measurement of ground speed, providing a ground truth measurement of height above terrain, and eliminating the dependency on the video and control telemetry. These problems were overcome with the introduction of a PC104 based control system onboard the RMAX helicopter.

### 6.4.4 Terrain Following using Onboard Processing

Terrain following experiments using all onboard processing were conducted on the RMAX helicopter. A sloping grass runway was used to provide the terrain for the test. The helicopter was first flown up and down the airstrip to confirm that the calculation of terrain height was sensible. Data from the optic flow ranging and the onboard LRF were logged in flight as the helicopter was flown at speeds up to 30 $\mathrm{km} / \mathrm{hr}$. Figure 6.6 shows an extract of the validation flights, showing a clear match between the two measures of terrain clearance.

For the test of closed loop control, a proportional controller was used to control height with the same feedback gain used to control hover. Cyclic control and yaw control were retained under pilot command. On handover of control of height, the current terrain clearance was set as the new reference terrain clearance. This was set to prevent a step change in height command at the moment of handover.

The pilot found the control of the helicopter close to the ground at speed quite difficult as even slight changes in collective pitch resulted in rapid changes in height. Handover of control in forward flight was therefore to be avoided in case a transient caused by the handover resulted in the helicopter getting too close to the ground.

Handover of control was, therefore, executed in hover and the speed gradually increased to the desired forward speed. In hover, the terrain clearance was obtained from the LRF. However, as the speed increased to above a certain threshold, the terrain height measured from optic flow began to be used. At low speeds the terrain height measured from optic flow is noisy and unreliable. As speed increases, the height measurement becomes more reliable. The test consisted of flying the helicopter up and down a sloping grass airstrip. As the helicopter needed to slow down at the end of the runway to turn around, an automatic transition from optic flow to LRF height control was implemented. Whenever transitions between modes of sensing terrain clearance were instigated by the autonomy software, a new datum terrain clearance was recorded using the new sensing mode at the instant of transition. This prevented step changes in terrain height that might occur if a significant mismatch between the two sensing modes existed. Hysteresis was used to prevent constant twitching between sensing modes that might occur if speed was near the switchover point between sensing modes. Specifically, as the helicopter accelerated to $2.2 \mathrm{~m} / \mathrm{s}$, control was switched from using LRF sensed height to optic flow sensed height. As the helicopter decelerated below $1.8 \mathrm{~m} / \mathrm{s}$, sensing from the LRF was again selected.

The helicopter was flown up and down the runway in closed loop. The ground track, groundspeed and terrain clearance results for one of the uphill runs is shown in figure 6.7. The top graph in the figure shows the height of the helicopter and the height of terrain on the same plot. The height of the terrain was calculated by adding the height above ground measured by the LRF to the absolute height measured by the differential GPS.

The helicopter can be seen to clearly respond to the terrain features in the runway. During the 100 second closed loop portion of the flight, shown in figure 6.7, the helicopter maintains $1.27 \pm 0.18(S D) \mathrm{m}$ clearance from the ground. There is, at times, a small lag evident between the helicopter response and flying over the disturbance. From the figure, this can be seen to equate to roughly 0.4 seconds. For this experiment, the height tracking accuracy would have been signficantly degraded by having a human pilot in the control loop. During the test, the pilot was responsible for controlling all axes except control of height. Owing to the chase vehicle being jerked around over the uneven ground, the pilot found it difficult to hold the aircraft at a steady speed and heading, causing cross-coupling disturbances to all of the control channels. Also, it was only possible to maintain a constant velocity for a few tens of seconds before it was neccessary to begin deccelerating the helicopter in preparation for reaching the end of the runway.

Small discrepancies between the terrain clearance estimates from the LRF and optic flow are present. This is not suprising since the ground was quite uneven in places and there were parts of the flight where the helicopter flew over long grass and tussocks. Also, the LRF beam was aimed at a slightly different part of the ground to the camera owing to the wider field of view of the camera and the geometric offset between the sensors. The pilot's efforts to control the helicopter in gusty conditions resulted in roll and pitch excursions from the datum value. The pitch and roll angles of the helicopter vary from their mean values by up to $5^{\circ}$ with a standard deviation
(a) Helicopter Height versus Terrain

(b) Comparison of LRF and Optic Flow Ranging

(c) Vertical Velocity

(d) Collective Pitch

(e) Roll Attitude

(h) Ground Track

(f) Pitch Attitude

(i) Ground Speed

(g) Heading

(j) Height Error


Figure 6.7: Terrain following results for RMAX helicopter using optic flow ranging: (a) Helicopter height measured by GPS versus the terrain height measured by GPS and LRF; (b) a comparison between the height from the LRF and from optic flow; (c) vertical velocity estimates derived from the GPS and from optic flow ranging; (d) PWM signal sent to the collective pitch servo; (e-g) attitude of the helicopter; (h-i) ground track and ground speed measured by GPS; (j) height difference between optic flow and LRF height estimates.


Figure 6.8: Use of lateral optic flow for control of drift. Left-hand diagram shows the lateral and longitudinal velocities as measured from optic flow. Right-hand diagram shows the ground track measured by GPS.
of $1.5^{\circ}$ and $2.0^{\circ}$ respectively. The attitude changes would introduce some errors in the laser range measurements as the ranges were not corrected for the tilt of the platform. The standard deviation of the error between the two sensors for the flight data presented was 9.5 cm .

### 6.4.5 Control of Lateral Motion using Optic Flow

The terrain following experiments described above were achieved using pilot control of roll. As shown in simulation, it should be possible to control the lateral velocity of the helicopter in addition to the terrain clearance using optic flow measurements. By setting the desired lateral optic flow to zero in forward flight, the helicopter should fly a path over the ground aligned with its longitudinal axis. Such a technique could be used to compensate for the sideways drift caused by wind or an out of trim condition. To test this theory, the RMAX helicopter was flown backwards and forwards over a level grass playing field with the reference roll attitude for the YACS inner loop controlled by proportional feedback from the lateral velocity derived from optic flow. The height above terrain was controlled using the existing optic flow ranging technique whilst the pilot controlled yaw and elevator manually. Unfortunately, less than one hundred metres of space was available for this experiment, and no chase vehicle could be used, so only a short burst of terrain following was possible. Figure 6.8 shows the lateral and longitudinal velocities versus time on the same graph for a 35 second segment of closed loop flight.

### 6.5 Discussion

The techniques presented in this chapter require an approximate measure of ground speed. GPS velocity is one suitable means of velocity measurement. In the absence of GPS, another measure of speed could be used such as airspeed from a pitotstatic air data system. However, airspeed needs to be corrected for the effect of wind, otherwise the calculated range would have a scale error. Using airpseed would result in a lower height above ground in a headwind and an increased height in a tailwind.

The use of optic flow for height control is really most practical where an aircraft is operating close to the ground. For example, a 747 cruising at Mach 0.86 and 40,000 feet generates an optic flow for a downwards looking camera of $1.5 \mathrm{deg} / \mathrm{sec}$. Whilst this flow is certainly measurable, even a $1 \%$ error in ranging would result in a 400 ft error in height which would make it too inaccurate for traffic separation. However, a $10 \%$ error in ranging for a terrain following aircraft flying at 400 ft would be acceptable since this would generate an acceptable 40 ft error, which is within the existing error margin associated with flying over trees and uneven ground. The scale, and hence accuracy, of the optic flow measurements could be optimised for various applications by varying the frame rate. For a situation where the flow is small, the frame rate could be reduced to increase the pixel shift and hence provide a more easily measured signal. Digital processing hardware continues to increase in speed, so it is concievable that it will soon, if not already, be possible to calculate optic flow at several hundred frames per second, which would be fast enough for any conceivable aircraft application.

As there is a lag between a change in terrain being sensed, and the response, some a priori knowledge of the upcoming terrain could be used to improve performance. The use of a downwards looking camera provides no anticipation of changes in terrain ahead of the helicopter's current position. By tilting the camera forward, however, some anticipation of bumps could be applied. In [35], Hrabar proposes that the optimum camera inclination for obstacle avoidance using optic flow is $45^{\circ}$. This would introduce some inaccuracy in the range measurement if the simple range equation (Eqn. (6.1)) was still used, since it assumes the camera axis is normal to the ground. However, as the control by telemetry experiment demonstrated, the method is sufficiently robust to cope with a $45^{\circ}$ forward tilt of the camera without any obvious side-effects, when using the average optic flow from the entire image.

In this work, the average optic flow over the whole image was used so that the camera acts as a point sensor. In a more general situation, optic flow vectors could be obtained and interpreted separately from different parts of the image. This would provide not only the range, but also the azimuth and elevation of obstacles that might be a threat to the vehicle. This would allow steering commands to be generated in addition to height control. A limitation of this technique is that objects in the direction of travel will not exhibit any translatory flow, so will not be detected. Measurement of image expansion or loom could be used to detect approaching obstacles at the front of an aircraft in forward flight, although this signal tends to be weaker than the optic flow from motion parallel to a surface.

Future work should investigate the use of complete flow fields for obstacle avoidance.

### 6.6 Summary

In this section, systems for maintaining terrain clearance in an unstable rotorcraft have been demonstrated and benchmarked against a LRF sensor. The system consists of a downwards looking or forward tilted camera and computational hardware to compute optic flow using the $I^{3} A$ algorithm. When combined with speed measurements from GPS, the approach was able to estimate terrain clearance to within $7.5 \%$ of the actual mean height at an average forward flight speed of $20 \mathrm{~km} / \mathrm{hr}$. Lateral optic flow was also shown to be able to be used in forward flight to correct sideways drift for a helicopter.

## Chapter 7

## Control using Artificial Neural Networks

### 7.1 Introduction

In keeping with a biological inspiration for the system architecture, a neural network based control system has been implemented. This technique has been applied to the case of altitude control and the control of lateral and longitudinal drift in hover. The aim of this chapter is to show that simple computational models can perform the task of sensor fusion and control without a detailed mathematical model of the plant being known. In biological systems, flight control and sensory processing systems are partly hardwired into the animal as a result of evolutionary adaption passed genetically and partly learnt from interaction with the environment. In this case, off-line neural computational models will be trained using recorded data from the real plant. Later work, beyond this thesis, could investigate the use of on-line adaption to adapt the neural controllers to changing conditions and to improve the control response.

Whilst we can use simplified mathematical models for designing controllers that will work on the helicopter, there are many unmodelled dynamics that a simple model will not incorporate. If we consider the collective control channel, for example, significant errors can arise from ignoring or simplifying any of the following: actuator kinematic non-linearities; ground effect; fuselage download; servo dynamics; rotor speed variation; sensor lag; and rotor inflow lag associated with the rate of change of collective pitch. These effects are virtually impossible to model exactly. The use of ANNs does not require an explicit analytical model of these effects, only raw flight test data from which a black-box model can be learnt. This simplifies the controller design significantly and allows the ANN controllers to be run on very simple hardware.

The 7 state EKF prediction and correction cycle used for fusing inertial and visual information discussed in Chapter 8 executes on PC104 hardware in approximately 1 ms . By comparison, a single layer feedforward ANN with eight hidden nodes used to fuse inertials and vision executes in about $10 \mu \mathrm{~s}$ on the same hardware. An ANN can be executed on much simpler processors than an EKF and may have other advantages in terms of robustness to unforeseen variations in sensor noise characteristics.

### 7.2 ANN Training

To make training easier, all of the input and output data was normalised to a Gaussian distribution of unit standard deviation and zero mean. Normalisation of the training data is known to speed up training [207] as the network biases and scale factors can be randomly chosen from an appropriate set of bounds when the network is initialised before training. Using normalised output data also has the advantage of providing a common scale to error metrics such as the mean square error (MSE) relating the difference between the network output and the training output.

All of the ANNs described in this thesis are feed-forward networks. A logsigmoidal transfer function was used for the hidden layers as shown in Equation (7.1). Linear output layers were used for the output layers.

$$
\begin{equation*}
\log \operatorname{sig}(n)=\frac{1}{1+e^{-n}} \tag{7.1}
\end{equation*}
$$

For the experiments described in this chapter, a neural network library was written by the author in the c programming language for use on both the helicopter and in SIMULINK ${ }^{\circledR}$. This library incorporates a dynamically allocated data structure to store the network weights, biases, and connectivity. The library has functions to load networks stored in files and to calculate the network outputs. Functions to train the network using simple backpropogation and Backpropogation Through Time (BPTT) were also written but found to be too slow to be practical and thus were abandoned. The Levenberg-Marquardt (LM) algorithm [134] implemented in the MATLAB ${ }^{\circledR}$ neural network toolbox was therefore used to train all of the networks discussed in this thesis. The LM algorithm was choosen specifically as it appears to be the fastest method for training ANN with up to several hundred weights [135]. A MATLAB ${ }^{\circledR}$ script was written that converts trained network objects in the MATLAB ${ }^{\circledR}$ workspace to an ASCII network definition file which stores the network structure and weights.

In SIMULINK ${ }^{\circledR}$ a user-defined S-function block was created to simulate each ANN. The SIMULINK ${ }^{\circledR}$ block calls subroutines in the neural network library file. At the start of the simulation the software loads a network from file and dynamically allocated memory to store the network parameters. A GUI dialogue box associated with the SIMULINK ${ }^{\circledR}$ block allows the user to set the filename for the network definition file and the time step.

Network definition files are transferred from MATLAB ${ }^{\circledR}$, once trained, using a USB memory stick. On boot of the PC104 computer, software is started automatically to load the network files into memory. The network data structure is accessed by a real-time implementation of the same neural network library used in simulation to run the networks as part of the main software thread running on the PC104. The outputs of the networks are logged by the PC104 and also sent to the MPC555 autopilot using the RS-232 link.

### 7.3 Control of Height using a Neural Network

In this section, the control of height using a neural network is first shown to be feasible using a simplified simulation of the helicopter dynamics. The approach is then shown to work on the real helicopter based on a plant model trained from real flight data. For this thesis, the helicopter height has been measured by a number of means including using a stereo camera, height from optic flow (only in forward flight), laser range finding and DGPS. For the work in this chapter, which was carried out on the Eagle helicopter, only DGPS height will be used, however the methods apply equally well to any of the other sensing modes.

The strategy used for control of height is a three-step process aiming at achieving an ANN implementation of an optimal controller. The process consists of training a neural network to represent the plant, developing a controller off-line using computationally intensive, but optimal methods, and then training another neural network to mimic the optimal controller. The technique has a number of advantages. Firstly, an optimal controller can be designed and tested in simulation using full and noisefree state data which may not be available in flight. The ANN can be trained to mimic the optimal controller but using only the data which it has available to it, which may be uncalibrated, corrupted by noise and affected by sensor lag. For example, the optimal controller might be executed with noise-free vertical velocity and height data but the ANN may only be given height data with no velocity information. Secondly, the optimisation process is computationally intensive and could not be achieved in real-time. The use of optimal control is of particular interest in this work, as biological entities will have optimised their control pathways through evolutionary process and also learned behavior during their individual lifetimes.

In order to develop an optimal controller for the helicopter, a means of capturing the dynamics of the helicopter is required. Certain aspects of the dynamics are known exactly. For example it is known from basic physics that the vertical acceleration can be integrated exactly to obtain vertical velocity and integrated again to obtain height. Furthermore, the dynamics of the collective servo have been measured directly and can be hard wired in to a model of the plant. The remaining dynamics involve the relationship between the vertical acceleration and the collective pitch, vertical velocity and ground effect. This relationship is affected by the kinematic relationship between the servo input and the blade pitch, the vertical drag on the fuselage caused by the downwash and rate of climb/descent and other effects. These non-linear dynamics are more difficult to predict analytically and would require a tedious experimental effort to model accurately. An ANN is therefore one means of capturing this dynamic behavior in one step without a detailed mathematical model.

The helicopter is fitted with a vertical accelerometer which allows the vertical acceleration to be recorded from flight test and used to train the ANN plant model. The accelerometer measures the local acceleration which is the sum of the component of gravitational acceleration (g) and the rate of change of velocity of the platform. The z-accelerometer output $a_{z}$ is actually just equal to the net aerodynamic zaxis force $(\mathrm{T})$ acting on the helicopter divided by the helicopter mass ( m ), so that
$a_{z}=T / M$. The force T is the net effect of the main rotor thrust and the vertical drag caused by the rotor downwash. No compensation for pitch and roll tilt is required as the accelerometer moves with the helicopter local axes. Although counter-intuitive this can be understood by considering a simple helicopter of mass ( m ) with no tail rotor side force so that it hovers perfectly level with thrust ( $T$ ) equal to weight (W). If the helicopter is in equilibrium hover, $T=W$ and the accelerometer would measure $a_{z}=W / m=T / m$. If the thrust of the helicopter suddenly changed to zero, the helicopter would be in free-fall and the accelerometer would be zero, so that still $a_{z}=T / m=0$. If the helicopter suddenly rolls to $45^{\circ}$ without changing its thrust, the helicopter would begin to fly sideways and descend since the vertical component of thrust is now less than the weight force. The accelerometer would measure the sum of the z-axis component of gravity and the rate of change of $z$-axis velocity which is $-g \cos 45^{\circ}+\left(m g \cos 45^{\circ}-T\right) / m=T / m$. In each case, the accelerometer still measures $T / M$ regardless of the effect of gravity and tilt. The significance of this is that all of the aerodynamic effects are encapsulated in the measurement of $a_{z}$, so that training a neural network to represent $a_{z}$ is the same as training a neural network to represent the aerodynamics of vertical flight. The dynamics of the vehicle, which is essentially integrating the acceleration to get velocity and position, are known exactly and for these purposes do not need to be represented by an ANN as they are very simply implemented. However, if desired, use of a recurrent neural network would allow the integration of acceleration to be carried out by the ANN also.

### 7.3.1 Simulation of Height Control using an ANN

The feasibility of using an ANN to control height was first tested in simulation using SIMULINK ${ }^{\circledR}$. The first objective was to train a neural network to mimic an analytical model of the helicopter and then see if this ANN could be used to develop an optimal controller to control the plant. The second objective was to see if another ANN could then be trained to mimic the optimal plant and to see how well this ANN could do the job of the optimal controller.

The vertical dynamic components of the eagle simulation developed in Chapter 3 were isolated to generate simulated helicopter data. The rotor induced flow model calculates thrust given collective pitch and vertical velocity as inputs. The weight of the helicopter is subtracted from the thrust and divided by mass to calculate the vertical acceleration. This acceleration is then integrated twice to get the velocity and height. For the purposes of this test, ground effect was not included in the vertical flight model. A servo model based on the measured dynamics was however included.

The objective of the simulation is to provide training data to allow an ANN to be trained to mimic the plant. A large training set was desired to adequately span the possible combinations of vertical velocity and collective pitch. The plant model was stimulated to provide this training set using a random step block in series with a rate limiter to generate a reference set of desired vertical velocities to drive the model. The random step block was set to output velocities using a step period


Figure 7.1: Ideal plant SIMULINK ${ }^{\circledR}$ model
ranging randomly from 0.2 sec to 2 sec and with amplitudes ranging randomly from $-2 \mathrm{~m} / \mathrm{s}$ to $2 \mathrm{~m} / \mathrm{s}$. Higher rates of descent were not used as the induced flow model does not take into account the complex aerodynamics of the vortex ring state which occur when the rate of descent reaches about half of the induced velocity $(2.1 \mathrm{~m} / \mathrm{s}$ for the Eagle). Furthermore, higher rates of climb and descent than about $\pm 2 \mathrm{~m} / \mathrm{s}$ are not desirable during actual flight tests due to the proximity to the ground and the inability of the pilot to regain control in the event of a system failure. The rate limiter was used to bound the maximum commanded acceleration of the plant to $\pm 10 \mathrm{~m} / \mathrm{s}$ so as to keep the training output data to a reasonable set of values. The use of random step lengths allows large collective pitch changes to be commanded when the step is small and the system has not had a chance to reach equilibrium. Conversely, when long steps occur, the system has the opportunity to reach near steady state and small collective pitch changes occur. The use of normalised inputs means that the fine-grain changes in collective used to accurately maintain height are well represented, whilst still providing a good coverage of outlying extreme collective values. A simple proportional feedback scheme was implemented in the SIMULINK ${ }^{\circledR}$ model to control the collective input in response to errors between the desired reference velocity and the actual vertical velocity. The helicopter vertical flight model and complete simulation are shown in figure 7.1. The simulation was used to generate 1000 seconds of data with a 50 Hz sample rate.

The match between the ideal plant data and the ANN representation had an MSE of $2.6 \times 10^{-6}$ with only a four hidden unit single layer network. Increasing the number of hidden units beyond four was found to have negligible benefit which would not justify the additional computational burden. The effect of $V_{z}$ and collective pitch on $a_{z}$ are compared for the ANN and the analytical plant models in figure 7.2. The chart confirms the expectation from Blade Element theory that the thrust of the main rotor is decreased by climb rate and increased by descent rate for the same collective pitch. The results are virtually indistinguishable, except for high collective values which are outside the range of training data provided to the network.

For this work, an optimal trajectory is defined in terms of a prescribed velocity profile versus distance remaining to achieve the desired height. The objective of the optimisation is to achieve the fastest correction of height errors whilst not exceeding


Figure 7.2: Ideal plant versus ANN. Variation of accelerometer output with varying collective and vertical velocity
safe limits. The use of a prescribed velocity profile is more stable than simply trying to minimise the error in height as the velocity will naturally be damped out as the helicopter approaches its target height.

The key factor in developing the optimal controller is the definition of what is considered to be an optimal trajectory. In terms of minimum time to resolve an error in height, the optimal trajectory is one where the body accelerates towards the desired position at the maximum permissible acceleration for the first half of the path and then decelerates at the same rate for the remaining half. This trajectory requires the velocity $\left(V_{z}\right)$ to be proportional to the square root of the error distance $(\Delta Z)$. However use of $V_{z}=k \sqrt{\Delta Z}$ is problematic in that the gradient of the trajectory $\left(d V_{z} / d \Delta Z=0.5 k / \sqrt{\Delta Z}\right)$ approaches infinity for small errors leading to limit cycle behavior. Initial attempts at using this trajectory equation confirmed the limit cycle behavior. For this reason, a more stable trajectory was selected that involves making the desired vertical velocity of the helicopter simply proportional to the observed error in height $V_{z}=k \Delta_{z}$. A value of k of 0.5 was chosen based on observations of how experienced human pilots correct for errors in altitude.

Figures 7.3 and 7.4 show the SIMULINK ${ }^{\circledR}$ model used to represent the helicopter in vertical flight. Figure 7.3 shows the model used for optimisation. This model is caused by a MATLAB ${ }^{\circledR}$ script used to optimise the collective pitch input. The model is initialised by the script to the values from the previous time step. The model outputs the various state variables which are written to a file. Figure 7.4 shows the ANN based rotor thrust model and the integrators used to update the vertical flight state variables. The plant model appears as a single subsystem block within the overall optimisation model shown in Figure 7.3. The SIMULINK ${ }^{\circledR}$ model


Figure 7.3: SIMULINK ${ }^{\circledR}$ model of optimal control loop. This model is called by the optimisation algorithm to calculate the vertical flight data over the look-ahead time using a linearly changing collective pitch. The slope of the collective pitch change defined in the ramp block is the variable optimised.


Figure 7.4: ANN vertical flight plant model. ANN outputs vertical acceleration $a_{z}$ which is then integrated explicitly to obtain $V_{z}$ and $Z$. The inputs to the ANN are collective pitch and $V_{z}$.
is updated with a fixed time step of 1millisecond using a fourth order Runge-Kutta solver.

The projected trajectory was calculated for a look-ahead time of 0.5 seconds. This time was chosen based on consideration of the time constant of the servo so that adequate time was given for the system to respond to any changes in collective. Smaller look-ahead times resulted in oscillations, whilst longer look-ahead times reduced the effectiveness of the controller. A linear change in collective was projected forward for the look-ahead time using a ramp input starting at the current value of collective. A linearly changing collective was found to provide smoother control inputs than trying to optimise a constant collective input over the look-ahead time. The linearly changing collective was bounded during optimisation so that the natural rate-limiting effect of the servo was never exceeded.

The SIMULINK ${ }^{\circledR}$ optimisation model calculates the error between the desired velocity profile and the actual velocity profile at each time step. The calling script passes the vector of the error to the MATLAB ${ }^{\circledR}$ Isqnonlin optimisation function which is part of the MATLAB ${ }^{\circledR}$ optimisation toolbox. The lsqnonlin algorithm is used to adjust the collective pitch slope over the look-ahead period to obtain the minimum sum of squares of the error. The algorithm uses an interior-reflective Newton method described in [208] to minimise the error.


Figure 7.5: ANN controller output after training to mimic optimal controller

Once the optimal collective slope is determined, the plant model is then run using this slope for a time of 0.02 seconds, corresponding to the update rate of the servo. The optimisation process is slow since to advance only 0.02 seconds the simulation will be called multiple times by the optimisation program, and each time the simulation is called, it will run for the look-ahead time of 0.5 seconds. After each 0.02 second epoch, the new collective pitch is calculated by propagating forward from the old collective pitch using the optimal collective slope. Then the pitch and all of the other state variables from the simulation, such as $z, V_{z}$ are stored in a vector to pass to the simulation to initialise the next run. The servo transfer function is represented in a state space form so that the current state can be stored and used to maintain continuity with the time history of the servo inputs.

The optimisation was run for 1000 seconds with data recorded at 0.02 second intervals. A new neural network was then trained to mimic the optimal controller, using this data set. The training inputs were the height error and $V_{z}$. The training output was the collective pitch produced by the optimisation process. The neural network was trained using 900 seconds of the optimisation data and the remaining 100 seconds of data was used for validation. Networks with up to 12 hidden nodes and either one or two hidden layers were tested. Once again, little benefit was gained by using more than four hidden nodes and a single hidden layer. Figure 7.5 shows an extract of the controller performance versus the optimal control data for a single hidden layer with four hidden nodes.

To test the optimal ANN, the SIMULINK ${ }^{\circledR}$ model shown in Figure 7.6 was created using the mimicking ANN as the controller and the ANN plant model as the plant. A random step was used to set a changing reference height that the ANN controller attempted to follow. The step parameters were changed to ensure that the test data experienced by the model was different to the training set provided by the actual optimal controller. To check the validity of the experiment, the ANN plant model was replaced by the original induced flow model and the experiment was rerun. No noticeable change in the results was observed. Figure 7.7 shows the response of the helicopter model to changes in reference height. The helicopter can be seen


Figure 7.6: SIMULINK ${ }^{\circledR}$ model used for testing ANN controller


Figure 7.7: Response of ANN controller and plant to change in reference height
to track the reference height accurately with minimal overshoot. The behavior of the ANN controller is very similar to the behavior of the optimal controller, except the ANN controller uses a trivial amount of processing and can be run in real-time onboard the helicopter whereas the optimal controller cannot.

A number of sensor errors were introduced in to the model to test their effect on the robustness of the controller. Firstly noise was added to the $V_{z}$ and $Z$ sensor data. The noise was increased beyond the expected levels from the DGPS. Noise with a standard deviation 10 times greater than that realised with the real sensor was applied to the controller with little effect on the outcome.

Offsets in velocity estimates are commonly present where inertial sensing is combined with another sensing mode such as GPS or vision. This is because of accelerometer drift due to thermal effects. Such drift can be minimised in real-time using an EKF as described in Chapter 8, however this incurs as significant processing overhead. In this case, the accelerometer drifts were minimised by thermal calibration of the accelerometers, but velocity offsets of up to $\pm 0.05 \mathrm{~m} / \mathrm{s}$ have been observed on the Eagle helicopter. To test the robustness of the controller to these effects, offsets in velocity sensing of up to $\pm 0.2 \mathrm{~m} / \mathrm{s}$ were simulated and the controller was found to remain stable and to still track the reference height albeit with
a tracking error in height. The effect of a $0.2 \mathrm{~m} / \mathrm{s}$ velocity offset was to cause a height tracking error of about 40 cm . Integral control could be added to the ANN controller to correct the effect of a velocity sensor offset, either by explicitly adding an integrator or by training the controller with data from on optimal controller trained in the presence of sensor offsets. In the later case, the ANN would have to be recurrent so that integrator state data could be retained within the network structure.

The final robustness check was to vary the servo transfer function from the value measured in the laboratory. This was important because variation in servo parameters are to be expected under different loads and supply voltages. Also I was mindful that a replacement servo part from a different manufacturer would have different characteristics and it would be undesirable to have to retrain the controller every time a servo was replaced. The servo was assumed to be able to be approximated by a first order transfer function in series with a rate limiter. The time constant of the transfer function was varied between 0.02 sec and 0.5 seconds. At all times, the controller remained stable with less than $5 \%$ variation in height error.

### 7.3.2 ANN Based Height Control on an Actual Helicopter

The helicopter was flown by the author for about 9 minutes in vertical flight to collect data to train a plant model. During that time the helicopter was aggressively maneouvered with the collective pitch control whilst the cyclic pitch and tail rotor controls were used sparingly to keep the helicopter in vertical flight only. The objective of the control inputs was to collect training data with as many combinations of $V_{z}$ and collective as possible. During the maneuvering, a range of $V_{z}$ between $-3.5 \mathrm{~m} / \mathrm{s}$ and $2.2 \mathrm{~m} / \mathrm{s}$ were achieved whilst the PWM control signals sent to the collective pitch servo were varied between extremes of $974 \mu \mathrm{~s}$ and $2124 \mu \mathrm{~s}$. The mean of the collective inputs was $1356 \mu \mathrm{~s}$ with a standard deviation of $164 \mu \mathrm{~s}$.

A SIMULINK ${ }^{\circledR}$ model was created to process the flight test data and convert it in to inputs that could be used to train the plant model ANN. The data obtained from the flight test was refined so that data points in the region of ground effect were ignored and points where full DGPS coverage was not present were ignored. Vertical velocity was obtained by sensor fusion of the DGPS vertical speed (at 20 Hz ) and accelerometer data using a simple complimentary filter. A side-effect of the onboard digital filtering of the accelerometers is that the data is lagged by 80 milliseconds at all frequencies owing to the linear phase response of the filter used. This time shift was easily corrected by lagging all of the ANN training inputs by 80 ms using a transport delay block. The SIMULINK ${ }^{\circledR}$ model saved the training inputs and outputs to separate binary data files.

As discussed in Chapter 3, the servo is modelled as a rate limiter combined with a first order transfer function. Initial attempts at modeling the ANN applied the servo model to the control inputs before training the network. However, this was found to produce slightly worse results than using no servo model at all, so the raw control inputs were used instead. The importance of the servo model is not


Figure 7.8: Validation of ANN vertical flight model
so significant under human control, as the pilot control inputs tend to be of a low bandwidth, which is not significantly changed by the servo rate limit or transfer function.

The best MSE obtained was approximately 0.1 and was not found to increase significantly when the number of hidden nodes was increased past four or the number of layers was increased beyond 1. Hence a single layer plant model of four hidden nodes was used. 340 seconds of data was used to train the network and 60 seconds of data were used for validation. Figure 7.8 shows the network output for the validation data. There are a number of causes of discrepancies in the results. The most significant of these is wind. Owing to the time of year that the flights were conducted, it was impossible to carry out the tests in nil-wind conditions. The helicopter experienced gusts of up to $30 \mathrm{~km} / \mathrm{hr}$ during the flight which caused a ballooning effect as the rotor rapidly transitions from a hover to forward flight regime. The wind speed fluctuations have significant spatial variation and it is not practical to measure them at the instantaneous position of the helicopter. This information is therefore unknown to the ANN plant model and it cannot learn the relationship. Secondly, the pilot needed to use cyclic pitch and tail rotor inputs to keep the helicopter stable. The cyclic pitch inputs tilt the TPP and cause the thrust vector to change. Tail rotor pitch inputs change the power requirements on the motor and cause excursions in rotor RPM and hence thrust for the same collective. The inaccuracies in the ANN model are assumed to be acceptable because the remaining dynamics of the system are known exactly.

Figure 7.9 is a chart showing the effect of collective and $V_{z}$ on the network. The chart clearly shows the non-linear nature of the relationship between collective, $V_{z}$ and $a_{z}$. For high climb rates and high collective values, the value of $a_{z}$ does not increase with increased collective and can actually decrease. The likely cause of this is stall of parts of the rotor owing to high angle of attack. Failure of the motor to maintain constant RPM in conditions of high blade drag may also contribute to this effect. Blade stall is a pronounced condition for a rotor with untwisted blades since stall is likely to occur at the tips of the blade first and the tips are where


Figure 7.9: Variation of accelerometer output with varying collective and vertical velocity for ANN trained from flight test data
most of the lift is produced for an untwisted blade. Based on blade element theory and taking into account blade tip losses, the expected angle of attack to produce an acceleration of $-12 \mathrm{~m} / \mathrm{s}^{2}$ with a $2 \mathrm{~m} / \mathrm{s}$ climb rate would be 14 degrees near the tip. This coincides closely with the stall angle of a typical aerofoil and would explain the behavior at high climb rates.

The same method for generating optimal control training data was used with the real plant model. During optimisation, bounds were set on $V_{z}$ and collective to prevent the helicopter entering into a stall or vortex ring regime. Furthermore, the bounds were set to prevent the ANN plant model from having to extrapolate past the training envelope it has experienced. The bounds of $V_{z}$ were set to $\left(V_{z}\right)_{\max }= \pm 1.5 \mathrm{~m} / \mathrm{s}$ and the maximum commanded acceleration was set to $1 \mathrm{~m} / \mathrm{s}^{2}$. In order to saturate the commanded velocity smoothly without violating the maximum acceleration, a hyberbolic tangent sigmoid function was used to limit the velocity to $\left(V_{z}\right)_{\text {limited }}$ as defined by Equation (7.2). Collective pitch PWM values were bounded between $1120 \mu s$ and $1680 \mu \mathrm{~s}$. Random steps in reference height of up to $\pm 3 m$ for durations ranging randomly from 1 second to 15 seconds were applied. The optimal controller was run for 3000 seconds and the data stored to file.

$$
\begin{equation*}
\left(V_{z}\right)_{\text {limited }}=\left(V_{z}\right)_{\max }\left(\frac{2}{1+e^{-2 n}}-1\right) \tag{7.2}
\end{equation*}
$$

ANNs with various numbers of hidden nodes were trained to mimic the optimal controller. An ANN comprising 12 nodes with an MSE of 0.024 was deemed to be a good compromise between the network complexity and the accuracy of the model. In comparison, a 10 node ANN had an MSE of 0.033 and a 20 node ANN was only


Figure 7.10: Tracking performance comparison between PID and ANN controllers for step changes in reference height
slightly better with an MSE of 0.018 . The 12 node network was saved to a file and loaded on to the Eagle helicopter flash card.

### 7.3.3 Comparison with a PD controller

A PD controller was tested with the ANN plant model to compare with the ANN optimal controller. Such a comparison is difficult to make objectively, since the gains of the PD controller need to be matched in a subjective way to the design parameters of the optimal controller. Gains for the PD controller were taken from a controller tuned in flight on the actual helicopter using the Ziegler and Nichols method [80] discussed in Chapter 3. As a further validation of the ANN plant model, the model reacted stably and in the same fashion as the real plant when the same PD gains were used as the actual helicopter.

A comparison of the controllers was conducted using the same random step heights, servo dynamics model, plant model and sensor noise parameters. An offset in collective trim pitch of $4 \mu s$ was used on both models. Figure 7.10 shows the response of the ANN and the PD controller on the same plot. The ANN exhibits a much faster response than the PD controller whilst maintaining minimal overshoot. The ANN also shows far less height offset error than the PD controller. Figure 7.11 shows the collective pitch from both controllers for the same test. Despite the better tracking performance, the ANN model uses less collective pitch with lower peaks in the collective pitch used. This places less stress on the servos, reducing mechanical wear and reducing power consumption. On the basis of the simulation, the ANN controller appears to be superior in all accounts to the PD controller.


Figure 7.11: Collective pitch comparison between PID and ANN controllers for step changes in reference height

### 7.3.4 Flight Test of Vertical Controller

Initial flight tests of the ANN were conducted in very gusty conditions and the tracking error was not as good as hoped. After reviewing the data, it was realised that the gain of the controller was actually quite low and that the acceleration saturation built in to the optimal controller was significantly reducing the ability of the controller to respond to gusts. Indeed, simulations of the induced flow model of the helicopter rotor show that when gusts of even just $20 \mathrm{~km} / \mathrm{hr}$ occur, the thrust of the rotor will vary by almost 60 N , corresponding to about $7 \mathrm{~m} / \mathrm{s}^{2}$, for the same collective pitch setting. The controller needs to be able to create accelerations of a comparative size to the gusts in order to compensate for their effect. The gain of the controller, defined by the constant $k$, in the optimal feedback equation $V_{z}=-k Z$, was therefore increased in stages up to 2.0. Each time the gain was increased, a new set of optimal training data was produced and a new ANN controller was trained. In order to accomodate the increased gain, the limit on vertical acceleration was increased to $\pm 3 \mathrm{~m} / \mathrm{s}^{2}$. At the highest gain, the helicopter was very stable and indications are that the gain could have been increased further.

Figure 7.12 shows the results of an 90 second hover using the ANN to keep the helicopter at a fixed height of about 1.8 m above the ground. The ANN tracks the height with a mean error of less than 8 cm . This error is mainly in the form of a fixed offset which can probably be attributed to the trim of the collective lever not being trimmed out perfectly on handover and the presence of wind. Figure 7.13 shows the ANN responding to a reference height that is varying with a square wave input of $\pm 40 \mathrm{~cm}$ with period 20 seconds. The ANN is seen to respond to the square wave very rapidly with minimal overshoot. As a comparison, figure 7.14 shows a PD


Figure 7.12: Response of ANN controller to steps in commanded height


Figure 7.13: Response of ANN controller to steps in commanded height
height controller responding to the same square wave. The response of the PD to the step inputs is similar to that of the ANN controller, however the PD controller has a larger offset. Based on the comparison of the two controllers performed in simulation, this offset can be expected to be more pronounced than for the ANN controller.

Deviations from the set height can be attributed to a number of environmental causes. Firstly, gusts of wind are responsible for rapidly changing the trim collective to hover and hence creating excursions in height. Secondly, a human pilot was controlling the aileron, elevator and rudder controls separately during the experiment and was introducing disturbances through cross-coupling effects as he struggled to keep the helicopter hovering over the same spot in wind.

### 7.4 Optic Flow Damped Hover using a Neural Network

I now turn to the problem of controlling the lateral and longitudinal motion of the helicopter using ANN. I have already demonstrated in Chapter 5 that the helicopter can be hovered using optic flow to control the sideways drift. In this section, I aim


Figure 7.14: Response of PID controller to steps in commanded height
to repeat the optic flow damped hover control experiment, except, this time the sensor fusion and control will both be done using neural networks.

Sensor fusion using ANN has the advantage that it allows raw uncalibrated data to be used by the network, eliminating the need for calibration, as the calibration process is built in to the training of the net. This could be used to eliminate tedious accelerometer, gyroscope, thermal drift and optic flow calibrations.

As the controller has already been implemented successfully, we have a good starting point for designing the architecture of the controller. Rather than attempting optimal control, the controller will be based on the same sort of feedback structure as the conventional PI outer loop velocity controller combined with a simple P type attitude inner loop. Combining the inner loop and outer loop, we arrive at Equations (7.3) and (7.4) for the pitch and roll control channels, where $K_{d}$ is the feedback from velocity, $K_{I}$ is integral feedback from velocity error and $K_{p}$ is proportional feedback from attitude.

$$
\begin{align*}
\delta_{l a t} & =K_{d}^{\text {lat }} V_{y}+K_{I}^{\text {lat }} \int V_{y} d t-K_{p}^{1 a t} \phi  \tag{7.3}\\
\delta_{l o n} & =K_{d}^{\text {lon }} V_{x}+K_{I}^{\text {lon }} \int V_{x} d t-K_{p}^{\text {lon }} \theta \tag{7.4}
\end{align*}
$$

A key aspect of the outer loop controller is an integral term to adjust the helicopter datum attitude to suit the environmental conditions. For example, in the presence of steady wind, the drag on the helicopter fuselage requires an additional tilt of the helicopter to maintain position. A recurrent neural network provides the ability to feedback outputs or hidden layer values to the input stage, so that temporal processing such as differentiation and integration can be achieved.


Figure 7.15: Longitudinal sensor fusion ANN

### 7.4.1 Sensor Fusion using ANN

A means of combining visual and inertial information using ANN is proposed. For the optic flow damped hover, velocities in the longitudinal and lateral directions are required. The available measurements are optic flow, accelerations, rotation rates and height. As outlined in Chapter 5, these inputs can be combined in a complimentary filter arrangement where the inertial data is propagated using the known state update equations. The resulting velocity data must then be corrected using measurements of velocity calculated from the product of optic flow and height, otherwise inertial sensor offsets will result in velocity drift. The correction and prediction steps can be combined in one recurrent ANN using the structure depicted in figure 7.15.

Whilst the eagle simulation developed in Chapter 3 could have been used to generate data for the sensor fusion ANNs, it is better to generate random combinations of the data as this provides a better coverage of the possible set of combinations of sensor values that could be expected. Uniform random number generator blocks were used inside SIMULINK ${ }^{\circledR}$ to generate the desired height, longitudinal velocity and lateral velocity to stimulate a range of combinations of optic flow, height, pitch rate and roll rate. The plant was used to generate 2000 seconds of data with a 50 Hz sample rate. For training, the lateral sensor fusion ANN was provided with five inputs: raw lateral optic flow; lateral acceleration $\left(a_{y}\right)$; roll rate $(p)$; height measurement $(Z)$; and the previous lateral velocity. The longitudinal sensor fusion ANN was provided with raw longitudinal optic flow; longitudinal acceleration $\left(V_{y}\right)$; pitch rate $(p)$; height measurement $(Z)$; and the previous longitudinal velocity. The ANNs were trained to mimic the velocity estimates from the existing complimentary filter described in section 5.3. A form of teaching forcing [129] was used to prevent instability during training. This means that the previous state estimate used for training was actually the real previous state rather than the ANN estimate of the state. Without doing this, the ANN is unable to learn how to make use of the old state estimate as before the network is trained, the ANN estimate of the state bears no relation to the real state.

Networks with five hidden units were found to be suitable for both $x$ and $y$ velocities and produced an MSE of less than $5 \times 10^{-8}$ for both networks. The ANNs


Figure 7.16: Validation of ANN sensor fusion


Figure 7.17: ANN cyclic pitch controller for roll
were validated in-flight to prove that the sensor fusion was working correctly. Figure 7.16 shows the $V_{x}$ and $V_{y}$ velocities estimated by the ANNs versus the corresponding state estimate calculated onboard the helicopter using GPS and inertial data.

### 7.4.2 Plant Training for Cyclic Pitch Controller

The roll cyclic controller has the structure shown in figure 7.17. The pitch cyclic controller has the same structure except the inputs are pitch attitude and longitudinal velocity instead of the lateral variables. The ANN has two outputs. The first output is the integrated velocity error which is fed back as an input through a delay of one sample time. This feedback makes the ANN recurrent and allows the network to integrate the velocity error. The second output is the cyclic pitch signal, consisting of gains multiplied by the angular error, drift velocity and the integrated velocity error.

As the function of the ANN is to mimic a simple linear controller where the gains have already been determined, the dynamics of the plant are not that relevant to the training data. The inputs to the controller are the current velocity, the integrated velocity error from the last time step and the current attitude. Random velocities in the range $-5 \mathrm{~m} / \mathrm{s}$ to $+5 \mathrm{~m} / \mathrm{s}$, random attitude angles ranging from $-30^{\circ}$ to $+30^{\circ}$ and random integrated velocity errors in the range -10 m to +10 m were simultaneously generated and collected for 2000 seconds. The cyclic pitch values produced by the classical controller used on the actual helicopter were then calculated for the given inputs and recorded. Two ANNs with five hidden nodes were trained separately for the lateral and longitudinal axes using these inputs. The ANNs were trained to mimic two outputs each: (a) the cyclic pitch value produced for the random inputs by the classical controller implemented on the helicopter, and (b) the integrated velocity error. An MSE of less than $1 \times 10^{-9}$ was obtained for each ANN after training.

### 7.4.3 Flight Test

The helicopter was flown with longitudinal and lateral velocity controlled by the two ANNs. Height was controlled by the ANN collective pitch controller. Sensor fusion of the optic flow with inertial information was carried out using the ANNs described previously. Figure 7.18 shows the flight path of the helicopter over 170 seconds of closed loop flight. During this time, the helicopter drifted 9 m North, 0.1 m East and stayed within 5 cm of the datum height set on handover. The performance of the controller is very similar to the performance of the classical optic flow damped hover controller introduced in Chapter 5.

### 7.5 Summary

In this chapter, we have seen that ANNs can be effectively used to control the flight of a helicopter in hover. Furthermore, the ability of ANNs to perform sensor fusion has been demonstrated using an in-flight experiment. Use of an ANN type structure has a number of advantages over more conventional control implementations. These include the ability to do optimal non-linear control in real-time, the elimination of the requirement for a complex analytical model and modest computational requirements that can be implemented on simple analog or digital hardware.

In vertical flight, a new technique has been explored for training a controller using a hybrid ANN plant model with an exact dynamics model. This combines the unknown relationship between acceleration and collective pitch to the exactly known dynamics relating acceleration, velocity and position. Such a technique should also be applicable to the lateral and longitudinal dynamics. In this case, an ANN could be trained to learn the relationship between angular accelerations and cyclic pitch inputs. A fully non-linear and exact set of motion equations could then be applied to calculate the angular rates and attitude. An optimal controller could be constructed in series with the hybrid plant model to generate data to train an ANN attitude controller.


Figure 7.18: Optic flow damped hover results using ANN for control of collective pitch, aileron and elevator: (a) Position as measured by DGPS; (b) Velocity measured by DGPS; (c) Attitude measured by inertial sensors; (d-f) Servo PWM values generated by ANN.

In this work I have not tested the limits of stability for the optimal controller. In future work, the gain $k$ in the feedback equation $V_{z}=-k z$ could be increased in steps to determine at what point the control loop becomes unstable or the effect of noise becomes too great. Increasing the gain will improve tracking performance. Steady state offsets in height due to collective pitch trim errors might be reduced further by introducing integral feedback in parallel with the ANN controller. The effect of the integral control on the stability of the ANN should be investigated in future work.

## Chapter 8

## Advanced Sensor Fusion

### 8.1 Introduction

The simple fusion techniques used so far in this thesis have been adequate for demonstrating closed loop control of the helicopters, but have had some shortcomings related to ignoring sensor errors. The main problem experienced is that the inertial sensors suffer drift, which arises because the sensor offsets change with temperature. In this chapter, I attempt to address these problems by exploring online sensor error estimation using Extended Kalman Filters (EKF).

### 8.1.1 Overview of the Discrete EKF Algorithm

A Kalman filter is a recursive filter that can provide an optimal estimate of the states of a linear dynamical system given noisy measurements of the system. The development of the Kalman filter is largely attributed to Rudolf Kalman [209] who the filter is named after. An Extended Kalman Filter (EKF), first proposed by Schmidt [210], is an extension of the Kalman filter where the state update equations and relationship between the measurements and the states is not linear. The EKF is an ad-hoc technique [211] which is not optimal owing to the linearisation that is used to propagate the filter. Both Kalman filters and EKF can be used to estimate states that are not directly measured. For example, in the case of vertical flight, an EKF could be used to estimate vertical velocity given only acceleration and height, but no direct measure of $w$. The downside of using an EKF is that it requires a higher level of computational overhead than a simple filter and in some cases may diverge or fail numerically.

A discrete EKF assumes a non-linear dynamic model defined by:

$$
\begin{equation*}
\boldsymbol{x}_{k}=f\left(\boldsymbol{x}_{k-1}, k-1\right)+\boldsymbol{w}_{k} \tag{8.1}
\end{equation*}
$$

where $\boldsymbol{x}_{k}$ is the $k_{t h}$ estimate of the state variables and $\boldsymbol{w}_{k}$ is process noise. The non-linear measurement model for the EKF is defined by:

$$
\begin{equation*}
\boldsymbol{z}_{k}=h\left(\boldsymbol{x}_{k}, k\right)+\boldsymbol{v}_{k} \tag{8.2}
\end{equation*}
$$

The measurement and plant noise $\boldsymbol{v}_{k}$ and $\boldsymbol{w}_{k}$ are assumed to be zero-mean Gaussian sequences. The EKF algorithm is implemented by solving the following set of equations recursively:

Computing the predicted state estimate:

$$
\begin{equation*}
\hat{\boldsymbol{x}}_{k}^{-}=f\left(\hat{\boldsymbol{x}}_{k-1}^{+}, k-1\right) \tag{8.3}
\end{equation*}
$$

Computing the predicted measurement:

$$
\begin{equation*}
\hat{\boldsymbol{z}}_{k}=h\left(\hat{\boldsymbol{x}}_{k}^{-}, k\right) \tag{8.4}
\end{equation*}
$$

Linear approximation equation:

$$
\begin{equation*}
\left.\boldsymbol{\Phi}_{k-1} \approx \frac{\partial f(\boldsymbol{x}, k-1)}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k}^{-}} \tag{8.5}
\end{equation*}
$$

Conditioning the predicted estimate on the measurement:

$$
\begin{gather*}
\hat{\boldsymbol{x}_{k}^{+}}=\hat{\boldsymbol{x}}_{k}^{-}+\boldsymbol{K}_{k}\left(\boldsymbol{z}_{k}-\hat{\boldsymbol{z}}_{k}\right)  \tag{8.6}\\
\left.\boldsymbol{H}_{k} \approx \frac{\partial h(\boldsymbol{x}, k)}{\partial \boldsymbol{x}}\right|_{\boldsymbol{z}=\hat{\boldsymbol{x}}_{k}^{-}} \tag{8.7}
\end{gather*}
$$

Computing the a priori covariance matrix:

$$
\begin{equation*}
\boldsymbol{P}_{k}^{-}=\boldsymbol{\Phi}_{k-1} \boldsymbol{P}_{k-1} \boldsymbol{\Phi}_{k-1}^{T}+\boldsymbol{Q}_{k-1} \tag{8.8}
\end{equation*}
$$

Computing the Kalman gain:

$$
\begin{equation*}
\boldsymbol{K}_{k}=\boldsymbol{P}_{k}^{-} \boldsymbol{H}_{k}^{T}\left[\boldsymbol{H}_{k} \boldsymbol{P}_{k}^{-} \boldsymbol{H}_{k}^{T}+\boldsymbol{R}_{k}\right]^{-1} \tag{8.9}
\end{equation*}
$$

Computing the a posteriori covariance matrix:

$$
\begin{equation*}
\boldsymbol{P}_{k}^{+}=\left(\boldsymbol{I}-\boldsymbol{K}_{k} \boldsymbol{H}_{k}\right) \boldsymbol{P}_{k}^{-} \tag{8.10}
\end{equation*}
$$

The $\boldsymbol{Q}$ and $\boldsymbol{R}$ matrices represent the process and measurement noise respectively. They are approximately represented by diagonal matrices having the diagonal terms set equal to the statistical variance of the noise.

In the EKFs described in this chapter, only first order approximations to the fundamental matrix $\boldsymbol{\Phi}$ will be used, since use of higher order approximations do not generally lead to an improvement in the performance of the EKF [212].

### 8.2 Attitude Estimation

The gyroscopes used on the Eagle helicopter are subject to thermal drift, which requires either: (a) constant zeroing in the field to eliminate the offsets, or (b) switching the avionics on and then waiting for up to an hour for the gyroscope temperatures to stabilise before use. A system that could estimate the gyroscope offset errors in real-time to provide a corrected gyroscope rate output would therefore
be very useful. Two methods of achieving this using EKF were devised and simulated for comparison. The aim of each EKF algorithm was to determine the gyroscope offsets dynamically, but not to estimate the gyroscope scale and misalignment errors as they could be removed by calibration as described in Appendix C and were not able to change appreciably post-calibration. Each EKF algorithm was tested using simulated sensor data for the helicopter in hover. The algorithm and results are described separately below.

### 8.2.1 EKF Algorithm One

This algorithm propogates a nine element state vector consisting of a unit vector $(\hat{\boldsymbol{g}})$ representing the direction of gravity, a unit vector $(\hat{\boldsymbol{b}})$ representing the earth's magnetic field and the three gyroscope offsets. The unit vectors define the directions relative to the helicopter body axes. The observations for this algorithm are the measured acceleration and magnetic vectors. Because both the observations of attitude and the representation of attitude are in vector form, the correction of the state estimate is already linear, and does not need to be linearised. This means that observations with large deviations from mean do not violate linearity assumptions and therefore very noisy observations can be tolerated. Also, because the magnetic and accelerometer vectors are independently stored, the algorithm can continue in a degraded mode when one sensor fails. The state vector for this algorithm is:

$$
\boldsymbol{x}=\left[\begin{array}{lllllllll}
g_{x} & g_{y} & g_{z} & b_{x} & b_{y} & b_{z} & \delta_{x} & \delta_{y} & \delta_{z} \tag{8.11}
\end{array}\right]^{T}
$$

where $\boldsymbol{g}=\left[g_{x} g_{y} g_{z}\right]^{T}, \boldsymbol{b}=\left[b_{x} b_{y} b_{z}\right]^{T}$ and $\boldsymbol{\delta}=\left[\delta_{x} \delta_{y} \delta_{z}\right]^{T}$ are the gravity direction unit vector, magnetic direction unit vector and gyroscope offset vectors respectively. The vectors are subject to body axis rotation rates defined by the symbols P,Q and $R$ which are equal to the sum of the measured gyroscope rates ( $\mathrm{p}, \mathrm{q}$ and r ) and the gyroscope offsets $\delta_{x}, \delta_{y}$ and $\delta_{z}$ as in Equation (8.12).

$$
\begin{equation*}
P=p+\delta_{p}, Q=q+\delta_{q}, R=r+\delta_{r} \tag{8.12}
\end{equation*}
$$

The change in the vectors due to $\mathrm{P}, \mathrm{Q}$ and R over a time step $\Delta T$ can be expressed as a rotation matrix. To determine this rotation matrix, first consider Equation (8.13) which describes the effect of these rotation rates on the attitude quaternion:

$$
\left[\begin{array}{c}
\dot{q_{0}}  \tag{8.13}\\
\dot{q}_{1} \\
\dot{q}_{2} \\
\dot{q}_{3}
\end{array}\right]=-\frac{1}{2}\left[\begin{array}{cccc}
0 & P & Q & R \\
-P & 0 & -R & Q \\
-Q & R & 0 & -P \\
-R & -Q & P & 0
\end{array}\right]^{T}\left[\begin{array}{l}
q_{0} \\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]
$$

or in more compact form

$$
\begin{equation*}
\frac{d q}{d t}=-\frac{1}{2} \Omega \boldsymbol{q} \tag{8.14}
\end{equation*}
$$

where $\boldsymbol{q}$ is the vector of quaternion paramaters $\left[q_{0} q_{1} q_{2} q_{3}\right]^{T}$ and $\boldsymbol{\Omega}$ is the angular velocity tensor. For sufficiently small $\Delta T$, this can be approximated with the first
order expression in Equation (8.15).

$$
\begin{equation*}
\Delta \boldsymbol{q} \approx-\frac{1}{2} \Omega \boldsymbol{q} \Delta T \tag{8.15}
\end{equation*}
$$

As we are only interested in the change of the vectors relative to the existing body axes then we can set the quaternion on the right hand side of Equation (8.15) to $\boldsymbol{q}=\left[\begin{array}{lll}1 & 0 & 0\end{array}\right]^{T}$ corresponding to a datum attitude. The change in attitude over the time step is then given by Equation (8.16).

$$
\left[\begin{array}{l}
\Delta q_{0}  \tag{8.16}\\
\Delta q_{1} \\
\Delta q_{2} \\
\Delta q_{3}
\end{array}\right]=\frac{1}{2}\left[\begin{array}{l}
0 \\
P \\
Q \\
R
\end{array}\right] \Delta T
$$

Hence the change in attitude over the time-step relative to the body axes is:

$$
\left[\begin{array}{l}
q_{0}  \tag{8.17}\\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]=\left[\begin{array}{c}
1 \\
P \Delta T / 2 \\
Q \Delta T / 2 \\
R \Delta T / 2
\end{array}\right]=\left[\begin{array}{c}
1 \\
\bar{P} \\
\bar{Q} \\
\bar{R}
\end{array}\right]
$$

where the definitions below have been used to simplify the expression:

$$
\begin{equation*}
\bar{P}=P \frac{\Delta T}{2}, \bar{Q}=Q \frac{\Delta T}{2}, \bar{R}=R \frac{\Delta T}{2} \tag{8.18}
\end{equation*}
$$

The effect of this change in attitude on the $\boldsymbol{g}$ and $\boldsymbol{b}$ vectors can be determined by a $3 \times 3$ rotation matrix $\boldsymbol{B}$. The complete state transition matrix is given below where $B_{i j}$ are the elements of $\boldsymbol{B}$.

$$
\left[\begin{array}{c}
g_{x}  \tag{8.19}\\
g_{y} \\
g_{z} \\
b_{x} \\
b_{y} \\
b_{z} \\
\delta_{p} \\
\delta_{q} \\
\delta_{r}
\end{array}\right]_{k+1}=F(\hat{\boldsymbol{x}}, k)=\left[\begin{array}{ccccccccc}
B_{11} & B_{12} & B_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\
B_{21} & B_{22} & B_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\
B_{31} & B_{32} & B_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & B_{11} & B_{12} & B_{13} & 0 & 0 & 0 \\
0 & 0 & 0 & B_{21} & B_{22} & B_{23} & 0 & 0 & 0 \\
0 & 0 & 0 & B_{31} & B_{32} & B_{13} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
g_{x} \\
g_{y} \\
g_{z} \\
b_{x} \\
b_{y} \\
b_{z} \\
\delta_{p} \\
\delta_{q} \\
\delta_{r}
\end{array}\right]_{k}
$$

The rotation matrix $\boldsymbol{B}$ is defined in terms of quaternion parameters as:

$$
\boldsymbol{B}=\left[\begin{array}{ccc}
q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{1} q_{2}+q_{0} q_{3}\right) & 2\left(q_{1} q_{2}+q_{0} q_{3}\right)  \tag{8.20}\\
2\left(q_{1} q_{2}-q_{0} q_{3}\right) & q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{2} q_{3}+q_{0} q_{1}\right) \\
2\left(q_{1} q_{3}+q_{1} q_{2}\right) & 2\left(q_{2} q_{3}-q_{0} q_{1}\right) & q_{0}^{2}-q_{1}^{2}-q_{2}^{2}+q_{3}^{2}
\end{array}\right]
$$

Using the quaternion attitude change in Equation (8.17), this becomes:

$$
\boldsymbol{B}=\left[\begin{array}{lll}
1 & 0 & 0  \tag{8.21}\\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]+\left[\begin{array}{ccc}
\bar{P}^{2}-\bar{Q}^{2}-\bar{R}^{2} & 2(\bar{P} \bar{Q}+\bar{R}) & 2(\bar{P} \bar{R}+\bar{Q}) \\
2(\bar{P} \bar{Q}+\bar{R}) & -\bar{P}^{2}+\bar{Q}^{2}-\bar{R}^{2} & 2(\bar{Q} \bar{R}+\bar{P}) \\
2(\bar{P} \bar{R}+\bar{Q}) & 2(\bar{Q} \bar{R}-\bar{P}) & -\bar{P}^{2}-\bar{Q}^{2}+\bar{R}^{2}
\end{array}\right]
$$

For the sake of brevity, only the state-update for the x -gravity component will be shown. In scalar form, the state update for $g_{x}$ can be found from Equation (8.19):

$$
\begin{align*}
g_{x}^{k+1} & =g_{x}^{k}+\frac{\Delta T^{2}}{4}\left((p+\delta p)^{2}-(q+\delta q)^{2}-(r+\delta r)^{2}-\right) g_{x}^{k} \\
& +\left(\frac{\Delta T^{2}}{2}(p+\delta p)(q+\delta q)+\Delta T(r+\delta r)\right) g_{y}^{k} \\
& +\left(\frac{\Delta T^{2}}{2}(p+\delta p)(r+\delta r)-\Delta T(q+\delta q)\right) g_{z}^{k} \tag{8.22}
\end{align*}
$$

In order to derive the fundamental matrix, $\boldsymbol{\Phi}$, this equation is differentiated with respect to each state variable resulting in Equations (8.23-8.28). Each partial derivative of $g_{x}$ forms a unique column of the first row of $\boldsymbol{\Phi}$.

$$
\begin{align*}
\frac{\partial g_{x}^{k+1}}{\partial g_{x}^{k}} & =1+\frac{\Delta T^{2}}{4}\left((p+\delta p)^{2}-(q+\delta q)^{2}-(r+\delta r)^{2}\right)  \tag{8.23}\\
\frac{\partial g_{x}^{k+1}}{\partial g_{y}^{k}} & =\frac{\Delta T^{2}}{2}\left(\frac{\Delta T^{2}}{2}(p+\delta p)(q+\delta q)+\Delta T(r+\delta r)\right)  \tag{8.24}\\
\frac{\partial g_{x}^{k+1}}{\partial g_{z}^{k}} & =\frac{\Delta T^{2}}{2}\left(\frac{\Delta T^{2}}{2}(p+\delta p)(r+\delta r)-\Delta T(q+\delta q)\right)  \tag{8.25}\\
\frac{\partial g_{x}^{k+1}}{\partial \delta_{p}^{k}} & =\frac{\Delta T^{2}}{2}(p+\delta p) g_{x}^{k}+\frac{\Delta T^{2}}{2}(q+\delta q) g_{y}^{k}+\frac{\Delta T^{2}}{2}(r+\delta r) g_{z}^{k}  \tag{8.26}\\
\frac{\partial g_{x}^{k+1}}{\partial \delta_{q}^{k}} & =-\frac{\Delta T^{2}}{2}(q+\delta q) g_{x}^{k}+\frac{\Delta T^{2}}{2}(p+\delta p) g_{y}^{k}-\Delta T g_{z}^{k}  \tag{8.27}\\
\frac{\partial g_{x}^{k+1}}{\partial \delta_{r}^{k}} & =-\frac{\Delta T^{2}}{2}(r+\delta r) g_{x}^{k}+\Delta T g_{y}^{k}+\frac{\Delta T^{2}}{2}(p+\delta p) g_{z}^{k} \tag{8.28}
\end{align*}
$$

Similarly, the scalar equation of $g_{y}$ is differentiated with respect to each state variable to form the second row of $\boldsymbol{\Phi}$. A row is created for each state variable to create the entire $\boldsymbol{\Phi}$ matrix.

The observations are the six normalised vector components so that the measurement matrix is linear and time invariant. Hence:

| Roll | Pitch | Yaw | $\delta_{p}$ | $\delta_{q}$ | $\delta_{r}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(d e g)$ | $(d e g)$ | $(d e g)$ | $(d e g / s e c)$ | $(d e g / s e c)$ | $(d e g / s e c)$ |
| 0.2250 | 0.2768 | 0.2640 | 0.005134 | 0.002363 | 0.007834 |

Table 8.1: Attitude EKF algorithm 1 results. Standard deviation of errors in attitude and rate offset estimates.

$$
\boldsymbol{H}=\left[\begin{array}{lllllllll}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0  \tag{8.29}\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}\right]
$$

## Attitude EKF Algorithm One Results

The algorithm was implemented as an S-function block in the Eagle simulation. The inertial inputs were simulated with the same lag and filtering present in the actual Eagle IMU. Constant gyroscope offsets of $\delta_{p}=-0.2^{\circ} / \mathrm{sec}, \delta_{q}=0.5^{\circ} / \mathrm{sec}$ and $\delta_{r}=1.2^{\circ} / \mathrm{sec}$ were applied. Gaussian noise of standard deviations $0.1 \mathrm{~m} / \mathrm{s}^{2}$, $0.05^{\circ} / \mathrm{sec}$ was added to the accelerometers and gyroscope rates respectively. The EKF was updated at 50 Hz .

The simulation was run for 100 seconds. The reference position was changed by a $2 m$ amplitude square wave of period 20 seconds and $50 \%$ duty cycle in both $x$ and $y$ axes. The results for algorithm 1 are shown in Figure 8.1. Convergence can be seen to take place in about 30 seconds. The estimates of attitude show an offset initially which results from the error in the gyroscope offset estimates. As the gyroscope offsets converge, the error in estimated attitude approaches zero-mean.

Noting that the EKF would normally have been allowed to settle before use, the section of data corresponding to the offsets converging was not included in calculating the performance of the algorithm. Instead, the last 50 seconds were used. The error standard deviation for each axis and offset is summarised in table 8.2.1.

### 8.2.2 EKF Algorithm Two

A practical solution often used in Inertial Navigation Systems is to combine the best features of both approaches: a fast attitude algorithm for computing attitude combined with an EKF for predicting the error model [211]. If the EKF fails for some reason, the fast attitude alogorithm can continue to function with zero or frozen error estimates, to provide a still functioning attitude system albeit in a degraded mode. This approach is tested in the second algorithm, which uses a seven element


Figure 8.1: EKF algorithm one results


Figure 8.2: Complimentary filter architecture
state vector consisting of a four-variable quaternion and the three gyroscope offsets.
The observations for this algorithm are the quaternion produced by the fast attitude algorithm. Because the observation and representation of attitude are in quaternion form, the correction is non-linear. This means that noisy quaternion observations would result in erroneous corrections due to the linearity assumption being violated. To prevent this from happening, the quaternion observations are taken from another simple filter arrangement as shown in Figure 8.2. The simple filter used in this chapter, was the same one used on the Eagle and described in Section 4.7.

The purpose of the low-pass filter is two fold. Firstly, the filter acts to smooth out noise in the offset values. Secondly, the filter acts to reduce system hunting. As the EKF changes its estimate of the gyroscope offsets, the attitude error of the simple filter changes. This causes a perceived rotation at the EKF input, which is not correlated to a change in gyroscope rate, causing the EKF to change its estimate of gyroscope offset. In turn, the change in offset estimate again causes the attitude error in the simple filter to change and the cycle repeats. The feedback between gyroscope offsets and observed attitude can cause oscillatory behavior if the rate of change is not limited. The state vector for this algorithm is:

$$
\boldsymbol{x}=\left[\begin{array}{lllllll}
q_{0} & q_{1} & q_{2} & q_{3} & \delta_{p} & \delta_{q} & \delta_{r} \tag{8.30}
\end{array}\right]^{T}
$$

The state update equation for this algorithm is based on the following first order approximation to Equation (8.13):

$$
\begin{align*}
q_{0}^{k+1} & =q_{0}^{k}-\frac{\Delta T}{2}\left(p+\delta_{p}\right) q_{1}^{k}-\frac{\Delta T}{2}\left(q+\delta_{q}\right) q_{2}^{k}-\frac{\Delta T}{2}\left(r+\delta_{r}\right) q_{3}^{k} \\
q_{1}^{k+1} & =q_{1}^{k}-\frac{\Delta T}{2}\left(p+\delta_{p}\right) q_{0}^{k}+\frac{\Delta T}{2}\left(q+\delta_{q}\right) q_{3}^{k}-\frac{\Delta T}{2}\left(r+\delta_{r}\right) q_{2}^{k} \\
q_{2}^{k+1} & =q_{2}^{k}-\frac{\Delta T}{2}\left(p+\delta_{p}\right) q_{3}^{k}-\frac{\Delta T}{2}\left(q+\delta_{q}\right) q_{0}^{k}-\frac{\Delta T}{2}\left(r+\delta_{r}\right) q_{1}^{k} \\
q_{3}^{k+1} & =q_{3}^{k}+\frac{\Delta T}{2}\left(p+\delta_{p}\right) q_{4}^{k}-\frac{\Delta T}{2}\left(q+\delta_{q}\right) q_{4}^{k}-\frac{\Delta T}{2}\left(r+\delta_{r}\right) q_{4}^{k} \\
\delta_{p}^{k+1} & =\delta_{p}^{k}, \delta_{q}^{k+1}=\delta_{q}^{k}, \delta_{r}^{k+1}=\delta_{r}^{k}, \tag{8.31}
\end{align*}
$$

|  | $\delta_{p}$ | $\delta_{q}$ | $\delta_{r}$ |
| :---: | :---: | :---: | :---: |
| Raw | $0.01657^{\circ} /$ sec | $0.02123^{\circ} /$ sec | $0.01755^{\circ} /$ sec |
| Filtered | $0.007746^{\circ} /$ sec | $0.006431^{\circ} /$ sec | $0.006557^{\circ} /$ sec |

Table 8.2: EKF algorithm 2 - standard deviation of errors in raw and filtered rate offset estimates.

| Roll | Pitch | Yaw |
| :---: | :---: | :---: |
| $0.3221^{\circ}$ | $0.4053^{\circ}$ | $0.3745^{\circ}$ |

Table 8.3: EKF algorithm 2 - standard deviation of errors in attitude.

Using the same process used for algorithm one the $\boldsymbol{\Phi}$ matrix can be derived. The measurement matrix $\boldsymbol{H}$ is:

$$
\boldsymbol{H}=\left[\begin{array}{lllllllll}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0  \tag{8.32}\\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

## Attitude EKF Algorithm Two Results

The second algorithm was tested using the same sensor errors and helicopter motions as for algorithm one. The results for algorithm two are shown in Figure 8.3. Convergence can be seen to have taken place in about 50 seconds, but the raw prediction of offsets is very noisy and not useable without filtering. As the rate of change of the gyroscope offsets is known to be very slow, heavy low pass filtering is possible. The black lines in Figure 8.3 show the filtered values of gyroscope offset after passing through a 2 -pole Butterworth filter with a time constant of 50 seconds.

### 8.3 Attitude EKF Discussion

Both EKFs tested showed an initial spike in the offset estimations of up to $5^{\circ} / \mathrm{sec}$. This spike occurs as the EKF is poorly converged after initialisation. This spike would not be so marked in normal operation because the gyroscope offsets would be zeroed prior to starting the EKF algorithm.

In both algorithms, the assumption is made that the average observation of the gravity vector is pointing straight down. This assumption is not true in the general case, as the vehicle is subject to g -force when maneovering and the accelerometers actually measure the sum of the earth's gravity and the vehicle acceleration. Hence, if the helicopter was placed in a sustained banked turn, the gravity vector measured by the accelerometers would, in fact, be tilted away from the direction of turn. Using either algorithm would result in a level attitude estimate despite the angle of bank.


Figure 8.3: EKF algorithm two results

The same errors will arise whenever the helicopter accelerates along one of its axes. For example, if the helicopter was to accelerate forwards and stay level, the pitch attitude estimate would indicate a nose-up attitude that did not exist. One solution to this problem would be to make use of GPS to determine the absolute accelerations, so that the GPS acceleration could be subtracted from the accelerometer reading before it was used as an observation [213-215].

The best performing filter in terms of estimate accuracy is algorithm one. This nine state algorithm uses a linear correction step, which gives it the greatest advantage in terms of robustness. This algorithm is the recommended choice for embedded application in the autopilot.

### 8.4 Velocity and Height Estimation

Optic flow provides the helicopter with observations of longitudinal and lateral velocity, $u$ and $v$. A measurement of height above terrain $(\mathcal{Z})$ may be obtained from a laser rangefinder, a downwards looking stereo camera, or by comparing GPS altitude with a known terrain height. Additional inertial data from accelerometers and gyroscopes can be used to update the velocity estimates. In order for stable control of height to be achieved, one unmeasured state, vertical velocity $w$, is required. The obvious choice of algorithm for fusing these sensor data and predicting the unmeasured state is the Extended Kalman Filter.

The accelerometer offset in the Eagle IMU can be mostly eliminated using accurate thermal calibration as each accelerometer is fitted with an on-chip temperature sensor. However, this may not be the case with all other IMU implementations. In addition, offset errors may still arise from errors in attitude due to IMU misalignment during installation. The effect of neglecting these errors is a biased velocity estimate. In the case of the optic flow damped hover controller, such a bias would cause the helicopter to continually drift in one direction. The solution to this problem may be to use an EKF to predict the biases in $x, y$ and $z$ acceleration measurements. The seven state representation chosen for the EKF, provided at Equation (8.33), includes the velocity estimates, acceleration offsets and height.

$$
\boldsymbol{x}=\left[\begin{array}{lllllll}
u & v & w & \delta_{x} & \delta_{y} & \delta_{z} & \mathcal{Z} \tag{8.33}
\end{array}\right]
$$

For a small enough time step, the state update equations can be written directly from the rigid body equations of motion using a first order approximation as:

$$
\begin{align*}
u^{k+1} & =u^{k}+\left(a_{x}+\delta_{x}+g B_{11}-q w+r v\right) \Delta T \\
v^{k+1} & =v^{k}+\left(a_{y}+\delta_{y}+g B_{12}-r u+p w\right) \Delta T \\
w^{k+1} & =w^{k}+\left(a_{z}+\delta_{z}+g B_{13}-p v+q u\right) \Delta T \\
\mathcal{Z}^{k+1} & =\mathcal{Z}^{k}+\left(B_{31} u+B_{32} v+B_{33} w\right) \Delta T \\
\delta_{x}{ }^{k+1} & =\delta_{x}{ }^{k} \\
\delta_{y}{ }^{k+1} & =\delta_{y}{ }^{k} \\
\delta_{z}{ }^{k+1} & =\delta_{z}{ }^{k} \tag{8.34}
\end{align*}
$$

The state transition matrix is defined by taking derivatives of the state update equations with respect to each state. The resulting state transition matrix is given by Equation (8.35).

$$
\boldsymbol{\Phi}=\left[\begin{array}{ccccccc}
1 & r \Delta T & -q \Delta T & \Delta T & 0 & 0 & 0  \tag{8.35}\\
-r \Delta T & 1 & p \Delta T & 0 & \Delta T & 0 & 0 \\
q \Delta T & -p \Delta T & 1 & 0 & 0 & \Delta T & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
B_{31} \Delta T & B_{32} \Delta T & B_{33} \Delta T & 0 & 0 & 0 & 1
\end{array}\right]
$$

The observations are the height $Z$ and the optic flows $Q_{x}$ and $Q_{y}$ in the longitudinal and lateral directions respectively. For a downwards looking camera, the optic flow is approximated by $Q_{x}=u / Z$ and $Q_{y}=v / Z$ so that the observation matrix $\boldsymbol{H}$ can be written as in Equation (8.36).

$$
\boldsymbol{H}=\left[\begin{array}{ccccccc}
\frac{1}{\mathcal{Z}} & 0 & 0 & 0 & 0 & 0 & -\frac{u}{\mathcal{Z}^{2}}  \tag{8.36}\\
0 & \frac{1}{Z} & 0 & 0 & 0 & 0 & -\frac{v}{\mathcal{Z}^{2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 1.0
\end{array}\right]
$$

## Velocity and Height Estimation EKF Results

The EKF was run as an S-function block inside the Eagle simulation. Gaussian noise was used to simulate errors in the sensors. Noise with standard deviations of $5.2^{\circ} / \mathrm{sec}$ for $Q_{x}$ and $9.5^{\circ} /$ sec for $Q_{y}$ were assumed (see Section 6.3 for an explanation). The height error measured by DGPS was assumed to have a standard deviation of 4 cm . The noise in the accelerometers due to vibration were based on the observed variance in accelerometers during hover after digital filtering. A conservative value of $0.2 \mathrm{~m} / \mathrm{s}^{2}$ standard deviation in acceleration noise was used for each axes. The diagonal of the measurement noise matrix $\boldsymbol{R}$ was set equal to the variances of the corresponding optic flow and height noise. Process noise was adjusted by trial and error in simulation to obtain a good compromise between the speed of convergence and filter stability.


Figure 8.4: Optic Flow and inertial EKF sensor fusion results. Note: The dotted lines on the bias chart represent the actual accelerometer biases.

| $V_{x}$ | $V_{y}$ | $V_{z}$ | $\mathcal{Z}$ | $\delta_{x}$ | $\delta_{y}$ | $\delta_{z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{~m} / \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ | $(\mathrm{m} / \mathrm{s})$ | $(\mathrm{m})$ | $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| 0.0023 | 0.0032 | 0.0037 | 0.0160 | 0.012 | 0.012 | 0.018 |

Table 8.4: Standard deviations of attitude error and accelerometer bias estimates

The simulated accelerometer biases were set to $\delta_{x}=0.3 \mathrm{~m} / \mathrm{s}^{2}, \delta_{y}=-0.4 \mathrm{~m} / \mathrm{s}^{2}$ and $\delta_{z}=-0.8 \mathrm{~m} / \mathrm{s}^{2}$. The simulated helicopter was subjected to a step in desired position of 5 m in $x$ and $y$ position, and 2 m in height at 20 seconds after the simulation was started. The results for 50 seconds are shown in Figure 8.4. Convergence of the biases takes place in less than 10 seconds.

Table 8.4 shows the standard deviations of the attitude errors and accelerometer bias estimates, calculated from the period of flight after the first 10 seconds when convergence is occuring. The errors in velocity and height, after convergence, are negligible and would be suitable for use in precision control of the helicopter in hover. The accelerometer biases are found to within $0.02 \mathrm{~m} / \mathrm{s}^{2}$ of their true values.

### 8.5 Summary

In this chapter, a number of means for estimating sensor offsets have been described. These techniques are not biologically inspired but could be used to augment an otherwise biologically inspired sensing and control system. Unfortunately, due to time constraints, flight testing of these EKF algorithms was not achieved but indications are that they would have worked on a real platform.

In future work, it may be possible to estimate the sensor error offsets using artificial neural networks instead of EKF, which would reduce computation time and may help to ensure stability. Future work on sensor fusion should include combining the attitude and velocity estimates into a single EKF. This would allow the errors determined from one sensing mode to be used to improve the estimates of the other. For example, the accelerometer biases found from optic flow could be used to reduce offset errors in the attitude.

## Chapter 9

## Conclusions and Recommendations

### 9.1 Summary of Achievements

In this thesis, techniques have been demonstrated for sensing and control for an autonomous helicopter in hover and forward flight using techniques inspired by biology. Optic flow combined with some measure of attitude from basic MEMS inertials was shown to be sufficient to contain the drift of a helicopter in hover, and to control the height of a helicopter in forward flight.

The use of the $I^{2} A$ algorithm for computing optic flow is robust in an outdoor environment and has been shown to work sufficiently well for precision flight control over unprepared surfaces such as tarmac, concrete and grass. The $I^{2} A$ algorithm appears to be more robust than classical machine vision techniques that involve feature tracking, owing to its ability to match over complete image patches rather than relying on identifying and correlating specific features.

I have presented a new technique for control using neural networks. Flight tests have confirmed that a neural network control system can achieve similar results to an optimal trajectory controller with orders of magnitude greater computational requirement. This makes a platform with complex open loop dynamics controllable using simple computational hardware. I have also shown that sensor fusion of visual and inertial information can be achieved using simple neural networks.

### 9.2 Areas for Future Study

### 9.2.1 Control of Flight using Optic Flow

Throughout the thesis other sensing modes have been combined with optic flow to determine either range given speed or speed given range. The problem of how vision might be used in a standalone mode without a secondary sensing mode has not been tackled. For example, is it possible to use optic flow to determine height above terrain when there is no available measure of ground speed? Preliminary experimental work by Baird and Boeddeker [216] at the Australian National University suggest that bees deliberately impart a lateral sinusoidal displacement of known amplitude upon their flight path in order to generate optic flow in a defined way (a notion
suggested on theoretical grounds in 1993 by Srinivasan et al [217]). As the velocities corresponding to the lateral wiggle are a function of the amplitude, the bees would be able to deduce range from the lateral optic flow signal and thereby control their height. Apparently, bees can also regulate their flight speed based on the longitudinal optic flow signal [218], which explains why bees can maintain a constant flight speed regardless of headwinds and tailwinds which may reach $50 \%$ of the bee's forward flight speed [219]. With range known from lateral optic flow and forward speed known from longitudinal optic flow, the bees are able to regulate both speed and height using the one sensory system.

One could imagine such an optic flow sensing scheme being implemented on a helicopter. An open loop oscillation in attitude could be applied about the trim roll reference attitude. Optic flow ranging from the assumed open loop lateral velocity would be used to control height. Sideways drift due to wind could be estimated and corrected based on the mean lateral optic flow over a complete lateral oscillation. This scheme could be used equally to control hover and forward flight of the helicopter. In the case of hover, the controller would simply act to maintain zero longitudinal optic flow. In forward flight, the controller would aim to achieve constant longitudinal optic flow, corresponding to the desired ground speed.

### 9.2.2 Computation of Optic Flow

In this thesis, optic flow was calculated with PC hardware implemented either on the ground using telemetry or using onboard PC104 flight computers. The control by telemetry approaches suffer problems in that telemetry degrades the quality of the imagery and may fail completely when line of sight is lost. Considerable research is underway towards developing autonomy for Micro Air Vehicles (MAV), which are defined by DARPA as flight vehicles with a maximum dimension of less than 15 cm [220]. PC104 hardware is comparatively heavy and much too heavy and power consuming for implementation on an MAV. A number of researchers have proposed using custom made integrated circuits as autopilot devices with optic flow sensing built in $[221-224]$. Ultimately, this approach would lead to a lightweight system, but the expense and long turn-around time is prohibitive in the short term.

Recent unpublished research at UNSW@ADFA has shown that use of the $I^{2} A$ on an FPGA can result in data at up to 50 frames per second, which is fast enough for flight control. FPGAs are integrated circuits containing millions of logic gates, which can be reconfigured through software to form any desired digital processing network. Due to their internal architecture, FPGA chips can process data in a massively parallel way rather than the sequential fashion normally present in computers, enabling real-time processing of high-bandwidth visual information. Some other groups $[225,226]$ have attempted optic flow on FPGA, but so far this has been done using slower algorithms than $I^{2} A$ and heavier electronics implementations, not suited for real-time control of a MAV. Research into development of an FPGA based optic flow system using $I^{2} A$ may result in a combined vision sensor and autopilot on a single chip which be suited to installation on an MAV payload.

### 9.2.3 Control of Position using Vision

One of the major problems of using optic flow for stabilisation in hover is that a near perfect hover will result in a very weak optic flow signal because motions are small. Further, optic flow does not provide for identification of landmark features that can be used to maintain station. There is some evidence that bees make use of a stored 2D snapshot to locate themselves [227, 228]. Cartwright and Collett [227] suggest that the direction in which the bees move at any moment is governed by the discrepancy between the snapshot and its current retinal image. A new algorithm is therefore proposed that uses a stored image of the ground, a snapshot, so to speak, taken of the ground directly under the helicopter. This snapshot would form a visual anchor point for the helicopter. By comparing subsequent frames with this snapshot using the $I^{2} A$ algorithm, it should be possible to calculate absolute translation from the datum.

For a practical implementation, sufficient overlap between the snapshot and the current image is neccessary. This means that the helicopter could not stray more than about 20 percent of the helicopter height away from where the snapshot was taken. Whenever the original image moves outside the image catchment area, a new snapshot would have to be taken. New snapshots might also be taken from time to time to compensate for long term environmental changes. To compensate for perspective distortion of the image as the craft moves, it may be necessary to transform the snapshot image according to the estimated position shift since the image was captured, prior to applying the interpolation. This should provide for greater accuracy and stability of the algorithm, although it is much more computationally demanding. A system using a stored snapshot may also need to compensate for changes in the lighting conditions that occur naturely, such as when the sun becomes occluded by a cloud. Work by Stürzl and Zeil [229], suggests that contrast normalisation may achieve this aim. A 2D version of the snapshot algorithm would need to make use of a secondary means of determining height above the ground, such as stereo vision. A 3D version of the snapshot hover might also be possible based on calculating the expansion and contraction of the image as the helicopter climbed and descended.

### 9.2.4 Control of Flight using ANN

In this thesis, sensor fusion using ANNs was demonstrated for the lateral and longitudinal velocities. The approach could easily be extended to the vertical motion of the helicopter and to the estimation of position. Future work might also yield methods for computing the sensor errors using ANN. For example, given optic flow and inertial data, the ANN might be able to be trained to predict the accelerometer offsets as well as the velocities. The estimates might also be improved by combining a feed-forward estimate of the vehicle motion, based on an ANN plant model provided with the control input history.

The use of ANN could be extended to flight regimes other than vertical flight. The dynamics of a helicopter change significantly at moderate to high forward flight
speeds. Having an additional forward flight speed input to the ANN controller would allow it to account for the effect of forward flight speed. An alternative strategem would be to allow the ANN to adapt to changes in flight conditions online.

### 9.3 Concluding Remarks

Vision provides a low-cost option for navigation and control of small vehicles which has many advantages over competing technologies such as GPS and laser-rangefinding. Visual sensing clearly plays a dominant part in the flight control of flying insects providing a strong suggestion for artificial flying machines to garner inspiration from biology. There are still issues of robustness to be tackled to make visually guided platforms practical in cluttered outdoor environments but there is certainly much promise that this is achievable.

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## Appendix A

## Summary of Simulation Model Parameters

| Parameter | Description | Value |
| :---: | :---: | :---: |
| $a_{m r}$ | Main rotor blade 2D lift curve slope | 5.7 |
| $a_{t r}$ | Tail rotor blade 2D lift curve slope | 4.0 |
| $A_{l a t}$ | Lateral cyclic to main rotor pitch ratio | -0.17 rad/ms |
| $B_{\text {lon }}$ | Longitudinal cyclic to main rotor pitch ratio | -0.19 rad/ms |
| $C_{\text {lon }}$ | Longitudinal cyclic to flybar pitch ratio | -1.58 rad/ms |
| $D_{l a t}$ | Lateral cyclic to flybar pitch ratio | -1.02 rad/ms |
| $c_{m r}$ | Main rotor blade chord | 0.058 m |
| $c_{t r}$ | Tail rotor blade chord | 0.026 m |
| $C_{D_{0}}^{m r}$ | Profile Drag Coefficient | 0.012 |
| $I_{x}$ | $2^{\text {nd }}$ Moment of Inertia about x-axis | $0.30 \mathrm{kgm}^{2}$ |
| $I_{y}$ | $2^{\text {nd }}$ Moment of Inertia about y -axis | $0.82 \mathrm{kgm}^{2}$ |
| $I_{z}$ | $2^{\text {nd }}$ Moment of Inertia about z-axis | $0.40 \mathrm{kgm}^{2}$ |
| $I_{x z}$ | Product of Inertia | $-0.01 \mathrm{kgm}^{2}$ |
| $k_{i} n d$ | Induced power correction factor | 1.2 |
| $K_{s}$ | Flybar to main rotor pitch mixing ratio | 0.8 |
| $K_{\beta}$ | Rotor Spring | $270 \mathrm{~N} / \mathrm{m}$ |
| $m$ | All up weight of helicopter | 8.2 kg |
| $N$ | Number of blades | 2 |
| $R$ | Rotor diameter | 0.76 m |
| $S_{\text {fus }}^{x}$ | Fuselage equivalent flat plate area in x-direction | $0.025 \mathrm{~m}^{2}$ |
| $S_{\text {fus }}^{y}$ | Fuselage equivalent flat plate area in y-direction | $0.084 \mathrm{~m}^{2}$ |
| $S_{\text {fus }}^{z}$ | Fuselage equivalent flat plate area in z-direction | $0.027 \mathrm{~m}^{2}$ |
| $\kappa$ | Profile drag power correction factor | 4.7 |
| $\Omega_{m r}$ | Main rotor angular velocity | $157.1 \mathrm{rad} / \mathrm{sec}$ |
| $\Omega_{t r}$ | Tail rotor angular velocity | $829.0 \mathrm{rad} / \mathrm{sec}$ |

## Eagle Simulation Subsytems



Figure B.1: Main simulation


Figure B.2: Dynamics simulation


Figure B.3: Controller


Figure B.4: Outer loop controller


Figure B.5: PID controller


Figure B.6: Servo subsystem


Figure B.7: Sensor subsystem


Figure B.8: IMU subsystem


Figure B.9: Optic flow sensor subsystem


Figure B.10: GPS sensor subsystem


Figure B.11: Display subsystem

## Appendix C

## Calibration Procedures

## C.0.1 Accelerometer Calibration

A scheme was devised to find the accelerometer offset, gain and alignment using the known acceleration of gravity. As the acceleration due to gravity has a known magnitude and direction, it provides a convenient reference load for calibration purposes. The scheme involves placing the IMU in six different orientations and measuring the accelerometer outputs in each orientation. Each orientation scenario involves making one of the IMU axes parallel to vertical. This is acheived using a flat reference surface which is levelled accurately using a precision spirit level with an accuracy of $0.1^{\circ}$. The reference plane used is a granite workbench with a precision ground surface. A special mechanical adaptor was made so that the IMU could be placed with each of its six faces parallel to the reference plane, including the face on which the connecting fittings were protruding. The six scenarios are numbered from (1) to (6) as follows:

- case 1: IMU resting on on its front face - gravity vector aligned with IMU positive x -axis.
- case 2 : IMU resting on its back face - gravity vector aligned with IMU negative x -axis.
- case 3 : IMU resting on its right hand face - gravity vector aligned with IMU positive y -axis.
- case 4: IMU resting on its left hand face - gravity vector aligned with IMU negative $y$-axis.
- case 5 : IMU resting in its normal orientation - gravity vector aligned with IMU positive z -axis.
- case 6 : IMU resting upside down on its top face - gravity vector aligned with IMU negative z-axis.

For each scenario, the output of each accelerometer is recorded after averaging for a few seconds. The output of each accelerometer depends on the gain, alignment and offset of the sensor. If the sensor is stationary, the only acceleration being measured is the local gravity vector. If the $i^{t h}$ accelerometer is aligned with the unit
vector $\hat{\boldsymbol{\zeta}}_{i}=\zeta_{i x} \hat{i}+\zeta_{i y} \hat{j}+\zeta_{i z} \hat{k}$, then the output $U_{i j}$ of the $i^{\text {th }}$ accelerometer for the $j^{\text {th }}$ scenario can be expressed by Equation (C.1).

$$
\begin{align*}
U_{i j} & =k_{i} \boldsymbol{\zeta}_{i} \cdot \boldsymbol{g} \\
& =|\boldsymbol{g}|\left(\zeta_{i x} g_{j x}+\zeta_{i y} g_{j y}+\zeta_{i z} g_{j z}\right)+\Upsilon_{i} \tag{C.1}
\end{align*}
$$

where $\boldsymbol{g}$ is the acceleration due to gravity, $k_{i}$ is the accelerometer sensor gain and $\Upsilon_{i}$ is the accelerometer offset.

Since the IMU is placed on a flat surface which is normal to gravity, for each scenario, only one of $g_{j x}, g_{j y}$ and $g_{j z}$ will be non-zero and equal to unity. This provides a means of finding the offset for each accelerometer, assuming that the offset does not change significantly during the calibration. The latter condition can be enforced by performing the calibration in a constant temperature environment and allowing the IMU to reach thermal equilibrium from internal heating before the calibration takes place. Consider the $i^{\text {th }}$ accelerometer subject to placement on each face:

$$
\begin{align*}
U_{i 1} & =k_{i} \zeta_{1 x}|\boldsymbol{g}|+\Upsilon_{i}  \tag{C.2}\\
U_{i 2} & =-k_{i} \zeta_{1 x}|\boldsymbol{g}|+\Upsilon_{i}  \tag{C.3}\\
U_{i 3} & =k_{i} \zeta_{1 y}|\boldsymbol{g}|+\Upsilon_{i}  \tag{C.4}\\
U_{i 4} & =-k_{i} \zeta_{1 y}|\boldsymbol{g}|+\Upsilon_{i}  \tag{C.5}\\
U_{i 5} & =k_{i} \zeta_{1 z}|\boldsymbol{g}|+\Upsilon_{i}  \tag{C.6}\\
U_{i 6} & =-k_{i} \zeta_{1 z}|\boldsymbol{g}|+\Upsilon_{i} \tag{C.7}
\end{align*}
$$

By adding consecutive equations above (e.g. adding Equation (C.2) to (C.3)), the offset $\Upsilon_{i}$ can be found. For better accuracy, the offset can be found from making use of all of the data as in Equation (C.8).

$$
\begin{equation*}
\Upsilon_{i}=\left(U_{i 1}+U_{i 2}+U_{i 3}+U_{i 4}+U_{i 5}+U_{i 6}\right) / 6 \tag{C.8}
\end{equation*}
$$

The offsets for all accelerometers can be found by repeating the process for each accelerometer. The gains of each accelerometer can be found next by subtracting consecutive equations, resulting in Equations (C.2-C.4). For example, subtracting Equation (C.3) from equation (C.2) eliminates the offset, resulting in Equation (C.9).

$$
\begin{align*}
& k_{i} \zeta_{i x}=\frac{U_{i 1}-U_{i 2}}{2|\boldsymbol{g}|}  \tag{C.9}\\
& k_{i} \zeta_{i y}=\frac{U_{i 3}-U_{i 4}}{2|\boldsymbol{g}|}  \tag{C.10}\\
& k_{i} \zeta_{i z}=\frac{U_{i 5}-U_{i 6}}{2|\boldsymbol{g}|} \tag{C.11}
\end{align*}
$$

By squaring both sides of Equations (C.9-C.11) and then adding the resultant terms, we arrive at Equation (C.12).

$$
\begin{equation*}
k_{i}^{2}\left(a_{i x}^{2}+a_{i y}^{2}+a_{i z}^{2}\right)=\left(\frac{U_{i 1}-U_{i 2}}{2|\boldsymbol{g}|}\right)^{2}+\left(\frac{U_{i 3}-U_{i 4}}{2|\boldsymbol{g}|}\right)^{2}+\left(\frac{U_{i 5}-U_{i 6}}{2|\boldsymbol{g}|}\right)^{2} \tag{C.12}
\end{equation*}
$$

But since the $a_{i j}$ terms are unit vectors, $a_{i x}^{2}+a_{i y}^{2}+a_{i z}^{2}=1$ and thus the gain can be found from Equation C.13.

$$
\begin{equation*}
k_{i}=\sqrt{\left(\frac{U_{i 1}-U_{i 2}}{2|\boldsymbol{g}|}\right)^{2}+\left(\frac{U_{i 3}-U_{i 4}}{2|\boldsymbol{g}|}\right)^{2}+\left(\frac{U_{i 5}-U_{i 6}}{2|\boldsymbol{g}|}\right)^{2}} \tag{C.13}
\end{equation*}
$$

Finally the alignment of the accelerometers can be found from Equations (C. 9 C.11), simply by dividing by the appropriate gains. This leads to the following set of equations:

$$
\begin{align*}
\zeta_{i x} & =\frac{U_{i 1}-U_{i 2}}{2 k_{i}|\boldsymbol{g}|}  \tag{C.14}\\
\zeta_{i y} & =\frac{U_{i 3}-U_{i 4}}{2 k_{i}|\boldsymbol{g}|}  \tag{C.15}\\
\zeta_{i z} & =\frac{U_{i 5}-U_{i 6}}{2 k_{i}|\boldsymbol{g}|} \tag{C.16}
\end{align*}
$$

Once the offset, gain and alignment of the accelerometers have been determined, the data must be converted into a form that can be used to provide the actual accelerations from the sensor outputs. In general use, the accelerometer outputs $U_{i}$ for the $i^{\text {th }}$ accelerometer can be expressed in terms of the actual accelerations [ $a_{x} a_{y} a_{z}$ ] in matrix form:

$$
\left[\begin{array}{c}
U_{1}  \tag{C.17}\\
U_{2} \\
U_{3}
\end{array}\right]=\left[\begin{array}{lll}
k_{1} \zeta_{1 x} & k_{1} \zeta_{1 y} & k_{1} \zeta_{1 z} \\
k_{2} \zeta_{2 x} & k_{2} \zeta_{2 y} & k_{2} \zeta_{3 z} \\
k_{3} \zeta_{3 x} & k_{3} \zeta_{3 y} & k_{3} \zeta_{3 z}
\end{array}\right]\left[\begin{array}{c}
a_{x} \\
a_{y} \\
a_{z}
\end{array}\right]+\left[\begin{array}{c}
\Upsilon_{1} \\
\Upsilon_{2} \\
\Upsilon_{3}
\end{array}\right]
$$

Equation (C.17) can be re-arranged to give Equation (C.18) which is implemented on the helicopter to calculate the accelerations. The matrix inversion is calculated offline as part of the calibration process. Software was written by the author to prompt the operator through the calibration process and gather the $U_{i j}$ values. This software then determines the inverted alignment matrix, offsets $\left(\Upsilon_{i}\right)$ and the reciprocal of the gains $\left(1 / k_{i}\right)$ in a form which can be readily transferred into non-volatile memory on the helicopter.

$$
\left[\begin{array}{l}
a_{x}  \tag{C.18}\\
a_{y} \\
a_{z}
\end{array}\right]=\left[\begin{array}{lll}
\zeta_{1 x} & \zeta_{1 y} & \zeta_{1 z} \\
\zeta_{2 x} & \zeta_{2 y} & \zeta_{3 z} \\
\zeta_{3 x} & \zeta_{3 y} & \zeta_{3 z}
\end{array}\right]^{-1}\left[\begin{array}{c}
\left(U_{1}-\Upsilon_{1}\right) / k_{1} \\
\left(U_{2}-\Upsilon_{2}\right) / k_{2} \\
\left(U_{3}-\Upsilon_{3}\right) / k_{3}
\end{array}\right]
$$

## C.0.2 Gyroscope Calibration

A system was developed by the author for calibrating gyroscopes using an accurate rotating turntable. A precision rate-of-turn table manufactured by Genisco Technology Corporation was acquired which is capable of rotation rates between $1^{\circ} / \mathrm{sec}$ and $3,200^{\circ} / \mathrm{sec}$ with a rate accuracy of $0.1 \%$ or $0.05^{\circ} / \mathrm{sec}$, whichever is greater. The calibration procedure determines the gain and alignments of the gyroscopes. The offset of the gyroscopes is found from a zeroing process on startup, based on the assumption that the helicopter is not moving when the IMU is first turned on. Dynamic estimation of the gyroscope offsets using an EKF is also described in Chapter 8.

The procedure involves fixing the IMU on the turntable and rotating at a number of known speeds within the range of the sensor. For each speed, several thousand gyroscope rate samples are recorded and averaged. This process is repeated for each axis by placing each of three orthogonal faces of the IMU flush with the turntable plate. Three scenarios are executed:

- case 1: IMU resting on on its front face - axis of rotation aligned with IMU positive x -axis.
- case 2: IMU resting on its right hand face - axis of rotation aligned with IMU positive $y$-axis.
- case 3: IMU resting in its normal orientation - axis of rotation aligned with IMU positive z-axis.

A line of best fit is made for each gyroscope for the sensor output versus speed. The slope of the line is the derivative $\mathcal{K}_{i j}=d U_{i j} / d|\boldsymbol{\Omega}|$. Figure C. 1 shows the data for one axis of the IMU.

The output of each gyroscope depends on the gain and alignment. The gyros are zeroed before each calibration run to ensure that no offset is present due to thermal drift. If the $i^{\text {th }}$ gyroscope is aligned with the unit vector $\zeta_{i}=\zeta_{i x} \hat{i}+\zeta_{i y} \hat{j}+\zeta_{i z} \hat{k}$, then the slope $\mathcal{K}_{i j}$ of the $i^{\text {th }}$ gyroscope for the $j^{\text {th }}$ scenario can be expressed by Equation (C.19).

$$
\begin{equation*}
\mathcal{K}_{i j}=k_{i}\left(\zeta_{i x} \Omega_{x}+\zeta_{i y} \Omega_{y}+\zeta_{i z} \Omega_{z}\right) \tag{C.19}
\end{equation*}
$$

where $k_{i}$ is the gain of the gyroscope. Since each IMU face is placed normal to the axis of rotation, for each scenario, only one of $\Omega_{x}, \Omega_{y}$ and $\Omega_{z}$ will be non-zero and equal to unity. The slopes for all three gyroscopes for all three scenarios can be expressed in matrix form as shown in Equation (C.20).

$$
\left[\begin{array}{lll}
\mathcal{K}_{11} & \mathcal{K}_{12} & \mathcal{K}_{13}  \tag{C.20}\\
\mathcal{K}_{21} & \mathcal{K}_{22} & \mathcal{K}_{23} \\
\mathcal{K}_{31} & \mathcal{K}_{32} & \mathcal{K}_{33}
\end{array}\right]=\left[\begin{array}{lll}
k_{1} \zeta_{1 x} & k_{1} \zeta_{1 y} & k_{1} \zeta_{1 z} \\
k_{2} \zeta_{2 x} & k_{2} \zeta_{2 y} & k_{2} \zeta_{2 z} \\
k_{3} \zeta_{3 x} & k_{3} \zeta_{3 y} & k_{1} \zeta_{3 z}
\end{array}\right]
$$

Noting that $\zeta_{i x}^{2}+\zeta_{i y}^{2}+\zeta_{i z}^{2}=1$, we can now derive the gains from Equation (C.20):


Figure C.1: Results of rotating IMU about the x-axis. Solid lines represent the lines of best fit to the data.

$$
\begin{align*}
k_{1} & =\sqrt{\mathcal{K}_{11}^{2}+\mathcal{K}_{12}^{2}+\mathcal{K}_{13}^{2}}  \tag{C.21}\\
k_{2} & =\sqrt{\mathcal{K}_{21}^{2}+\mathcal{K}_{22}^{2}+\mathcal{K}_{23}^{2}}  \tag{C.22}\\
k_{3} & =\sqrt{\mathcal{K}_{31}^{2}+\mathcal{K}_{32}^{2}+\mathcal{K}_{33}^{2}} \tag{C.23}
\end{align*}
$$

And once the gains are known, the alignment coefficients can be found from:

$$
\left[\begin{array}{lll}
\zeta_{1 x} & \zeta_{1 y} & \zeta_{1 z}  \tag{C.24}\\
\zeta_{2 x} & \zeta_{2 y} & \zeta_{2 z} \\
\zeta_{3 x} & \zeta_{3 y} & \zeta_{3 z}
\end{array}\right]=\left[\begin{array}{ccc}
\mathcal{K}_{11} / k_{1} & \mathcal{K}_{12} / k_{1} & \mathcal{K}_{13} / k_{1} \\
\mathcal{K}_{21} / k_{2} & \mathcal{K}_{22} / k_{2} & \mathcal{K}_{23} / k_{2} \\
\mathcal{K}_{31} / k_{3} & \mathcal{K}_{32} / k_{3} & \mathcal{K}_{33} / k_{3}
\end{array}\right]
$$

Now, the output of the $i^{\text {th }}$ gyroscope, $U_{i}$ can be written in terms of the actual rotation rate $[p q r]$ and sensor parameters in matrix form:

$$
\left[\begin{array}{c}
U_{1}  \tag{C.25}\\
U_{2} \\
U_{3}
\end{array}\right]=\left[\begin{array}{lll}
k_{1} \zeta_{1 x} & k_{1} \zeta_{1 y} & k_{1} \zeta_{1 z} \\
k_{2} \zeta_{2 x} & k_{2} \zeta_{2 y} & k_{2} \zeta_{2 z} \\
k_{3} \zeta_{3 x} & k_{3} \zeta_{3 y} & k_{3} \zeta_{3 z}
\end{array}\right]\left[\begin{array}{c}
p \\
q \\
r
\end{array}\right]
$$

Finally, Equation (C.25) can be re-arranged to give the equations implemented on the helicopter to find the rotation rates $[p q r]$ from the gyroscope measurements:

$$
\left[\begin{array}{c}
p  \tag{C.26}\\
q \\
r
\end{array}\right]=\left[\begin{array}{lll}
\zeta_{1 x} & \zeta_{1 y} & \zeta_{1 z} \\
\zeta_{2 x} & \zeta_{2 y} & \zeta_{2 z} \\
\zeta_{3 x} & \zeta_{3 y} & \zeta_{3 z}
\end{array}\right]^{-1}\left[\begin{array}{c}
U_{1} / k_{1} \\
U_{2} / k_{2} \\
U_{3} / k_{3}
\end{array}\right]
$$

## C.0.3 Magnetometer Calibration

The magnetometer parameters to be determined by calibration are offset, sensor gain and alignment. Unlike gravity, the variation of the magnetic field over the earth's surface is significant. Indeed, the earth's magnetic field vector has a horizontal and vertical component. The angle between the field vector and vertical is known as magnetic dip and varies from $0^{\circ}$ at the equator to $90^{\circ}$ at the North and South magnetic poles. In the Canberra region, the local magnetic dip is approximately $65^{\circ}$ such that the magnetic field vector actually has a stronger vertical component than horizontal component. A primary objective when designing the calibration process was to develop an algorithm that could be executed without apriori knowledge of the local magnetic field strength, direction or dip angle. This would simplify the process and make it less susceptible to unexpected variations in the local field properties.

The procedure developed involves rotating the IMU about each of its three axes and recording the outputs from each magnetometer for each of the three cases. The resulting data is in the form of three sine waves for each magnetometer. By considering the magnitude and DC offset of each sine wave, it is possible to determine the alignment and gain of each sensor. The calibration apparatus constructed for magnetometer calibration is shown in Figure C.2. The apparatus consists of a stepper motor with 400 steps per revolution which drives a mounting pedestal for the IMU via a long shaft. The stepper motor is controlled through the parallel port of a PC. The same PC is interfaced to the IMU via an RS232 cable connected to a spare serial port. The shaft is rotated in steps of $0.9^{\circ}$ at a time. After each step, a delay is enforced by the calibration software to allow transients due to the analog filters to settle. A number of samples are taken at each step for each magnetometer, averaged and then recorded on the PC. Once data for one complete revolution is recorded a least squares method is used to fit a sine wave to the data. The amplitude and DC offset for each sine wave is stored.

The shaft and mounting bracket is made of non-ferrous metals to preserve the earth's natural magnetic field properties. Figure C. 3 is a graph showing the typical variation of a single magnetometer as it is rotated about each axis of the IMU. Calibration is conducted outdoors away from large metal objects, power conductors and at a minimum height above soil.

We begin our analysis by defining unit vectors $\hat{i}, \hat{j}, \hat{k}$ aligned with the calibration apparatus rotational axis, such that $\hat{i}$ is parallel to the axes of the machine, $\hat{j}$ is horizontal to the right when looking from the motor side of the device and $\hat{k}$ is pointing vertically down in accordance with the right hand rule. The earth's magnetic field can then be expressed in terms of components $\lambda_{x}, \lambda_{y}$ and $\lambda_{z}$ along these unit vector directions:


Figure C.2: Magnetometer calibration apparatus


Figure C.3: Magnetometer calibration data

$$
\begin{equation*}
\boldsymbol{b}=|\boldsymbol{b}|\left(\lambda_{x} \hat{i}+\lambda_{y} \hat{j}+\lambda_{z} \hat{k}\right) \tag{C.27}
\end{equation*}
$$

The output of each magnetometer depends on the alignment of the sensor with the earth's local magnetic field and the gain of the sensor. If the $i^{t} h$ magnetometer is aligned with the unit vector $\hat{\boldsymbol{m}}_{i}=m_{i x} \hat{i}+m_{i y} \hat{j}+m_{i z} \hat{k}$, then the output $U_{i}$ of the $i^{\text {th }}$ magnetometer can be expressed by the dot product:

$$
\begin{align*}
U_{i} & =k_{i} \boldsymbol{m}_{i} \cdot \boldsymbol{b} \\
& =|\boldsymbol{b}|\left(m_{i x} \lambda_{x}+m_{i y} \lambda_{y}+m_{i z} \lambda_{z}\right) \tag{C.28}
\end{align*}
$$

The orientation unit vectors for the magnetometer $m_{i x}, m_{i y}, m_{i z}$ are numerically equal to the cosine of the included angles between the $i^{\text {th }}$ magnetometer and the IMU $x, y$ and $z$ reference axes.

Now consider the IMU oriented so that its reference $x$-axis is aligned with the shaft axis whilst the other axes lie in an arbitrary orientation. To aid the determination of the sense of the magnetometer axes, the $\lambda_{x}$ axes of the calibration rig can be pointed roughly North. By pointing the rig somewhere between, say NE and NW, we can then safely assume that the value of $\lambda_{x}$ is positive and not a small number. This only requires a very rudimentary knowledge of the local geography. If the IMU is now rotated an angle $\psi$ about the shaft, the orientation of the magnetometer axes will be transformed to the new unit vector $\hat{\boldsymbol{m}}_{i}^{\prime}$ according to Equation (C.29).

$$
\begin{equation*}
\hat{\boldsymbol{m}}_{i}^{\prime}=\left[\boldsymbol{\mathcal { R }}_{\psi}\right] \hat{\boldsymbol{m}}_{i} \tag{C.29}
\end{equation*}
$$

where

$$
\left[\boldsymbol{\mathcal { R }}_{\psi}\right]=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{C.30}\\
0 & \cos (\psi) & -\sin (\psi) \\
0 & \sin (\psi) & \cos (\psi)
\end{array}\right]
$$

The output of the $i^{\text {th }}$ magnetometer is therefore:

$$
\begin{align*}
U_{i}= & k_{i}|\boldsymbol{b}|\left[m_{i x} \lambda_{x}+\cos (\psi) m_{i y} \lambda_{y}-\sin (\psi) m_{i z} \lambda_{y}+\right. \\
& \left.\sin (\psi) m_{i y} \lambda_{z}+\cos (\psi) m_{i z} \lambda_{z}\right] \\
= & k_{i}|\boldsymbol{b}|\left[m_{i x} \lambda_{x}+\cos (\psi)\left(m_{i y} \lambda_{y}+m_{i z} \lambda_{z}\right)+\right. \\
& \left.\sin (\psi)\left(m_{i y} \lambda_{z}-m_{i z} \lambda_{y}\right)\right] \\
= & k_{i}|\boldsymbol{b}|\left[m_{i x} \lambda_{x}+A_{i x} \cos (\psi+\phi)\right] \tag{C.31}
\end{align*}
$$

where $A_{i x}$ and $\phi$ are constants. Clearly as the shaft is rotated the output of the sensor traces out a sinusoidal path with a mean value of $\overline{U_{i x}}=k_{i}|\boldsymbol{b}|\left(m_{i x} \lambda_{x}\right)$ and a magnitude of A. The mean value is equal to the product of the $\hat{i}$ component of the magnetometer and the $\hat{i}$ component of the earth's magnetic field. Through a similar process, it can be shown that the same will hold true for rotating the magnetometer
about any of the axes making up the box. Hence for the $i^{t h}$ sensor, rotated about each axis of the box,

$$
\begin{align*}
& \overline{U_{i x}}=k_{i}|\boldsymbol{b}| m_{i x} \lambda_{x} \\
& \overline{U_{i y}}=k_{i}|\boldsymbol{b}| m_{i y} \lambda_{x} \\
& \overline{U_{i z}}=k_{i}|\boldsymbol{b}| m_{i z} \lambda_{x} \tag{C.32}
\end{align*}
$$

Note that only the component of the magnetic field aligned with the shaft affects the average value of the output. The shaft axis should therefore be selected to lie in an orientation which is more parallel than perpendicular to the magnetic field, so that the magnitude of the results is optimised.

Rearranging components of Equation (C.32),

$$
\begin{equation*}
k_{i}|\boldsymbol{b}| \lambda_{x}=\frac{\bar{U}_{i x}}{m_{i x}}=\frac{\bar{U}_{i y}}{m_{i y}}=\frac{\bar{U}_{i z}}{m_{i z}} \tag{C.33}
\end{equation*}
$$

Inverting,

$$
\begin{equation*}
\frac{m_{i x}}{\overline{U_{i x}}}=\frac{m_{i y}}{\bar{U}_{i y}}=\frac{m_{i z}}{\bar{U}_{i z}} \tag{C.34}
\end{equation*}
$$

Since $m_{i x}{ }^{2}+m_{i y}{ }^{2}+m_{i z}{ }^{2}=1$,

$$
\begin{align*}
\frac{1}{m_{i x}} & =\sqrt{1+\left(\frac{\bar{U}_{i y}}{\bar{U}_{i x}}\right)^{2}+\left(\frac{\bar{U}_{i z}}{\bar{U}_{i x}}\right)^{2}} \\
\frac{1}{m_{i y}} & =\sqrt{1+\left(\frac{\bar{U}_{i x}}{\bar{U}_{i y}}\right)^{2}+\left(\frac{\bar{U}_{i z}}{\bar{U}_{i y}}\right)^{2}} \\
\frac{1}{m_{i z}} & =\sqrt{1+\left(\frac{\bar{U}_{i x}}{\bar{U}_{i z}}\right)^{2}+\left(\frac{\bar{U}_{i y}}{\bar{U}_{i z}}\right)^{2}} \tag{C.35}
\end{align*}
$$

The equations in (C.35) can be used to determine the magnitude of the magnetometer unit vectors, but not the sense. Also, numeric difficulties can be experienced if either $\bar{U}_{i x}, \bar{U}_{i y}$ or $\bar{U}_{i z}$ are small numbers. Thus a more robust strategy is to choose the equation from (C.35) with the largest denominator and use this to find one of the unit vector magnitudes. Equation (C.33) is then used to find the remaining unit vectors. The sense of the magnitude can also be found from Equation (C.33) based on the convention that $k_{i}|\boldsymbol{b}| \lambda_{x}$ is a positive number.

The IMU can be oriented with any of its three axes aligned with the axis of rotation of the calibration apparatus. The pedestal of the rig has a face which is normal to the shaft, so that by clamping one of the faces of the IMU flush to the pedestal, one of the IMU axes can be aligned with the rotational axes. We denote the magnitude of the varying part of the sine wave as $A_{i x}, A_{i y}, A_{i z}$, for rotations about the IMU $x, y$ and $z$ axes respectively. The magnitude of the varying part of Equation (C.31), resulting from rotating the IMU about its x -axis is:

$$
\begin{align*}
A_{i x}^{2} & =k_{i}^{2}|\boldsymbol{b}|^{2}\left(m_{i y} \lambda_{y}+m_{i z} \lambda_{z}\right)^{2}+\left(m_{i y} \lambda_{z}-m_{i z} \lambda_{y}\right)^{2} \\
& =k_{i}^{2}|\boldsymbol{b}|^{2}\left(m_{i y}^{2}+m_{i z}^{2}\right)\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right) \\
& =k_{i}|\boldsymbol{b}|^{2}\left(1-m_{i x}^{2}\right)\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right) \tag{C.36}
\end{align*}
$$

The same derivation applied to each axis in turn results in the following set of three equations:

$$
\begin{align*}
A_{i x}^{2} & =k_{i}^{2}|\boldsymbol{b}|^{2}\left(1-m_{i x}^{2}\right)\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right) \\
A_{i y}^{2} & =k_{i}^{2}|\boldsymbol{b}|^{2}\left(1-m_{i y}^{2}\right)\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right) \\
A_{i z}^{2} & =k_{i}^{2}|\boldsymbol{b}|^{2}\left(1-m_{i z}^{2}\right)\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right) \tag{C.37}
\end{align*}
$$

The gain $k_{i}$ for each magnetometer may now be calculated by combining Equations (C.32) and (C.37). Consider first the Equation (C.36) for rotation about the IMU x-axis:

$$
\begin{equation*}
k_{i}^{2}|\boldsymbol{b}|^{2}\left(\lambda_{y}^{2}+\lambda_{z}^{2}\right)=\frac{A_{i x}^{2}}{1-m_{i x}^{2}} \tag{C.38}
\end{equation*}
$$

And from equation C.32:

$$
\begin{equation*}
k_{i}^{2}|\boldsymbol{b}|^{2} \lambda_{x}^{2}=\left(\frac{\bar{U}_{i x}}{m_{i x}}\right)^{2} \tag{C.39}
\end{equation*}
$$

Adding Equations (C.38) and (C.39),

$$
\begin{equation*}
k_{i}^{2}|\boldsymbol{b}|^{2}\left(\lambda_{x}^{2}+\lambda_{y}^{2}+\lambda_{z}^{2}\right)=\left(\frac{\bar{U}_{i x}}{m_{i x}}\right)^{2}+\left(\frac{A_{i x}^{2}}{1-m_{i x}^{2}}\right) \tag{C.40}
\end{equation*}
$$

Since $\lambda_{x}^{2}+\lambda_{y}^{2}+\lambda_{z}^{2}=1$

$$
\begin{equation*}
k_{i}=\frac{1}{|\boldsymbol{b}|} \sqrt{\left(\frac{\bar{U}_{i x}}{m_{i x}}\right)^{2}+\left(\frac{A_{i x}^{2}}{1-m_{i x}^{2}}\right)} \tag{C.41}
\end{equation*}
$$

Alternately the gain of each magnetometer can be found from the IMU rotations about the IMU y and z axes. The resulting Equations are:

$$
\begin{align*}
& k_{i}=\frac{1}{|\boldsymbol{b}|} \sqrt{\left(\frac{\bar{U}_{i x}}{m_{i x}}\right)^{2}+\left(\frac{A_{i y}^{2}}{1-m_{i y}^{2}}\right)}  \tag{C.42}\\
& k_{i}=\frac{1}{|\boldsymbol{b}|} \sqrt{\left(\frac{\bar{U}_{i x}}{m_{i x}}\right)^{2}+\left(\frac{A_{i z}^{2}}{1-m_{i z}^{2}}\right)} \tag{C.43}
\end{align*}
$$

The final step in the calibration process is to develop a calibration matrix to convert the raw output of the magnetometers into components of the relative field vector which are properly aligned with the helicopter body axes. This matrix is
found by inverting the matrix of unit vectors and gains. Since the output $U_{i}$ of the 3 magnetometers is given by,

$$
\left[\begin{array}{c}
U_{1}  \tag{C.44}\\
U_{2} \\
U_{3}
\end{array}\right]=\left[\begin{array}{lll}
k_{1} & k_{2} & k_{3}
\end{array}\right]\left[\begin{array}{lll}
m_{1 x} & m_{1 y} & m_{1 z} \\
m_{2 x} & m_{2 y} & m_{2 z} \\
m_{3 x} & m_{3 y} & m_{3 z}
\end{array}\right]\left[\begin{array}{c}
B_{x} \\
B_{y} \\
B_{z}
\end{array}\right]=[K]^{T}[M] \boldsymbol{b}
$$

Then the magnetic field vector can be determined from Equation (C.45).

$$
\left[\begin{array}{c}
B_{x}  \tag{C.45}\\
B_{y} \\
B_{z}
\end{array}\right]=\left[\begin{array}{lll}
\frac{1}{k_{1}} & \frac{1}{k_{2}} & \frac{1}{k_{3}}
\end{array}\right]\left[\begin{array}{lll}
m_{1 x} & m_{1 y} & m_{1 z} \\
m_{2 x} & m_{2 y} & m_{2 z} \\
m_{3 x} & m_{3 y} & m_{3 z}
\end{array}\right]^{-1}\left[\begin{array}{c}
U_{1} \\
U_{2} \\
U_{3}
\end{array}\right]
$$

## C.0.4 Thermal Calibration

All of the sensors in the IMU were found to drift with temperature. The accelerometers and the Analog Devices gyroscopes have on-chip temperature sensors. The Murata gyroscopes do not have built-in temperature sensors and hence are subject to unchecked thermal drift. Initially, it was hoped to use the temperature sensors built into the accelerometers. Unfortunately, experiments have shown that, during even moderate rates of temperature change, significant variation in temperature between adjacent sensors can occur. For this reason, the on-chip accelerometer temperature sensors are only really useful for calibrating the accelerometers.

Calibration of the sensors is carried out by cooling the sensors down to approximately $0^{\circ} \mathrm{C}$ and then warming them to about $50^{\circ} \mathrm{C}$ over a period of one hour. The variation in sensor output while at rest during these temperature changes was recorded and a second order polynomial fit was made to the data using a least squares approximation. The resulting polynomials were incorporated into the helicopter control software to dynamically correct for temperature variations in flight. A second order correction polynomial for one of the accelerometers is shown as a fit to thermal drift samples at Figure C.4.


Figure C.4: Accelerometer thermal calibration

