

Spin entanglement, decoherence and Bohm's EPR paradox

E. G. Cavalcanti¹, P. D. Drummond², H. A. Bachor³, M. D. Reid²

¹Centre for Quantum Dynamics, Griffith University, Brisbane QLD 4111, Australia,

²ARC Centre of Excellence for Quantum-Atom Optics, Swinburne University of Technology,
Melbourne VIC 3122, Australia,

³ARC Centre of Excellence for Quantum-Atom Optics, Building 38, The Australian National
University, Canberra ACT 0200, Australia

pdrummond@swin.edu.au

Abstract: We obtain criteria for entanglement and the EPR paradox for spin-entangled particles and analyse the effects of decoherence caused by absorption and state purity errors. For a two qubit photonic state, entanglement can occur for all transmission efficiencies. In this case, the state preparation purity must be above a threshold value. However, Bohm's spin EPR paradox can be achieved only above a critical level of loss. We calculate a required efficiency of 58%, which appears achievable with current quantum optical technologies. For a macroscopic number of particles prepared in a correlated state, spin entanglement and the EPR paradox can be demonstrated using our criteria for efficiencies $\eta > 1/3$ and $\eta > 2/3$ respectively. This indicates a surprising insensitivity to loss decoherence, in a macroscopic system of ultra-cold atoms or photons.

© 2009 Optical Society of America

OCIS codes: (270.5585) Quantum information and processing; (270.6570) Squeezed states; (020.1475) Bose-Einstein condensates.

References and links

1. A. Einstein, B. Podolsky and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," *Phys. Rev.* **47**, 777-780 (1935).
2. J. S. Bell, "On the Einstein-Podolsky-Rosen paradox," *Physics* **1**, 195-201 (1965).
3. A. Peres, "Separability Criterion for Density Matrices," *Phys. Rev. Lett.* **77**, 1413-1415 (1996).
4. M. Horodecki, P. Horodecki, and R. Horodecki, "Mixed-State Entanglement and Distillation: Is there a "Bound" Entanglement in Nature?," *Phys. Rev. Lett.* **80**, 5239-5242 (1998).
5. K. Wootters, "Entanglement of Formation of an Arbitrary State of Two Qubits," *Phys. Rev. Lett.* **80**, 2245-2248 (1998).
6. G. Vidal and R. F. Werner, "Computable measure of entanglement," *Phys. Rev. A* **65**, 032314 (2002).
7. T. Yu and J. H. Eberly, "Sudden Death of Entanglement," *Science* **323**, 598-601, (2009).
8. W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," *Rev. Mod. Phys.* **75**, 715-775 (2003).
9. T. Yu and J. H. Eberly, "Finite-Time Disentanglement Via Spontaneous Emission," *Phys. Rev. Lett.* **93**, 140404 (2004).
10. Z. Ficek and R. Tanas, "Dark periods and revivals of entanglement in a two-qubit system," *Phys. Rev. A* **74**, 024304 (2006).
11. M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, "Environment-Induced Sudden Death of Entanglement," *Science* **316**, 579-582 (2007).
12. A. M. Steane, "Error Correcting Codes in Quantum Theory," *Phys. Rev. Lett.* **77**, 793-797 (1996).
13. C. H. Bennett et. al., "Mixed-state entanglement and quantum error correction," *Phys. Rev. A* **54**, 3824-3851 (1996).
14. P. W. Shor, "Scheme for reducing decoherence in quantum computer memory," *Phys. Rev. A* **52**, R2493- R2496 (1995).

15. D. Bohm, *Quantum Theory* (Englewood Cliffs, NJ, Prentice-Hall, 1951).
16. J. Larsson, "Modeling the singlet state with local variables," *Phys. Lett. A* **256**, 245-252 (1999).
17. R. F. Werner, "Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model," *Phys. Rev. A* **40**, 4277-4281 (1989).
18. M. Barberi, F. De Martini, G. Di Nepi, and P. Matalon, "Generation and Characterization of Werner States and Maximally Entangled Mixed States by a Universal Source of Entanglement," *Phys. Rev. Lett.* **92**, 177901 (2004).
19. P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, "New High-Intensity Source of Polarization-Entangled Photon Pairs," *Phys. Rev. Lett.* **75**, 4337-4341 (1995).
20. A. Fedrizzi, T. Herbst, A. Poppe, T. Jennewein, and A. Zeilinger, "A wavelength-tunable fiber-coupled source of narrowband entangled photons," *Opt. Express* **15**, 15377-15386 (2007).
21. A. Garg and N. D. Mermin, "Detector inefficiencies in the Einstein-Podolsky-Rosen experiment," *Phys. Rev. D* **35**, 3831-3835 (1987).
22. B. Yurke and D. Stoler, "Generating quantum mechanical superpositions of macroscopically distinguishable states via amplitude dispersion," *Phys. Rev. Lett.* **57**, 13-16 (1986).
23. P. Skwara, H. Kampermann, M. Kleinmann, and D. Bruss, "Entanglement witnesses and a loophole problem," *Phys. Rev. A* **76**, 012312 (2007).
24. A. Aspect, J. Dalibard, and G. Roger, "Experimental Test of Bell's Inequalities Using Time-Varying Analyzers," *Phys. Rev. Lett.* **49**, 1804-1807 (1982), among others.
25. H. F. Hofmann and S. Takeuchi, "Violation of local uncertainty relations as a signature of entanglement," *Phys. Rev. A* **68**, 032103 (2003).
26. M. D. Reid, "Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification," *Phys. Rev. A* **40**, 913-923 (1989).
27. E. G. Cavalcanti and M. D. Reid, "Uncertainty relations for the realisation of macroscopic quantum superpositions and EPR paradoxes," *Journ. Mod. Opt.* **54**, 2373-2380 (2007).
28. G. Toth, "Entanglement detection in optical lattices of bosonic atoms with collective measurements," *Phys. Rev. A* **69**, 052327 (2004).
29. N. Korolkova, G. Leuchs, R. Loudon, T. C. Ralph, and C. Silberhorn, "Polarization squeezing and continuous-variable polarization entanglement," *Phys. Rev. A* **65**, 052306 (2002).
30. W. P. Bowen, R. Schnabel, H.-A. Bachor, and P. K. Lam, "Polarization Squeezing of Continuous Variable Stokes Parameters," *Phys. Rev. Lett.* **88**, 093601 (2002).
31. W. H. Furry, "Note on the Quantum-Mechanical Theory of Measurement," *Phys. Rev.* **49**, 393-399 (1936).
32. W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, "Towards Quantum Superpositions of a Mirror," *Phys. Rev. Lett.* **91**, 130401 (2003).
33. N. D. Mermin, "Quantum mechanics vs local realism near the classical limit: A Bell inequality for spin s ," *Phys. Rev. D* **22**, 356-361 (1980).
34. P. D. Drummond, "Violations of Bell's Inequality in Cooperative States," *Phys. Rev. Lett.* **50**, 1407-1410 (1983).
35. M. D. Reid, W. J. Munro, and F. De Martini, "Violation of multiparticle Bell inequalities for low- and high-flux parametric amplification using both vacuum and entangled input states," *Phys. Rev. A* **66**, 033801 (2002).
36. H. M. Wiseman, S. J. Jones, and A. C. Doherty, "Steering, Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox," *Phys. Rev. Lett.* **98**, 140402 (2007).
37. D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, "Bell Inequalities for Arbitrarily High-Dimensional Systems," *Phys. Rev. Lett.* **88**, 040404 (2002).
38. W. P. Bowen, R. Schnabel, P. K. Lam, and T. C. Ralph, "Experimental Investigation of Criteria for Continuous Variable Entanglement," *Phys. Rev. Lett.* **90**, 043601 (2003).
39. B. Julsgaard, A. Kozhekin, and E. S. Polzik, "Experimental long-lived entanglement of two macroscopic objects," *Nature* **413**, 400-403 (2001).
40. J. Esteve, C. Gross, A. Weller, S. Giovanazzi, and M. K. Oberthaler, "Squeezing and entanglement in a Bose-Einstein condensate," *Nature* **455**, 1216-1219 (2008).
41. A. B. U'Ren, C. Silberhorn, K. Banaszek, and I. A. Walmsley, "Conditional preparation of single photons for scalable quantum-optical networking," *Phys. Rev. Lett.* **93**, 093601 (2004).
42. A. E. Lita, A. J. Miller, and S. W. Nam, "Counting near-infrared single-photons with 95% efficiency," *Opt. Express* **16**, 3032-3040 (2008).
43. Y. Takeno, M. Yukawa, H. Yonezawa, and A. Furusawa, "Observation of -9 dB quadrature squeezing with improvement of phase stability in homodyne measurement," *Opt. Express* **15**, 4321-4327 (2007).
44. H. Vahlbruch, M. Mehmet, S. Chelkowski, B. Hage, A. Franzen, N. Lastzka, S. Goler, K. Danzmann, and R. Schnabel, "Observation of Squeezed Light with 10-dB Quantum-Noise Reduction," *Phys. Rev. Lett.* **100**, 033602 (2008).

1. Introduction

In the development of quantum information science, entanglement is central. It is at the heart of the Einstein-Podolsky-Rosen (EPR) paradox[1] and Bell's theorem[2], which draw a clear delineation between local realistic and quantum theories. Entanglement is also considered a vital resource for future quantum technologies, both for photonic systems and for applications involving ultra-cold quantum gases.

Crucial to the generation and detection[3, 4, 5, 6] of entanglement is decoherence[7], which is the degradation of a pure state into a mixed state due to coupling with the environment. Decoherence can degrade or even destroy entanglement. Sensitivity to decoherence, particularly for large systems, is thought to explain the transition from the quantum to classical regime[8]. However, decoherence can be caused by many different physical mechanisms, including particle loss, phase errors, and mixing with uncorrelated particles.

This leads to the following fundamental question. When is entanglement and EPR correlation preserved between two N particle systems, if each is independently decohered? Yu and Eberly[9, 10] studied the issue for $N = 1$, for a certain type of decoherence, and showed that entanglement can be destroyed at a finite time. This is termed "entanglement sudden death" (ESD). The existence of ESD – which is experimentally verified[11] – has far reaching implications for quantum information, since error correcting protocols may restore a degraded but nonzero entanglement[12, 13, 14]. However, these questions have not been investigated in detail for EPR correlations, which are more sensitive to decoherence than entanglement per se.

In this communication, we obtain quantitative criteria applicable to Bohm's original two-particle spin realization[15] of the EPR state, and generalize these to $2N$ -particle states which display spin entanglement. We investigate both EPR correlations and entanglement for these correlated multi-particle states. Two distinct types of decoherence are investigated. We show that entanglement can resist decoherence even for the case of large N . To understand this feature, we distinguish between *noise* and *loss* decoherence. With *noise* decoherence, the wrong information ("up" instead of "down") is given, while *loss* decoherence causes an absence of information and broadening[16] (e.g. the changing of a qubit into a qutrit in the lossy situation of section 3) of the measured Hilbert space.

The decoherence causing entanglement sudden death is essentially *noise*. It leads to density matrices that are mixtures with random states, with state purity $R = \text{Tr} [\rho^2] < 1$. In the Werner state[17, 18], for example, the entangled two qubit Bell state is mixed with a random state, and complete disentanglement occurs once $R \leq 1/3$ [3].

Loss decoherence gives a completely different result. In the case of the photonic two qubit Bell state[18, 19, 20], used extensively in many seminal experiments and applications, loss arises from absorption of photons into the environment, and is the major source of decoherence. This loss causes an absence of a "count", the latter arising with probability η .

The key result of the present analysis is that loss in the case of a Bell state only *reduces* the entanglement, which remains detectable at all $\eta > 0$. We extend this to treat EPR correlations, which are more sensitive to decoherence. Both results are more experimentally accessible than the violation of Bell inequalities, which require high efficiencies ($\eta > .83$)[21, 16]. We also treat systems with macroscopic particle number. Such cases have significance both as fundamental tests of quantum measurement theory, and as potentially important quantum technologies for measurements both with photons and with ultra-cold bosonic atoms in correlated states.

2. Bell state entanglement with losses

We start with Bohm's gedanken-experiment — often called the Bell[2] singlet state. This could involve any particles having internal degrees of freedom, for example photons or atoms. In the laboratory, there is noise in the form of randomly polarised particles, and various forms of loss

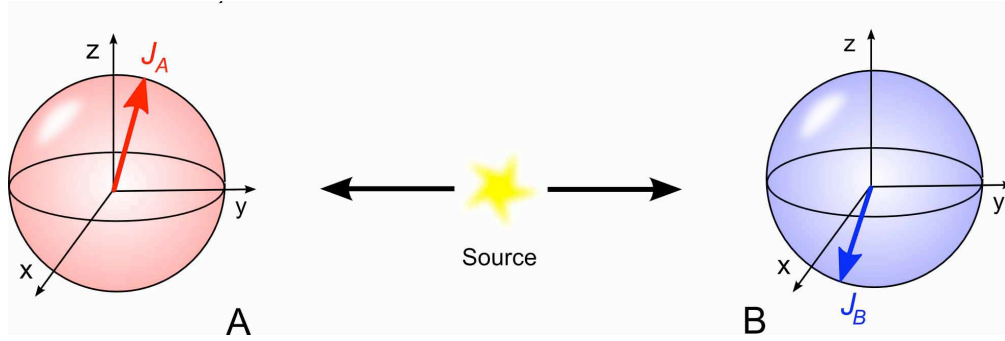


Fig. 1. Schematic diagram of Bohm's EPR experiment with correlated spins at spatially-separated locations A and B.

that cause only one or zero particles to be detected, instead of two. We describe noise using a Werner[17] state. This is composed of a singlet Bell state (Fig. 1),

$$|\Psi\rangle_S = \frac{1}{\sqrt{2}} (|1\rangle_A |-1\rangle_B - |-1\rangle_A |1\rangle_B) \quad (1)$$

with probability p , and a state with a particle of random spin at each detector, with probability $1 - p$. In the photonic case, $|\pm 1\rangle_A$ indicates a photon with positive or negative helicity detected at A, or more generally simply two distinct spin states of any quantum field. The overall Werner state is then $\rho_W = p\rho_S + (1 - p)I_4/4$. Here I_4 indicates an identity operator on the two-particle subspace of the four-mode Hilbert space. It includes all states with one particle at A, and one at B, irrespective of their spin. The purity of the Werner state is $R_W = \text{Tr}[\rho_W^2] = (3p^2 + 1)/4$.

We account for all loss that occurs prior to the measurement of the "spin" (polarization) of each particle[23], by defining the overall efficiency as η . Thus at each detector, three outcomes are possible: +1 (spin "up"); -1, (spin "down"); and 0, (no detection). The detection subspace corresponds to a qutrit. To determine the density matrix, we derive the full matrix based on a beam splitter model of loss[22], in which the initial state is represented as $\rho_W \rho_{vac}$. Here $\rho_{vac} = |0\rangle\langle 0|$ is the multimode vacuum state for four field modes ($a_{\pm,vac}$ and $b_{\pm,vac}$) that collect lost photons. Thus, we assume the standard quantum description of losses.

It is useful to adopt the Schwinger representation of the Werner state, for which $|1\rangle \equiv |1,0\rangle_{A/B}$ and $|-1\rangle_{A/B} \equiv |0,1\rangle_{A/B}$ where $|i,j\rangle_{A/B}$ means i and j quanta in two distinguishable field modes at A that have spin labels +1 and -1, and for which a_{\pm}^{\dagger} are the creation operators respectively. States at B for modes b_{\pm} are defined similarly. We call the A and B measurements Alice's and Bob's respectively.

The effect of the beam splitter model is to couple the field and vacuum modes. After loss, the modes are transformed as $a_{\pm} \rightarrow \sqrt{\eta}a_{\pm} + \sqrt{1-\eta}a_{\pm,vac}$ and $b_{\pm} \rightarrow \sqrt{\eta}b_{\pm} + \sqrt{1-\eta}b_{\pm,vac}$. We derive ρ_F , the matrix for the detected system obtained by taking the trace over the lost photon modes. The 9 basis states are in three groups, categorised as 2, 1, 0, by the number of particles: $u_{1-4} = |\pm 1\rangle_A |\pm 1\rangle_B$; $u_{5,6} = |\pm 1\rangle_A |0\rangle_B$, $u_{7,8} = |0\rangle_A |\pm 1\rangle_B$; and $u_9 = |0\rangle_A |0\rangle_B$. We find that:

$$\rho_F = \begin{bmatrix} \eta^2 \rho_W & 0 & 0 \\ 0 & (\eta/2)(1-\eta)I_4 & 0 \\ 0 & 0 & (1-\eta)^2 \end{bmatrix}. \quad (2)$$

Whether the decohered system ρ_F is entangled is readily determined using the PPT criterion[3, 4], that if a partial transpose of the density matrix has a negative eigenvalue, then

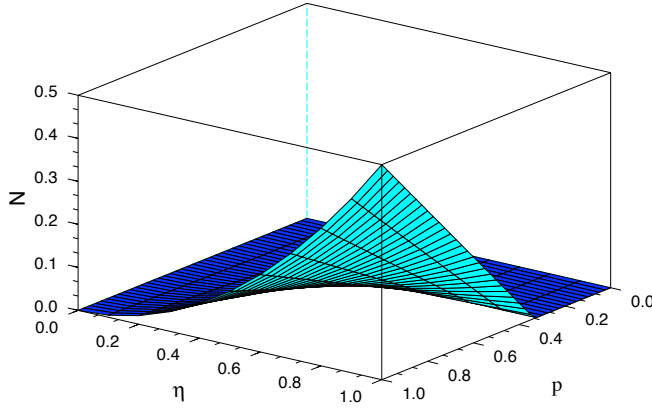


Fig. 2. Negativity of decohered Bell state to show entanglement sudden death for *noise* decoherence at $p < 1/3$, but continuous suppression of entanglement for *loss* decoherence at all η .

the state must be entangled. Calculation of the eigenvalues reveals entanglement for all $\eta > 0$ and $p > 1/3$. A plot of the negativity (magnitude of the smallest negative eigenvalue of the density matrix) $\eta^2(3p - 1)/4$ is shown in Fig. 2. This could be experimentally determined using quantum state tomography.

We emphasize that here we proceed by taking a standard quantum theoretic approach to loss. Within our quantum treatment, η incorporates the effect of all generation and detection losses. It is also possible to take a black box approach, and consider a completely unknown cause of measurement errors[23], which leads to a different efficiency threshold.

3. Projected entanglement measures

We now show that a more practical criterion to confirm entanglement is obtained using operators which project onto the subspace in which no photons are lost. Experimentally, this amounts to the procedure of measuring the “qubit” at each detector A and B only where one obtains a count at each detector. This procedure is exploited in many seminal experiments[24] that infer the properties of the photonic Bell state. We now formally validate this approach when inferring entanglement, for the case where photons are lost prior to measurement[23].

We use the Schwinger representation approach to define spin operators in terms of the particle numbers detected at each location[23]: $J_x^A = (a_+^\dagger a_- + a_-^\dagger a_+)/2$, $J_y^A = i(a_-^\dagger a_+ - a_+^\dagger a_-)/2$, $J_z^A = (a_+^\dagger a_+ - a_-^\dagger a_-)/2$. The total observed particle number operator at Alice’s location A is $N^A = a_+^\dagger a_+ + a_-^\dagger a_-$. At Bob’s location, B , J_x^B , J_y^B , J_z^B and N^B are defined in terms of b_\pm . Since $N^{A,B} = 0, 1$ these are also projectors $P^{A,B}$ on the subspace of particles detected at A, B respectively. Results for J_Z at A can be $\pm 1, 0$, where 0 is only achieved where there is no photon counted. We now suppose Alice measures the local projection $s_z^A = J_z^A P^A$. She thus defines her spin operators after projection onto the spin-1/2 subspace: $s_x^A = J_x^A P^A = (1/2)[|10\rangle\langle 01| + |01\rangle\langle 10|]_A$, $s_y^A = J_y^A P^A = (1/2i)[|10\rangle\langle 01| - |01\rangle\langle 10|]_A$, $s_z^A = (1/2)[|10\rangle\langle 10| - |01\rangle\langle 01|]_A$ and $s_A^2 = s_x^2 + s_y^2 + s_z^2 = (3/4)N^A$. Similar operators are defined for Bob.

Alice and Bob can also measure polarisations on a doubly projected subspace, corresponding to coincident counts. Their measurements then become nonlocal, given by the set

$S_{\theta}^{A|B} = J_{\theta}^A P^A P^B = s_{\theta}^A P^B$ and $S_{A|B}^2 = (3/4) P^A P^B$ where $\theta \in \{x, y, z\}$. We have already defined the full 9x9 density matrix ρ . The projected 4x4 one is $\rho_{proj} = P^A P^B \rho P^A P^B / \text{Tr}[\rho P^A P^B]$. This projected density matrix, as one might expect intuitively, is just the Werner density matrix that we started with.

We wish to prove that the projected measurements are sufficient to prove entanglement over the full density matrix. This follows using the fact that local projections or their products cannot induce entanglement[4]. Hence, a criterion sufficient for entanglement on the projected density matrix is also sufficient over the complete density matrix. As an example, we consider the entanglement criterion of Hofmann and Takeuchi[25], and define a sum of spin measurements for Alice and Bob as $s_{\theta} = s_{\theta}^A + s_{\theta}^B$, where $\theta \in \{x, y, z\}$. It is therefore sufficient to measure $\Delta^2 s_x + \Delta^2 s_y + \Delta^2 s_z < 1$ over the two-photon subspace to confirm entanglement on the full space. This is always possible[25] provided $p > 1/3$.

It is perhaps rather obvious that by losing some particles one can still retain entanglement. After all, if one has several copies of an entangled state, but some of them are lost, the resulting ensemble should still be entangled. However, it might also seem “obvious” that if one has an ensemble of entangled states and some of them are replaced by mixed states, the ensemble should still retain some entanglement. In reality this is actually false. The interest of the apparently “obvious” (and true) result about entanglement with loss must be contrasted with the falsity of the also apparently “obvious” result about noise.

Our conclusion is therefore that as far as entanglement is concerned, the effect of particle losses on the Bell state can be ignored, if one simply makes measurements conditioned on observing two-particle coincidences. This is, of course, a key difference between the two kinds of decoherence: the effect of loss can be filtered out by post-selecting the subset of measurements in which all expected detections occur, whereas this cannot be done for noise. In other words, *loss* decoherence has no effect on entanglement — there is no ESD here — while *noise* decoherence has a stronger effect, causing a decoherence threshold which is equivalent to the ESD phenomenon. In both cases, the total density matrix prior to measurement is in a mixed state caused by the decoherence.

4. Spin EPR paradox

Next, we turn to the EPR paradox. This is much more challenging experimentally. The paradox shows that local realism (LR) is inconsistent with the completeness of quantum mechanics, which is a stronger result than entanglement. As a first requirement, since EPR’s no “action-at-a-distance” is crucial to the local realism part of the EPR argument, one must have causal separation between Alice’s and Bob’s measurements. Thus, for EPR, we *must rule out the non-local procedure of projections* onto the two-photon subspace. Alice’s and Bob’s measurements must be *local*. Second, the measurements at Alice’s location must allow a local state to be inferred at Bob’s location — assuming LR. If this inferred state has a lower uncertainty than allowed by quantum mechanics, the EPR paradox is obtained. Thus LR is false (local prediction does not imply a local element of reality) *or* quantum mechanics is incomplete (it fails to fully describe the inferred state, since this violates the uncertainty principle). This logic is central to the EPR argument applied to real experiments.

It is important here to recognise the difference between the EPR and Bell arguments. Following Einstein, we explicitly assume that quantum theory correctly describes our measurements. The EPR logic does not require any alternative theory to quantum theory. It simply deals with the question of whether the completeness of quantum mechanics is compatible with local realism. Therefore, there is no need here to consider Bell’s investigation into local hidden variable theories, which may have an arbitrary treatment of loss. This means that we can use a standard quantum treatment of loss.

Our route to a signature of an unambiguous EPR paradox[26, 27] is via an inference argument together with the known quantum uncertainty principle $\Delta^2 J_x + \Delta^2 J_y + \Delta^2 J_z \geq \langle N \rangle / 2$ for spins — the same uncertainty principle used in the derivation of the entanglement criterion of [25, 28], given as

$$\Delta^2 J_x + \Delta^2 J_y + \Delta^2 J_z < \langle N^A + N^B \rangle / 2, \quad (3)$$

where $J_\theta = J_\theta^A + J_\theta^B$. Our EPR criterion simply requires that the inferred variance of Bob's measurements must be less than that for any possible quantum state; that is:

$$\Delta_{inf}^2 J_x^B + \Delta_{inf}^2 J_y^B + \Delta_{inf}^2 J_z^B < \langle N^B \rangle / 2. \quad (4)$$

Measurement schemes for all quantities in (4) have been demonstrated in recent polarisation-squeezing experiments[29, 30]. Here $\Delta_{inf}^2 J_x^B = \sum_{J_x^A} P(J_x^A) \Delta^2(J_x^B | J_x^A)$ is the average, over J_x^A , of the conditional variances $\Delta^2(J_x^B | J_x^A)$, for a measurement J_x^B given an outcome J_x^A . This inferred uncertainty is the average error associated with the inferred result for a remote measurement J_x^B , given measurement of J_x^A . To prove the EPR criterion (4), one considers the conditional distributions as predictions for B given A [26, 27]. If LR holds, the predetermined prediction for J_x^B means there is a corresponding localised state ρ_B at B . This is because if the systems are causally separated, according to LR, the measurement at B does not induce immediate change to A . EPR called such predetermined states “elements of reality”.

In the case of Bohm's EPR paradox, the assumption of LR means that elements of reality exist for each of the spins J_x^B, J_y^B, J_z^B . The variances associated with the prediction for each of them are respectively, $\Delta_{inf}^2 J_x^B, \Delta_{inf}^2 J_y^B, \Delta_{inf}^2 J_z^B$. Where we satisfy (4), EPR's elements of reality defy the quantum uncertainty relation for B . That is, it is impossible to represent Einstein's proposed element of reality as a quantum state ρ_B . In this way the EPR paradox is able to be experimentally demonstrated. This is an important conceptual boundary, which demonstrates the inadequacy of the classical concept of local realism in dealing with quantum states.

When loss is included, we find that it increases the uncertainties associated with the inference of measurements at Bob's location. As before, we take the Werner state and calculate the EPR inequality with loss included. The RHS of (4) is $\eta/2$ while $\Delta_{inf}^2 J_x^B = \Delta_{inf}^2 J_y^B = \Delta_{inf}^2 J_z^B = \eta(1 - \eta^2 p^2)/4$. The EPR criterion is then satisfied for $\eta p > 1/\sqrt{3}$. This implies that, unlike the entanglement case, both loss and noise have a similar effect on the EPR paradox. The reason is simply that the EPR paradox is related to causality. Nonlocal projections cannot be used, as in the entanglement case, to obtain a smaller ensemble for conditional measurement. This is also the same reason why one cannot use the term EPR paradox or Bell inequality unless there is a clear causal separation between the measurement events.

Although the required efficiency is greater than in any reported Bell state measurement to date, it is within reach of current photo-detectors. It would be an interesting challenge to demonstrate the EPR paradox for spatially separated, correlated particles. This would resolve Furry's question[31] about the possibility of entanglement decay for separated massive particles, which was an early proposal to resolve the EPR paradox.

5. Macroscopic EPR entanglement

Finally, we consider entanglement and EPR for macroscopic states with *more* than one particle per mode. This implies that we now consider only bosonic fields, like photons or ultra-cold BEC experiments. These correlated states would give a much more powerful test of quantum measurement theory, testing features of quantum reality in domains that become meso- or macroscopic. In this domain, a number of alternatives to quantum mechanics have been suggested, where quantum superpositions are prevented from forming via novel mechanisms such as couplings to gravitational fields[32]. If gravitational effects are involved, it seems clear that

one must test the relevant quantum predictions for massive particles, in order to allow for a strong enough gravitational coupling to occur.

To test for quantum effects in such macroscopic cases, we first consider the way in which the relevant states would be generated in practice. We consider a macroscopic version of the Bell state (1), using the Schwinger representation

$$|\psi_N\rangle = \frac{1}{N!\sqrt{N+1}}(a_+^\dagger b_-^\dagger - a_-^\dagger b_+^\dagger)^N|0\rangle. \quad (5)$$

We can generate the states of Eq. (5) using two parametric amplifiers[29, 30] as modeled by the interaction Hamiltonian

$$H = i\hbar\kappa(a_+^\dagger b_-^\dagger - a_-^\dagger b_+^\dagger) - i\hbar\kappa(a_+ b_- - a_- b_+). \quad (6)$$

With an initial vacuum state, the solution after a time t is a superposition of the $|\psi_N\rangle$. In the regime of large $\langle N^B \rangle$, higher photodiode detection efficiencies ($\eta \approx 0.9$) can be achieved, although a precise photon count, which would enable a test of Bell's inequality[33, 34, 35], is difficult. The solutions are readily obtained to give

$$a_\pm = a_\pm(0) \cosh(r) \pm b_\mp^\dagger(0) \sinh(r) \quad (7)$$

$$b_\pm = b_\pm(0) \cosh(r) \mp a_\mp^\dagger(0) \sinh(r), \quad (8)$$

where $a_\pm(0)$ represent vacuum initial states and $r = |\kappa|t$. The effect of loss is analysed using a standard beam splitter model[22] which adds vacuum terms so that final outputs after loss become $a_\pm^L = \sqrt{\eta}a_\pm + \sqrt{1-\eta}a_{\pm,0}$ and $b_\pm^L = \sqrt{\eta}b_\pm + \sqrt{1-\eta}b_{\pm,0}$ where the $a_{\pm,0}$ and $b_{\pm,0}$ represent independent vacuum inputs. With this we get

$$\langle (J_Z^A)^2 \rangle = (1/2)\eta \sinh^2(r)(1 + \eta \sinh^2(r)) \quad (9)$$

$$\langle J_Z^A J_Z^B \rangle = -(1/2)\eta^2 \cosh^2(r) \sinh^2(r), \quad (10)$$

which gives a final result for the spin uncertainties of:

$$\Delta^2 J_x = \Delta^2 J_y = \Delta^2 J_z = \eta(1 - \eta) \sinh^2(r), \quad (11)$$

and $\langle N \rangle = 4\eta \sinh^2(r)$. For all N , efficiencies $\eta > 1/3$ are enough to demonstrate entanglement.

A similar result is obtained for the spin EPR correlations. Here the minimum efficiency required to satisfy (4) approaches $\eta = 2/3$ for infinite $\langle N^B \rangle$. We calculate the conditional variances for this Gaussian system using a linear regression approach[26, 27], where the estimate for the result of the remote measurement J_x^B is simply $J_{\theta,est}^B = gJ_\theta^A$, so that the average inference variance is $\Delta_{inf}^2 J_\theta^B = \langle (J_\theta^B - gJ_\theta^A)^2 \rangle$. We calculate the linear inference variance for (6) by selecting $g = -\langle J_\theta^B J_\theta^A \rangle / \langle J_\theta^A J_\theta^A \rangle$ to minimise $\Delta_{inf}^2 J_\theta^B$:

$$\begin{aligned} \Delta_{inf}^2 J_\theta^B &= \langle (J_\theta^B)^2 \rangle - \langle J_\theta^B J_\theta^A \rangle^2 / \langle (J_\theta^A)^2 \rangle \\ &= \frac{\eta \sinh^2 r (1 - \eta^2 + 2\eta(1 - \eta) \sinh^2 r)}{2(1 + \eta \sinh^2 r)} \end{aligned} \quad (12)$$

and $\langle N^B \rangle = 2\eta \sinh^2 r$. Figure 3 plots the threshold efficiency η for satisfaction of (4), to indicate a test of macroscopic EPR for large $\langle N^B \rangle$ and $\eta > 2/3$.

Note that the calculations in this section refer to *loss* decoherence. A calculation including a model of the effect of *noise* in the entanglement of this many-particle system would of course be important before experimental realisation of this proposal. As for the two-qubit case, we expect that some finite amount of noise will lead to the elimination of entanglement as well as the EPR paradox, with the precise value depending on the type of noise affecting the system.

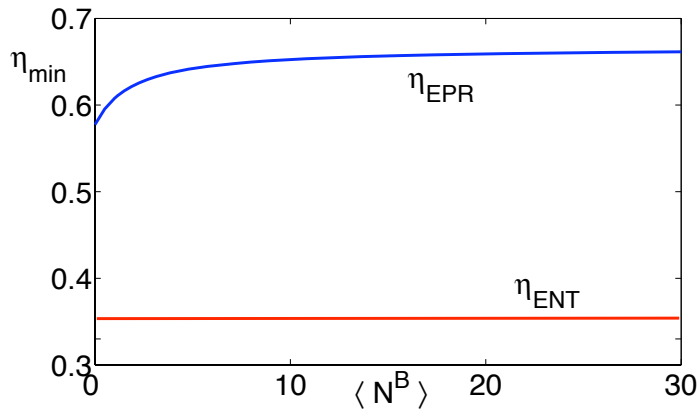


Fig. 3. (a) Threshold detection efficiency η_{min} required (by (6)) to confirm entanglement and the spin EPR paradox via (3) and (4), for a given mean photon number $\langle N^B \rangle$.

6. Conclusion

We have shown that it is possible to demonstrate two qubit entanglement for any value of loss, although ESD occurs when there is noise decoherence. Demonstrating Bohm's two qubit spin EPR paradox[15] is more difficult. With our criterion, this is only possible above a critical detection efficiency $\eta > 1/\sqrt{3}$. This is still more accessible than a loophole-free demonstration of Bell nonlocality, which has an even higher efficiency threshold.

The significance of progressively testing for stronger forms of nonlocality, from entanglement to the EPR paradox through to Bell's theorem, has been outlined recently by Wiseman *et al.*[36], who report a cut-off of the EPR paradox for Werner states, at $p \leq 0.5$.

We then progress to examine the resilience of the nonlocality of *macroscopic* systems to decoherence. We report that the entanglement and Bohm's spin EPR paradox are preserved for $\eta > 1/3$ and $\eta > 2/3$ respectively, even for higher qubit systems with arbitrarily large N . This is a surprising result that contradicts the popular view that sensitivity of entanglement to decoherence increases with the "largeness" of bodies entangled[7, 22].

Our prediction that entanglement between macroscopic (arbitrary $\langle N^B \rangle$) systems is preserved up to a large and fixed loss appears to counter previous results regarding macroscopic decoherence [8, 22, 9, 10]. Yet, the result is consistent with recent predictions for decoherence based on mixing with noisy states[37], and reports of experimental measurement of entanglement between large, lossy systems[38, 29, 30, 39]. The prediction can be tested with either photonic or massive atomic systems[32, 40], leading both to new understandings and tests of quantum mechanics, and the possibility of novel quantum technologies.

In current optical experiments the best quantum efficiency achievable is about 75%, which is typically a combination of losses in the apparatus (85%)[41], mode matching (95%) and efficiency of the detectors (95%)[42]. Further refinements, like those employed in the best optical squeezing experiments [43, 44], will bring the total efficiency closer to 90%. This is well above our calculated benchmark efficiency of 58% for an EPR test, provided other technical noise sources can be suppressed sufficiently well. In summary, an unambiguous experimental test of this EPR criterion is not impossible.

Acknowledgments

This work was funded by the Australian Research Council Center of Excellence for Quantum-Atom Optics, a Griffith University Postdoctoral Research Fellowship, an ARC Postdoctoral Research Fellowship and an ARC Professorial Fellowship .