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# 1 Saltwater-freshwater mixing fluctuation in shallow 2 beach aquifers

3 Qiang Han<sup>1, 2</sup>, Daoyi Chen<sup>1, 2</sup>, Yakun Guo<sup>2, 3</sup>, Wulong Hu<sup>4</sup>

## 4 Abstract

5 Field measurements and numerical simulations demonstrate the existence of an upper  
6 saline plume in tidally dominated beaches. The effect of tides on the  
7 saltwater-freshwater mixing occurring at both the upper saline plume and lower salt  
8 wedge is well understood. However, it is poorly understood whether the tidal driven  
9 force acts equally on the mixing behaviours of above two regions and what factors  
10 control the mixing fluctuation features. In this study, variable-density,  
11 saturated-unsaturated, transient groundwater flow and solute transport numerical  
12 models are proposed and performed for saltwater-freshwater mixing subject to tidal  
13 forcing on a sloping beach. A range of tidal amplitude, fresh groundwater flux,  
14 hydraulic conductivity, beach slope and dispersivity anisotropy are simulated. Based  
15 on time sequential salinity data, the gross mixing features are quantified by computing  
16 the spatial moments in three different aspects, namely, the centre point, length and  
17 width, and the volume (or area in a two-dimensional case). Simulated salinity  
18 distribution varies significantly at saltwater-freshwater interfaces. Mixing

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19 characteristics of the upper saline plume greatly differ from those in the salt wedge for  
20 both the transient and quasi-steady state. The mixing of the upper saline plume largely  
21 inherits the fluctuation characteristics of the sea tide in both the transverse and  
22 longitudinal directions when the quasi-steady state is reached. On the other hand, the  
23 mixing in the salt wedge is relatively steady and shows little fluctuation. The  
24 normalized mixing width and length, mixing volume and the fluctuation amplitude of  
25 the mass centre in the upper saline plume are, in general, one-magnitude-order larger  
26 than those in the salt wedge region. In the longitudinal direction, tidal amplitude, fresh  
27 groundwater flux, hydraulic conductivity and beach slope are significant control  
28 factors of fluctuation amplitude. In the transverse direction, tidal amplitude and beach  
29 slope are the main control parameters. Very small dispersivity anisotropy (e.g.,  
30  $\alpha_L/\alpha_T < 5$ ) could greatly suppress mixing fluctuation in the longitudinal direction.  
31 This work underlines the close connection between the sea tides and the upper saline  
32 plume in the aspect of mixing, thereby enhancing understanding of the interplay  
33 between tidal oscillations and mixing mechanisms in tidally dominated sloping beach  
34 systems.

35 *Keywords: Coastal aquifer; Saltwater-freshwater mixing fluctuation; Tidal effects;*  
36 *Mixing quantification; Upper saline plume; Subterranean estuary*

## 37 1 Introduction

38 Submarine groundwater discharge (SGD) represents an important transport  
39 pathway for terrestrial nutrients, carbon, metals, anthropogenic substances and  
40 persistent organic pollutants into the coastal ocean [Burnett *et al.*, 2001; Moore, 2010].  
41 Discharge from unconfined near-shore aquifers to the ocean is a significant  
42 component of SGD, including discharges of fresh groundwater, recirculating seawater,  
43 and mixed salt and fresh-water. These discharges are dynamic processes, controlled  
44 by tidal fluctuations, wave set-up, storm events, winds, seasonal changes and density

45 dependent flow along the saltwater-freshwater mixing zone [Guo *et al.*, 2009; Santos  
46 *et al.*, 2012; Xin *et al.*, 2015; Itugha *et al.*, 2016]. Saltwater-freshwater mixing  
47 processes have been proven to accelerate chemical flux from aquifers to the ocean,  
48 alter geochemical conditions, change habitat species and affect biogeochemical  
49 reactions in aquifers [Slomp and Van Cappellen, 2004; Anschutz *et al.*, 2009; Moore,  
50 2010; Charbonnier *et al.*, 2013].

51 Saltwater-freshwater mixing in beach aquifers has long been thought to occur  
52 mainly in the salt wedge (see Figure 1) dispersion zone [Cooper *et al.*, 1964;  
53 Robinson *et al.*, 2007a; Pool *et al.*, 2014]. However, integrated field salinity  
54 measurements [Robinson *et al.*, 1998; Urish and McKenna, 2004; Michael *et al.*,  
55 2005; Robinson *et al.*, 2007b; Vandenbohede and Lebbe, 2006, 2007; Abarca *et al.*,  
56 2013; Hesis and Michael, 2014] and electrical resistivity tomography profiles [Turner  
57 and Acworth, 2004; Morrow *et al.*, 2010; Befus *et al.*, 2013; Buquet *et al.*, 2016] in  
58 intertidal regions, confirm the existence of another important mixing zone, the upper  
59 saline plume (USP) (see Figure 1), where saltwater-freshwater mixing is faster than  
60 that in the classical salt wedge. USP is an inverted structure with dense saltwater  
61 above light freshwater in shallow beach aquifers [Ataie-Ashtiani *et al.*, 1999;  
62 Boufadel, 2000; Mango *et al.*, 2004]. In the USP, pore-water has faster flow rates and  
63 significantly lower transit times than that in the dispersion zone of the classical salt  
64 wedge [Robinson *et al.*, 2007a]. Therefore, this plume represents a potentially more  
65 dynamic zone for mixing and reaction than the salt wedge dispersion zone and may  
66 play a crucial role in geochemical transformation and coastal ecosystems [Charette  
67 and Sholkovitz, 2002; Moore, 2010].

68 Four essential conditions, namely, large enough oceanic oscillations, suitable  
69 terrestrial fresh groundwater flux, appropriate sloping intertidal topography, and  
70 moderate heterogeneity, are considered to be responsible for the formation of USP.  
71 Evans and Wilson [2016] noted that the development of an USP under a beach  
72 requires high rates of recirculation to create strong salinity gradients. This requires the  
73 infiltration of sufficient volumes of seawater into the beach aquifer and necessitates

74 that the groundwater flush through the beach is not too rapid. Tides, which generate  
75 the main oceanic oscillations, have three main possible effects on saline recirculation  
76 in a beach: (i) tidal forcing making seawater intrude inland during flood phases and  
77 making brackish water percolate during ebb phases [Nielsen, 1990; Li et al., 1997;  
78 Cartwright et al., 2004]; (ii) widening the zone of dispersion [Ataie-Ashtiani et al.,  
79 1999; Li et al., 2008; Kuan et al., 2012]; and (iii) causing seawater to infiltrate into a  
80 beach directly from wave run-up [Kang et al., 1995; Bakhtyar et al., 2013; Geng and  
81 Boufadel, 2015]. The slope of a beach would intensify (e.g., a flatter slope) or weaken  
82 (e.g., a steeper slope) the infiltration of saltwater [Ataie-Ashtiani et al., 1999; Li et al.,  
83 2008]. High heterogeneity of a sloping beach will increase both the spatial  
84 connectivity and effective permeability in a porous beach to reduce the degree of  
85 mixing as well as the extent of USP [Fiori and Jankovic, 2012].

86 Tidal fluctuation will cause hydraulic head fluctuation and pore water salinity  
87 oscillation [Erskine, 1991; Abdollahi-Nasab et al., 2010; Hesis and Michael, 2014;  
88 Elad et al., 2017]. Tide-induced hydraulic gradients can result in a transient increase  
89 of the solute transfer rate more than 20 times higher than the average rate in the  
90 aquifers that undergo saltwater intrusion [Li et al., 1999] and likely contribute to  
91 fluctuations in submarine seepage rates [Burnett et al., 2002]. However, the transient  
92 behaviour of solute migration and the associated mixing processes under highly  
93 variable groundwater flow have remained largely unexplored so far. Several studies  
94 have been conducted to investigate some aspects of tidal fluctuations on the mixing  
95 patterns in shallow unconfined coastal aquifers. Numerical studies have concluded  
96 that salinity distribution does not fluctuate significantly over the tidal cycle  
97 [Ataie-Ashtiani et al., 1999; Mao et al., 2006; Robinson et al., 2007a; Pool et al.,  
98 2014]. However, the conceptual models suggest that the width of the freshwater  
99 discharge tunnel (FDT) contracts and expands over the semi-diurnal period [Barry  
100 and Parlange, 2004; Urish and McKenna, 2004], which is supported by laboratory  
101 experiments and field investigation [Boufadel, 2000; Robinson et al., 2006, Shalev et  
102 al., 2009; Kuan et al., 2012, Abarca et al., 2013; Hesis and Michael, 2014]. Such

103 discrepancy shows that there still exists a knowledge gap for a full understanding of  
104 the behaviour of solute migration and the associated mixing processes under  
105 tide-dominated groundwater systems. Moreover, the magnitudes of mixing between  
106 seawater and fresh groundwater in the USP have not been properly quantified. This is  
107 partly due to the difficulties in measuring and quantifying the mixing process in  
108 groundwater systems under complex spatiotemporal flow variations. When tidal  
109 conditions include variable-density flows in a saturated-unsaturated aquifer with  
110 unpredictable seepage face, the strong non-linearity makes full understanding of the  
111 beach hydraulics a challenging task, even for spatially and temporally varying  
112 saltwater-freshwater mixing [Bear, 1972; Simmons *et al.*, 2001; Smith, 2004;  
113 Boufadel *et al.*, 2011]. The recently developed mesh-free numerical modelling  
114 technique, such as SPH [Shao, 2012], could be an effective method for simulating  
115 saltwater-freshwater mixing.

116 This study investigates the fluctuation of salinity distribution, examines whether  
117 the tidal driven force acts equally in the mixing behaviours at the upper saline plume  
118 and lower salt wedge, and explores the factors controlling the mixing fluctuation  
119 features, thus filling the knowledge gap in this field. To this end, we constructed  
120 variable-density, saturated-unsaturated, transient groundwater flow and solute  
121 transport models for a wide range of major hydrogeological parameters. The  
122 objectives of this study are (i) to evaluate the transient location and shape of the  
123 salinity distribution in the aquifer, considering tidal fluctuation effects,  
124 saturated-unsaturated flow and the seepage face developed at the aquifer-air interface  
125 and (ii) to further gain a quantitative understanding of the mixing behaviour under the  
126 above conditions, including the mixing pathway and mixing fluctuation characteristics.  
127 The results are analysed using a redefined spatial moment scheme for both the  
128 transient and the quasi-steady state to investigate the interaction between tidal  
129 fluctuations and mixing mechanisms.

## 130 2 Concepts and Methods

### 131 2.1 Problem Statement

132 A two-dimensional (2D) domain with a slope is set up to represent a cross-shore  
133 transect of an unconfined coastal aquifer with a length of  $L_x$  [L], a depth of  $L_y$  [L], a  
134 constant thickness of  $B$  [L] and a terrestrial freshwater flux of  $q_f$  [ $L^3 T^{-1} L^{-1}$ ]. The  
135 beach slope with an angle  $\theta$  is located at the upper part of the seaward boundary. The  
136 aquifer is assumed to be anisotropic and homogeneous. A Cartesian coordinate system  
137 is established with the x-axis pointing seaward orthogonally to the shoreline and the  
138 y-axis pointing vertically upward (see Figure 2).

### 139 2.2 Dimensional Analysis

140 The shallow coastal aquifer is parameterized for generic application as well as  
141 for better understanding the factors controlling seawater recirculation in the presence  
142 of tidal forcing. To this end, the aquifer depth  $B$  [L], vertical hydraulic conductivity  
143  $K_V$  [ $L T^{-1}$ ], freshwater density  $\rho_f$  [ $M L^{-3}$ ], transverse dispersivity  $\alpha_T$  [L] are used to  
144 normalize the key control parameters and the dimensionless parameters are defined as  
145 follows:

$$\delta = \frac{B}{A}, \quad q_f^* = \frac{BK_H}{q_f}, \quad K^* = \frac{K_H}{K_V}, \quad \rho^* = \frac{\rho_s - \rho_f}{\rho_f}, \quad \alpha^* = \frac{\alpha_L}{\alpha_T}, \quad D^* = \frac{B}{\alpha_T} \quad (1)$$

146 where  $\delta$  is the tidal spatial ratio of the aquifer depth  $B$  relative to the tidal amplitude  $A$   
147 [L], representing the strength of tidal forcing,  $q_f^*$  is the ratio of aquifer transmissivity  
148 ( $K_H B$ ) to inland groundwater discharge flux, representing the inland hydraulic  
149 gradient,  $K^*$  is the ratio of horizontal hydraulic conductivity  $K_H$  to  $K_V$ , representing the  
150 anisotropy of the aquifer,  $\rho^*$  is the buoyancy factor,  $\alpha^*$  is dispersivity anisotropy, and  
151  $D^*$  is the dispersion parameter. The combination of  $\delta$ ,  $q_f^*$ ,  $K^*$  and  $\rho^*$  controls the  
152 location and shape of the transition zone, while  $\rho^*$ ,  $\alpha^*$  and  $D^*$  control the rate of  
153 density-driven convection.

## 154 2.3 Numerical Methodology

155 In this study, the density-dependent variable-saturated groundwater flow and  
156 solute transport model SUTRA Version 2.2 [Voss and Provost, 2010] is employed to  
157 perform the numerical simulation. Though details of SUTRA 2.2 can be found in Voss  
158 and Provost [2010], a brief description of the model is provided here for completeness  
159 and convenience.

### 160 2.3.1 Governing equations

161 Neglecting subsidence and compaction, the model solves the *Richards* equation  
162 [Richards, 1931] with hydraulic conductivity for variable density groundwater flow.  
163 The governing equations for describing variable-density saturated-unsaturated  
164 groundwater flow are [Voss and Provost, 2010]

$$\frac{\partial(\varepsilon S_w \rho)}{\partial t} - \nabla \cdot [\rho k_r K \cdot \nabla h] = Q_p + \gamma, \quad (2)$$

$$h = h_p + ELEVATION \quad (3)$$

165 where  $\varepsilon$  [-] is the porosity of beach soil;  $S_w$  [-] is the sediment water saturation;  $\rho$  [M  
166 L<sup>-3</sup>] is the fluid density;  $t$  [T] is time;  $k_r$  [-] is relative permeability to fluid flow;  $K$  [L  
167 s<sup>-1</sup>] is the hydraulic conductivity;  $h$  [L] is the hydraulic head;  $h_p$  [L] is pressure head;  
168  $Q_p$  [M L<sup>-3</sup> s<sup>-1</sup>] is fluid mass source term, accounting for external additions of fluid  
169 including pure water mass plus the mass of any solute dissolved in the source fluid;  
170 and  $\gamma$  [M L<sup>-3</sup> s<sup>-1</sup>] is the pure solute mass source term, accounting for external additions  
171 of pure mass not associated with a fluid source, e.g., dissolution of solid matrix or  
172 desorption.

173 The variable saturation can be described by the *van Genuchten* formulas [van  
174 Genuchten, 1980; van Genuchten and Nielsen, 1985].

$$S_w = 1, k_r = 1 \quad \text{saturated flow} \quad (4)$$

$$k_r = S_w^{*1/2} \left\{ 1 - \left[ 1 - S_w^{* \left( \frac{n}{n-1} \right)} \right]^{\frac{n-1}{n}} \right\}^2, \quad \text{unsaturated flow} \quad (5)$$



$$S_w^* = \frac{S_w - S_{wres}}{1 - S_{wres}} = \frac{\theta_w - \theta_{wres}}{\theta_s - \theta_{wres}}$$

$$= \left[ \frac{1}{1 + (\alpha p_c / \rho g)^n} \right]^{\left( \frac{n-1}{n} \right)},$$

175 where  $S_{wres}$  [-] is residual saturation below which saturation is not expected to fall;  $S_w^*$   
 176 [-] is effective saturation;  $\theta_w$  [-] is the volumetric water content;  $\theta_s$  [-] is the  
 177 saturated volumetric water content;  $\theta_{wres}$  [-] is the residual volumetric water content;  
 178  $\alpha$  [ $L^{-1}$ ] is the capillary fringe parameter;  $n$  is the grain size distribution parameter; and  
 179  $p_c$  [ $M L^{-1} s^{-2}$ ] is capillary pressure.

180 The solute transport model including advection, mechanical dispersion or  
 181 molecular or ionic diffusion is described by

$$\frac{\partial(\varepsilon S_w \rho C)}{\partial t} = \nabla \cdot [S_w \rho K C \cdot \nabla h] + \nabla \cdot [\varepsilon S_w \rho (D + D_m I) \cdot \nabla C] + Q_p C^*, \quad (6)$$

182 where  $C$  [ $M M^{-1}$ ] is dissolved solute concentration (mass fraction);  $D$  [ $L^2 s^{-1}$ ] is  
 183 dispersion tensor;  $D_m$  [ $L^2 s^{-1}$ ] is apparent molecular diffusivity of solute in solution in  
 184 a porous medium including tortuosity effects;  $I$  is identity tensor; and  $C^*$  [ $M M^{-1}$ ] is  
 185 solute concentration of fluid sources (mass fraction).

186 The relationship between fluid density  $\rho$  and the mass fraction  $C$  is calculated by  
 187 using a linear equation, and assuming that only the salt mass fraction effect is  
 188 considered:

$$\rho = \rho_0 + \frac{\partial \rho}{\partial C} (C - C_0), \quad (7)$$

189 where  $\rho_0$  is the fluid density at a base salt mass fraction  $C_0$ , and  $\partial \rho / \partial C$  is a constant  
 190 coefficient of density variability due to a linearity assumption.

### 191 2.3.2 Boundary and initial conditions

192 We directly apply the tidal fluctuation at the seaward slope rather than  
 193 introducing another zone as done by Robinson et al. [2007a]. A sinusoidal tidal  
 194 fluctuation is implemented by applying temporal pressures or an equivalent  
 195 freshwater head,  $H(y, t)$ , along the aquifer-ocean interface.

$$H(y, t) = h_{msl} + A \cos(2\pi t/\tau), \quad (8)$$

196 where  $H(y, t)$  is the tidal level (L) at time  $t$ ,  $h_{msl}$  is the mean sea level (set at 0 m),  $\tau$  is  
197 the tidal period (T), and  $\pi = 3.1415927$ .

198 A seepage face boundary is introduced based on the approach presented by Park  
199 and Aral [2008] and Abdollahi-Nasab et al. [2010]. In other words, a Dirichlet  
200 boundary condition is set for the aquifer-ocean interface below the water level  
201 regardless if the flow enters or leaves the interface. The value of the pressure is set  
202 according to time-varying hydraulic heads and the salinity is equal to that in the  
203 seawater. No-flow and no dispersion flux boundary conditions are specified for the  
204 aquifer-ocean interface above the seepage face. The seaward vertical boundary is set  
205 as a no-flow boundary. The left boundary of the domain is set using a uniformly  
206 distributed constant flux, representing the fresh groundwater discharge into the  
207 domain. No water flow and solute transport occurs across the bottom boundary,  
208 considered as impermeable, i.e.,  $\partial h/\partial n = 0$  and  $\partial c/\partial n = 0$ . As precipitation and  
209 evapotranspiration are negligible, the upper boundary is set as an unsaturated free  
210 surface.

### 211 2.3.3 Numerical implementation

212 To quantify the effects of tides on solute mixing, models use a step-wise  
213 approach [Robinson et al., 2007a] in which models are initially run using a constant  
214 mean sea level at the seaward boundary until a steady state is reached. Tidal  
215 oscillations are then superimposed using the first-step results (i.e., salinity and  
216 hydraulic heads) as the initial condition and models are run until the dynamic  
217 quasi-steady state with respect to both heads and salinity is reached (i.e., the relative  
218 differences of hydraulic heads and salinity at the same cycle stage are within 1%).  
219 Simulations are performed by independently varying the tidal amplitude, the tidal  
220 period, the fresh groundwater flux, the hydraulic conductivity, the specific storage  
221 coefficient and the ratio of longitudinal and transverse dispersivities. Table 1  
222 summarizes the range of parameters used in the numerical simulation performed in

223 this study.

224 In different simulation scenarios, one parameter is systematically varied, while  
225 all others are held constant to examine the effect of this parameter on the simulation.  
226 The parameters for the base simulation are  $\delta = 0.1$ ,  $q_f^* = 0.01$ ,  $K^* = 1$ ,  $\rho^* = 0.025$ ,  $\alpha^*$   
227  $= 10$  and  $D^* = 300$ . These values represent a typical tidally affected coastal aquifer  
228 system. In the vertical direction the model domain extends from 15 m below to 3 m  
229 above mean sea level. This aquifer depth is chosen to effectively perform the  
230 numerical computation and to ensure that  $D^*$  is realistic and does not create excessive  
231 backward dispersion across the aquifer-ocean interface.

232 SUTRA 2.2 is an unstructured grid model using a quadrilateral type of element.  
233 In the current study, a total of 37,000 quadrilateral elements and 37631 nodes are used  
234 for spatial discretization of the study area. Since the spatial discretization schemes  
235 play an important role in simulation accuracy, a non-uniform scheme is used and  
236 ensures a mesh Péclet number ( $Pe_x = \Delta x/\alpha_L$ ,  $Pe_y = \Delta y/\alpha_T$ ) less than 2 [Voss and Souza,  
237 1987]. The horizontal grid-spacing ranges from 1 m in the most inland position to 0.5  
238 m near the sloping beach and seaside boundaries. Finer grids ( $\Delta x = 1/3$  m and  $\Delta y =$   
239 0.1 m) are used within the intertidal region where flow and mixing is stronger.

## 240 2.4 Quantification method

241 Previous studies using iso-salinity contours to demonstrate results only provide  
242 qualitative results showing the relative size and spread of the USP and salt wedge  
243 penetration. In this study, spatial moments of salt mass fractions are calculated to  
244 quantify the solute mixing and spreading processes. Four measurable parameters,  
245 namely, the centre of mass, the transverse mixing width and longitudinal mixing  
246 length, and the mixing volume, are used to demonstrate and quantify three key aspects  
247 of the problem under investigation, namely, the mass centre position of the mixing  
248 zone, the mixing range, and the accumulation of mixing. Though details for  
249 calculating these four parameters can be found in Pool et al. [2014, 2015], we present  
250 a brief description of their calculation here for convenience and completeness.

251 2.4.1 The mass centre position of the mixing zone

252 The mass centre position  $(X_c, Y_c)(t)$  is defined by ratio of the zeroth and the first  
253 moments of the salinity distribution,  $M^{(0)}(t)$  and  $M^{(1)}(t)$ , respectively,

$$X_c(t) = \frac{M_x^{(1)}(t)}{M^{(0)}(t)} \quad (9)$$

$$Y_c(t) = \frac{M_y^{(1)}(t)}{M^{(0)}(t)} \quad (10)$$

$$M^{(0)}(t) = \iint \omega(x, y, t) dx dy \quad (11)$$

$$M_x^{(1)}(t) = \iint \omega(x, y, t) x dx dy \quad (12)$$

$$M_y^{(1)}(t) = \iint \omega(x, y, t) y dx dy, \quad (13)$$

254 where subscript  $c$  indicates centre, subscript  $x$  indicates the horizontal direction,  
255 subscript  $y$  indicates the vertical direction, and  $\omega$  is the salinity. Note that the location  
256 variance contains two points: one in the USP, the other in the SW.

257 2.4.2 The mixing range

258 Actual solute mixing is produced by the interaction of advectively created  
259 concentration contrasts and mass transfer due to diffusion. The mixing width needs to  
260 measure the increase of the extension of the mean concentration distribution and not  
261 the increase of the mean extension of the plume [Dagan, 1988; Dentz et al., 2011].

262 The mixing range is defined in horizontal and vertical directions using the  
263 rigorous method of Pool et al. [2014, 2015]. The longitudinal mixing length is derived  
264 by calculating the spatial moments of  $g_x(x|y, t)$ , which is defined by the scaled and  
265 normalized salinity distribution as following:

$$g_x(x|y, t) = \frac{[\omega(x, y, t) - \omega_f][\omega_s - \omega(x, y, t)]}{\int [\omega(x, y, t) - \omega_f][\omega_s - \omega(x, y, t)] dx} \quad (14)$$

266 where  $\omega_s$  is the seawater salinity, and  $\omega_f$  is the inland groundwater salinity. The  
 267 transverse mixing width is quantified by the spatial moments of  $g_y(y/x, t)$ , which can  
 268 be defined similar to (14).

269 The spatial variance  $\sigma_x^2(y, t)$  in the horizontal direction is derived from the first  
 270 and second moments of  $g_x(x/y, t)$ ,

$$\sigma_x^2(y, t) = \{m_x^{(2)}(y, t) - m_x^{(1)}(y, t)m_x^{(1)}(y, t)\} \quad (15)$$

$$m_x^{(1)}(y, t) = \int g_x(x|y, t)x dx \quad (16)$$

$$m_x^{(2)}(y, t) = \int g_x(x|y, t)x^2 dx. \quad (17)$$

271 The spatial variance  $\sigma_y^2(x, t)$  in the vertical direction can be defined similarly to  
 272  $\sigma_x^2(y, t)$ . The longitudinal mixing length and the transverse mixing width of the  
 273 transition zone are characterized by the standard deviations,  $\sigma_x(y, t)$  and  $\sigma_y(x, t)$ ,  
 274 respectively. Because the transverse mixing width and the longitudinal mixing length  
 275 are not constant along the interface, the averaged values, namely,  $\langle\sigma_y\rangle(t)$  and  $\langle\sigma_x\rangle(t)$ ,  
 276 are calculated respectively along the vertical and horizontal directions of the mixing  
 277 zone for both the USP and SW.

#### 278 2.4.3 The accumulation of mixing

279 The accumulation of mixing is calculated using the mixing fluid volume. Based  
 280 on the iso-salinity contour, the mixing volume  $V_m(t)$  [ $L^3 L^{-1}$ ] represents the  
 281 mixing-zone volume per unit length of coastline bounded by a 0.1 ( $\omega_{min}$ ) and 0.9  
 282 ( $\omega_{max}$ ) normalized salinity contour line [Pool et al., 2014] and is defined as

$$V_m(t) = \int_V H[\omega(x, y, t) - \omega_{min}]H[\omega_{max} - \omega(x, y, t)]\epsilon dx dy, \quad (18)$$

$$H(\omega) = \begin{cases} 1, & \omega \geq 0 \\ 0, & otherwise \end{cases} \quad (19)$$

283 where  $V$  is the bulk aquifer volume, and  $H(\omega)$  is the Heaviside step function.

## 284 3 Results and discussions

### 285 3.1 Transient Behaviour

#### 286 3.1.1 Qualitative analysis

287 Figure 3 displays the impact of tides on the temporal evolution of salt mass  
288 distribution. The simulations show that tidal forcing across the sloping beach surface  
289 can introduce the formation of USP, SW and FDT. Fresh groundwater discharge  
290 occurs through the tunnel between USP and SW. The saltwater-freshwater mixes little  
291 at the centre of the tunnel (which is apparent from the fact that salinity is still the  
292 same as freshwater at this position); however, at both tunnel sides, the mixing  
293 intensifies. A mixing gradient always points to the mass centre along the groundwater  
294 discharge pathway.

295 Tidal forcing can significantly affect the pattern of salinity distribution, including  
296 affecting the position where saltwater-freshwater mixing happens. In particular, the  
297 location and extent of the USP, the developing process of saltwater-freshwater  
298 interface and the transient behaviours of the salt wedge movement along the basement  
299 of aquifer are largely related to tidal forcing. Increase of the tidal amplitude can cause  
300 the USP to migrate deeper and wider, while the salt wedge movement towards the  
301 land is inhibited (see the 600 days results in the last line of Figure 3). This finding is  
302 consistent with the sand flume experiments of Kuan et al. [2012].

303 Under the action of tidal forcing, the equilibrium under constant mean sea level  
304 is broken as saltwater intrudes into the aquifer from the seaward boundary at the high  
305 tidal level. It is seen from Figure 3 that at the very early stage (e.g., the 1st day), the  
306 aquifer hydraulic head responds quickly, prior to the solute in the aquifer, to water  
307 level rising at the seaward boundary. The tide driven recirculation occurs faster than  
308 the density-driven recirculation. USP forms prior to the development of the salt wedge.  
309 Analysis of the simulation results reveals that the maximum Darcy velocity always  
310 appears at the intersection of the water table and the beach surface, while the offshore  
311 beach groundwater is almost stagnant compared with the onshore groundwater flow.

312 This asymmetry leads to a time-averaged groundwater circulation [*Mango et al.*,  
313 2004]. This confirms the previous finding of numerical work [*Ataie-Ashtiani et al.*,  
314 1999; *Boufadel, 2000*; *Mao et al., 2006*; *Robinson et al., 2007a*; *Li et al., 2008*] and  
315 sand flume experiments [*Kuan et al., 2012*].

316 Three different evolution patterns of DRZ can be identified by carefully  
317 examining the temporal evolution of the saline distributions for various tidal  
318 amplitudes. For a small tidal amplitude (e.g.,  $A = 0.5$  m), the DRZ begins to develop  
319 from the upper position of the aquifer rather than from the toe along the bottom. The  
320 infiltrated saline water into the aquifer first appears at the position below the low tidal  
321 mark, then gradually migrates downward. This migration enhances the salt wedge and  
322 makes it sufficiently strong to spread along the bottom to landward. Thus, the salt  
323 wedge first moves landwards at the shallow region then evolves into the deeper region.  
324 This dynamic may be ascribed to the fact that a small tidal amplitude does not have  
325 sufficient energy to affect the mixing behaviour deep in the aquifer. For a small tidal  
326 amplitude (e.g.,  $A = 0.5$  m), saltwater tends to intrude into the upper aquifer when  
327 water level rises to near high tidal level mark [*Vandenbohede and Lebbe, 2006*;  
328 *Robinson et al., 2007a, c*; *Abarca et al., 2013*].

329 In contrast, for a larger tidal amplitude (e.g.,  $A = 1.0, 1.5$  m), the development of  
330 DRZ is mainly restricted by the growth of the USP. For strong tidal forcing (i.e., a  
331 large tidal amplitude), the USP develops significantly and extends in both horizontal  
332 and vertical directions, which suppresses the spreading of the salt wedge at the middle  
333 to deep positions of the aquifer (see figures in the right column of Figure 3). This  
334 means that a higher tidal amplitude generates a deeper wide mixing layer. On the  
335 other hand, the large growth of the USP reduces the horizontal extent of the mixing at  
336 the upper part of the salt wedge. A crescent-shaped interface is formed after  
337 approximately 30 days of simulation. The simulation shows that the wide mixing at  
338 the middle position always passes downward to the bottom layer, causing salt to mix  
339 at bottom aquifer over the long term (e.g., over 300 days).

340           When the tidal amplitude increases to 2.0 metres, the development of DRZ  
341 seems to have the two above patterns. The simulation shows that the infiltration is  
342 dominant at the beginning (e.g., the first few days), then the USP pushing works at the  
343 later stage (e.g., after about one week). Though the infiltration dominates at the early  
344 stage for both the 0.5 m and 2.0 m tidal amplitude cases, the evolution processes are  
345 very different. As the USP grows much faster and larger for the 2-m tidal amplitude  
346 than that for the 0.5-m tidal amplitude case, the initial infiltrated saline water tends to  
347 mix quickly to form a wider mixing zone at the upper salt wedge for the larger tide. In  
348 other words, the quick development of the USP tends to intensify the mixing locally  
349 rather than push it into the deeper aquifer in the large tidal amplitude case. This  
350 demonstrates that a wider mixing zone develops in the upper aquifer for the large tidal  
351 amplitude. As a result, the position of the toe for large tidal amplitude is seaward,  
352 while it is landward for the smaller tidal amplitude. This suggests that the large tidal  
353 amplitude tends to develop the USP rather than the DRZ.

354           As the unsaturated part is included in the model, it allows us to investigate the  
355 coupling-decoupling processes at the saturated-unsaturated interface during the tidal  
356 flood and ebb phases. It is seen from the second row in Figure 3 (1 day figures) that  
357 saltwater diffuses into the aquifer when the saltwater level rises to near the high tidal  
358 level mark. The saltwater level then falls during the ebb phase while the decoupling  
359 process occurs. The water table slopes seaward during the ebb tide, some of the  
360 seawater that previously infiltrated the sediment progressively seeps out of the aquifer  
361 along the beach surface, while some remains at the unsaturated layer as residual water  
362 in the pore space. The maximum height of the seepage face always appears when the  
363 sea level reaches its lowest level. Figure 3 also shows that as the USP develops  
364 downwards, the mixing of plume fluid and fresh groundwater flow fluid occurs at its  
365 interface. The mixing event dilutes the plume fluid at its boundary layer (as seen in  
366 Figure 3), generating stratification within the USP.

367           The local salinity gradients provide direct evidence that saltwater-freshwater  
368 mixing occurs in both the USP and SW. The temporal evolution shows that the



369 salinity distribution is a balance between the USP and SW. The growth of the USP  
370 makes the FDT move along the beach face towards the seaside. The width of the FDT  
371 gradually shrinks and then contracts and expands over the semi-diurnal period for all  
372 four tidal range cases (see Figure 3). This is in agreement with previous conceptual  
373 models [Urish and McKenna, 2004], but the USP does not vanish with the ebb tide,  
374 which accords with the previous numerical models [Ataie-Ashtiani et al., 1999; Mao  
375 et al., 2006; Robinson et al., 2007a]. This can be explained by the phase-averaged  
376 effect of tides, which can also be considered as an accumulative mass transport and  
377 storage effect.

### 378 3.1.2 Quantitative analysis

379 The above qualitative analysis shows the different evolution patterns of salinity  
380 distribution corresponding to various tidal amplitudes. This section will conduct  
381 quantitative investigation to further understand the problem under study. The temporal  
382 behaviours of mass centre position and the mixing range are extracted from the  
383 simulation results to characterize the problem under the action of four tidal  
384 amplitudes.

385 Figure 4 is the temporal variation of the mass centre of the USP and DRZ.  
386 Different stages of the centre migration are seen in the transient state. In general, the  
387 mass centre of the USP has a sharp movement downwards and towards offshore  
388 except the case of small tidal forcing (e.g.,  $A = 0.5$  m), in which the mass centre  
389 movement is gradual in both directions throughout the computational period. After a  
390 short transition, the position of the mass centre of the USP oscillates horizontally and  
391 vertically around an asymptotic value at approximately 30 days (see also Figure 3),  
392 indicating that a periodic quasi-steady state is reached. Figure 4 also shows that the  
393 highest tidal amplitude generates the lowest position of the mass centre of the USP.  
394 This may be ascribed to the fact that the density and size of the USP generated by a  
395 large tide is larger than that generated by a small tide (see also Figure 3).

396 On the other hand, the movement of the mass centre of the DRZ shows a

397 different evolution pattern. A sharp onshore and downwards movement of the mass  
398 centre is seen to occur at the early stage. When the mass centre of the DRZ reaches  
399 the lowest vertical position, it bounces back to stabilize a little above its lowest  
400 position. The movement of the mass centre of the DRZ in the horizontal direction  
401 slightly differs from its vertical movement. After the mass centre reaches the furthest  
402 onshore position at approximately 100 days, there is little horizontal and vertical  
403 movement observed for the mass centre. Figure 4 also demonstrates that the vertical  
404 movement of the mass centre of the DRZ is much smaller than its horizontal  
405 movement. It is seen that the vertical height of the mass centre of the DRZ decreases  
406 with the increase of the tidal amplitude, while the horizontal onshore position of the  
407 mass centre of the DRZ increases with the decrease of the tidal amplitude.

408 Unlike the oscillation feature of the mass centre of the USP, the mass centre of  
409 the DRZ does not have significant fluctuation around its asymptotic value after it  
410 reaches this asymptotic value. The time lag between the USP and DRZ to reach the  
411 asymptotic value is approximately 90 days. This result implies that the energy to  
412 maintain the tidal fluctuation is mostly passed to the USP rather than to the DRZ  
413 when considering beach morphology with a slope.

414 Figure 5 displays the transient behaviour of the transverse mixing width and the  
415 longitudinal mixing length in the USP and DRZ, respectively. A sharp increase of the  
416 transverse mixing width and longitudinal mixing length of the USP is seen to occur,  
417 although the extent of the increase largely depends on the tidal forcing. This sharp  
418 increase reaches its maximum at approximately 30 days, and then, a quasi-steady state  
419 is reached for all the tidal amplitudes simulated. This finding is consistent with the  
420 results shown in Figures 3 and 4. Figures 5a and 5c show that both the transverse  
421 mixing width and longitudinal mixing length of the USP increases with the increase of  
422 the tidal amplitude. This is because the USP generated by a larger tidal amplitude has  
423 a larger initial potential energy and hydrostatic pressure which in turn generates a  
424 larger penetration velocity (see the slope of  $\langle\sigma_y\rangle(t)$  and  $\langle\sigma_x\rangle(t)$  in Figures 5a and 5c).  
425 The higher velocity generates strong mixing, resulting in a wider and deeper mixing

426 zone. It is also seen that the longitudinal mixing length is approximately 4 times the  
427 transverse mixing width, indicating that the mixing of the USP mainly occurs in the  
428 horizontal direction. This finding agrees with that by Boufadel et al. [2011] who found  
429 that dispersivity did not affect the travel time of the peak of solute concentration at a  
430 specific location but did affect its magnitude.

431 Similarly, Figures 5b and 5d show that an initial very short oscillation for both  
432 the transverse mixing width and longitudinal mixing length of the DRZ exists. A sharp  
433 increase of the transverse mixing width and the longitudinal mixing length occurs  
434 before a quasi-steady state is reached. Unlike the USP, the time for reaching a  
435 quasi-steady state depends on the tidal forcing. The larger the tidal amplitude, the  
436 shorter the time required for the transverse mixing width and the longitudinal mixing  
437 length of the DRZ to reach the steady state.

438 Before the quasi-steady state is reached, as expected, the tidal fluctuation mainly  
439 exhibits in the USP rather than in the DRZ, except in the case of  $A = 0.5$  m. Unlike the  
440 transient behaviour of the mass of centre, the temporal evolution of the transverse  
441 mixing width for the  $A = 0.5$  m case shows appreciable oscillation to reach the  
442 quasi-steady state. This can be explained by the transient evolution discussed in  
443 Figure 3 in which the slow development of the USP leads to a periodic saltwater  
444 intrusion into the shallow aquifer from the upper DRZ which itself has the feature of  
445 tidal oscillation. On the other hand, a small USP causes significant unbalance from  
446 landward to neutralize the periodic oscillation at the seaward boundary, making the  
447 DRZ partially oscillate with the tidal fluctuation.

448 Figures 5b and 5d also show a different increase pattern comparing with the USP.  
449 Both the transverse mixing width and longitudinal mixing length of the DRZ  
450 increases with the decrease of the tidal amplitude (see also the fourth line of Figure 3,  
451 600 days). This can be explained as follows. A small tidal amplitude makes the DRZ  
452 migrate longer a distance landward than the larger tidal amplitude does. This is  
453 because a longer and broader USP generated by large tidal forcing develops from a

454 sloped beach and tends to suppress the DRZ evolution landwards. In addition, a  
455 smaller tidal amplitude (e.g.,  $A = 0.5$  m) causes a more seawards USP than a larger  
456 tidal amplitude (e.g.,  $A = 2.0$  m).

457 Another quantitative parameter to describe the mixing event is the mixing  
458 volume, which represents the accumulation of mixing fluid in the aquifer. Figure 6  
459 shows the temporal evolution of the mixing volume. It is seen that the development of  
460 the mixing fluid volume sharply increases with time at the early stage and then  
461 reaches a quasi-steady state at 70 days for the USP (Figure 6a) and 130 days for the  
462 DRZ (Figure 6b). The mixing fluid volume of the USP significantly increases with the  
463 increase of the amplitude of the tidal forcing. Simulation shows that the mixing fluid  
464 volume at the quasi-steady state for  $A = 2.0$  m is approximately 45 times that for  $A =$   
465  $0.5$  m. Figure 6 reveals that the mixing fluid volume of the DRZ decreases slightly  
466 with an increase of the tidal amplitude, indicating that the effect of the tidal forcing on  
467 the DRZ is marginal.

### 468 3.2 Steady State Behaviour

469 Previous studies suggest that once the quasi-steady state is reached, the salinity  
470 distribution in coastal aquifers will not fluctuate significantly over a tidal cycle [see,  
471 e.g., *Ataie-Ashtiani et al., 1999; Dentz and Carrera, 2003; Pool et al., 2014*]. The  
472 results obtained from this study show that behaviour of mass centre position, mixing  
473 width and length, and mixing volume of the DRZ, but not those of the USP, agree  
474 well with previous studies. These findings imply that there may exist another  
475 mechanism that governs the migration of the salt mass and the mixing behaviour of  
476 the USP when a quasi-steady state is reached. To further investigate this, the mass  
477 centre position and the mixing behaviour around its mean position over a daily cycle  
478 are analysed for the quasi-steady state.

479 Figure 7 illustrates the temporal variation of centre of mass around its mean  
480 position ( $\bar{X}_c, \bar{Y}_c$ ) over two tidal cycles ( $t = 24$  h) once the quasi-steady state is

481 reached. It is seen that the fluctuation amplitude of the mass centre in the USP  
482 increases with increase of tidal amplitude. In particular, when tidal amplitude  
483 increases from 0.5 m to 1.0 m, the fluctuation extent increases almost 5 times. When  
484 tidal amplitude continues to increase (e.g., from 1.0 m to 2.0 m), the increase of  
485 fluctuation becomes gradual, e.g., the fluctuation response to an increase of tidal  
486 amplitude from 1.0 m to 2.0 m is only 1/5. Figures 7a and 7c also demonstrate that  
487 there is no significant phase lag of vertical mass centre oscillation between tidal signal  
488 and USP oscillation, while more than a  $5\pi/4$  phase lag is observed for horizontal  
489 component of mass centre of the USP for tidal amplitude increases from 1.0 m to 2.0  
490 m. It is evident that the phase-resolved effect of tides is more significant for salt  
491 transport in horizontal directions of aquifers. Furthermore, for all four tidal cases,  
492 tidal fluctuations cause the horizontal component of the mass centre of the USP to be  
493 more displaced back than forth around its mean position. For the vertical component,  
494 it becomes more up than down. That is, for both components, the behaviour of the  
495 mass centre is asymmetric.

496 The fluctuation of both the horizontal and vertical mass centres of the DRZ  
497 around its mean location is very small (see also Figure 4), indicating that a very  
498 steady state for the mass centre of the DRZ is reached. However, the phase lag needs  
499 to be mentioned, for the horizontal component of the mass centre of the DRZ (see  
500 Figure 7b) there is almost a  $\pi/2$  phase difference between tidal signal and DRZ  
501 oscillation. For the vertical component of the mass centre of the DRZ (see Figure 7d),  
502 the situation becomes complicated; four different tidal amplitudes cause four kinds of  
503 responses (see Figure 7d). This finding implies that the tidal effect on the evolution of  
504 the vertical component of the DRZ is complex.

505 Figure 8 shows the oscillation of the mixing width and length is around its  
506 averaged value when a quasi-steady state is reached. For the transverse mixing width  
507 of the USP, there is almost no phase lag at the first half tidal cycle, while phase lag is  
508 seen at the other half tidal cycle (see Figure 8a), which seems to increase with the  
509 increase of tidal amplitude. For a small tidal amplitude (e.g.,  $A = 0.5$  m), two peak

510 values for the longitudinal mixing length of the USP exist over a tidal cycle, and they  
511 occur roughly at the highest and lowest tidal levels for the 1.0 m case, and both occur  
512 at the highest tidal level for the 0.5 m case. This phenomenon is also observed in the  
513 study assuming a vertical beach face [*Pool et al., 2014*], which supposed the  
514 phenomenon was attributable to the unsynchronized movement between the saline  
515 end and the freshwater end of the mixing zone. On the other hand, for larger tidal  
516 amplitudes, the longitudinal mixing length of the USP behaves normally, only  
517 displaying an increase of phase lag with an increase of tidal amplitude. The  
518 asymmetric oscillation still exists for both the transverse mixing width and the  
519 longitudinal mixing length of the USP (see Figures 8a, 8c).

520 The fluctuation of both the transverse mixing width and longitudinal length of  
521 the DRZ around its mean value  $\overline{\langle\sigma\rangle}$  is very small (see also Figure 5) and it may be  
522 concluded that a very steady state for the transverse mixing width and longitudinal  
523 length of the DRZ is reached.

524 Comparing the transverse mixing width to the longitudinal mixing length of the  
525 USP and DRZ demonstrates that the temporal variations of flow at the seaward  
526 boundary enhance longitudinal dispersivity as well as transverse dispersivity in the  
527 USP, while effects on both longitudinal dispersivity and transverse dispersivity of the  
528 DRZ are small.

529 Figure 9 shows the tidal influence on the mixing volume of the USP and the  
530 DRZ in a daily cycle. The results show that a large mixing fluid volume oscillation  
531 around its averaged value exists for the USP, while insignificant oscillation of the  
532 mixing fluid volume of the DRZ around its mean value is observed after the  
533 quasi-steady state is reached. The mixing volume oscillation amplitude of the USP is  
534 more than 10 times that of the DRZ. This implies that the tide not only accelerates the  
535 formation of the USP but also maintains its fluctuation around a relative constant  
536 value when a quasi-steady state is reached.

537 To examine the relative importance of different parameters, the fluctuation

538 amplitude of the mass centre, mixing width and length, and mixing volume are  
539 calculated under six dimensionless parameters. To avoid repetition, here, we only  
540 show the results of the fluctuation amplitude of longitudinal mixing length and the  
541 transverse mixing width. As shown in Figure 10, the fluctuation amplitude of both the  
542 longitudinal mixing length and the transverse mixing width in the USP are at least one  
543 order magnitude larger than those in the DRZ. This finding again emphasizes that the  
544 USP is the dominant mixing region in tidally dominated beach aquifers with a sloping  
545 beach face.

546       There are differences when comparing six different dimensionless parameters. In  
547 the transverse direction (see Figure 10c), the strength of tidal forcing and the slope of  
548 the beach can change the fluctuation amplitude of mixing. The remaining four  
549 parameters rarely have any contribution to the fluctuation of the transverse mixing  
550 width. The fluctuation amplitude of the transverse mixing width in the intertidal zone  
551 increases with the increase of the beach slope. The simulation shows that such  
552 fluctuation amplitude for the steep slope (e.g., beach slope = 0.3) is approximately 50%  
553 larger than that of the mild slope beach (e.g., beach slope = 0.1). In the longitudinal  
554 direction (see Figure 10a), the strength of tidal forcing, the inland hydraulic gradient,  
555 hydraulic conductivity and the slope of the beach collectively control the fluctuation  
556 amplitude. A steep slope tends to decrease the fluctuation amplitude of the  
557 longitudinal mixing length. This is because the steep beach face is too high to prevent  
558 strong salinity gradients from developing [Evans and Wilson, 2016]. The anisotropy  
559 of hydraulic conductivity is a finite regulator to adjust the amplitude of mixing on the  
560 basis of tidal oscillation. It is interesting to note that when the anisotropy of  
561 dispersivity is small (e.g.,  $\alpha_L/\alpha_T < 5$ ), the aquifer cannot prevent seawater intrusion  
562 but can eliminate the fluctuation of the longitudinal mixing length.

## 563 4 Conclusion

564       Saltwater-freshwater mixing in the near shore region is significant for chemical  
565 fluxes from the aquifer to the ocean. The upper saline plume in the intertidal region

566 has proven to be a more active zone for mixing and reaction than the classical salt  
567 wedge. This study examines tidal effects on saltwater-freshwater mixing fluctuations  
568 in unconfined coastal aquifers using two-dimensional numerical models for a wide  
569 range of hydrogeological parameters. The mixing characteristics of the USP and DRZ  
570 are investigated and quantified using three measurable parameters to quantify three  
571 aspects of the problem under investigation, namely, the mass centre position of the  
572 plume, the mixing range, and the accumulation of mixing. The effect of tidally driven  
573 dynamic circulation on the mixing and spreading behaviour of the USP and the DRZ  
574 has been investigated and discussed.

575 Results suggest that tidal forcing can greatly affect the USP yet has an  
576 insignificant effect on the DRZ. The tidal amplitude not only affects the saline plume  
577 size but also affects the process of the plume reaching a quasi-steady state. The results  
578 clearly demonstrate that an increase in the tidal amplitude causes the mass centre of  
579 the USP to spread deeper and wider, as well as inhibits the mass centre of the DRZ  
580 from moving further into the land. Furthermore, tidal forcing causes an increase of  
581 both the transverse mixing width and longitudinal mixing length of the USP. In  
582 contrast, an increase of the tidal amplitude decreases the growth of the transverse  
583 mixing width and longitudinal mixing length of the DRZ. The fluctuation amplitude  
584 of the mass centre, mixing width and length, and mixing volume in the USP are, in  
585 general, one-magnitude-order larger than those in the DRZ.

586 The mixing in the USP largely inherits the fluctuation characteristics of tides in  
587 both transverse and longitudinal directions. Meanwhile, the mixing in the DRZ is  
588 relatively steady and shows little fluctuation. The effect of beach slope on mixing in  
589 the USP is reversed in the longitudinal and transverse directions. A steep slope tends  
590 to promote mixing fluctuation in the transverse direction and suppress mixing  
591 oscillation in the longitudinal direction.

592 This study has only examined the impact of sinusoidal fluctuations of the sea  
593 level on the salt mass spreading, mixing and quasi-steady distributions in shallow



594 unconfined coastal aquifers. However, real tides have multiple harmonic constituent  
595 signals and long time scale variability leading to various flow patterns and instabilities  
596 that will induce significant variations in the intertidal salinity structure [*Robinson et*  
597 *al., 2007b; Abarca et al., 2013; Hesis and Michael, 2014*]. Further study is required to  
598 take into account time scales, dynamic spatial distributions, and the frequency and  
599 amplitude of mixing fluctuation to simulate the actual situation under real tides.

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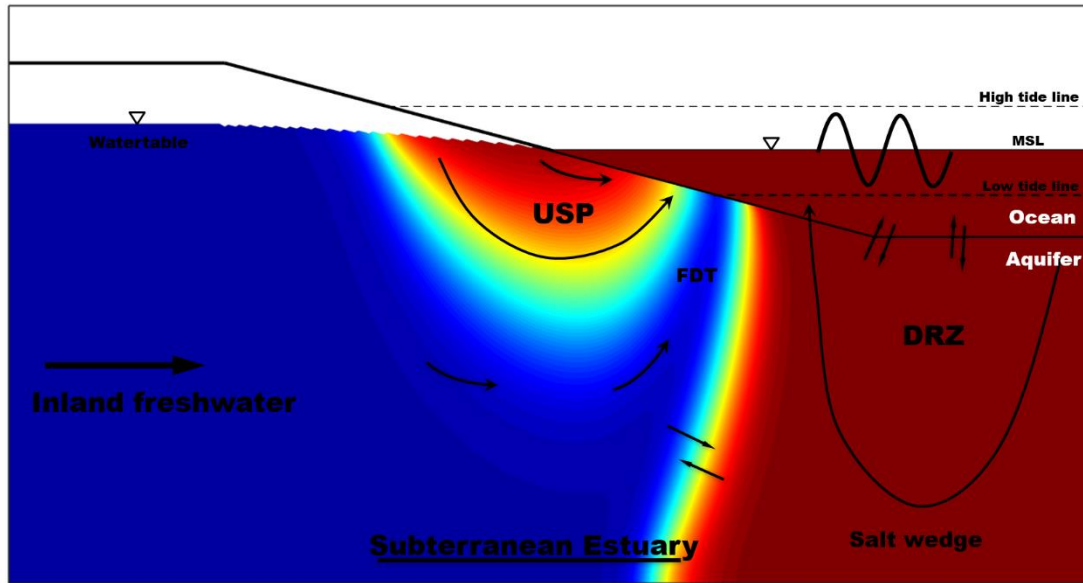
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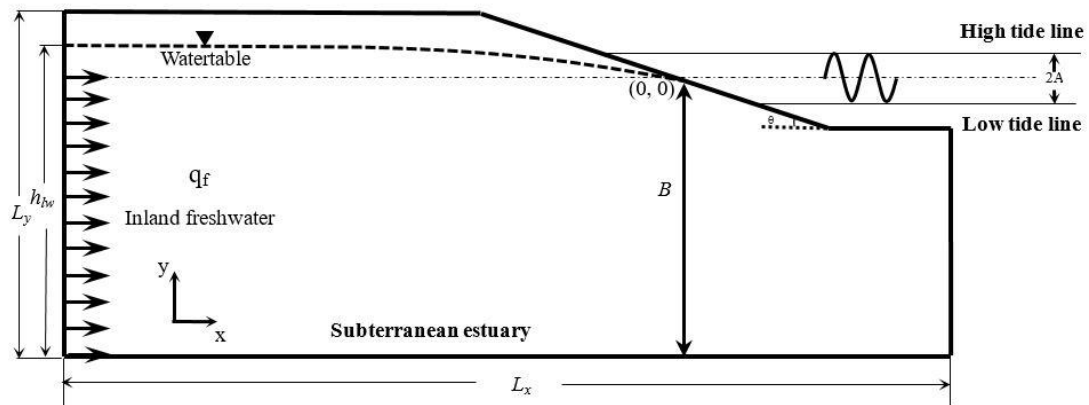


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768 Figure 1. Conceptual diagram of subterranean estuary including major nearshore circulations. Showing  
 769 upper saline plume (USP), freshwater discharge tube (FDT), salt wedge (SW) and density driven  
 770 recirculation zone (DRZ).

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774 Figure 2. Sketch of the model geometry (2-D vertical cross-section) and boundary conditions. The

775 dashed line represents the beach watertable, and the dash-dot line represents the mean sea level (MSL).

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778 Table 1. Parameters Used in Numerical Simulations

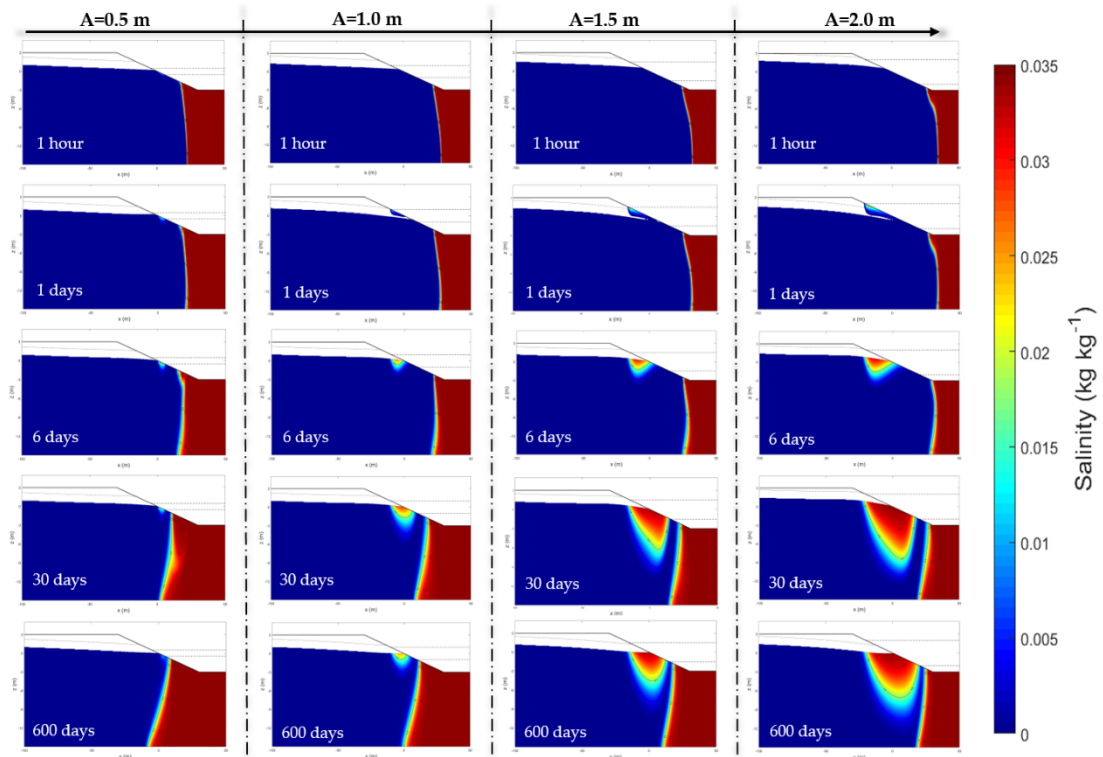
Symbol	Parameter	value
$K_H$	Hydraulic conductivity [ $\text{m d}^{-1}$ ]	7.6-152.6
$S_s$	Specific storativity [ $\text{m}^{-1}$ ]	$7.36 \times 10^{-2}$
$q_f$	Terrestrial fresh groundwater flux [ $\text{m}^3 \text{d}^{-1} \text{m}^{-1}$ shoreline]	0-4.0
$\varepsilon$	Effective porosity [-]	0.25
$B$	Aquifer thickness [m]	15
$S_b$	Beach slope [-]	0.1
$\alpha_L$	Longitudinal dispersivity [m]	0.5
$\alpha_T$	Transverse dispersivity [m]	0.005-0.5
$\rho_f$	Freshwater density [ $\text{kg m}^{-3}$ ]	1000
$\rho_s$	Seawater density [ $\text{kg m}^{-3}$ ]	1024.99
$D_m$	Molecular diffusivity [ $\text{m}^2 \text{s}^{-1}$ ]	$1.0 \times 10^{-9}$
$\mu$	Fluid viscosity [ $\text{kg m}^{-1} \text{s}^{-1}$ ]	0.001
$S_{wres}$	Residual saturation [-]	0.2
$\alpha$	Van Genuchten capillary fringe parameter [ $\text{m}^{-1}$ ]	0.8
$n$	Van Genuchten grain size distribution parameter [-]	3.0
$A$	Tidal amplitude [m]	0.5-2.0
$\tau$	Tidal period [d]	0.5

779 \* values in parentheses with hyphens are the minimum and maximum.

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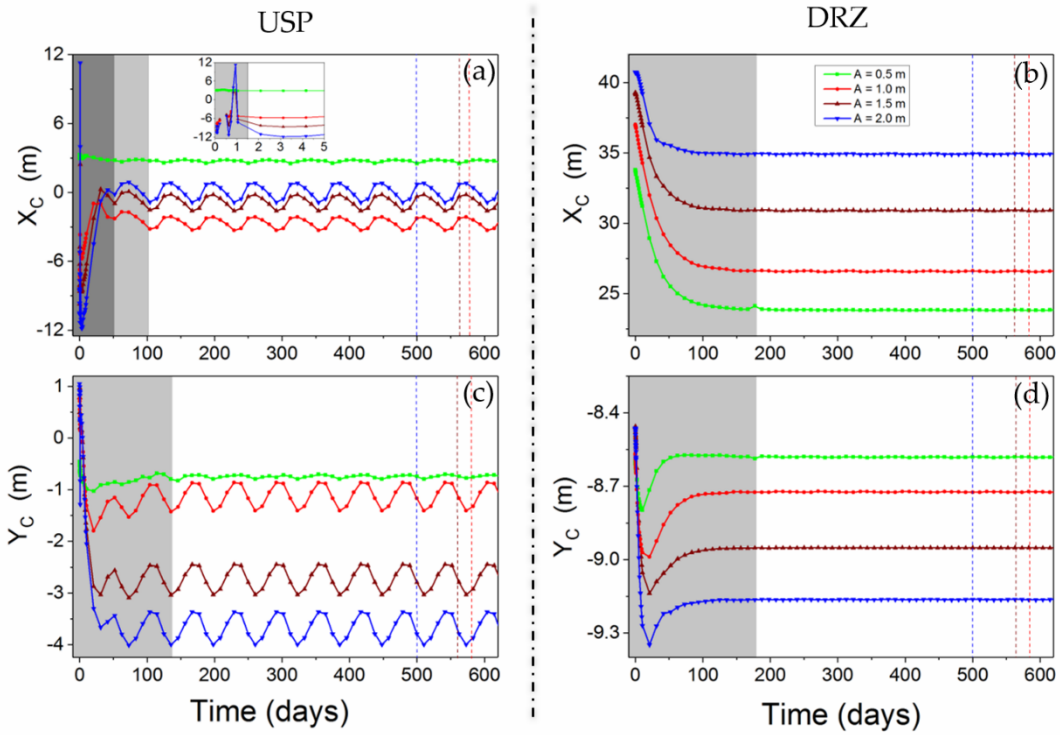
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784 Figure 3. Temporal evolution of the saline distributions for various tidal amplitudes, showing the USP

785 and DRZ growing processes, and the distinct coupling-decoupling of saturated-unsaturated layer. Two

786 parallel dashed lines at the seaward boundary represent the tidal range.

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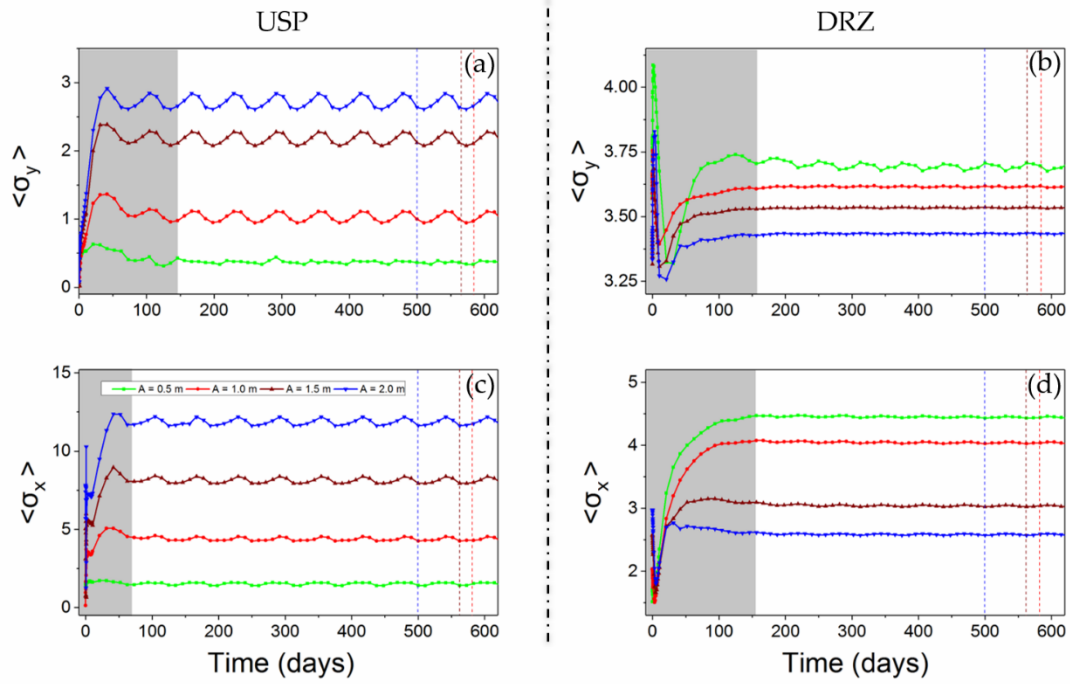


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790 Fig. 4 Transient behavior of the horizontal (a, c) and vertical (b, d) components of the USP and DRZ  
 791 mass center position for various tidal fluctuations. Squares:  $A = 0.5$  m; dots:  $A = 1.0$  m; triangles:  $A =$   
 792  $1.5$  m; inverted triangle:  $A = 2.0$  m. The dashed line indicates the time for 1% difference between two  
 793 adjacent time steps.

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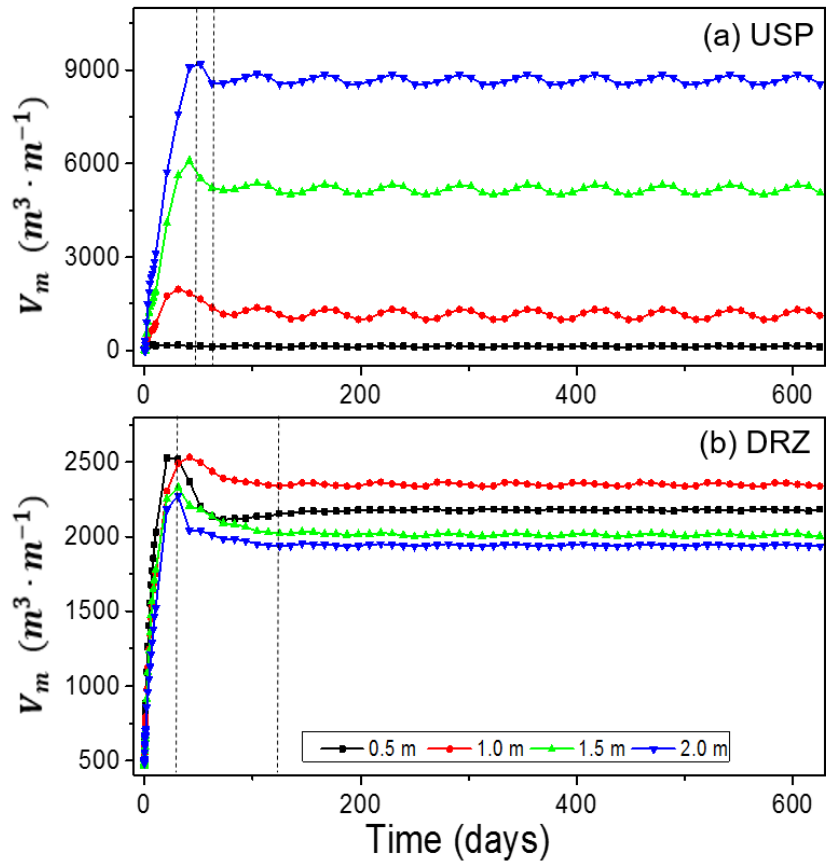


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797 Figure 5. Transient behavior of the transverse mixing width (a, b) and the longitudinal mixing length (c,  
798 d) of the USP (a, c) and DRZ (b, d), for various tidal fluctuations. Squares:  $A = 0.5$  m; dots:  $A = 1.0$  m;  
799 triangles:  $A = 1.5$  m; inverted triangle:  $A = 2.0$  m. The dashed line indicates the time for 1% difference  
800 between two adjacent time steps.

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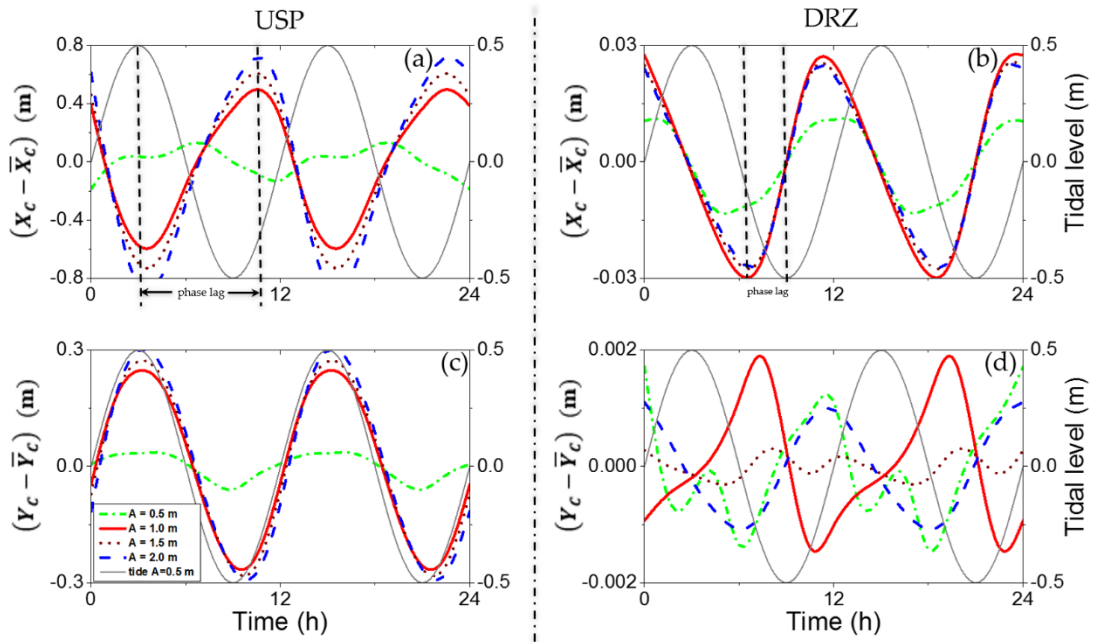
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804 Fig. 6 Transient behavior of the mixing volume of the (a) USP and (b) DRZ for various tidal

805 fluctuations. Squares:  $A = 0.5$  m; dots:  $A = 1.0$  m; triangles:  $A = 1.5$  m; inverted triangle:  $A = 2.0$  m.

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809 Figure 7. Daily cycle of the horizontal (a, b) and vertical (c, d) components of the USP (a, c) and DRZ

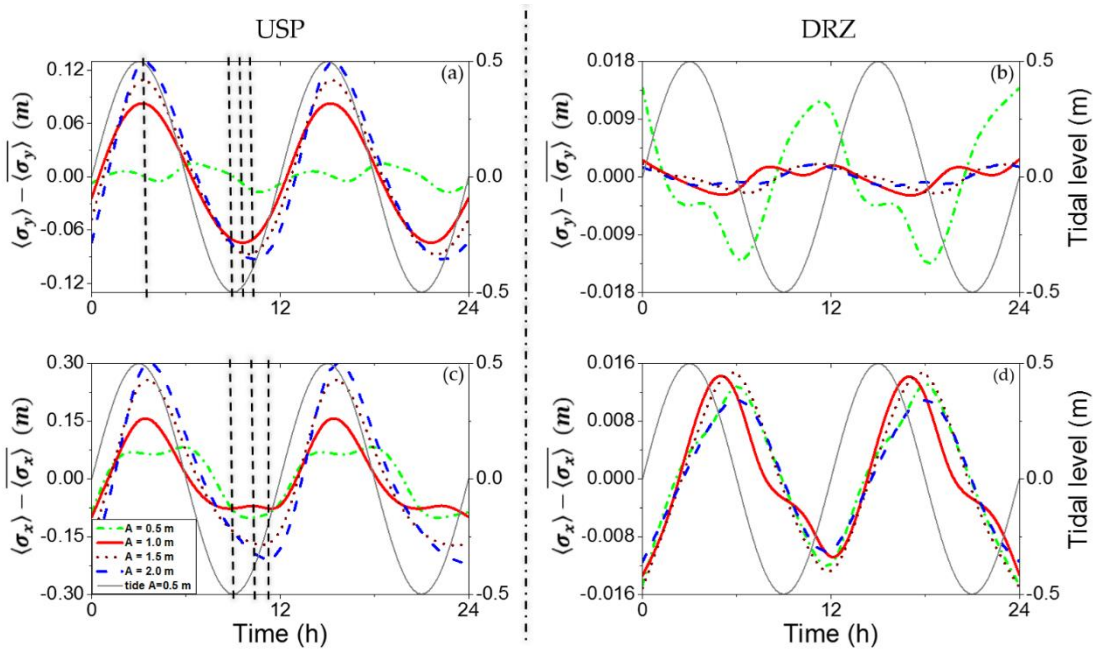
810 (b, d) mass center position once the periodic quasi-steady state is reached, respectively, for different

811 tidal amplitudes cases. The left vertical scale is for the mass center position, and the right vertical scale

812 is for tidal level.

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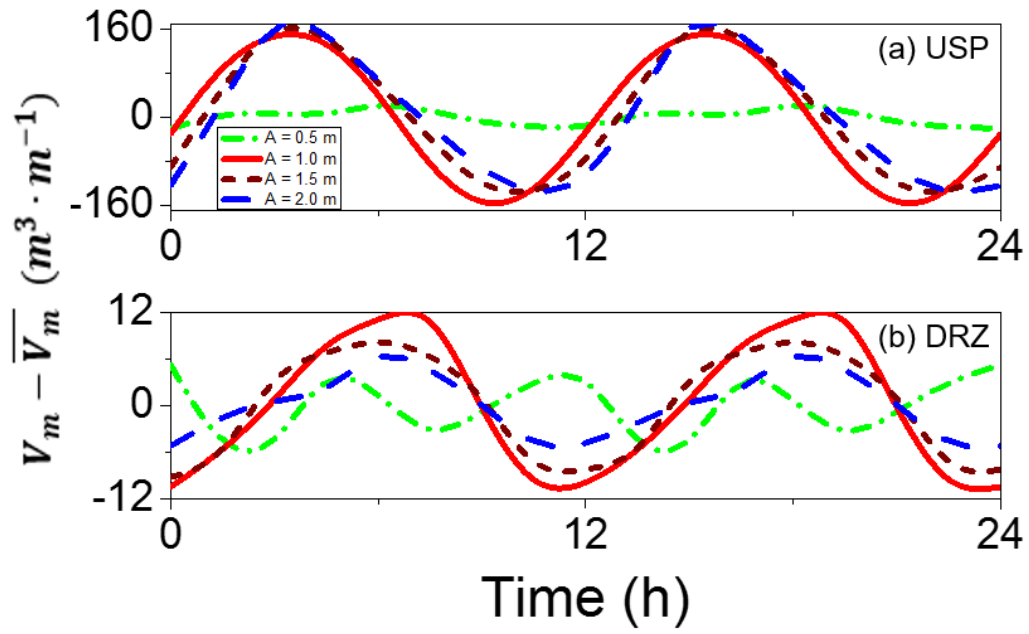


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816 Figure 8. Daily cycle of the transverse mixing width (a, b) and the longitudinal mixing length (c, d) of  
817 the USP (a, c) and DRZ (b, d) around their mean value  $\overline{\langle \sigma \rangle}$  once the periodic quasi-steady state is  
818 reached, respectively, for different tidal amplitude. The left vertical scale is for the mass center position,  
819 and the right vertical scale is for tidal level.

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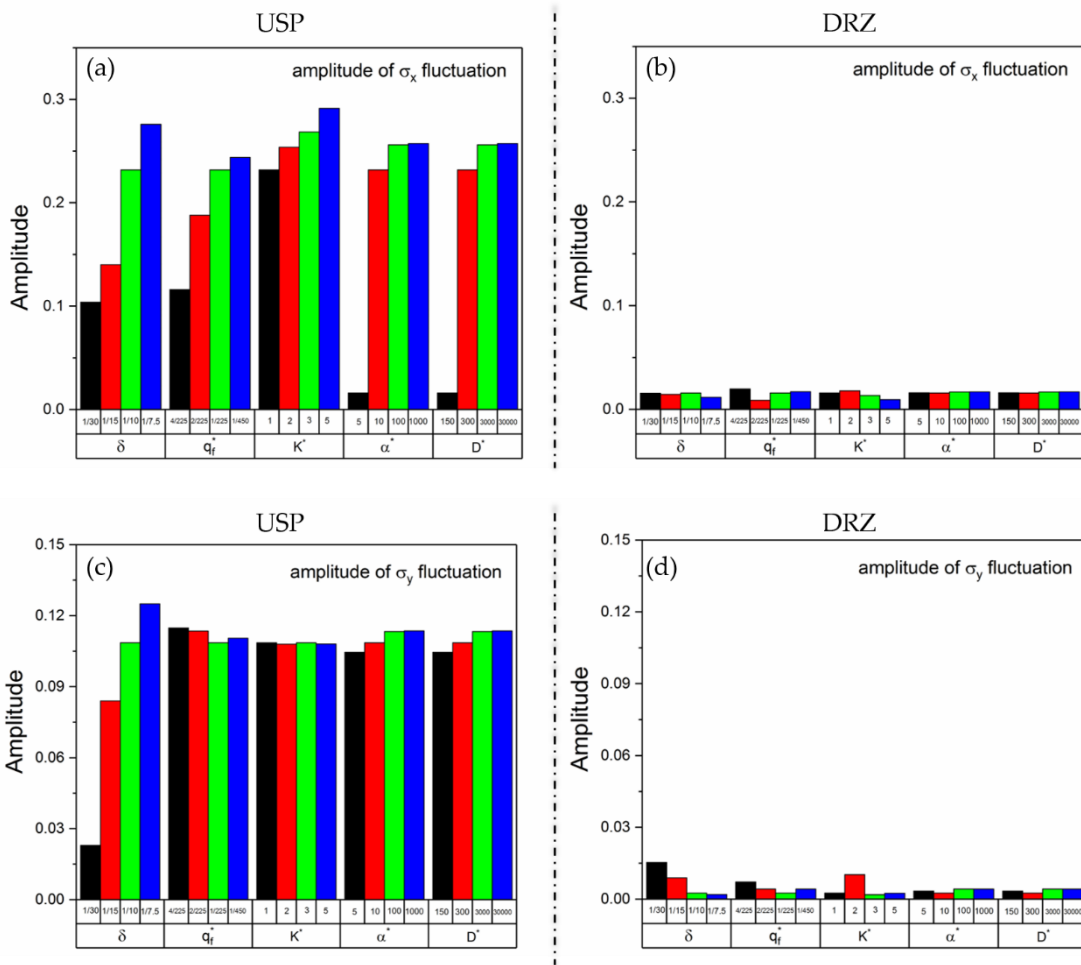
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823 Figure 9. Daily cycle of the mixing volume of (a) USP and (b) DRZ once the periodic quasi-steady

824 state is reached, respectively, for different tidal amplitude.

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829 Figure 10. Compare of the fluctuation amplitude of longitudinal mixing length (a, b) and the fluctuation  
 830 amplitude of transverse mixing width (c, d) of the USP (a, c) and DRZ (b, d) calculated based on the  
 831 quasi-steady state results.

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