Comment on "Influence of Noise on Force Measurements"

In a recent Letter [1], Volpe *et al.* describe experiments on a colloidal particle near a wall in the presence of a gravitational field for which they study the influence of noise on the measurement of force. Their central result is a striking discrepancy between the forces derived from experimental drift measurements via their Eq. (1), and from the equilibrium distribution. From this discrepancy they infer the stochastic calculus realized in the system.

We comment, however, that (a) Eq. (1) does not hold for space-dependent diffusion, and corrections should be introduced, and (b) the "force" derived from the drift need not coincide with the "force" obtained from the equilibrium distribution.

The problem of what should be the "correct" stochastic calculus was tackled in the early 1980s. The consensus was that, for a model in the form of a stochastic differential equation, the calculus to be used, e.g., in a simulation, is part of the model itself. Correspondingly, starting from measured data, what we observe is a distribution function, but in the absence of further information and/or specific models, we cannot infer the underlying stochastic calculus [2]. For a continuous physical system, with noise of (inevitably) finite bandwidth, we expect the Stratonovich calculus to apply [3].

From the stochastic differential equation (3) of [1],

$$dz = f(z)dt + g(z)dW = \frac{F(z)}{\gamma(z)}dt + \sqrt{2D_\perp(z)}dW, \quad (1)$$

we obtain the family of Fokker-Planck equations

$$\frac{\partial P(z,t)}{\partial t} = \frac{\partial}{\partial z}\left\{-f(z) - \alpha g(z)g'(z) + \frac{1}{2}\frac{\partial}{\partial z}g^2(z)\right\}P(z,t), \quad (2)$$

where $g'(z) = \partial g(z)/\partial z$ and α is 0 or 1/2 for the Ito or Stratonovich stochastic calculi, respectively. In an experiment the diffusion [related to $g^2(z)$] and the drift [related to $f(z) + \alpha g(z)g'(z)$] can be measured. To infer α, however, additional information [e.g., knowledge of $f(z)$] is needed. From (2), the drift velocity is

$$\bar{v}_d = \frac{dz}{dt} = f(z) + \alpha g(z)g'(z) = \frac{F(z)}{\gamma(z)} + \alpha\frac{dD_\perp(z)}{dz}. \quad (3)$$

This relation does not coincide with Eq. (1) of [1] because the nonlinearity of the diffusion coefficient enters the drift. Hence, it is impossible to derive the force $F(z)$ from a measurement of the drift velocity, as in Eq. (1) of [1], where it was assumed that $F(z) = \gamma(z)\bar{v}_d(z)$.

In [1] the force is also computed from $F_e(z) = -dU(z)/dz$, where the potential $U(z) = -k_BT\ln(P(z))$ is obtained from the equilibrium distribution $P(z)$. From (2) we obtain $U(z) = -k_BT\int\frac{f(z)+(\alpha-1)g(z)g'(z)}{g^2(z)/2}dz$,

$$\frac{F_e(z)}{\gamma(z)} = -\frac{1}{\gamma(z)}\frac{dU(z)}{dz} = f(z) + (\alpha-1)g(z)g'(z). \quad (4)$$

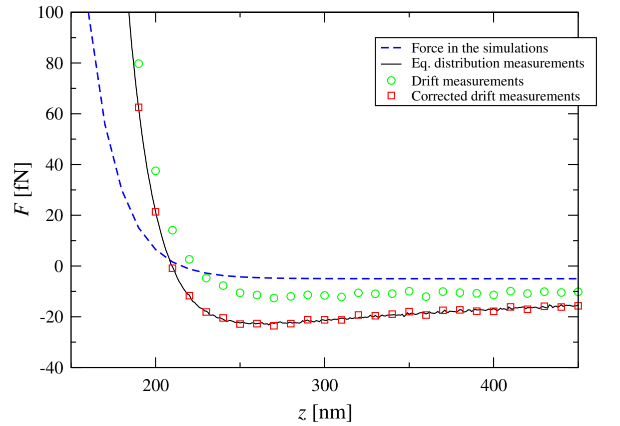

FIG. 1 (color online). Force computed from a simulation of (1) using the Stratonovich calculus. Consistent with the experiment, we took $f(z) = Be^{-kz} + C$ with $B = 770$ pN, $C = -5$ fN, $k = (18\text{ nm})^{-1}$, and $D_\perp(z) = D_\infty z/(z+a)$ with z in nm, $a = 700$ nm, $D_\infty = k_BT/6\pi\eta R$, $2R = 1.31$ nm, $T = 300$ K, $\eta = 8.5\times10^{-3}$ Pa s.

Equations (3) and (4) differ by $-g(z)g'(z) = -\frac{dD_\perp(z)}{dz}$, which is *exactly* the experimental discrepancy reported in [1]; this difference is independent of α, i.e., independent of the stochastic calculus used to describe the physical system.

As a demonstration, we simulated (1) numerically for the Stratonovich calculus with the same definitions as [1], computing the average forces from the drift and equilibrium distribution of the time sequence $z(t)$. Figure 1 shows that the force from the drift [$\bar{v}_d\gamma(z)$, Eq. (3), circles] differs from that from the equilibrium distribution [$F_e(z)$, Eq. (4), full curve]. When the drift result is corrected by the additional term $-g(z)g'(z)$, however, we recover the equilibrium distribution result. Thus the discrepancy reported in [1] has nothing to do with different stochastic calculi: it is simply a consequence of having two different definitions of force. Neither of them corresponds to the true microscopic force, and they coincide only where the diffusion coefficient happens to be constant.

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