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# Financial Markets where Traders Neglect the Informational Content of Prices

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**Abstract:** We model a financial market where some traders of a risky asset do not fully appreciate what prices convey about others' private information. Markets comprising solely such "cursed" traders generate more trade than those comprising solely rationals. Because rationals arbitrage distortions caused by cursed traders, mixed markets can generate even more trade. Per-trader volume in cursed markets increases with market size; volume may instead disappear when traders infer others' information from prices, even when they dismiss it as noisier than their own. Making private information public raises rational and "dismissive" volume, but lowers cursed volume given moderate non-informational trading motives.

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# 1 Introduction

Ever since Milgrom and Stokey (1982) and Tirole (1982), researchers have understood that common knowledge of rationality combined with a common prior precludes purely speculative trade. Of course, people might rationally trade financial assets for a variety of non-speculative motives, such as portfolio rebalancing and liquidity. Yet even in settings where the presence of non-speculative motives allows for speculative trade, a rational understanding of the adverse-selection problem causes the overall volume of trade to be constrained by non-speculative motives. In many people's estimation, trading volume in financial markets greatly exceeds what can be plausibly explained by models applying rational-expectations equilibrium (REE).<sup>1</sup>

Researchers have sought to explain excessive trading volume by relaxing the common-prior assumption. Harrison and Kreps (1978) show how non-common priors about an asset's payoff generate volume in a dynamic model where risk-neutral traders cannot sell the asset short. Scheinkman and Xiong (2003) use Harrison and Kreps' framework to explore traders who are "overconfident": all traders observe all signals about the payoff, yet certain traders overestimate the information content of certain signals.<sup>2</sup> In these models without private information, trade derives from traders agreeing to disagree about the relationship between payoff and public information; the lack of private information disencumbers traders from the need to invert market prices.<sup>3</sup> A second approach incorporates non-common priors into incomplete-information models by assuming that privately informed traders agree to disagree about the precision of traders' private information. Daniel, Hirshleifer, and Subrahmanyam (1998) and Odean (1998), for example, show how traders' over-

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<sup>1</sup>For example, in his presidential address to the American Finance Association, French (2008) notes that the capitalized cost of trading exceeds 10% of market capitalization, and turnover in 2007 was 215%, creating a puzzle that "from the perspective of the negative-sum game, it is hard to understand why equity investors pay to turn their aggregate portfolio over more than two times in 2007" (page 1552).

<sup>2</sup>Hong, Scheinkman, and Xiong (2006) model overconfidence similarly, allowing also for heterogenous priors, in a model where the number of shares of a risky asset increases over time.

<sup>3</sup>Other models of trade deriving from differences in beliefs include Lintner (1969) and Varian (1985), where traders have different subjective priors, De Long, Shleifer, Summers, and Waldmann (1990), where symmetrically informed traders disagree because some of them ("noise traders") misperceive next-period prices for exogenous reasons, Harris and Raviv (1993) and Kandel and Pearson (1995), where traders disagree about the informativeness of public signals, and Biais and Bossaerts (1998), where traders are uncertain about others' belief hierarchy. Hong and Stein (2007) summarize this literature. Eyster and Piccione (2013) model traders with incomplete theories of price formation, also in a complete-information setting.

confidence about the precision of their private information can increase trading volume. Similarly, Odean (1998), Banerjee, Kaniel, and Kremer (2009), Banerjee, Kaniel, and Kremer (2010), and Banerjee (2011) show that when traders downplay the precision of one another’s private signals—which we call *dismissiveness*—volume also increases.<sup>4</sup> In this second class of agreeing-to-disagree models, the presence of private information infuses market prices with information content, and traders are assumed to fully invert market prices to uncover others’ information. Both types of agreeing-to-disagree models depict traders who recognize their disagreements in beliefs—and trade because of these recognized disagreements.

This paper proposes a different conceptual approach to explaining speculative trade: people trade because they *neglect* disagreements in beliefs. We capture this idea in a simple and tractable model where some or all traders, when choosing their demands, do not fully invert prices to uncover others’ information. This approach builds on extensive evidence, reviewed in Section 5, that people do not sufficiently heed the information content of others’ behavior, even in the absence of intrinsic disagreements.

Not inferring information from prices may appear observationally similar to inferring and then dismissing that information. We show that the implications for prices are indeed similar, but the implications for trading volume can differ sharply. In particular, disagreement neglect generates large volume in settings where overconfidence and some natural forms of dismissiveness do not. Disagreement neglect also “enables” overconfidence and other biases to have large effects on volume, while the effects would be small in its absence.

Section 2 introduces our formal set-up, based on Grossman (1976), Hellwig (1980), and Diamond and Verrecchia (1981). We consider a market in which traders can exchange a risky asset for a riskless asset over one period. Each trader observes a public and a private signal about the risky asset’s payoff, with all signals being independent conditional on that payoff. Each trader also receives a random endowment, whose covariance with the asset payoff he is the only one to observe. Random endowments furnish traders with a non-speculative (hedging) motive to trade. We define

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<sup>4</sup>Banerjee, Kaniel, and Kremer (2009), Banerjee, Kaniel, and Kremer (2010), and Banerjee (2011) use the term “differences of opinion” to describe the heterogeneity in beliefs that drives their models. We use the term dismissiveness instead, to distinguish it from overconfidence and other disagreements about signal structures that also create differences of opinion.

*cursed-expectations equilibrium (CEE)* by the assumption that some traders do not infer information from the asset price. We call traders who do not extract any information *fully cursed* and traders who extract some information *partially cursed*. CEE is the competitive-markets analogue of the game-theoretic concept of cursed equilibrium, defined by Eyster and Rabin (2005) and reviewed in Appendix A.<sup>5</sup> For tractability, we assume that traders have constant-absolute-risk-aversion (CARA) preferences and that all relevant probability distributions are normal.

Section 3 derives the main predictions of CEE in a simple version of our model, where traders are symmetric in private-signal precision, risk aversion, and cursedness, and there are no random endowments or public signals. The most important implication of CEE is also the most basic: cursedness produces substantial trade, with aggregate volume approaching infinity as the number of traders grows large. We show additionally that per-trader volume increases with the number of traders. This is because the discrepancy between each private signal and the average of all signals increases with the number of traders, and volume is proportional to this discrepancy since each cursed trader gives a constant positive weight to his own signal, failing to realize that the price reveals the average signal. Cursedness distorts not only volume, but also prices: because traders do not fully infer others' information from the price, the price under-reacts to private signals, and hence price changes are positively autocorrelated.

Section 3 next contrasts the implications of cursedness to those of overconfidence and dismissiveness. Following Odean (1998), we model overconfident traders as exaggerating the precision of their own private signal, and dismissive traders as under-estimating the precision of other traders' private signals. We allow for an additional form of dismissiveness, introduced by Banerjee (2011): traders treat the noise in others' signals as correlated (while in fact it is independent), hence under-estimating the collective precision of others' signals. As in previous literature, we assume that overconfident or dismissive traders fully understand the mapping between the price and other traders' private information.

Because overconfident traders overweight their own signals and estimate correctly the preci-

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<sup>5</sup>In the special case where all agents are fully cursed, cursed equilibrium coincides with a version of the analogy-based-expectations equilibrium (ABEE) defined by Jehiel (2005) and extended to incomplete-information games by Jehiel and Frederic (2008). In this sense, CEE with fully cursed agents can be regarded as a competitive-market analog of ABEE.

sion of others' signals, the price over-reacts to private signals. When traders are dismissive, the price instead under-reacts to private signals. Hence, dismissiveness has similar implications to cursedness for prices, while overconfidence has opposite implications. Our over- and under-reaction results for overconfidence and dismissiveness are similar to those in, e.g., Daniel, Hirshleifer, and Subrahmanyam (1998), Odean (1998), and Banerjee, Kaniel, and Kremer (2009).

The implications of cursedness differ sharply from those of dismissiveness and overconfidence for the question of trading volume. While per-trader volume increases with the number of traders under cursedness, it converges to zero under overconfidence. Intuitively, even though each overconfident trader thinks that he knows more than he does, he understands that the total amount of "valid" information revealed by the price in a large market swamps his own information. Hence, the same no-trade logic that prevails in REE also prevails in large markets of overconfident traders. The same is true for dismissive traders when they understand correctly that others' private signals are conditionally independent. When dismissive traders mistakenly assume some correlation, per-trader volume converges to a positive limit, and hence aggregate volume converges to infinity. Yet, per-trader volume can be decreasing, hump-shaped or increasing in the number of traders, while it is always increasing under cursedness.

Additional differences concern the behavior of volume when private information is revealed publicly. Public revelation of traders' private signals does not change overconfident or dismissive volume because such volume is generated by disagreements about signal precisions, which persist even when signals are made public. By contrast, cursed volume shrinks to zero because it stems from traders' failure to infer the signals from the price, and this failure becomes irrelevant when signals are public.

In addition to generating large trading volume in the absence of other errors, cursedness enables overconfidence to have large effects on volume. Indeed, *cursed* overconfident traders fail to infer the average signal from the price, so they trade even in a large market—and more so the more overconfident they are. In this sense, cursedness and overconfidence work as complements, and cursedness helps vindicate the basic intuition from the literature that overconfidence can be a significant source of trading volume. Cursedness may similarly exacerbate other biases as well, as

we argue in Section 6, where we conclude the paper.

In Section 4, we extend the model in three different directions. First, we allow traders to observe a public signal. Whereas private signals continue to affect price less than in REE, the public signal influences price *more* than it does in REE. This is because cursed traders use fewer signals than rational traders, so they attach larger weight to each signal that they do use. Because observing the public signal induces traders to attach less weight to their private signals, cursed volume decreases. The latter result is in the spirit of public revelation of private signals reducing cursed trade.

Our second extension is to allow for random endowments. As in, e.g., Akerlof (1970) and Hirshleifer (1971), asymmetric information about asset values impedes non-speculative trade because traders worry that others' trades reflect such information rather than hedging needs. Conversely, public revelation of traders' private signals causes volume to increase. Asymmetric information also impedes dismissive trade, but stimulates cursed trade when the variance of hedging needs is small.

Our final extension is to allow traders to differ in signal precision, risk aversion, and cursedness. When some traders are cursed and some are rational, the latter exploit the positive autocorrelation of price changes induced by the former. Because of the predictability-induced trading by rationals, volume is larger in markets that include both rational and cursed traders than in those comprising solely cursed traders.

The link between non-inference from price and positively autocorrelated price changes was first shown in Hong and Stein (1999). Some or all of their traders are “newswatchers,” assumed to trade based on signals or news they watch without inverting price to infer unwatched news. Hong and Stein show that prices move predictably when information diffuses gradually, yielding positive autocorrelation. A key difference between our work and theirs is that we analyze trading volume and compare its level to that predicted by alternative models such as overconfidence and dismissiveness. Vives and Yang (2017) propose an optimal-inattention-style variant of partial cursedness in which each trader observes the price but employs a noisy signal of it to infer the information that it contains and can pay a cost to reduce the noise. They endogenize traders' sophistication levels and show that sophistication acquisition can exhibit complementarities.<sup>6</sup>

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<sup>6</sup>Kondor and Köszegi (2017) apply cursedness to financial innovation. Issuers in their model design securities using their payoff-relevant private information, which investors fail to infer. Investors are worse off when issuers

## 2 Model and Equilibrium Concept

We begin this section by defining cursed-expectations equilibrium in a general version of our model. We then make more specific assumptions on traders' utility functions and the distribution of their information that allow us to derive analytically tractable, linear equilibria.

There are two periods, 1 and 2, and two assets that pay off in terms of a consumption good in Period 2. One asset is riskless and pays off one unit of the consumption good with certainty. The other asset is risky and pays  $d = \bar{d} + \epsilon + \zeta$  units, where  $\bar{d}$  is a constant and  $(\epsilon, \zeta)$  are random variables with mean zero. We use the riskless asset as the numeraire, and denote by  $p$  the price of the risky asset in Period 1. Our choice of numeraire implies that the price of the risky asset in Period 2 is  $d$  and the riskless rate is zero. We assume that the risky asset is in zero supply.

There are  $N$  traders who can exchange the two assets in Period 1. Trader  $i = 1, \dots, N$  observes the private signal

$$s_i = \epsilon + \eta_i, \tag{1}$$

as well as the public signal

$$s = \epsilon + \theta, \tag{2}$$

which is also observed by all other traders. The random variables  $(\{\eta_i\}_{i=1, \dots, N}, \theta)$  have mean zero. The signals are observed in Period 1. They provide information about the component  $\epsilon$  of the risky asset's payoff but not about  $\zeta$ .

Trader  $i$  starts with a zero endowment of the riskless and the risky assets, and receives an endowment  $z_i d$  of the consumption good in Period 2. We refer to  $z_i$  as the endowment shock, and assume that it is observed privately by trader  $i$  in Period 1 and has mean zero. Through its correlation with  $d$ , the endowment generates a hedging motive to trade. When  $z_i > 0$ , for example, trader  $i$  is exposed to the risk that  $d$  will be low and wishes to hedge by selling the risky asset. We

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can securitize a larger pool of underlying assets or can create more securities out of the pool—a result reversing the traditional logic that diversification benefits investors.



assume that the variables  $(\epsilon, \zeta, \{\eta_i\}_{i=1,\dots,N}, \theta, \{z_i\}_{i=1,\dots,N})$  are mutually independent.

The budget constraint of trader  $i$  is

$$c_i = x_i(d - p) + z_i d, \quad (3)$$

where  $x_i$  denotes the number of shares of the risky asset that the trader buys in Period 1 and  $c_i$  denotes the trader's consumption in Period 2. Negative values of  $x_i$  correspond to shares sold. We impose no portfolio constraints, allowing  $x_i$  to take any value in  $\mathbb{R}$ .

Traders maximize expected utility of consumption. We denote by  $u_i(c_i)$  the utility that trader  $i$  derives from consumption in Period 2. If the trader is rational, then he maximizes the expected utility

$$\mathbb{E}[u_i(x_i(d - p) + z_i d) | \{s_i, s, z_i, p\}]$$

in Period 1, where we use (3) to substitute for  $c_i$ . A rational trader conditions his estimate of the asset payoff  $d$  on his private signal, the public signal, the endowment shock, and the price. If instead the trader is *fully cursed*, then he completely neglects the relationship between the price and other traders' information, and maximizes the expected utility

$$\mathbb{E}[u_i(x_i(d - p) + z_i d) | \{s_i, s, z_i\}],$$

which differs from the rational utility because there is no conditioning on the price. Full cursedness can be viewed as a form of inattention: a fully cursed trader neglects to think through the information that the market price conveys. We also allow for behavior that lies between rationality and full cursedness. If a trader is *partially cursed*, then he infers the information conveyed by price partially but not fully, and maximizes the utility

$$\mathbb{E}[u_i(x_i(d - p) + z_i d) | \{s_i, s, z_i, p\}]^{1-\chi_i} \mathbb{E}[u_i(x_i(d - p) + z_i d) | \{s_i, s, z_i\}]^{\chi_i},$$

which is a geometric average of the rational expected utility with weight  $1 - \chi_i$  and the fully cursed

expected utility with weight  $\chi_i$ . The parameter  $\chi_i \in [0, 1]$  measures the extent of cursedness:  $\chi_i = 0$  corresponds to rationality,  $\chi_i = 1$  to full cursedness, and  $\chi_i \in (0, 1)$  to partial cursedness. We employ the geometric average of utilities rather than the arithmetic average as in Eyster and Rabin (2005) for tractability.

The objective function of partially cursed traders involves two information sets, the one under rationality and the one under full cursedness. Hence, these traders may appear to have two conflicted selves, a rational and a fully cursed one. Indeed, to the extent that they actively interrogate others' trading motivations, traders may discern the information content in prices; but to the extent that they dwell upon their own private information, traders may overlook that connection. Consequently, a trader's demand may vary with his focus. Under that interpretation, a partially cursed trader who reaches two different conclusions about demand when thinking about the problem in two different ways simply averages the two demand functions. Alternatively (and in a somewhat similar spirit) we can interpret the partially cursed traders' objective function as an "as if" one: this objective function captures in a compact way the idea that traders partially neglect the information conveyed by price. Consistent with this interpretation, the maximization of the partially cursed traders' objective yields a demand function that always lies between the rational and the fully cursed one.

One could alternatively conceptualize partially cursed traders as perceiving the price correctly for budgeting while simultaneously overestimating its noisiness for the purpose of inference. That is, partially cursed traders observe  $p$ —and understand that the risky asset costs  $p$ —but for inference believe that they instead observe  $p + \phi$ , where  $\phi$  is noise. This alternative model maps closely to ours. In particular, rational behavior corresponds to zero variance of  $\phi$ , fully cursed behavior to infinite variance, and partially cursed behavior to intermediate values. Vives and Yang (2017) analyze a model in a similar spirit. They assume that when traders infer from the price, they evaluate a noisy signal of price,  $p + \phi$ , rather than price  $p$ . (In the interpretation sketched above, by contrast, traders treat the market price  $p$  as if it were  $p + \phi$ .)

Our definition of cursed-expectations equilibrium (CEE) combines utility maximization under cursed expectations with market clearing. As in the case of rational-expectations equilibrium

(REE), the equilibrium involves a price function  $p$  that depends on all the random variables in the model. These are the private signals  $\{s_i\}_{i=1,\dots,N}$ , the public signal  $s$ , and the endowment shocks  $\{z_i\}_{i=1,\dots,N}$ .

**Definition 1** A price function  $p(\{s_i\}_{i=1,\dots,N}, s, \{z_i\}_{i=1,\dots,N})$  and demand functions  $\{x_i(s_i, s, z_i, p)\}_{i=1,\dots,N}$  are a cursed-expectations-equilibrium (CEE) if:

(i) (**Optimization**) For each trader  $i = 1, \dots, N$ , and each  $(s_i, s, z_i, p)$ ,

$$x_i \in \arg \max_x \{E[u_i(x(d-p) + z_i d) | \{s_i, s, z_i, p\}]^{1-\chi_i} E[u_i(x(d-p) + z_i d) | \{s_i, s, z_i\}]^{\chi_i}\}, \quad (4)$$

(ii) (**Market Clearing**) For each  $(\{s_i\}_{i=1,\dots,N}, s, \{z_i\}_{i=1,\dots,N})$ ,

$$\sum_{i=1}^N x_i = 0. \quad (5)$$

We next specialize our analysis by making two assumptions that allow us to derive tractable linear equilibria. First, the variables  $(\epsilon, \zeta, \{\eta_i\}_{i=1,\dots,N}, \theta, \{z_i\}_{i=1,\dots,N})$  follow normal distributions, with variances denoted by  $(\sigma_\epsilon^2, \sigma_\zeta^2, \{\sigma_{\eta_i}^2\}_{i=1,\dots,N}, \sigma_\theta^2, \{\sigma_{z_i}^2\}_{i=1,\dots,N})$  and precisions, i.e., the inverses of the variances, denoted by  $(\tau_\epsilon, \tau_\zeta, \{\tau_{\eta_i}\}_{i=1,\dots,N}, \tau_\theta, \{\tau_{z_i}\}_{i=1,\dots,N})$ . Second, traders have negative exponential, or constant absolute risk aversion (CARA), utility functions:  $u_i(c_i) = -\exp(-\alpha_i c_i)$ , where  $\alpha_i$  is the coefficient of absolute risk aversion.

A linear CEE price function has the form

$$p = \bar{d} + \sum_{i=1}^N A_i s_i + B s - \sum_{i=1}^N C_i z_i, \quad (6)$$

for coefficients  $(\{A_i\}_{i=1,\dots,N}, B, \{C_i\}_{i=1,\dots,N})$ . For CARA utility, we can write the expectations in (4) as

$$E[u_i(x(d-p) + z_i d) | I_i] = -\exp \left[ -\alpha_i \left( x_i (E(d|I_i) - p) + z_i E(d|I_i) - \frac{1}{2} \alpha_i (x_i + z_i)^2 \text{Var}(d|I_i) \right) \right], \quad (7)$$

where the information set  $I_i$  is equal to  $I_{ir} \equiv \{s_i, s, z_i, p\}$  for the rational expected utility and to  $I_{ic} \equiv \{s_i, s, z_i\}$  for the fully cursed expected utility. The second step in (7) follows because all variables are normally distributed. Substituting (7) into (4) and maximizing, we find the demand function

$$x_i = \frac{(1 - \chi_i)\mathbf{E}(d|I_{ir}) + \chi_i\mathbf{E}(d|I_{ic}) - p}{\alpha_i[(1 - \chi_i)\text{Var}(d|I_{ir}) + \chi_i\text{Var}(d|I_{ic})]} - z_i. \quad (8)$$

The demand function is the solution to a mean-variance problem. The conditional expectation of the asset payoff in that problem is the weighted average of the rational expectation with weight  $1 - \chi_i$  and the fully cursed expectation with weight  $\chi_i$ . The conditional variance of the asset payoff is the same weighted average of the rational and fully cursed variances. The geometric average formulation of utilities ensures that traders' optimization problems retain a tractable mean-variance structure even under partial cursedness. Combining (8) with the market-clearing condition (5), we derive conditions in Proposition 1 so that (6) is an equilibrium price. Proposition 1 does not show existence or uniqueness of  $(\{A_i\}_{i=1,\dots,N}, B, \{C_i\}_{i=1,\dots,N})$  satisfying these conditions, both of which are instead demonstrated in the special cases studied in subsequent sections.

To state Proposition 1, we introduce some notation. From the perspective of a rational trader  $i$ , the price (6) includes information on  $(s_i, s, z_i)$ , which the trader knows, and on  $(\{s_i\}_{j \neq i}, \{z_i\}_{j \neq i})$ , which he does not. The latter information is summarized in the signal

$$\frac{\sum_{j \neq i} A_j s_j - \sum_{j \neq i} C_j z_j}{\sum_{j \neq i} A_j}, \quad (9)$$

which the trader can extract from the price. Using (1), we can write this signal as  $\epsilon + \xi_i$ , where

$$\xi_i \equiv \frac{\sum_{j \neq i} A_j \eta_j - \sum_{j \neq i} C_j z_j}{\sum_{j \neq i} A_j}. \quad (10)$$

We denote the variance of  $\xi_i$  by  $\sigma_{\xi_i}^2$  and its precision by  $\tau_{\xi_i}$ .

**Proposition 1** *The price (6) is an equilibrium price if and only if  $(\{A_i\}_{i=1,\dots,N}, B, \{C_i\}_{i=1,\dots,N})$*

satisfy the conditions

$$\begin{aligned}
& \frac{\tau_{\eta_i}(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i}) - (1 - \chi_i) \tau_{\xi_i} \frac{A_i}{\sum_{k \neq i} A_k} (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)}{\alpha_i [(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})]} \\
&= A_i \sum_{j=1}^N \frac{(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j}) - (1 - \chi_j) \tau_{\xi_j} \frac{1}{\sum_{k \neq j} A_k} (\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)}{\alpha_j [(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \chi_j \tau_{\xi_j})]}, \tag{11}
\end{aligned}$$

$$\begin{aligned}
& \sum_{i=1}^N \frac{\tau_\theta (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})}{\alpha_i [(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})]} \\
&= B \sum_{j=1}^N \frac{(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j})}{\alpha_j [(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \chi_j \tau_{\xi_j})]}, \tag{12}
\end{aligned}$$

$$C_i = A_i \alpha_i \frac{(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})}{\tau_\zeta \tau_{\eta_i} (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})}. \tag{13}$$

In addition to the price, we are interested in trading volume. We define the volume generated by trader  $i$  as the absolute value of the number  $x_i$  of shares of the risky asset that trader  $i$  buys in equilibrium, or sells if  $x_i$  is negative. The aggregate volume is the sum of the volume generated by each trader. We compute expected volume, defined as the unconditional expectation of volume over the realizations of all random variables in the model.

### 3 Equilibrium

In this section, we solve for the equilibrium in the baseline case where traders are symmetric, receive no random endowments, and observe only their private signals and not the public one. We compute the price and trading volume, and compare cursedness to overconfidence and dismissiveness.

To specialize the equilibrium conditions derived in Proposition 1 to symmetric traders, we set private-signal precisions  $\tau_{\eta_i}$ , risk-aversion coefficients  $\alpha_i$ , and cursedness parameters  $\chi_i$  to values  $(\tau_\eta, \alpha, \chi)$  common for all traders. To dispense with random endowments, we set the variances  $\{\sigma_{z_i}^2\}_{i=1, \dots, N}$  to zero, so that the endowment shocks are equal to their mean which is zero. To eliminate the public signal, we set its precision  $\tau_\theta$  to zero. We relax all these restrictions in Section 4.

### 3.1 Price and Trading Volume

With symmetric traders, no random endowments, and no public signal, the price (6) simplifies to

$$p = \bar{d} + A \sum_{i=1}^N s_i. \quad (14)$$

**Proposition 2** *Suppose that traders are symmetric with cursedness parameter  $\chi$ , receive no random endowments, and observe only their private signals. The price (14) is an equilibrium price if and only if*

$$A = \frac{\tau_\eta ([N - \chi(N - 1)] \tau_\epsilon + N\tau_\eta)}{N(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + N\tau_\eta)}. \quad (15)$$

*The coefficient  $A$  decreases in  $\chi$ . For  $\chi > 0$ , price changes exhibit positive autocorrelation: the regression*

$$d - p = \beta(p - \bar{d}) + \nu, \quad (16)$$

*yields coefficient  $\beta > 0$ . The expected volume that each trader generates is*

$$\frac{\chi\tau_\zeta(\tau_\epsilon + N\tau_\eta)\sqrt{2(N-1)\tau_\eta}}{\alpha[\tau_\zeta(\tau_\epsilon + [1 + \chi(N-1)]\tau_\eta) + (\tau_\epsilon + \tau_\eta)(\tau_\epsilon + N\tau_\eta)]\sqrt{\pi N}}. \quad (17)$$

*Volume increases in  $\chi$  and  $N$ .*

When traders are rational ( $\chi = 0$ ), the price equals the expected value of the asset payoff  $d$  conditional on all the private signals. The result that the price aggregates the private signals efficiently is as in Grossman (1976). Moreover, trading volume is zero, consistent with the no-trade theorem of Milgrom and Stokey (1982) and Tirole (1982). The no-trade theorem applies because traders start with zero endowments in the risky asset and receive no random endowments, so no-trade is a Pareto-efficient allocation.

When traders are fully cursed ( $\chi = 1$ ), they do not condition on the price, and hence the private signal  $s_i$  of a trader  $i$  receives no weight in other traders' conditional expectations of the

asset payoff. As a consequence, the weight of  $s_i$  on the price, i.e., the coefficient  $A$ , is smaller in the fully cursed case than in the rational case. The same logic carries through to partial cursedness:  $A$  is smaller when traders are partially cursed than when they are rational, and decreases in  $\chi$ , i.e., is smaller when traders are more cursed. Since  $A$  is smaller than in the rational case, the price under-reacts to the private signals.

The price under-reaction implies positively autocorrelated price changes. The positive autocorrelation is reflected in the regression (16). The dependent variable in (16) is the price change between Period 1, in which the asset trades at  $p$ , and Period 2, in which the asset pays off  $d$ . The independent variable is the price change between a Period 0, in which private signals have not yet been revealed and the asset trades at the unconditional expectation  $\bar{d}$  of its payoff, and Period 1. The regression coefficient  $\beta$  is positive, meaning that a price rise in Period 1 predicts a further price rise, and vice-versa for a price drop.

Since fully cursed traders do not learn others' signals from the price, they trade with each other even without random endowments. Moreover, the expected volume that each generates increases in market size as measured by the number  $N$  of traders. To explain the intuition for the latter result, we recall the demand function  $x_i$  of a trader  $i$ , given by (8). With fully cursed traders and no random endowments, (8) implies that the volume  $|x_i|$  generated by trader  $i$  is proportional to the discrepancy  $|E(d|I_{ic}) - p|$  between the trader's conditional expectation of the asset payoff and the price. The conditional expectation is

$$E(d|I_{ic}) = E(d|s_i) = \bar{d} + \frac{\tau_\epsilon}{\tau_\epsilon + \tau_\eta} s_i,$$

and the price is the average of traders' conditional expectations because of symmetry and no random endowments. Therefore,

$$|E(d|I_{ic}) - p| = \left| \frac{\tau_\epsilon}{\tau_\epsilon + \tau_\eta} \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right) \right|.$$

As  $N$  increases, the signal  $s_i$  of trader  $i$  becomes more discordant with the average signal  $\frac{\sum_{j=1}^N s_j}{N}$ . Hence, the discrepancy  $|E(d|I_{ic}) - p|$  increases, and so does the volume that trader  $i$  generates.<sup>7</sup> Since per-trader volume increases in  $N$ , aggregate volume converges to infinity when  $N$  becomes large. Hence, cursedness produces large volume in large markets with dispersed private information.

The result that per-trader volume increases with  $N$  extends to partially cursed traders. Indeed, (8) implies that the volume  $|x_i|$  that a partially cursed trader  $i$  generates is proportional to

$$|(1 - \chi)E(d|I_{ir}) + \chi E(d|I_{ic}) - p| = \chi |E(d|I_{ic}) - p|,$$

where the equality follows from  $E(d|I_{ir}) = p$ . The discrepancy between conditional expectation and price is therefore proportional to that for a fully cursed trader, with proportionality coefficient  $\chi$ . As  $\chi$  increases, so does volume.

Since there are no aggregate gains from trade and traders are symmetric, they are all made worse off by trading. Traders take on excessive risk: they hold risky positions while in fact they should be bearing no risk.

### 3.2 Comparison to Overconfidence and Dismissiveness

In this subsection, we examine the relationship between cursedness and other theories that have been used in the literature to explain large trading volume. Under all the alternative theories that we consider, traders exaggerate the precision of their own signals relative to the precision of others' signals. Such beliefs have often been described as overconfidence, but we distinguish between different forms of overconfidence and use different terms to describe them.

We reserve the term *overconfidence* for its seemingly most direct form, whereby traders exaggerate the precision of their private signal. With symmetric traders, this means that each trader  $i$  perceives the precision of his own signal  $s_i$  to be  $\kappa\tau_\eta$  for  $\kappa > 1$ . When trader  $i$  is merely overconfident, he correctly perceives the precision of all other traders' signals  $s_j$ ,  $j \neq i$ , to be  $\tau_\eta$ .

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<sup>7</sup>Formally,  $E\left|s_i - \frac{\sum_{j=1}^N s_j}{N}\right|$  increases in  $N$  because it is equal to  $E\left|\eta_i - \frac{\sum_{j=1}^N \eta_j}{N}\right|$ , as implied by (1), and because the noise terms  $\{\eta_j\}_{j=1,\dots,N}$  are independent.



We use the term *dismissiveness* for beliefs under which traders underestimate the precision of others' signals. With symmetric traders, this means that each trader  $i$  incorrectly perceives the precision of all other traders' signals  $s_j$ ,  $j \neq i$ , to be  $\gamma\tau_\eta$  for  $\gamma \in [0, 1)$ . When trader  $i$  is merely dismissive, he correctly perceives the precision of his own private signal  $s_i$  to be  $\tau_\eta$ .

We allow dismissive traders to not only underestimate the precision of others' signals but to also overestimate the correlation of the noise terms. That is, trader  $i$  can perceive incorrectly that the noise terms  $\eta_j$  and  $\eta_{j'}$  for  $j, j' \neq i$  are positively correlated with coefficient  $\rho > 0$ , while in fact they are independent. That traders perceive some non-existent positive correlation is a form of dismissiveness because it causes them to underestimate the information content of the collection of others' signals (rather than of each signal separately).

We distinguish between overconfidence and dismissiveness because they are conceptually different and yield different equilibrium properties. We consider dismissive beliefs over both precision and correlation because equilibrium properties also can differ. We assume that the beliefs of overconfident or dismissive traders about the probability distribution of signals are common knowledge, and hence traders agree to disagree. For example, it is common knowledge that each overconfident trader thinks that he is better informed than all other traders think he is.

We nest overconfidence and dismissiveness in a single model, i.e., each trader can be both overconfident and dismissive, and his dismissive beliefs can concern both precision and correlation. In our nested model, trader  $i$  believes that private signals are given by (1) with  $\text{Var}(\eta_i) = \frac{1}{\kappa\tau_\eta}$ ,  $\text{Var}(\eta_j) = \frac{1}{\gamma\tau_\eta}$  for  $j \neq i$ ,  $\text{Cov}(\eta_i, \eta_j) = 0$  for  $j \neq i$ , and  $\text{Cov}(\eta_j, \eta_{j'}) = \frac{\rho}{\gamma\tau_\eta}$  for  $j \neq j'$  and  $j, j' \neq i$ . Our nested model allows us to isolate the effects of each bias by setting the parameters corresponding to the other biases to their values under rational expectations:  $\kappa$  and  $\gamma$  to one, and  $\rho$  to zero.

Our modelling of overconfidence follows Odean (1998), whom we also follow in modelling dismissiveness as underestimation of the precision of others' signals. Modelling dismissiveness as overestimation of correlations is in the spirit of Banerjee (2011), who also allows for underestimation of precisions. In Banerjee, each trader  $i$  observes a private signal  $s_i = \epsilon + \eta_i$  and assumes that

the signal of each other trader  $j \neq i$  is

$$s_j = \hat{\rho}\epsilon + \sqrt{(1 - \hat{\rho}^2)}\phi_i + \eta_j,$$

where  $\hat{\rho} \in [0, 1]$  and  $\phi_i$  is a random variable that is independent of  $\epsilon$ . Trader  $i$  further assumes that  $\phi_i$  has the same distribution as  $\epsilon$ , and perceives correctly the precisions of  $\epsilon$  and  $\eta_j$  (within his misspecified model for  $s_j$ ). If  $\hat{\rho} < 1$ , then trader  $i$  underestimates the precision of trader  $j$ 's signal because he assumes that it includes the additional noise term  $\sqrt{(1 - \hat{\rho}^2)}\phi_i$ . Because that term is independent of  $j$ , trader  $i$  also overestimates the correlation of the noise in others' signals. Note that the parameter  $\hat{\rho}$  in Banerjee plays an inverse role to  $\rho$  in our model: trader  $i$ 's estimated correlation of the noise in others' signals is decreasing in  $\hat{\rho}$  in Banerjee but is equal to (and hence increasing in)  $\rho$  in our model.<sup>8</sup>

**Proposition 3** *Suppose that traders are symmetric and not cursed, receive no random endowments, and observe only their private signals. Suppose also that each trader perceives the precision of his private signal to be  $\kappa\tau_\eta$  for  $\kappa \geq 1$ , the precision of every other trader's signal to be  $\gamma\tau_\eta$  for  $\gamma \in [0, 1]$ , and the correlation between the noise terms in others' signals to be  $\rho \in [0, 1]$ . The price (14) is an equilibrium price if and only if*

$$A = \frac{\left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right)\tau_\eta}{N \left[\tau_\epsilon + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right)\tau_\eta\right]}. \quad (18)$$

The coefficient  $A$  increases in  $\kappa$  and  $\gamma$ , and decreases in  $\rho$ . The expected volume that each trader generates is

$$\frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right)\tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \left[\tau_\epsilon + \tau_\zeta + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right)\tau_\eta\right] \sqrt{\pi N}}. \quad (19)$$

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<sup>8</sup>Banerjee does not require the noise terms  $\eta_i$  for  $i = 1, \dots, N$  to be independent, as we do in our model. An additional difference between our specification and his is that we assume that  $\epsilon$  enters with a unit coefficient in  $s_j$  as perceived by trader  $i$ , while the coefficient is  $\hat{\rho}$  in his model.

Volume increases in  $\kappa$  and  $\rho$ , and decreases in  $\gamma$ . Volume decreases in  $N$  if

$$\frac{\gamma(\kappa - \gamma)\tau_\eta}{\tau_\epsilon + \tau_\eta + 2\kappa\tau_\eta} - \gamma\rho > \frac{(\kappa - \gamma)[\tau_\epsilon + \tau_\zeta + (\kappa + \gamma)\tau_\eta]}{4(\tau_\epsilon + \tau_\eta + 2\kappa\tau_\eta)}, \quad (20)$$

increases in  $N$  if

$$\frac{\kappa\rho[\rho(\tau_\epsilon + \tau_\zeta) + (\kappa\rho + \gamma)\tau_\eta]}{2(\tau_\epsilon + \tau_\eta + 2\kappa\tau_\eta)} > \frac{\gamma(\kappa - \gamma)\tau_\eta}{\tau_\epsilon + \tau_\eta + 2\kappa\tau_\eta} - \gamma\rho, \quad (21)$$

and is hump-shaped in  $N$  for values of  $\frac{\gamma(\kappa - \gamma)\tau_\eta}{\tau_\epsilon + \tau_\eta + 2\kappa\tau_\eta} - \gamma\rho$  in the intermediate region. If  $\gamma > 0$  and  $\rho = 0$ , then volume converges to zero as the number  $N$  of traders grows large, and aggregate volume, summed across traders, converges to a positive limit. If  $\gamma = 0$  or  $\rho > 0$ , then volume converges to a positive limit as  $N$  grows large, and aggregate volume converges to infinity.

Overconfidence and dismissiveness have opposite effects on the price. Fixing the dismissiveness parameters  $(\gamma, \rho)$ , more overconfidence (larger  $\kappa$ ) causes traders to attach larger weight to their own private signals. As a consequence, the weight of the signals on the price, i.e., the coefficient  $A$ , increases. Fixing instead the overconfidence parameter  $\kappa$ , more dismissiveness (smaller  $\gamma$  or larger  $\rho$ ) causes traders to attach smaller weight to other traders' private signals, as revealed by the price. This causes  $A$  to decrease.

The effect of dismissiveness on the price goes in the same direction as that of cursedness. Indeed, in both cases the coefficient  $A$  decreases relative to the rational case, and this happens because traders underweight others' signals. Cursed traders underweight others' signals because they fail to infer them from the price. Dismissive traders infer those signals from the price, but view them as less informative than they actually are. In both cases the price under-reacts to the signals, and price changes are positively autocorrelated.

Cursedness and dismissiveness have different implications for trading volume. The differences are sharpest when  $\gamma > 0$  and  $\rho = 0$ , i.e., dismissive traders do not treat others' signals as pure noise and perceive correctly that the noise terms in those signals are independent. Recall from Proposition 2 that per-trader volume under cursedness increases as the number  $N$  of traders increases. Hence, when  $N$  grows large, per-trader volume converges to a positive limit and aggregate volume

converges to infinity. Proposition 3 shows instead that per-trader volume under overconfidence or dismissiveness converges to zero, and aggregate volume converges to a finite limit. Thus, overconfidence does not generate large aggregate volume in large markets with dispersed information, in contrast to cursedness. Dismissiveness does not generate large volume either, when  $\gamma > 0$  and  $\rho = 0$ .

The ability of overconfident or dismissive traders to infer others' signals from the price is key to why they trade little in large markets. Indeed, such traders realize that the price fully reveals the average signal of all other traders. And while they underestimate the precision of others' signals relative to their own signal, they understand that their own signal carries much less information than the average of a large number of other, even less precise, signals. In large markets, therefore, overconfident or dismissive traders base their expectations about the asset payoff almost exclusively on the price. As a result, the difference between any two traders' expectations converges to zero, and so does per-trader volume. By contrast, cursed traders do not fully realize that the price reveals the average signal of other traders. Hence, they give their signal non-negligible weight even in large markets when forming their expectations about the asset payoff, and per-trader volume does not converge to zero.

The different implications that cursedness and dismissiveness have for trading volume concern not only the large  $N$  limit but also the comparative statics with respect to  $N$ . The differences in comparative statics for large  $N$  follow directly from previous results. Since per-trader volume under overconfidence and dismissiveness converges to zero when  $N$  grows large, it decreases with  $N$  for large  $N$ . By contrast, per-trader volume under cursedness increases in  $N$  for all values of  $N$ , so for large  $N$  changes in  $N$  have opposite effects on volume. These differences carry through to all values of  $N$  if signals are precise enough ( $\tau_\eta$  large) and overconfidence and dismissiveness are not too extreme ( $\gamma$  is not close to zero and  $\kappa$  is not much larger than one). Indeed, Proposition 3 shows that overconfident and dismissive volume are decreasing in  $N$  if  $(3\gamma - \kappa)\tau_\eta > \tau_\epsilon + \tau_\zeta$  and are hump-shaped in  $N$  otherwise.

Cursed and dismissive volume become more similar when  $\gamma = 0$  or  $\rho > 0$ . When  $\gamma = 0$ , dismissive traders perceive others' signals as being pure noise, and hence ignore them completely when forming their expectations of the asset payoff. This is observationally equivalent, in the

context of our model, to fully cursed traders failing to infer the signals from price. (As we note below, however, the observational equivalence breaks down when private signals are revealed publicly to all traders.) In particular, price and trading volume are identical when  $\chi = 1$  (full cursedness) and when  $\kappa = 1$  and  $\gamma = 0$  (no overconfidence and extreme dismissiveness). Hence, per-trader volume increases in  $N$ , and aggregate volume converges to infinity when  $N$  grows large. The result that per-trader volume under dismissiveness converges to zero, shown for  $\gamma > 0$  and  $\rho = 0$ , breaks down because traders view the average of pure-noise signals also as pure noise.

When  $\rho > 0$ , dismissive traders perceive incorrectly that the noise terms in others' signals are correlated, and hence do not view the average of a large number of such signals as much more informative than their own signal. As a result, per-trader volume under dismissiveness does not converge to zero when  $N$  grows large, but converges instead to a positive limit that is increasing in  $\rho$ . Proposition 3 also implies that volume is decreasing or hump-shaped in  $N$  when  $\rho$  is small but becomes increasing in  $N$  when  $\rho$  is close to one.

The assumptions  $\gamma = 0$  and  $\rho > 0$  are somewhat strong: under  $\gamma = 0$  each trader treats informative signals as pure noise, and under  $\rho > 0$  he treats independent errors by others as correlated and assumes that he is the only one to avoid the common error. Since these assumptions are required for dismissive volume to be large in large markets with dispersed information, cursedness may be a more plausible explanation for large volume.<sup>9</sup>

Even when  $\gamma = 0$  or  $\rho > 0$ , cursedness and dismissiveness can be distinguished in terms of their implications for trading volume. Suppose that private signals are revealed publicly to all traders. Cursed traders would then learn those signals, and their failure to infer from the price would be inconsequential because the price would not contain any additional information. Hence, cursed volume would decline to zero. By contrast, dismissive volume would remain the same. Indeed, dismissive traders infer others' signals from the price, and trade because they view them as less informative than they actually are. Revealing the signals publicly would not change their

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<sup>9</sup>Alternatively, overconfident or dismissive volume could be large in large markets if information dispersion is limited and does not increase with market size. Suppose that there is a fixed number  $M$  of signals that does not increase with the number  $N$  of traders, and that different groups of traders, of size  $N/M$  each, observe a different signal. Increasing market size would not make the average of the signals more informative because the number of distinct signals in the average would not change. Assumptions along these lines are made, for example, in Odean (1998), Scheinkman and Xiong (2003), and Banerjee, Kaniel, and Kremer (2010).

information. Corollary 1 confirms these results.<sup>10 11</sup>

**Corollary 1** *Suppose that traders are symmetric, receive no random endowments, and observe only their private signals. Public revelation of all private signals would impact volume as follows:*

- *If all traders are cursed, then volume would decline to zero.*
- *If all traders are overconfident or dismissive, then volume would not change.*

### 3.3 Cursedness as an Enabling Bias

Cursedness not only generates large volume in large markets, but can also act as an “enabling bias,” amplifying the effects that other biases may have on volume. Recall from Proposition 3 that per-trader volume when overconfidence is the only bias ( $\kappa \geq 1$ ,  $\chi = 0$ ,  $\gamma = 1$ ,  $\rho = 0$ ) converges to zero as market size  $N$  grows large. Key to this result is that while overconfident traders exaggerate the information content of their signal, they realize that the average signal of all other traders, as revealed by the price, conveys much more information. This effect is suppressed when overconfident traders are also cursed, because there is no learning from the price. Hence, traders who are both overconfident and cursed give their signal non-negligible weight even in large markets, and that weight increases with the extent of overconfidence. Accordingly, per-trader volume in markets with such traders converges to a positive limit as  $N$  grows large, and that limit is larger when traders are more overconfident. Cursedness and overconfidence work as complements in generating trade: overconfidence on its own does not generate large volume in large markets but does so in the presence of cursedness.

**Proposition 4** *Suppose that traders are symmetric with cursedness parameter  $\chi$ , receive no random endowments, and observe only their private signals. Suppose also that each trader perceives*

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<sup>10</sup>Corollary 1 would hold even if the revealed information were the average of traders’ signals rather than each and every signal. This is because with symmetry and normality, the average is a sufficient statistic for all the signals.

<sup>11</sup>An important difference between cursedness on the one hand, and overconfidence and dismissiveness on the other, concerns the type of statistical relationships that people misperceive. Overconfident or dismissive people disagree about the correlations between exogenous variables (private signals and asset payoff), and this creates disagreement about the relationship between endogenous and exogenous variables (price and asset payoff). Cursed people, by contrast, share common beliefs about correlations between exogenous variables and hold opposing beliefs only about the relationship between endogenous and exogenous variables (price and asset payoff). This difference drives Corollary 1. It is important, in particular, that cursed traders can interpret others’ signals correctly when revealed.

the precision of his private signal to be  $\kappa\tau_\eta$  for  $\kappa \geq 1$ , the precision of every other trader's signal to be  $\gamma\tau_\eta$  for  $\gamma \in [0, 1]$ , and the correlation between the noise terms in others' signals to be  $\rho \in [0, 1]$ . The expected volume that each trader generates is

$$\frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho} + \frac{\chi\gamma(\tau_\epsilon + N\kappa\tau_\eta)}{[1+(N-2)\rho](\tau_\epsilon + \kappa\tau_\eta)}\right) \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \left[\tau_\epsilon + \tau_\zeta + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta + \frac{(N-1)\chi\gamma\tau_\zeta\tau_\eta}{[1+(N-2)\rho](\tau_\epsilon + \kappa\tau_\eta)}\right] \sqrt{\pi N}}. \quad (22)$$

If  $\chi > 0$ , then volume converges to a positive limit as the number  $N$  of traders grows large, and that limit increases in the overconfidence parameter  $\kappa$ .

## 4 Extensions

### 4.1 Public Signal

In this section we re-introduce the public signal  $s = \epsilon + \theta$  that was allowed for in our general model but excluded from Section 3. We maintain the other assumptions of Section 3 that traders receive no endowment shocks and are symmetric. The price (6) takes the form

$$p = \bar{d} + A \sum_{i=1}^N s_i + Bs. \quad (23)$$

**Proposition 5** *Suppose that traders are symmetric with cursedness parameter  $\chi$ , receive no random endowments, and observe their private signals and the public signal. The price (23) is an equilibrium price if and only if*

$$A = \frac{\tau_\eta ([N - (N-1)\chi] (\tau_\epsilon + \tau_\theta) + N\tau_\eta)}{N(\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)}, \quad (24)$$

$$B = \frac{\tau_\theta (\tau_\epsilon + [1 + (N-1)\chi] \tau_\eta + \tau_\theta)}{(\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)}. \quad (25)$$

The coefficient  $A$  decreases in  $\chi$  and the coefficient  $B$  increases in  $\chi$ . For  $\chi > 0$ , the regression

$$d - p = \beta_1(p - \bar{d}) + \beta_2 s + \nu, \quad (26)$$

yields coefficients  $\beta_1 > 0$  and  $\beta_2 < 0$ . The expected volume that each trader generates is

$$\frac{\chi\tau_\zeta(\tau_\epsilon + N\tau_\eta + \tau_\theta)\sqrt{2(N-1)\tau_\eta}}{\alpha[\tau_\zeta(\tau_\epsilon + [1 + \chi(N-1)]\tau_\eta + \tau_\theta) + (\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)]\sqrt{\pi N}}, \quad (27)$$

and is lower than when traders do not observe the public signal.

As in Section 3, traders' private signals enter the price with a smaller weight than in the rational case. The public signal, however, enters the price with a larger weight. The intuition is easier to understand in the case where traders are fully cursed. Since they form their conditional expectations of the asset payoff using fewer signals than rational traders, they attach larger weight to each signal they use. The public signal thus receives larger weight in each trader's conditional expectation, and enters the price with a larger weight. The same logic carries through to partial cursedness:  $B$  is larger when traders are partially cursed than when they are rational, and increases in  $\chi$ , i.e., is larger when traders are more cursed.

Because the public signal enters the price with a larger weight than in the rational case, it predicts future price changes negatively. This predictability is revealed from a bivariate regression of the price change between Periods 1 and 2 on the public signal and on the price change between Periods 0 and 1. The regression coefficient  $\beta_2$  on the public signal is negative. The coefficient becomes zero, however, if the price change between Periods 0 and 1 is not controlled for. This is because cursed traders understand the relationship between the public signal and the asset payoff, so if they were to condition their expectation of the payoff on the public signal alone, they would do so correctly.

The last result of Proposition 5 is that observing the public signal lowers volume. The intuition is that cursed traders trade with each other because they observe different private signals and do not learn others' signals from the price. When they also observe the public signal, they give their private signals less weight and hence trade less. This result is in the spirit of Corollary 1 that public revelation of private signals reduces cursed trade.

Observing the public signal lowers volume not only under cursedness but also under overconfidence and dismissiveness, and the intuition is the same. We show this result in the appendix, where



we compute the equilibrium with overconfident and dismissive traders who observe a public signal (Proposition B.1). We assume that overconfident and dismissive traders agree on the precision of the public signal (as do cursed traders in this section). If instead, they disagree, volume could remain the same, as shown in Corollary 1 for the case where private signals are publicly revealed, or perhaps increase.

## 4.2 Random Endowments

In this section we re-introduce the random endowments that were allowed for in our general model but excluded from Section 3. We maintain the other assumptions of Section 3 that traders observe no public signal and are symmetric. We assume that the symmetry extends to the precision of endowment shocks  $\{z_i\}_{i=1,\dots,N}$ , which takes a value  $\tau_z$  common to all traders. The price (6) takes the form

$$p = \bar{d} + A \sum_{i=1}^N s_i - C \sum_{i=1}^N z_i. \quad (28)$$

**Proposition 6** *Suppose that traders are symmetric with cursedness parameter  $\chi$ , receive random endowments, and observe only their private signals and endowment shocks. The price (28) is an equilibrium price if and only if*

$$A = \frac{\tau_\eta \left( \tau_\epsilon + \tau_\eta + \chi \frac{(N-1)\tau_\eta\tau_z}{\tau_z + \frac{C^2}{A^2}\tau_\eta} \right) + (1-\chi)(\tau_\epsilon + \tau_\eta) \frac{(N-1)\tau_\eta\tau_z}{\tau_z + \frac{C^2}{A^2}\tau_\eta}}{N(\tau_\epsilon + \tau_\eta) \left( \tau_\epsilon + \tau_\eta + \frac{(N-1)\tau_\eta\tau_z}{\tau_z + \frac{C^2}{A^2}\tau_\eta} \right)}, \quad (29)$$

and  $\frac{C}{A} > 0$  is the unique solution to the cubic equation

$$\left( \tau_z + \frac{C^2}{A^2}\tau_\eta \right) \left( \frac{C}{A}\tau_\zeta\tau_\eta - \alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta) \right) + \frac{(N-1)\tau_\eta\tau_z}{\tau_\epsilon + \tau_\eta} \left[ \chi \frac{C}{A}\tau_\zeta\tau_\eta - \alpha(\tau_\epsilon + \chi\tau_\zeta + \tau_\eta) \right] = 0. \quad (30)$$

The expected volume that each trader generates is

$$\frac{\left[ \frac{C^2}{A^2} \tau_\eta (\tau_\epsilon + \tau_\eta) + \chi (\tau_\epsilon + N \tau_\eta) \tau_z \right] \sqrt{2(N-1) \left( \tau_z + \frac{C^2}{A^2} \tau_\eta \right)}}{\left[ \left( \tau_z + \frac{C^2}{A^2} \tau_\eta \right) (\tau_\epsilon + \tau_\eta) + \chi (N-1) \tau_\eta \tau_z \right] \frac{C}{A} \sqrt{\pi N \tau_\eta \tau_z}}. \quad (31)$$

Volume increases in  $N$ , for  $\chi \in \{0, 1\}$ .

Random endowments generate trade even among rational traders. This can be confirmed by setting  $\chi = 0$  in (31): when there are no random endowments ( $\tau_z = \infty$ ) rational volume is zero consistent with Proposition 2, and when there are random endowments ( $\tau_z$  finite) rational volume is positive. Eq. (31) implies additionally that per-trader volume in the rational case increases in the number  $N$  of traders. Hence aggregate volume goes to infinity when  $N$  grows large.

Since rational volume is generated by random endowments, Proposition 6 suggests that these endowments should generate large aggregate volume in large markets in all the cases that we consider: rationality, cursedness, overconfidence, and dismissiveness.<sup>12</sup> Eq. (31) indeed implies that per-trader volume converges to a positive limit for all  $\chi \in [0, 1]$ , and hence aggregate volume in the rational and cursed cases is large in large markets. The same result holds for overconfidence and dismissiveness, as we show in the appendix, where we compute the equilibrium with overconfident and dismissive traders who receive random endowments (Proposition B.2).

While the limit behavior of volume when traders receive random endowments is the same under cursedness and under dismissiveness, other properties of volume can differ. Section 3 emphasizes two such properties in the absence of random endowments: the dependence of volume on  $N$ , and the effect of revealing private signals publicly. The differences on how cursed and dismissive volume depend on  $N$ , shown in Section 3, extend to small endowment shocks by continuity. Corollary 2 examines how cursed and dismissive volume change when private signals are revealed publicly. Continuity does not pin down the effect on dismissive volume because there is no effect in the absence of random endowments. Continuity also does not pin down the effect on rational volume

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<sup>12</sup>Although rational traders trade both because of random endowments and private information, random endowments generate rational volume in the sense that volume would be zero in their absence. The contribution of private information to rational volume is, in fact, negative, as shown in Corollary 2: when private signals are revealed publicly, traders trade only because of random endowments and volume goes up.

because that volume is zero in the endowments' absence.

**Corollary 2** *Suppose that traders are symmetric, receive random endowments, and observe only their private signals and endowment shocks. Public revelation of all private signals would impact volume as follows:*

- *If all traders are rational, then volume would increase.*
- *If all traders are cursed, then volume would increase when  $\chi$  is close to zero and decrease when  $\chi$  is close to one.*
- *If all traders are non-fully dismissive ( $\kappa = 1$ ,  $\gamma \in (0, 1)$ , and  $\rho \geq 0$ ), then volume would increase. Volume could decrease, however, if in addition traders are overconfident ( $\kappa > 1$ ).*
- *If all traders are fully dismissive ( $\gamma = 0$ ) or fully overconfident ( $\kappa = \infty$ ), volume would not change.*

Recall from Corollary 1 that in the absence of random endowments, cursed volume drops to zero if signals are publicly revealed because traders learn the average signal and no longer trade on their own signal. In the presence of random endowments, a similar effect appears for both rational and cursed traders: public revelation of the signals induces traders to no longer trade on their own signal because they learn the average signal, rather than a noisy version of it from the price. We term this the *information-trading effect*. At the same time, a new effect appears: public revelation of the signals induces traders to trade more aggressively when the price moves in response to endowment shocks because they are not worried that these movements may instead be due to information. We term this the *risk-sharing effect*.

When traders are rational, the risk-sharing effect dominates the information-trading effect, and public revelation of the signals raises volume. This is a standard result in adverse-selection models (e.g., Akerlof (1970), Hirshleifer (1971)). When traders are fully cursed, the risk-sharing effect is not present, and public revelation of the signals lowers volume. Put differently, not revealing information and keeping it asymmetric impedes trade between rational traders but stimulates trade between fully cursed ones. The case of partial cursedness is in-between the two extremes: public

revelation of the signals causes volume to increase when  $\chi$  is close to zero and to decrease when  $\chi$  is close to one.

The information-trading and risk-sharing effects are also at play under overconfidence and dismissiveness. Recall from Corollary 1 that in the absence of random endowments, public revelation of the signals has no effect on overconfident or dismissive volume because traders can infer others' signals from the price even when they do not observe them. This neutrality result continues to hold with random endowments only in the extreme cases where traders are fully dismissive ( $\gamma = 0$ ) or fully overconfident ( $\kappa = \infty$ ). This is because in both cases they believe that they do not learn useful new information (for  $\gamma = 0$  they view others' signals as noise, and for  $\kappa = \infty$  they believe that they observe a perfectly informative signal). Between the two extreme cases, the information-trading and risk-sharing effects come into play and neutrality does not hold. When traders are non-fully dismissive, the latter effect dominates, and public revelation of the signals increases volume. The former effect instead dominates when traders are also sufficiently overconfident.

Corollary 2 implies that the contrast between cursed and dismissive volume is sharpest when the variance of endowment shocks is small. In that case, public revelation of information lowers cursed volume for most values of  $\chi$  (all values when the variance of endowment shocks is zero) but raises dismissive volume.

### 4.3 Heterogeneous Traders

In this section we allow traders to be asymmetric in terms of their private-signal precision, risk-aversion coefficient, and cursedness parameter. We maintain the other assumptions of Section 3 that traders observe no public signal and receive no random endowments.

We start by allowing traders to differ in their cursedness parameter  $\chi_i$ , and for analytical simplicity assume that some are rational ( $\chi_i = 0$ ) and the rest are fully cursed ( $\chi_i = 1$ ). We denote by  $N_r$  and  $N_c = N - N_r$ , respectively, the numbers of rational and fully cursed traders, and by  $R$  and  $C$  the sets of these traders. The price (6) takes the form

$$p = \bar{d} + A_r \sum_{i \in R} s_i + A_c \sum_{i \in C} s_i. \tag{32}$$

**Proposition 7** *Suppose that  $N_r$  traders are rational and  $N_c = N - N_r$  traders are fully cursed. Traders are otherwise symmetric, receive no random endowments, and observe only their private signals. The price (32) is an equilibrium price if and only if*

$$A_r = qA_c, \quad (33)$$

$$A_c = \frac{\frac{N_r \tau_\xi}{[(N_r - 1)q + N_c](\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi)} + \frac{\tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta}}{\frac{N_r(\tau_\epsilon + \tau_\eta + \tau_\xi)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c(\tau_\epsilon + \tau_\eta)}{\tau_\epsilon + \tau_\zeta + \tau_\eta}}, \quad (34)$$

where

$$\tau_\xi \equiv \frac{[(N_r - 1)q + N_c]^2 \tau_\eta}{(N_r - 1)q^2 + N_c}, \quad (35)$$

and  $q \in (0, 1)$  is the unique solution of

$$q = \frac{N_c(1 - q)(\tau_\epsilon + \tau_\zeta + \tau_\eta)}{[(N_r - 1)q^2 + N_c](\tau_\epsilon + \tau_\zeta + \tau_\eta) + [(N_r - 1)q + N_c]^2 \tau_\eta}. \quad (36)$$

When both rational and fully cursed traders are present in the market ( $1 \leq N_r \leq N - 1$ ), the former trade in the direction of price movements and the latter in the opposite direction: the regression

$$x_i = \beta_i(p - \bar{d}) + \nu, \quad (37)$$

yields coefficient  $\beta_i > 0$  for  $i \in R$  and  $\beta_i < 0$  for  $i \in C$ . Expected aggregate volume, viewed as a function of  $N_r$ , is maximum at an interior point  $1 \leq N_r \leq N - 1$  if

$$\frac{1}{N - 1} \left( \frac{\tau_\epsilon + N\tau_\eta}{\tau_\epsilon + \tau_\eta} \right)^2 + \left[ \frac{2(\tau_\epsilon + \tau_\zeta + N\tau_\eta)}{(N - 1)(\tau_\epsilon + \tau_\zeta + \tau_\eta)} - 1 \right] \frac{\tau_\epsilon + N\tau_\eta}{\tau_\epsilon + \tau_\eta} + \left( \frac{\tau_\epsilon + \tau_\zeta + N\tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \right)^2 < 0. \quad (38)$$

When the shock  $\zeta$  has zero variance, (38) holds if  $N$  exceeds a threshold  $\bar{N}$ .

As in Section 3, the price under-reacts to traders' private signals. When traders differ in their cursedness parameter, price inefficiency takes an additional form. While the price should give the same weight to all signals because they all have the same precision, it overweights the signals of

the fully cursed traders relative to those of the rational traders ( $q = \frac{A_r}{A_c} < 1$ ). This is because rational traders give weight both to their signals and to those of cursed traders when forming their expectations about the asset payoff, while fully cursed traders give weight to their signals only.

The price under-reaction implies positively autocorrelated price changes. Rational traders exploit this predictability by buying in Period 1 if the price rises relative to Period 0, and selling if the price drops. This is reflected in a positive coefficient  $\beta$  in the regression (37) of signed volume  $x_i$  on the price change between Periods 0 and 1. Conversely, the coefficient is negative for cursed traders, who are on the losing side of this trade.

The expected utility of rational traders is higher than that of cursed traders because they learn from the price. In addition, because rational traders have the option not to trade, they are better off relative to not trading. Cursed traders are instead worse off because there are no aggregate gains from trade. Cursed traders are thus “exploited” by rational traders.

Because the predictability-induced trading by rational traders adds to trading volume, a market in which some traders are rational and some are fully cursed can have higher volume than an otherwise identical market where all traders are fully cursed. To show this result, we hold constant the total number  $N$  of traders and change the number  $N_r$  of rational traders. When (38) holds, volume increases when some rational traders enter the market ( $N_r > 0$ ). A sufficient condition for (38) to hold is that the total number  $N$  of traders is large: with a large number of cursed traders, the predictability of price changes induces rational entrants to engage in a sizeable amount of trading.

We next allow traders to differ in their risk aversion coefficient  $\alpha_i$  and private-signal precision  $\tau_{\eta_i}$ . For analytical simplicity, we assume that all traders are fully cursed. The price (6) takes the form

$$p = \bar{d} + \sum_{i=1}^N A_i s_i. \tag{39}$$

**Proposition 8** *Suppose that traders differ in their risk-aversion coefficients  $\alpha_i$  and private-signal precisions  $\tau_{\eta_i}$ , are fully cursed, receive no random endowments, and observe only their private*

signals. The price (39) is an equilibrium price if and only if

$$A_i = \frac{\frac{\tau_{\eta_i}}{\alpha_i(\tau_\epsilon + \tau_\zeta + \tau_{\eta_i})}}{\sum_{j=1}^N \frac{\tau_\epsilon + \tau_{\eta_j}}{\alpha_j(\tau_\epsilon + \tau_\zeta + \tau_{\eta_j})}}. \quad (40)$$

If all traders have the same risk-aversion coefficient  $\alpha$  and the shock  $\zeta$  has zero variance, then the expected trading volume generated by trader  $i$  is

$$\frac{\sqrt{2 [\tau_\epsilon^2 [\tau_\eta + (N-2)\tau_{\eta_i}] + \tau_\epsilon [N\tau_\eta^2 + (N-2)\tau_{\eta_i}^2] + (N\tau_\eta - \tau_{\eta_i})\tau_\eta\tau_{\eta_i}]}}{\alpha(\tau_\epsilon + \tau_\eta)\sqrt{\pi N}}, \quad (41)$$

where  $\tau_\eta$  denotes the average precision of private signals. Trader  $i$  generates more volume than trader  $j$  if and only if he observes a more precise private signal ( $\tau_{\eta_i} > \tau_{\eta_j}$ ).

As in Proposition 7, the price is inefficient both because it under-reacts to traders' private signals and because it does not give the correct relative weights to the signals. In the rational case, where the price equals the expected value of the asset payoff  $d$  conditional on the signals, the weight of a signal  $i$  is proportional to its precision  $\tau_{\eta_i}$  and does not depend on any other characteristic of trader  $i$  (Grossman (1976)). Proposition 8 shows that when traders are fully cursed, the weight is increasing in  $\tau_{\eta_i}$  but not proportionately, and depends on trader  $i$ 's risk aversion coefficient  $\alpha_i$ . In particular, a trader who is less risk averse trades more aggressively on his signal, failing to realize that he trades against others' signals and that his trading activity causes his signal to be overweighed.

Proposition 8 shows additionally that traders with more precise signals trade more. One may conjecture that these traders are better off relative to those with less precise signals, in the same way that rational traders are better off than cursed traders. Surprisingly, however, this conjecture turns out not to be always true, as shown in an earlier version of this paper (Eyster, Rabin, and Vayanos (2015)). On the one hand, cursed traders with more precise signals do not lose as much by trading against others' signals because their signal aligns better with the asset value, e.g., is more likely to be negative when others' signals are negative. On the other hand, they can overtrade, taking on excessive risk, and this effect can dominate.

## 5 Evidence on Cursedness

Cursed equilibrium, as defined by Eyster and Rabin (2005), captures the psychology behind the winner’s curse in common-value auctions—the average price paid by the auction winner exceeds the average value of the object being auctioned—in a manner sufficiently general to be applied across strategic settings. It assumes that people fail to correctly infer other people’s private information from those other people’s actions. In the context of common-value auctions, bidders fail to fully appreciate the bad news inherent in winning, namely that their opponents have found it optimal to bid lower. The winner’s curse has been documented empirically as well as in controlled laboratory settings. Capen, Clapp, and Campbell (1971) is an early empirical study in the context of auctions for oil-drilling rights. Roll (1986) documents the winner’s curse in corporate takeovers. Kagel and Levin (2002) review the voluminous laboratory evidence on the winner’s curse.

The same kind of failure of inference that characterizes bidding in common-values auctions has been uncovered in other strategic settings. Esponda and Vespa (2014) report on a laboratory experiment on voting in which people fail to draw the correct inference from the event that their vote is pivotal. Samuelson and Bazerman (1985) and Holt and Sherman (1994) find that people under-infer each other’s private information in laboratory experiments on positive-sum bilateral trade. Carrillo and Palfrey (2011) find the same in zero-sum bilateral-trade experiments. Failure of inference in bilateral-trade settings implies excessive trade. Weizsäcker (2010) presents a meta-study of a social-learning experiment that documents that people do not learn as much as they should from their predecessors’ choices.

More closely related to our paper are experimental papers that have tested for REE. Plott and Sunder (1988) devise an experimental asset market in which an asset’s payoff takes one of three possible values:  $v \in \{50, 240, 490\}$ . Given true value  $v$ , one-half of the subjects learn that the value is not  $v' \neq v$ , and the other half learn that the value is not  $v'' \neq v, v'$ . For example, when  $v = 50$ , one-half of the people learn  $v \neq 240$ , and the other half that  $v \neq 490$ ; collectively, people’s private information reveals the state. Plott and Sunder show that after several experimental rounds, the prices generated by an oral double auction closely approximate REE prices, namely the true value. Biais, Hilton, Mazurier, and Pouget (2005) essentially replicate Plott and Sunder’s design but find



substantial deviations from REE. Corgnet, DeSantis, and Porter (2017) also replicate the same design and identify prices very different from REE prices. They show that CEE with fully cursed traders, fits their own data as well as the data of Biais et al. better than REE.<sup>13</sup>

Magnani and Oprea (2017) conduct an experiment intended to identify whether cursedness or dismissiveness drives trade. They estimate that 80% of subjects employ cursed reasoning, but also argue that most subjects are dismissive of others' private information.

Evidence that investors do not sufficiently heed the information content of asset prices comes not only from laboratory experiments but also from actual markets. Chague, De-Losso, and Giovannetti (2018) study the behavior of individual investors around "fictitious price falls," defined as events when stock prices drop without a change in company value. One type of such events are dividend payments: on ex-dividend dates, prices fall mechanically by the dividend amount, which is announced well in advance. Chague et al. find that individuals buy stocks on ex-dividend dates, and the more so the larger is the dividend. They also find that a trading strategy exploiting individuals' behavior earns abnormally high returns. These findings are consistent with CEE if some cursed traders do not observe (or not fully understand) news on dividend payments.<sup>14</sup>

Another instance where individual investors appear to be giving undue positive weight to low prices concerns "lottery-like" stocks. Kumar (2009) and Eraker and Ready (2015) find that individuals overinvest in low-priced stocks even though these stocks yield abnormally low returns. To explain their findings, they hypothesize that individuals view low-priced stocks as lottery tickets because of their positively-skewed payoffs (Barberis and Huang (2008)). Using data from the options market, however, Birru and Wang (2016) estimate that investors exaggerate the positive skewness of low-priced stocks, because they fail to fully appreciate that low prices may signal low future value. Among other findings, they show that skewness expectations increase on dates when

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<sup>13</sup>Biais, Hilton, Mazurier, and Pouget (2005) convincingly argue that overconfidence in one's private information should play no role in the information structure that they consider: how could someone who learns that  $v \neq 240$  be overconfident about that information? In the same way, dismissiveness does not seem a likely explanation for non-REE prices. Would a subject who learns that  $v \neq 240$  and hears from the experimenter that one-half of the other subjects learn either  $v \neq 50$  or  $v \neq 490$  really believe that despite the experimenter's instructions other subjects hold no payoff-relevant information?

<sup>14</sup>The findings of Chague, De-Losso, and Giovannetti (2018) are not driven by taxes because they extend to non-taxable dividends. They do not extend to institutional investors, suggesting that only individual investors succumb to the fallacy.

prices fall mechanically because of stock splits.

## 6 Conclusion

In this paper, we propose a new market equilibrium concept, *cursed expectations equilibrium (CEE)*, in which traders fail to fully infer information from market prices. Unlike agreeing-to-disagree models in which traders have differences of opinion about the informativeness of exogenous private signals but correctly infer others' private signals from the price, cursed traders correctly perceive the relationship between all exogenous variables and simply misperceive the relationship between the endogenous price and traders' exogenous private signals.

Cursed traders trade significant quantities and take on excessive risk. We show that cursed volume per trader grows with the size of the market, whereas per-trader volume under overconfidence or dismissiveness may decline to zero. Absent endowment shocks, revealing all private signals would not affect trade due to overconfidence or dismissiveness, but would eliminate cursed trade. Cursedness amplifies trading volume due to overconfidence, thus enabling that bias to have a more significant effect. Markets comprising entirely cursed traders generate more trade than those comprising entirely rational traders; mixed markets can generate more trade still, because rationals exploit the predictability of returns caused by cursed traders.

In Section 3, we showed the necessity in some settings of cursedness to “enable” overconfidence to explain appreciable per-trader volume of trade. We conclude by speculating how cursedness may similarly enable the study of various other biases in asset markets. Researchers have recently proposed that a number of statistical errors may be relevant for financial decisions, including over-inference from small samples (see Rabin (2002) and Rabin and Vayanos (2010)) and non-belief in the law of large numbers (see Benjamin, Rabin, and Raymond (2016)). Predicting the consequences of these and other biases for markets where traders extract information from prices requires additional assumptions about traders' theories of one another's errors. Yet relatively little is known about how people reason about others' errors. In its extreme, cursedness provides a simple assumption about what people think of others' errors: they don't think about them at all. If models of errors are instead closed by assuming that people do agree to disagree about the meaning of private signals,

then, much like with overconfidence in Section 3.3, we suspect that the per-trader volume of trade will be small in information-rich settings where each trader values the sum total of others' private information far more heavily than his own private signal.

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## ONLINE APPENDIX

### A Cursed Equilibrium

Based on evidence from strategic situations, Eyster and Rabin (2005) define *cursed equilibrium* in Bayesian games by the requirement that every player correctly predicts the behavior of others but fails to fully attend to its informational content. In this appendix, we define cursed equilibrium and illustrate its workings in a simple zero-sum game of speculative trade.

Cursed equilibrium is defined in finite Bayesian games of the form

$$(\{A_i\}_{i=1,\dots,N}, \{T_i\}_{i=0,\dots,N}, p, \{u_i\}_{i=1,\dots,N}).$$

For each player  $i = 1, \dots, N$ ,  $A_i$  is a finite set of available actions and  $T_i$  is a finite set of types, including one,  $T_0$ , for nature. We denote the set of action profiles by  $A \equiv \prod_{i=1,\dots,N} A_i$  and the set of type profiles by  $T \equiv \prod_{i=0,\dots,N} T_i$ . We assume that all players share the common prior probability distribution  $p$  over  $T$ . Player  $i$ 's utility function is  $u_i : A \times T \rightarrow \mathbb{R}$ .

A strategy for player  $i$ ,  $\sigma_i : T_i \rightarrow \Delta A_i$ , specifies a probability distribution over actions for each type. We denote by  $\sigma_i(a_i|t_i)$  the probability that type  $t_i$  plays action  $a_i$  when he follows strategy  $\sigma_i$ . We denote the set of action profiles for players other than  $i$  by  $A_{-i} \equiv \prod_{j \neq i} A_j$ , and the set of type profiles for nature and players other than  $i$  by  $T_{-i} \equiv \prod_{j \neq i} T_j$ . We denote by  $a_{-i}$  and  $t_{-i}$  generic elements of these sets. We denote by  $\sigma_{-i}(a_{-i}|t_{-i})$  the probability that types  $t_{-i}$  play action profile  $a_{-i}$  when they follow strategy  $\sigma_{-i} \equiv \{\sigma_j\}_{j \neq 0,i}$ . Finally, we denote by  $p(t_{-i}|t_i)$  the distribution of player  $i$ 's beliefs about other players' types conditional on his own type  $t_i$ . The standard solution concept for these games is Bayesian Nash equilibrium.

**Definition 2** *A strategy profile  $\sigma$  is a Bayesian Nash equilibrium if for each player  $i$ , each type  $t_i \in T_i$ , and each  $a_i^*$  such that  $\sigma_i(a_i^*|t_i) > 0$ :*

$$a_i^* \in \arg \max_{a_i \in A_i} \sum_{t_{-i}} p(t_{-i}|t_i) \left( \sum_{a_{-i}} \sigma_{-i}(a_{-i}|t_{-i}) u_i(a_i, a_{-i}; t_i, t_{-i}) \right). \quad (\text{A.1})$$

To define cursed equilibrium, we compute for each type of each player the average strategy of other players, averaged over the other players' types. For type  $t_i$  of player  $i$  we define

$$\bar{\sigma}_{-i}(a_{-i}|t_i) \equiv \sum_{t_{-i} \in T_{-i}} p_i(t_{-i}|t_i) \cdot \sigma_{-i}(a_{-i}|t_{-i}).$$

This is the marginal probability that other players play action profile  $a_{-i}$ , and is derived by averaging over type profiles  $t_{-i}$  the probabilities  $\sigma_{-i}(a_{-i}|t_{-i})$  that other players play  $a_{-i}$  conditional on  $t_{-i}$ . We associate to each player  $i$  a cursedness parameter  $\chi_i \in [0, 1]$ .

**Definition 3** *A strategy profile  $\sigma$  is a cursed equilibrium if for each player  $i$ , each type  $t_i \in T_i$ , and each  $a_i^*$  such that  $\sigma_i(a_i^*|t_i) > 0$ :*

$$a_i^* \in \arg \max_{a_i \in A_i} \sum_{t_{-i}} p(t_{-i}|t_i) \left( \sum_{a_{-i}} (1 - \chi_i) \sigma_{-i}(a_{-i}|t_{-i}) u_i(a_i, a_{-i}; t_i, t_{-i}) + \chi_i \bar{\sigma}_{-i}(a_{-i}|t_i) u_i(a_i, a_{-i}; t_i, t_{-i}) \right). \quad (\text{A.2})$$

Player  $i$  best-responds to beliefs that with probability  $1 - \chi_i$  the other players' actions depend on their types (the probability of action profile  $a_{-i}$  in (A.2) is conditional on type profile  $t_{-i}$ ) and with probability  $\chi_i$  actions do not depend on types (the probability of  $a_{-i}$  in (A.2) is the marginal). When  $\chi_i = 0$ , player  $i$  is rational, and his objective is as in Bayesian Nash equilibrium (Eq. (A.1)). When  $\chi_i = 1$ , player  $i$  is fully cursed, and neglects entirely the relationship between the other players' actions and their types. Note that while cursed players fail to map actions to types, they assess correctly the probability distribution of other players' actions.

To illustrate the concept, consider the following trading game. A seller owns an asset that he knows to be worth  $s$  both to himself and to a potential buyer. The buyer does not know  $s$ , but believes that it is randomly drawn from  $[0, 1]$  with a cumulative distribution function  $F$ . The buyer makes the seller a take-it-or-leave-it offer  $p$  for the asset.

The seller's optimal strategy is to accept the buyer's offer  $p$  if and only if  $s \leq p$ . In a Bayesian Nash equilibrium the buyer understands this, and so chooses  $p$  to maximize  $F(p) \times (E[s|s \leq p] - p)$ .

This objective is the probability  $F(p)$  that the seller accepts the offer  $p$ , times the buyer's expected surplus from acquiring the asset conditional on seller acceptance. Because  $E[s|s \leq p] < p$  for each  $p > 0$ , the buyer's optimal offer is  $p^* = 0$ . Thus, no trade occurs, consistent with the no-trade theorems of Milgrom and Stokey (1982) and Tirole (1982).

In a cursed equilibrium players fail to appreciate the informational content of others' behavior. This does not matter for the seller, who knows  $s$  perfectly and hence has nothing to learn, but matters for the buyer. A buyer who is *fully cursed* completely neglects the relationship between the seller's willingness to sell at price  $p$  and the seller's private information  $s$ , but correctly predicts the probability distribution over the seller's actions. As a consequence, a fully cursed buyer perceives the expected value of an asset traded at price  $p$  to be its unconditional expectation,  $E[s]$ . A fully cursed buyer thus chooses  $p$  to maximize  $F(p) \times (E[s] - p)$ . A *partially cursed* buyer appreciates that the seller's willingness to sell correlates with his private information but underestimates that relationship. A buyer who is partially cursed with coefficient  $\chi$  perceives the expected value of an asset traded at price  $p$  to be  $(1 - \chi)E[s|s \leq p] + \chi E[s]$ . This is the weighted average of the rational belief with weight  $1 - \chi$  and the fully cursed belief with weight  $\chi$ . In effect, the buyer believes that with probability  $1 - \chi$  the seller's decision to sell conveys information about the asset, and with probability  $\chi$  it does not. The coefficient  $\chi$  measures the buyer's naivety:  $\chi = 0$  corresponds to full rationality, while  $\chi = 1$  corresponds to full cursedness. A  $\chi$ -cursed buyer thus chooses  $p$  to maximize  $F(p) \times ((1 - \chi)E[s|s \leq p] + \chi E[s] - p)$ . Since  $E[s] > 0$ , the buyer's optimal offer exceeds zero for any  $\chi > 0$ . Moreover, since the buyer's objective function is supermodular in  $(p, \chi)$  for  $p \in [0, E[s]]$ , Topkis' Theorem implies that  $p^*$  increases in  $\chi$ . In summary, cursedness produces trade in no-trade settings, and the more cursed the buyer, the higher the volume of trade.

## B Proofs

We first prove the following lemma, which we use for proving Proposition 1.

**Lemma B.1** *Suppose that the variables  $(x, \{y_i\}_{i=1, \dots, K})$  are normal, independent, with mean zero and precisions  $(\tau_x, \{\tau_{y_i}\}_{i=1, \dots, K})$ . Then, the distribution of  $x$  conditional on  $\{x + y_i\}_{i=1, \dots, K}$  is normal*

with mean

$$\mathbb{E}(x | \{x + y_i\}_{i=1, \dots, K}) = \sum_{i=1}^K \frac{\tau_{y_i}}{\tau_x + \sum_{j=1}^K \tau_{y_j}} (x + y_i) \quad (\text{B.1})$$

and precision

$$\tau(x | \{x + y_i\}_{i=1, \dots, K}) = \tau_x + \sum_{i=1}^K \tau_{y_i}. \quad (\text{B.2})$$

**Proof.** The conditional mean and variance can be computed from the regression

$$x = \sum_{i=1}^K \beta_i (x + y_i) + e, \quad (\text{B.3})$$

where  $\{\beta_i\}_{i=1, \dots, K}$  are the regression coefficients and  $e$  is the error term. Taking covariances of both sides of (B.3) with  $x + y_i$  and noting that  $(x, \{y_i\}_{i=1, \dots, K}, e)$  are independent, we find

$$\begin{aligned} \text{Cov}(x, x + y_i) &= \sum_{j=1}^K \beta_j \text{Cov}(x + y_j, x + y_i) \\ \Rightarrow \frac{1}{\tau_x} &= \beta_i \left( \frac{1}{\tau_x} + \frac{1}{\tau_{y_i}} \right) + \sum_{j \neq i} \beta_j \frac{1}{\tau_x} \\ \Rightarrow \beta_i &= \frac{\tau_{y_i}}{\tau_x} \left( 1 - \sum_{j=1}^K \beta_j \right). \end{aligned} \quad (\text{B.4})$$

Summing (B.4) across  $i$  and solving for  $\sum_{j=1}^K \beta_j$ , we find

$$\sum_{j=1}^K \beta_j = \frac{\sum_{j=1}^K \tau_{y_j}}{\tau_x + \sum_{j=1}^K \tau_{y_j}}. \quad (\text{B.5})$$

Substituting  $\sum_{j=1}^K \beta_j$  from (B.5) into (B.4), we find

$$\beta_i = \frac{\tau_{y_i}}{\tau_x + \sum_{j=1}^K \tau_{y_j}}. \quad (\text{B.6})$$

Since

$$E(x | \{x + y_i\}_{i=1, \dots, K}) = \sum_{i=1}^K \beta_i (x + y_i),$$

(B.6) implies (B.1). Taking variances of both sides of (B.3) and noting that  $(x, \{y_i\}_{i=1, \dots, K}, e)$  are independent, we find

$$\begin{aligned} \text{Var}(x) &= \left( \sum_{i=1}^K \beta_i \right)^2 \text{Var}(x) + \sum_{i=1}^K \beta_i^2 \text{Var}(y_i) + \text{Var}(e) \\ \Rightarrow \frac{1}{\tau_e} &= \frac{1}{\tau_x} \left[ 1 - \left( \frac{\sum_{i=1}^K \tau_{y_i}}{\tau_x + \sum_{i=1}^K \tau_{y_i}} \right)^2 \right] - \sum_{i=1}^K \frac{1}{\tau_{y_i}} \left( \frac{\tau_{y_i}}{\tau_x + \sum_{j=1}^K \tau_{y_j}} \right)^2 \\ \Rightarrow \frac{1}{\tau_e} &= \frac{1}{\tau_x + \sum_{i=1}^K \tau_{y_i}}, \end{aligned} \tag{B.7}$$

where the second step follows from (B.5) and (B.6). Since

$$\tau(x | \{x + y_i\}_{i=1, \dots, K}) = \tau_e,$$

(B.7) implies (B.2). ■

**Proof of Proposition 1.** We first determine traders' demand functions using (8). Since  $d = \bar{d} + \epsilon + \zeta$  and  $\zeta$  is independent of traders' information  $I_i$ ,

$$E(d|I_i) = \bar{d} + E(\epsilon|I_i), \tag{B.8}$$

$$\text{Var}(d|I_i) = \text{Var}(\epsilon|I_i) + \text{Var}(\zeta) = \frac{1}{\tau(\epsilon|I_i)} + \frac{1}{\tau_\zeta}. \tag{B.9}$$

Using Lemma B.1 with  $x = \epsilon$ ,  $K = 3$  and  $\{y_j\}_{j=1,2,3} = (\eta_i, \eta, \xi_i)$ , we find

$$E(d|I_{ir}) = \bar{d} + \frac{\tau_{\eta_i}}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} s_i + \frac{\tau_\theta}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} s + \frac{\tau_{\xi_i}}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} (\epsilon + \xi_i), \tag{B.10}$$

$$\text{Var}(d|I_{ir}) = \frac{1}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} + \frac{1}{\tau_\zeta}. \tag{B.11}$$

Using Lemma B.1 with  $x = \epsilon$ ,  $K = 2$  and  $\{y_j\}_{j=1,2} = (\eta_i, \eta)$ , we find

$$E(d|I_{ic}) = \bar{d} + \frac{\tau_{\eta_i}}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta} s_i + \frac{\tau_\theta}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta} s, \quad (\text{B.12})$$

$$\text{Var}(d|I_{ic}) = \frac{1}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta} + \frac{1}{\tau_\zeta}. \quad (\text{B.13})$$

Substituting (B.10), (B.11), (B.12) and (B.13) into (8), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + (1 - \chi_i) \frac{\tau_{\eta_i} s_i + \tau_\theta s + \tau_{\xi_i} (\epsilon + \xi_i)}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} + \chi_i \frac{\tau_{\eta_i} s_i + \tau_\theta s}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta} - p}{\alpha_i \left[ (1 - \chi_i) \frac{1}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}} + \chi_i \frac{1}{\tau_\epsilon + \tau_{\eta_i} + \tau_\theta} + \frac{1}{\tau_\zeta} \right]} - z_i. \quad (\text{B.14})$$

We next substitute (B.14) into the market-clearing condition (5), use (6) to write  $p$  in terms of  $(\{s_i\}_{i=1,\dots,N}, s, \{z_i\}_{i=1,\dots,N})$ , and use (9) to write  $\epsilon + \xi_i$  in terms of  $(\{s_i\}_{j \neq i}, \{z_i\}_{j \neq i})$ . This yields an equation that is linear in  $(\{s_i\}_{i=1,\dots,N}, s, \{z_i\}_{i=1,\dots,N})$ . Identifying terms in  $s_i$  yields (11). Identifying terms of  $s$  yields (12). Identifying terms in  $z_i$  yields

$$\begin{aligned} & 1 - \frac{(1 - \chi_i) \tau_\zeta \tau_{\xi_i} \frac{C_i}{\sum_{k \neq i} A_k} (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)}{\alpha_i [(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \tau_{\xi_i}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_i} + \tau_\theta + \chi_i \tau_{\xi_i})]} \\ & = C_i \tau_\zeta \sum_{j=1}^N \frac{(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j}) - (1 - \chi_j) \tau_{\xi_j} \frac{1}{\sum_{k \neq j} A_k} (\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)}{\alpha_j [(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta)(\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \tau_{\xi_j}) + \tau_\zeta (\tau_\epsilon + \tau_{\eta_j} + \tau_\theta + \chi_j \tau_{\xi_j})]}. \end{aligned} \quad (\text{B.15})$$

Combining (B.15) with (11) yields (13). ■

**Proof of Proposition 2.** Eq. (10) implies that

$$\tau_{\xi_i} = \frac{\left( \sum_{j \neq i} A_j \right)^2}{\text{Var} \left( \sum_{j \neq i} A_j \eta_j \right)} = \frac{1}{\text{Var} \left( \frac{\sum_{j \neq i} \eta_j}{N-1} \right)} = (N-1) \tau_\eta, \quad (\text{B.16})$$

where the first step follows because  $z_j = 0$  for all  $j$ , the second because  $A_j = A$  for all  $j$ , and the third because  $\{\eta_j\}_{j=1,\dots,N}$  are i.i.d. with precision  $\tau_\eta$ . Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i) = (\chi, \alpha, \tau_\eta, 0, (N-1)\tau_\eta, A)$  for all  $i$  in (11), we find (15). Eq. (15) implies that  $A$  decreases in  $\chi$ .

The coefficient  $\beta$  in the regression (16) is proportional to

$$\begin{aligned}
& \text{Cov}(d - p, p - \bar{d}) \\
&= (1 - NA)NA\sigma_\epsilon^2 - NA^2\sigma_\eta^2 \\
&= NA [(1 - NA)\sigma_\epsilon^2 - A\sigma_\eta^2] \\
&= NA \frac{(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + N\tau_\eta) - \tau_\eta ([N - \chi(N - 1)]\tau_\epsilon + N\tau_\eta) - \frac{\tau_\epsilon}{N} ([N - \chi(N - 1)]\tau_\epsilon + N\tau_\eta)}{\tau_\epsilon(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + N\tau_\eta)} \\
&= A \frac{\chi(N - 1)(\tau_\epsilon + N\tau_\eta)}{(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + N\tau_\eta)} > 0,
\end{aligned}$$

where the first step follows from (1) and (14), and the third from (15).

Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, z_i) = (\chi, \alpha, \tau_\eta, 0, (N - 1)\tau_\eta, A, 0)$  in (B.14), we can write the demand of trader  $i$  as

$$\begin{aligned}
x_i &= \frac{\bar{d} + (1 - \chi) \frac{\tau_\eta s_i + \tau_\eta \frac{p - \bar{d} - A s_i}{A}}{\tau_\epsilon + N\tau_\eta} + \chi \frac{\tau_\eta s_i}{\tau_\epsilon + \tau_\eta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]} \\
&= \frac{\bar{d} + (1 - \chi) \frac{\tau_\eta \frac{p - \bar{d}}{A}}{\tau_\epsilon + N\tau_\eta} + \chi \frac{\tau_\eta s_i}{\tau_\epsilon + \tau_\eta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]}, \tag{B.17}
\end{aligned}$$

where we use (9) and (14) to write  $\epsilon + \xi_i$  as a function of  $p$ . Summing over  $i$  and dividing by  $N$ , we find

$$\frac{\sum_{i=1}^N x_i}{N} = \frac{\bar{d} + (1 - \chi) \frac{\tau_\eta \frac{p - \bar{d}}{A}}{\tau_\epsilon + N\tau_\eta} + \chi \frac{\tau_\eta \frac{\sum_{i=1}^N s_i}{N}}{\tau_\epsilon + \tau_\eta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]}. \tag{B.18}$$

Subtracting (B.18) from (B.17), and using the market-clearing condition (5), we find

$$\begin{aligned}
x_i &= \frac{\chi \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]} \\
&= \frac{\chi \tau_\zeta \tau_\eta (\tau_\epsilon + N\tau_\eta) \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha [\tau_\zeta (\tau_\epsilon + [1 + \chi(N - 1)] \tau_\eta) + (\tau_\epsilon + \tau_\eta) (\tau_\epsilon + N\tau_\eta)]}.
\end{aligned} \tag{B.19}$$

Since  $x_i$  is normal with mean zero

$$E(|x_i|) = \sqrt{\frac{2\text{Var}(x_i)}{\pi}}. \tag{B.20}$$

Substituting  $x_i$  from (B.19) into (B.20), and noting that symmetry and (1) imply that

$$\text{Var} \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right) = \frac{N - 1}{N\tau_\eta}, \tag{B.21}$$

we find (17). Eq. (17) implies that  $E(|x_i|)$  increases in  $\chi$ . It also implies that  $E(|x_i|)$  increases in  $N$ , as can be seen by noting that

$$E(|x_i|) = \frac{\chi \tau_\zeta \sqrt{2\tau_\eta}}{\alpha \left[ \tau_\zeta \frac{\tau_\epsilon + [1 + \chi(N - 1)] \tau_\eta}{\tau_\epsilon + N\tau_\eta} + \tau_\epsilon + \tau_\eta \right] \sqrt{\pi}} \sqrt{\frac{N - 1}{N}}$$

and that  $\frac{\tau_\epsilon + [1 + \chi(N - 1)] \tau_\eta}{\tau_\epsilon + N\tau_\eta}$  decreases in  $N$  and  $\frac{N - 1}{N}$  increases in  $N$ . ■

**Proof of Proposition 3.** The coefficient  $A$  can be deduced from (11) by setting  $(\chi_i, \alpha_i, \tau_\theta, A_i) = (0, \alpha, 0, A)$  for all  $i$  and deriving  $(\tau_{\eta_i}, \tau_{\xi_i})$  based on traders' subjective assessments of the precision of private signals. Overconfidence implies that  $\tau_{\eta_i} = \kappa \tau_\eta$  for all  $i$ . Dismissiveness, combined with symmetry and no random endowments, implies that

$$\tau_{\xi_i} = \frac{1}{\text{Var} \left( \frac{\sum_{j \neq i} \eta_j}{N - 1} \right)} = \frac{1}{\frac{1}{(N - 1)\gamma\tau_\eta} + \frac{(N - 2)\rho}{(N - 1)\gamma\tau_\eta}} = \frac{(N - 1)\gamma\tau_\eta}{1 + (N - 2)\rho}.$$



Substituting into (11), we find (18). Eq. (18) implies that  $A$  increases in  $\kappa$  and  $\gamma$ , and decreases in  $\rho$ .

Making the same substitutions, as well as  $z_i = 0$ , in (B.14), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + \frac{\kappa\tau_\eta s_i + \frac{\gamma\tau_\eta}{1+(N-2)\rho} \frac{p-\bar{d}-As_i}{A}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.22})$$

Following the same steps as in the proof of Proposition 2, we can write (B.22) as

$$\begin{aligned} x_i &= \frac{\left( \kappa - \frac{\gamma}{1+(N-2)\rho} \right) \tau_\eta \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}}}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]} \\ &= \frac{\left( \kappa - \frac{\gamma}{1+(N-2)\rho} \right) \tau_\zeta \tau_\eta \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ \tau_\epsilon + \tau_\zeta + \left( \kappa + \frac{(N-1)\gamma}{1+(N-2)\rho} \right) \tau_\eta \right]}. \end{aligned} \quad (\text{B.23})$$

Substituting  $x_i$  from (B.23) into (B.20), and using (B.21) (which remains valid under overconfidence and dismissiveness because it concerns the true distribution of signals), we find (19). Eq. (19) implies that  $E(|x_i|)$  increases in  $\kappa$  and  $\rho$ , and decreases in  $\gamma$ . Eq. (19) also implies that the asymptotic behavior of  $E(|x_i|)$  and of aggregate volume  $NE(|x_i|)$  when  $N$  grows large is as described in the proposition.

To derive the comparative statics of  $E(|x_i|)$  with respect to  $N$ , we treat  $N$  as a continuous variable and differentiate:

$$\begin{aligned}
\frac{dE(|x_i|)}{dN} &= \frac{d}{dN} \left[ \frac{\left( \kappa - \frac{\gamma}{1+(N-2)\rho} \right) \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \left[ \tau_\epsilon + \tau_\zeta + \left( \kappa + \frac{(N-1)\gamma}{1+(N-2)\rho} \right) \tau_\eta \right] \sqrt{\pi N}} \right] \\
&= \frac{d}{dN} \left[ \frac{\{ \kappa [1 + (N-2)\rho] - \gamma \} \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \{ [1 + (N-2)\rho] (\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta) + (N-1)\gamma\tau_\eta \} \sqrt{\pi N}} \right] \\
&= \frac{[\gamma\rho(\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta) - \gamma(\kappa - \gamma)\tau_\eta] \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \{ [1 + (N-2)\rho] (\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta) + (N-1)\gamma\tau_\eta \}^2 \sqrt{\pi N}} \\
&\quad + \frac{\{ \kappa [1 + (N-2)\rho] - \gamma \} \tau_\zeta \sqrt{\tau_\eta}}{\alpha \{ [1 + (N-2)\rho] (\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta) + (N-1)\gamma\tau_\eta \} N \sqrt{2\pi(N-1)N}} \tag{B.24}
\end{aligned}$$

where the third equality follows by writing  $E(|x_i|)$  as the product of  $\sqrt{\frac{N-1}{N}}$  and the remaining terms, and differentiating using the product rule. Eq. (B.24) implies that  $\frac{dE(|x_i|)}{dN}$  has the same sign as

$$\begin{aligned}
G(N) &\equiv 2N(N-1) [\gamma\rho(\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta) - \gamma(\kappa - \gamma)\tau_\eta] \\
&\quad + \{ [1 + (N-2)\rho] (\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta) + (N-1)\gamma\tau_\eta \} \{ \kappa [1 + (N-2)\rho] - \gamma \}.
\end{aligned}$$

The function  $G(N)$  is quadratic in  $N$ . To determine its sign, we distinguish cases according to the sign of  $\rho(\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta) - (\kappa - \gamma)\tau_\eta$ .

Suppose first that

$$\rho(\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta) - (\kappa - \gamma)\tau_\eta < 0. \tag{B.25}$$

For  $N \in [0, 2]$ ,

$$\begin{aligned}
H(N) &\equiv \kappa [1 + (N - 2)\rho] - \gamma \\
&\geq \kappa \left[ 1 + \frac{(N - 2)(\kappa - \gamma)\tau_\eta}{\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta} \right] - \gamma \\
&= \frac{(\kappa - \gamma)[\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta + (N - 2)\tau_\eta]}{\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta} > 0,
\end{aligned} \tag{B.26}$$

where the first inequality follows from (B.25) and  $N \leq 2$ , and the second is strict because (B.25) implies  $\kappa > \gamma$ . Eq. (B.26) implies, in particular, that  $H(1) > 0$ . Since, in addition,  $\rho \leq 1$ ,  $G(1) \geq 0$ . We next show that  $G(N_0) < 0$  for some  $N_0 < 0$ . If  $\rho = 0$ , then the existence of  $N_0$  follows from  $\lim_{N \rightarrow -\infty} G(N) = -\infty$ . If  $\rho > 0$ , then we define  $N_0$  by  $H(N_0) = 0$ . Since  $H(N)$  is linear,  $N_0$  is uniquely defined, and is negative because  $H(N) > 0$  for  $N > 2$  and because (B.26) implies that  $H(N) > 0$  for  $N \in [0, 2]$ . Since  $N(N - 1) > 0$  for  $N < 0$ ,  $G(N_0) < 0$ . Since  $G(N)$  is quadratic in  $N$ , negative for  $N = N_0 < 0$ , and non-negative for  $N = 1$ , its sign for  $N \geq 2$  is as follows:

- If  $\lim_{N \rightarrow \infty} G(N) = \infty$ , then  $G(N) > 0$  for  $N \geq 2$ .
- If  $\lim_{N \rightarrow \infty} G(N) = -\infty$  and  $G(2) > 0$ , then  $G(N)$  is positive for  $N \in [2, N_1)$  and negative for  $N > N_1$  for some  $N_1 > 2$ .
- If  $G(2) < 0$ , then  $G(N) < 0$  for  $N \geq 2$ .

The condition for  $\lim_{N \rightarrow \infty} G(N) = \infty$  is (21), and it implies that  $E(|x_i|)$  increases in  $N$ . The condition for  $G(2) < 0$  is (20), and it implies that  $E(|x_i|)$  decreases in  $N$ . When these inequalities are strict in the opposite direction, then  $\lim_{N \rightarrow \infty} G(N) = -\infty$  and  $G(2) > 0$ , and hence  $E(|x_i|)$  is hump-shaped in  $N$ .

Suppose next that

$$\rho(\tau_\epsilon + \tau_\zeta + 2\kappa\tau_\eta) - (\kappa - \gamma)\tau_\eta \geq 0. \tag{B.27}$$

Since  $H(N) > 0$  for  $N \geq 2$ ,  $G(N) > 0$  for  $N \geq 2$ , and hence  $E(|x_i|)$  increases in  $N$ . Since (B.27) implies that the right-hand side of (21) is non-positive, this case is covered by (21). ■

**Proof of Corollary 1.** Consider first the case where traders are cursed. When all private signals are publicly revealed, the information sets  $I_{ir}$  and  $I_{ic}$  coincide, and are also equivalent to  $\left\{\sum_{j=1}^N s_j\right\}$  because symmetry and normality imply that the sum of the signals is a sufficient statistic for all of them. Using Lemma B.1 with  $x = \epsilon$ ,  $K = 1$  and  $y_1 = \frac{\sum_{j=1}^N \eta_j}{N}$ , we find

$$E(d|I_{i\phi}) = \bar{d} + \frac{N\tau_\eta}{\tau_\epsilon + N\tau_\eta} \frac{\sum_{j=1}^N s_j}{N}, \quad (\text{B.28})$$

$$\text{Var}(d|I_{i\phi}) = \frac{1}{\tau_\epsilon + N\tau_\eta} + \frac{1}{\tau_\zeta}, \quad (\text{B.29})$$

for  $\phi = r, c$ . Substituting (B.28) and (B.29) into (8), and setting  $z_i = 0$ , we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + \frac{N\tau_\eta}{\tau_\epsilon + N\tau_\eta} \frac{\sum_{j=1}^N s_j}{N} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + N\tau_\eta} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.30})$$

Since all traders have the same demand, there is no trade.

Consider next the case where traders are overconfident or dismissive. When all private signals are publicly revealed, trader  $i$ 's information set is equivalent to  $\left\{s_i, \sum_{j \neq i} s_j\right\}$ . This is because trader  $i$  treats the signals of the other traders as symmetric, but not symmetric with his own signal. Using Lemma B.1 with  $x = \epsilon$ ,  $K = 2$  and  $\{y_j\}_{j=1,2} = \left(\eta_i, \frac{\sum_{j \neq i} \eta_j}{N-1}\right)$ , and trader  $i$ 's subjective assessments of precision, we find

$$E(d|I_i) = \bar{d} + \frac{\kappa\tau_\eta}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} s_i + \frac{\frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} \frac{\sum_{j \neq i} s_j}{N-1}, \quad (\text{B.31})$$

$$\text{Var}(d|I_i) = \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta}. \quad (\text{B.32})$$

Substituting (B.31) and (B.32) into (8), and setting  $z_i = 0$ , we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + \frac{\kappa\tau_\eta}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} s_i + \frac{\frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} \frac{\sum_{j \neq i} s_j}{N-1} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.33})$$

Summing over  $i$  and dividing by  $N$ , we find

$$\frac{\sum_{i=1}^N x_i}{N} = \frac{\bar{d} + \left[ \frac{\kappa\tau_\eta}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{\frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} \right] \frac{\sum_{i=1}^N s_i}{N} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.34})$$

Subtracting (B.34) from (B.33), and using the market-clearing condition (5), we find

$$x_i = \frac{\frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right)\tau_\eta}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.35})$$

Eq. (B.35) is identical to (B.23), and hence volume is the same as when the private signals are not publicly revealed. ■

**Proof of Proposition 4.** We proceed as in the proof of Proposition 3, except that we set  $\chi_i$  to  $\chi$  instead of 0. Making the substitutions in (B.14), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + (1 - \chi) \frac{\kappa\tau_\eta s_i + \frac{\gamma\tau_\eta}{1+(N-2)\rho} \frac{p - \bar{d} - A s_i}{A}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \chi \frac{\kappa\tau_\eta s_i}{\tau_\epsilon + \kappa\tau_\eta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}} + \chi \frac{1}{\tau_\epsilon + \kappa\tau_\eta} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.36})$$

Following the same steps as in the proof of Proposition 2, we can write (B.36) as

$$\begin{aligned}
x_i &= \frac{\left[ (1 - \chi) \frac{\left( \kappa - \frac{\gamma}{1 + (N-2)\rho} \right) \tau_\eta}{\tau_\epsilon + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1 + (N-2)\rho}} + \chi \frac{\kappa \tau_\eta}{\tau_\epsilon + \kappa \tau_\eta} \right] \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1 + (N-2)\rho}} + \chi \frac{1}{\tau_\epsilon + \kappa \tau_\eta} + \frac{1}{\tau_\zeta} \right]} \\
&= \frac{\left[ \kappa - \frac{\gamma}{1 + (N-2)\rho} + \frac{\chi \gamma (\tau_\epsilon + N \kappa \tau_\eta)}{[1 + (N-2)\rho](\tau_\epsilon + \kappa \tau_\eta)} \right] \tau_\zeta \tau_\eta \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ \tau_\epsilon + \tau_\zeta + \left( \kappa + \frac{(N-1)\gamma}{1 + (N-2)\rho} \right) \tau_\eta + \frac{(N-1)\chi \gamma \tau_\zeta \tau_\eta}{[1 + (N-2)\rho](\tau_\epsilon + \kappa \tau_\eta)} \right]}. \tag{B.37}
\end{aligned}$$

Substituting  $x_i$  from (B.37) into (B.20), and using (B.21), we find (22). Eq. (22) implies that when  $\rho = 0$

$$\lim_{N \rightarrow \infty} E(|x_i|) = \frac{\chi \kappa \tau_\zeta \sqrt{2\tau_\eta}}{\alpha (\tau_\epsilon + \kappa \tau_\eta + \chi \tau_\zeta) \sqrt{\pi}},$$

and when  $\rho > 0$

$$\lim_{N \rightarrow \infty} E(|x_i|) = \frac{[\kappa \rho (\tau_\epsilon + \kappa \tau_\eta) + \chi \gamma \kappa \tau_\eta] \tau_\zeta \sqrt{2\tau_\eta}}{\alpha ([\rho (\tau_\epsilon + \tau_\zeta) + (\kappa \rho + \gamma) \tau_\eta] (\tau_\epsilon + \kappa \tau_\eta) + \chi \gamma \tau_\eta \tau_\zeta) \sqrt{\pi}}.$$

In both cases the limit is positive and increasing in  $\kappa$ . ■

**Proof of Proposition 5.** We proceed as in the proof of Proposition 2, except that we do not set  $\tau_\theta$  to 0. Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_{\xi_i}, A_i) = (\chi, \alpha, \tau_\eta, (N-1)\tau_\eta, A)$  for all  $i$  in (11) and (12), we find (24) and (25), respectively. Eqs. (24) and (25) imply, respectively, that  $A$  decreases in  $\chi$  and  $B$  increases in  $\chi$ .

The coefficients  $(\beta_1, \beta_2)$  in the regression (26) can be derived by taking covariances of both sides with  $p - \bar{d}$  and  $s$ :

$$\text{Cov}(d - p, p - \bar{d}) = \beta_1 \text{Var}(p - \bar{d}) + \beta_2 \text{Cov}(s, p - \bar{d}), \tag{B.38}$$

$$\text{Cov}(d - p, s) = \beta_1 \text{Cov}(p - \bar{d}, s) + \beta_2 \text{Var}(s). \tag{B.39}$$

Eqs. (B.38) and (B.39) form a linear system in  $(\beta_1, \beta_2)$ . Its solution is

$$\beta_1 = \frac{\text{Cov}(d-p, p-\bar{d})\text{Var}(s) - \text{Cov}(d-p, s)\text{Cov}(s, p-\bar{d})}{\text{Var}(p-\bar{d})\text{Var}(s) - \text{Cov}(s, p-\bar{d})^2}, \quad (\text{B.40})$$

$$\beta_2 = -\frac{\text{Cov}(d-p, p-\bar{d})\text{Cov}(p-\bar{d}, s) - \text{Cov}(d-p, s)\text{Var}(p-\bar{d})}{\text{Var}(p-\bar{d})\text{Var}(s) - \text{Cov}(s, p-\bar{d})^2}. \quad (\text{B.41})$$

Eqs. (1), (2) and (23) imply that

$$\text{Cov}(d-p, p-\bar{d}) = (1-NA-B)(NA+B)\sigma_\epsilon^2 - NA^2\sigma_\eta^2 - B^2\sigma_\theta^2, \quad (\text{B.42})$$

$$\text{Cov}(d-p, s) = (1-NA-B)\sigma_\epsilon^2 - B\sigma_\theta^2. \quad (\text{B.43})$$

Using (24) and (25), we find

$$(1-NA-B)\sigma_\epsilon^2 - B\sigma_\theta^2 = \frac{\tau_\epsilon + [1+(N-1)\chi]\tau_\eta + \tau_\theta}{(\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)} - \frac{\tau_\epsilon + [1+(N-1)\chi]\tau_\eta + \tau_\theta}{(\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)} = 0.$$

Hence, (B.43) implies that  $\text{Cov}(d-p, s) = 0$ , and (B.42) implies that

$$\begin{aligned} \text{Cov}(d-p, p-\bar{d}) &= (1-NA-B)NA\sigma_\epsilon^2 - NA^2\sigma_\eta^2 \\ &= NA[(1-NA-B)\sigma_\epsilon^2 - A\sigma_\eta^2] \\ &= A\frac{\chi(N-1)(\tau_\epsilon + N\tau_\eta + \tau_\theta)}{(\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)} > 0, \end{aligned}$$

where the third equality follows from (24) and (25). Since  $\text{Cov}(d-p, s) = 0$ ,  $\text{Cov}(d-p, p-\bar{d}) > 0$  and  $\text{Var}(s) > 0$ , (B.40) implies that  $\beta_1 > 0$ . Since  $\text{Cov}(d-p, s) = 0$ ,  $\text{Cov}(d-p, p-\bar{d}) > 0$  and  $\text{Cov}(p-\bar{d}, s) > 0$ , (B.41) implies that  $\beta_2 < 0$ .

Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_{\xi_i}, A_i, z_i) = (\chi, \alpha, \tau_\eta, (N-1)\tau_\eta, A, 0)$  in (B.14), we can write the demand of trader  $i$  as

$$\begin{aligned}
x_i &= \frac{\bar{d} + (1 - \chi) \frac{\tau_\eta s_i + \tau_\theta s + \tau_\eta \frac{p - \bar{d} - A s_i - B s}{A}}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \chi \frac{\tau_\eta s_i + \tau_\theta s}{\tau_\epsilon + \tau_\eta + \tau_\theta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\theta} + \frac{1}{\tau_\zeta} \right]} \\
&= \frac{\bar{d} + (1 - \chi) \frac{\tau_\theta s + \tau_\eta \frac{p - \bar{d} - B s}{A}}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \chi \frac{\tau_\eta s_i + \tau_\theta s}{\tau_\epsilon + \tau_\eta + \tau_\theta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\theta} + \frac{1}{\tau_\zeta} \right]}, \tag{B.44}
\end{aligned}$$

where we use (9) and (23) to write  $\epsilon + \xi_i$  as a function of  $p$ . Following the same steps as in the proof of Proposition 2, we can write (B.44) as

$$\begin{aligned}
x_i &= \frac{\chi \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta + \tau_\theta} \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \chi \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\theta} + \frac{1}{\tau_\zeta} \right]} \\
&= \frac{\chi \tau_\zeta \tau_\eta (\tau_\epsilon + N\tau_\eta + \tau_\theta) \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha [\tau_\zeta (\tau_\epsilon + [1 + \chi(N-1)]\tau_\eta + \tau_\theta) + (\tau_\epsilon + \tau_\eta + \tau_\theta)(\tau_\epsilon + N\tau_\eta + \tau_\theta)]}. \tag{B.45}
\end{aligned}$$

Substituting  $x_i$  from (B.45) into (B.20), and using (B.21), we find (27). Eq. (27) implies that  $E(|x_i|)$  decreases in  $\tau_\theta$ , as can be seen by noting that

$$E(|x_i|) = \frac{\chi \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \left[ \tau_\zeta \frac{\tau_\epsilon + [1 + \chi(N-1)]\tau_\eta + \tau_\theta}{\tau_\epsilon + N\tau_\eta + \tau_\theta} + \tau_\epsilon + \tau_\eta + \tau_\theta \right] \sqrt{\pi N}}$$

and that  $\frac{\tau_\epsilon + [1 + \chi(N-1)]\tau_\eta + \tau_\theta}{\tau_\epsilon + N\tau_\eta + \tau_\theta}$  increases in  $\tau_\theta$ . ■

**Proposition B.1** *Suppose that traders are symmetric and not cursed, receive no random endowments, and observe their private signals and the public signal. Suppose also that each trader perceives the precision of his private signal to be  $\kappa \times \tau_\eta$  for  $\kappa \geq 1$ , the precision of every other trader's signal to be  $\gamma \times \tau_\eta$  for  $\gamma \in [0, 1]$ , and the correlation between the noise terms in others' signals to*



be  $\rho \in [0, 1]$ . The price (23) is an equilibrium price if and only if

$$A = \frac{\left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta}{N \left[\tau_\epsilon + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta + \tau_\theta\right]}, \quad (\text{B.46})$$

$$B = \frac{\tau_\theta}{\tau_\epsilon + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta + \tau_\theta}. \quad (\text{B.47})$$

The expected volume that each trader generates is

$$\frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right) \tau_\zeta \sqrt{2(N-1)\tau_\eta}}{\alpha \left[\tau_\epsilon + \tau_\zeta + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta + \tau_\theta\right] \sqrt{\pi N}}, \quad (\text{B.48})$$

and is lower than when traders do not observe the public signal.

**Proof of Proposition B.1.** The coefficients  $(A, B)$  can be deduced from Proposition 1 with the same substitutions as in Proposition 3 except that we do not set  $\tau_\theta$  to 0. Making the same substitutions, as well as  $z_i = 0$ , in (B.14), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + \frac{\kappa\tau_\eta s_i + \tau_\theta s + \frac{\gamma\tau_\eta}{1+(N-2)\rho} \frac{p-\bar{d}-As_i-Bs}{A}}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho} + \tau_\theta} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho} + \tau_\theta} + \frac{1}{\tau_\zeta} \right]}. \quad (\text{B.49})$$

Following the same steps as in the proof of Proposition 2, we can write (B.49) as

$$x_i = \frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right) \tau_\zeta \tau_\eta \left(s_i - \frac{\sum_{j=1}^N s_j}{N}\right)}{\alpha \left[\tau_\epsilon + \tau_\zeta + \left(\kappa + \frac{(N-1)\gamma}{1+(N-2)\rho}\right) \tau_\eta + \tau_\theta\right]}. \quad (\text{B.50})$$

Substituting  $x_i$  from (B.50) into (B.20), and using (B.21), we find (B.48). Eq. (B.48) implies that  $E(|x_i|)$  decreases in  $\tau_\theta$ . ■

**Proof of Proposition 6.** With random endowments, (10) implies that

$$\begin{aligned}
\tau_{\xi_i} &= \frac{\left(\sum_{j \neq i} A_j\right)^2}{\text{Var}\left(\sum_{j \neq i} A_j \eta_j - \sum_{j \neq i} C_j z_j\right)} \\
&= \frac{1}{\text{Var}\left(\frac{\sum_{j \neq i} \eta_j}{N-1}\right) + \frac{C^2}{A^2} \text{Var}\left(\frac{\sum_{j \neq i} z_j}{N-1}\right)} \\
&= \frac{1}{\frac{1}{(N-1)\tau_\eta} + \frac{C^2}{A^2} \frac{1}{(N-1)\tau_z}} \\
&= \frac{(N-1)\tau_\eta \tau_z}{\tau_z + \frac{C^2}{A^2} \tau_\eta}, \tag{B.51}
\end{aligned}$$

where the second step follows because  $\{\eta_j\}_{j=1,\dots,N}$  are independent of  $\{z_j\}_{j=1,\dots,N}$ , and the third step because  $\{\eta_j\}_{j=1,\dots,N}$  are i.i.d. with precision  $\tau_\eta$  and  $\{z_j\}_{j=1,\dots,N}$  are i.i.d. with precision  $\tau_z$ . Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, C_i) = (\chi, \alpha, \tau_\eta, 0, \frac{(N-1)\tau_\eta \tau_z}{\tau_z + \frac{C^2}{A^2} \tau_\eta}, A, C)$  for all  $i$  in (11) and (13), we find (29) and (30), respectively. Eq. (30) is cubic in  $\frac{C}{A}$ , and hence has at least one solution. Any of its solutions satisfies

$$\frac{C}{A} \tau_\zeta \tau_\eta - \alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta) \geq 0 \Rightarrow \frac{C}{A} > 0. \tag{B.52}$$

Indeed, if  $\frac{C}{A} \tau_\zeta \tau_\eta - \alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta) < 0$ , then  $\chi \frac{C}{A} \tau_\zeta \tau_\eta - \alpha(\tau_\epsilon + \chi \tau_\zeta + \tau_\eta) < 0$ . Hence, the left-hand side of (30) would be negative rather than zero, a contradiction. Because of (B.52), the derivative of the left-hand side of (30) with respect to  $\frac{C}{A}$  is positive at any solution of (30). Hence, (30) has a unique solution.

Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, C_i) = (\chi, \alpha, \tau_\eta, 0, \tau_\xi, A, C)$  in (B.14), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + (1 - \chi) \frac{\tau_\eta s_i + \tau_\xi \frac{p - \bar{d} - A s_i + C z_i}{(N-1)A}}{\tau_\epsilon + \tau_\eta + \tau_\xi} + \chi \frac{\tau_\eta s_i}{\tau_\epsilon + \tau_\eta} - p}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\xi} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]} - z_i \tag{B.53}$$

where we use (9) and (28) to write  $\epsilon + \xi_i$  as a function of  $p$ . Following the same steps as in the proof of Proposition 2, we can write (B.53) as

$$\begin{aligned}
x_i &= \frac{\left( (1 - \chi) \frac{\tau_\eta - \frac{\tau_\xi}{N-1}}{\tau_\epsilon + \tau_\eta + \tau_\xi} + \chi \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} \right) \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\xi} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]} \\
&\quad - \left( 1 - \frac{(1 - \chi) \frac{\frac{C}{A} \frac{\tau_\xi}{N-1}}{\tau_\epsilon + \tau_\eta + \tau_\xi}}{\alpha \left[ (1 - \chi) \frac{1}{\tau_\epsilon + \tau_\eta + \tau_\xi} + \chi \frac{1}{\tau_\epsilon + \tau_\eta} + \frac{1}{\tau_\zeta} \right]} \right) \left( z_i - \frac{\sum_{i=1}^N z_i}{N} \right) \\
&= \frac{\tau_\zeta \left[ (1 - \chi)(\tau_\epsilon + \tau_\eta) \left( \tau_\eta - \frac{\tau_\xi}{N-1} \right) + \chi \tau_\eta (\tau_\epsilon + \tau_\eta + \tau_\xi) \right] \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left[ \tau_\zeta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi) + (\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\eta + \tau_\xi) \right]} \\
&\quad - \left( 1 - \frac{(1 - \chi) \frac{C}{A} \tau_\zeta (\tau_\epsilon + \tau_\eta) \frac{\tau_\xi}{N-1}}{\alpha \left[ \tau_\zeta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi) + (\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\eta + \tau_\xi) \right]} \right) \left( z_i - \frac{\sum_{j=1}^N z_j}{N} \right). \tag{B.54}
\end{aligned}$$

Since for  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, C_i) = (\chi, \alpha, \tau_\eta, 0, \tau_\xi, A, C)$ , (13) implies that

$$\alpha \left[ \tau_\zeta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi) + (\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\eta + \tau_\xi) \right] = \frac{C}{A} \tau_\zeta \tau_\eta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi),$$

we can write (B.54) as

$$\begin{aligned}
x_i &= \frac{\left[ (1 - \chi)(\tau_\epsilon + \tau_\eta) \left( \tau_\eta - \frac{\tau_\xi}{N-1} \right) + \chi \tau_\eta (\tau_\epsilon + \tau_\eta + \tau_\xi) \right] \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\frac{C}{A} \tau_\eta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi)} \\
&\quad - \left( 1 - \frac{(1 - \chi)(\tau_\epsilon + \tau_\eta) \frac{\tau_\xi}{N-1}}{\tau_\eta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi)} \right) \left( z_i - \frac{\sum_{j=1}^N z_j}{N} \right) \\
&= \frac{\left[ (\tau_\epsilon + \tau_\eta) \left( \tau_\eta - \frac{\tau_\xi}{N-1} \right) + \chi \tau_\xi \frac{\tau_\epsilon + N \tau_\eta}{N-1} \right] \left[ \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right) - \frac{C}{A} \left( z_i - \frac{\sum_{j=1}^N z_j}{N} \right) \right]}{\frac{C}{A} \tau_\eta (\tau_\epsilon + \tau_\eta + \chi \tau_\xi)}. \tag{B.55}
\end{aligned}$$

Substituting  $x_i$  from (B.55) into (B.20), and using (B.51), (B.21) and its counterpart for  $\{z_i\}_{i=1, \dots, N}$ , and the independence between  $\{s_i\}_{i=1, \dots, N}$  and  $\{z_i\}_{i=1, \dots, N}$ , we find (31).

For  $\chi = 0$ , (31) becomes

$$\frac{\frac{C}{A}\sqrt{2(N-1)\tau_\eta}}{\sqrt{\pi N\left(\tau_z + \frac{C^2}{A^2}\tau_\eta\right)\tau_z}}. \quad (\text{B.56})$$

Eq. (B.56) implies that  $E(|x_i|)$  increases in  $N$  if  $\frac{C}{A}$  increases in  $N$ . The result that  $\frac{C}{A}$  increases in  $N$  holds for all  $\chi \in [0, 1)$ , as can be seen from differentiating (30) implicitly. Indeed, the derivative of the left-hand side of (30) with respect to  $\frac{C}{A}$  is positive at any solution of (30). Moreover, the argument establishing (B.52) implies that this inequality is strict for all  $\chi \in [0, 1)$ . Hence,  $\chi\frac{C}{A}\tau_\zeta\tau_\eta - \alpha(\tau_\epsilon + \chi\tau_\zeta + \tau_\eta) < 0$  at any solution of (30), and the derivative of the left-hand side of (30) with respect to  $N$  is also negative.

For  $\chi = 1$ , (31) becomes

$$\frac{\sqrt{2(N-1)\left(\tau_z + \frac{C^2}{A^2}\tau_\eta\right)}}{\frac{C}{A}\sqrt{\pi N\tau_\eta\tau_z}}. \quad (\text{B.57})$$

Eq. (30) implies that  $\frac{C}{A}$  is equal to  $\frac{\alpha(\tau_\epsilon + \tau_\eta + \tau_\zeta)}{\tau_\zeta\tau_\eta}$ . Since  $\frac{C}{A}$  is independent of  $N$ , (B.57) implies that  $E(|x_i|)$  increases in  $N$ . ■

**Proposition B.2** *Suppose that traders are symmetric and not cursed, receive random endowments, and observe only their private signals and endowment shocks. Suppose also that each trader perceives the precision of his private signal to be  $\kappa \times \tau_\eta$  for  $\kappa \geq 1$ , the precision of every other trader's signal to be  $\gamma \times \tau_\eta$  for  $\gamma \in [0, 1]$ , and the correlation between the noise terms in others' signals to be  $\rho \in [0, 1]$ . The price (28) is an equilibrium price if and only if*

$$A = \frac{\left(\kappa + \frac{(N-1)\gamma\tau_z}{[1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta}\right)\tau_\eta}{N\left[\tau_\epsilon + \left(\kappa + \frac{(N-1)\gamma\tau_z}{[1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta}\right)\tau_\eta\right]}, \quad (\text{B.58})$$

and  $\frac{C}{A} > 0$  is the unique solution to the cubic equation

$$\left( [1 + (N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta \right) \left( \frac{C}{A}\kappa\tau_\zeta\tau_\eta - \alpha(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta) \right) - (N-1)\alpha\gamma\tau_\eta\tau_z = 0. \quad (\text{B.59})$$

The expected volume that each trader generates is

$$\frac{\left( 1 - \frac{\gamma\tau_z}{\kappa([1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta)} \right) \sqrt{2(N-1) \left( \tau_z + \frac{C^2}{A^2}\tau_\eta \right)}}{\frac{C}{A}\sqrt{\pi N\tau_\eta\tau_z}}. \quad (\text{B.60})$$

**Proof of Proposition B.2.** The coefficients  $(A, C)$  can be deduced from Proposition 1 with the same substitutions as in Proposition 3 except for

$$\begin{aligned} \tau_{\xi_i} &= \frac{1}{\text{Var} \left( \frac{\sum_{j \neq i} \eta_j}{N-1} \right) + \frac{C^2}{A^2} \text{Var} \left( \frac{\sum_{j \neq i} z_j}{N-1} \right)} \\ &= \frac{1}{\frac{1}{(N-1)\gamma\tau_\eta} + \frac{(N-2)\rho}{(N-1)\gamma\tau_\eta} + \frac{C^2}{A^2} \frac{1}{(N-1)\tau_z}} \\ &= \frac{(N-1)\gamma\tau_\eta\tau_z}{[1 + (N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta}. \end{aligned} \quad (\text{B.61})$$

Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, C_i) = (0, \alpha, \kappa\tau_\eta, 0, \tau_\xi, A, C)$  in (B.14), we can write the demand of trader  $i$  as

$$x_i = \frac{\bar{d} + \frac{\kappa\tau_\eta s_i + \tau_\xi \frac{p - \bar{d} - A s_i + C z_i}{(N-1)A}}{\tau_\epsilon + \kappa\tau_\eta + \tau_\xi} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa\tau_\eta + \tau_\xi} + \frac{1}{\tau_\zeta} \right]} - z_i. \quad (\text{B.62})$$

Following the same steps as in the proof of Proposition 2, we can write (B.62) as

$$x_i = \frac{\tau_\zeta \left( \kappa\tau_\eta - \frac{\tau_\xi}{N-1} \right) \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \tau_\xi)} - \left( 1 - \frac{\frac{C}{A}\tau_\zeta\tau_\xi}{\alpha(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \tau_\xi)} \right) \left( z_i - \frac{\sum_{j=1}^N z_j}{N} \right), \quad (\text{B.63})$$

which is the same as (B.54) except that  $\chi$  is set to zero and  $\tau_\eta$  is replaced by  $\kappa\tau_\eta$ . Since for  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, \tau_{\xi_i}, A_i, C_i) = (0, \alpha, \kappa\tau_\eta, 0, \tau_\xi, A, C)$ , (13) implies that

$$\alpha(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \tau_\xi) = \frac{C}{A}\kappa\tau_\zeta\tau_\eta, \quad (\text{B.64})$$

we can write (B.63) as

$$x_i = \frac{\left(\kappa\tau_\eta - \frac{\tau_\xi}{N-1}\right) \left[ \left(s_i - \frac{\sum_{j=1}^N s_j}{N}\right) - \frac{C}{A} \left(z_i - \frac{\sum_{j=1}^N z_j}{N}\right) \right]}{\frac{C}{A}\kappa\tau_\eta}. \quad (\text{B.65})$$

Substituting  $x_i$  from (B.65) into (B.20), and using (B.61), (B.21) and its counterpart for  $\{z_i\}_{i=1,\dots,N}$ , and the independence between  $\{s_i\}_{i=1,\dots,N}$  and  $\{z_i\}_{i=1,\dots,N}$ , we find (B.60). For  $\rho = 0$ , (B.59) implies that  $\frac{C}{A}$  converges to  $\infty$  when  $N$  grows large, and is of order  $N^{\frac{1}{3}}$ . For  $\rho > 0$ , (B.59) implies that  $\frac{C}{A}$  converges to a positive limit. In both cases, (B.60) implies that  $E(|x_i|)$  converges to a positive limit. ■

**Proof of Corollary 2.** Consider first the case where traders are cursed. Proceeding as in the proof of Corollary 1, we find that the demand of trader  $i$  is

$$x_i = \frac{\bar{d} + \frac{N\tau_\eta}{\tau_\epsilon + N\tau_\eta} \frac{\sum_{j=1}^N s_j}{N} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + N\tau_\eta} + \frac{1}{\tau_\zeta} \right]} - z_i. \quad (\text{B.66})$$

Summing over  $i$  and dividing by  $N$ , we find

$$\frac{\sum_{i=1}^N x_i}{N} = \frac{\bar{d} + \frac{N\tau_\eta}{\tau_\epsilon + N\tau_\eta} \frac{\sum_{i=1}^N s_i}{N} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + N\tau_\eta} + \frac{1}{\tau_\zeta} \right]} - \frac{\sum_{i=1}^N z_i}{N}. \quad (\text{B.67})$$

Subtracting (B.67) from (B.66), and using the market-clearing condition (5), we find

$$x_i = - \left( z_i - \frac{\sum_{i=1}^N z_i}{N} \right). \quad (\text{B.68})$$

Substituting  $x_i$  from (B.68) into (B.20), and using the counterpart of (B.21) for  $\{z_i\}_{i=1,\dots,N}$ , we find

$$E(|x_i|) = \sqrt{\frac{2(N-1)}{\pi N \tau_z}}. \quad (\text{B.69})$$

Eqs. (31) and (B.69) imply that volume increases when all private signals are publicly revealed if and only if

$$1 > \frac{\left[ \frac{C^2}{A^2} \tau_\eta (\tau_\epsilon + \tau_\eta) + \chi (\tau_\epsilon + N \tau_\eta) \tau_z \right] \sqrt{\tau_z + \frac{C^2}{A^2} \tau_\eta}}{\left[ \left( \tau_z + \frac{C^2}{A^2} \tau_\eta \right) (\tau_\epsilon + \tau_\eta) + \chi (N-1) \tau_\eta \tau_z \right] \frac{C}{A} \sqrt{\tau_\eta}}. \quad (\text{B.70})$$

When  $\chi = 0$ , (B.70) becomes

$$1 > \frac{\frac{C^2}{A^2} \tau_\eta \sqrt{\tau_z + \frac{C^2}{A^2} \tau_\eta}}{\left( \tau_z + \frac{C^2}{A^2} \tau_\eta \right) \frac{C}{A} \sqrt{\tau_\eta}} \Leftrightarrow \sqrt{\tau_z + \frac{C^2}{A^2} \tau_\eta} > \frac{C}{A} \sqrt{\tau_\eta},$$

and holds. By continuity, it also holds when  $\chi$  is close to zero. When  $\chi = 1$ , (B.70) becomes

$$1 > \frac{\left[ \frac{C^2}{A^2} \tau_\eta (\tau_\epsilon + \tau_\eta) + (\tau_\epsilon + N \tau_\eta) \tau_z \right] \sqrt{\tau_z + \frac{C^2}{A^2} \tau_\eta}}{\left[ \left( \tau_z + \frac{C^2}{A^2} \tau_\eta \right) (\tau_\epsilon + \tau_\eta) + (N-1) \tau_\eta \tau_z \right] \frac{C}{A} \sqrt{\tau_\eta}} \Leftrightarrow 1 > \frac{\sqrt{\tau_z + \frac{C^2}{A^2} \tau_\eta}}{\frac{C}{A} \sqrt{\tau_\eta}},$$

and does not hold. By continuity, it also does not hold when  $\chi$  is close to one. This establishes the results in the corollary for rational ( $\chi = 0$ ) and cursed traders.

Consider next the case where traders are overconfident or dismissive. Proceeding as in the proof of Corollary 1, we find that the demand of trader  $i$  is

$$\begin{aligned} x_i &= \frac{\bar{d} + \frac{\kappa \tau_\eta}{\tau_\epsilon + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1+(N-2)\rho}} s_i + \frac{\frac{(N-1)\gamma \tau_\eta}{1+(N-2)\rho} \sum_{j \neq i} s_j}{\tau_\epsilon + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1+(N-2)\rho}} - p}{\alpha \left[ \frac{1}{\tau_\epsilon + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1+(N-2)\rho}} + \frac{1}{\tau_\zeta} \right]} - z_i \\ &= \frac{\left( \kappa - \frac{\gamma}{1+(N-2)\rho} \right) \tau_\zeta \tau_\eta \left( s_i - \frac{\sum_{j=1}^N s_j}{N} \right)}{\alpha \left( \tau_\epsilon + \tau_\zeta + \kappa \tau_\eta + \frac{(N-1)\gamma \tau_\eta}{1+(N-2)\rho} \right)} - \left( z_i - \frac{\sum_{i=1}^N z_i}{N} \right). \end{aligned} \quad (\text{B.71})$$

Substituting  $x_i$  from (B.65) into (B.20), and using (B.21) and its counterpart for  $\{z_i\}_{i=1,\dots,N}$ , as well as the independence between  $\{s_i\}_{i=1,\dots,N}$  and  $\{z_i\}_{i=1,\dots,N}$ , we find

$$E(|x_i|) = \sqrt{\left[ \frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right)^2 \tau_\zeta^2 \tau_\eta}{\alpha^2 \left(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}\right)^2} + \frac{1}{\tau_z} \right] \frac{2(N-1)}{\pi N}}. \quad (\text{B.72})$$

Eqs. (B.60) and (B.72) imply that volume increases when all private signals are publicly revealed if and only if

$$\begin{aligned} & \sqrt{\left[ \frac{\left(\kappa - \frac{\gamma}{1+(N-2)\rho}\right)^2 \tau_\zeta^2 \tau_\eta \tau_z}{\alpha^2 \left(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}\right)^2} + 1 \right]} > \frac{\left(1 - \frac{\gamma\tau_z}{\kappa([1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta)}\right) \sqrt{\left(\tau_z + \frac{C^2}{A^2}\tau_\eta\right)}}{\frac{C}{A}\sqrt{\tau_\eta}} \\ & \Leftrightarrow \sqrt{\left[ \frac{\left(1 - \frac{\gamma}{\kappa[1+(N-2)\rho]}\right)^2 \tau_z \left(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{[1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta}\right)^2}{\frac{C^2}{A^2}\tau_\eta \left(\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{(N-1)\gamma\tau_\eta}{1+(N-2)\rho}\right)^2} + 1 \right]} \\ & > \frac{\left(1 - \frac{\gamma\tau_z}{\kappa([1+(N-2)\rho]\tau_z + \frac{C^2}{A^2}\gamma\tau_\eta)}\right) \sqrt{\left(\tau_z + \frac{C^2}{A^2}\tau_\eta\right)}}{\frac{C}{A}\sqrt{\tau_\eta}}, \end{aligned} \quad (\text{B.73})$$

where the second step follows from (B.61) and (B.64). Setting

$$f(X) \equiv \frac{\gamma\tau_z}{[1+(N-2)\rho]\tau_z + X^2\gamma\tau_\eta},$$



we can write (B.73) as

$$\begin{aligned}
& \sqrt{\left[ \frac{\left(1 - \frac{f(0)}{\kappa}\right)^2 \tau_z [\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f\left(\frac{C}{A}\right)\tau_\eta]^2}{\frac{C^2}{A^2}\tau_\eta [\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2} + 1 \right]} > \frac{\left(1 - \frac{f\left(\frac{C}{A}\right)}{\kappa}\right) \sqrt{\left(\tau_z + \frac{C^2}{A^2}\tau_\eta\right)}}{\frac{C}{A}\sqrt{\tau_\eta}} \\
& \Leftrightarrow \frac{[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f\left(\frac{C}{A}\right)\tau_\eta]^2}{[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2} - \frac{\left(1 - \frac{f\left(\frac{C}{A}\right)}{\kappa}\right)^2}{\left(1 - \frac{f(0)}{\kappa}\right)^2} + \frac{\frac{C^2}{A^2}\tau_\eta}{\left(1 - \frac{f(0)}{\kappa}\right)^2 \tau_z} \left[1 - \left(1 - \frac{f\left(\frac{C}{A}\right)}{\kappa}\right)^2\right] > 0,
\end{aligned} \tag{B.74}$$

where the second step follows by squaring both sides and rearranging.

When  $\gamma = 0$ , (B.74) holds as an equality because  $f(X) = 0$ . In the limit when  $\kappa$  goes to  $\infty$ , (B.74) also holds as an equality because (B.59) implies that  $\frac{C}{A}$  takes a positive value. This establishes the results in the corollary for fully dismissive and fully overconfident traders.

Using

$$\begin{aligned}
& \frac{[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f\left(\frac{C}{A}\right)\tau_\eta]^2}{[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2} - \frac{\left(1 - \frac{f\left(\frac{C}{A}\right)}{\kappa}\right)^2}{\left(1 - \frac{f(0)}{\kappa}\right)^2} \\
& = \frac{\frac{2}{\kappa} \left(f\left(\frac{C}{A}\right) - f(0)\right) (\tau_\epsilon + \tau_\zeta + N\kappa\tau_\eta)}{[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2 \left(1 - \frac{f(0)}{\kappa}\right)^2} \\
& \quad \times \left[ \tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{f\left(\frac{C}{A}\right) + f(0)}{2\kappa} [(N-2)\kappa\tau_\eta - (\tau_\epsilon + \tau_\zeta)] - \frac{N-1}{\kappa} f(0)f\left(\frac{C}{A}\right)\tau_\eta \right]
\end{aligned}$$

and

$$f\left(\frac{C}{A}\right) - f(0) = -\frac{\frac{C^2}{A^2}\tau_\eta}{\tau_z} f(0)f\left(\frac{C}{A}\right),$$

we next simplify (B.74) to

$$\begin{aligned}
& -f(0)(\tau_\epsilon + \tau_\zeta + N\kappa\tau_\eta) \left[ \tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{f\left(\frac{C}{A}\right) + f(0)}{2\kappa} [(N-2)\kappa\tau_\eta - (\tau_\epsilon + \tau_\zeta)] \right. \\
& \left. - \frac{N-1}{\kappa} f(0) f\left(\frac{C}{A}\right) \tau_\eta \right] + \left( 1 - \frac{f\left(\frac{C}{A}\right)}{2\kappa} \right) [\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2 > 0. \tag{B.75}
\end{aligned}$$

(This simplification involves eliminating terms in  $\frac{1}{\kappa}$  and  $f\left(\frac{C}{A}\right)$ , which are zero when  $\kappa = \infty$  and  $\gamma = 0$ , respectively, and which make (B.74) hold as an equality in those cases.) The left-hand side of (B.75) is linear in  $f\left(\frac{C}{A}\right)$ , which varies from  $f(0)$  to 0 as  $\frac{C}{A}$  varies from 0 to  $\infty$ . For  $f\left(\frac{C}{A}\right) = f(0)$ , we can write the left-hand side of (B.75) as  $[\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]$  times

$$-f(0) \left( 1 - \frac{f(0)}{\kappa} \right) (\tau_\epsilon + \tau_\zeta + N\kappa\tau_\eta) + \left( 1 - \frac{f(0)}{2\kappa} \right) [\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]. \tag{B.76}$$

For  $f\left(\frac{C}{A}\right) = 0$ , we can write the left-hand side of (B.75) as

$$-f(0)(\tau_\epsilon + \tau_\zeta + N\kappa\tau_\eta) \left[ \tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + \frac{f(0)}{2\kappa} [(N-2)\kappa\tau_\eta - (\tau_\epsilon + \tau_\zeta)] \right] + [\tau_\epsilon + \tau_\zeta + \kappa\tau_\eta + (N-1)f(0)\tau_\eta]^2. \tag{B.77}$$

For  $\kappa = 1$ , (B.76) is equal to

$$\begin{aligned}
& -f(0)(1 - f(0))(\tau_\epsilon + \tau_\zeta + N\tau_\eta) + \left( 1 - \frac{f(0)}{2} \right) [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta] \\
& > (1 - f(0)) [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta - f(0)(\tau_\epsilon + \tau_\zeta + N\tau_\eta)] \\
& = (1 - f(0))^2 (\tau_\epsilon + \tau_\zeta + \tau_\eta) > 0,
\end{aligned}$$

and (B.77) is equal to

$$\begin{aligned}
& -f(0)(\tau_\epsilon + \tau_\zeta + N\tau_\eta) \left[ \tau_\epsilon + \tau_\zeta + \tau_\eta + \frac{f(0)}{2} [(N-2)\tau_\eta - (\tau_\epsilon + \tau_\zeta)] \right] + [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta]^2 \\
& > [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta] \{-f(0)(\tau_\epsilon + \tau_\zeta + N\tau_\eta) + [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta]\} \\
& = (1-f(0)) [\tau_\epsilon + \tau_\zeta + \tau_\eta + (N-1)f(0)\tau_\eta] (\tau_\epsilon + \tau_\zeta + \tau_\eta) > 0.
\end{aligned}$$

Since the left-hand side of (B.75) is positive at both ends of the interval, it is positive for all values of  $\frac{C}{A}$ , and hence (B.75) holds. For  $\kappa$  large, the largest term in (B.76) is  $\kappa[1-Nf(0)]\tau_\eta$  and the largest term in (B.77) is  $\kappa^2[1-Nf(0)]\tau_\eta^2$ . Both are negative if

$$1 - Nf(0) = \frac{1 + (N-2)\rho - N\gamma}{1 + (N-2)\rho} < 0.$$

Hence, for  $\kappa$  large and  $1 + (N-2)\rho - N\gamma < 0$ , (B.75) does not hold. This establishes the results in the corollary for non-fully dismissive and non-fully overconfident traders. ■

**Proof of Proposition 7.** Setting  $\tau_{\eta_i} = \tau_\eta$  for all  $i$ ,  $A_i = A_r$  for  $i \in R$ , and  $A_i = A_c$  for  $i \in C$  in the first equation in (B.16), we find

$$\tau_{\xi_i} = \frac{[(N_r-1)A_r + N_cA_c]^2}{\text{Var}\left(A_r \sum_{j \in R, j \neq i} \eta_j + A_c \sum_{j \in C} \eta_j\right)} = \frac{[(N_r-1)A_r + N_cA_c]^2 \tau_\eta}{(N_r-1)A_r^2 + N_cA_c^2} \equiv \tau_\xi. \quad (\text{B.78})$$

for  $i \in R$ . Setting  $(\alpha_i, \tau_{\eta_i}) = (\alpha, \tau_\eta)$  for all  $i$ ,  $(\chi_i, \tau_{\xi_i}, A_i) = (0, \tau_\xi, A_r)$  for  $i \in R$ , and  $(\chi_i, A_i) = (1, A_c)$  for  $i \in C$  in (11), we find

$$\frac{\tau_\eta - \tau_\xi \frac{A_r}{(N_r-1)A_r + N_cA_c}}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} = A_r \left[ \frac{N_r \left( \tau_\epsilon + \tau_\eta + \tau_\xi - \tau_\xi \frac{1}{(N_r-1)A_r + N_cA_c} \right)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c(\tau_\epsilon + \tau_\eta)}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \right] \quad (\text{B.79})$$

for a rational trader, and

$$\frac{\tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} = A_c \left[ \frac{N_r \left( \tau_\epsilon + \tau_\eta + \tau_\xi - \tau_\xi \frac{1}{(N_r-1)A_r + N_cA_c} \right)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c(\tau_\epsilon + \tau_\eta)}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \right] \quad (\text{B.80})$$

for a fully cursed trader. The system of (B.78), (B.79) and (B.80) can be reduced into one equation in the unknown  $q \equiv \frac{A_r}{A_c}$ . Indeed, using  $q$ , we can write (B.78) and (B.80) as (35) and (34), respectively. Moreover, dividing (B.79) by (B.80), we find

$$\frac{\tau_\eta - \tau_\xi \frac{q}{(N_r-1)q+N_c}}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} \frac{\tau_\epsilon + \tau_\zeta + \tau_\eta}{\tau_\eta} = q \Rightarrow q = \frac{\left(1 - \frac{q[(N_r-1)q+N_c]}{(N_r-1)q^2+N_c}\right) (\tau_\epsilon + \tau_\zeta + \tau_\eta)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \frac{[(N_r-1)q+N_c]^2 \tau_\eta}{(N_r-1)q^2+N_c}}, \quad (\text{B.81})$$

where the second step follows by (35). Equation (B.81) yields (36).

For  $q \leq 0$ , the left-hand side of (36) is non-positive and the right-hand side is positive. For  $q \geq 1$  the left-hand side of (36) is positive and the right-hand side is non-positive. Therefore, a solution of (36) must belong to  $(0, 1)$ . For  $q \in (0, 1)$ , the left-hand side of (36) is increasing in  $q$  and the right-hand side is decreasing in  $q$  (because the numerator is decreasing, the denominator is increasing, and both are positive). Since the left-hand side is zero at  $q = 0$ , and the right-hand side is zero at  $q = 1$ , a solution of (36) exists and is unique.

We next determine the sign of the coefficient  $\beta_i$  in the regression (37). Because of symmetry,  $\beta_i$  is equal to a common value  $\beta_r$  for all rational traders and to a common value  $\beta_c$  for all fully cursed traders. We show that  $\beta_c < 0$ ; this will imply that  $\beta_r > 0$  because market clearing (5) implies that  $N_r \beta_r + N_c \beta_c = 0$ .

The coefficient  $\beta_c$  is proportional to  $\text{Cov}(x_i, p - \bar{d})$ , which in turn is proportional to

$$\begin{aligned} & \text{Cov} \left( \bar{d} + \frac{\tau_\eta s_i}{\tau_\epsilon + \tau_\eta} - p, p - \bar{d} \right) \\ &= \left( \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - N_r A_r - N_c A_c \right) (N_r A_r + N_c A_c) \sigma_\epsilon^2 + \left( \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - N_c A_c \right) A_c \sigma_\eta^2 - N_r A_r^2 \sigma_\eta^2, \end{aligned} \quad (\text{B.82})$$

where the first step follows by setting  $(\chi_i, \tau_{\eta_i}, z_i) = (1, \tau_\eta, 0)$  in (B.14), and the second from (1) and (32). To determine the sign of (B.82), we compute some of the terms in that equation.

Multiplying (B.79) by  $N_r$ , (B.80) by  $N_c$ , and adding the resulting equations, we find

$$\begin{aligned} (N_r A_r + N_c A_c) \mathcal{D} &= \frac{N_r \tau_\eta + N_r \tau_\xi \frac{(N_r-1)A_r + N_c A_c}{(N_r-1)A_r + N_c A_c}}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \\ &= \frac{N_r(\tau_\eta + \tau_\xi)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta}, \end{aligned}$$

where

$$\mathcal{D} \equiv \frac{N_r(\tau_\epsilon + \tau_\eta + \tau_\xi)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c(\tau_\epsilon + \tau_\eta)}{\tau_\epsilon + \tau_\zeta + \tau_\eta}.$$

Therefore,

$$\begin{aligned} \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - N_r A_r - N_c A_c &= \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - \frac{N_r(\tau_\eta + \tau_\xi)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \\ &= \frac{\frac{N_r \tau_\eta (\tau_\epsilon + \tau_\eta + \tau_\xi)}{\tau_\epsilon + \tau_\eta} - N_r(\tau_\eta + \tau_\xi)}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi) \mathcal{D}} \\ &= -\frac{N_r \tau_\epsilon \tau_\xi}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi)(\tau_\epsilon + \tau_\eta) \mathcal{D}}. \end{aligned} \tag{B.83}$$

Multiplying (B.79) by  $N_r A_r$ , (B.80) by  $N_c A_c$ , and adding the resulting equations, we find

$$\begin{aligned} (N_r A_r^2 + N_c A_c^2) \mathcal{D} &= \frac{N_r A_r \tau_\eta + N_r \tau_\xi \frac{(N_r-1)A_r^2 + N_c A_c^2}{(N_r-1)A_r + N_c A_c}}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c A_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \\ &= \frac{N_r A_r \tau_\eta + N_r [(N_r-1)A_r + N_c A_c] \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c A_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \\ &= \frac{N_r(N_r A_r + N_c A_c) \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c A_c \tau_\eta}{\tau_\epsilon + \tau_\zeta + \tau_\eta}, \end{aligned}$$

where the second step follows from (B.78). Therefore,

$$\begin{aligned} \left( \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - N_c A_c \right) A_c \sigma_\eta^2 - N_r A_r^2 \sigma_\eta^2 &= \frac{A_c}{\tau_\epsilon + \tau_\eta} - \frac{N_r(N_r A_r + N_c A_c)}{\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi} + \frac{N_c A_c}{\tau_\epsilon + \tau_\zeta + \tau_\eta} \\ &= \frac{\frac{N_r(\tau_\epsilon + \tau_\eta + \tau_\xi) A_c}{\tau_\epsilon + \tau_\eta} - N_r(N_r A_r + N_c A_c)}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi) \mathcal{D}}. \end{aligned} \tag{B.84}$$

Equations (B.83) and (B.84) imply that (B.82) is equal to

$$\begin{aligned} & - \frac{N_r \tau_\xi (N_r A_r + N_c A_c)}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi)(\tau_\epsilon + \tau_\eta) \mathcal{D}} + \frac{\frac{N_r (\tau_\epsilon + \tau_\eta + \tau_\xi) A_c}{\tau_\epsilon + \tau_\eta} - N_r (N_r A_r + N_c A_c)}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi) \mathcal{D}} \\ & = \frac{N_r (\tau_\epsilon + \tau_\eta + \tau_\xi) (A_c - N_r A_r - N_c A_c)}{(\tau_\epsilon + \tau_\zeta + \tau_\eta + \tau_\xi)(\tau_\epsilon + \tau_\eta) \mathcal{D}}. \end{aligned}$$

This is negative because  $N_c \geq 1$ ,  $A_r > 0$  and  $A_c > 0$ .

We finally show that if (38) holds, then expected aggregate volume when all  $N$  traders are fully cursed is larger than when  $N - 1$  traders are fully cursed and one trader is rational. This will establish that expected aggregate volume is maximum at an interior point if (38) holds, because volume when all  $N$  traders are rational is zero.

When all  $N$  traders are fully cursed, expected volume per trader can be derived from (17) by setting  $\chi = 1$ , and expected aggregate volume can be derived by multiplying by  $N$ :

$$NE(|x_i|) = \frac{N \tau_\zeta \sqrt{2(N-1) \tau_\eta}}{\alpha (\tau_\epsilon + \tau_\zeta + \tau_\eta) \sqrt{\pi N}}. \quad (\text{B.85})$$

To compute expected aggregate volume when  $N - 1$  traders are fully cursed and one trader is rational, we start by computing the expected volume that one fully cursed trader generates. Setting  $(\chi_i, \alpha_i, \tau_{\eta_i}, \tau_\theta, z_i) = (1, \alpha, \tau_\eta, 0, 0)$  in (B.14), substituting  $p$  from (32), and denoting the rational trader by  $i_r$ , we can write the quantity that trader  $i \neq i_r$  trades in equilibrium as

$$x_i = \frac{\tau_\zeta (\tau_\epsilon + \tau_\eta)}{\alpha (\tau_\epsilon + \tau_\zeta + \tau_\eta)} \sum_{j=1}^N a_{ij} s_j, \quad (\text{B.86})$$

where

$$a_{ii} \equiv \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - A_c, \quad (\text{B.87})$$

$$a_{ii_r} \equiv -A_r, \quad (\text{B.88})$$

$$a_{ij} \equiv -A_c \quad \text{for } j \neq i, i_r. \quad (\text{B.89})$$

Using (1), we can write (B.86) as

$$x_i = \frac{\tau_\zeta(\tau_\epsilon + \tau_\eta)}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)} \left[ \left( \sum_{j=1}^N a_{ij} \right) \epsilon + \sum_{j=1}^N a_{ij} \eta_j \right]. \quad (\text{B.90})$$

Substituting  $x_i$  from (B.90) into (B.20), and noting that the  $N + 1$  variables  $(\epsilon, \{\eta_i\}_{i=1,\dots,N})$  are mutually independent, we find

$$E(|x_i|) = \frac{\tau_\zeta(\tau_\epsilon + \tau_\eta)}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)} \sqrt{\frac{2}{\pi} \left[ \left( \sum_{j=1}^N a_{ij} \right)^2 \frac{1}{\tau_\epsilon} + \sum_{j=1}^N a_{ij}^2 \frac{1}{\tau_\eta} \right]}. \quad (\text{B.91})$$

For  $N_r = 1$  and  $N_c = N - 1$ , (35) implies that  $\tau_\xi = (N - 1)\tau_\eta$ , (36) implies that

$$x = \frac{\tau_\epsilon + \tau_\zeta + \tau_\eta}{2(\tau_\epsilon + \tau_\zeta) + (N + 1)\tau_\eta},$$

and (33) and (34) become

$$A_r = \frac{\tau_\eta(\tau_\epsilon + \tau_\zeta + \tau_\eta)}{\mathcal{G}}, \quad (\text{B.92})$$

$$A_c = \frac{\tau_\eta [2(\tau_\epsilon + \tau_\zeta) + (N + 1)\tau_\eta]}{\mathcal{G}}, \quad (\text{B.93})$$

respectively, where

$$\mathcal{G} \equiv (N - 1)(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\zeta + N\tau_\eta) + (\tau_\epsilon + N\tau_\eta)(\tau_\epsilon + \tau_\zeta + \tau_\eta).$$

Substituting  $(A_r, A_c)$  from (B.92) and (B.93) into (B.87)-(B.89), we find

$$a_{ii} = \frac{\tau_\eta [(N-2)(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\zeta + N\tau_\eta) + (N-1)\tau_\eta(\tau_\epsilon + \tau_\zeta + \tau_\eta)]}{(\tau_\epsilon + \tau_\eta)\mathcal{G}}, \quad (\text{B.94})$$

$$\sum_{j=1}^N a_{ij} = -\frac{(N-1)\tau_\epsilon\tau_\eta(\tau_\epsilon + \tau_\zeta + \tau_\eta)}{(\tau_\epsilon + \tau_\eta)\mathcal{G}}, \quad (\text{B.95})$$

$$\begin{aligned} \sum_{j \neq i}^N a_{ij}^2 &= \frac{\tau_\eta^2}{\mathcal{G}^2} [(N-1)(\tau_\epsilon + \tau_\zeta + \tau_\eta)^2 \\ &\quad + 2(N-2)(\tau_\epsilon + \tau_\zeta + \tau_\eta)(\tau_\epsilon + \tau_\zeta + N\tau_\eta) + (N-2)(\tau_\epsilon + \tau_\zeta + N\tau_\eta)^2]. \end{aligned} \quad (\text{B.96})$$

Using (B.94), (B.95) and (B.96), we can write (B.91) as

$$E(|x_i|) = \frac{\tau_\zeta \sqrt{2\tau_\eta(\tau_\epsilon + \tau_\eta)\mathcal{N}_c}}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)\mathcal{G}\sqrt{\pi}}, \quad (\text{B.97})$$

where

$$\begin{aligned} \mathcal{N}_c &\equiv (\tau_\epsilon + N\tau_\eta)(\tau_\epsilon + \tau_\zeta + \tau_\eta) [(N-1)(\tau_\epsilon + \tau_\zeta + \tau_\eta) + 2(N-2)(\tau_\epsilon + \tau_\zeta + N\tau_\eta)] \\ &\quad + (N-2)(N-1)(\tau_\epsilon + \tau_\eta)(\tau_\epsilon + \tau_\zeta + N\tau_\eta)^2. \end{aligned}$$

We next compute the expected trading volume that the rational trader  $i_r$  generates. The market-clearing equation (5) implies that

$$x_{i_r} = -\sum_{j \neq i_r} x_j = -\frac{\tau_\zeta(\tau_\epsilon + \tau_\eta)}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)} \sum_{j \neq i_r} \sum_{k=1}^N a_{jk} s_k = -\frac{\tau_\zeta(\tau_\epsilon + \tau_\eta)}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)} \sum_{k=1}^N \sum_{j \neq i_r} a_{jk} s_k, \quad (\text{B.98})$$

where the second step follows from (B.86), and the third by inverting the order of summation. Using (B.98) and proceeding as in the derivation of (B.91), we find

$$E(|x_{i_r}|) = \frac{\tau_\zeta(\tau_\epsilon + \tau_\eta)}{\alpha(\tau_\epsilon + \tau_\zeta + \tau_\eta)} \sqrt{\frac{2}{\pi} \left[ \left( \sum_{k=1}^N \sum_{j \neq i_r} a_{jk} \right)^2 \frac{1}{\tau_\epsilon} + \sum_{k=1}^N \left( \sum_{j \neq i_r} a_{jk} \right)^2 \frac{1}{\tau_\eta} \right]}. \quad (\text{B.99})$$



Eq. (B.95) implies that

$$\sum_{k=1}^N \sum_{j \neq i_r} a_{jk} = \sum_{j \neq i_r} \sum_{k=1}^N a_{jk} = -\frac{(N-1)^2 \tau_\epsilon \tau_\eta (\tau_\epsilon + \tau_\zeta + \tau_\eta)}{(\tau_\epsilon + \tau_\eta) \mathcal{G}}. \quad (\text{B.100})$$

Equations (B.88) and (B.89) imply that

$$\begin{aligned} \sum_{k=1}^N \left( \sum_{j \neq i_r} a_{jk} \right)^2 &= (N-1)^2 A_r^2 + (N-1) \left( \frac{\tau_\eta}{\tau_\epsilon + \tau_\eta} - (N-1) A_c \right)^2 \\ &= \frac{(N-1) \tau_\eta^2 (\tau_\epsilon + \tau_\zeta + \tau_\eta)^2 \left[ (N-1) (\tau_\epsilon + \tau_\eta)^2 + [\tau_\eta - (N-2) \tau_\epsilon]^2 \right]}{(\tau_\epsilon + \tau_\eta)^2 \mathcal{G}^2}, \end{aligned} \quad (\text{B.101})$$

where the second step follows from (B.92) and (B.93). Using (B.100) and (B.101), we can write (B.99) as

$$E(|x_{i_r}|) = \frac{\tau_\zeta \sqrt{2(N-1) \tau_\eta (\tau_\epsilon + N \tau_\eta) \mathcal{N}_r}}{\alpha \mathcal{G} \sqrt{\pi}}, \quad (\text{B.102})$$

where

$$\mathcal{N}_r \equiv (N-1)^2 (\tau_\epsilon + \tau_\eta) - (N-2) (\tau_\epsilon + N \tau_\eta).$$

Combining (B.85) with (B.97) and (B.102), we find that expected aggregate trading volume is larger when one trader is rational and  $N-1$  traders are fully cursed than when all  $N$  traders are fully cursed if and only if

$$\begin{aligned} N \frac{\sqrt{N-1}}{(\tau_\epsilon + \tau_\zeta + \tau_\eta) \sqrt{N}} &< (N-1) \frac{\sqrt{(\tau_\epsilon + \tau_\eta) \mathcal{N}_c}}{(\tau_\epsilon + \tau_\zeta + \tau_\eta) \mathcal{G}} + \frac{\sqrt{(N-1) (\tau_\epsilon + N \tau_\eta) \mathcal{N}_r}}{\mathcal{G}} \\ \Leftrightarrow \sqrt{N} &< \frac{\sqrt{(N-1) (\tau_\epsilon + \tau_\eta) \mathcal{N}_c}}{\mathcal{G}} + \frac{(\tau_\epsilon + \tau_\zeta + \tau_\eta) \sqrt{(\tau_\epsilon + N \tau_\eta) \mathcal{N}_r}}{\mathcal{G}}. \end{aligned} \quad (\text{B.103})$$

Equation (B.103) holds under the sufficient condition

$$N < \frac{(N-1) (\tau_\epsilon + \tau_\eta) \mathcal{N}_c}{\mathcal{G}^2} + \frac{(\tau_\epsilon + \tau_\zeta + \tau_\eta)^2 (\tau_\epsilon + N \tau_\eta) \mathcal{N}_r}{\mathcal{G}^2}. \quad (\text{B.104})$$

Multiplying both sides by  $\mathcal{G}^2$ , using the definitions of  $(\mathcal{G}, \mathcal{N}_c, \mathcal{N}_r)$ , and rearranging, we can write (B.104) as (38).

When  $\tau_\zeta = \infty$ , (38) becomes

$$\frac{1}{N-1} \left( \frac{\tau_\epsilon + N\tau_\eta}{\tau_\epsilon + \tau_\eta} \right)^2 + \left( \frac{2}{N-1} - 1 \right) \frac{\tau_\epsilon + N\tau_\eta}{\tau_\epsilon + \tau_\eta} + 1 < 0. \quad (\text{B.105})$$

Setting  $y \equiv \frac{\tau_\eta}{\tau_\epsilon}$ , we can write (B.105) as

$$\begin{aligned} \frac{1}{N-1} \left( \frac{1+Ny}{1+y} \right)^2 - \frac{(N-3)(1+Ny)}{(N-1)(1+y)} + 1 < 0 \\ 3 - (N^2 - 6N - 1)y + (4N - 1)y^2 < 0. \end{aligned} \quad (\text{B.106})$$

The left-hand side of (B.106) converges to  $-\infty$  when  $N$  goes to  $\infty$ . Hence, (B.106) holds for  $N$  large enough. ■

**Proof of Proposition 8.** Setting  $(\chi_i, \tau_\theta) = (1, 0)$  for all  $i$  in (11), we find (40). Setting  $(\chi_i, \tau_\theta, z_i) = (1, 0, 0)$  in (B.14), and substituting  $p$  from (39), we find

$$x_i = \frac{\tau_\zeta(\tau_\epsilon + \tau_{\eta_i})}{\alpha_i(\tau_\epsilon + \tau_\zeta + \tau_{\eta_i})} \sum_{j=1}^N a_{ij} s_j, \quad (\text{B.107})$$

where

$$a_{ii} \equiv \frac{\tau_{\eta_i}}{\tau_\epsilon + \tau_{\eta_i}} - A_i, \quad (\text{B.108})$$

$$a_{ij} \equiv -A_j \quad \text{for } j \neq i. \quad (\text{B.109})$$

Proceeding as in the proof of Proposition 7, we find

$$E(|x_i|) = \frac{\tau_\zeta(\tau_\epsilon + \tau_{\eta_i})}{\alpha_i(\tau_\epsilon + \tau_\zeta + \tau_{\eta_i})} \sqrt{\frac{2}{\pi} \left[ \left( \sum_{j=1}^N a_{ij} \right)^2 \frac{1}{\tau_\epsilon} + \sum_{j=1}^N a_{ij}^2 \frac{1}{\tau_{\eta_j}} \right]}. \quad (\text{B.110})$$

When  $\alpha_i = \alpha$  for all  $i$  and  $\tau_\zeta = \infty$ , (40) implies that  $A_i = \frac{\tau_{\eta_i}}{N(\tau_\epsilon + \tau_\eta)}$ . Substituting into (B.108) and (B.109), we find

$$\begin{aligned} a_{ii} &= \frac{[(N-1)\tau_\epsilon + N\tau_\eta - \tau_{\eta_i}]\tau_{\eta_i}}{N(\tau_\epsilon + \tau_{\eta_i})(\tau_\epsilon + \tau_\eta)}, \\ a_{ij} &= -\frac{\tau_{\eta_j}}{N(\tau_\epsilon + \tau_\eta)} \quad \text{for } j \neq i, \\ \sum_{j=1}^N a_{ij} &= \frac{(\tau_{\eta_i} - \tau_\eta)\tau_\epsilon}{(\tau_\epsilon + \tau_{\eta_i})(\tau_\epsilon + \tau_\eta)}. \end{aligned}$$

Substituting into (B.110), and using again  $\alpha_i = \alpha$  for all  $i$  and  $\tau_\zeta = \infty$ , we find

$$\begin{aligned} E(|x_i|) &= \frac{\sqrt{2 \left[ (\tau_{\eta_i} - \tau_\eta)^2 \tau_\epsilon + \frac{[(N-1)\tau_\epsilon + N\tau_\eta - \tau_{\eta_i}]^2 \tau_{\eta_i}}{N^2} + \sum_{j \neq i} \frac{(\tau_\epsilon + \tau_{\eta_i})^2 \tau_{\eta_j}}{N^2} \right]}}{\alpha(\tau_\epsilon + \tau_\eta)\sqrt{\pi}} \\ &= \frac{\sqrt{2 \left[ (\tau_{\eta_i} - \tau_\eta)^2 \tau_\epsilon + \frac{[(N-1)\tau_\epsilon + N\tau_\eta - \tau_{\eta_i}]^2 \tau_{\eta_i}}{N^2} + \frac{(\tau_\epsilon + \tau_{\eta_i})^2 (N\tau_\eta - \tau_{\eta_i})}{N^2} \right]}}{\alpha(\tau_\epsilon + \tau_\eta)\sqrt{\pi}}, \end{aligned} \quad (\text{B.111})$$

where the second step follows from the definition of  $\tau_\eta$ . Eq. (41) follows from (B.111) by separating quadratic, linear and constant terms in  $\tau_\epsilon$ . Trader  $i$  generates more volume than trader  $j$  if and only if the difference between the term inside the squared root in (41) and the corresponding term for  $j$  is positive. The difference is

$$[(N-2)\tau_\epsilon^2 + (N-2)\tau_\epsilon(\tau_{\eta_i} + \tau_{\eta_j}) + (N\tau_\eta - \tau_{\eta_i} - \tau_{\eta_j})\tau_\eta](\tau_{\eta_i} - \tau_{\eta_j}).$$

Since

$$N\tau_\eta - \tau_{\eta_i} - \tau_{\eta_j} = \sum_{k=1}^N \tau_{\eta_k} - \tau_{\eta_i} - \tau_{\eta_j} = \sum_{k \neq i, j} \tau_{\eta_k} > 0,$$

the difference is positive if and only if  $\tau_{\eta_i} > \tau_{\eta_j}$ . ■