# Cross-curricular Mathematics: A case study 

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#### Abstract

Educational and academic success of the various subjects is not yet a won battle. For that reason, organizations continue to (re)invent methods which may contribute to that success. More recently, it is strongly supported that the Curricula should be articulated: i) vertically, throughout all the school years, and ii) horizontally, among themselves in each different year. Research studies show, however, this measure is not being systematically implemented or even carried out in the best way. Therefore, a research was developed to explore how the process of inter-year and inter-cycle Cross Curricular Mathematics is interpreted, planned and experienced, as set out at a ministerial level, within a vertical grouping of schools. A qualitative and interpretative case study was chosen. The participants were 11 teachers, some with management positions. For data collection, a document analysis, observation and inquiry were included. The data show that teachers unanimously believed in the potentialities of collaborative work; and that, despite all efforts and measures introduced, there continues to be a difference between the scholar cycles. In fact, there are horizontal articulations regarding the several years of the same cycle but, in what concerns vertical articulation, there is a gap between the 1st and the others cycles.


Key words: Vertical grouping of schools, autonomy, cross-curricular mathematics, teacher's work, collaborative culture and collegiality, teacher participation.

## Introduction

Several sources have been repeatedly denouncing the "general crisis" system in which we have been living in. In several countries political-educational panorama, including Portugal, this situation lead to continuous reforms, which translated in legislative measures that intended to resolve problems resulting, namely, from the mass generalization of Education. As a consequence, a school network structure was implemented, organized in School Groups and more recently, in mega School Groups, which lead to an increased autonomy of those schools. This allowed them to create their own development project at a strategic, pedagogic, administrative, financial and organizational level. Such autonomy also legitimizes the increase of autonomous participation of teachers. Therefore, the management structures they belong to are given the power and the means to define their own school policies and to establish their development plans. It is assumed the collegial spirit promotes efficient and innovative collaborative practices, based on a constant reflexive confront of ideas, experiences and knowledge among the teachers within the same or different school cycles and/or years.
Within this framework, it is pertinent to understand in particular the way different school structures are organized in order to accomplish the vertical articulation, between school years and cycles. And Mathematics was elected as one of the most problematic subjects at the basic level.
Therefore, the defined main research question was: how is interpreted, planned and experienced the process of vertical Cross Curricular Mathematics at the basic school level, in a School Group?
From this starting point, seven research objectives were outlined:

- To identify the process of student characterization and to assess the impact on the teaching planning;
- To learn more about the organization of Curriculum Department and its influence on teachers' work;
- To understand the process of inter-years and inter-cycle Cross Curricular Mathematics;
- To identify the impact of the school Project - Mathematics Plan - in the teachers' work;
- To identify the difficulties and benefits of collaborative work among the Mathematics teachers;
- To obtain teachers' points of view on what it means to be a mathematics teacher today;
- To obtain teachers' points of view on what the work of a mathematics teacher entails - the link between cycles and school years.


## Theoretical framework

This study's theoretical framework is based on three main connected themes: vertical grouping of schools, curricular articulation and teachers work.
Current demands require a more integrating and inclusive school, one which does not promote the compartmentalization of knowledge, but assures a comprehensive training of all basic level students. One of the government solutions for this issue translated in the creation of Vertical School Groupings. This united all independent institutions towards a common project. Management structures were merged making the most of each one. An autonomy perspective emerges and assumes a conception of School with its very own identity. Teachers interact and their participation is valued and promoted. This allowed a better management of resources and, in consequence, a better educational public service performance (Day, 2001; Simões 2005).
Therefore, all teachers should mobilize and assume themselves as co-creators of a more pertinent curriculum for their schools or school groupings, as they should take responsibility for the promotion and structuring of the learning process of their students (NCTM, 2007; Barnes 2011, Morgado 2013). Such curricular management is essentially linked with the way teachers interpret and shape the curriculum in two levels: a macro level, which is related to the planning of the teaching practice, and a micro level, which corresponds to the class room and the execution of the teaching practice (Barnes, 2011; Roldão, 2005).

Taking into account a periodic evaluation and reflection of its professional practices, the curriculum is always subject to adjustments and should be based on a true curricular articulation, which, in a logical and sequential way, assures the continuity between the school years and cycles - "The Mathematics School Curriculum should provide a sort of map, which should help the teachers to lead their students towards increased levels of complexity of knowledge. This guidance requires a well-articulated curriculum allowing the teachers, in each level, to understand the mathematics learned by their students in the previous level, as well as contents in which they should focus in the following levels." (NCTM, 2007: 17).

In fact, the curricular articulation, either at a vertical level, between school cycles and years, or at a horizontal level, between subjects and non-subject curricular areas in the same school year, seem to be the keyword at least in what mandatory basic education is concerned. Such articulation is clear in the teachers' work, which should be based in collaborative behavior. These behaviors are currently promoted and developed by the pedagogic structures, particularly the Curricular Departments. In these structures, the Subject Groupings and Areas are represented according to the courses (for example, "Mathematics and Experimental Sciences" or "Languages"), the number of teachers by subjects and the dynamics which should be developed by the School.
Taking into consideration the Portuguese Basic Education Mathematics Program (PMEB), we realized its intention in promoting the vertical and horizontal articulation (Ponte at al, 2007). On one hand, it argues that past, present and future learning should be integrated, and on the other it states that connections should be made within different school years. This way, a progressive continuity line between cycles and learning levels is ensured. Sequentially, teachers should assume the role of connecting the cycles and school years, since their work is one of the elements which most influence the quality of education and learning (Thurler, 2000; Bolivar, 2000; Day, 2001; Ponte et al, 2007; NCTM, 2007).
Teachers reflections on their conceptions and their practices is also an indispensable requirement for the development and change of practices (Loyd, 2002; Hart, 2002; Champman, 2002; Ernest, 1989; Wilson e Cooney, 2002).
Such demanding task will certainly be facilitated and fostered if developed in collaboration with their pears (Fullan e Hargreaves, 2000; Day, 2001). In fact, team work promotes the resolution problems and provides mutual support for their professional development, targeting innovation and education quality and efficiency (Hargreaves, 1998; Bolivar, 2000; Minnett 2003). It is through participation that teachers can make decisions, together and actively, present, confront and share ideas. Particularly they can develop curricular management interpreting and promoting the curriculum taking into account the specific characteristics of their students, existing resources, school conditions and social-economic and educational framework (NCTM, 2007). By getting involved in reflection processes, this participation will allow the existence of a critical debate about their tasks, their problems and the way to resolve them. In short, it will allow them to intervene fully in all education activities e contribute actively for their professional development (Ernest, 1989; Thompson, 1992; Wilson e Cooney, 2002; Llinares, 2002; Sowder, 2007).
In order to fully defeat the new and great challenges, they will have to overcome constraints and obstacles. They will also have to (re)invent other conditions and factors which promote and optimize the
work of the teacher, mathematics in particular, namely, within a framework of curricular articulation between school cycles and years at a Basic Education level.

## Research methodology

In this item, we start to explain the methodology options and participants of this research work. We proceed with a brief presentation of the chosen information gathering tools and techniques for each research phase and a summary description of the case study. Finally, we explain the method used for data processing and its presentation.

## Methodology options and participants

Taking into account the research objectives, it was decided to undertake a qualitative study, based on a constructivist paradigm, and to follow the single study strategy, micro ethnographic study form (Bogdan \& Biklen, 1994; Stake, 1995; Gomez, Flores and Jimenez, 1996; Crosswell, 2003).
The study focused on a specific Vertical Grouping of Schools, selected due to the schools' accessibility, since it was geographically close to the researcher residence area, as well as for the schools' voluntary participation to implement a project of actual vertical curricular articulation.
At the macro level, this study had the participation of the President of the Executive Council and the president of the Pedagogic Council. At the meso level, the study counted on the participation of the coordinator of the Curricular Department of the Basic Education $2^{\text {nd }}$ and $3^{\text {rd }}$ Cycles (from the $5^{\text {th }}$ to the $9^{\text {th }}$ Grade), which integrates the Mathematics subject, and the coordinator of the Curricular Department of the $1^{\text {st }}$ Cycle (from the $1^{\text {st }}$ to the $4^{\text {th }}$ Grade). Finally, at the micro level participated three math teachers from the $3^{\text {rd }}$ Cycle, three from the $2^{\text {nd }}$ Cycle a one teacher from the $1^{\text {st }}$ Cycle. Nine of the participants were females and two were males, ranging from 8 to 39 years of teaching experience.

## Information gathering tools and techniques and study description

The information gathering techniques used in this study were the document analysis, which was based on formal records at an exo and macro level, the inquiry, using script-oriented semi-structured interviews, and the direct observation, supported by field note registration and logbook, which permitted a complete perspective of the studied phenomena.
This research occurred during the 2008/2009 school year in a Vertical Grouping of Schools of the Central Coastal Region of Portugal. This School Grouping assembles nine $1^{\text {st }}$ Cycle schools (all geographically separated from each other and from the Head School) and one $2^{\text {nd }}$ and $3^{\text {rd }}$ Cycle school. The $1^{\text {st }}$ Cycle schools are built according to traditional architecture models, although well preserved e globally well equipped. In the Head-School facility there is an exclusive classroom for the Mathematics subject.
The empiric study was organized in three distinct phases. The first phase consisted in the planning of the study in what concerned the theoretical framework, method definition of the research, question preparation, setting of research objectives, selection of School Grouping and participants. After the acceptance of all participants, a guided visit to the $2^{\text {nd }}$ and $3^{\text {rd }}$ Cycle School and some of the 1 st Cycles Schools facilities was carried out. During this guided tour we were able to observe all areas and collect some field notes. At that same time, the School Grouping documents were gathered: School Grouping Curricular Project, School Grouping Education Project and Internal Regulation Documents. It was then created the interviews script, which were promptly individually carried out onsite. In the last phase, we proceeded to the sorting and analysis of the collected data.

## Data processing and presentation

The data was processed through content analysis and sorted by categories, which were created taking into account the research purposes. The data was then presented using a descriptive approach, transcribing some of the most relevant statements.

## Data analysis and discussion

In a Vertical School Grouping, the students are considered 9 year residents - from the $1^{\text {st }}$ to the $9^{\text {th }}$ grade. Their Individual File gathers information such as their social-economic context, family background and their school path. However, these elements are not enough to have a general perspective of their development stage comprehensively and individually, as far as Mathematics is concerned. Therefore, to complete this characterization it was taken into account diagnostic tests, results from the national exams
and other records from previous school years, and also the questionnaires filled in the classrooms, which completed the information recorded in the Class Curricular Project. A participant from the $1^{\text {st }}$ Cycle further stated that Individual Student Files "are assembled throughout each school year. I have a chart composed of specific student evaluation items, which ranges from problem solving, to memorizing and communication skills in Mathematics, which I use to record the related values" ( $1^{\text {st }}$ Cycle teacher).
Other source used to characterize the students was the meetings carried out at the beginning and at the end of the school year. These meetings' agenda included the execution of the curricular management and involved the three Basic Education Cycles - "In the first meetings we prepare and organize the entire school year and the final meetings are used to evaluate the execution of the programs and to improve some aspects of the work carried out during the year and to plan specific activities for the beginning of the following year" ( $2{ }^{\text {nd }}$ Cycle teacher). According to the interviewees, the collected information influenced, at a meso level, the management of the Subject Group activities and, at a micro level, it affected the planning and organization of the class tasks. The $1^{\text {st }}$ Cycle coordinator stated that the student characterization influenced the work of the group of teachers who taught in the School Grouping.
In what concerns the curricular departments, we must single out the Mathematics and Experimental Sciences Departments, which are composed by $2^{\text {nd }}$ and $3{ }^{\text {rd }}$ Cycle Mathematics, Nature Sciences, Natural Sciences and Physic-Chemistry Science teachers, as well as the $1^{\text {st }}$ Cycle Curricular Department. These departments were responsible for the curricular development based on a collaborative culture. Together, teachers were responsible for the standardization of the work to be undertaken in the future, shared and crossed ideas and experiences, and searched for adequate activities for their students' needs, as stated a $3^{\text {rd }}$ Cycle teacher: "This is the opportunity for us to act in a more or less uniform fashion, with mutual help and sharing collaborative work." The $1^{\text {st }}$ Cycle Math teacher emphasized that the Curricular Department meetings, held on a mandatory monthly basis, were useful to outline new challenges and new projects. However, she expressed the number of participants was too high and the meeting duration too short. She also referred to the legislation changes, which altered the Department's composition. More Subject Areas were associated, which reveilles to be less productive. However, it was unanimously recognized that this is the only formal moment of group work, in what concerns the model of curricular management developed on a collaborative culture basis.
After analyzing the School Grouping Education Project, it has emerged the curricular articulation as an element to improve, as referred by all the Curricular Departments. In that Project, measures were taken, namely: i) not attributing a single school level to each teacher; ii) the school defines the pedagogic organization guidelines, assuring the continuity between school levels and years; iii) appealing to a participation of all teachers.
In what concerns the $2^{\text {nd }}$ and $3^{\text {rd }}$ Cycles, the Math teachers benefit from the School Project - Mathematics Plan (implemented by the Government). This sub-group of teachers held 90 minutes weekly meetings to develop their work, promoting horizontal and vertical articulations, intra and inter-years and school cycles. According to the general opinion, that Project brought many positive aspects for the teaching and learning of Math.
These teachers underlined, on one hand:

- the possibility to work as a team and to obtain new (and more) materials, namely, computer and technological resources, and, in particular, interactive boards;
- the better teaching articulation and equity in the curricular development and in the evaluation moments;
- the assignment of more teaching time for the Math subject, making the most of curricular areas of nonrelated Math subjects, such as Guided Study and, in the $6^{\text {th }}$ Grade, the School offer also oriented for its teaching;
-the achievement of better results from students.
None of the interviewed teachers mentioned any negative sides associated to the Mathematics Plan. They only denounced the lack of work among pears in the classrooms, similar to what was found in other studies (Day, 2011). They also stated the implementation of the project justified an increase of funding, as well as more physical resources, since they considered them to be scarce. As far as the $1^{\text {st }}$ Cycle is concerned, the time destined for non-related teaching activities were all used with study support activities, supervision and meetings. Nonetheless, the teachers held regular meetings, although isolated from the other cycles, as it's stressed by a $3^{\text {rd }}$ cycle teacher: "In this school, the $3^{\text {rd }}$ cycle teachers work in articulation with the $2^{\text {nd }}$ and $3^{\text {rd }}$ cycles, whereas the $1^{\text {st }}$ cycle is a bit distant. Nevertheless, it's possible that next year we will be able to improve the articulation between the cycles. We are thinking about it".
In what concerns factors which make vertical curricular articulation more difficult, the interviewed teachers also underlined the incompatibility of schedules and the lack of culture for this kind of work, to which the great workload, beyond the scheduled school timetable, is no stranger (which is confirmed by Nóvoa, 2007; Brites, 2002; Day 2001). They also stressed that the number of teachers participating in the
workgroup influenced the outcome of their work. Additional difficulties come from the lack of physical spaces, which has negative implications in the teachers' timetables. For example, if they wanted to hold their group meetings beyond the weekly 90 minutes determined by their Management there would not be any rooms available.
The system hiatus are starting to be resolved since the creation of weekly or daily periods and the appreciation for collaborative work praised by the interviewed teachers, as it's also perceived the Official Authority intends to implement a mechanism at an exo level, by conceding greater autonomy to School Groupings.
About the perception of what it means to be a mathematics teacher today, the interviewees admitted that it is a challenge, a difficult task, sometimes discouraging. In opposition, they understand it to be enriching when it comes to sharing the knowledge of science. These opinions correspond to records from other studies (Groenwald e Nunes, 2007) and are related, on one hand, to the students' attitudes towards the subject and their motivation; with the level of knowledge they are able to acquire throughout their school path; with the belief mathematics will influence their professional future (Ponte et al, 2007; NCTM, 2007). On the other hand, emphasis is given to the new information emerging and to the new challenges introduced by the Official Authorities, namely concerning innovative methodologies to be implemented and assessment tests. Some of these issues were already raised by Morgado (2013).
To this regard, the $1^{\text {st }}$ Cycle Curricular Department Coordinator stated: "I believe the first change happened with the assessment tests. This shook the class and teachers started to realize they had to change methods and teaching material and even their own training. I can add that, in our School Grouping, all teachers are currently in training".
The interviewees further recognized the importance of their role in the society, associated to an increase tendency for the subject (Ponte et al, 2007). As stated by Putnam \& Borko (2000), they argue that being a Math teacher today demands continuing training and a constant exchange of professional experiences gathered through many years of work. They also valued collective practices in opposition to individual and hallow actions in what concerns learning and final outcomes.
All interviewed teachers underlined the work developed by their math colleagues and placed the link between cycles at the level of curricular articulation and collaborative work. They admitted that the taste and motivation for mathematics is "born" in the $1^{\text {st }}$ cycle and used terms such as "this is how we build a house" and "it's a snow ball" to justify the connection between cycles, in a perspective of continuity and sequential progress. Even though they understand the importance of their role as links between cycles, the teachers referred to the existence of some obstacles in that articulation, namely between the $1^{\text {st }}$ and $2^{\text {nd }}$ cycles, starting from the fact of their physical separation.
In short, from the several collected statements, it has stood out a feeling of hope in a greater student involvement with mathematics. The teachers recognized unanimously that the Mathematics Plan opens new horizons for teaching, but also new and increased responsibilities in its interpretation and application. The changes introduced in the education system, mainly through the Mathematics Plan, were greatly emphasized by the teachers, now integrated in a new concept of school, where the involvement of parents, students and teachers is faced as increasingly important and decisive, now and in the future; in other words, an open and modern school, open to the community, free of barriers and obstacles of all sorts.


## Conclusion

The study allowed us to understand better a certain reality and obtain several significant conclusions about the constraints faced by the mathematics teachers of the basic education while developing their work, in particular, in the creation of links inter-cycles. It also permitted us to assess which measures should be taken to surpass the obstacles, and on the other hand, to identify which conditions and factors promote the desirable vertical articulation.

Considering the raised questions at the beginning of this study, we can conclude:

## How were the students characterized and what impact did it have in the teaching planning?

Student characterization was carefully made, having resorted to a wide range of instruments. And, in fact, it did influence de management of the programmed initial activities, in order to correspond to the true students learning needs, as confirmed by the research made by Monteiro (2011).

How is the Curricular Department structured and how does it influence the work of teachers?

This structure was recognized as the collaborative workplace, par excellence. Together, teachers standardized their work, shared and crossed ideas and pedagogic experiences. They also sought to define the activities to undertake, as argued by Pereira 2012.

How is inter-cycles and inter-school years Cross Curricular Mathematics processed?
The curricular articulation is recognized as one of the features to improve. To this achievement, the collaborative attitudes among the teachers should be strengthen (as stated in the study undertaken by Morgado, 2013), therefore making the most of the curricular departments. They, however, underlined the necessity of seeking different spaces, which would allow a better articulation between the $1^{\text {st }}$ Cycle and the following cycles.

What was the School Project - Mathematics Plan impact in the teachers' work?
The teachers argued the Mathematics Plan brought, undoubtedly, advantages, mainly because it instigated a more regular collaborative work, as is insistently underlined by Little, 1990; Hargreaves, 1998; Fullan e Hargreaves, 2000; Thurler, 2000. This had had an impact in the students learning (Day, 2001 e Boavida e Ponte, 2002).

What are the difficulties and the benefits of the collaborative work, appointed by the Mathematics teachers?
All teachers admitted the collaborative work facilitates a better management of the curriculum and of the different subject contents, as it is stated by Minnett (2003). However, they also acknowledged this collaborative culture is not yet dominant in our education system, as denounced by Pereira 2012.

What is the opinion of teachers about what it means to be a Mathematics teacher nowadays?
The teachers confessed that, nowadays, being a Mathematics teacher is a huge and exciting challenge. It demands continuing training and updates, since Mathematics influences the highly unpredictable future life of students, both personally and professionally.

What is the opinion of teachers about the work of Mathematics teachers - link between cycles?
The teachers revealed to have a clear notion their work is crucial to the articulation between cycles and school years. That would only be reinforced when a real collaboration between teachers of different cycles of the same subjects occurs, as stated by Roldão (2007).

As a final footnote, we have to convey that only an actual connection between cycles and school years will create of a global vision on Mathematics throughout the school path, which is indispensable to assure a quality teaching-learning process. In order for this to happen, there is an urgency in the creation of a true collaborative work culture, focused on the essence of the teachers' work - teaching.

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