Performance Evaluation of Multicell Coordinated Beamforming Approaches for OFDM Systems

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Abstract* - In this paper we propose and evaluate multicell coordinated beamforming schemes for the downlink of MISO-OFDM systems. The precoders are designed in two phases: first the precoder vectors are computed in a distributed manner at each BS considering two criteria, namely distributed zero-forcing and virtual signal-to-interference noise ratio. Then the system is optimized through distributed power allocation under per-BS power constraint. The proposed power allocation scheme is designed based on minimization of the average bit error rate over all the available subcarriers. Both the precoder vectors and the power allocation are computed by assuming that the BSs have only knowledge of local channel state information and do not share the data symbols. The performance of the proposed schemes are evaluated, considering typical pedestrian scenarios based on LTE specifications. The results have shown that the proposed distributed power allocation scheme outperform the equal power allocation approach.

Keywords-component; distributed precoding, distributed power allocation, multicell systems, OFDM and LTE.

I. Introduction

Multicell cooperation is one of the fastest growing areas of research, and it is a promising solution for cellular wireless systems to mitigate intercell interference, improving system fairness and increasing capacity in the years to come. This technology is already under study in LTE-Advanced under the coordinated multipoint (CoMP) concept.

There are several CoMP approaches depending on the amount of information shared by the transmitters through the backhaul network and where the processing takes place, i.e., centralized if the processing takes place at the central unit (CU) or distributed if it takes at the different transmitters. Coordinated centralized beamforming approaches, where transmitters exchange both data and channel state information (CSI) for joint signal processing at the CU, promise larger spectral efficiency gains than distributed interference coordination techniques, but typically at the price of larger backhaul requirements and more severe synchronization requirements. Two centralized multicell precoding schemes based on the waterfilling technique have been proposed in [1]. It was shown that these techniques achieve a performance, in terms of weighted sum rate, very close to the optimal. In [2] a clustered BS coordination is enabled through a multicell block diagonalization (BD) strategy to mitigate the effects of interference in multicell MIMO systems. A new BD cooperative multicell scheme has been proposed in [3], to maximize the weighted sum-rate achievable for all the user terminals (UTs).

Distributed precoding approaches, where the precoder vectors are computed at each BS in a distributed fashion, have been proposed in [4]. It is assumed that each base station has only the knowledge of local CSI and based on that a parameterization of the beamforming vectors used to achieve the outer boundary of the achievable rate region was derived. In [5], distributed precoding schemes based on zero-forcing criterion with several centralized power allocation based on minimization of the average BER and sum of inverse of signal-to-noise ratio (SNIR) have been derived.

In the previous approaches, it was assumed that the transmitters (or BSs) share the entire data of all UTs. However, there are distributed beamforming approaches where the transmitters do not share the data, which fall into the interference channel (IC) framework. The local CSI, i.e. the CSI between a given BS and all UTs, is used by transmitters to design individual precoders to transmit exclusively to the users within their own cell [6], [7]. This approach, known as inter-cell interference nulling (ICIN), in which each BS transmits in the null-space of the interference it is causing to neighboring cells, has been discussed in the 3GPP long term evolution advanced (LTE-A) literature. The authors of [8] proposed a non-iterative distributed solution to design precoding matrices for multi-cell systems, which maximizes the sum-rates for only a two-cell system at high SNR. In [9], a coordinated beamforming approach based on the virtual SINR framework, for a special case of two transmitters, has been proposed.

The aim of this work is to propose and evaluate coordinated beamforming for the downlink of multicell MISO-OFDM systems. It is assumed that the BSs have only knowledge of local CSI and do not share the data symbols. The precoder is designed in two phases: first the precoder vectors are computed based on distributed zero-forcing (DZF), and distributed virtual signal-to-interference noise ratio (DVSINR). Then the system is further optimized by proposing a novel distributed power allocation algorithm, based on minimization of the average bit error rate (BER) over the available subcarriers. With the proposed strategy both the precoder vectors and the power allocation are computed at each BS in a distributed manner. The considered criterion for power allocation essentially lead to a redistribution of powers among subcarriers, and therefore provide data symbols fairness, which in practical cellular systems may be for the operators a goal as important as throughput maximization.

The remainder of the paper is organized as follows: section II presents the multicell MISO-OFDM system model. Section III briefly describes the considered distributed precoder vectors. In Section IV the novel distributed power allocation scheme is derived. Section V presents the main simulation results. The conclusions will be drawn in section VI.

II. SYSTEM MODEL

Throughout this paper, we will use the following notations. Lowercase letters, boldface lowercase letters and boldface uppercase letters are used for scalars, vectors and matrices, respectively. $(.)^H$ represents the conjugate transpose operators, $\mathrm{E}[.]$ represents the expectation operator, $\mathbf{I}_N$ is the identity matrix of size $N \times N$, $\mathcal{CN}(.,.)$ denotes a circular symmetric complex Gaussian vector and χ_n^2 denotes the chi-square random variable with n degrees of freedom.

We consider the MISO interference channel where B BSs, each equipped with N_{t_b} antennas, transmit to B single antenna UTs, as shown in Fig. 1. Also, we assume an OFDM based system with N_c available subcarriers. Under the assumption of linear precoding, the signal transmitted by the BS b on subcarrier l is given by,

$$\mathbf{x}_{b,l} = \sqrt{p_{b,l}}\mathbf{w}_{b,l}s_{b,l} \tag{1}$$

where $p_{b,l}$ represents the transmitted power allocated to sub-carrier l at BS b, $\mathbf{w}_{b,l} \in \mathbb{C}^{N_{tb}\times 1}$ is the precoder at BS b on sub-carrier l with unit norms, i.e., $\|\mathbf{w}_{b,l}\| = 1,\ b = 1,...,B,\ l = 1,...,N_c$. The data symbol $s_{b,l}$, with $\mathrm{E}\left[\left|s_{b,l}\right|^2\right] = 1$, is intended for UT b. The average power transmitted by the BS b is then given by,

$$\mathrm{E}\left[\|\mathbf{x}_b\|^2\right] = \sum_{l=1}^{N_c} p_{b,l} \tag{2}$$

where $\mathbf{x}_b$ is the signal transmitted over the N_c subcarriers.

The received signal at the UT b on sub-carrier l, $y_{b,l} \in \mathbb{C}^{1\times 1}$, can be expressed by,

$$y_{b,l} = \sum_{j=1}^{B} \sqrt{p_j}\mathbf{h}_{j,b,l}^H \mathbf{w}_{j,l}s_{j,l} + n_{b,l} \tag{3}$$

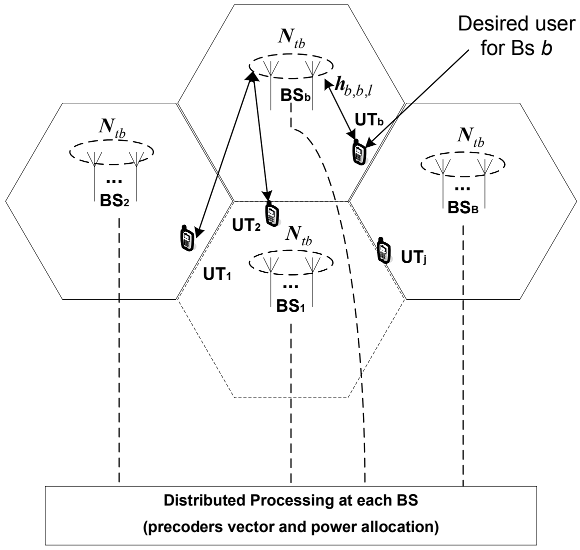

Fig. 1: System Model considered.

where $\mathbf{h}_{j,b,l} \sim \mathcal{CN}\left(0, \rho_{j,b}\mathbf{I}_{N_{t_b}}\right)$ of size $N_{t_b} \times 1$, represents the channel between user b and BS j on subcarrier l and $\rho_{j,b}$ is the long-term channel power gain between BS j, and UT b and $n_{b,l} \sim \mathcal{CN}\left(0, \sigma^2\right)$ is the noise.

From (1) and (3) the received signal at UT b on sub-carrier l can be decomposed in,

$$y_{b,l} = \underbrace{\sqrt{p_{b,l}}\mathbf{h}_{b,b,l}^H \mathbf{w}_{b,l}s_{b,l}}_{Desired\ Signal} + \underbrace{\sum_{\substack{j=1\\ j\neq b}}^{B} \sqrt{p_{j,l}}\mathbf{h}_{j,b,l}^H \mathbf{w}_{j,l}s_{j,l}}_{Multiuser\ Multicell\ Interference} + \underbrace{n_{b,l}}_{Noise} \tag{4}$$

and from (4) the instantaneous SINR of user b on sub-carrier l can be written as,

$$\mathrm{SINR}_{b,l} = \frac{\left|\sqrt{p_{b,l}}\mathbf{h}_{b,b,l}^H \mathbf{w}_{b,l}^{(type)}\right|^2}{\sum_{\substack{j=1\\ j\neq b}}^{B}\left|\sqrt{p_{j,l}}\mathbf{h}_{j,b,l}^H \mathbf{w}_{j,l}^{(type)}\right|^2 + \sigma^2} \tag{5}$$

where type = {DZF,DVSINR}. Assuming M-ary QAM constellations, the instantaneous probability of error for user b and data symbol transmitted on subcarrier l is given by [10],

$$P_{e,b,l} = \psi Q\left(\sqrt{\beta SINR_{b,l}}\right) \tag{6}$$

where $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^{\infty} e^{-\left(t^2/2\right)}dt$, $\beta = 3/\left(M-1\right)$ and $\psi = \left(4/\log_2 M\right)\left(1-1/\sqrt{M}\right)$.

III. Distributed Precoder Vectors

In this section we describe the distributed precoding vectors, namely DZF and DVSINR. To design the distributed precoder vector we assume that the BSs have only knowledge of local CSI and its own data symbols, i.e., BS b knows the instantaneous channel vectors $\mathbf{h}_{b,j,l}, \forall j, l$, and only the data symbols $s_{b,l}, l = 1, \ldots, N_c$ reducing the feedback load over the backhaul network as compared with the data and/or CSI sharing beamforming approaches.

A. Distributed Zero Forcing (DZF)

Zero forcing is considered a classic beamforming strategy which removes the co-terminal interference. We derive a distributed ZF transmission scheme with the phase of the received signal at each UT aligned. In this case, $\mathbf{w}_{b,l}^{(DZF)}$ in (5) is a unit-norm zero forcing vector orthogonal to $B-1$ channel vectors $\left\{\mathbf{h}_{b,j,l}^{H}\right\}_{j \neq b}$. By using such precoding vectors, the multicell interference is canceled and the data symbol at each BS on each subcarrier is only transmitted to its intended UT. The SVD of $\left\{\mathbf{h}_{b,j,l}^{H}\right\}_{j \neq b}$ can be portioned as follows,

$$\left\{\mathbf{h}_{b,j,l}^{H}\right\}_{j \neq b} = \mathbf{U}_{b,l} \Omega_{b,l} \left[\ddot{\mathbf{W}}_{b,l} \quad \bar{\mathbf{W}}_{b,l}\right] \tag{7}$$

where $\bar{\mathbf{W}}_{b,l} \in \mathrm{C}^{N_{t_b} \times \left(N_{t_b} - B + 1\right)}$ holds the $\left(N_{t_b} - B + 1\right)$ singular vectors in the null space of $\left\{\mathbf{h}_{b,j,l}^{H}\right\}_{j \neq b}$.The columns of $\bar{\mathbf{W}}_{b,l}$ are candidates for b's precoding vector since they will produce zero interference at the other UTs. It can be shown that an optimal linear combination of these vectors can be given by [5],

$$\mathbf{w}_{b,l}^{(DZF)} = \bar{\mathbf{W}}_{b,l} \frac{\left(\mathbf{h}_{b,b,l}^{H} \bar{\mathbf{W}}_{b,l}\right)^{H}}{\left\|\mathbf{h}_{b,b,l}^{H} \bar{\mathbf{W}}_{b,l}\right\|} \tag{8}$$

Also, it can be shown that $\mathbf{h}_{b,b,l}^{H} \mathbf{w}_{b,l}^{(DZF)} \sim \chi^2_{2\left(N_{t_b} - K + 1\right)}$.

B. Distributed Virtual SINR (DVSINR)

Intuitively, the maximal ratio combining (MRT) is the asymptotically optimal strategy at low SNR, while ZF has good performance at high SNR or as the number of antennas increase. As discussed in [4][9], the optimal strategy lies in between these two precoders and cannot be determined without global CSI. However, inspired by the uplink-downlink duality for broadcast channels, the authors of [4] have derived a novel distributed virtual SINR precoder. The precoder vectors are achieved by maximizing the SINR-like expression in (9) where the signal power that BS b generates at UT b is balanced against the noise and interference power generated at all other UTs. It was named DVSINR as it originates from the dual virtual uplink and does not directly represent the SINR of any of the links in the downlink.

$$\mathbf{w}_{b,l}^{(DVSINR)} = \underset{\|\mathbf{w}\|^2 = 1}{\arg\max} \frac{\left|\mathbf{h}_{b,b,l}^{H} \mathbf{w}\right|^2}{\sum_{j \neq b} \left|\mathbf{h}_{b,j,l}^{H} \mathbf{w}\right| + \frac{\sigma^2}{P_{t_b}}} \tag{9}$$

where P_{t_b} is the per-BS power constraint. The solution to (9) is not unique, since the virtual SINR is unaffected by the phase shifts in $\mathbf{w}$. One possible solution can be written as [4],

$$\mathbf{w}_{b,l}^{(DVSINR)} = \frac{\mathbf{C}_{b,l}^{-1} \mathbf{h}_{b,b,l}}{\left\|\mathbf{C}_{b,l}^{-1} \mathbf{h}_{b,b,l}\right\|} \tag{10}$$

where

$$\mathbf{C}_{b,l}^{-1} = \frac{\sigma^2}{P_{t_b}} \mathbf{I}_{N_{t_b}} + \sum_{j \neq b} \mathbf{h}_{b,j,l} \mathbf{h}_{b,j,l}^{H} \tag{11}$$

IV. Power Allocation Strategy

In this section we design a novel distributed power allocation algorithm, based on minimization of the average BER over the available subcarriers. The criteria used to design distributed power allocation essentially lead to a redistribution of powers among subcarriers. To derive the power allocation for both precoders, we assume that the interference is negligible at both low and high SNR, even for the VSINR precoder.

The above precoders were specifically designed to make the equivalent channels, given by $h_{b,b,l}^{eq} = \mathbf{h}_{b,b,l}^{H} \mathbf{w}_{b,l}^{(type)}$, positive and real valued. Under free interference assumption the SINR defined in (5) reduces to,

$$\mathrm{SNR}_{b,l} = \frac{\left|\sqrt{p_{b,l}}\, h_{b,b,l}^{eq}\right|^2}{\sigma^2} \tag{12}$$

The above expression can be used to derive distributed power allocation because it only contains the local channel gains at BS b. Based on (6) and (12) we define the average BER as,

$$P_{av,b} = \frac{\psi}{N_c} \sum_{l=1}^{N_c} Q\left(\sqrt{\beta \, \mathrm{SNR}_{b,l}}\right) \tag{13}$$

The power allocation problem at each BS b, with per-BS power constraint, can be formulated as,

$$\min_{\{p_{b,l}\geq 0\}}\left(\frac{\psi}{N_c}\sum_{l=1}^{N_c}Q\left(\sqrt{\beta\,\mathrm{SNR}_{b,l}}\right)\right)\ \text{s.t.}\ \left\{\sum_{l=1}^{N_c}p_{b,l}\leq P_{t_b},\forall b\right. \quad (14)$$

The Lagrangian associated with this problem is given by,

$$L\left(p_{b,l},\mu\right)=\frac{\psi}{N_c}\sum_{l=1}^{N_c}Q\left(\sqrt{\beta\mathrm{SNR}_{b,l}}\right)+\mu\left(\sum_{l=1}^{N_c}p_{b,l}-P_{t_b}\right) \quad (15)$$

where $\mu\geq 0$ is the Lagrange multiplier [11]. Since the objective function is convex in $p_{b,l}$, and the constraint functions are linear, this is a convex optimization problem. Thus, it is necessary and sufficient to solve the Karush–Kuhn–Tucker (KKT) conditions, given by,

$$\begin{cases}\dfrac{\partial L}{\partial p_{b,l}}=\dfrac{-1}{N_c}\dfrac{\psi\beta\dfrac{h_{b,b,l}^{eq}}{\sigma}e^{-\frac{1}{2}\left(\beta\frac{h_{b,b,l}^{eq}}{\sigma}\sqrt{p_{b,l}}\right)^2}}{2\sqrt{2\pi}\sqrt{p_{b,l}}}+\mu=0 \\ \dfrac{\partial L}{\partial \mu}=\displaystyle\sum_{l=1}^{N_c}p_{b,l}-P_{t_b}=0\end{cases} \quad (16)$$

It can be shown that the powers $p_{b,l}$ as function of the Lagrange multiplier μ are given by,

$$p_{b,l}=\frac{\sigma^2}{\beta\left(h_{b,b,l}^{eq}\right)^2}W_0\left(\frac{\psi^2\beta^2\left(h_{b,b,l}^{eq}\right)^4}{8\pi\sigma^4N_c^2\mu^2}\right) \quad (17)$$

where W_0 stands for Lambert's W function of index 0 [12]. This function $W_0(x)$ is an increasing function with $W_0(x)=0,\ x=0$ and $W_0(x)>0,\ x>0$. Therefore, μ^2 can be easily determined iteratively to satisfy $\sum_{l=1}^{N_c}p_{b,l}=P_{t_b}$, by using the bisection method. This scheme is referred as DZF *virtual* minimum BER power allocation (DZF MBER PA) or VSINR minimum BER power allocation (VSINR MBER PA) when DZF or VSINR precoders are considered, respectively.

V. NUMERICAL RESULTS

In this section, the performance of the coordinated beamforming approaches with the proposed distributed power allocation scheme will be illustrated numerically. The scenario consists of 4 uniformly distributed single antenna UTs in a square with BSs in each of the corners. The power decay is proportional to $1/r^4$, where r is the distance from a transmitter. We define the SNR at the cell edge as $\mathrm{SNR}=\mathrm{P}_{t_b}\rho_c/N_c\sigma^2$, where the ρ_c represents the long term channel power in the center of the square. This represents a scenario where terminals are moving around in the area covered by 4 base stations.

The main parameters used in the simulations are based on LTE standard [14]: FFT size of 1024; number of available subcarriers set to 128; sampling frequency set to 15.36 MHz; useful symbol duration is 66.6 μs, cyclic prefix duration is 5.21 μs; overall OFDM symbol duration is 71.86 μs; sub-carrier separation is 15 kHz, and modulation is QPSK. We used the ITU pedestrian channel model B, with the modified taps delays according to the sampling frequency defined by LTE standard.

We compare the performance results of the proposed distributed power allocation schemes, DZF MBER PA and DVSINR MBER PA. Also, these schemes are compared with equal power allocation approach, i.e., the power available at each BS is equally divided by the subcarriers, $p_{b,l}=P_{t_b}/N_c\ ,\forall(b,l)$, referred as DZF EPA and DVSINR EPA for DZF and VSINR, respectively. The results are presented in terms of the average BER as a function of Cell-edge SNR defined above.

From Fig. 2, we can see that the performance of the proposed distributed power allocation scheme, for the two precoders, outperforms their equal power i.e. the DZF EPA and DVSINR EPA approaches. This is because they redistribute the powers across the different subchannels more efficiently. As can be seen in this figure, the gains of the proposed power allocation schemes, DZF MBER PA and DVSINR MBER PA) against the equal power approaches are approximately, 8 and 6 dB (at target BER of 10^{-3}), respectively. Also, we can observe that the performance of the DZF MBER PA tends to the DVSINR MBER one as the SNR increases.

Fig. 3 shows the performance results when one more antenna is added to each BS. In this scenario the DoF of the equivalent channels variables, given by $2\left(N_{t_b}-K+1\right)$, increases from 2 (scenario one) to 4. It can be observed that

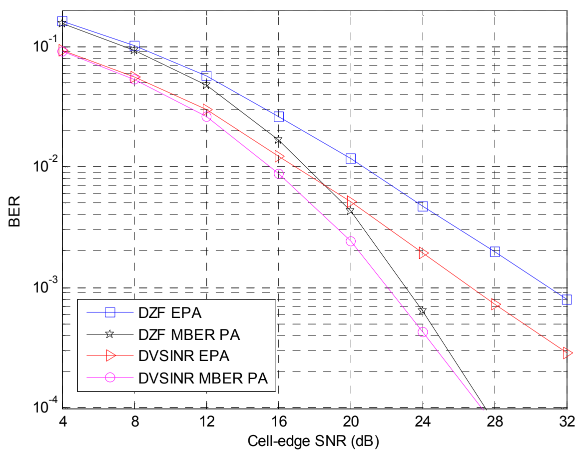

Fig. 2: Performance evaluation of the distributed precoding schemes for $N_{t_b}=4$.

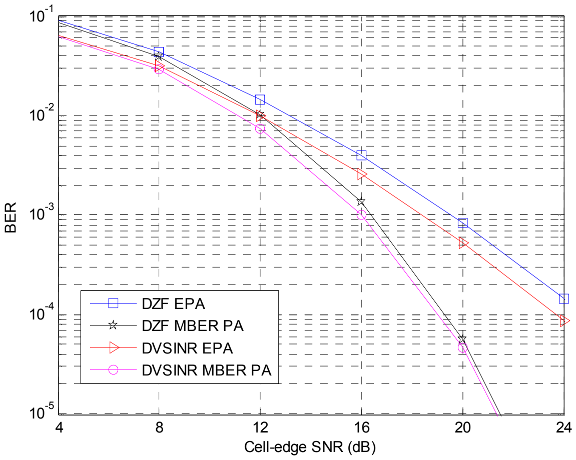

Fig. 3: Performance evaluation of the distributed precoding schemes for $N_{t_b} = 5$.

increasing the DoF, the DZF tends to the DVSINR. This behaviour is similar to the single cell systems where the precoders based on ZF criterion tends to the ones based on MMSE as the number of transmit antennas (or DoF) increases or at high SNR. From the results we can see that the gains obtained with power allocation schemes are lower, as compared with equal power approaches, than in the previous scenario.

VI. CONCLUSIONS

We proposed a novel distributed power allocation scheme for distributed precoding schemes, namely DZF and DVSINR, and for the downlink MISO-OFDM based systems. Both the precoders and power allocation were computed at each base station just by assuming the knowledge of local CSI without data sharing.

The results have shown that the proposed distributed power allocation schemes outperform the equal power ones. Also, the performance of the DZF based approaches tend to the DVSINR ones when the number of DoF increases or at high SNR.

It is clear from the presented results that the proposed distributed precoding schemes present significant interest for next generation wireless networks for which cooperation between BSs is anticipated.

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