

# Tuned liquid dampers simulation for earthquake response control of buildings

**T. Novo, H. Varum, F. Teixeira-Dias & H. Rodrigues**

*Universidade de Aveiro, Aveiro*

**M.J. Falcão Silva & A. Campos Costa**

*Laboratório Nacional de Engenharia Civil, Lisboa*

**L. Guerreiro**

*Instituto Superior Técnico, Lisboa*



## SUMMARY:

This paper is focused on the study of an earthquake protection system, the Tuned Liquid Damper (TLD), which can, if adequately designed, reduce earthquake demands on buildings. This positive effect is accomplished taking into account the oscillation of the free surface of a fluid inside a tank (sloshing). The behaviour of an isolated Tuned Liquid Damper, subjected to a sinusoidal excitation at its base, with different displacement amplitudes, was studied by finite element analysis. The efficiency of the TLD in improving the seismic response of an existing building, representative of modern architecture buildings in southern European countries was also evaluated based on linear dynamic analyses.

*Keywords: Tuned Liquid Damper, TLD, earthquake protection systems, energy dissipation, numerical simulation, sloshing.*

## 1. INTRODUCTION

Recent earthquakes have dramatically revealed that research in earthquake engineering should be directed towards the evaluation of the vulnerability of existing buildings, generally devoid of adequate structural characteristics. Its strengthening should be made aiming at the reduction of its vulnerabilities, consequently reducing the associated risk to acceptable levels. The study and development of new strengthening techniques and/or the improvement of seismic performance is fundamental so as to avoid significant economic losses, as well as human lives, in future events.

Tuned Liquid Dampers (TLDs) have been used as passive or semi-active control devices in a wide range of applications, such as tall buildings and high-rise structures. TLDs are passive energy absorbing devices that have been suggested for controlling vibrations of structures under different dynamic loading conditions. A TLD consist on a rigid tank with a shallow fluid. This fluid, that can be water or another fluid, is inside the tank that is rigidly connected to the structure. Tuning the fundamental sloshing frequency of the TLD to the structure's natural frequency causes sloshing and breaking waves at the resonant frequencies of the combined TLD-structure system which allows the dissipation of significant amounts of energy.

TLDs have already been studied by other research groups for application as earthquake response controlling device in buildings. In fact, a rectangular TLD can allow sloshing to occur in any direction and, when properly designed, its inherent behaviour properties under dynamic loading can make it a good solution in the reduction of bi-directional earthquake demands in buildings.

The global mechanism of controlling vibrations in structures with TLDs is based on the phenomena of fluid sloshing and wave breaking. These phenomena dissipate part of the energy induced by earthquakes. The fundamental sloshing frequency of the fluid in the TLD should be close to the natural frequency of the structure if the TLD is to dissipate energy efficiently.

The recent growing interest in liquid dampers for application in large structures can be associated to their potential advantages, such as: low cost when compared to other solutions, easy installation in existing building structures, minimal maintenance costs, may be employed for temporary use, benefit

dynamic effect in uni- or bi-directional earthquake excitations, and effectiveness even for small amplitude vibrations.

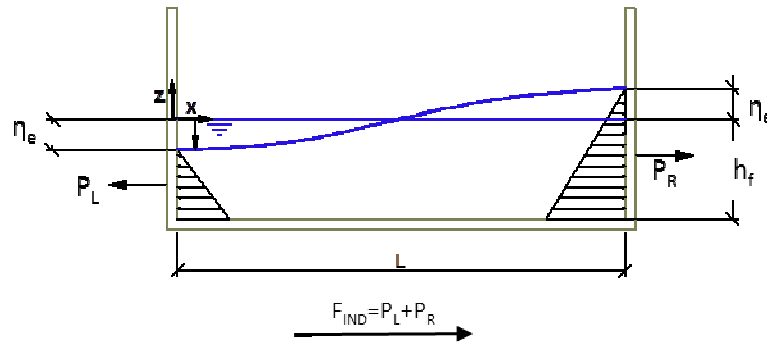
## 2. PROBLEM FORMULATION

The inclusion of TLDs in structures can improve its behaviour, especially when subjected to dynamic actions, such as wind and earthquakes. When TLDs are rigidly linked to a building structure, which is subjected to a dynamic action, fluid sloshing originates pressures that change the dynamic characteristics of the structure and its response to dynamic loads. The hydrodynamic horizontal impulses applied by the steady state fluid on the tank side walls act as an auto-equilibrated system, neglecting higher-order terms and shear forces on the structure. When the structure is subjected to dynamic actions, fluid sloshing induces non-equilibrated impulsive loads in both tank side walls corresponding to the development of lateral shear forces in the structure of the building,  $F_{ind}$ . These shear forces can be calculated, at any time instant  $t$ , as a function of the hydrostatic pressure on the tank side walls perpendicular to the direction of ground motion (equivalent to hydrostatic pressure at the same instant). Therefore, these forces depend on the height of fluid close to the tank side walls [Sun, 1991].

The building structure's shear force induced by fluid sloshing can be expressed as:

$$F_{ind} = \frac{\rho \times g \times b}{2} (h_r^2 - h_l^2) = \frac{\rho \times g \times b}{2} ((h_f + \eta_e)^2 - (h_f - \eta_e)^2) \quad (2.1)$$

where  $\rho$  is the density of the fluid,  $b$  is the tank width and,  $h_r$  and  $h_l$  are the surface elevations at the right and left side walls of the tank, respectively.



**Figure. 2.1.** Schematic representation of the forces induced to the structure due to fluid sloshing in a TLD [Novo, 2008].

The lateral shear forces induced to the structure can be considered to be dependent only on the hydrostatic pressure, because this is significantly higher to the inertial loads originated by the horizontal acceleration of fluid sloshing in the tank and by friction forces acting on the tank side walls and bottom. Therefore, in this study, these two loads were neglected. It is then considered that liquid-induced horizontal loads can be calculated from the sum of total pressures acting on the right and left side walls of the tank. According to Fig. 2.1 the hydrostatic pressures are dependent on the free surface height of the fluid at each side wall of the TLD [Sun, 1991].

## 3. FLUID MOVEMENT MATHEMATICAL MODEL

Liquid sloshing movement into a horizontal and low depth surface involves non-linearities. Non-linear models, supported by the shallow water wave theory and based on partial differential equations under the specified initial and boundary conditions, can be solved using numerical methods. However, these

approaches frequently lead to high numerical instabilities. These equations are the result of both continuity relations and two-dimensional Navier-Stokes equations [Sun et. al, 1992].

A linear analytical model is applied in this work, where the linear solution for liquid sloshing in the tank is simulated applying the long wave theory, based on dissipative and dispersive non-linear study-cases. This theory is used associated with integration techniques, in order to obtain the transference's function expression in a simple analytical form.

As previously presented Fig. 2.1. shows the liquid sloshing in a tank, which is subjected to a horizontal motion  $x$ , in  $O_x$  direction. The local Cartesian coordinate system ( $O_{xz}$ ) has to be considered above the steady free fluid's surface and its origin is placed at the left side wall of the tank. The length of the tank is designated by  $L$  and the average liquid depth is  $h_f$ . In an one-dimensional approach, liquid sloshing in the tank can be expressed by two non-dimensional partial differential equations (PDE), where a straight line is applied and the  $u(x,t)$  variable is eliminated, and so the expression for the elevation of the free surface becomes [Lepelletier e Reichelen, 1988]:

$$\eta(x,t) = -\text{Re} \left\{ \frac{1}{k} \cdot \frac{\sin \left[ \kappa \cdot \left( x - \frac{1}{2} \right) \right]}{\cos \left( \frac{\kappa}{2} \right)} \cdot e^{i \cdot \sigma \cdot t} \right\} - 2 \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \sin \left[ a_n \cdot \left( x - \frac{1}{2} \right) \right] \cdot \text{Re} \left[ f_n \cdot e^{S_n \cdot t} \right] \quad (3.1)$$

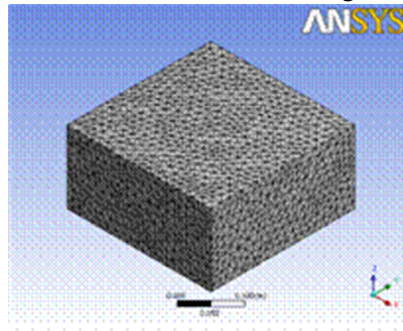
The fundamental linear sloshing frequency of the TLD ( $\omega_f$ ) can be given by [Fujino et. al., 1992]:

$$\omega_f = \sqrt{\frac{\pi \cdot g}{L} \cdot \tanh \left( \frac{\pi \cdot h_f}{L} \right)} \quad (3.2)$$

where  $L$  is the length of the tank,  $h_f$  is the liquid height and  $g$  is the acceleration of gravity.

#### 4. NUMERICAL SIMULATION OF A TLD

The finite element method (FEM) was used to evaluate the efficiency of the proposed tuned liquid damper model and to study the characteristics of the liquid sloshing mechanism in a TLD when subjected to a sinusoidal dynamic excitation at its base for different displacement amplitudes. ANSYS CFX™ was the finite element analysis tool used to test the efficiency of the TLD. The finite element mesh of the tank was generated and the problem boundary conditions and the particular properties of the fluid were defined [Novo, 2008]. The optimized finite element mesh (Fig. 4.1) adopted in these analyses has 69131 tetrahedral elements with an element average dimension of 18 mm.



**Figure 4.1.** Finite element mesh of the TLD (with 69131 tetrahedral elements) [Novo, 2008].

The same tank dimensions and TLD parameters were used for all the analyses. The dynamic load applied to the tank was imposed by a sinusoidal motion law of the accelerations applied directly on the tank base. The sinusoidal acceleration imposed along the  $O_x$  direction, which simulates the direction of ground motion, is given by:

$$a_y = K \times g \times \sin(2 \times \pi \times f \times t) \quad (4.1)$$

where  $K$  is the constant acceleration amplitude and  $f$  is the natural frequency of the building. Gravity acceleration is considered along the  $O_Z$  direction ( $a_z=9.81 \text{ m/s}^2$ ) and no acceleration is considered along the  $O_Y$  direction ( $a_x=0 \text{ m/s}^2$ ). Considering the acceleration law:

$$\ddot{x}(t) = A_c \cdot \sin(2 \cdot \pi \cdot f \cdot t) \quad (4.2)$$

and integrating twice, leads to:

$$x(t) = \int \dot{x}(t) = -\frac{1}{4 \cdot \pi^2 \cdot f^2} \cdot A_c \cdot \sin(2 \cdot \pi \cdot f \cdot t) \quad (4.3)$$

Thus, the displacement amplitude can be calculated with the expression:

$$D = \frac{A_c}{4 \cdot \pi^2 \cdot f^2} \Leftrightarrow A_c = D \cdot (4 \cdot \pi^2 \cdot f^2) \quad (4.4)$$

The values of  $A_c$  and  $K$  are listed in Table 4.1. According to equations (4.1) and (4.2):

$$K \cdot g = A_c \quad (4.5)$$

**Table 4.1.** Amplitude values for  $A_c$  and  $K$

$D$ [m]	$A_c$ [m/s <sup>2</sup> ]	$K$
0.002	0.092	0.009
0.005	0.230	0.023
0.010	0.460	0.046
0.015	0.690	0.070
0.030	1.355	0.138
0.040	1.806	0.184
0.100	4.517	0.460

The dissipated energy and corresponding equivalent damping were calculated to evaluate the efficiency of the tuned liquid damper. The dissipated energy for all simulations was calculated and compared to the input energy of the system. For one specific load cycle, the energy introduced in system is calculated with the following expression [Uang e Bertero, 1998]:

$$E_i = -\int m \ddot{x}(t) dt \quad (4.6)$$

The energy dissipated per cycle can be evaluated determining the interior area of a cyclic response of the TLD in terms of lateral force versus horizontal displacement. For one specific cycle of the TLD's response, the energy dissipation is determined by the following expression [Uang e Bertero, 1998]:

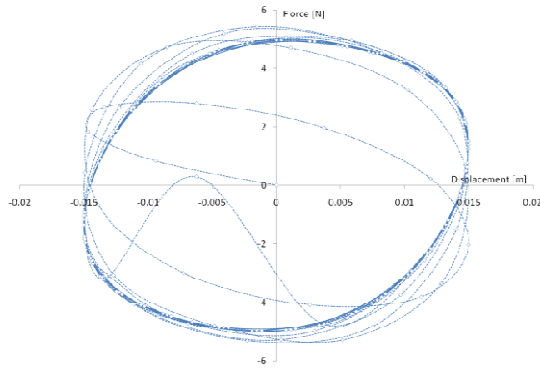
$$E_d = \int F x(t) dt \quad (4.7)$$

For a chosen cycle, it is possible to estimate the equivalent damping coefficient through the following expression:

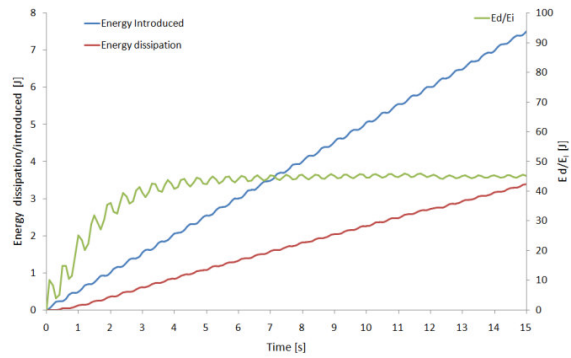
$$\zeta_{eq} = \frac{\text{Area of the cycle}}{2 \cdot \pi \cdot F_{max} \cdot D_{max}} \quad (4.8)$$

where  $F_{max}$  is the maximum force (due to hydrostatic pressure) and  $D_{max}$  is the maximum horizontal displacement of the tank.

In what concerns the obtained results, Fig. 4.2 shows the force-displacement response of the TLD for cyclic loading amplitude of 15 mm. Fig. 4.3 shows the evolution of the input and dissipated energy components for a cyclic loading corresponding to an imposed displacement amplitude of 15 mm.

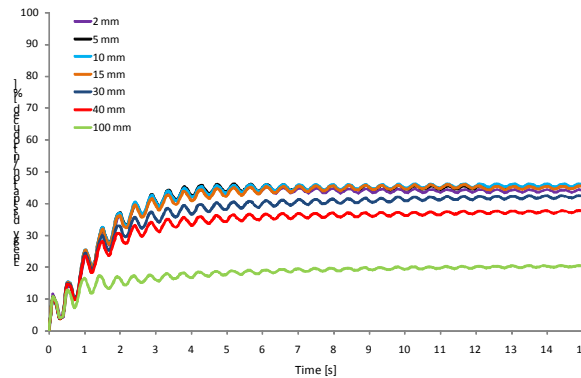


**Figure 4.2.** Force-displacement behaviour for a cyclic loading amplitude of 15 mm.

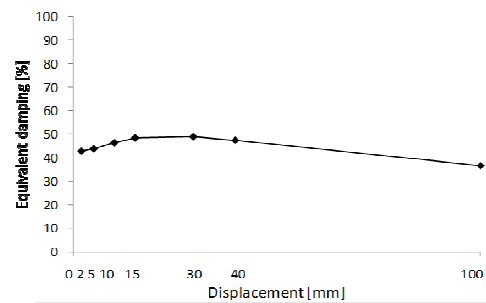
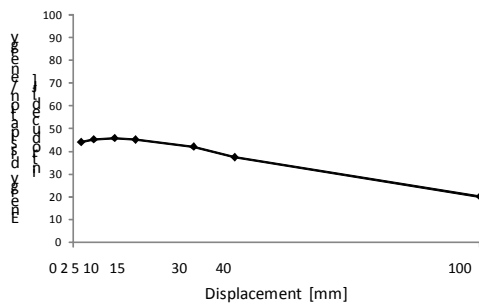


**Figure 4.3.** Input and dissipated energy evolutions for a cyclic loading corresponding to an imposed displacement amplitude of 15 mm.

For all the analyses performed, corresponding to displacements imposed in the 2-100 mm amplitude range, the evolution of the ratio between the input energy and the dissipated energy is shown in Fig. 4.4. In Fig. 4.5. are plotted, for each displacement amplitude the maximum ratio between the dissipated energy and the input energy (Fig. 4.5a), as well as the estimated equivalent damping, which gives an indication of the TLD efficiency.



**Figure 4.4.** Ratio between the input and dissipated energy.



**Figure 4.5.** Efficiency of the TLD as a function of the cyclic loading amplitude, in terms of: (a) maximum ratio between the dissipated energy and the input energy and (b) the estimated equivalent damping.

## 5. DESIGN OF A TLD FOR AN EXISTING BUILDING

For the applications carried out in the scope of this paper, and having in mind a specific building, the ground motion considered was defined only along one horizontal direction. The building chosen for this study is representative of the modern Portuguese architecture, namely the buildings designed and built in the 1950s, when earthquake design was still not considered in the national standards. This representative building has nine-storeys. The option for this case-study is justified by the moderate to high local seismic hazard of the Lisbon region and by the significant number of buildings with this typology designed and built in Southern European cities in that period. The procedure adopted for the design of the tuned liquid dampers for the studied building was as follows:

- i) The building first natural frequency in the longitudinal direction measured in-situ [Miranda et. al, 2005] was  $f=1.08$  Hz. The sloshing frequency of the fluid is equalled to the first natural frequency of the building:

$$f = \frac{1}{T} = \frac{\omega}{2 \cdot \pi} \quad (5.1)$$

where  $T$  is the natural period of the structure and  $\omega$  is the corresponding angular frequency.

- ii) In terms of linear frequency (in Hz) the equation of sloshing frequency of the fluid (equation 3.3), can be rewritten as follows:

$$f_f = \frac{1}{2} \cdot \sqrt{\frac{g}{\pi \cdot L} \cdot \tanh\left(\frac{\pi \cdot h_f}{L}\right)} \quad (5.2)$$

Imposing a ratio between the height of the fluid,  $h_f$ , and the length of the tank,  $L$ , equal to 0.15, [Fujino et al., 1992], and substituting the values for the structure under analysis in equation (5), the following non-linear system of equations can be obtained as a function of  $h_f$  and  $L$ :

$$1.08 = \frac{1}{2} \cdot \sqrt{\frac{9.81}{\pi \cdot L} \cdot \tanh\left(\frac{\pi \cdot h_f}{L}\right)} \quad \text{and} \quad \frac{h_f}{L} = 0.15 \quad (5.3)$$

In the scope of this work, it was assumed that the width  $b$  of the TLD is equal to its length  $L$ , leading to a quadrangular geometry in plan. This geometry guarantees that the TLD will behave equally on both horizontal directions, as opposed to a rectangular tank. From the literature review, no rule or proposal was found for the height limitation of the TLDs. Therefore, on the present case-study it was considered,  $L=0.3\text{m}$  and  $h_f=0.0441$  m.

## 6. CASE STUDY

The efficiency of the TLD in the improvement of the seismic response of an existing building was evaluated and the main results are presented in the next. The studied building was presented in Section 5. The linear analyses were performed with a finite element analysis program (SAP 2000<sup>TM</sup>).

According to the building dimensions, the structure properties and the TLD dimensions and its characteristics, the number of TLDs needed to guarantee the required performance was estimated, in terms of energy dissipation of the structure-TLD system. The mass of the fluid in each tank is  $m_f = \rho \cdot h_f \cdot b \cdot L$ , where  $\rho$  is the mass density of the fluid and  $h_f$ ,  $b$  and  $L$  are the tank dimensions. The number of tanks required,  $N$ , was determined with the following relation [Banerji et. al, 2000]:

$$N = \frac{\mu \cdot m_e}{m_f} \quad (6.1)$$

where  $\mu$  is the mass ratio between the liquid and the structure (typically varying between 1 and 5%),  $m_s$  is the mass of the structure and  $m_f$  is the mass of the fluid in each tank.

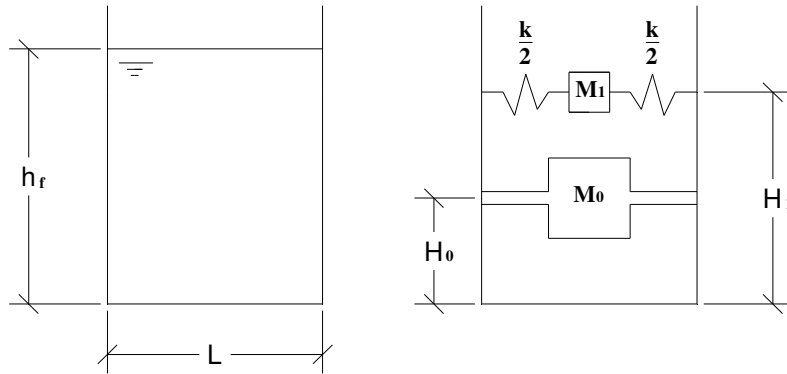
The fluid chosen for this analysis is water. The total mass of the building is 3863 ton. Consequently, according to equation (6.1), the number of tanks needed for each mass ratio considered in this study is listed in Table 6.1.

**Table 6.1.** Amplitude values for  $A_c$  and  $K$

$\mu$ (%)	Number of tanks
1.0	9600
2.5	24000
5.0	48000

## 6.1. Lumped mass method

Simplified methods can be found in the literature to simulate the behaviour of tuned liquid dampers. The lumped mass method and the linear wave theory are such methods. With the lumped mass method [Housner and Brady, 1963] method, the wall of the TLD is assumed to be rigid. Hydrodynamic pressure caused by liquid sloshing in the tank due to the dynamic loading is considered separately as impulsive pressure and sloshing pressure. The impulsive pressure is proportional to the tank acceleration, but with opposite direction. The sloshing pressure is related to the height of the wave and to the sloshing frequency of the liquid. Therefore, both hydraulic pressures can be simulated by two equivalent masses linked to the tank. Fig. 6.1. shows a schematic representation of the lumped mass model, in which  $M_0$  is the mass rigidly connected to the tank at an elevation  $H_0$  above its base and  $M_1$  represents the impulsive mass attached to springs with stiffness  $k$  at elevation  $H_1$ .



**Figure 6.1.** Lumped mass model for a rectangular TLD.

For rectangular tanks, these parameters can be estimated by [Newmark and Rosenblueth, 1971]:

$$M_0 = \frac{\tanh(\sqrt{3} \cdot (L/2)/h_f)}{\sqrt{3} \cdot (L/2)/h_f} \cdot m_f \quad (6.2)$$

$$M_1 = \frac{0.83 \cdot \tanh(1.6 \cdot h_f/(L/2))}{1.6 \cdot h_f/(L/2)} \cdot M \quad (6.3)$$

$$H_0 = 0.38 \cdot h_f \cdot \left[ 1 + \alpha \left( \frac{m_f}{M_0} - 1 \right) \right] \quad (6.4)$$

$$H_1 = h_f \cdot \left[ 1 - 0.33 \cdot \frac{m_f}{M_1} \cdot \left( \frac{(L/2)}{H_f} \right)^2 + 0.63 \cdot \beta \cdot \frac{(L/2)}{h_f} \cdot \sqrt{0.28 \cdot \left( \frac{(m_f \cdot (L/2))^2}{M_1 \cdot h_f} \right) - 1} \right] \quad (6.5)$$

$$k = \frac{3 \cdot g \cdot M_1^2 \cdot h_f}{m_f \cdot L^2} \quad (6.6)$$

where  $M$  is the total mass of the contained fluid,  $\beta=2.0$  and  $\alpha=1.33$  are parameters due to the hydrodynamic moment on the tank base and  $h_f$  is the liquid height.

## 6.2. Damping of surface wave in a rectangular tank

The damping of surface waves in a rectangular tank was studied by Miles [Miles, 1967], who suggested that the dissipation coefficient can be corrected by a factor  $1+2h/b+S$ , where  $b$  is the tank width. With this correction factor, the energy dissipation due to the fluid friction on the lateral side walls and the liquid surface contamination is taken into account. Friction due to the side wall boundary layer is assumed to be the same as that of the bottom boundary layer.  $S$  is a surface contamination factor that can vary between 0 and 2.  $S=1$  will be used in this study, which corresponds to a fully contaminated surface [Sun, 1991]

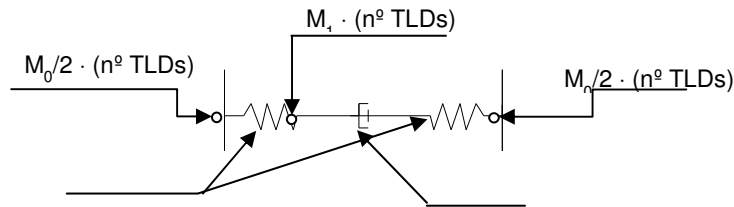
$$\zeta_f = \sqrt{\frac{v_f \cdot \omega_f}{2}} \cdot \left[ 1 + \left( \frac{2 \cdot h_f}{b} \right) + S \right] \cdot \frac{L}{h_f \cdot \sqrt{g \cdot h_f}} \quad (6.7)$$

## 6.3. Proposed mechanical model

The steps adopted for the simplified simulation of the passive energy dissipation system based on TLDs in the Structural Analysis Program [Novo, 2008] are briefly described in the next paragraphs. Each TLD group is simulated by a macro-model, as follows:

- i) A damper type link was defined in order to simulate the equivalent damping ( $\zeta$ ) properties of the TLD;
- ii) Each link is connected to the structure with bar elements with stiffness  $k$  (according to expression 6.6); and
- iii) Concentrated masses were introduced at the ends of the macro-model to simulate the static mass and at the centre to simulate the dynamic mass.

To simplify the analysis, and due to the large number of TLDs needed for the passive energy dissipation system designed in this study, each macro-model corresponds to a set of TLDs. The schematic representation of the macro-model developed in SAP to simulate a group of TLDs is shown in Fig.6.2.



**Figure 6.2.** Schematic representation of the macro-model used to represent a set of TLDs with the proposed model [Novo, 2008].

## 6.4. Ground motion

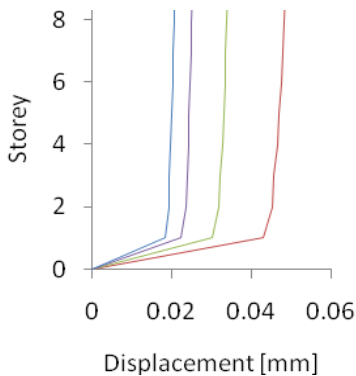
This study was performed for a set of synthetic ground motions artificially generated, for a moderate



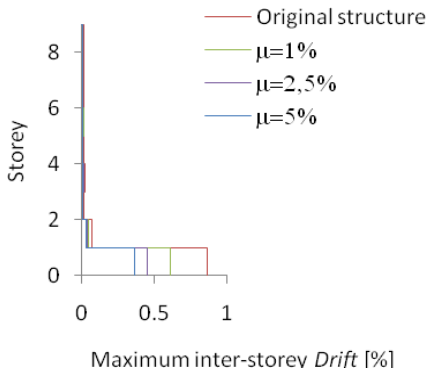
risk scenario, according to a non-stationary stochastic finite fault seismological simulation model based on random vibration [Rodrigues et. al, 2007]. The considered ground motions reflect both the close and distant earthquake scenarios and were defined for several return periods. Distant earthquake scenarios were considered for this study and three artificial earthquakes were generated for each return period [Carvalho et. al, 2008].

**6.5. Numerical results**

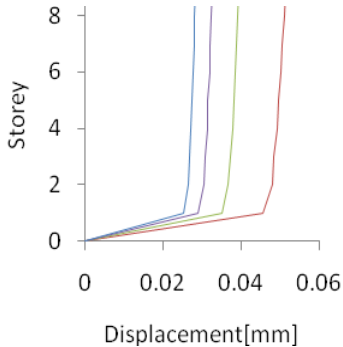
The envelope lateral displacement profiles and the maximum inter-storey drift profiles for 475 and 975 years return period are shown in Fig. 6.3 to Fig. 6.6. Each profile represents the average, for the different earthquakes, of the results for each return period considered.



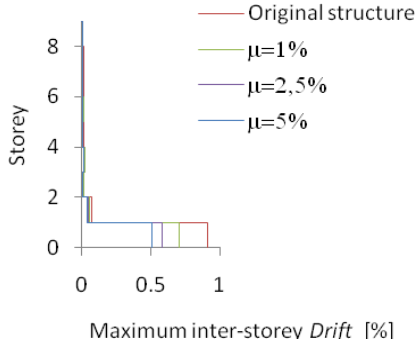
**Figure 6.3.** Envelope lateral displacement (475yrp).



**Figure 6.4.** Maximum inter-storey drift profiles (475yrp).



**Figure 6.5.** Envelope lateral displacement (975yrp).



**Figure 6.6.** Maximum inter-storey drift profiles (975yrp).

**7. CONCLUDING REMARKS**

The main objective of this paper was to analyse the efficiency of quadrangular tuned liquid dampers in controlling the response of building structures when subjected to earthquake ground motions. To characterize the TLDs, as well as the liquid sloshing mechanism, a TLD was studied using a refined finite element model. The TLD studied was designed for a frequency of 1.08 Hz and subjected to sinusoidal excitations at its base with displacement amplitudes between 2 and 100 mm. From the analysis of the obtained results it can be concluded that, for a sinusoidal cyclic loading, the behaviour of the studied TLD is globally efficient, significantly increasing the energy dissipation and the corresponding equivalent damping of the global system. The TLD is more effective in reducing the

structural demands for structures with lower natural periods, as the building structure here studied with a natural period close to 1s, representative of a large number of existing buildings in many countries around the world.

Finally, it can be concluded that TLDs can be adopted as effective measures for reducing building structural demands to earthquake input motions. However, these innovative structural protection systems have to be further studied by shaking table experimental tests and in-situ assessment of their performance on existing structures. Additionally, the architectonic implications associated to this type of rehabilitation measures should be considered in each specific case.

Concerning the assessment of the performance of TLDs by experimental shaking table testing, it was developed at LNEC facility a study that aimed to define TLDs linear and non-linear behavior when subjected to random vibrations of increasing amplitude [Falcão Silva, 2010]. Besides that, a considerable effort was also made in what concerns numerical simulations of the behaviour of TLDs when isolated or included in SDOF and MDOF structural systems..

## REFERENCES

- Banerji, P.; Murudi, M.; Popplewell, (2000) N., Tuned Liquid Dampers for controlling earthquake response of structures, *Earthquake Engineering and Structural Dynamics*; **v. 29**, pp. 507-602.
- Carvalho, A.; Zonno, G.; Franceschina, G.; Bilé Serra J.; Campos Costa, A., (2008) Earthquake shaking scenarios for the metropolitan area of Lisbon, *Soil Dynamics and Earthquake Engineering*, **v. 28, n. 5**, pp. 347-364.
- Falcão Silva, M.J. (2010). Sistemas de Protecção Sísmica: Uma abordagem baseada no desempenho de Amortecedores de Líquido Sintonizados, PhD Thesis, Instituto Superior Técnico, Universidade Técnica de Lisboa.
- Fujino, Y.; Sun, L.M.; Pacheco, B.M.; Chaiseri, P., (1992). Tuned Liquid Damper (TLD) for Suppressing Horizontal Motion of Structures, *Journal of Engineering Mechanics*, **v. 118**, n. 10, pp. 2017-2030.
- Housner, G.W.; Brady, A.G. (1963). Natural Periods of Vibrations of Buildings, *Proceedings of the American Society of Civil Engineering*, **89 (EM4)**, pp. 31-35.
- Lepelletier, T.G.; Raichlen, F., (1988). Nonlinear oscillations in Rectangular Tanks, *Journal of Engineering Mechanics*, **v. 114, n. 1**, pp. 1-23.
- Miles, J.W., (1967). Surface Wave Damping in Closed Basins, *Proceedings of the Royal Society of London*, **A 297**, pp. 459-475, 1967.
- Miranda, L.; Rodrigues, H.; Fonseca, J.; Costa, A. G. (2005). Relatório de inspeção, Faculdade de Engenharia da Universidade do Porto
- Newmark, M.N.; Rosenbluth, E., (1971). *Fundamentals of Earthquake*, Prentice-Hall, Inc.
- Novo, T., (2008). Melhoramento da resposta sísmica de edifícios com recurso a TLDs, MSc Thesis, University of Aveiro.
- Rodrigues, H.; Varum, H.; Costa, A., (2007). Avaliação da vulnerabilidade e reforço sísmico de um edifício representativo da arquitectura moderna em Portugal, *SÍSMICA 2007 - 7º Congresso de Sismologia e Engenharia Sísmica*.
- Sun, L.M. (1991). Semi-Analytical Modelling of Tuned Liquid Damper (TLD) with Emphasis on Damping of Liquid Sloshing, PhD, University of Tokyo.
- Sun, L.M.; Fujino, Y.; Pacheco, B.M., (1992). Modelling of Tuned Liquid Damper (TLD), *Journal of Wind Engineering and Industrial Aerodynamics*, **v. 41-44**, pp. 1883-1894, 1992.
- Uang, C.M.; Bertero, V.V., (1998). Use of energy as a design criterion in earthquake-resistant design, College of Engineering, University of California at Berkeley.