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Patterns in the city: a mathematics project

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One aspect of mathematics education, in this century, is the lack of scientific culture and curiosity of young people as well as the low mathematics knowledge of students. These students' low mathematics skills translate into lack of motivation for learning this subject, traditionally identified as a hard task. It is our belief that it is important to show the invisible face of mathematics around us to get students and teachers more motivated. In this presentation we introduce an overview of this project whose first goal is to promote the mathematics culture of elementary pre-service teachers and students through the observation and exploration of the urban environment while designing mathematics materials for elementary education. We will present the description of some of the tasks, in particular those connected with patterns.

Un aspect de l'enseignement des mathématiques, dans ce siècle, c'est le manque de culture scientifique et de curiosité des jeunes et leur faible connaissance des mathématiques. Les faibles compétences des élèves en mathématiques se traduisent par l'absence de motivation pour l'apprentissage de cette discipline, traditionnellement reconnue comme difficile. C'est notre forte conviction qu'il est important de montrer la face invisible des mathématiques autour de nous pour que plus d'étudiants et d'enseignants en soient motivés. Dans cette communication, nous présenterons brièvement ce projet dont le premier objectif c'est de devélopper la culture des mathématiques élémentaires chez les enseignants et les étudiants par le biais de l'observation et l'exploration de l'environnement urbain tant bien que la conception de matériaux pour l'enseignement des mathématiques. Nous présenterons aussi la description de certaines tâches, en particulier celles liées aux padrons.

Between 2006-2008 we developed a project[1] that was oriented by the following reasons: As teachers educators we have been noticing the lack of scientific culture and curiosity of young people that is one of the most important features in mathematics education in this century. This happens in short anywhere around the world, but in particular in Portugal, since the recent international studies (e.g. TIMSS, PISA) have shown that our students have weak performance in Mathematics tests. These students' low mathematics skills translate into a lack of motivation for learning this subject, traditionally identified as a hard task.

When students enroll in school they carry, with them, a negative attitude towards mathematics, subject about which they have already heard, most of the times in an inexact way, but about which they have a limited knowledge yet. This situation provokes in the teachers despondency, lack of motivation and of alternative strategies to break this vicious cycle in which mathematics teaching have been transformed. This happens not only at schools but it is a socially installed attitude towards mathematics.

It is our strong belief that mathematics is accessible to everybody and is present everywhere around us. Only actions that show these features of mathematics and conduct to a great aware-

[1] This project, *MatCid*, refª CV/33-2006, was supported by Ciência Viva Agency

ness of people in general and students in particular, will allow us to recover the delay and invert general unmotivation.

With this project, we intended to promote the contact with a contextualized mathematics, starting from the daily life features, walking through and analyzing the city where we live in, connecting some of its details with exploration and investigation tasks in school mathematics. Its aim is to promote the mathematics culture of elementary (pre-service and in-service) teachers and students through the observation and exploration of the urban environment, while designing mathematics curriculum for elementary education. At the same time they learn social and historical events related with what they saw, contributing for a more civic, historical and cultural knowledge of the place where they live.

Background

Teaching a contextualized, applied, experienced, visual, intuitive mathematics to all students, promoting peer and teacher/students interaction, using didactical materials, and communicating their own ideas can contribute to a more significant learning of mathematics. In this perspective, learning requires an active and reflexive student engagement in significant and diversified tasks. In addition, most of mathematics students' failures derive from the affective classroom environment, once it can seriously compromise their initial expectations and motivations. After all we must develop and stimulate creative thinking in the mathematics classroom.

Teachers have a determinant role in the teaching process, so according to that perspective, teacher education should promote a new vision about mathematics knowledge and its teaching. Teacher education must create opportunities either for pre-service or for in-service teachers to explore their world and discover that mathematics is everywhere, connecting mathematical ideas to real world interests, experiences, and empowerment. Teacher education must as well develop teachers' competences such us to be aware, critic and more confident in their mathematical abilities, but most of all teachers have to be alert to know and to discover the "invisible" mathematics that surrounds us.

Among the different mathematical tasks that we use in mathematics classes, those involving problem solving play an important role in the learners' lifes. To look for a pattern is a powerful problem solving strategy and mathematics may be defined as the *science of patterns* (e.g. Devlin, 1999; Orton, 1999). Pattern study has a growing importance in particular on algebra, since "algebraic thinking" has become a catch-all phrase for the mathematics teaching and learning that will prepare students with the critical thinking skills needed to fully participate in society and for successful experiences in algebra. Algebra as the study and generalization of patterns means that students should be able to observe a pattern, form a general algebraic rule and then be able to justify that rule (e.g. Lannin et al., 2006; Mason, 1996; Rivera & Becker, 2005). Moreover patterns are everywhere; we see them in nature, in architecture and in art. In this presentation we will give a special attention to different types of patterns: those related to numbers and algebra and those related to tessellations and friezes, where students can generalize in numerical or geometric contexts.

The project – methodology and procedures

The aim of this project is to awake elementary pre-service teachers, students and population in general to the beauty and utility of mathematics in daily life, discovering it, unveiling it, and exploring the multiples features of the city where we live. Our main goals are: to promote mathematics culture of population; to contribute to the architectural, natural and historic knowledge of the city; to contribute to a more positive view of mathematics; to promote the mathematics culture of elementary teachers and students through the observation and exploration of the urban environment while designing mathematics curriculum for elementary education; to promote mathematic and transversal competences of students; and to contribute to the professional development of teachers.

According to the aims of this project we adopted an exploratory methodology where the participants were pre-service elementary teachers of mathematics; in-service teachers; and elementary students of grades 1-6. The project had different phases, but it started in the classes of teacher education at a School of Education. During the classes of Didactics of Mathematics some themes were selected of different features of the city, such as: documents; traffic and signs; gardens; monuments; buildings; windows; forged iron; tiles; tessellations; and regional embroideries, clothes, *palmitos*, pottery, gold. Then those students, future teachers, had to walk around the city of Viana do Castelo looking for different elements, according to a previous theme and work it mathematically at an elementary level proposing adequate tasks for children (grades 1-6). These tasks were used, with some adaptations, in the posters presented in the exhibition.

The different data was collected and analyzed by the project team according to the previous objectives and the expected products, supported by literature on teaching and learning mathematics education for elementary levels and teacher training.

The products

The main products/tasks created within this project were:

(1) One booklet of tasks exploring mathematic details of particular itineraries of the city of Viana do Castelo. This booklet is for using by common citizens that enjoy mathematics while visiting our city, in teacher training (pre-service and in-service) and in elementary school mathematics;

(2) One CD-ROM/DVD and a Website with information, materials and selected resources for students and teachers of elementary education;

(3) An exhibition – ExpoMatCid. This exhibition was and can be presented in schools and places of general public access. It includes posters with mathematical tasks grounded on several aspects of the city of Viana do Castelo. These tasks are to be solved by people with elementary mathematics knowledge and also intend to motivate young students to mathematics;

and

(4) A mathematical walk through the city, where students of grades 1-6 solve mathematics tasks during a narrative path through some streets of Viana do Castelo and was integrated in the Summer Courses of our institution.

This task was replicated with in-service teachers and after with their own students.

Some conclusions

At the end of this project we hope to have contributed to: the promotion of science; a more positive attitude towards mathematics; and to broaden the vision of the possible connections that can be established between mathematics and the world around us. To design all the tasks in an adequate way to general population with basic mathematics knowledge wasn't always easy. A project that involved students till the secondary level would more and more provide to deepen mathematics subjects and would allow the exploration of different tasks.

Patterns are indeed a powerful resource to develop mathematics concepts and to establish several connections among different subjects and school levels. In general the intervenient in this project find the more obvious connections of patterns those related to geometry (geometric motions - flips, slides and turns – friezes, tessellations) because they are more visual and familiar. The new connection for these intervenient was that related with the development of the algebraic thinking through generalization of patterns. This approach it was less familiar because it wasn't in the Portuguese mathematics curriculum of school. To conclude we can say that the implementation of a project such as MatCid-*Mathematics in the City* has endless chances of exploration.

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