

8. Patterns in the first school years – main constraints

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Abstract

Authors suggest that several mathematical patterns must be addressed, in a systematic way, since an early age, given their key role in the development of algebraic thinking, amongst other aspects. Following this perspective, we developed several activities focusing on patterns of various kinds, involving children from 3 years of age. These activities revealed a set of tasks which are extremely complex and difficult for children. In this paper, by focusing on the pupils' own productions, we will discuss some of above-mentioned didactic proposals and the implication of the results.

Introduction

The starting point of the study we intend to present was a wider research project which gave way to a Master's dissertation (Ventura, 2008).

The problematic of such a research touches the confluence of three main vectors: i) children should live, since an early stage, mathematical experiences with patterns which are not worked much in Portuguese preschools; ii) information technology is ever more representative in society, even in children's daily lives, who, from an early age, familiarize themselves with these means. This is a process in which school hasn't had a significant role so far iii) multiculturalism is another reality which characterizes the Portuguese society in schools. Nevertheless, schools do not know how to handle this problem. This research admitted CD-ROM "*Images of Interculturalism in Preschool Education – Us and the Others The Others and Us*" ("*Imagens de Interculturalidade na Educação de Infância – Nós e os Outros*") (Cabrita & Moderno, 2003) as the object of study. The CD-ROM focuses on several aspects: a) A literacy logic with and for the media; b) characteristics from different social realities such as the Portuguese, the PALOP (African countries having Portuguese as official language), gypsies and east European countries; c) didactic suggestions, namely about patterns, which cross the diverse domains in the communication and expression area – mathematics included. The main objective of this Master dissertation was to evaluate the impact of exploring this CD-ROM in the construction and/or development of technological competence, of a more effective multicultural conscience and intercultural sensitiveness and, finally, along with development of a sense of pattern in a group of preschool children.

This article will focus on patterns, more specifically, situations of success and constraints faced by children in preschool when solving tasks which involve the identification, reproduction, creation, continuation or gap-filling of patterns.

Theoretical framework

By using mathematics to organize and systematize our ideas about pattern, we discover a great secret: nature patterns do not exist solely for the rules which govern natural processes. (Stewart, 1996: 11, our translation).

Working with patterns and finding the generalizing law(s) allows us to recognize order and to know and organize the world around us (NCTM, 2007). In a dynamic process, it also allows us to better and more effectively understand a series of other mathematical phenomena.

On the other hand, such activities demand a formal method of thinking, which they also help to develop. The principles of mathematical thinking emerge even before the child goes to school (Crucio & Schwartz, 1997), but it is unquestionable that there is a rapid development of mathematical reasoning when children go into a preschool classroom which provides them with adequate learning experiences. The discovery of regularities, replication, continuation, gap-filling of listening, visual, motor patterns, ..., activities which children should verbalize, represent and discuss, are part of this group (Papic & Mulligan, 2005; Waters, 2005).

The term “pattern” is used when we refer to a *disposition* or *arrangement* of numbers, forms, colours or sounds where we can find *regularities*. Such dispositions or arrangements have *logical rules* subjacent to them (Barros & Palhares, 1997; ME, 1997; Vale et al., 2006; Vale et al, 2009). More formally, when applying a *law of transformation* to a *module*, a *motive* or to a *term*, we obtain a *sequence* of the respective math objects in which it is possible to determine, at least, one *regularity*. The relationship between the several components of the pattern constitute the structure of the pattern which is in the heart of school mathematics (Papic & Mulligan, 2005).

Importance of preschool patterns

According to Hardy (2002), the study and construction of patterns attract and motivate children, and develop their creativity by appealing strongly to their aesthetic sense.

Several authors also argue that when children recognize and work with patterns, they learn to see and establish intra mathematical connections, as well as between maths and their daily lives and other areas (Barros & Palhares, 1997; ME, 1997; NCTM, 1998 e 2007; Palhares & Mamede, 2002).

On the other hand, the work with and about patterns facilitates acquiring and developing transversal mathematical capacities namely those related with representation, problem-solving, math communication, thinking and reasoning (Copley 2000; NCTM, 2007; Nummela & Rosengren, 1986; Payne & Huinker, 1993; Ponte et al, 2007; Vale et al, 2009).

In this aspect, Fox (2006) states that activities which involve patterns and understanding them, offer preschool children access to “elements of mathematical thinking which are not available to them in any other maths environment” (126, our translation). According to Barros and Palhares (1997), talking about relationships and forecasting what will happen develops thinking and reasoning, especially the logical one. Nickson (2000), on the other hand, values exploring patterns in the development of geometrical thinking.

According to Vale & Pimentel (2005) and Vale et al. (2009), working with patterns in preschool serves as a support to learning Algebra and to the development of algebraic thinking, namely by introducing and manipulating symbols instead of the

most varied math objects. Still according to Papic and Mulligan (2005), algebraic thinking in preschool may involve developing abilities where symbols are used to describe patterns. Lee and Freiman (2006) corroborate this idea saying that children, who understand mathematical patterns from an early stage, are in an excellent position to learn algebraic language and produce algebraic activities. Fox (2006), too, defends that activities involving patterns and relationships constitute a nucleus of acquisition of mathematical competencies, especially those related with algebra and functions. On the same line of thought, NCTM (2007) refers that algebraic concepts may improve and develop from preschool up to the second year of primary school.

Learning experiences with patterns

“Adequate mathematical experiences stimulate children in exploring ideas related with patterns, forms, numbers and space, with increasing levels of profoundness” (NCTM, 2007: 83, our translation).

The first experiences given to children should involve classification, allowing them to act on the things which surround them, “relating and combining them according to any criteria, be it at the beginning within a criteria given by the characteristics of the objects (square, hard, smooth, red, etc.), as well as later, within an abstract criteria (democratic attitudes, ethical behaviour, etc.)” (Favéro, 2005: 108). Children should also be given tasks which allow them to display objects according to a certain order – series – by identifying differences and variations which characterize them (Hohmann & Weikart, 1997). Waters (2004) highlights the importance of recognizing and pre-viewing sequences and of using the associated language.

Other learning experiences should be focused on identifying the regularities of the occurrences, forms, drawings and sets of numbers (NCTM, 1991). The work may begin by inviting the child to reproduce a pattern physically – jump/jump/stop, jump/jump/stop, ...) or by reproducing a sequence of sounds – high/low, high/low and then trying to express it in different ways (Palhares & Mamede, 2002). ‘Reading’ a pattern shown to a child using simple vocabulary (circle – square – circle – square – ...), asking children to reproduce that reading and to keep records of it take notes also allows them to acquire a proper vocabulary on patterns while becoming acquainted with the corresponding concepts. An activity room is the perfect environment to help children recognize, describe and reproduce regularities and patterns. Copying patterns made up of bead, pasta, cubes or other manipulative materials may help children to better understand the underlying concepts (Spodek & Saracho, 1998).

Tasks involving the continuation of patterns should also be considered. Forecasting what will come next are competencies the child should acquire from an early stage. But continuing forward is just as important as doing it backwards. Gap-filling in both directions is a more demanding task, but it should not be forgotten.

Besides the discovery of the logical rule which underlies a certain pattern, it is also very important for children to develop patterns from their own imagination. (Barros & Palhares, 1997; Threlfall, 1999). Hohmann & Weikart (1997) consider this to be an activity which, additionally, the child enjoys performing – “they enjoy arranging things into sets and patterns in order to build something they want or need, such as a necklace made of alternated coloured buttons, or a row of cubes forming a ladder” (704, our translation).

These may develop into increasing or decreasing patterns which can be explored with seeds, cubes or other materials and dimensions. This kind of pattern is initiated

with an object, which represents the first term of the pattern, to which another object of the same species, bigger or smaller according to the established pattern, is added (Copley, 2000). This activity is a lot more demanding since it requires the creation of a module, a rule and its consistent application to that module.

It is very important for children to talk about what they are thinking and doing; so, educators should ask for their cooperation and invite them to share their ideas so that they may develop competencies in math communication. They should begin by using their own language, representations and symbols – forms of representation which are meaningful to them (Bay-Williams, 2001). And each pattern should, in turn, allow several representations (pictorial, symbolic, graphic, ...) so that the child may easily identify the properties of such a configuration – it is a way of “helping children learn how to generalize and recognize patterns in ampler contexts” (NCTM, 1998: 1, our translation).

It is most important to perform the work with real and diverse material (Steen, 1990; Fox, 2006; NCTM, 2007). The educator has the responsibility of providing the child with books and stories with numbers and patterns; rhythmic music which have instructions of the “upward, downward”, “inward, outward” type; games with rules, and problem-solving activities.

If children live the kind of experiences mentioned in this point in preschool, they shall be much better prepared to safely face the challenges which lie ahead of them at school later on, where it is evident that care is taken to involve students in identifying and exploring patterns (DEB, 2001; Ponte et al, 2007).

Method

A case-study was opted due to the objectives set by the research. It was developed under an investigation-action context as proposed and considered adequate namely by Bogdan & Biklen (1994), Cohen & Manion (1990), Coutinho & Chaves (2002), Lessard-Hébert et al (1994), Stake (1995) and Vale (2004). Given the scarce studies on this theme, in the above-mentioned context and theoretical framework, the study also assumed an exploratory nature, just as justified by Cabrita (1998).

The investigator who underwent the study was not the preschool teacher. Five children were chosen, between the ages of 3 and 4. Later, some parts of the study related to patterns were replicated under the same conditions, and an extension was introduced to other groups of children in preschool, 28 on the whole, slightly older and coming from various countries/realities. In this phase, undergraduate students from a degree in Preschool Education from a Portuguese university were involved in the experience.

The study was structured in two phases: 1) familiarization between the investigator and the group, through didactic experiences introducing the theme to be studied and 2) empiric phase, involving learning experiences centred namely on patterns, inspired on the afore-mentioned CD-ROM. These experiences were developed within a natural environment in the same kindergarten the children were attending. The investigator proposed the tasks, made sure the children understood them, and then the children developed them individual and autonomously.

Collection of the investigating material was essentially made, as suggested by Erlandson et al. (1993), Yin (1989), Jorgensen (1989), Pardal & Correia (1995) and Soares (2006), through:

- Direct and participative observation registered in a logbook, photographed and video-taped with all the children’s behaviour and reactions to the proposals during the several phases of the process;

- Documents and artefacts produced by the children;
- More or less formal conversations held with them.

The material which was collected was subject to a content analysis oriented by categories coming from the objectives proposed by the study.

Regarding situations of success or difficulties involving the performance of the 16 task on patterns, we analysed:

- reproduction of patterns – orally, with manipulative materials, on paper;
- creation of patterns – without limitation of space, with delineation of space;
- continuation of patterns – frontwards.

In the extension of the study, some tasks were adapted and others created in order to explore situations which the main study revealed as being potentially interesting. Such was the case of reproducing patterns which had been presented orally, continuing backward and gap-filling.

Results

Following are some of the main results, structured according to pre-defined categories.

Success situations

Overall, the children did not reveal difficulty in reproducing patterns orally. In relation to the following sequence and after attributing a sound to each animal, the children verbalized for example, “*Mic-mic, mic-mic, moo, mic-mic, mic-mic, moo, mic-mic, mic-mic, moo*”.



Figure 1: Example of a sequence to be verbalized

Reproducing sequences through the use of manipulative material was also a success, as may be verified in the following example where pasta (in a sequence of two yellow and one blue) and beads (in a sequence of one blue and one white) were used.

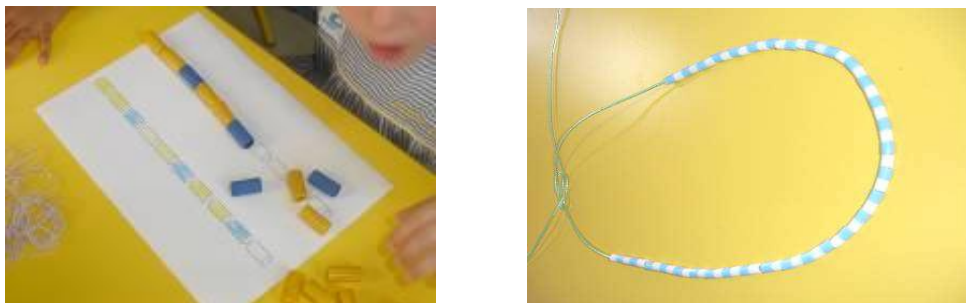


Figure 2: Reproducing patterns with manipulative materials

An identical situation was verified with a reproduction on paper (or cloth), as may be illustrated in the following figure, even involving complex tasks (see the 3rd image).



Figure 3: Reproducing patterns on paper

Most of the children did not reveal any difficulty in building patterns without a limited space, and some were very creative. See the first image with a pattern type AB-BABBABB... made with dominoes placed in an unusual position; the second with a pattern type AABBAABBAABB made of cards with animals; the third with the decoration of a mask cut out on coloured paper, type ABABABABABAB, and the last one with a drawing of a pattern type ABBBABBABBABB.



Figure 4: Creating patterns without a delineation of space

Curiously, gypsy children seem more creative in inventing patterns with beads to make necklaces.

Some children were also successful in creating patterns with a limited space, namely with 10 cylindrical configurations; with 11 circumferences and in a table of 7 columns by 5 lines.

The first image that follows shows a pattern type ABABABABAB ending in B. The second image though, shows a pattern type AABBAABBAA, where the child had no difficulty in finishing with AA.

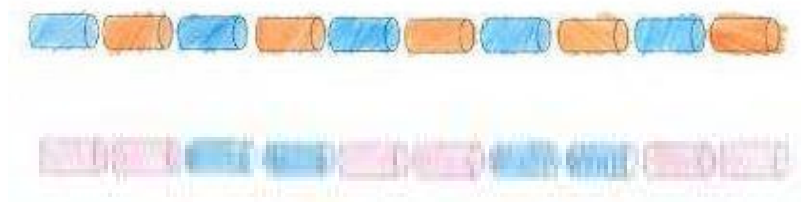


Figure 5: Creating patterns in 10 cylindrical spaces

In the following image, the child also did not have any problem in ending the sequence type ABABAB... with A. Less frequent was the solution presented in the following table. Actually, the child respected the law of formation by repeating the module type AB, even when he or she changed line.

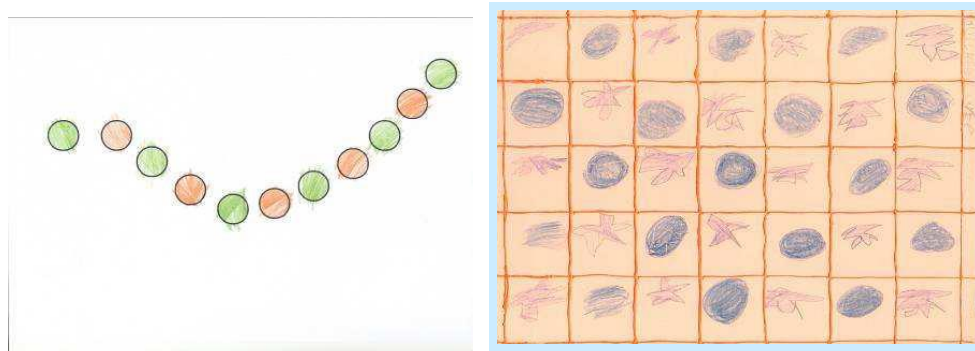


Figure 6: Creating patterns in 11 circular spaces and inside a 7x5 grid

Continuing patterns forward, even the more complex such as the case of the 3rd and 4th rulers, did not reveal being difficult to the majority of the children.

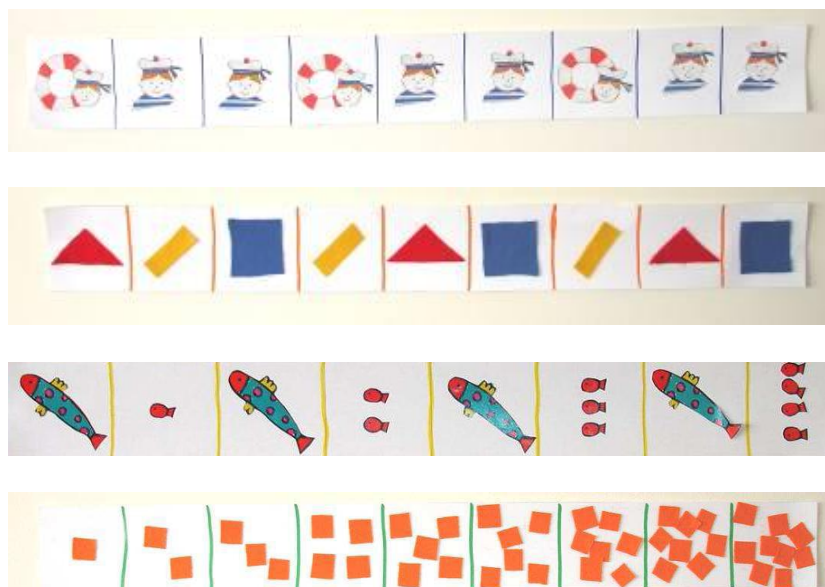


Figure 7: Continuing patterns forward

Main constraints

Although most children revealed a positive performance, even surpassing our expectations, some activities revealed themselves complex and difficult. One of those activities has to do with the creation of patterns with delineation of space. For example, in relation to the model with 10 cylinders, some children left a space for painting. Note that the number of cylinders is not a multiple of the number of elements of the module (3) type AAB.

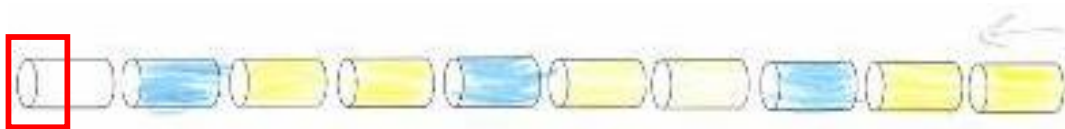


Figure 8: Difficulty in creating patterns in 10 cylindrical spaces

Also in relation to the model with 11 circles, some children added a circle, remaining with a multiple number of the number of elements of the module – “I had to end in red because it is blue yellow red, blue yellow red” (first image of the following figure). Others coloured over the “missing” circles (second image), without being able to explain why they did it and why they did not colour over the green only, as had been expected, in order to be able to “close” the pattern whose module is violet, green, green.

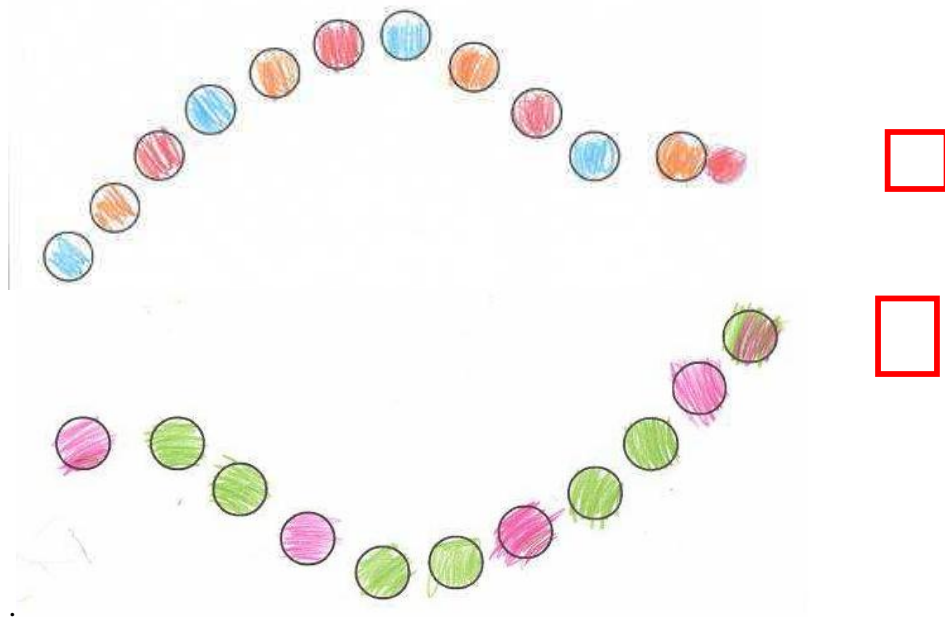


Figure 9: Difficulty in creating patterns in 11 circular spaces

In the reproduction and continuation of the pattern created before, with 20 plasticine balls, the children also “invented” some interesting solutions:

- Either the children abandon the balls which do not allow them to “close” the pattern – for example, in the first images of the following figure, the modules have, respectively, six elements (three green balls and three yellow balls) and three elements (one red ball and two green balls); as the nearest multiple to 20 is 18, two balls remain;

- Or they “invent” new balls – in the 3rd image, they “close” the pattern with 21 balls because the module has three elements.

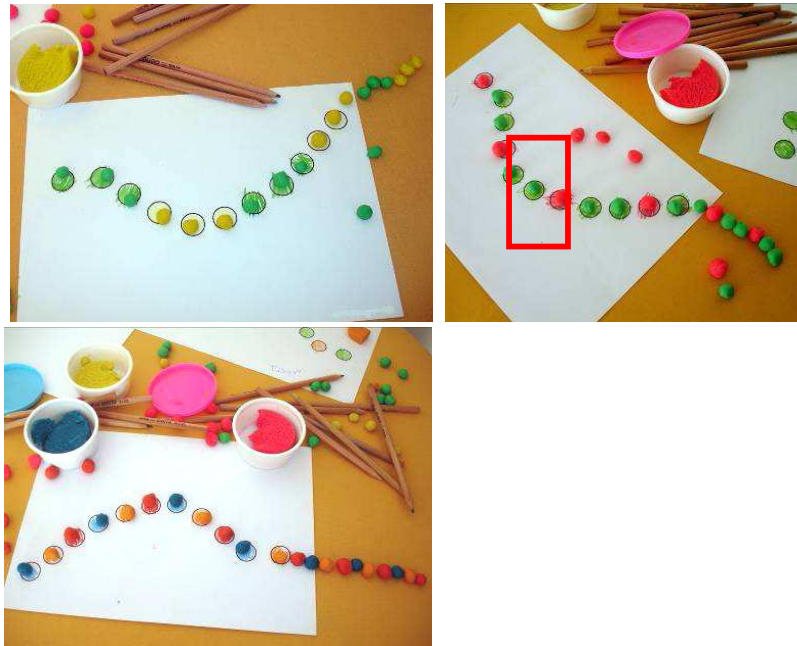


Figure 10: Difficulty in reproducing and continuing the former pattern with 20 plasticine balls

It is still interesting to verify the “creativity” of some children when faced with a 7x5 table. Some abandoned the pattern and made regularities in column; others “invented” a new column where the last element of the initial module would fit (following figure).

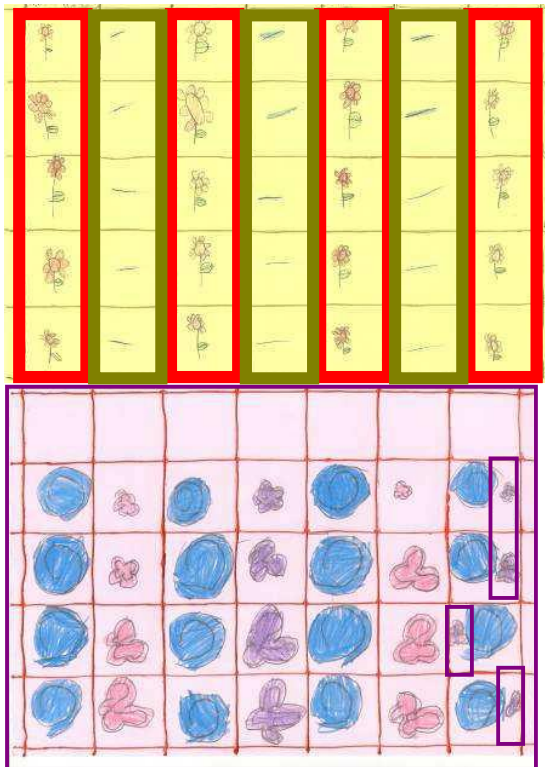


Figure 11: Difficulty in creating a pattern in a 7x5 table

They also appear to have difficulty in reproducing a pattern presented orally with sounds. For example, they heard *crack, clap, clap, crack, clap, clap, crack, clap, clap...* and they could not reproduce it with cards. Curiously, children from the PA-LOP seem to be more at ease in dealing with musical patterns.



Figure 12: Difficulty in reproducing a pattern presented orally

In what continuing backward is concerned, some children reproduce fully the pattern they read forward. That is the case represented in the following figure – *It's one blue, one red and one yellow.*



Figure 13: Difficulty in continuing the pattern backward

Finally, regarding the gap-filling of the empty spaces, such “logic” does not seem to apply to all directions. Actually, the following figure shows us that, from the 3 central images back, some children do not consider the ‘*boy boat*’ but, instead, ‘*whale boy*’, continuing the two first images. Even when asked to “read” the pattern orally, they thought it to be correct.

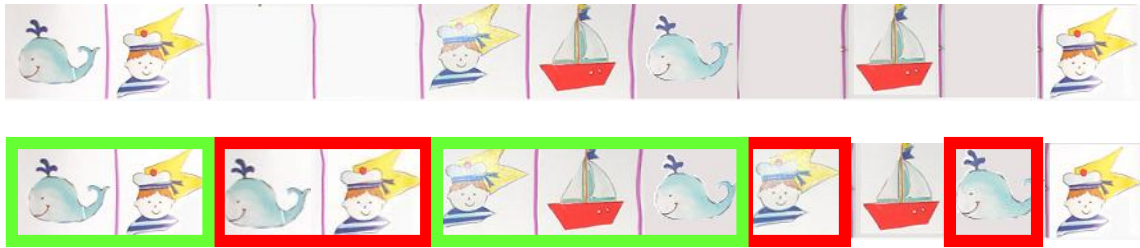


Figure 14: Difficulty in completing the pattern

Discussion of results and recommendations for future investigations

The investigation and experience undergone with the replication and extension of this study, allowed us to corroborate the findings of Hohmann & Weikart (1997), which state the work with patterns is greatly appreciated by children – “they enjoy arranging things into sets and patterns in order to build something they want or need, such as a necklace made of alternated coloured buttons or a row of cubes forming a ladder” (704, our translation).

The diversity of the material provided to children may also have contributed to this, as defended by several authors and/or organisms (Barros & Palhares, 1997; DEB, 2001; Fox, 2006; Moreira & Oliveira, 2003; NCTM, 2007; Palhares & Mamede, 2002).

It was also possible to confirm that the children already bring a great deal of mathematical knowledge to kindergarten (see, for example, Clements & Sarama, 2007; DEB, 2001; Fox, 2006; Moreira & Oliveira, 2003; NCTM, 2007), which allow them, with the least didactic intervention, to successfully and creatively solve a variety of tasks involving patterns. Some of these tasks include reading, reproduction (namely oral) and continuing frontward without a delineation of space. This did not only happen with repetitive patterns but also with growth patterns generally considered to be a lot more complex (Copley, 2000).

Concerning children’s successful performance, this study revealed a very curious aspect we wish to highlight and which was not revealed in any other study – gypsy children seem to be more creative in making bead necklaces. The reason for this performance may be of a cultural nature, an issue which should be further studied. In fact, ornaments such as necklaces and bracelets are highly appreciated and worn by people from this ethnic group, a fact explored in the CD-ROM which served as basis for the investigation. (Cabrita & Moderno, 2003).

Still related to the cultural dimension, we consider it is also pertinent to investigate further whether, in fact, the easiness in reproducing patterns which were presented orally is extensive to other children from the PALOP, besides those whom we worked with. It is worth mentioning that music has always held an important space within these people’s lives. Noteworthy is also the fact that this task revealed to have been very difficult for the majority of the children whom we worked with, and who came from other realities. Once again, we did not see these aspects referred nor discussed when revising the literature. We, therefore, recommend an investigation on this issue.

Another added value which came from this study is the fact that it allowed us to mention tasks which revealed themselves of a greater complexity, mainly:

- The continuation of patterns frontward, with delineation of space, when the number of such spaces is not a multiple of the number of elements of the module. Actually, the majority of the children only considered the pattern to be correct when

it “closed” with the last element of the module. When there is no space for that to happen, they come up with the most incredible ways of overcoming the difficulties they face. As for the particular case of tables, most of the children forget the pattern and only worry about keeping the regularities within the column;

- Continuing the pattern backwards. Most of the children place the elements of the module backward and in sequence, as though they were handling symmetry through reflection;
- Gap-filling. The majority of the children repeated the elements which came before and functioned as modules, inside the blank spaces. It was as though they were handling different patterns which only needed to be continued forward.

The results bring up a vast number of possibilities and a need for a more systematic and endured investigation concerning mathematical issues in education. Patterns should be studied more specifically both from a scientific as well as a cultural point of view, along with their clear repercussions at a curricular level. Actually, since patterns play a key role in maths education, namely in the development of algebraic and geometric thinking, their treatment is imperative both at an initial and on a continuous formation level. We emphasise that particular attention should be given to the crucial aspects brought up by this research.

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