

A Multi-Objective Genetic Algorithm for the Vehicle Routing with Time Windows and Loading Problem

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Abstract. This work presents the Vehicle Routing with Time Windows and Loading Problem (VRTWLP) as a multi-objective optimization problem, implemented within a Genetic Algorithm. Specifically, the three dimensions of the problem to be optimized – the number of vehicles, the total travel distance and volume utilization – are considered to be separated dimensions of a multi-objective space. The quality of the solution obtained using this approach is evaluated and compared with results of other heuristic approaches previously developed by the author. The most significant contribution of this work is our interpretation of VRTWLP as a Multi-objective Optimization Problem.

Keywords: Vehicle Routing Problem with Time Windows, Container Loading Problem, Multi-objective Optimization Problem.

1 Introduction

Managing the distribution of goods is a vital operation for many companies which realize that distribution has a major economic impact. However, clients' satisfaction depends mainly on meeting their demand as effectively as possible. This is commonly described as providing a service to a client. Usually a service is a combination of different distribution characteristics, for example: product availability, delivery time, delivery programming and good conditions after delivery. One of the most important areas in serving clients is said to be the transportation of goods. With an adequate transportation, items arrive on time, undamaged and in the desired quantities. Indeed, those are the three main client's demands to be achieved and for that, the integration of route planning and vehicle packing is essential.

When solving a Vehicle Routing Problem with Time Windows (VRPTW), the solutions ensure that items arrive to the client within the time window. To ensure they do not suffer any damage during transportation, a stable load is necessary which is achieved by solving the Container Loading Problem (CLP). To ensure that all the items of each demand are delivered to a client, one must solve the VRPTW and CLP in an integrated way. Indeed, the classical model of vehicle routing ensures that total client demand, assigned to one vehicle, does not exceed the vehicle capacity restrictions in terms of weight (or other scalar measure). However it is not certain that the

cargo can be physically loaded and arranged inside the vehicle/container. So, a cargo, which in terms of weight can be packed in a vehicle, can exceed its volume capacity, or vice versa. To deal with this drawback we propose the resolution of the VRTWLP as a Multi-objective Optimization Problem.

Ideally the client items are packed in the vehicle considering a LIFO (Last-in-First-Out) strategy. Some authors ([10], [25]) present a variation of the travelling salesman problem with pickup and delivery in which the loading and unloading operations are executed in a LIFO order. [10] presents three local search operators embedded within a variable neighbourhood search metaheuristic. [25] introduces the double travelling salesman problem with multiple stacks and presents three metaheuristic approaches where repacking is not allowed. The items are packed in several rows inside the container and each row is considered a LIFO stack.

The container loading problem is a three-dimensional problem that establishes arrangements of items in a container. Usually, the CLP aims to maximize loading efficiency – that is, the container space usage. For instance, [16], [5], [6], [7], [14], [3] and [21] deal with the container loading problem considering specifically the efficiency of the loading arrangements.

The other problem discussed in this paper is the VRPTW. In the VRPTW, clients have to be served within a period of time [27]. In literature there are four goals that are usually considered: (i) minimize the number of vehicles; (ii) minimize the total travel distance; (iii) minimize the total time; and (iv) minimize the vehicles total waiting time at clients. Some approaches use one of these goals and others combine two (or more) of them. [1], [8], [9], [11], [13] and [22] are some examples of recently published work where original algorithms for the VRPTW are presented.

Very few papers approach the vehicle routing problem and the two-dimensional bin packing problem integration, all very recent. [18] presents a special case of the symmetric capacitated vehicle routing problem and proposes an exact approach based on branching algorithms. [15] presents a taboo search heuristic to solve the routing problem with three-dimensional loading constraints. [19] presents a framework to integrate the VRPTW and CLP using two different solution methods. The first one treats the problem in a sequential approach while the second uses a hierarchical approach applying a GRASP algorithm.

In the next sections of this paper, the VRTWLP as a multi-objective optimization problem implemented with Genetic Algorithm (GA) is proposed. [23] applied a hybrid search based on GA and Tabu Search to soft VRPTW where the multi-objective VRPTW is dealt like a single-objective optimization. In a more recent work [24], the same authors present a multi-objective genetic approach to VRPTW in which the two objective dimensions are the number of vehicles and the total distance.

In section 2 a VRTWLP specification and an overview of multi-objective optimization search is provided. A Multi-objective Genetic Algorithm and some experimental details are presented in section 3. Section 4 reports the results and makes comparisons with related works while section 5 concludes the paper with a general discussion.

2 Description of the Vehicle Routing with Time Windows and Loading Problem

Capacity constraints of vehicles in the vehicle routing problem are often improperly used when real-world applications are considered. The capacity constraints are not only related to admissible weight but also to the vehicle's volume dimensions. The routes designed for a given vehicle capacity, in terms of weight limits, can lose their admissibility due to incompatibility of cargo dimensions, and vice versa. To address loading issues in more detail in routing problems, a more complex model is required. Loading constraints may seriously affect the nature of the problem. The integration of routing and loading problem calls for tailored resolution procedures. This integration results in the VRTWLP – The Vehicle Routing with Time Windows and Loading Problem.

Let us consider, for this problem a set of clients defined by their geographical coordinates and a fleet of homogeneous vehicles. There is only one depot from where the vehicles start to visit the clients and return at the end of the delivery. Each client has a demand to be satisfied by a single vehicle and a time window that must be respected. All the clients' demand must be satisfied even if another vehicle has to be used. Preferably the vehicle loading order is the inverse of the clients visit order (LIFO strategy) and the demand of each client should be packed together inside the vehicle in order to increase the efficiency of the unloading operations. Vehicle capacity, defined in terms of weight and width, height and length of the loading volume must not be exceeded. A demand is composed by a set of different box types. This means that for each box type one two or three dimensions are allowed to face upwards. If only one dimension is admissible as height then we have the "This Side Up" constraint. Each client is defined by geographical coordinates, time window, demand (type of boxes and related quantities per type), total weight of demand and service time.

The interdependency between the VRPTW and the CLP is greater when the number of clients visited by each vehicle is small. This means that each client's demand takes an important portion of the container. Therefore, the decision to include or exclude a client in a route has a major impact on the CLP, and may cause the route to become unfeasible. On the other hand, a solution which provides good volume utilization may lead to long and unfeasible routes. When we have many clients per vehicle, the routing aspects dominate the loading aspects, as the choice of clients to visit influences the routes much more than the loading efficiency. Conversely, when one client completely fills a vehicle, the only problem is how to load the cargo and the CLP dominates the VRPTW (that may end up just in one client per container). It is when we have a relatively small number of clients per vehicle, and a weakly heterogeneous cargo, the integrated resolution of the two problems becomes rather important for the final solution quality. The relevance of the integrated resolution of the VRPTW and CLP problems is also dependent on the density of the goods to transport. If the goods are very heavy and of small size, the usual weight constraint will be the active constraint and there is no need to consider the CLP. Considering these assumptions, we consider the following constraints and goals to the problem:

1. Clients and depot time windows. All clients must be visited within a certain period of time and the vehicle has a maximum travel time (time to visit all clients and return to the depot).
2. Homogeneous fleet. Identical vehicle capacity in terms of weight, volume and dimensions (length, width and height). The vehicle's capacity must always be respected.
3. Cargo's orientation. The client's demand consists of parallelepiped boxes that may have to satisfy orientation constraints: for example, the "This side up" sign.
4. Client's demands are heterogeneous and the total demand of each client in terms of volume and weight does not fill a vehicle. Every demand must be satisfied by a single vehicle.
5. The density of the cargo¹ is such that the maximum weight of the container is not a constraint for the problem.
6. Cargo positioning inside the vehicle. Each client's demand should be packed together in order to make unloading easier, even though this is not strictly necessary to ensure compatibility between the routes and the loading pattern. A LIFO policy will be used so that when a client is visited, it must be possible to unload all items of his demand without unloading boxes of other clients (see [15]). The loading order is the reverse order of the client's visits order.
7. Cargo stability. To ensure that the load cannot move significantly during transport, the cargo must be packed in such a way that it remains stable. Also an unstable load can have important safety implications for loading and unloading operations. From a stability point of view, two different measurements are considered. The first one is the full support of each item from below. This measurement does not indicate the potential for lateral movement of a box, though. The second measurement is the average percentage of boxes not surrounded by at least three sides ([4] and [21]).

The VRTWLP goals, as for the VRPTW, are to minimize the number of vehicles and the total travel distance. From the CLP point of view, the objective function is to maximize the container's volume utilization. Considering multiple vehicles for the CLP, this objective function can be seen as packing all the available cargo in the vehicles. Thus, the minimization of the number of needed vehicles is also implicit. So, in short, the VRTWLP's basic idea is trying to serve the greatest possible number of clients with each vehicle and pack their demand in a feasible way while considering also the minimization of the travel distance for each route.

2.1 Multi-objective optimization

This work studies the VRTWLP as a multi-objective optimization problem (MOP). A MOP is a problem in which two or more objectives contribute to the final result. The three dimensions of the VRTWLP to be optimized are: number of vehicles, total tra-

¹ Density of a cargo is a measure of mass per unit volume. For example, an object made from a comparatively dense material (such as iron) will have more mass than an equal-sized object made from some less dense substance (such as aluminum).

velled distance and vehicles volume utilization are considered to be separated dimensions of a multi-objective space. As with all MOP's, an immediate advantage is that it is not necessary to use a weighted coefficient for each objective function's component. We do not specify that either the number of vehicles, the travel distance or the vehicles volume utilization take priority. Using the Pareto ranking procedure, each of these problem characteristics is kept separate and there is no attempt to unify them.

The instances of the VRTWLP may have more than one locally optimal solution (multimodal solutions) where some solutions may minimize the number of vehicles and by inference the vehicles wasted volume at the expense of distance. On the other hand, other solutions minimize the distance while necessarily increasing the vehicles number and the wasted space. Looking to this problem as a MOP the objective components that are mutually exclusive (number of vehicles and wasted volume utilization), contribute to the overall result and these objective components affect one another in nonlinear ways. The challenge is to find a set of values for them and an underlying solution which yield an optimization of the overall problem.

Genetic algorithms are suitable search engines for multi-objective problems because of their population-based approach. A multi-objective evolutionary algorithm (MOEA) is capable of supporting diverse, simultaneous, solutions in the search environment. Considering the success of applying MOEA in finding good solutions to problems and knowing that the GA are suitable search engines for multi-objective problems primarily because of their population-based approach, a Multi-objective Genetic Algorithm (MOGA) is presented in section 3 in order to solve the VRTWLP.

2.2 Multi-objective ranking

In order to evaluate generated VRTWLP solutions they are represented with a vector describing its performance across the set of criteria. This vector must be transformed into a scalar value for the purposes of the GA. This process is achieved by ranking the population of solutions relative to each other, and then assigning fitness based on rank. Individual solutions are compared in terms of Pareto dominance. The MOGA developed in this work uses Pareto-ranking (often used in MOGA, like in [24]) as a means of comparing solutions across the multiple objectives. The Pareto-optimal set or non-dominated set [12] consists of all those vectors for which components cannot be simultaneously improved without having a detrimental effect on at least one of the remaining components.

The Pareto ranking scheme is easily incorporated into the fitness evaluation process within a GA, by replacing the raw fitness scores with Pareto ranks. Each of the problem objectives is kept separated and there is no attempt to unify them. These ranks stratify the population into preference categories and the lower ranks are preferable. The individuals on each rank set represent solutions that are incomparable with one another. The Pareto ranking only differentiates individuals that are superior to others in at least one dimension and not inferior in all other dimensions.

3 Multi-Objective Genetic Algorithm for VRTWLP

A GA is a programming technique that imitates biological evolution as a problem-solving strategy. Given a specific problem to solve, the input to the GA is a set of potential solutions to that problem, encoded in some fashion (chromosomes). Then, a fitness function is defined in order to allow each candidate to be quantitatively evaluated. The algorithm then applies genetic operators such as mutation and crossover to “evolve” the solutions in order to find the best one(s). The promising candidates are kept and allowed to reproduce. These offspring then go on to the next generation, forming a new pool of candidate solutions, and are subjected to a second round of fitness evaluation. Those candidate solutions which were not improved by the changes are not considered for the final solution.

In this section some details of the VRTWLP solutions representation, fitness evaluation, Pareto strategy and other GA features used are described.

3.1 Initial population

In order to generate the initial population for the GA, an approach developed for VRTWLP and presented in [20] is used. In this approach, for each vehicle, a route and the container loading are planned until no more clients and demands to deliver exist.

For the GA, the initial population is composed by a number of individuals (i.e. solutions), each being generated by performing a constructive algorithm. A brief description follows.

The problem data is represented as a list named Sequential Candidate List (SCL). A candidate in a SCL is composed by a client and a single box type of his demand. The number of candidates (n - size of the SCL) is the total number of combinations of clients and box types (Fig. 1). For each solution generation, the candidates are randomly placed in the SCL.

Candidate 1: Client 1; Box Type 1
Candidate 2: Client 1; Box Type 3
Candidate 3: Client 2; Box Type 2
Candidate 4: Client 2; Box Type 5
.....
Candidate n: Client 25; Box Type 2

Fig. 1. Sequential Candidate List.

In a solution the demand of a client could be physically separated (grouped by box types) in one vehicle or in more than one vehicle. In this case, two of the presented problem constraints (Section 2) could be relaxed. The first one is related to the cargo’s position in the vehicle (constraint 6) and the second specifies that every client must be visited only by one vehicle (constraint 4). To build a solution a constructive algorithm is applied to the SCL. The first candidate of SCL is chosen and the related client is inserted in a route and the box type is loaded in a free space of the vehicle. If all the restrictions of VRTWLP are satisfied, the solution is accepted and the algo-

rithm selects the next candidate of the SCL. If one of the problem restrictions is violated the chosen candidate is removed from the solution and inserted at the end of the SCL. When a vehicle is full or the depot time windows exceeded another vehicle must be used. The algorithm stops when the SCL is empty.

As mentioned before, a solution could have one client served by more than one vehicle. For example, client 2 (Fig. 1) could be placed in 3rd, 4th and (n-2)th position in the SCL (could correspond to candidate 3, 4 and n-2) if his demand is composed by three different box types. Candidates 3 and 4 could be assigned to vehicle k and candidate n-2 to a different one. So, in this example, the same client is visited by two different vehicles.

For this same reason, in this approach it is allowed that each client's demand might be spread in the vehicle. When this happens a client could have its demand not directly accessible and in this case one of the two following alternatives must be chosen:

1. the cargo that blocks the access to those boxes must be unloaded and reloaded at the client's location so that the complete demand is unloaded;
2. or, the client could be later revisited by the same vehicle.

The choice between these two alternatives is made by evaluating them in terms of total route time. If the first alternative is chosen the route must be rebuilt because the vehicle does not need to revisit the client that has the demand split up and an additional time for unloading and reloading the blocking cargo is considered. In each case the algorithm computes a cost and the alternative with smaller cost is chosen.

Each time a client is inserted in a route, the boxes of his demand are loaded in the vehicle using a 3D packing algorithm. The constructive algorithm of the approach described in [21] was followed. It is an improvement of the George and Robinson heuristic [16] for solving the Container Loading Problem. This wall-building constructive heuristic packs boxes in a container ensuring cargo stability. All boxes are fully supported and all the columns of boxes have at least three sides supported. For each type of box the free space in the container is filled with the best possible arrangement. The arrangements are found by simulating all choices of possible orientations of the box types, and by computing the corresponding volume utilization. Then the best arrangement is chosen and packed in the vehicle's free space.

3.2 Chromosome representation

Each potentially non-dominated solution (Section 3.3) must be encoded and thus originate a chromosome, in order to apply the GA. The structure of each chromosome (set of genes) is a string of equal length to the SCL that originated the potentially non-dominated solution. A single chromosome comprises all the information of one complete solution to the VRTWLP. A gene of a given chromosome is a candidate (of the SCL) and the sequence of the genes in the chromosome defines the visiting order of each vehicle. An example of a chromosome that represents a potentially non-dominated solution is as follows (Fig. 2):

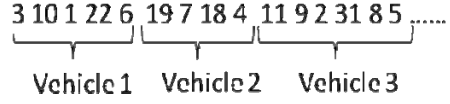


Fig. 2. Chromosome representation.

In order to select the potentially non-dominated solution to be encoded, the chromosome fitness function is evaluated using a Pareto ranking procedure. The Pareto ranking is incorporated into a GA by replacing the chromosome fitness with Pareto ranks. This procedure and the selection phase are described in the next section.

3.3 Selection phase and Pareto ranking procedure

According to VRTWLP objective function, the three dimensions of the problem: number of vehicles, travel distance and volume waste, must be minimized. When the number of vehicles is minimized the volume waste is also minimized and this affects vehicles and labour costs. Minimizing the travel distance affects the time of each route and the fuel resources. The two objectives components mutually dependent, number of vehicles and distance travelled, are used to evaluate the solutions. Each candidate of the initial population is associated to a vector $\vec{v} = (n, d)$, where n is the number of vehicles and d the total distance. These two dimensions are used by the Pareto ranking procedure (Fig. 3).

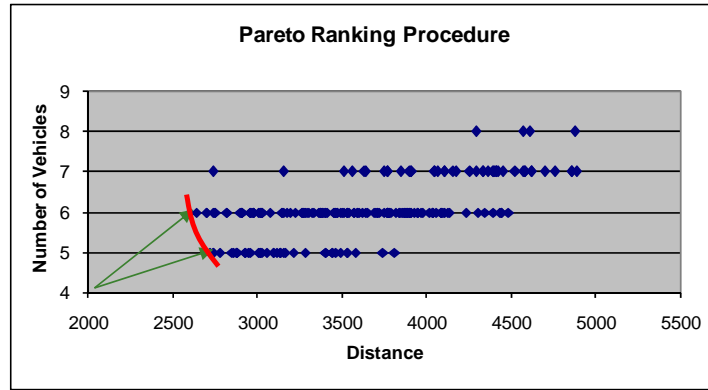


Fig. 3. Pareto Ranking Procedure.

The Pareto ranking procedure is applied to the vectors of the population and a set of solutions is ranked according to the following procedure. A list of potential non-dominated solutions is built. The size of this list is computed according to the following criteria (Equation 1):

$$\text{MaxVS} = \text{Max} - \alpha * (\text{Max} - \text{Min}) \quad (1)$$

Where:

- MaxVS is the maximum admitted size of the solution vector (computed as the size of the vectors in Figure 3).
- Max is the maximum vector of Pareto front
- Min is the minimum vector of Pareto front
- α is a parameter that defines the MaxVS value.

The list of potentially non-dominated solutions (LS) is composed by all solutions with vector size smaller or equal to MaxVS.

The two potentially non-dominated solutions to the crossover procedure are randomly selected from LS. The size of this list is important for the reproduction because the main idea is to select the best individuals of the population in order to guarantee that the best solution (produced by the best chromosome) can never deteriorate from one generation to the next. The value of α is crucial to accomplish this, because if α is equal to 0 all the potentially non-dominated solutions belong to the list. Making a random choice, a bad solution could be selected for reproduction. When α is equal to 1 only the two best potentially non-dominated solutions belong to the list. However, the best results are achieved when α is equal to 0.8.

3.4 Recombination phase and Mutation operator

As the VRTWLP constraints must be always satisfied, the crossover operator must not result in an unfeasible solution. In the recombination phase an approximation of the Best Cost Route Crossover (BCRC) is used. [23] and [24] applied the BCRC to Vehicle Routing Problem with Time Windows and [18] also applied the BCRC to the Dynamic Vehicle Routing Problem. The BCRC aims at minimizing the number of vehicles and distance simultaneously while checking feasibility constraints. In the next example (Fig. 4) the dynamic of BCRC is explained.

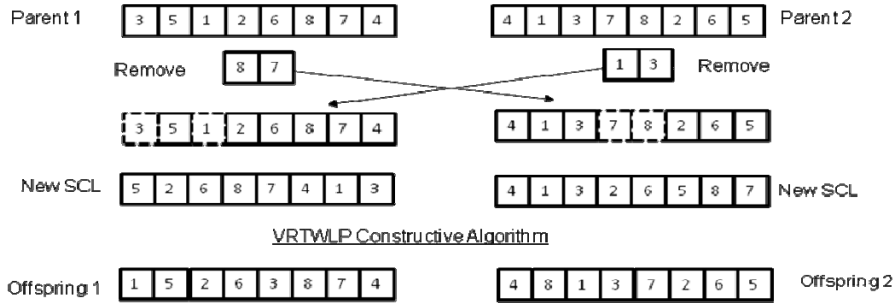


Fig. 4. Best Cost Route Crossover Operator.

In Fig. 4 the generation of two offspring using an approximation of best cost route crossover operator is presented. From the LS the algorithm two parents are randomly chosen. The two encoded parents correspond to a sequence of genes (candidates). This sequence defines a solution that is the order by each candidate was inserted in the solution and the order by each vehicle visits the clients. So, each parent corresponds to a SCL (Section 3.2) that originates this particular solution. From this solu-

tion two contiguous genes are randomly selected and removed from the other parent. The two removed candidates are inserted in the end of the encoded sequence creating a new SCL. With this two new SCL the VRTWLP constructive algorithm is applied and two new offspring are generated. The feasibility of the offspring is always guaranteed by the inherent characteristics of the constructive algorithm.

The mutation procedure aids a genetic algorithm to achieve any point of the search space. Nevertheless, mutation could destruct very good solutions and in this particular problem the existence of time windows and loading constraints could easily lead to an unfeasible solution. Only a few offspring are chosen for mutation with a probability of ten percent. The mutation is a small change of the chromosome (Fig. 5).

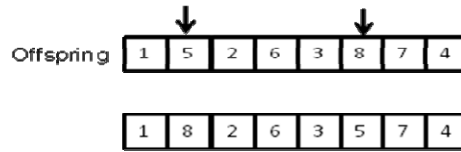


Fig. 5. Mutation operator.

If an offspring is selected to the mutation procedure, two random candidates of the SCL that originates the solution are selected and swapped. Then with the new SCL the VRTWLP constructive algorithm is applied. When the algorithm tries to insert the swapped candidates in a route and the client demand in the vehicle, if any restriction is violated the solution becomes unfeasible and the new offspring is not considered.

4 Test Problems and Computational Results

This approach was tested using some problem instances early developed by the author in [20] and available on the ESICUP web site (<http://paginas.fe.up.pt/~esicup/tiki-index.php>). These problems combine the standard test problems characteristics available in the literature for the VRPTW and CLP. The VRPTW test problems used are the R1 and R2 (with 25 clients) from [27] and the BR2 from [4] for the CLP test problems. Two problem classes were tested: I1 (R1 and BR2) and I2 (R2 and BR2). Each problem class has two different groups:

- Group I – Clients' demand varies from 30 to 80 boxes, with 1 to 5 box types per client and an average of 42 boxes per demand. The total number of boxes is 1050;
- Group II – Clients' demand varies from 50 to 100 boxes, with 1 to 5 box types per client and an average of 62 boxes per demand. The total number of boxes is 1550.

In total, there are forty six test problem instances covering the four combinations of classes and groups (around 12 instances per combination). The next tables present the results achieved by the MOGA approach and the best results achieved with other approaches developed by the author to solve VRTWLP and published in [20]. The first one uses a GRASP approach while the second one uses a hierarchical approach. The hierarchical approach first builds the routes and then loads the demands inside the

vehicles, this means, the VRPTW dominates the CLP. In the constructive phase the two different approaches use five different ranking schemes. To evaluate the solutions those approaches use a weighted coefficient for each objective function's component and the sum of these three coefficients is equal to 1. The actual values of these weights will depend on the practical application under consideration and on the importance given by the decision-maker to each component. The best results of these two approaches are achieved with the time windows ranking - clients whose time window is smaller and starts earlier are inserted first [20]. For this reason, such are the results that we compare with those obtained using the MOGA approach.

The MOGA algorithm stops when after ten consecutive runs with no better solution (offspring) is found. In each run, in order to build the initial population the constructive algorithm is performed 100 times and each time with a different SCL. Because of the problem characteristics the LS is usually relatively small (around four potentially non-dominated solutions). Depending on the size of the LS, the following procedure is repeated (number of repetitions is equal to half of the LS size plus one): selection of parents, crossover (with a rate of 0,90), mutation (with a rate of 0,10) and offspring evaluation.

The running time for the Integrated GRASP Heuristic and the Hierarchical Approach is less than one minute. For the MOP Genetic Algorithm the running time is, on average, four minutes. Times were obtained using a Centrino Core2Duo T7100@1.80 GHz.

Tab. 1 and Tab. 2 present the results of problem instances of Class I1. Those problems have a short planning horizon. The depot has a small time window, which implies many vehicles per problem and routes with a small number of clients.

Tab. 1. Group I Class I1 problem instances.

GI/I1	Integrated GRASP Heuristic	Hierarchical Approach	MOGA
Instance	Vehicle / Distance Time windows ordering	Vehicle / Distance Time windows ordering	Vehicle / Distance
1	10 / 1711.34	9 / 762.59	12/1817.87
2	9 / 1617.70	11 / 944.44	11/986.35
3	6 / 1250.86	8 / 754.27	7/1332.38
4	6 / 1212.51	6 / 804.14	6/1094.40
5	9 / 1562.02	10 / 815.36	11/1750.46
6	7 / 1273.26	7 / 757.08	5/1040.47
7	6 / 1227.40	7 / 901.80	6/1215.17
8	6 / 1158.76	6 / 785.95	6/1277.07
9	7 / 1270.07	7 / 820.17	10/1008.82
10	6 / 1098.71	7 / 753.01	7/1211.96
11	7 / 1198.02	7 / 851.45	8/1559.14
12	6 / 1070.44	6 / 803.62	6/803.62

Tab. 2. Group II Class I1 problem instances.

GII/I1	Integrated GRASP Heuristic	Hierarchical Approach	MOGA
Instance	Vehicle / Distance Time windows ordering	Vehicle / Distance Time windows ordering	Vehicle / Distance
1	13 / 2089.69	9 / 823.04	13/2089.69
2	9 / 1622.59	11 / 927.17	9/1622.59
3	8 / 1380.53	10 / 970.26	8/1287.10
4	8 / 1405.80	8 / 844.66	7/1201.75
5	10 / 1675.86	12 / 864.48	10/1860.28
6	8 / 1476.54	14 / 1109.74	8/2006.00
7	8 / 1381.41	9 / 944.26	9/913.83
8	7 / 1303.89	9 / 1035.37	8/1431.84
9	8 / 1359.87	15 / 1202.91	11/1711.96
10	8 / 1298.79	8 / 673.16	8/1135.79
11	8 / 1502.61	10 / 1023.30	7/1431.58
12	8 / 1377.93	8 / 844.40	8/1298.77

Tab. 3 and Tab. 4 include the results of Class I2 problem instances. These instances have a long planning horizon. The depot has a large time window, which implies few vehicles per problem and routes with a big number of clients.

Tab. 3. Group I Class I2 problem instances

GI/I2	Integrated GRASP Heuristic	Hierarchical Approach	MOGA
Instance	Vehicle / Distance Time windows ordering	Vehicle / Distance Time windows ordering	Vehicle / Distance
1	5 / 2668.55	14 / 1105.19	5/2660.50
2	5 / 2555.26	12 / 987.27	5/2699.64
3	5 / 2526.11	11 / 1038.27	5/2645.46
4	5 / 1953.67	10 / 1091.51	5/2071.06
5	5 / 2647.03	17 / 1129.44	5/2745.34
6	5 / 2394.25	14 / 1128.91	5/3211.54
7	5 / 2187.27	11 / 997.21	5/2604.00
8	5 / 1804.70	9 / 903.80	5/1962.85
9	5 / 2351.13	13 / 1073.35	5/3102.49
10	5 / 3063.39	13 / 1058.32	5/3033.20
11	5 / 2076.89	13 / 1005.83	5/1573.34

One advantage of interpreting the VRTWLP as a MOP using Pareto ranking as opposed to using weighted sum is that we have two or more solutions provided to the decision maker. In every instance of the four different problems, the MOGA never achieves a solution which is better both in vehicle number and total distance. For example, in Tab. 2, instance 7, the Integrated GRASP approach is better than the MOGA in vehicle number, but the total distance achieved by MOGA outperforms the total distance achieved with the other two approaches. Nevertheless some results obtained with the MOGA could be considered good results because the difference in the number of vehicles is considerable, for example Tab. 1 instance 6.

Tab. 4. Group II Class I2 problem instances.

GII/I2	Integrated GRASP Heuristic	Hierarchical Approach	MOGA
Instance	Vehicle / Distance Time windows ordering	Vehicle / Distance Time windows ordering	Vehicle / Distance
1	7 / 3740.55	17 / 1168.68	7/3816.58
2	7 / 3496.39	15 / 1069.66	8/4001.70
3	7 / 3134.62	10 / 923.09	7/3409.78
4	6 / 3814.29	12 / 1051.02	7/3442.50
5	7 / 3180.35	19 / 1204.52	7/3405.67
6	7 / 3115.18	16 / 1133.04	7/3615.44
7	7 / 2740.03	14 / 1070.33	7/3368.93
8	7 / 2330.75	11 / 953.54	7/2078.06
9	7 / 3076.78	18 / 1189.13	8/3687.12
10	7 / 4081.19	16 / 1108.39	7/4076.12
11	6 / 2631.39	15 / 1055.11	7/2662.07

5 Conclusions

This work presents a Multi-objective Genetic Algorithm approach to the Vehicle Routing with Time Windows and Loading Problem. This problem is an integration of the two well known problems VRPTW and CLP. The solutions obtained with the MOGA are competitive when compared with the other two approaches developed to this problem. However, the constructive algorithm used to generate the initial population is based on an already published algorithm developed by the author and named Integrated GRASP Heuristic, where the two problems (VRPTW and CLP) are dealt at the same level. Due to this fact, the total distances achieved with MOGA approach are very similar to the distances achieved with the Integrated GRASP Heuristic. In the Hierarchical approach, the VRPTW is the main problem and the CLP is the subsidiary problem. For this reason the Hierarchical approach has always presented much better results for total distances.

With MOGA approach, in some instances a reduction of the number of vehicles was achieved. Comparing the total distance achieved by the MOGA approach with the Integrated GRASP heuristic in some instances, the solution was improved. However, with the Hierarchical approach the difference in the number of vehicles is significant.

The choice of the preferable solution must be made by the decision-maker because with these three approaches we have a range of possible feasible solutions. Thus, it is not adequate to state that one particular algorithm provides better results than one other, or even claim one solution outperforms the others.

Admitting that there is no advantage in giving priority to a given objective function component, because from a theoretical point of view neither is more important than the other, we can conclude that the most significant contribution of this paper is the interpretation of VRTWLP as a MOP.

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