# Structural optimisation problem in support to building retrofitting decision

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ABSTRACT: Various analysis methods, either linear elastic or non-linear, static or dynamic, are available for the performance analysis of existing buildings. Despite its advantages, it must be admitted that non-linear time history analysis can frequently become overly complex and impractical for general use as a first assessment. Simplified models, as the Capacity Spectrum Method, are frequently not able to accurately assess irregular structures. Considering these limitations, it is proposed and evaluated a simplified MDOF non-linear dynamic model, accounting for non-linear storey behaviour and storey damping. Based on the MDOF non-linear dynamic model, were developed optimization algorithms for the redesign of existing non-seismically designed structures. The optimization procedure searches for the optimum storey strengthening distribution (strength, stiffness or damping) in order to meet specific performance requirements, in terms of maximum inter-storey drift for a given seismic demand level. Numerical examples are presented in order to illustrate the capability of methodology.

Keywords: optimum strengthening, performance based design, existing RC buildings

## 1 INTRODUCTION

In Europe, many structures are potentially seismically vulnerable due to the late introduction of seismic loading into building codes. Therefore, there is an evident need to investigate the seismic behaviour of existing reinforced concrete (RC) buildings, in order to assess their seismic vulnerability and ultimately to design optimum retrofitting solutions.

Various analysis methods, either linear elastic or non-linear, static or dynamic, are available for the performance analysis of existing buildings. Despite its advantages, it must be admitted that non-linear time history analysis can frequently become overly complex and impractical for general use as a first assessment. Simplified models, as the Capacity Spectrum Method, are frequently not able to accurately assess irregular structures.

Based on a MDOF non-linear dynamic model, were developed optimization algorithms for the redesign of existing non-seismically designed structures. The procedure searches for the optimum storey strengthening distribution (strength, stiffness or damping) in order to meet specific performance requirements, in terms of maximum inter-storey drift for a given seismic demand level. A four-storey full-scale building was tested pseudo-dynamically at the ELSA laboratory, at the Joint Research Centre, in Italy. Numerical strengthening design examples, based on the tested structure, are presented in order to illustrate the capability of the methodology.

## 2 DESCRIPTION OF THE BUILDING FRAME MODEL, MATERIALS, VERTICAL LOADS AND SEISMIC INPUT MOTION

Figure 1 shows the general layout of the building frame model. It is a reinforced concrete 4-storey full-scale frame with three bays, two of 5 m span and one of 2.5 m span. The inter-storey height is 2.7 m and a 0.15 m thick slab of 2 m on each side is cast together with the beams (Fig. 2). Equal beams (geometry and reinforcement) were considered at all floors. The columns, all but the wider interior one, have equal geometric characteristics along the height of the structure. A comprehensive description of the frames, tests on material samples used in the construction (steel reinforcement and concrete) and PsD test results can be found in Pinto et al. (1999).

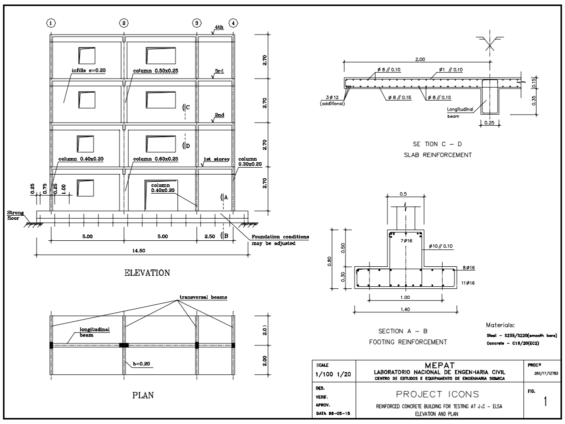


Figure 1. Plan and elevation views for the frame

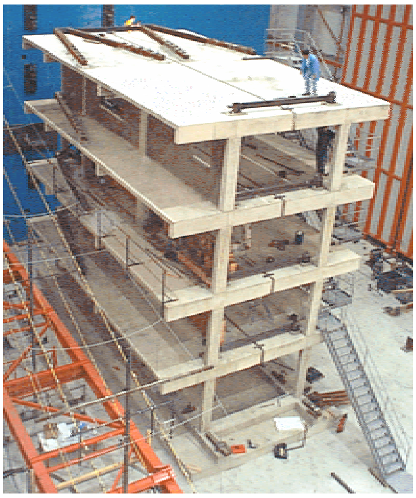

Figure 2. Models in the ELSA laboratory

The materials considered at the design phase (Carvalho et al. 1999) were a low strength concrete, class C16/20 (Eurocode 2) and smooth reinforcing steel (round smooth bars) of class FeB22k (Italian standards). Vertical distributed loads on beams and concentrated loads on the column nodes were considered in order to simulate the dead load other than the self-weight of the frame. These correspond to the following vertical loads: weight of slab 25 × 0.15 = 3.75 kN/m$^2$, weight of finishings 0.75 kN/m$^2$, weight of transverse beams 2.5 kN/m, weight of masonry infills 1.1 kN/m$^2$ of wall area, and live load 1.0 kN/m$^2$ (quasi-permanent value).

The input seismic motions were defined in order to be representative of a moderate-high European seismic hazard scenario Campos-Costa & Pinto (1999). Hazard consistent acceleration time series (15 seconds duration) were artificially generated yielding a set of uniform hazard response spectra for increasing return periods. Acceleration time histories for 475, 975 and 2000 years return periods (yrp) were used in the tests (PGA of 218, 288 and 373 cm/s$^2$, respectively).

## 3 EQUIVALENT DAMPING

To perform a structural assessment, it is essential to define accurately the damping as a function of the deformation demand. In the literature, there are some proposals of damping functions for new buildings, but not for existing structures. In this study, it was estimated a damping function for the tested frame, representative of existing RC structures (Varum 2003).

The structural equivalent viscous damping was calculated. Firstly, at storey level from the curves inter-storey drift versus storey shear. Subsequently, the equivalent damping of the global structure was computed as a function of the damping at storey level, weighted by the storey potential energy. The best-fit logarithmic curve obtained, in terms of storey equivalent damping, as a function of the maximum inter-storey drift, is plotted in Figure 3.

Even for considerable deformation levels, for existing structures, a low value of damping was estimated (maximum value less than 11%), which confirms that existing structures, with reinforcing plain bars, have a small energy dissipation capacity.

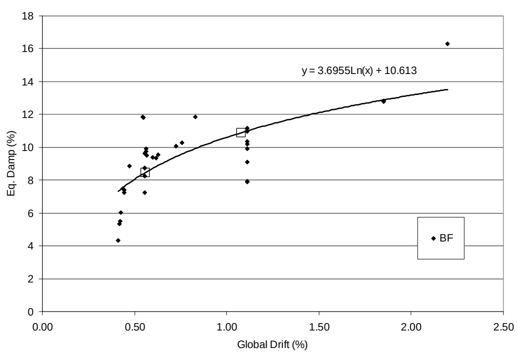


Figure 3. Equivalent global damping versus global drift for the existing structure

## 4 IMPROVED MDOF NON-LINEAR DYNAMIC MODEL FOR STRUCTURAL ASSESSMENT

### 4.1 *Generalities*

Simplified non-linear static models considering just one DOF (such the Capacity Spectrum Method) are frequently not able to assess accurately irregular structural systems.

As remarked by Peter & Badoux (2000), the seismic evaluation of buildings requires the prediction of the seismic performance, and, in consequence, the prediction of the inelastic deformations of the RC structures.

Despite the advantages of a refined non-linear dynamic structural analysis, as fibre modelling, it must be admitted that this approach can frequently become elaborated and costly. This fact sustains the development of less complicated structural models without debasing the essential features of dynamic response.

### 4.2 *Model description*

The procedure is based on a generalization of the substitute-structure method. Linear spectral analysis and multi-modal response methods with quadratic combination are applied. The proposed model is applied to simulate the results of a series of PsD tests on full-scale regular and irregular structures. The results obtained are in good agreement with experimental ones, even for the irregular structure. This non-linear displacement-based model can be an efficient numerical tool for seismic vulnerability assessment, which could allow for parametric studies and rapid screening of existing building classes.

A simplified non-linear MDOF dynamic procedure, for structural assessment is here proposed and evaluated. The model accounts for two levels of non-linearities, namely: a) storey behaviour in terms of shear-drift; and, b) damping as a function of deformation. The procedure assumes that a non-linear MDOF system can be represented by an equivalent linearized system with element stiffness given the secant stiffness. Consequently, linear spectral analysis can be used and multi-modal response methods can be applied. The procedure is based on a generalization of the substitute-structure method, proposed by Shibata & Sozen (1976), which states that the response of a non-linear SDOF system can be accurately approximated by the response of an equivalent linear system with an equivalent period corresponding to the secant stiffness.

The non-linear damping relationships can be modelled in two different ways, namely: a) variable (with damping functions defined for different structural components, e.g. for each DOF, storey); and, b) modal (global structural level). It was included the possibility of participation of several natural modes (multi-mode) for the structural response, with their quadratic combination.

The building structure is idealised as a bi-dimensional (2D) cantilever model (shear building), with a number of horizontal translational DOF's equal to the number of storeys. The structural model is fixed at the base, as represented in Figure 4, and the rotation of each node is fixed against rotation. The shear force-displacement relationship of each beam-element represents the curves storey shear versus inter-storey drift.

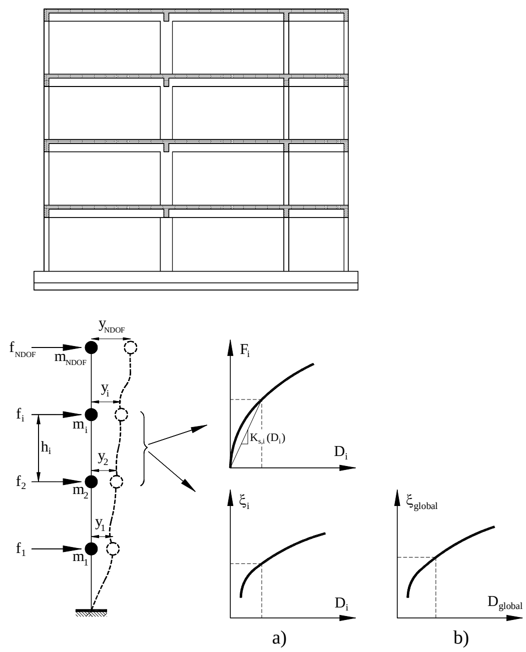


Figure 4. MDOF simplified model with concentrated masses at storey levels being connected by shear beam elements: a) damping defined for each storey, b) global first mode damping

In this model, represented schematically in Figure 4, the mass distribution of the building is defined for each floor level accounting for the mid-height storey masses and lumped at floor level (equivalent total storey masses). Therefore, the *i*-th storey mass ($m_i$) concentrates the total storey mass at node (storey) *i*, and these nodes are connected by shear-beam elements. The storey damping is labelled $\xi_i$. The force vector $\{F\}$ is expressed in terms of the shear forces acting on the beam elements (storey shear), and the relative inter-node displacement vector $\{D\}$ is expressed in terms of lateral deformation of the beam element (inter-storey drift). The storey shear force ($F_i$) acting on a beam element and the inter-storey drift ($D_i$) are related by the non-linear $F_i$-$D_i$ curve.

In the iterative step-by-step procedure, for each step, the calculations are made with constant secant stiffness and damping at the storey levels.

The required mechanical non-linear relationships can be obtained and calibrated from one or more of the following: a) experimental tests on structural specimens; b) simplified empirical expressions; and c) analytical calculations from a detailed structural model (pushover numerical analysis).

The proposed simplified MDOF non-linear dynamic method for assessment of multi-storey building structures calls for a relatively small number of DOF's (one per floor), compared to a detailed FE model. Evident advantages come out, for example, fast parametric studies with a good level of confidence can be carried out with the model. A practical application of this simplified method is made in the next section. The method is described in the next section.

### 4.3 *Description of the implemented algorithm*

The basic steps of the proposed MDOF non-linear dynamic assessment iterative step-by-step procedure with two levels of non-linearities are:

*1st step*: data, initial model and demand parameters (structure geometry; non-linear storey shear-drift constitutive behaviour laws and damping curves at storey, or global level; storey masses; elastic seismic demand).

*2nd step*: starting point (number of fundamental modes to be considered in the structural response; select the initial values for the storey secant stiffness and for the storey or global damping coefficient, on the basis of the constitutive relations).

*3rd step*: determine the seismic response (compute and assemble the stiffness matrix and the diagonal mass matrix of the MDOF system; compute the structural natural periods and modal shapes; compute the structural effective damping; compute the reduced elastic seismic response spectra according to EC8, with the damping correction factor; determine the structural response from the modal analysis with quadratic combination; determine storey shear forces and the inter-storey drifts).

*4th step*: check for convergence at two levels (storey shear-drift and damping). If convergence is not satisfied, prepare new values for the next iteration point (for the secant stiffness and/or damping, on the basis of the constitutive relations and deformation demand), and return to step 3.

*5th step*: graphical output of the converged response (graphical representation of the storey shear-drift response point, inter-storey drift profile, and damping).

### 4.4 *Verification of the model with earthquake test results*

In order to calibrate and verify the method in predicting global parameters (such as top-displacement, maximum inter-storey drift, maximum storey shear, and equivalent damping), the proposed MDOF non-linear dynamic seismic analysis methodology (described in Sections 4.2 and 4.3) is applied to simulate the PsD tests performed on the bare and strengthened structures.

The structure was analysed for input motions corresponding to the maximum accelerations of the earthquakes considered in the tests, namely 218 and 288 cm/s$^2$ for the BF (corresponding to 475 and 975-yrp). The description of the structure can be found in Section 2.

For the structure under analysis, four DOF are considered, being the storey masses considered for the first three storeys ($m_1$, $m_2$ and $m_3$) 44.6 ton, and for the fourth storey ($m_4$) 40.0 ton. The envelope storey shear-drift behaviour curves, obtained from the PsD earthquake tests for the structure, were here adopted as capacity curves. The storey shear-drift envelope curves of the PsD test are in good agreement with the storey behaviour curves obtained with the pushover analysis. In these numerical analyses, it was considered the structural damping at storey level (Fig. 3).

The inter-storey drift profile obtained from the numerical analyses performed with the proposed simplified MDOF non-linear dynamic method is plotted in Figure 5, for the BF structure. In this figure, it is also plotted, for comparison, the maximum inter-storey drift profile observed in the corresponding PsD test.

The structural response was estimated considering the participation of one and four natural modes of the equivalent linear system, in order to analyse the influence of the number of natural modes in the global response. These two situations is also represented in the graphic.

A good estimation of the maximum response was achieved, with the simplified non-linear dynamic model, considering a small number of DOF (4 versus 372 DOF's for the refined 2D FE model). Therefore, this displacement-based methodology can be an effective numerical tool to perform fast non-linear analyses, which could allow for parametric studies and rapid screening (seismic vulnerability assessment) of existing building classes.

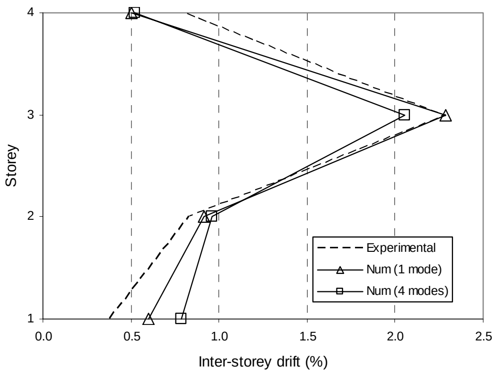


Figure 5. Inter-storey drift profile computed and PsD test results for the BF structure and for the earthquake input action 975-yrp

## 5 STRUCTURAL OPTIMIZATION PROBLEMS

### 5.1 *Introduction*

These optimization algorithms deal with non-linear objective functions and allow to impose constrains on the design variables (strength, stiffness or damping) and on any other response variable depending on the design variables, such as inter-storey drift, top-displacement, etc.

The optimization procedure can be a useful design tool, as a preliminary step, in the global structural strengthening decision, for one or multiple performance objectives.

Structural optimization problems consist on determining the configurations of structures that obey assigned constraints, and produce an extremum for a chosen objective function. In order to solve them, they are normally transformed into a mathematical form that can be solved by general optimization tools. Since structural optimization problems are characterized by computationally expensive function evaluations, it is common to generate a sequence of convex, separable sub-problems, which are then solved iteratively (Chickermane & Gea 1996).

It is therefore judged appropriate to have a methodology that can address the strengthening design of MDOF structural systems, generating optimal distribution (location) of the strengthening in the structure components (at storey level).

In this study, three methodologies for optimum redesign of existing structures are proposed and programmed. The optimization algorithms are based on the convex approximation methods, such as the Convex Linearization Method developed by Fleury (1989, 1979) and Braibant (1985), and the Method of Moving Asymptotes. These optimization algorithms can deal with non-linear objective functions (minimum cost of intervention) and allows to impose constrains on the design variables (strength, stiffness or damping) and on any other response variable depending on the design variables, such as inter-storey drift, top-displacement, etc.

The optimization procedure requires several structural response evaluations, namely of the objective function, of constraints, and of their derivatives. The calculation of the structural response is required many times during the optimization process, which would be unfeasible with a refined FE model. The simplified model allows for spectral analysis, which constitutes a great advantage over the multi-series analyses. The model is able to estimate the response of irregular structures those we address with the optimization of the retrofit. Therefore, the simplified MDOF dynamic method, presented in Section 4, was incorporated in the redesign optimization algorithms here proposed.

In these three structural optimization problems, the design variables, or control variables, are defined at storey level, and they are:

- The additional strength (Problem I);
- The additional pre-yielding stiffness (Problem II);
- The yielding strength of the energy dissipation device (Problem III).

In the next are revised the theoretical concepts related with the optimization problem. One of the implemented optimization problems is explained. Strengthening design examples based on the structure under analysis are used to illustrate the capability of the proposed methodology.

### 5.2 *Structural strengthening optimization problems' formulation*

For the optimization problems here proposed, it is assumed that the behaviour of a multi-storey RC existing building (non-seismically designed) subjected to a certain earthquake action level can be represented by the multi-modal model proposed in Section 3. Buildings are modelled with one DOF per storey, linked by beam elements that represent the storey behaviour. The beam elements have an equivalent secant stiffness corresponding to the maximum deformation point in the non-linear storey constitutive curve. Furthermore, response spectra modal analysis with concentrated and/or distributed damping is used to compute the seismic response for each step of the optimization procedure.

The optimization procedure requires previous identification of simplified (bilinear) storey shear-drift constitutive relations made on the basis of pushover analysis, as represented in Figure 6.

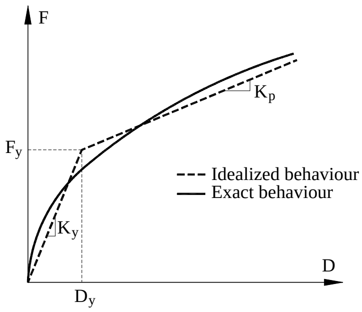


Figure 6. Lateral storey shear versus inter-storey drift behaviour (exact and idealized bilinear behaviour)

A seismic performance objective is formed by combining a desired building performance level (a damage limit-state) with a given earthquake ground motion (level of hazard). The objective of this analysis is to find the optimum retrofitting solution in order to comply with a certain seismic demand-level defined for each limit-state.

The optimization problem, in generic terms, is to minimise the total strengthening requirements in the structure, whilst satisfying the upper limits for the inter-storey drifts and strengthening at each storey.

The objective function for each problem is the sum of the control variables (additional strengthening costs) at each storey level. The inequality constraints are upper inter-storey drift limits (to restrain the damage at storey level) and upper storey strengthening limits (to restrict the strengthening within acceptable values).

Three design optimization structural strengthening problems were established in this work. They were conceptually based on the strengthening strategies commonly used in practice, which call for the control variables: the strength (controlled by the yielding shear force, $\Delta F_y$), the pre-yielding stiffness ($\Delta K_y$), and the yielding strength of the energy dissipator devices ($F_y^{dev}$), as will be explained in the subsequent sections.

## 5.3 *Problem I: storey yielding strength*

Problem I (control variables: strength, $\Delta F_y$, Fig. 7) can be described in the following mathematical form:

Find

$$\Delta F_y = \left\{\Delta F_{y,1};\Delta F_{y,2};...;\Delta F_{y,NDOF}\right\} \quad (1)$$

Minimise

$$Cost\left(\Delta F_y\right) = \sum_{i=1}^{NDOF} \Delta F_{y,i} = \sum_{i=1}^{NDOF} \left(F_{y,i} - F_{y,i}^{0}\right) \quad (2)$$

Subject to

$$\begin{cases} \Delta F_{y,i}^{\min} \le \Delta F_{y,i} \le \Delta F_{y,i}^{\max} & i=1,2,...,NDOF \\ D_i \le D_i^{\max} & i=1,2,...,NDOF \end{cases} \quad (3)$$

in which: $F^0_{y,i}$, $\Delta F_{y,i}$ and $F_{y,i}$ are the initial, incremental and total yielding strength of the storey $i$, respectively. NDOF represents the number of degrees of freedom of the problem, i.e. number of storeys. $\Delta F^{min}_{y,i}$ and $\Delta F^{max}_{y,i}$ are the lower and upper bound limits for each control variable ($\Delta F_{y,i}$). $D_i$ is the inter-storey drift at storey-level $i$. $D_i^{max}$ is the maximum admissible inter-storey drift for each storey-level.

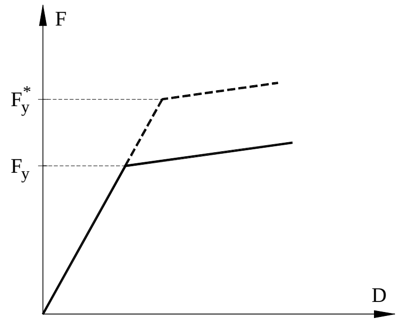


Figure 7. Control variable: strength (yielding shear force - $F_y$)

## 5.4 *Implementation of the optimization problems*

The optimization problems were implemented in the finite element code CASTEM (Millard 1993). The philosophy of CASTEM is modular. Therefore, the optimization methodology was implemented in separate modules for pre-processing, structural analyses, optimization, pos-processing and graphical results visualisation. The basic steps of the iterative optimization process implemented can be summarised as follows:

*1st step*: select the design control variables, i.e. strengthening intervention strategy (strength, stiffness, or damping).

*2nd step*: define the structure geometry (NDOF storeys and inter-storey heights, $h_i$), the original bilinear storey shear-drift behaviour curves, $F_i(D_i)$, and variable damping curves at storey, $\xi_i(D_i)$, or global level, $\xi_{Global}(D_{Global})$. Set storey masses, $m_i$.

*3rd step*: define design performance objective (design seismic demand, $S_a(T, \xi_0)$, and inter-storey drift limit, $D_i^{max}$, for each storey-level), based on commonly accepted values for exceedance probabilities, as for example the given by ATC-40 (1996) or VISON-2000 (SEAOC 1995).

*4th step*: choose a starting point $\{x^0\}$ and let the iteration index $k = 0$.

*5th step*: given an iteration point $\{x^k\}$ calculate the first order derivatives of the objective and constraint functions with respect to the design variables.

*6th step*: generate the approximated sub-problems using one of the approximation methods available in CASTEM. Then the convex optimization problem is formulated and solved iteratively.

*7th step*: get optimum design variables for the limit-state considered: $\Delta F_y$ for additional yielding strength, $\Delta K_y$ for additional yielding stiffness, or $F_y^{dev}$ for K-bracing with dissipator device intervention.

*8th step*: with the obtained optimal design point $\{x^*\}$, the convergence is verified. If the solution does not converge, this solution is used as the next iteration point. The iteration index $k$ is increased by one and the iterative process continues (go to step 5).

*9th step*: with converged solution (which minimizes the strengthening costs for a specific limit-state requirement), graphical output of the solution is prepared.

For the numerical optimization problem, the value of the objective function and of the restrictions as well as the respective first order derivatives at the starting point $\{x^0\}$ are computed and given to start the optimization algorithm. At any design point, the implemented algorithm calculates the first order derivative information to formulate the approximation. The points chosen to calculate the derivatives in the vicinity of the current design point have to give a good approximation of the functions in the vicinity of the design point.

## 5.5 *Illustrative example: existing structure*

A numerical example is herein presented in order to illustrate the proposed optimal retrofit design methodology.

From the experimental tests performed on the original bare frame, it were calculated the envelope curves of storey shear versus inter-storey drift and approximate for the best-fit idealized bi-linear curves. The original storey shear-drift curves were approximate for the idealized bi-linear curves, maintaining the dissipated energy and the maximum shear load. The adopted storey shear-drift curves are plotted in Figure 8 and are used in the optimization analyses.

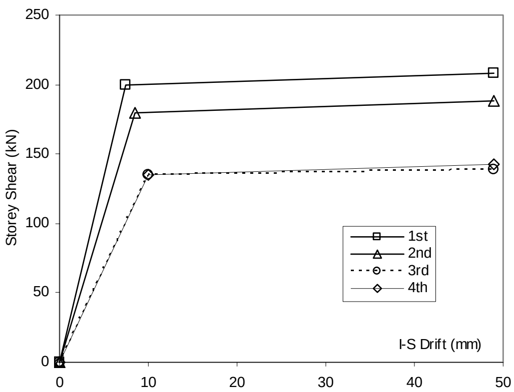


Figure 8. Storey shear-drift curves adopted from the experimental tests

## 5.6 *Optimum redesign of the existing structure*

The example of an optimization problem presented in this section assumes as control variables the additional storey strength. The structure under analysis is the four-storey RC building non-seismically designed. The objective function to be minimised is the total structural additional strength, i.e. the sum of the storeys additional strength. It is intended to find the optimal distribution of strengthening in the building, whilst satisfying the restrictions in terms of maximum storey strengthening and maximum allowable inter-storey drift. The problem can be mathematically described as in the follows

Find

$$\Delta F_y = \left\{\Delta F_{y,1};\Delta F_{y,2};\Delta F_{y,3};\Delta F_{y,4}\right\} \quad (4)$$

Minimise

$$Cost\left(\Delta F_y\right) = \sum_{i=1}^{4} \Delta F_{y,i} \quad (5)$$

Subject to

$$\begin{cases} 0 \le \Delta F_{y,i} \le 500\,kN & i=1,2,3,4 \\ D_i \le 3cm & i=1,2,3,4 \end{cases} \quad (6)$$

The start design point for each storey consisted on storey strength 1.4 times higher than the initial yielding strength in the existing structure.

The constraint conditions for this structural optimization problem are: a) maximum admissible drift of 3.0 cm (1.1%), for every storey; and, b) upper limit of 500 kN for each storey additional strength, that do not restraint the solution, and minimum zero (not additional strength).

The pre-yielding stiffness is assumed to be constant, i.e. the strengthened storey has higher strength, but the same pre-yielding stiffness. The pos-yielding stiffness is assumed constant.

The optimization problem converges after 12 iterations. In Figure 9 are represented the storey strength profiles of the original structure and of the optimum strengthening, to accomplish with a performance objective corresponding to an earthquake of 975-yrp and a drift limit of 3.0 cm.

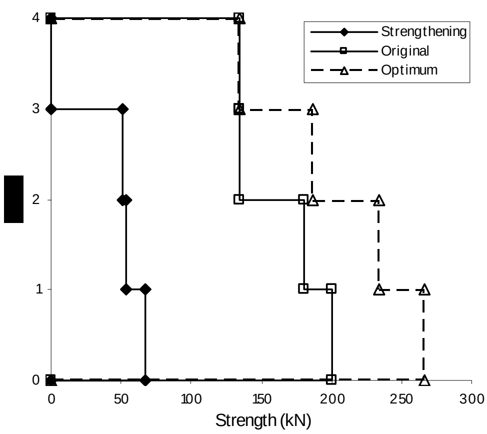


Figure 9. Storey yielding strength of the existing structure and optimum strengthening distribution

## 5.7 *Multiple optimum strengthening design*

To illustrate the methodology proposed, a series of retrofitting design examples based on the non-seismic designed existing structure under analysis are presented here. The strength is the control variable used (Problem I). In this problem it was considered constant yielding displacement and constant pos-yielding stiffness. Regarding the damping, the curve storey damping-drift, presented in Figure 3 was used. Additionally, it was considered that the storey damping functions do not change with the strengthening.

The optimal retrofit was calculated for a vast series of performance objectives (multiple performance objectives). Particularly, for the input motion, it were considered the seismic hazard levels corresponding to return periods of 73, 475, 975 and 2500 years (corresponding to the 'Serviceability Earthquake', SE, 'Design Earthquake', DE, 'Maximum Earthquake', ME, and, 'Maximum Considered Earthquake', MCE, as recommended in the ATC-40 (1996). For the inter-storey drift limit (limit-states) it were considered several values. In this analysis, no upper limits were imposed for the strengthening.

In Figure 10 are represented for each redesign performance objective the results in terms of total strengthening. For all redesign performance objectives, the obtained optimum strengthening distribution does not involve strengthening at the 4th storey level.

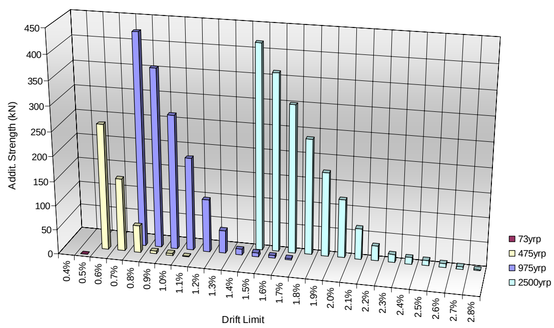


Figure 10. Total additional strength

## 6 FINAL REMARKS

The proposed optimization methodology involves reduced computational costs. As observed in the studied strengthening structural optimization problems, this methodology leads to fast convergence.

The methodology can be a useful design tool, as a preliminary step, in the global structural strengthening decision, generating the optimum strengthening (strength, stiffness or damping) storey distribution, for one or multiple performance objectives.

With this optimization procedure, it is possible to define optimum strengthening needs for different limit-states ('Fully Operational', 'Operational', 'Life Safe' and 'Collapse Prevention') as well as to achieve probabilistic sensitivity functions for specific limit-states, in terms of the basic design variables (storey strength, storey stiffness or additional damping). This procedure leads to a retrofit design, for each limit-state considered, which requires further considerations and possibly recourse to life-cycle cost analyses to identify the optimum design (see for example, the procedure proposed by Pinto 1998).

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