

A MECHANICAL METHOD FOR THE VULNERABILITY ASSESSMENT OF MASONRY BUILDINGS

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ABSTRACT :

This paper discusses a mechanical model for the vulnerability assessment of masonry buildings that takes into account the uncertainties inherent to the structural parameters and the limit states. At first a bilinear model for the capacity spectrum for masonry buildings is derived as an analytical function of a few number of geometrical and mechanical parameters. Applying a suitable procedure for the uncertainty propagation, the statistical moments of the structural capacity is obtained as a function of the statistical moments of the input parameters, showing the role of each in the results. Using the capacity spectrum method formulated in the so called N2 procedure, vulnerability analysis is carried out with respect to a certain number of random limit states which depend, in turn, on the building parameters. Fragility curves are derived taking into account the uncertainties of each quantity involved.

KEYWORDS: Masonry buildings, seismic vulnerability, capacity spectrum, uncertainties

1. INTRODUCTION

Mechanical methods based on inelastic static analysis are gaining a central role for the vulnerability assessment of buildings. Among these, the capacity spectrum method (Fajfar, 1999) is becoming very popular for its ready-to-use format. In engineering practice, seismic verifications usually assign structural parameters and limit states according to deterministic quantities, making use of all-inclusive coefficients which take into account all the uncertainties involved. Starting from a non linear model of the mechanical behaviour of masonry buildings under seismic excitation (Cattari *et al.*, 2004), in the first part of the paper an analytical formulation of the capacity spectrum is derived which leaves free the geometrical and mechanical parameters. Applying suitable procedures for the uncertainty propagation, a statistical description of the model is given as a function of the statistical moments of the input parameters. The second part of the paper carries out vulnerability analysis in the acceleration displacement domain with respect to each random limit states defined. Fragility curves are therefore derived taking into account the uncertainties inherent to each quantity involved (which are different when considering a single building or aggregates).

2. MECHANICAL MODEL

Consider a masonry building of height H . It is represented by a “stick model” (Shibata and Sozen, 1976) with vertical axis coincident with axis ZZ ; XX is coincident with the longitudinal dimension, parallel with main plan dimension, YY is coincident with the transversal direction. It is described by N nodes characterized by the lumped mass m_i at level $z_i=i \times h$, being h the inter-storey height. It has $N-1$ elements characterized by the area A_i and the inertia moment J_i of the resistant walls in the direction considered. Let be Ψ the vector which lists the N components ψ_i of the fundamental mode shape, here assumed as shear type. The vibration period for the equivalent single degree of freedom system ($SDOF$) is given by:

$$T = 2\pi \times \sqrt{M^*/K^*} \quad \text{with} \quad M^* = \sum_{i=1}^N m_i \times \psi_i^2; \quad K^* = h \sum_{i=1}^N GA_i \times \psi_i^2 \quad (2.1)$$

where M^* and K^* are the generalized modal mass and stiffness for the equivalent $SDOF$ system. The yielding acceleration capacity (strength) is given by:

$$A_y = \frac{F_y}{m^* \times \Gamma} \quad (2.2)$$

where F_y is the yielding lateral force of the building, m^* is the equivalent mass, Γ is the transformation factor to the equivalent $SDOF$ system (Vidic et al, 1994):

$$m^* = \sum_{i=1}^N m_i \times \psi_i; \quad \Gamma = \frac{m^*}{\sum_{i=1}^N m_i \times \psi_i^2} \quad (2.3)$$

The displacement associated to the yield capacity, D_y , is related to A_y and T by the relationship:

$$D_y = \frac{A_y}{(2\pi)^2} \times T \quad (2.4)$$

For masonry buildings, F_y is basically related to the sole base shear capacity of the walls at the first floor level:

$$F_y = \zeta \times A_f \times \tau_u; \quad \tau_u = \tau_k \times \sqrt{1 + \frac{\sigma_o}{1.5 \times \tau_k}} \quad (2.5)$$

ζ is a coefficient which takes into account the non-uniform response of the masonry panels, considered initially as a shear mechanism, τ_u is the ultimate shear strength of the masonry (Turnšek and Čačovič, 1971), where τ_k is the characteristic shear strength and σ_o is the compression stress installed at ground floor level:

$$\sigma_o = \frac{\sum_{i=1}^N m_i \times g}{A_f} \quad \text{with} \quad g = 9.81 \text{ m/s}^2 \quad (2.6)$$

The resistant wall area $A_{dir,i}$ for the direction considered ($dir=XX$ or YY) and the floor area A_p (considered constant) are expressed as a function of the resistant area at the top floor level $A_{dir,N}$, in the same direction by:

$$A_{dir,i} = \beta_{dir,i} \times A_{dir,N}; \quad A_{dir,N} = \alpha_{dir} \times A_p \quad (2.7)$$

where $\beta_{dir,i}$, α_{dir} are suitable dimensionless coefficients. Therefore the coefficient $\beta'_{dir,i}$ can be defined by:

$$\beta'_{dir,i} = \frac{1}{2} (\beta_{dir,i} + \beta_{dir,i+1}), \quad i=1, \dots, N-1; \quad \beta'_{dir,i} = \frac{1}{2} \beta_{dir,i}, \quad i=N \quad (2.8)$$

The distribution of wall resistant area along each floor can be schematized according to two different conditions. The first condition applies when the resistant wall area decreases linearly with the increasing of the

building height. The second condition applies when the resistant walls are characterized by large openings at the first level. This is a typical feature of the façade walls along the longitudinal direction of historical buildings; in this case a bilinear distribution should be used.

It is to be noted that, like the wall resistant area $A_{dir,i}$, also $\beta_{dir,i}$, $\beta'_{dir,i}$ depend on the direction. In the following, XX or YY are used to specifically refer to X - Y orthogonal directions. Assuming A_{xi} , A_{yi} the resistant wall area in each specific direction, γ is the specific weight of the masonry, q is the floor loading (permanent and live load). Then the i -th mass related to i level can be expressed by:

$$m_i = (A_{xi} + A_{yi}) \times \gamma \times h + A_p \times q \quad (2.9)$$

The wall loading at ground floor level can therefore be expressed by:

$$\sigma_0 = g \times \gamma \times h \times \frac{\sum_{i=1}^N \beta_{dir,i}}{\beta_{dir,1}} + \frac{N \times q \times g}{\alpha_{dir} \times \beta_{dir,1}} \times \delta_{dir} \quad (2.10)$$

being δ_{dir} a boolean type coefficient, depending on the load path of the floors onto the masonry walls. The ultimate displacement capacity, D_u , can be calculated in function of parameters discussed above. For a uniform collapse or a soft-storey mechanism, the ultimate displacement are respectively given by (Cattari *et al.*, 2004):

$$D_u = \delta_u \times \frac{N \times h}{\Gamma} \quad ; \quad D_u = \delta_u \times h + D_y \times \left(1 - \frac{\Gamma}{N}\right) \quad (2.11)$$

δ_u is the ultimate drift of a masonry panel (varying from 0.004 to 0.01). When the structure responds according to a linear shape mode, then $\psi_1 = 1/N$. Using the definitions in Eqn. 2.7 to Eqn. 2.9 into Eqn. 2.1, the fundamental period of vibration T_{dir} is:

$$T_{dir} = 2\pi \times \sqrt{\frac{h}{G \times \alpha_{dir} \times \sum_{i=1}^N \beta_{dir,i}} \times \left[\gamma \times h \left(\alpha_x \times \sum_{i=1}^N \beta'_{xi} \times i^2 + \alpha_y \times \sum_{i=1}^N \beta'_{yi} \times i^2 \right) + q \times \sum_{i=1}^N i^2 \right]} \quad (2.12)$$

Substituting in Eqn. 2.2 the Eqn. 2.3 to Eqn. 2.5, the yielding acceleration capacity $A_{y,dir}$, can be expressed by:

$$A_{y,dir} = \frac{\beta_{dir,1} \times \zeta \times \tau_k \times \sqrt{1 + \frac{g}{1.5 \times \tau_k \times \beta_{dir,1}} \times \left(\gamma \times h \times \sum_{i=1}^N \beta_{dir,i} + \frac{N \times q \times \delta_{dir}}{\alpha_{dir}} \right)}}{\kappa} \quad (2.13)$$

with:

$$\kappa = \frac{1}{\alpha_y} \times \frac{\left[\gamma \times h \times \left(\alpha_x \times \sum_{i=1}^N \beta'_{xi} \times i + \alpha_y \times \sum_{i=1}^N \beta'_{yi} \times i \right) + q \times \sum_{i=1}^N i \right]^2}{\gamma \times h \times \left(\alpha_x \times \sum_{i=1}^N \beta'_{xi} \times i^2 + \alpha_y \times \sum_{i=1}^N \beta'_{yi} \times i^2 \right) + q \times \sum_{i=1}^N i^2} \quad (2.14)$$

The displacement D_u is given by Eqn. 2.11 where Γ is obtained by Eqn. 2.3 and 2.9:

$$\Gamma = N \times \frac{\gamma \times h \times \left(\alpha_x \sum_{i=1}^N \beta'_{xi} \times i + \alpha_y \sum_{i=1}^N \beta'_{yi} \times i \right) + q \times \sum_{i=1}^N i}{\gamma \times h \times \left(\alpha_x \sum_{i=1}^N \beta'_{xi} \times i^2 + \alpha_y \sum_{i=1}^N \beta'_{yi} \times i^2 \right) + q \times \sum_{i=1}^N i^2} \quad (2.15)$$

For a soft-storey mode, then $\psi_i=1$. Simplified capacity curves can be derived in a similar way.

3. MODEL UNCERTAINTIES

The mechanical and geometrical parameters on which the capacity spectrum depends constitute random variables. When dealing with single building assessment, uncertainties are mainly due to the lack of expertise knowledge of the building features. In this case, an accurate survey can reduce the scattering in the model parameters. When dealing with vulnerability assessment at large scale, *i.e.* buildings grouped in respect to their structural typology or in the case of urban aggregates, all parameters have a strong randomness. Since the calculation model too is affected by errors, the capacity spectrum is a inherent random function of random parameters.

Consider $\mathbf{P}=\{\alpha_x, \beta_{x,i}, \alpha_y, \beta_{y,i}, h, \gamma, \tau_k, G, \xi, q\}$; it lists the uncertain parameters on which the capacity curve depends. Consider $R=T_{dir}$ or $R=A_{y,dir}$ or $R=D_u$; it is a random function of random parameters \mathbf{P} . Applying the Taylor series expansion around the mean value, it is obtained:

$$R(\mathbf{P}) = R|^{P_0} + \sum_i (P_i - P_{i0}) \times \left. \frac{\partial R}{\partial P_i} \right|^{P_0} + \frac{1}{2} \sum_{ij} (P_i - P_{i0})(P_j - P_{j0}) \times \left. \frac{\partial^2 R}{\partial P_i \partial P_j} \right|^{P_0} + \dots \quad (3.1)$$

where $\mathbf{P}_0=E[\mathbf{P}]$, being $E[\cdot]$ is the expected value, P_i, P_j are the i, j -th terms of \mathbf{P} ; P_{i0}, P_{j0} are the i, j -th terms of \mathbf{P}_0 ; the superscript $|^{P_0}$ denotes quantities evaluated in correspondence of \mathbf{P}_0 . Applying statistical operators to $R(\mathbf{P})$, Eqn. 3.1, approximations of the statistical moments of R may be obtained when the statistical moments of \mathbf{P} are available (Haldar and Madhavan, 1999):

$$E[R] \approx R|^{P_0}; \quad V[R] \approx V[R]_{par} + V[R]_e \quad (3.2)$$

where:

$$V[R]_{par} = \sum_i \sum_j \left. \frac{\partial R}{\partial P_i} \right|^{P_0} \left. \frac{\partial R}{\partial P_j} \right|^{P_0} \times Cov[P_i P_j]; \quad V[R]_e = V[\varepsilon_R] \quad (3.3)$$

ε_R is the error of the calculation model, $V[R]_{par}, V[R]_e$ are terms associated to the parameter variability and to the model error, $Cov[\cdot]$ is the covariance.

3.1 Damage limit states and their distribution

The seismic effects over the buildings are compared with four damage limit states (HAZUS, 1999). Each k -th damage limit state (*dls*) is related to a spectral displacement value $S_{d,k}$ of the equivalent *SDOF* capacity curve:

$$S_{d,1}^{NV} = 0.7 \times D_y; \quad S_{d,2}^{NV} = 1.5 \times D_y; \quad S_{d,3}^{NV} = 0.5 \times (D_y + D_u); \quad S_{d,4}^{NV} = D_u \quad (3.4)$$

the superscript ^{NV} indicates nominal value. As a matter of fact, each dls is definable as a random quantity, for which the probability density function (pdf) is assumed to be:

$$\begin{cases} p_{S_{d,k}}(S_{d,k}) = \lambda & \text{for } S_{d,k}^{NV} - \theta_{L,k} \leq S_{d,k} < S_{d,k}^{NV} + \theta_{U,k} \\ p_{S_{d,k}}(S_{d,k}) = 0 & \text{outside the range} \end{cases} \quad (3.5)$$

where λ is a suitable constant, $\theta_{L,k}$, $\theta_{U,k}$ are the lower and upper bound of the distribution. It is assumed that such limits lie on the mean point between nominal values of $S_{d,k}$ and $S_{d,k+1}$. As in the previous case, approximate statistical moments of $S_{d,k}$ can be obtained according Eqn. 3.2, taking $R=S_{d,k}$. Unlike in the previous case, its inherent randomness distribution is given (Eqn. 3.5).

3.2 Formulation for structural reliability

The seismic risk assessment and loss estimation is carried out according to the N2 procedure (Fajfar, 1999); the performance point (S_d) is defined comparing the capacity spectrum and the response spectrum in the acceleration-displacement domain (spectral coordinates). In this context, uncertainties arise from the calculation model, dls definition and from the parameters \mathbf{P} . The seismic demand is taken as a deterministic scenario at this stage. Therefore, a combination of the aforementioned uncertainties is encountered in each step of the process contributing significantly to the variability of the results.

Reliability analyses involve the comparison of the limit state $S_{d,k}$, representative of the structural failure, with the performance point S_d for the seismic action considered. Since both $S_{d,k}$ and S_d are random, a proper reliability evaluation involves the knowledge of the distribution of all these quantities. The failure probability P_f is the threshold exceeding probability (Haldar and Madhavan, 1999):

$$P_f = \int_{\Omega_f} f_{S_d}(r) dr \quad (3.6)$$

where Ω_f is the failure domain where $S_d > S_{d,k}$; r is the state variable of the pdf of f_{S_d} . Except for some very simple situations, the integral in Eqn. 3.6 is very difficult to evaluate. Therefore, it is usually referred to simplified procedures. The safety margin function for the k -th dls can be defined by:

$$M_k = \ln S_{d,k} - \ln S_d \quad (3.7)$$

M_k is a inherent random quantity which depends on \mathbf{P} . The failure probability related to the k -th dls is given by:

$$P_{f,k} = P(M_k < 0) \quad (3.8)$$

Assuming that S_d , $S_{d,k}$ are lognormal distributed random variables, thus $\ln S_d$, $\ln S_{d,k}$ are normal. Therefore, also M_k is normal. $P_{f,k}$ is given by:

$$P_{f,k} = \Phi \left[-\frac{E[M_k]}{\sqrt{V[M_k]}} \right]; \beta_k = \sqrt{V[M_k]} \quad (3.9)$$

Eqn. 3.9 coincides with the conditional probability of exceeding a given dls , generally expressed in the form of fragility curves (HAZUS, 1999):

$$P[ds_k | S_d] = \phi \left[\frac{1}{\beta_k} \times \ln \left(\frac{S_d}{S_{d,k}} \right) \right] \quad (3.10)$$

where $S_d, \bar{S}_{d,k}$ are mean values of the performance point and of the damage limit state threshold; β_k (the standard deviation of the natural logarithm of the spectral displacement normalized to the displacement for the k -th limit state) traduces the variability of each damage state, ds_k .

Following the same procedure described by Eqn. 3.1 and 3.2, assuming $R=M_k$, $P=\{h, \beta_{x,i}, \alpha_x, \beta_{y,i}, \alpha_y, \gamma, q, \tau_k, G\}$, approximate estimates of the statistical moments of M_k can be obtained. In this case, the model error concerns both the mechanical model and the procedure for the evaluation of the performance point; moreover, a further source of uncertainties related to the dls must be taken into account. Eqn. 3.2 becomes:

$$E[R] \approx R^{P_0}; V[R] \approx V[R]_{par} + V[R]_{\epsilon} + V[R]_{dls} \quad (3.11)$$

Therefore, β_k can be expressed as the sum of three contributions, associated respectively to the uncertainties in the model parameters, to the model error and to the randomness of the dls :

$$\beta_k = \sqrt{\beta_{k,par}^2 + \beta_{k,\epsilon}^2 + \beta_{k,dls}^2}, \quad \beta_{k,par}^2 = V[M_k]_{par}; \quad \beta_{k,\epsilon}^2 = V[M_k]_{\epsilon}, \quad \beta_{k,dls}^2 = V[M_k]_{dls} \quad (3.12)$$

4. NUMERICAL APPLICATION

The procedure is applied to a 4 storey building; $h=3.1\text{m}$, $\beta_x=2.04$, $\beta_y=1.52$, $\alpha_x=0.02$, $\beta_y=0.05$, $q=310 \text{ kg/m}^2$, $\gamma=2200 \text{ kg/m}^3$, $G=2 \times 10^8 \text{ N/m}^2$, $\tau_k=60000 \text{ N/m}^2$. The following values for the error in the estimate of parameters are assumed: $Var[h]=0.05$, $Var[\beta_x]=0.10$, $Var[\beta_y]=0.10$, $Var[\alpha_x]=0.10$, $Var[\alpha_y]=0.10$, $Var[q]=0.15$, $Var[\gamma]=0.10$, $Var[G]=0.10$, $Var[\tau_k]=0.15$, $Var[.]$ being the coefficient of variation; $Cov[\beta_x, \beta_y] / \sqrt{V[\beta_x]V[\beta_y]} = -0.4$; $Cov[\alpha_x, \beta_y] / \sqrt{V[\alpha_x]V[\beta_y]} = -0.3$; $Cov[G, \gamma] / \sqrt{V[G]V[\gamma]} = 1$. The other parameters are considered uncorrelated. The building is studied for the uniform mode shape and it is assumed that the wall resistant area decreases linearly.

4.1 Uncertainty propagation

The first step of the analysis propagates the uncertainties of $P=\{h, \beta_x, \beta_y, \alpha_x, \alpha_y, \tau_k, \gamma, G\}$ over the capacity curve for the transversal (YY) direction. The effects associated to each parameter are obtained by modeling as random variables just one parameter at a time, taking the remaining ones as coincident with their mean values. Applying the procedure shown in section 3, first order approximations of the mean and variance of each quantity defining the capacity curve and damage limit states, *i.e.* $R=T_{dir}$, $R=A_{y,dir}$, $R=S_{d,1}^{NV}$, $R=S_{d,2}^{NV}$, $R=S_{d,3}^{NV}$, $R=S_{d,4}^{NV}$, are obtained using Eqn. 3.2. Figure 1 shows the variation of the capacity curve associated to each of the most influent parameters. Parametric diagrams correspond to the mean capacity curve ($E[T_{dir}]=0.273 \text{ s}$, $E[A_{y,dir}]=0.32\text{g}$, $E[D_y]=0.006 \text{ m}$), plus and minus a standard deviation. The analysis of Figure 1 reveals that each parameter affects differently the period, T_{dir} and the acceleration yielding capacity, $A_{y,dir}$. On one hand, uncertainty associated to h , q and G affects the period in a more significant matter, on the other hand, τ_k , α_y and β_y affect the strength. The results for α_x , β_x and γ reveal low importance in this direction (YY). The inter-storey height (h) is not source of relevant variability, therefore approximate evaluation of this dimension can be acceptable. The same applies for the specific weight (γ). The shear modulus (G) influences exclusively the elastic range of the capacity curve. The floor loading (q) reveals high influence in the overall structural behaviour. The most significant parameters are the ones related to the resistant wall area in the direction

considered, *i.e.* β_y , α_y , and above all the characteristic shear strength, τ_k , over which uncertainty is very high, even when dealing with a single building assessment. Results for the soft-storey mode, not reported here, are similar in what concerns the role of parameter uncertainties. Capacity, in terms of strength and displacement, is quite smaller in this case.

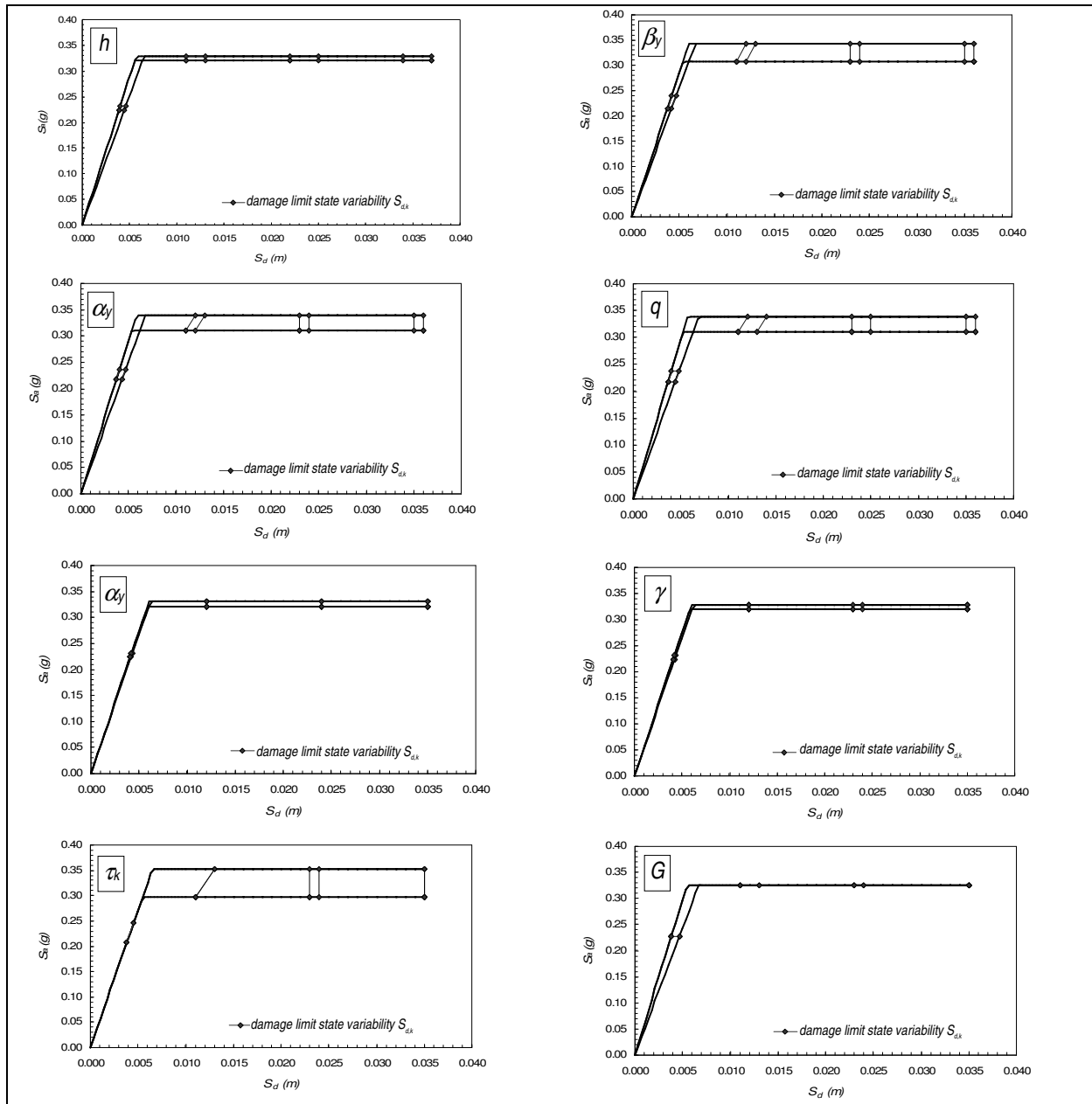


Figure 1 Propagation of uncertainties for the uniform mode shape due to h , β_y , α_y , q , τ_k , G

4.2 Fragility curves

Fragility curves for the damage assessment are determined for each limit state on the basis of Eqn. 3.12. Since the procedure is completely analytical, using a symbolic calculation tool the variance of the safety margin (M_k) is obtained numerically. It is pursued by the following steps: 1) the performance point and the damage limit state are obtained as an analytical function of \mathbf{P} ; the former is a deterministic function; the latter is a random function; 2) the *performance point* spectral displacement is obtained by the modified capacity spectrum

procedure (Fajfar, 1999) and is a analytical function of P ; 3) M_k is defined through Eqn. 3.7 for each limit state considered; it is also an analytical function of P . The mean value and standard deviation of M_k are obtained developing the marginal function in Taylor series (Eqn. 3.2). The exceeding threshold probability and the coefficient β_k are given by Eqn. 3.9; 4) β_k can be related to the different contributions, see Eqn. 3.12. Figure 2 lists the main results, exposing the different contributions due to parameters ($\beta_{k,par}$) and inherent randomness of the dls ($\beta_{k,rms}$). Due to lack of knowledge in the model error, it is omitted in this application.

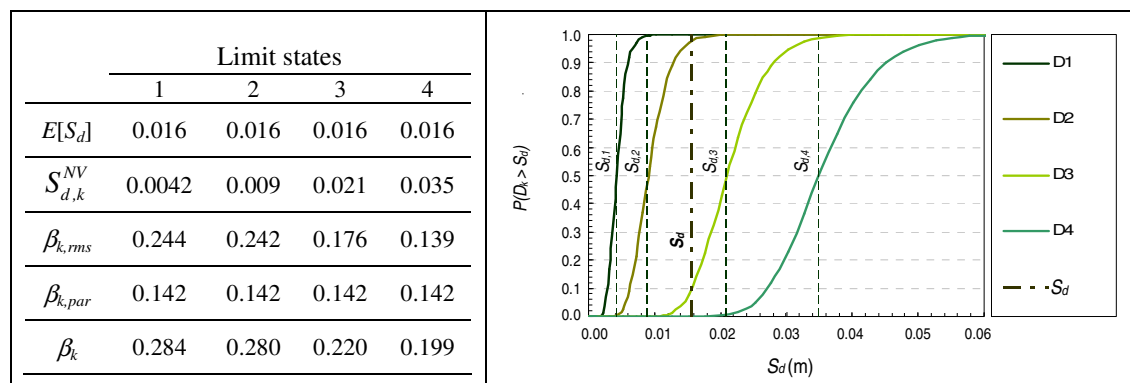


Figure 2 coefficient β variability analysis and the fragility curves

5. CONCLUSIONS

This research has presented a mechanical model for the vulnerability assessment of masonry buildings. The analytical formulation has allowed the discussion and interpretation of uncertainties that affect the capacity spectrum and the damage limit states. The numerical application has been carried out for the seismic assessment of a single building.

The capacity spectrum method on which the proposed model is based represents a strong alternative to other vulnerability simplified methodologies when a good survey of the building stock is available. In this case, parameters are derived from a statistical treatment of a database. The implementation of an automatic procedure is still undergoing to create fragility curves for different building typologies within the masonry type. Future development will include moreover the introduction of the uncertainty associated to the response spectrum and the analysis of the model error contributions.

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