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# COMPARISON OF DIFFERENT MODELLING STRATEGIES FOR THE REPRESENTATION OF NON LINEAR RESPONSE OF RC COLUMNS SUBJECTED TO BIAXIAL LOADING

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**Key words:** RC columns; non-linear behaviour; biaxial bending; fibre modelling.

**Summary.** Aiming at studying the performance of different non-linear modelling strategies for RC columns subjected to axial load and cyclic biaxial horizontal loading, several analyses were developed using the computer code SeismoStruct. This study comprised the simulation of 24 cantilever RC columns quasi-statically tested under constant axial force and cyclic uniaxial and biaxial displacements imposed with different loading paths. The results show that all adopted modelling strategies (distributed-inelasticity and lumped-plasticity) lead to similar accuracy in terms of restoring shear-force evolution. Generically, they were found suitable for accurately predicting the cyclic response of RC columns, with standard material parameters, although the models may not succeed in representing the strength degradation for cyclic loading.

## 1 INTRODUCTION

Since the columns are key structural elements for the seismic performance of buildings,

special attention should be given to their structural response under load reversals. Moreover, earthquake effects generally require the inclusion of two horizontal component loads that are recognized to be more damaging than one direction actions. The interest in the inelastic response of axially loaded members under biaxial bending is relatively recent and the available experimental results are limited. Possibly, this is partially due to the uncertainty of combining bending moments' histories in the two orthogonal directions that adds considerable complication to the problem. The practical result is that our present-day knowledge of the inelastic behaviour of RC columns under biaxial cyclic moments is very much behind our understanding of the behaviour under 1D cyclic bending with axial load.

Similarly, in what concerns modelling of RC members under cyclic 2D bending combined with axial load, the present knowledge is still far behind than that for 1D bending.

A vast number of models for the material nonlinear behaviour of beam-column members have been developed, ranging from concentrated plasticity formulations to distributed plasticity formulations based on finite-element method [1].

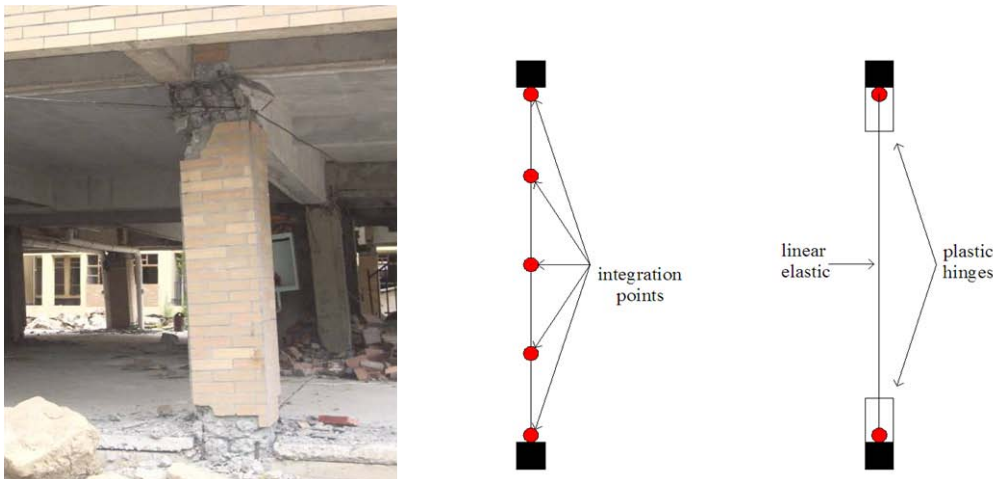


Figure 1: Element model approaches for non-linear numerical modelling of RC beam-columns

The lumped-plasticity models assume the concentration of the non-linear behaviour to be concentrated or lumped at predetermined sections. The non-linear behaviour is often controlled at the centre of the plastic hinge zones, which are generally located at each end of RC elements [2]. Several lumped-plasticity models have been proposed in the last decades, starting by the first studies by Clough *et al.* [3], subsequently developing with the consideration of the bending and axial force interaction [4-5], biaxial bending interaction [6-7] and bending and shear interaction [5, 8].

The lumped-plasticity models are simplifications of the real behaviour (see Figure 1), for which they present some deficiencies. The first is related with the assumption of the concentrated inelasticity in the hinge, thus ignoring the damage spread along the element, which takes higher importance in large resisting elements like RC walls [9], wherein the combination of plasticity concentration in the element ends (mainly due to bending) with a significant contribution of shear can induce major inclined cracks spreading into the element

inner region; clearly, for such cases, the consideration of concentrated plasticity in the member ends is not an accurate option [10]. Furthermore, lumped models simplify some aspects of the hysteretic response of RC members since the section behaviour characteristics are defined *a priori*. The principal proposals can be found in the literature, namely those resumed by Otani [11], Fardis [12], Filippou and Fenves [13].

Another approach of modelling at element level is associated with the distribution of the nonlinearity along the element (see Figure 1), providing the element with a certain number of control sections, where the inelastic behaviour is integrated to obtain the global inelasticity of the element. This concept was first introduced by Otani [14] and the major advantage of these models is the nonexistence of a predetermined length where the inelasticity can occur, because all the sections can have incursions in the non-linear response. While this approach is a closer approximation to reality, it also requires more computational capacity [15]. Exhaustive reviews describing these formulations can be found in the works of Taucer *et al.* [16], Spacone *et al.* [17] and Arede [18].

At the section level, different strategies have been proposed for the simulation of the biaxial cyclic behaviour of RC elements with axial force. Besides the fibre models [16-17, 19], other analytical proposals are available, namely those following the concepts of classical plasticity [20], the Mroz multisurface plasticity model [6, 21-22], the Bouc-Wen model [23], the hysteresis modelling [24-27], the bounding surface plasticity model [7, 28-29], or lumped damage models [30-32], among others.

With the present study it is intended to evaluate and to compare the adequacy of different non-linear modelling strategies in the representation of RC columns' response when subjected to axial force combined with cyclic biaxial horizontal loading.

## 2 EXPERIMENTAL TESTING CAMPAIGN

The experimental campaign comprised twenty-four rectangular RC columns, with different types of geometric characteristic and reinforcement detailing, which were cyclically tested for different loading histories with constant axial force under displacement controlled conditions. The column specimens are 1.70m high and cast in strong square concrete foundation blocks. The cross-section dimensions and the reinforcement detailing are presented in Figure 2. The materials considered at the specimen design phase were regular C35/45 and C30/35 concrete classes, respectively for N01-N04 and for N05-N24 columns, and A400NR-SD reinforcement steel grade; the average concrete strength obtained from real samples' tests are summarised in Table 1.

Figure 2 shows the setup adopted for the experimental testing. The system includes two independent horizontal actuators to apply the lateral loads on the column specimen, one with 500kN force capacity and +/-150mm stroke and another with 200kN maximum force and +/-100mm stroke. A 700kN capacity actuator was used to apply the vertical axial load. Two steel frames and a concrete wall provided the reaction system for the three actuators. The column specimens and the reaction frames were fixed to the laboratory strong floor with prestressed steel bars to avoid sliding or overturning during the tests. Since this system enforces the axial load actuator to remain in the same position during the test while the column specimen laterally deflects, a very low friction sliding device is used between the top-column and the

actuator; however, spurious friction effects are dully measured by a load cell attached to that device. As stated previously, for each of the tested specimens, a constant axial force was imposed taking the values included in Table 1, for both absolute and normalised axial force.

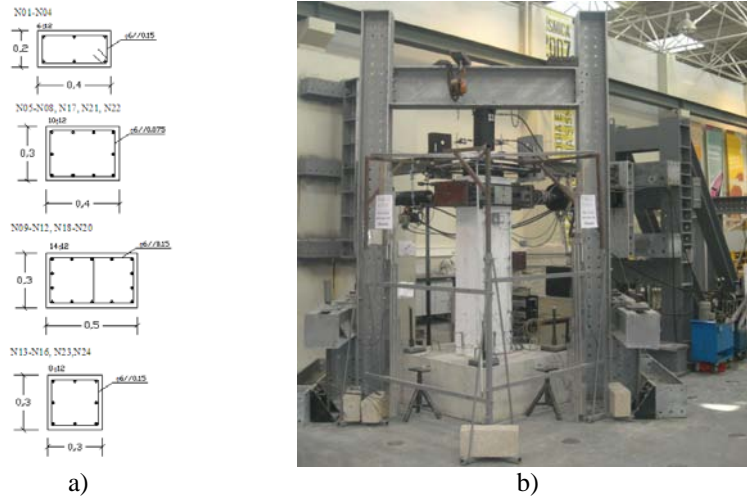


Figure 2: a) RC column cross-sections details; b) General view of the testing setup

Table 1: Specimen specifications and loading characteristics [33]

Series	Column	Geometry [cmxcm]	$f_{cm}$ [MPa]	N [kN]	$v$ $N/(A_c \cdot f_{cm})$	Displacement path type
1	PB01-N01	20x40	48.35	170	0.04	Uniaxial Strong
	PB02-N02					Uniaxial Weak
	PB12-N03					Cruciform
	PB12-N04					Rhombus
2	PB01-N05	30x40	21.40	300	0.12	Uniaxial Strong
	PB02-N06					Uniaxial Weak
	PB12-N07		Rhombus			
	PB12-N08		Quadrangular			
			36.30	510		Circular
3	PB01-N09	30x50	24.39	300	0.08	Uniaxial Strong
	PB02-N10					Uniaxial Weak
	PB12-N11		Rhombus			
	PB12-N12		Quadrangular			
			36.30	440		Circular
4	PB01-N13	30x30	21.57	210	0.1	Uniaxial Strong
	PB12-N14					Rhombus
	PB12-N15					Quadrangular
	PB12-N16					Circular
5	PB12-N19	30x50	43.14	300	0.045	Rhombus
	PB12-N20			600	0.09	Rhombus
6	PB12-N21	30x40	43.14	620	0.12	Rhombus
	PB12-N22					Quadrangular
7	PB12-N23	30x30	36.30	650	0.2	Rhombus
	PB12-N24					Quadrangular

### 3 SUMMARY OF THE EXPERIMENTAL RESULTS

A detailed information about the force-displacement results can be found in [33]; however from the analysis of the measured displacement and shear force paths (see the example in Figure 3) some conclusions can be drawn as described in the next paragraphs.

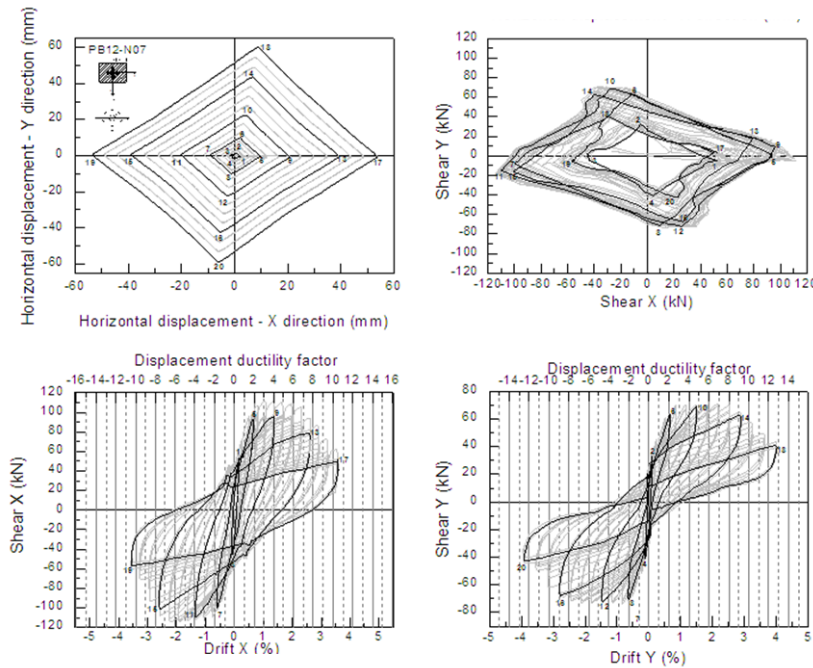


Figure 3: Global results of rectangular column PB12-N07 for rhombus load path [33]

- From the observation of the shear-drift curves, four main branches can be identified in their envelopes, corresponding to: i) pre-cracking response; ii) post-cracking until the reinforcement steel yields; iii) a plateau or post-yield hardening zone and iv) a softening phase. These four stages are clear in both the uniaxial and biaxial tests. However, in the biaxial tests the plateau tends to be shorter and the softening is more pronounced, i.e. a more abrupt decay of the column strength is observed with increasing lateral deformation demands.
- The initial column stiffness in both directions it is not significantly affected by the biaxial load path.
- As expected, by comparing the maximum strength in one specific direction of the columns for each biaxial test against the corresponding uniaxial test, lower values are obtained for all biaxial tests than for uniaxial ones. The biaxial loading induces a 20-30% reduction of the maximum strength of the columns in their weak direction, Y, while reductions of 8-15% are found for the stronger direction, X.
- The ultimate ductility is significantly reduced in columns subjected to biaxial load paths.
- The strength degradation is practically zero, in the first loading cycles, increasing after

displacement ductility demands of about 3. From the strength degradation analysis, more pronounced strength degradation was observed for biaxial tests when compared with corresponding uniaxial tests.

- It was found that biaxial loading (circular, rhombus and cruciform load paths) can introduce higher energy dissipation than uniaxial loading, as previously recognised by other authors. It was confirmed that the energy dissipation also depends on the cross-section geometry. For a given imposed maximum drift, among all the load paths considered in this study, the circular path was shown to be the most dissipative, while the quadrangular load path evidenced less energy dissipation.
- In the first cycle of each peak displacement level, larger energy dissipation is observed than in the subsequent cycles for the same peak displacement. This effect is more pronounced in the biaxial loading tests. After reaching the column conventional failure, the dissipated energy exhibits lower increasing rate.

#### 4 MODELLING STRATEGIES

The numerical analyses carried out within this work, and described in the present paper with different non-linear modelling strategies, were performed using the computer code SeismoStruct [34]. The program includes models for the representation of spatial frames' behaviour under static and/or dynamic loading, considering both material and geometric non-linearities. The software permits performing seven types of analyses, namely: dynamic and static time-history, conventional and adaptive pushover, incremental dynamic analysis, modal analysis and static analysis (possibly non-linear) under quasi-permanent loading. The software allows using elements with distributed inelasticity (force or displacement-based formulations) and elements with lumped-plasticity (with fixed length, i.e. the so-called plastic-hinge). Fibre discretization is adopted to represent the behaviour at section level where each fibre is associated with a uniaxial stress-strain law. The sectional moment-curvature state of the beam and column elements is then obtained through the integration of the non-linear uniaxial stress-strain response of the individual fibres into which the section has been subdivided.

This work comprises the comparison of three non-linear modelling strategies based on: *i*) elements with lumped-plasticity (Figure 4a); *ii*) elements with distributed inelasticity and force-based formulation (Figure 4b) and *iii*) elements with distributed inelasticity and displacement based formulation (Figure 4c).

For the applications made with the different modelling strategies, decisions for each specific strategy were taken based on the results of parametric studies performed by other authors [15-16, 35].

Different studies have proposed expressions to estimate the plastic hinge length ( $L_p$ ) of RC elements to be adopted in lumped plasticity models [36]. Priestley and Park [37] proposed a formulation that estimates the plastic hinge length based on the distance of the critical section to the contraflexure point and on the diameter of longitudinal rebars. Based on that work, Paulay and Priestley [38] reported that, for typical RC columns, the plastic hinge length is approximately equal to half of the cross-section depth, which was herein considered for the uniaxial tests' simulations. Based on experimental evidence for biaxial tests, other authors have concluded that the plastic hinge length is not strongly affected by 2D loading [39]; in

particular, the plastic hinge length for rectangular columns under biaxial loading was evidenced approximately equal to the length observed in a similar column tested uniaxially in its strong direction [33]. Thus, in the biaxial analyses performed and described in this paper, half of the larger dimension of the cross-section was taken for the plastic hinge length.

For the force based formulation, seven integration points were considered, based on the results of Calabrese *et al.* [35]. These authors state that at least six integration sections are needed in order to obtain a completely stabilized prediction of the local response.

For displacement based formulations, according to the same authors, a good approximation to a cantilever column response can be obtained with a mesh discretization of at least four equal length elements, with two Gauss-Legendre points per element. Considering this and taking into account the concentration of the non-linear response close to the column fixed end (plastic hinge length), the column discretization was adopted with six elements sized according to Figure 4-c.

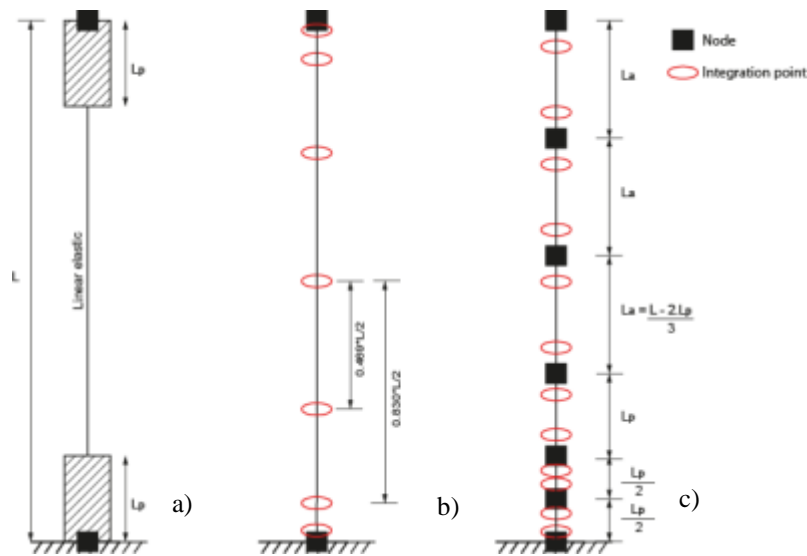


Figure 4: Modelling strategies with indication of the control section points: a) lumped-plasticity element; b) distributed inelasticity element with force-based formulation; c) distributed inelasticity element with displacement-based formulation

## 5 MATERIALS SECTION MODELS

This section reports on the constitutive models used for concrete and reinforcing steel. For the former, it was adopted the Madas [40] uniaxial model, which follows the constitutive law proposed by Mander *et al.* [41]. The cyclic rules included in the model for the confined and unconfined concrete were proposed by Martinez-Rueda and Elnashai [42]. The confinement effects provided by the transverse reinforcement were considered through the rules proposed by Mander *et al.* [41], whereby constant confining pressure is assumed throughout the entire stress-strain range, conveyed by the increase of the compression strength peak value and the stiffness of the unloading branch [34]. Material mechanical parameters were defined based on test results on samples [33].

The uniaxial model proposed by Menegotto and Pinto [43], coupled with the isotropic



hardening rules proposed by Filippou *et al.* [44], was adopted for the steel reinforcement simulation in these analyses. This steel model does not represent the yielding plateau characteristic of the mild steel virgin curve, but it takes into account the Bauschinger effect, which is relevant for the representation of the columns' stiffness degradation under cyclic loading. The input parameters of the model are: the yield strength ( $f_y$ ); the elastic Young modulus ( $E_s$ ); the strain-hardening ratio ( $r$ ) and five parameters to describe the transition from elastic to plastic branches ( $R_0$ ,  $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$ ). All the adopted values are in accordance with the properties obtained in the material test samples [33].

## 6 COMPARISON BETWEEN MODELLING STRATEGIES

### 6.1 Envelopes

The experimental shear-drift envelope curve obtained for each column is compared with the results of the non-linear models in terms of initial stiffness and in terms of evolutions of tangent and secant stiffness.

For example, Figure 5 presents the shear-drift envelopes in each direction (X and Y) for the columns with  $30 \times 50 \text{ cm}^2$  cross-section for rhombus biaxial tests (column N11). From the analysis of all shear-drift envelopes, a reasonably good agreement was found between the numerical simulation and the experimentally measured response. The differences found between the modelling strategies are briefly discussed next.

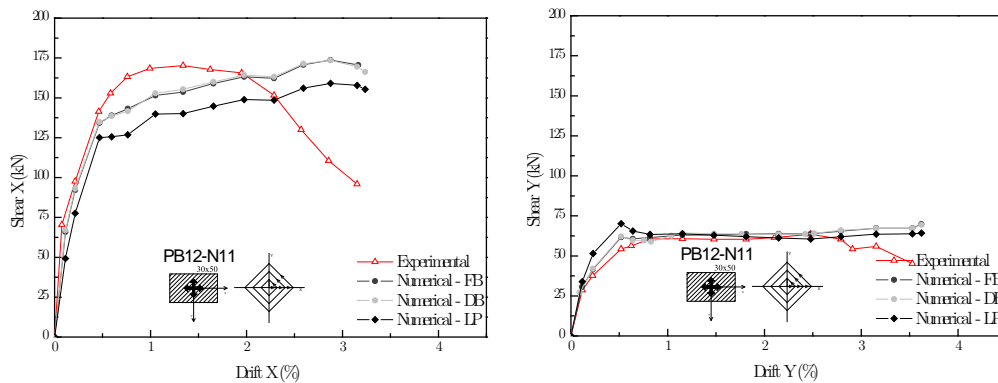


Figure 5: Shear-drift envelopes for  $30 \times 50 \text{ cm}^2$  columns (measured and calculated)

In order to compare the envelopes from the experimental results and with those obtained from numerical models, the traditional correlation coefficient ( $R^2$ ) was calculated. As usual, the statistical measure  $R^2$  assumes the value of 1.0 when a perfect correlation is found between the numerical and experimental data. In these comparisons, a conventional threshold of 0.75 is assumed as the minimum  $R^2$  value for a good fit. The obtained  $R^2$  results for all analyses are shown in Figure 6, plotted in circular charts with diameter proportional to the  $R^2$  value). According to these plots, it can be concluded that correlation coefficients higher than 0.75 were obtained for most of the numerical analyses, indicating a good representation of the experimental envelopes. For each studied column, similar correlation coefficients were found with the three modelling strategies, but the lumped plasticity modelling strategy always



exhibited the lowest correlation factors.

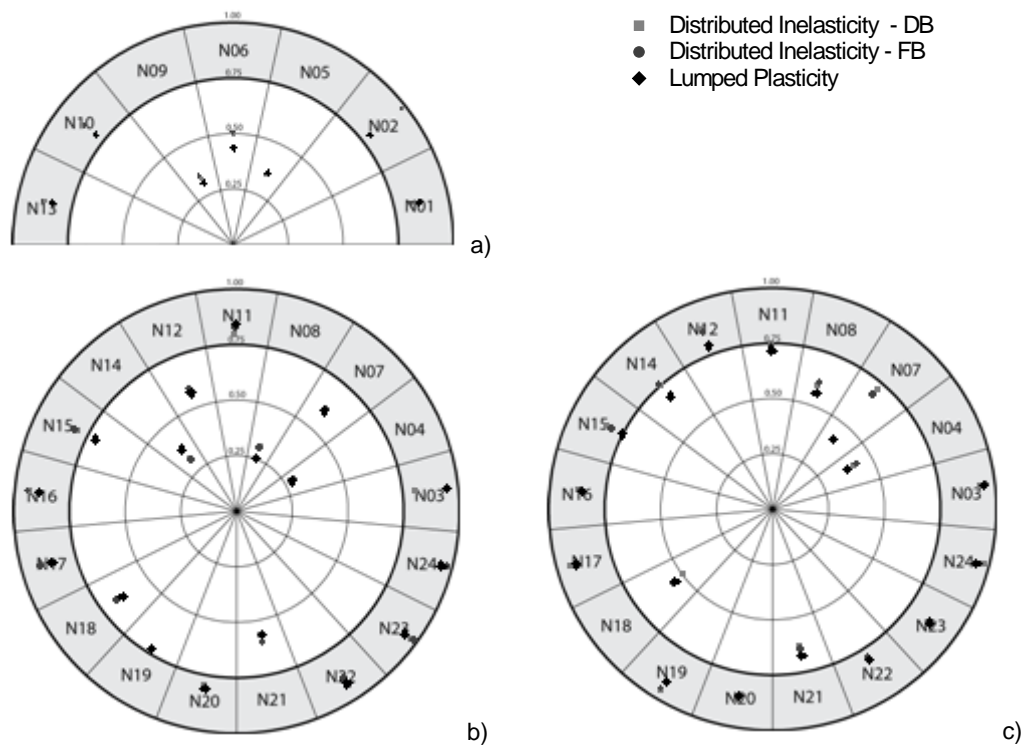


Figure 6: Correlation coefficients of shear-drift envelopes between experimental and numerical results ( $R^2$ ): a) uniaxial tests; b) biaxial tests – strong direction (X); c) biaxial tests – weak direction (Y)

## 6.2 Cyclic response

This section focuses on the evaluation of the numerical models' accuracy for the cyclic response simulation of the RC columns under study. The general analysis of the whole cyclic columns' response obtained with the three modelling strategies and the comparison with the corresponding experimental results allowed concluding that even when a good representation of the global columns' response is achieved, for the largest deformation demands the numerical response may not capture well the strength degradation, which is mainly associated to the buckling of the reinforcing steel bars. Additionally, in many cases, the unloading-reloading phase of the numerically obtained cyclic response does not accurately follow the experimentally observed pinching effect, which can be due to the models' limitations in simulating the longitudinal reinforcing steel slippage.

### 6.2.1 Shear evolution

In order to evaluate the accuracy of the adopted numerical models for the representation of the experimentally obtained shear force evolution, the frequency domain error (FDE) index [45-46] was calculated. The FDE index [46] measures the deviation between two waveforms, in this case the shear force measured in the test and the corresponding calculated shear force for each modelling strategy. The FDE index quantifies amplitude and phase deviations

between two signals, giving an error factor with a value between 0 and 1 (0 means no deviation and 1 corresponds to full deviation). In this analysis, it is considered that a FDE index larger than 0.75 represents a poor correlation and a value below 0.25 represents a very good correlation. These limits were proposed in [45].

The FDE indices obtained for all analyses are represented in Figure 7. From the analysis of the results, it can be observed that for all columns, with uniaxial and biaxial loading, the three developed numerical models give good estimates of shear force evolutions recorded in the experimental tests. In fact, the calculated FDE index was slightly larger than 0.25 and no significant differences were found in the FDE index for the three modelling strategies applied to each column tested. For all columns analysed under biaxial demands, the FDE index is lower in the weak direction (Y) than in the strong direction (X), which is in accordance with the larger differences identified in this direction in terms of stiffness evolution and strength degradation.

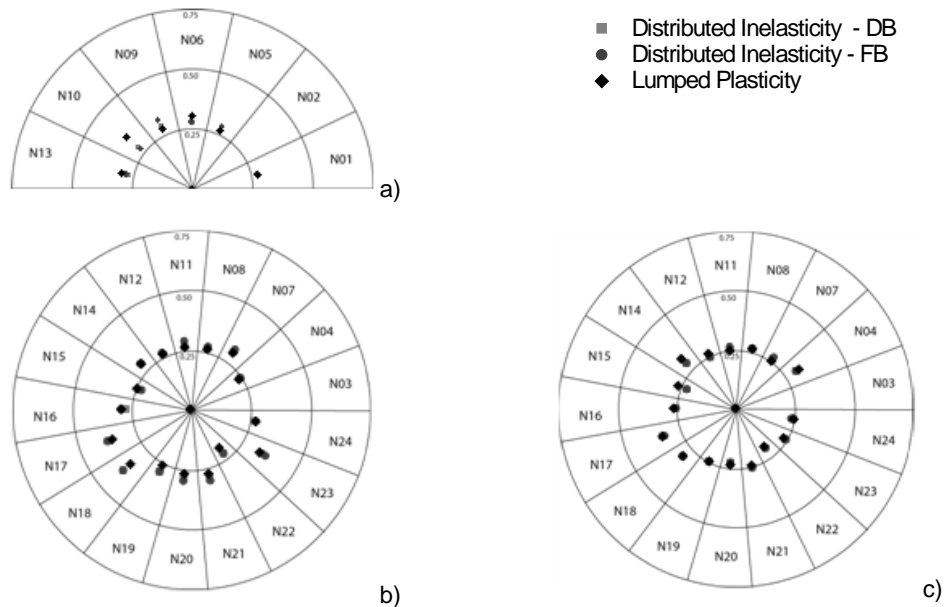


Figure 7: FDE index obtained from the comparison of numerical and experimental shear force evolutions: a) uniaxial tests; b) biaxial tests – strong direction (X); c) biaxial tests – weak direction (Y)

## 6.2.2 Energy dissipation

The accuracy of the non-linear models is characterized here in terms of the total dissipated energy, comparing the calculated values from numerical models with the values obtained from column tests. Figure 8 shows the ratio between the numerical and experimental values of the total dissipated energy.

Based on the obtained results, it can be observed that, for some columns analysed under uniaxial loading conditions, a considerable overestimation of the total dissipated energy was obtained with the numerical models (Figure 8a), which can be justified by the deficiencies of numerical models concerning the strength degradation representation. However, this dissipated energy overestimation was not observed in any column under biaxial loading. The

results for the columns with biaxial loading, analysed for each direction independently, show a variation in the total dissipated energy between 25% and 50% (Figure 8c and 8d) relative to the experimental measurements, being larger for the weak direction (Y). In addition, the sum of the dissipated energy at each direction (Figure 8b), conducts generally to an overestimation of the total energy obtained with the adopted numerical strategies. But, for each column under biaxial loading conditions, similar ratios were obtained with the three modelling strategies. However, for the weak direction (Figure 8d) the model with distributed plasticity with a force-based formulation presents larger differences in the total dissipated energy. Comparing the total dissipated energy obtained as the sum of the energy dissipated for each direction, it is observed that the lumped plasticity modelling strategy presents the largest deviations.

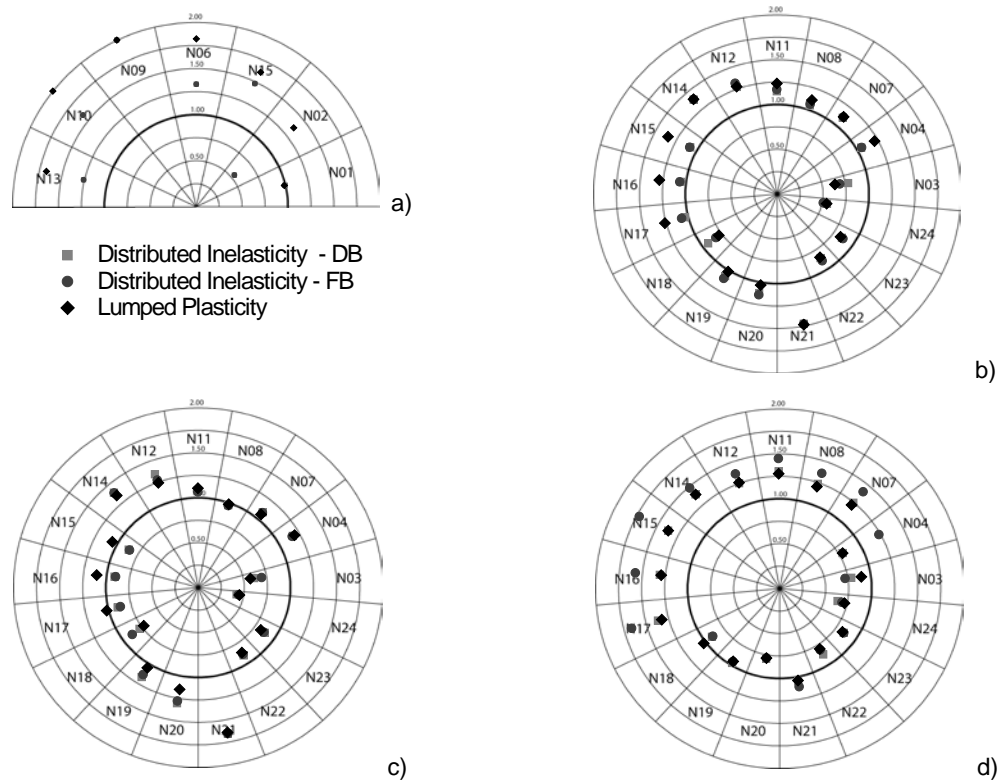


Figure 8: Ratio between the total dissipated energy obtained with the numerical models and the experimental value: a) uniaxial tests; b) biaxial tests – total (X + Y); c) biaxial tests – strong direction (X) only; d) biaxial tests – weak direction (Y) only

## 6 FINAL CONSIDERATIONS

In this work, the adequacy of three modelling strategies in the representation of RC columns' responses, under uniaxial and biaxial loading conditions, was analysed.

From the analysis of the obtained results, similar levels of adequacy were achieved using the distributed inelasticity (force and displacement formulations) and the lumped plasticity modelling strategies. The response obtained with the three modelling strategies for all columns was found to be satisfactory, but difficulties were found in capturing the strength degradation for the higher drift demands. Additionally, most of the models show limitations

in representing the pinching effect in the unloading-reloading stage.

Finally, it is worth emphasizing that many questions are still open in the modelling of the biaxial response of RC columns. The adopted strategies can provide reasonably accurate simulations of the columns' cyclic response until the strength degradation begins. Although also evidenced for uniaxial loading, this aspect is not so relevant because, for that case, the strength degradation starts for larger drift demands in comparison with biaxial loadings.

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