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THE FORCE METHOD TO CALCULATE STRESS INTENSITY FACTORS FOR ARBITRARY MESHES

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Summary. The force method is a simple and accurate technique to obtain the stress intensity factors (SIF) for both modes I, II and also mixed I+II modes of fracture. The method uses the summation of internal nodal forces in the vicinity of the crack tip to compute SIFs. Recently, de Morais[1] showed that the force method is able to yield accurate SIF values from FE models constructed with regular meshes of linear elements. In this paper, the force method is applied successfully to general finite element meshes, in such a way that it can be used on crack propagation algorithms with arbitrary crack paths.

1 INTRODUCTION

Stress Intensity Factors (SIFs) are commonly used to predict crack propagation in structural materials. One of the most accurate methods for SIF calculation is based on its relation with the strain energy release rate (G)[2]. In fact, in the framework of linear elastic fracture mechanics (LEFM), the latter is equal the well known J-integral[3], whose calculation is provided by most FE codes. Recently, de Morais[1] showed that the force method can give very accurate results, even when linear reduced integration elements are used. The method consists of obtaining SIF estimates from the forces along a ligament in the vicinity of a crack tip. In a certain range of radial distances from the crack tip, the SIF estimates versus r-plot can be approximated by a line. Therefore, the SIF can be computed by linear extrapolation of SIF estimates to the crack tip (r = 0). Obviously, in FE models, forces along r-distances are computed from nodal force summation. De Morais[1] applied the force method to regular meshes, which have element faces along the x-axis (Figure 1), tangent to the crack tip. However, arbitrary meshes and crack growth simulations are required in general fracture problems. This creates additional difficulties in obtaining the required forces, as internal node forces are zero at equilibrium.

In this paper, application of the force method is extended to arbitrary meshes of quadrilateral elements. An algorithm splits them in triangular elements, which allow evaluation of nodal forces along the x-axis. Forces along x_c-ligaments result automatically from assembling the internal node forces of the newly created triangles. The proposed method is general and valid for distorted meshes and is thus appropriate for general arbitrary crack path propagation algorithms.

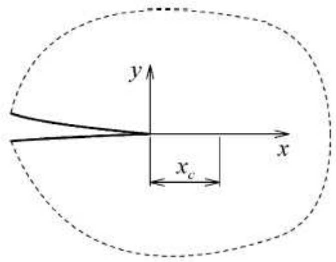

Figure 1: Crack tip and local coordinate system at crack tip.

2 CALCULATION OF STRESS INTENSITY FACTORS FOR ARBITRARY MESHES

For the crack tip local coordinate system of Figure 1, the stresses close to the crack tip along the *x*-axis can be related to mode I and mode II stress intensity factors in the following way:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi x}} \quad ; \quad \tau_{xy} = \frac{K_{II}}{\sqrt{2\pi x}} \tag{1}$$

Then, the total internal forces along the xc–ligament are:

$$F_y = \int_0^{x_c} \sigma_y dx = K_I \sqrt{\frac{2x_c}{\pi}} \quad ; \quad F_x = \int_0^{x_c} \tau_{xy} dx = K_{II} \sqrt{\frac{2x_c}{\pi}} \tag{2}$$

The values of F_y and F_x are obtained directly from the internal nodal forces of the finite element model. As can be seen in Figure 2, the original method imposes that finite element nodes must be located on the *x*-axis for a correct evaluation of the internal nodal forces.

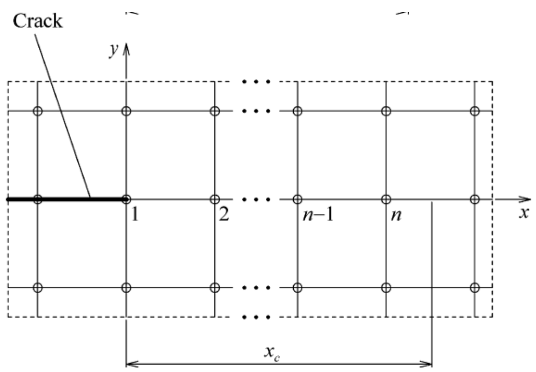

Figure 2: The force method for regular mesh.

Figure 3 illustrates how the new proposed method works for arbitrary meshes. The algorithm searches quadrilateral elements that are intersected by the x-axis line. For these elements, it constructs the intersection point and, if necessary, creates new nodes (see bold nodes in Figure 3). Then, new triangular elements are formed on the remaining quadrilateral element's area located along the positive $y > 0$ axis line.

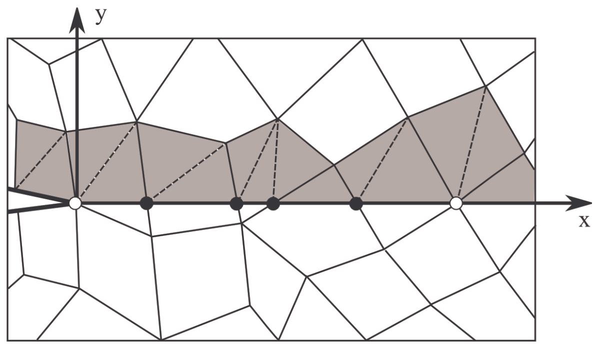

Figure 3: Subdivision of quadrilaterals into triangular elements.

The stresses assigned to these new created triangular elements are transferred from the quadrilateral elements. This turns out to be simple if reduced in-plane quadrilateral elements are used[4]. The internal nodal forces for the nodes are thus obtained from assembling the triangular nodal forces. This procedure is equivalent to the calculation of reaction forces along the x-axis line, overcoming on this way the zero internal forces at equilibrium configurations that results from the use of the original method.

Having the correct internal nodal forces, the SIF estimation can be formulated from:

$$K'_I = \sqrt{\frac{\pi}{2x_c}} \sum_{i=1}^{n} F_{y,i} \quad ; \quad K'_{II} = \sqrt{\frac{\pi}{2x_c}} \sum_{i=1}^{n} F_{x,i} \tag{3}$$

The results from the new proposed method will be evaluated for different meshes.

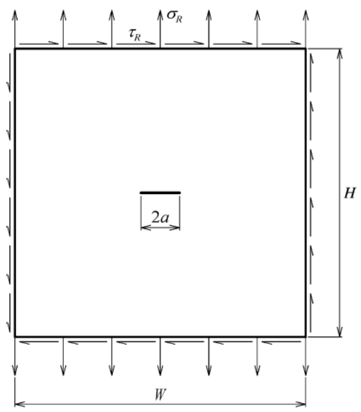

Figure 4: The centre-cracked specimen for finite element analysis.

3 RESULTS

We applied the new proposed method to a cracked plate, with H = W = 200 mm and 2a = 10 mm, as showed in Figure 4. The material properties are: E = 70 GPa and $\upsilon = 0.3$.

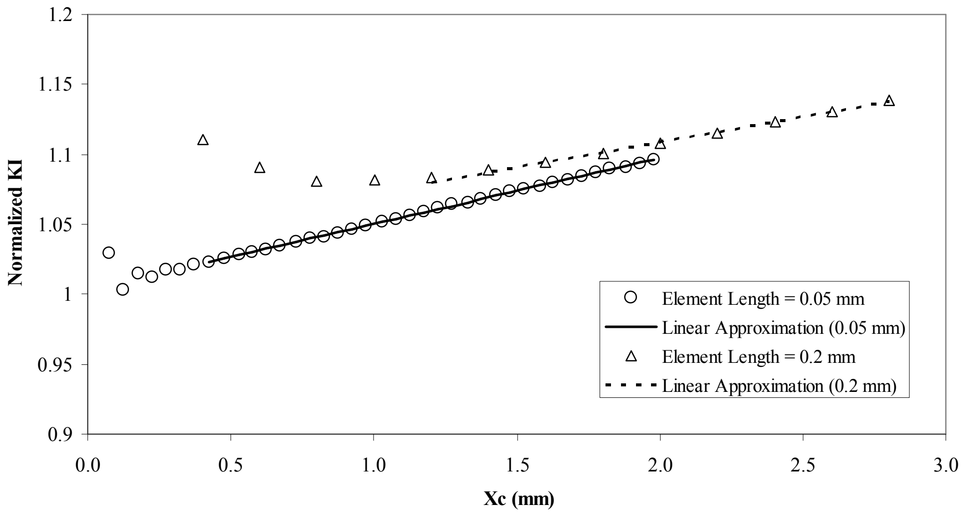

Figure 5: Normalized SIF for mode-I.

The meshes used had two different refinement levels within a 3 mm zone around the crack tip: one with element length of 0.2 mm and another with 0.05 mm. The exact analytical solution is: $K_I = \sigma_R \sqrt{\pi a}$. As can be seen in Figure 5, the method converges exactly to the normalized solution for a local element size near the crack tip of 0.05 mm. Nevertheless, for the coarser 0.2 mm element size mesh, the method still yields a good conservative result i.e. error of 3.7%.

4 CONCLUSIONS

The proposed algorithm to extract SIFs for general arbitrary meshes was validated in this study for two different mesh refinements. The algorithm is robust and adapts easily to any kind of mesh system. In this way, it can be used on linear elastic fracture mechanics analysis, including crack propagation with arbitrary crack paths.

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