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Dissertação apresentada à Universidade de Aveiro para cumprimento dos requisitos necessários à obtenção do grau de Mestre em Economia, realizada sob a orientação científica do Doutor António Manuel da Mota Freitas Martins, Professor Auxiliar da Faculdade de Economia da Universidade do Porto e co-orientação do Professor Doutor Joaquim Carlos da Costa Pinho, Professor Auxiliar do Departamento de Economia, Gestão e Engenharia Industrial da Universidade de Aveiro.

"A journey of a thousand miles begins with a single step."

Old Chinese Proverb

o júri

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palavras-chave

Sincronização de Ciclos Económicos, Integração Europeia, Modelos State-Space, Markov-Switching.

resumo

Esta dissertação estuda os padrões de sincronização de ciclos económicos numa amostra composta por 18 países desenvolvidos e a Zona Euro ao longo do período 1970:1-2008:1.

Para realizar este estudo, propomos um novo modelo de componentes não observáveis multivariado com markov-switching e interdependência de estados variável no tempo, no qual a sincronização é modelizada como uma componente comum variável no tempo entre os ciclos económicos. Para estimar o modelo, desenvolvemos um filtro de Kalman adequado, que permite a projecção das componentes não observáveis e a estimação dos hiperparâmetros por máxima verosimilhança. Propomos também um novo *full-sample smoother* para recalcular as componentes não observáveis do modelo com base em toda a informação amostral.

Usamos este modelo para testar 3 hipóteses: se a criação da União Monetária Europeia promoveu um aumento na sincronização dos ciclos económicos entre os seus membros; se a integração promoveu uma mudança na filiação cíclica com o ciclo económico dos EUA; se existe o surgimento de um ciclo económico agregado da Zona Euro.

Os resultados mostram que a sincronização cíclica dos países da Zona Euro com a Zona Euro agregada foi superior à dos restantes países. No entanto, para a maioria dos países da Zona Euro, a sincronização com a Zona agregada aumentou até ao início da década de 90, e diminuiu a partir desse período. Apesar de existir um ligeiro aumento na sincronização com a Zona Euro agregada para algumas economias participantes em torno do momento da introdução da moeda única, não somos capazes de detectar um "*efeito Euro"* claro. Por outro lado, para a maioria das economias, a introdução da moeda única é coincidente com uma redução na sincronização com o ciclo dos EUA. Finalmente, não encontramos evidência do surgimento de um ciclo económico agregado da Zona Euro.

keywords

Business Cycle Synchronization, European Integration, State-Space Models, Markov-Switching.

abstract

This dissertation studies the patterns of business cycle synchronization across a sample of 18 developed countries and the aggregate Euro Area over the period 1970:1-2008:1.

To perform this study, we propose a novel multivariate unobserved-components model with markov-switching and time-varying state interdependence, in which synchronization is modelled as a time-varying common component between the business cycles. To estimate the model, we develop an adequate Kalman filter, which allows the projection of the unobserved components and the estimation of the hyperparameters by maximum likelihood. We also propose a new full-sample smoother to re-compute the unobserved components of the model based on all in-sample information.

We use this model to test 3 hypothesis: whether the creation of the European Monetary Union promoted an increase in business cycle synchronization among its members; whether the integration has promoted a change in the cyclical affiliation with the US business cycle; and whether there is an emergence of an aggregate Euro Area business cycle.

The results show that synchronization between the Euro Area countries with the aggregate Euro Area has been higher than for the remaining countries. Nevertheless, for the majority of the Euro Area countries, synchronization with the aggregate Area increased until the beginning of the 1990s, and dropped from that period onwards. Moreover, despite the existence of a slight increase in synchronization with the aggregate Euro Area for some participant economies around the timing of the introduction of the common currency, we are not able to uncover a clear "*Euro effect*". On the other hand, for most of the economies, the introduction of the common currency is shown to be coincident with a drop in synchronization with the US business cycle. Finally, we do not find evidence of the emergence of an aggregate Euro-Area business cycle.

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1 Introduction

This thesis studies the patterns of business cycle synchronization between 17 industrialized economies and the aggregate Euro Area, on one hand, and the US, on the other hand, over the period 1970-2008. Our motivation is twofold: first, given the efforts of economic integration in Europe, we want to assess whether they have impacted the participating economies' cyclical synchronization; second, we further aim at assessing whether those efforts altered the cyclical position of the European economies with respect to the US, which is arguably a world-wide "*cyclical attractor*". Hence, the European integration is the central motivation for this research, which aims at providing an analysis of its cyclical impacts with respect to the two worldwide main currency areas, the aggregate Euro Area and the US.

To understand our argument, one must however take account on some specific monetary episodes of the past century. By the end of the 1960s, the *Bretton-Woods* system of fixed but adjustable exchange rates was showing signs of fragility in maintaining macroeconomic stability, as a result of significant divergences in economic policy across its members. This promoted the interest in enlarging the already ongoing process of integration within Europe, expanding it from a customs union into a fully-integrated monetary system. The disruption of the *Bretton-Woods* in the beginning of the 1970s marked the start of monetary integration within Europe, with the implementation of an exchange-rates system with narrow fluctuation bands. Under this system, the Benelux countries, Sweden and Norway locked their currencies to the Deutsche Mark.

With the creation of the European Monetary System (EMS hereafter) in March 1979, this process was further deepened, with the participation of some European Economic Community members in a system of fixed exchange rates, based on a grid of parities against the European Currency Unit (ECU). The introduction of the Common Market rules in 1986, preconizing free circulation of people, goods and capital, was yet another factor which contributed to widening economic integration and promoting policy cooperation. Nevertheless, the full potential of these rules was not achieved during the first years of their implementation, mainly due to the still existence of persistent currency misalignments and transaction costs which tended to affect intra-community trade.

This set the stage for the adoption of convergence rules which would prepare each member economy for the adoption of a common currency by the end of the 1990s. These rules embodied clear guidelines for the conduction of fiscal and monetary policy, and potentially created sound macroeconomic stability and harmonization in the run-up to the common currency. Most importantly, the introduction of the Euro, created a new macroeconomic block, with similar geographical magnitude as the US, and a new currency which would potentially be a rival to the US dollar. All this process of integration created conditions to increase cyclical affiliations across European countries, particularly for those that adopted the Euro[1].

As is well known, the existence of an efficient *one-size-fits all* monetary policy requires that business cycles are homogeneous and move together, such that the response of policy to shocks suits all countries. In fact, a set of countries can only benefit from a common monetary policy if, at each point in time they are in the exact same state of the business cycle. Hence, the Euro Area will only be a well- functioning currency area under a very high synchronization of business cycles across its member states. This clearly shows the importance of the analysis of business cycle synchronization in Europe.

One advantage of our study is that we do not rely on a restricted set of economies, as the G7. We perform our analysis by focusing our attention on three main economic-geographic blocks: the first comprises Europe (embodying countries belonging to the Euro Area and also non-participants); a North-American block; and Japan. Several studies on business cycle synchronization often try to test the emergence of an Eurozone business cycle, but sometimes rely on a restricted set

[1] A deeper analysis to the European integration is carried out in Scheller (2004).

of countries belonging to the currency union, namely Canova et al. (2007), Stock and Watson (2005). We overcome this problem not only by considering all countries which constitute the Euro Area up to 2001, but also, by considering countries outside the Euro. As such, this not only allows evaluating hypothesis concerning directly to the currency union but also, as we also include the complete set of G7 countries, document stylized facts on the cyclical connection between these countries.

Our main motivations allows us to establish a broad set of goals for this research. First, we test whether European integration and particularly, the creation of the European Common Currency Area has promoted an increase in business cycle synchronization. This, leads us to the second goal for this study namely, we test whether the integration process has changed the cyclical affiliation of European countries with respect to the US. Most of the findings presented by the literature favor the existence of an increase in business cycle synchronization within Europe, as a consequence of the process of political and economic integration. Nevertheless, since the US was viewed as the central economy up to the start of the integration process, we are interested in testing whether the cyclical affiliations have changed. Third, we analyze the importance of common fluctuations with the Euro Area and the US, contrasting them with country-specific fluctuations. Hence, we are also interested in testing the hypothesis that an Euro Area business cycles has emerged in the recent years. If this hypothesis is true, this should come up as as increase in common fluctuations with the Euro Area.

To carry out this analysis, we propose a new multivariate unobserved-components model with changes in regime, based on the trend-cycle decomposition proposed by Lam (1990), assuming the existence of a common component capturing common variability in business cycles which may "naturally" model the comovements across business cycles. Our model incorporates into a state-space model the idea of state interdependence proposed by Camacho and Perez-Quiros (2001), as we assume that when countries have synchronized business cycles the dynamic system must be in the same state. To estimate this new model, we propose a novel nonlinear filter which allows us to obtain projections of the unobserved components and estimates of the hyperparameters, and a new fixed-interval smoother to obtain the unobserved components taking into account all the in-sample information. Our approach is particularly flexible and provides a rich set of information regarding the cyclical links between economies. First, on the basis of a common component, we construct a time-varying measure of business cycle synchronization. Secondly, our approach delivers a probabilistic inference on whether countries are in the same state (recession and expansion) at each point in time. Third, our model allows measuring the importance of common fluctuations for the business cycle component of each country's output.

The inclusion of countries outside the Euro Area, and specifically, the results we document for cyclical affiliations with the US, also serve the purpose of evaluating the ability of the model to successfully capture the comovements between business cycles. Consistent with our motivations and objectives outlined above, we focus on bivariate systems with the aggregate Euro Area and the US.

The following main results emerge from our investigation. First, the estimates suggest that business cycle synchronization of the Euro Area countries with the aggregate Area is higher than the synchronization for the remaining countries. Our estimates also suggest that, for the majority of the Euro Area economies, synchronization increases during the 1970s till the beginning of the 1990s and decreases from that period onwards. An interpretation of this result would lead us to consider the introduction of the Single Market rules and also, the adoption of nominal convergence rules adopted by some of the European countries during the preparation for the adoption of the common currency, as factors which fostered desynchronization between the business cycles. Notwithstanding the drop in comovements, we are able to detect a tenuous increase in synchronization preceding the introduction of the common currency but rapidly reverted after this event. Hence, we are not able to uncover a clear effect of the introduction of the Euro in synchronization.

Second, it is shown that France is the country with higher synchronization with the aggregate Euro Area, but our method estimates a drop in comovements in the 2000s, in line with the remaining countries. In contrast, the Japanese economy is shown to be loosely locked to the aggregate Euro Area, with comovements dropping during the 1990s (mainly due to the well known depression).

Third, we show that synchronization of Eurozone countries is higher with the aggregate Euro Area than with the US economy. We provide evidence that for the majority of economies of the Euro Area, the synchronization with the business cycle of the US economy, either remained relatively stable across the entire sample or dropped. In contrast, an interesting characteristic of the estimates for some European countries is that the level of synchronization with the US business cycle increased until the end of the 1990s, and then, around 1999-2000, began decreasing; the decrease persisted until the end of the sample, and we argued that it may be interpreted as a break in synchronization with the US resulting from introduction of the Euro.

Our discussion of the results confirms the ability of our model to robustly identify the patterns of synchronization in cross-country real fluctuations, and largely validates our findings described in summary above, suggesting also that common fluctuations with the Euro Area are not very important in explaining real fluctuations in most of the Euro Area countries, with the same happening for the US. Hence, we find no support for the evidence of the emergence of an Euro Area business cycle, as suggested by Canova et al. (2007).

The rest of the thesis proceeds as follows. Chapter 2 reviews the literature on the theoretical foundations of business cycle synchronization and on recent empirical contributions, both assuming linear and nonlinear business cycles. Chapter 3 presents the model, the new nonlinear filter and the new fixed-interval smoother used in the estimation of the model. Chapter 4 presents the data used in this study, the results and our discussion of the findings. Chapter 5 concludes. An appendix is included, presenting additional graphics not presented in the main body of the text.

2 Business Cycle Synchronization: a Brief Overview of the Literature

This chapter provides a review of the literature that defines the roots for the analysis of business cycle synchronization, looking at its importance, main determinants and results, both theoretically and empirically. At the empirical level, we divide the description into two bodies of the literature: one which assumes that cyclical fluctuations are in all periods linear and, on the other hand, one that assumes that business cycles are nonlinear.

2.1 The Theoretical Analysis of Business Cycle Synchronization

The theoretical motivation for the analysis of business cycle synchronization comes from two different fields of economic thought: (i) the literature that analyses the optimality of currency areas and, (ii) the literature that studies the effects of international trade, financial links, international transmission of shocks, and the emergence of international business cycles. This section is devoted to frame our analysis within these theoretical frameworks, explicitly addressing the main differences between the two fields.

The optimality of exchange rate systems has been one of the most active areas of research in international macroeconomic theory in the recent decades. Usually, a flexible exchange rate regime is defended on grounds that an increase in the rate of unemployment and/or external deficit is corrected by spontaneous depreciations of the exchange rate, while a spontaneous appreciation of the exchange rate provides the necessary correction when the economy faces an increase in the rate of inflation or an excessive external surplus. However, the ability of exchange rate oscillations to correct external and/or internal imbalances depends on the economic domain in question. This argument was originally put forward by Mundell (1961) in his Optimum Currency Areas (OCAs hereafter) theory. In summary, the theory of OCAs suggests that if two countries are persistently hit by asymmetric shocks (shocks that improve the economic performance of one of the economies and cause a downturn in the other) then a currency area between the two economies would imply severe welfare costs and a flexible exchange rate between both would be optimal.

Following the ambiguity in Mundell (1961) on the definition of an OCA, whether it is constituted by a fixed exchange rate system or, by a common currency area, McKinnon (1963) proposes to define an OCA as a common currency area where economic policies aim to promote internal and external equilibrium, as well as price stability. This motivates his suggestion that the higher the trade intensity between countries, the higher will be the benefits from deepening the anchoring of the exchange rate or of forming a currency union. In fact, the higher the bilateral trade the less desirable are exchange rate fluctuations since, (i) it increases the probability of large price movements and (ii), introduces exchange rate uncertainty and transaction costs[2].

The theory developed by Mundell (1961) and McKinnon (1963) has put business synchronization at the forefront of international monetary economics. On the one hand, the existence of asymmetric shocks directly implies that the degree of business cycle synchronization is low. On the other hand, the existence of high bilateral trade increases business cycle synchronization, since trade increases the degree of integration between economies. Hence, the nature and dimension of asymmetric shocks, as well as trade, prove to be critical for business cycle synchronization and to the desirability of international monetary integration, i.e., to the optimality of a common currency.

This point has been further developed by Frankel and Rose (1998) who suggested that the optimality of a currency area, even when not verified *ex ante* could (and most likely would) be observed *ex post*, i.e. OCAs are endogenous to the adoption of the common currency. In short, the introduction of a common currency fosters trade integration (due to the reduction of exchange rate risks and transaction costs) and consequently, increases business cycle synchronization.

The theory of endogenous OCAs provides an important motivation for the analysis of business

[2] A formal operationalization of the OCAs theory is provided in Ricci (1997).

cycle synchronicity, especially between the countries that participate in the Euro Area. If such theory is right, then an increase in business cycle synchronization between Euro area countries is expected. Moreover, this theory motivates the study of business cycle synchronization in Euro Area within a time-varying framework: even if synchronization is low before the formation of the EMU, we should expect it to increase after the inception of the common currency.

A contrary hypothesis was provided by Krugman (1993). According to his view, the increased integration resulting from lower transaction costs and higher trade tends to foster specialization and geographical concentration of industries. The increase in specialization increases the likelihood of industry-specific shocks and hence, country-specific shocks which would lead to growth divergence across countries and promote less cyclical synchronization.

The theory of OCAs is rooted on the Keynesian assumptions of rigid wages and prices and of imperfect labor mobility. However, different assumptions about the general equilibrium adjustment of the economy led Backus et al. (1992, 1995) to analyze the nature of international comovements in aggregate variables, using a multi-country model nested in the theory of real business cycles, i.e., a model in which the economy features a sufficient degree of flexibility in prices, so that they systematically attain the Walrasian equilibrium in a dynamic stochastic general equilibrium framework. Their model seems however unable to match the size of international comovements of economic variables, as they find that consumption tends to be more correlated across countries than real output (a finding clearly at odds with the data). Kehoe and Perri (2002) use a similar framework to analyze the effects on international comovements of incomplete financial markets and imperfect risk sharing between economies. These assumptions help to reconcile some of the anomalies of the Backus et al. (1992) model but still fail to give a full account of international synchronization of business cycles since the correlation between the cyclical components of real output are systematically lower than the ones produced by the data.

Despite setting a general framework for policy analysis, the international real business cycle literature proposed by Backus et al. (1992) hasn't been originally devoted to analyze the effects of common monetary policy on international comovements of business cycles. However, this topic is of great importance since the creation of the Euro Area and the introduction of common monetary policy. The international output comovement have also been studied more recently, in the New Keynesian framework[3]. Within this framework, Corsetti and Pesenti (2002) and Faia (2007) filled this gap and provided an analysis of the effects of common monetary policy on business cycle correlations through Multi-Country New Keynesian models of the business cycle. Overall, the literature finds that output correlation will always be higher under a monetary union than under flexible exchange rates despite delivering lower welfare. Faia (2007) includes in her analysis the effects of differences in financial structures. The results suggest that the correlation of business cycles decrease with the increase in financial heterogeneity. Hence, at least theoretically, it is expected that (i) the elimination of capital controls during the run-up to the EMU and (ii) the introduction of the Euro, have led to an increase in the comovements of the Eurozone economies and to a decrease in the potential costs of giving up national currencies, as a tool for the stabilization of each economy.

2.2 The Empirical Evidence on Business Cycle Synchronization

2.2.1 Linear Models of the Business Cycle and Synchronization

In the recent years, a large body of empirical research has investigated the main determinants of comovements of real output across countries and addressed the question of whether post-war business cycles have become more synchronized and similar. Different studies have explored different causes for synchronization such as globalization, lower incidence of idiosyncratic shocks and the prominence of global shocks, monetary integration, higher coordination of stabilization poli-

[3]This framework has been popularized by the New Open Economy Macroeconomics advanced by Obstfeld and Rogoff (1996).

cies. The results are rather mixed, as are the samples and frequency of the data, the methods employed to extract business cycles from the original data and the methods for the analysis of synchronization. Here, we review some relevant contributions to the empirics of business cycle synchronization[4].

The analysis of the main determinants of business cycle synchronization has been largely connected to the theory of OCAs and, as such, has looked largely at the relation between the asymmetry of shocks, trade intensity, the convergence of economic policies and business cycle correlations.

Following the work of Frankel and Rose (1998), several authors have provided robust evidence for the relation between trade and business cycle comovements, e.g. , Baxter and Kouparitsas (2005), Böwer and Guillemineau (2006), Furcery and Karras (2007), Schiavo (2008) and Inklaar et al. (2008). In general, they have found a positive effect of trade intensity on business cycle synchronization, somehow validating the endogenous nature of the optimality of a currency area. Yet, Inklaar et al. (2008) argue that the effect of trade on comovements is lower than that predicted by Frankel and Rose (1998), because the relation is not constant along the cross-section of countries, i.e., for counties with already highly synchronized business cycles, the effects of trade tend to be lower. Moreover, they've shown that the estimates tend to be biased upwards when one doesn't control for policy variables.

Other than trade intensity, many other variables have been scrutinized to test whether they constitute determinants of an OCA and thus, of business cycle correlations. For example, Imbs (2004) analyzes the effects of financial integration between 24 countries over the 80s and the 90s, and uncovers an indirect negative effect of financial integration working through trade specialization and a direct positive effect on business cycle synchronization. Baxter and Kouparitsas (2005) analyze (among others) the effects of trade similarity, industrial structure similarity, factor

[4]An extensive survey of the literature of business cycle synchronization within the Euro Area is provided in de Haan et al. (2008).

endowments, population differentials, distance between countries and common borders on business cycle synchronization for 100 developed and developing countries[5]. While bilateral trade intensity, similarity in development patterns and distance between countries seem to be significant determinants of cyclical correlations, other (raw) measures of trade and industrial similarity, factor endowments, population differentials, and common borders do not seem to explain cyclical synchronization. Their results also suggest that currency unions do not seem to be a clear determinant of business cycle comovements (a prediction also found by Schiavo (2008)). A similar approach has been pursued by Böwer and Guillemineau (2006) for the Euro Area countries for the period 1980-2004, considering the effects of financial integration and differences in fiscal positions, accounting for possible structural changes in the relation between these indicators and cyclical synchronization. Financial integration is shown to be a robust determinant of business cycle synchronization while fiscal deficit differentials have decreased their importance over time.

More recently, Silva (2009) analyzed the effects of fiscal similarity, labor market flexibility, differences in business cycle volatility and common borders on business cycle correlations for a sample of 26 European countries. He estimates a positive effect of fiscal similarity and common borders on business cycle synchronization while divergence in volatility substantially reduces comovements. According to his results, labor market flexibility does not seem to have a significant impact on business cycle correlations[6].

Despite its empirical focus, this literature provides theoretical insights for the importance of business cycle synchronization. This importance emerges here by the interactions it plays with

[5]The effects of common borders on business cycle correlations was previously studied by Clark and van Wincoop (2001) who found that the correlations tend to be higher between regions within countries than between neighbour countries. A somewhat contrasting picture had been provided by Fatás (1997) who found that within-countries correlations of employment growth rates have decreased over the period 1966-1992 while cross-countries correlations increased.

[6]Fonseca et al. (2007) assess the impact of labor market institutions on output correlations. They show that divergence in employment policies and institutions causes countries to diverge cyclically when hit by common shocks. Additionally, their empirical results suggest that differences in employment protection do not impact significantly on cyclical comovements.

other determinants of the optimality of a given currency area. In fact, economic features such as fiscal similarity, heterogeneity in labor market flexibility and financial integration, are themselves crucial for the optimality of a common monetary policy. Thus, if business cycle synchronization depends on these factors then, they are a necessary condition for the well functioning of a monetary union[7].

In a different vein of the literature, Bayoumi and Eichengreen (1993) have compared the exposure of European countries and US regions to asymmetric shocks. Estimating supply and demand shocks from a (Blanchard-Quah) Structural-VAR on output growth and inflation, they find that shocks were more correlated across US regions than within Europe during the period 1963-1988. In line with the OCAs theory, this should predict high costs to the adoption of a common currency.

A separate line of literature has not tried to find the determinants of cyclical comovements, but focused on the detection of time-series patterns of synchronization across economies using increasingly sophisticated methods. Lumsdaine and Prasad (2003) estimated an index of global fluctuations by aggregating monthly industrial production indexes for a set of 17 OECD countries with time-varying weights constructed through the conditional variances of individual countries' fluctuations. They argue that the high correlation between the estimated common component and the growth rates of industrial production of individual countries over the period 1963-1994 is a symptom that global shocks are relevant for economic fluctuations. Moreover, their results suggest that fluctuations in the post-1973 period have become more synchronized, confirming the emergence of a world business cycle. Interestingly, despite using pre-Eurozone data, they provide evidence in favor of a European Business Cycle. Canova et al. (2008) estimate a Panel-BVAR for 10 European countries and also find evidence of the emergence of an European cycle during the period 1970-2007.

[7]Camacho et al. (2008) argue that despite being a necessary condition for assessing the costs of monetary integration, it is not a sufficient condition. They propose a careful analysis to the shape of economic fluctuations across countries namely, their length, depth and excess growth/contraction to complement that of synchronization.

These results contrast with the ones provided by Canova et al. (2007), who using a Panel-BVAR, find no evidence of the emergence of an Euro Area cycle significantly different from the G7 cycle. Moreover, they find some evidence of an increase in synchronization for the UK, France and Italy, while the US, Canada and Germany's business cycles have progressively become less synchronized. These results are highly relevant since Germany is usually regarded as the main "engine" behind the fluctuations in the Euro Area. Afonso and Furceri (2007) also show that France increased its affiliation with the EMU aggregate cycle, while Germany displays the opposite.

A specific strand of literature models comovements through trend-cycle decompositions within an unobserved-components framework. Luginbuhl and Koopman (2003) model the convergence in business cycles through a gradual reduction in the eigenvalues of the covariance matrix of cycles within a multivariate unobserved components model of real per-capita GDP for 5 European countries for the period 1970-2001. Their results suggest that Germany and France play the role of attractors in relation to the other countries' cycles and that full convergence is estimated to have been attained at the beginning of the 1990's. Convergence analysis between the cycles of real per capita GDP was also undertaken by Carvalho and Harvey (2005) through a multivariate unobserved components model with *similar cycles* and convergence. Their results suggest that convergence took place mainly during the 1990's for France, Germany, Belgium, The Netherlands and Italy.

Recently, Koopman and Azevedo (2008) proposed an alternative approach based on the introduction of phase-shifts to analyze synchronization (a method previously proposed by Rünstler (2004)) and of deterministically-varying correlation to model the convergence among stochastic cycles. They estimate an increase in synchronization with the Eurozone cycle for Germany, France, Italy, UK and The Netherlands. Interestingly, convergence between Italy and the Eurozone cycle decreased despite remaining at high correlation levels. Similarly to the results obtained by Luginbuhl and Koopman (2003), the increase in correlations was most significant in the beginning of the 1980's (close to the start-up of the European Exchange Rate Mechanism) as well as during the mid-nineties (after the introduction of the European Single Market).

These results suggest that the political and institutional events of integration in Europe such as the creation of the European Monetary System, the introduction of the Single Market, the ratification of the Maastricht treaty and the introduction of the Euro may have contributed to an increase in the cyclical comovements inside Europe. Canova et al. (2008) tested if the Maastricht treaty and the creation of the ECB contributed to an increase in business cycle synchronization. Despite finding an increase in comovements between 10 European countries over the period 1970-2007, they haven't found an explicit connection with those two institutional events.

A different branch of the literature follows Stock and Watson (1988) and estimates dynamic factor models (DFMs hereafter) to recover common components of international fluctuations. Kose et al. (2003) assess whether globalization has affected the synchronization of business cycles of 21 industrialized and 55 developing economies in the period 1960-1999 using a DFM to analyze the relative importance of common factors over time. Their results suggest that comovements have increased for developed countries during the 70's, 80's and the 90's, but have decreased over time for developing countries. Del Negro and Otrok (2008) propose a DFM with time-varying parameters and GARCH effects on the variances of the components and estimate the model for 19 industrial economies over the period 1970-2005. Their approach is particularly flexible and allows the computation of a measure of correlation. For the G7 economies, they found a sharp decrease in the degree of synchronization since the 1970's, while regarding the Euro Area, they found a very low and declining cyclical synchronization.

This result contrasts with evidence presented in previous literature such as Lumsdaine and Prasad (2003), Luginbuhl and Koopman (2003), Carvalho and Harvey (2005) and Koopman and Azevedo (2008). However Stock and Watson (2005) also provide extensive evidence of low synchronization between G7 countries using quarterly data over the period from 1960:1 to 2002:4.

They propose the view that the moderation of common international shocks contributed not only to the moderation of business cycle volatility but also to the inexistence of an upward trend in correlations during the period[8]. Yet, they find evidence of the existence of an Eurozone group, distinct from an English-speaking group, the first comprising Germany, France and Italy while the second composed by the US, Canada and the UK. Further evidence on this topic had been previously provided by Camacho and Perez-Quiros (2001).

The relevance of the enlargement of the EU has also been taken into account by the literature. Eickmeier and Breitung (2006) consider whether new EU member states are ready to enter the Euro Area. They condition their preliminary analysis first on descriptive statistics for structural similarity and investment flows intensity and second on the dynamic correlation statistic developed by Croux et a. (2001) computed over the period 1993:1-2005:2 for GDP growth. They show the existence of high synchronization between Hungary, Slovenia and Estonia and the aggregate Euro Area, and also, the fact that correlations between new member states and the Euroland are higher then between the later and some of the current member states. Moreover, they estimate Structural-DFMs to analyze the existence of common shocks and their transmission to the new member states. For most of these countries, they suggest that the transmission is quite similar to the current member states.

Darvas and Szapry (2008) analyze the cyclical links between 8 Central and Eastern European Countries and 10 countries belonging to the EU using quarterly data covering the period 1983-2002. They compute several indicators of synchronization namely, correlation coefficients (contemporaneous and intertemporal), volatility, persistence and responses to an Euro Area shock. Their results are similar to the ones provided by Eickmeier and Breitung (2006) in that Hungary, Poland and Slovenia and highly correlated with the Euro Area. Moreover, they find that with the exception

[8]A continuous decrease in the variance of business cycles commonly denominated as *Great Moderation* has been repeatedly reported by the literature, for example, McConnell and Perez-Quiros (2000), Doyle and Faust (2002,2005), Stock and Watson (2005) and Del Negro and Otrok (2008).

Finland, Ireland, Portugal and Spain, the remaining EU countries are strongly connected with the aggregate Euro Area. To control for countries outside the Eurozone, they include 8 additional economies outside the Eurozone. They argue in favor of the emergence of a world business cycle.

Overall, the empirical evidence on cyclical comovements is mixed and the differences between the models proposed in the literature makes the comparison of results a difficult task. First, some literature does not model synchronization directly or uses rather naive measures of synchronization. Canova et al. (2007) is an example where correlation analysis is made posterior to the estimation of the business cycle components. Second, some literature disregards the time-varying nature of synchronization, for example Croux et al. (2001) and Azevedo (2002). As mentioned before, the possible existence of endogeneity in the synchronization of business cycles, as proposed by the OCA theory, provides sufficient reasons to avoid such assumption. Third, some literature presents point estimates of correlations or related measures of synchronization without a proper statistical testing of their significance, for example Croux et al. (2001), Karras and Stokes (2001), Azevedo (2002), Furcery and Karras (2007). If there are significant fluctuations in the variability of the business cycles this turns a correlation analysis of questionable relevance.

The literature has used many forms of taking into account the possible variation of the degree of business cycle synchronization. For example, Luginbuhl and Koopman (2003), Carvalho and Harvey (2005), Koopman and Azevedo (2008), Del Negro and Otrok (2008) use models that allow modeling time-varying synchronization or convergence between business cycles. Doyle and Faust (2002,2005) and Stock and Watson (2005) use rolling correlations as an indicator of varying synchronization while Karras and Stokes (2001), Belo (2001), Stock and Watson (2005), Furceri and Karras (2007) and Silva (2009) split the samples to detect increases or decreases in synchronization.

Regarding statistical testing, several studies have suggested formal tests of synchronization for example, Belo (2001) tests for convergence between the Euro Area cycle and the cycle of 17 industrial economies, finding evidence of convergence between the Euro Area countries and the aggregate Euro Area cycle. Bodman and Crosby (2005) use a parametric test for the null of independence of chronologies of business cycles and find evidence of dependence between the business cycle regimes across the G7 economies, Doyle and Faust (2005), derive a formal test for changes in the correlation and covariance and present evidence of significant decreases in the covariance between the growth rates of G7 countries and no significant increases in correlation coefficients. Silva (2009), using the same test as Belo (2001) has found a significant increase in synchronization between the Euro Area cycle and the countries belonging to the monetary union.

Among the problems in the literature just described, there is, in our view, a special benefit to be expected from modelling the degree of cyclical synchronization as a time-varying phenomenon, as in Luginbuhl and Koopman (2003), Carvalho and Harvey (2005), Koopman and Azevedo (2008), Del Negro and Otrok (2008).

2.2.2 Nonlinear Business Cycles and Synchronization

The literature reviewed in the previous section assumes that the business cycle can be well approximated by a linear time series process. However, the recognition of asymmetries in business fluctuations goes back at least to Keynes (1936, page 314) who wrote:

"*(...) the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no sharp turning-point when an upward is substituted for a downward tendency.*"

In the recent decades, the hypothesis of nonlinear business cycles has been revived by the seminal paper of Neftçy (1984), who decisively contributed to the establishment of the stylized fact of the existence of nonlinearities in post-war business cycles. Simultaneously, throughout the years, much of the business cycle literature has focused on the identification and measurement of the many types of asymmetries present in real fluctuations as well as on their causes.

The issue of cyclical nonlinearities is highly relevant as the shape of economic fluctuations is very important for determining the optimality of a unique monetary policy in a currency union. This argument has been proposed by Camacho et al. (2008) who argue that the similarity of business cycles is an important determinant of the costs and benefits arising from joining a currency union. If countries have similar patterns of cyclical nonlinearities, then, the costs of adopting a common currency could decrease, since other things equal, this would be an additional factor contributing to an increase in business cycle synchronization.

The definition of business cycle asymmetry established by the literature, involves a set of characteristics that may be summarized as (i) deepness (usually referring to troughs being more deep that peaks, see Sichel (1993)), (ii) steepness (recessions steeper than expansions, see Sichel (1993)), (iii) duration (expansions longer than recessions) [9]. Overall, these features of the business cycle imply a cyclical behavior with different patterns in the two possible states - expansion versus recession. This calls for theoretical and empirical models that are flexible enough to capture nonlinearities of this kind.

Several theoretical justifications have been advanced in order to build models that endogenously produce asymmetric business cycles. Motivated by the evidence that the amplitude of a recession is highly correlated with that of the preceding expansion, Friedman (1993) proposed the Plucking model of the business cycle, where output is randomly plucked downwards during recessions, subsequently reverting to the trend. This theory implies the existence of a ceiling level for real output. Acemoglu and Scott (1997) incorporate intertemporal increasing returns in individual firms' investment decisions to create persistence in economic fluctuations and sharpness in turning points. Their arguments rest on the idea that maintenance reduces the costs of adopting new technologies and increases the productivity of existing ones. Hence the probability of investing increases if firms have already invested in the recent past. Hansen and Prescott (2005) propose the incorporation of capacity constraints at the plant level to account for deepness differentials

[9] McKay and Reis (2008) also find that contractions in employment are briefer and more violent that expansions.

between expansions and recessions. Recently, McKay and Reis (2008) introduce asymmetric costs on labor adjustment by firms to account for the fact that contractions in employment are briefer and more violent that expansions. Asymmetry in costs arises since firms need to train newly hired workers while the marginal costs of firing a worker are approximately independent of the amount of workers fired. Overall, theoretical models seem to indicate that the behavior of firms seem to be one key element in explaining asymmetries in business cycles.

Chalkley and Lee (1998) suggest that risk aversion and noisy aggregate information may well be a source of the difference between the violence of contractions and the slow adjustment during expansions. At peaks, information about the state of the economy is precise and agents react immediately to shocks; at troughs however, there is higher uncertainty, which delays agent's responses during the recovery.

Camacho et al. (2008) also present evidence of significant nonlinearities in 30 industrialized economies. Moreover, they argue that differences in the shape of economic fluctuations of European countries haven't decrease and this was also accompanied by a lack of convergence in business cycles.

The recent development of time series models able to reflect an asymmetric behavior of business cycles has been crucial. In this literature, the seminal paper by Hamilton (1989) in which economic fluctuations are generated by a markov-switching model has been particularly important[10]. The baseline markov-switching model assumes that real GDP growth switches stochastically between two different states, expansion and recession, where switches are governed by a first-order markov-

[10]Several other time-series models were proposed by the literature to account for the asymmetric patterns of real fluctuations. Examples are the threshold autoregressive (TAR) model proposed by Tsay (1989) and Tiao and Tsay (1994), the smooth transition autoregressive (STAR) model proposed by Teräsvirta and Anderson (1992), the self-exciting threshold autoregressive (SETAR) model used by Potter (1995) to test for asymmetries in US real GNP and the current depth of the recession (CDR) model analyzed in Jansen and Oh (1999) and also used in Altissimo and Violante (2001). A review of the STAR family of time series models is provided in van Dijk et al. (2002) while a general review of nonlinear models is provided by Potter (1999). Despite being quite vast, here we only cover in depth the markov switching literature since it will guide us through the rest of the thesis.

chain[11]. Hamilton (1989) successfully applied a standard autoregressive model with switching mean to US real GNP over the period 1952:1-1984:4 and showed that the model is able to replicate the NBER business cycle chronologies, the transitions and the expected durations of both the expansionary and recessionary state. However, the baseline model proposed by Hamilton (1989) assumes, unrealistically, that the business cycle is generated by a process with a unit-root, which is at odds with standard business cycle literature (see Baxter and King (1999)) and the structural representation of real output assumed by the unobserved-components literature (see Kim and Nelson (1999)).

To account for this drawback, Lam (1990) proposed a generalized trend-cycle decomposition with regime-switching for real GNP and an exact filter to evaluate the likelihood function without assuming a nonstationary process for business cycles, while Kim (1994) presented an approximate filter which considerably reduces the computational costs of estimation[12].

The flexibility of the markov-switching model proposed by Hamilton (1989) and Lam (1990) associated with the algorithm presented in Kim (1994) has stimulated several applications of the model. For example, Phillips (1991) proposes a multi-country model of business cycles with regime switching to gauge the patterns of international transmission of shocks; Kim and Nelson (1999) tested the plucking model of the business cycle by assuming that the cyclical dynamics of real output follows a regime-switching process in an unobserved components model, and were not able to reject the hypothesis of the existence of a ceiling level for real output. Kim and Murray (2002) tested for the existence of peak-reversion in recessions, and estimate that between 77% and 96%

[11] Interestingly, the increasing returns theory proposed by Acemoglu and Scott (1997) assumes that business cycles may well follow a markov-switching process. However, they also take into account the case in which fluctuations are generated by a smooth transition regression (STR) similar to the ones analyzed in Teräsvirta and Anderson (1992).

[12] The usefulness of nonlinear models to describe the business cycle is questioned in Engel et al. (2005). They assert that markov-switching models tend to generate expansions with longer duration than they are in reality. However, the wide application of the markov-switching model well documented in the literature, has proved its usefulness as a tool to summarize cyclical information.

of the variance of recessions are attributed to transitory shocks.

The hypothesis of markov-switching has also important implications for the forecasting of economic time series. As argued in Hamilton (1989) if a given series follows a markov-switching process, the forecast produced by a linear model will be suboptimal. Moreover, the claim that the standard DFM proposed by Stock and Watson (1988) fails to capture the 1990's recession in the US real GDP led Chauvet (1998) to propose a DFM with regime-switching on the common factors which delivers a good fit to post-war US data[13].

A number of recent sophistications of the markov-switching regime model have proved to be interesting for business cycle analysis. For example, recent literature has tested for the presence of duration-dependence in expansions and recessions. Examples are Durland and McCurdy (1994) and Lam (2004), who both find evidence of positive duration dependence in recessions, i.e., as the recession ages the probability that it will end increases. This implies that the age of a recession helps to forecast the succeeding state of the economy.

Despite being relatively scarce, there is some literature testing the existence of synchronization between international business cycles in the framework of markov switching models. Camacho and Perez-Quiros (2001) propose a multivariate markov-switching model to estimate the degree of synchronization between the G7 business cycles, finding that the level of comovement is higher than what is commonly suggested in the literature. Their results confirm the existence of an English-speaking and an European group, with high within-group comovements and low between-group comovements. Dueker and Wesche (2001) propose an augmented Probit model with Markov-Switching to compute business cycle indexes. They detect a substantial increase in the correlation between the indexes of Germany, France, Italy and an European Index. A substantial decrease in synchronization is observed for the US during the 90's and for the all sample for the UK[14]. Girardin

[13]Diebold and Rudebusch (1996) have previously pointed this direction but were unable to estimate the model due to the inexistence of econometric methods that allowed the estimation.

[14]It should be noted that their evidence is based on the computation of correlation coefficients between indexes

(2002) tests for the presence of common states and common cycles between 4 Asian countries and generally rejects this hypothesis. Smith and Summers (2005) test for the existence of contagion and common markov states with a cointegrated markov-switching VAR for 6 industrialized countries. Despite the fact that the model provide a poor fit to the data, there is evidence of significant transmission of states between countries, albeit little evidence of synchronization.

after their estimation as in Canova et al. (2007). This can be a rather naive assumption, compared to approaches in which correlation is modeled and estimated directly (see our text above).

3 An Unobserved-Components Model with Markov-Switching and Time-Varying State Interdependence

3.1 The Model

In this section, we suggest a model for the analysis of business cycle synchronization. The model builds on the stylized facts presented in for example, Hamilton (1989) and Lam (1990) namely, that business cycles have asymmetric dynamics that can be well approximated by a model with stochastic shifts between different states. The model provides a novel method to account for cross-country comovements between cyclical fluctuations. The framework outlined here does not suffer from the criticism put forward in section 2.2.1 regarding some recent literature, as we assume that synchronization is a time-varying process (as suggested by the theoretical underpinnings brought by the endogeneity of the OCAs) and as we model the comovements directly within the structural form of the model (rather than conduct any post-filtering analysis of synchronization).

The model here proposed is in the spirit of Smith and Summers (2005) and Camacho and Perez-Quiros (2001), who use multivariate markov-switching models to analyze the extent to which business cycles are synchronized[15]. The former uses a cointegration approach to test the existence of a common markov state and long-run cointegrating vectors; the latter uses a multivariate version of Hamilton's (1989) model and explicitly takes into account the idea that when business cycles synchronize they must be in the same phase of the business cycle, i.e., the idea of state interdependence. Both have, however, some shortcomings that we try to solve with our model. They assume that the degree of business cycle synchronization is constant, which may not be an appealing assumption, as business cycles tend to synchronize with increased trade integration, financial liberalization and monetary unification and these clearly evolve with time in most his-

[15]Bengoechea et al. (2006) use the same model to forecast Euro Area business cycle phases.

torical episodes. Smith and Summers (2005), in spite of incorporating common stochastic trends in the model, obtained rather poor results, both in capturing the main business cycle features and the patterns of comovements between business cycles.

In this dissertation, in contrast, we propose modeling the cyclical dynamics of a vector Y_t by a variant of the unobserved-components model with markov-switching proposed by Lam (1990). Here, Y_t is regarded as a $N \times 1$ vector of real outputs $[y_{1,t}, y_{2,t}, ..., y_{N,t}]'$ for each country $c = 1, ..., N$ under analysis. The model is parametrically represented by:

$$Y_t = \tau_t + x_t \tag{1}$$

$$\tau_t = \tau_{t-1} + \Gamma_{S_t} \tag{2}$$

$$\Gamma_{S_t} = \Gamma_0 + \Gamma_1 S_t \tag{3}$$

$$x_t = \sum_{w=1}^{W} \Phi_w x_{t-w} + \vartheta_{t-1}\mathbf{1} + \epsilon_t \tag{4}$$

$$\vartheta_t = \rho\vartheta_{t-1} + \zeta_t \tag{5}$$

$$\epsilon_t \sim \mathcal{N}(0, \Lambda) \tag{6}$$

$$\zeta_t \sim \mathcal{N}\left(0, \sigma_\zeta^2\right) \tag{7}$$

It is thus assumed that for each country $c = 1, ..., N$, real output can be decomposed into a trend $\tau_{c,t}$ and a cycle $x_{c,t}$, where the trend follows a random-walk with drift Γ_{S_t} which undergoes switches between two different states, recession and expansion, according to a first-order markov-switching process as proposed by Hamilton (1989), Lam (1990) and Kim (1994)[16]. Thus, for each

[16]It is implicit in this formulation that we define the business cycle as the deviation of real output along its trend or potential of the economy. Harding and Pagan (2001,2002) analyze its implications for the empirics of the business cycle namely, its shape, duration and amplitude, and provides a comparison with the classical definition.

country, we explicitly assume two different parameterizations for the trend, defined by equation (2), according to the two different phases of the business cycle. While inside each state the trend is deterministic, the shifts between states are stochastic. S_t is a $N \times 1$ vector composed of the states of the economy of each country $[s_{1,t}, ..., s_{N,t}]' \in \{0, 1\}$. $\Gamma_0 = [\gamma_{0,1}, ..., \gamma_{0,N}]'$ corresponds to a $N \times 1$ vector of mean growth rates when the economies are in recession while Γ_1 is a $N \times N$ matrix of mean growth rates when the economies are in expansion. Hence, when a given economy c is in expansion, its growth rate is represented by $\gamma_{0,c} + \gamma_{1,c}$. However, the design of Γ_1 assumes the possibility that the mean growth rate of each country c is driven by its own markov-switching process and possibly, by the other countries' processes. In this case, the mean growth rate during the expansion phase will be $\gamma_{0,c} + \gamma_{1,c} + ... + \gamma_{0,N} + \gamma_{1,N} s_{N,t}$, which is a setup similar to that previously used in Smith and Summers (2005)[17].

Following Hamilton et al. (2007) and as described above, we normalize the stochastic dynamics of the markov-switching variable by assuming that $S_t = 0$ is associated with a recession period while $S_t = 1$ is associated with an expansion; we further impose that:

$$diag\,(\Gamma_1) > \Gamma_0 \tag{8}$$

where $diag\,(\mathcal{A})$ represents the diagonal of a matrix $\mathcal{A}$. This imposes that during expansions output growth must be higher than during recessions. Similar normalization schemes were applied by Hamilton (1989) and Lam (2004). [18]

[17] Morley and Piger (2008) propose an alternative trend-cycle decomposition with regime switching exploring the well-known Beveridge-Nelson procedure.

[18] Note that one could assume more generally that recessions are also transmitted across countries. In that case, Γ_0 would be a $N \times N$ matrix of mean growth rates during recessions and one would have to redefine the mean growth rate equation as:

$$\Gamma_{S_t} = \Gamma_0 \left(\mathbf{1} - S_t\right) + \Gamma_1 S_t$$

A normalization scheme equivalent to the one above is:

$$diag\,(\Gamma_1) > diag\,(\Gamma_0)$$

Transitions between states of the markov-switching variables $s_{c,t}$ are governed by constant transition probabilities defined by:

$$P\left(s_{c,t}=1|s_{c,t-1}=1\right)=p_{11} \quad , \quad P\left(s_{c,t}=0|s_{c,t-1}=0\right)=p_{00} \ , \ \ c=1,...N \tag{9}$$

Introduction of varying transition probabilities as in Durland and McCurdy (1994), Filardo and Gordon (1998) and Lam (2004) would be straightforward in this setup. However, it is not clear if the benefits from such an option would balance the costs associated to the increase in the dimension of the model. Also, the computational costs associated with the algorithm for estimation, to be presented in the next section, would increase substantially. As such, we decided for a simpler approach as in Hamilton (1989), Lam (1990), Kim (1994) and many others, and assume constant transition probabilities.

The $N\times1$ vector of cycles x_t is assumed to follow a stationary VAR(W) process with a common component ϑ_t. In order to maintain the stationarity assumption of x_t we assume that ϑ_t follows a simple AR(1) with $|\rho|<1$. If this was not the case and the common component followed an integrated process, as pointed by Kim and Nelson (1999), it would not be possible to identify the cycle since both cycle and trend would follow integrated processes. As a consequence, the model would collapse into a version of the baseline Hamilton (1989) model with a unit root on the cyclical component. Note also that in this general model, fluctuations in one country can spread to other countries in the case where $\phi_{ij,w}\neq0$ for $i,j=1,...,N$ and $i\neq j$. Also, $\mathbf{1}$ is a $N\times1$ vector of ones which allows incorporating the common component in the VAR process.

As regards innovations, ϵ_t is a $N\times1$ vector of innovations with mean zero and a $N\times N$ covariance matrix Λ, while ζ_t is an innovation with mean zero and variance σ_{ζ}^{2}. It is assumed that all innovations are mutually orthogonal at all points in time.

This leaves unrestricted the off-diagonal parameters, which measure the effects of transmission of fluctuations across countries. For the empirical application here considered this turns out to be irrelevant since, as discussed below, we will impose some restrictions on these matrices.

The application of the filter outlined below, requires the definition of a measure of comovements, δ_t. A natural candidate is the common component of fluctuations which we interpret here as capturing common cyclical variability. However, as we will describe, this measure must in every period be defined over the interval $[0, 1]$ hence, we postulate the following transformation:

$$\delta_t = 1 - \frac{1}{2}\left(1 - \frac{\vartheta_t}{\sqrt{1 + \vartheta_t^2}}\right) \tag{10}$$

Hence, an increase in ϑ_t makes δ_t approach 1 and, on the other hand, when ϑ_t decreases δ_t approaches 0. We interpret the first case as an increase in synchronization while the second case we view as a decrease in synchronization.

In view of the well known unit root in real GDP, it is also useful to consider the model in its stationary representation. Following Lam (1990) and Kim (1994) we can take first differences to obtain:

$$\Delta Y_t = \Gamma_0 + \Gamma_1 S_t + x_t - x_{t-1} \tag{11}$$

$$x_t = \sum_{w=1}^{W} \Phi_w x_{t-w} + \vartheta_{t-1}\mathbf{1} + \epsilon_t \tag{12}$$

$$\vartheta_t = \rho\vartheta_{t-1} + \zeta_t \tag{13}$$

Where ϵ_t and ζ_t have the same properties as outlined above. This process achieves higher simplicity if we note that S_t can be thought of as a multivariate state of nature composed by each of the realizations of the univariate states. This is the model which we estimate to analyze synchronization. The advantages of this redefinition will be highlighted in the next section where a generalized filter to estimate multivariate unobserved-components models with markov switching is presented.

3.2 Class of Models

This section describes the main assumptions and characteristics underlying the class of models that are considered in this thesis (namely, the model presented in the previous section). Assume that a multivariate unobserved-components model with markov-switching can be cast in the following state-space representation:

$$y_t = H_{S_t}\theta_t + A_{S_t}Z_t + \omega_t \tag{14}$$

$$\theta_t = \widetilde{\mu}_{S_t} + F_{S_t}\theta_{t-1} + \nu_t \tag{15}$$

$$\omega_t \sim \mathcal{N}\left(0, \mathcal{G}\right) \tag{16}$$

$$\nu_t \sim \mathcal{N}\left(0, Q\right) \tag{17}$$

where y_t is a $N \times 1$ vector of observed stationary endogenous variables, each with size T, θ_t is a $R \times 1$ vector of unobserved components, Z_t is a $K \times 1$ vector of exogenous variables, ω_t is an $N \times 1$ vector of innovations in the measurement equation with a $N \times N$ covariance matrix $\mathcal{G}$ and orthogonal to the $R \times 1$ vector of innovations ν_t in the transition equation, which has a $R \times R$ covariance matrix Q. The remaining system matrices H_{S_t}, A_{S_t}, $\widetilde{\mu}_{S_t}$ and F_{S_t} are of size $N \times R$, $N \times K$, $R \times 1$ and $R \times R$ respectively. The subscripts S_t in the matrices $H_{S_t}, A_{S_t}, \widetilde{\mu}_{S_t}, F_{S_t}$ implies that some of the parameters in the state-space are subject to markov-switching. For such a model, the assumption of normality is regarded as strong due to the existence of stochastic discrete shocks in the state-space originated in the markov-switching variable that generates non-normality in the innovations. However, it allows the development of an approximate recursive filter based on the standard Kalman filter.

The model presented in section 3.1 (equations (1)-(7)) is related to this general class of unobserved components models in the sense that we decompose a vector of real GDP's Y_t, the endogenous variables, into an unobserved vector of trends τ_t and an unobserved vector of cycles x_t. The vector of cycles also incorporate an unobserved common component ϑ_t. Hence, these unobserved variables will constitute the state vector θ_t. Markov-switching regime is introduced in the trend equation (2) which makes the empirical model under analysis, a version of the general state-space representation (14)-(17).

The same argument could be made to the stationary formulation represented by eqs. (11)-(13). In this case, the unobserved variables in θ_t are the cycles x_t and the common component ϑ_t. The difference operator makes the markov-switching variables now appear in the canonical equation (11), which implies that the markov-switching regime variable will enter the measurement equation (14) through the state-dependent matrix A_{S_t}.

Assume that in each period, there are N realizations of the markov-switching variable, consistent with the existence of N series in the vector y_t, and which may assume one of the M possible states. The model here considered is then assumed to be multivariate in a sense that $N > 1$ [19]. As such, the application of the filter proposed here, is conditioned on the existence of more than two markov-switching variables and also two endogenous variables. This can be seen as a necessary and sufficient condition for the applicability of the filter. The multivariate state in period t is regarded as a set $S_t = (s_{1t}, s_{2t}, ..., s_{Nt})$, which has M^N possible realizations in each period of time depending on the realizations of the univariate states $s_{c,t}$[20].

Assumption 1 *Each of the realizations of $s_{c,t}$ is generated by a reducible first-order markov-chain with constant transition probabilities, which, for given univariate states $i, j = 1, ..., M$ can be specified as:*

[19] Without loss of generality, we will assume that the number of markov-switching variables equals exactly the number of endogenous variables. It could be the case that some endogenous variable is subject to more than one markov-switching process. However, since we will use the markov-switching variable as source of comovements, this simplifies the exposition of the filter.

[20] Multivariate states, as S_t, will be denoted hereafter by capital letters.

$$P\left(s_t = i | s_{t-1} = j\right) = p_{j,i}, \quad \text{with} \quad \sum_{i=1}^{M} p_{j,i} = 1 \tag{18}$$

where $p_{j,i}$ represent the probabilities of transition from state j to state i. It is common to write the transition probabilities in a $M \times M$ matrix that in the univariate case takes the form:

$$\boldsymbol{p}_{(M \times M)} = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{M1} \\ p_{12} & p_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ p_{1M} & \cdots & \cdots & p_{MM} \end{pmatrix} \tag{19}$$

As described in Phillips (1991), it is possible to show that the multivariate transition probabilities are constructed through linear combinations of the univariate transition probabilities $p_{j,i}$. Hence, we can specify a $M^N \times M^N$ transition probability matrix for the multivariate case as:

$$\Psi_{(M^N \times M^N)} = \begin{pmatrix} \psi_{11} & \psi_{21} & \cdots & \psi_{M^N 1} \\ \psi_{12} & \psi_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{1M^N} & \cdots & \cdots & \psi_{M^N M^N} \end{pmatrix} \tag{20}$$

where $\psi_{k,l} = P\left(S_t = l | S_{t-1} = k\right)$ denotes the probability of transition from multivariate state k to multivariate state l which is computed through combinations of the univariate ones. In this dissertation our interest amounts to two possible extreme cases: the first, in which the univariate states of nature are assumed to be completely independent; and the second, in which they are completely dependent. This implies different parameterizations for the matrix of multivariate transition probabilities. For example, without loss of generality, if we assume two endogenous variables, $N = 2$, and two states of nature, $M = 2$, we could write the transition probability matrix in the case of statistical independence and dependence respectively as:

$$\Psi^{I}_{(4\times4)} = \begin{pmatrix} p^1_{11}p^2_{11} & p^1_{12}p^2_{11} & p^1_{11}p^2_{12} & p^1_{12}p^2_{12} \\ p^1_{21}p^2_{11} & p^1_{22}p^2_{11} & p^1_{21}p^2_{12} & p^1_{22}p^2_{12} \\ p^1_{11}p^2_{21} & p^1_{12}p^2_{21} & p^1_{11}p^2_{22} & p^1_{12}p^2_{22} \\ p^1_{21}p^2_{21} & p^1_{22}p^2_{21} & p^1_{21}p^2_{22} & p^1_{22}p^2_{22} \end{pmatrix} \tag{21}$$

$$\Psi^{D}_{(4\times4)} = \begin{pmatrix} p^1_{11} & 0 & 0 & p^1_{12} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ p^1_{21} & 0 & 0 & p^1_{22} \end{pmatrix} \tag{22}$$

As the number of states M and endogenous variables N increases, these matrices become more complex to construct, hence, despite setting the operational framework for the estimation algorithm proposed in the next section, we follow a different strategy that avoids that complexity. Since the unobserved components model is assumed to be multivariate, then, the multivariate markov states should guide the estimation of the parameters present in each of the system matrices.

Several methods have been presented in the literature to estimate the class of models here considered. Shumway and Stoffer (1991) propose a state-space model where the system matrices of the measurement equation switch according to a serially uncorrelated unobservable discrete variable. However, the assumption of serial independence may not be realistic as argued by Kim (1994) and moreover, the existence of markov-switching only in the measurement equation is highly restrictive. Kim (1994) proposes a a state-space model with markov-switching on both measurement and transition equations, where the discrete switching variables are guided by a markov-chain. However, the filter proposed by Kim (1994) requires that when two or more markov-switching variables drive the model, they must be independent for the filter to be operable. Both approaches are then highly restrictive, either because (1) the variables present in the state-space representation may show some degree of comovement, which turns implausible the assumption that the states of nature described by the markov-switching variables are statistically independent or because (2) the degree of interdependence may vary in time. Hence, if these two features characterize the data under analysis, a new model-based Kalman filter must be developed[21].

Camacho and Perez-Quiros (2001) provide the necessary motivation for the existence of interdependence of states by using a multivariate autoregressive markov-switching model to analyze business cycle synchronization in the G7 countries. However, they consider the case in which synchronization is constant in time, which may not capture accurately the dynamics of comovements between business cycles. In fact, as seen in chapter 2, it is possible that synchronization increases as economies become more integrated, as argued for example by Frankel and Rose (1998) and Schiavo (2008).

3.3 A General Nonlinear Filter and Quasi-Maximum Likelihood Estimation

This section proposes a new method for signal extraction in the state-space model with markov-switching embodying time-varying state interdependence that we have setup in section 3.1 above. The model belongs to a class of models that may be used to describe the dynamics of business cycle synchronization.

Our strategy for estimation consists in two stages: first, we decompose the multivariate state-space representation into simpler univariate state-space representations for each endogenous variable in y_t and use them to compute probabilistic inferences about the states of nature for each variable; second, we then use those inferences to construct multivariate probabilistic inferences and

[21] Carlin et al. (1992) proposed a general framework, where nonlinearity is introduced by scale distributions and a Gibbs-sampler approach to estimate the unobserved components and the parameters. Their approach does not encompass the markov-switching model proposed by Hamilton (1989) and Lam (1990).

estimates of the parameters of the multivariate model through quasi-maximum likelihood. The advantage of our approach is that it fully exploits the possible comovements between the variables and uses it to construct inferences about the set of states of nature S_t. It is operational, since both univariate and multivariate unobserved components models may be cast in the state-space form defined in equations (14) and (15).

In our empirical framework, the multivariate model is represented by eqs. (11)- (13) together with eqs. (6)-(7). Univariate models are obtained by setting $\vartheta_t = 0$ for $t = 1, \ldots, T$ and $c = 1$, where c indexes the number of countries, which amounts to the trend-cycle decomposition proposed by Lam (1990). This univariate trend-cycle decomposition will be used to construct the probabilities that each country c is in the state of expansion and recession, which will allow the construction of multivariate probabilities through a mechanism introduced by Camacho and Perez-Quiros (2001). Multivariate probabilities are then used to evaluate the log-likelihood and estimate the parameters of the model.

Since the framework here described can suffer from the problem of dimensionality, we assume that the parameters in the univariate model are the same as those in the multivariate model, namely, the autoregressive process for the business cycles and the state-dependent mean growth rates of real output.

Assumption 2 *The $\kappa \times 1$ vector $\alpha \in \Theta^{\kappa}$, collects all the hyperparameters to be estimated, where $\Theta^{\kappa} \subset \Re^{\kappa}$ defines the permissible parameter space as a subset of the Euclidean space.*

Here, the set Θ^{κ} should respect the normalization and identification principles outlined in Hamilton et al. (2007). The problem of normalization in estimation of markov-switching models, but in the context of Bayesian estimation is also addressed in Frühwirth-Schnatter (2001).

Let $\left(\widetilde{j}, j\right)$ denote the transition from the univariate state $\widetilde{j}$ to the univariate state j, (k, l) denote the transition from the multivariate state k to the multivariate state l, define $\Omega_{c,t-1} =$

$(y_{c,t-1}, y_{c,t-2}, ..., y_{c,1}, Z_{c,t-1}, Z_{c,t-2}, ..., Z_{c,1})$ and $\Omega_{t-1} = (y_{t-1}, y_{t-2}, ..., y_1, Z_{t-1}, Z_{t-2}, ..., Z_1)$ as the information available up to time $t-1$ in the univariate and multivariate cases respectively, $s^*_{c,t} = (s_{c,t}, s_{c,t-1})$ and $S^*_t = (S_t, S_{t-1})$ the sets containing the information on the univariate and multivariate states of nature at time t and $t-1$. The recursions used can then be obtained similarly to the standard Kalman filter. Conditional on the transitions above, the vector of hyperparameters α and the information sets above, projections and updates of the state vector θ_t can be computed as in Kim (1994):

- *Step 1*: Compute a projection of the state vector $E\left(\theta_{c,t}|\Omega_{c,t-1}, s^*_{c,t}\right) = \theta_{c,t|t-1}$ using information up to $t-1$ and the Mean-Squared-Error matrix $E\left[\left(\theta_{c,t} - \theta_{c,t|t-1}\right)\left(\theta_{c,t} - \theta_{c,t|t-1}\right)^{'} |\Omega_{c,t-1}, s^*_{c,t}\right]$ $= \mathcal{P}_{c,t|t-1}$ for the univariate state-space model:

$$\theta^{(\tilde{j},j)}_{c,t|t-1} = \widetilde{\mu}_{s_t} + F_j \theta^{\tilde{j}}_{c,t-1|t-1} \tag{23}$$

$$\mathcal{P}^{(\tilde{j},j)}_{c,t|t-1} = F_j \mathcal{P}^{\tilde{j}}_{c,t-1|t-1} F^{'}_j + Q_c \tag{24}$$

- *Step 2*: Evaluate the one-step-ahead prediction error $y_{c,t} - y_{c,t|t-1} = \eta_{c,t|t-1}$ and compute the associated MSE, $E\left[\left(y_{c,t} - y_{c,t|t-1}\right)\left(y_{c,t} - y_{c,t|t-1}\right)^{'}\right] = \mathcal{F}_{c,t|t-1}$, using the projection of the state vector $\theta_{c,t|t-1}$ for the univariate model:

$$\eta^{(\tilde{j},j)}_{c,t|t-1} = y_{c,t} - H_j \theta^{(\tilde{j},j)}_{c,t|t-1} - A_j Z_{c,t} \tag{25}$$

$$\mathcal{F}^{(\tilde{j},j)}_{c,t|t-1} = H_j \mathcal{P}^{(\tilde{j},j)}_{c,t|t-1} H^{'}_j + \mathcal{G}_c \tag{26}$$

- *Step 3*: Update the univariate state vector using information up to period t:

$$\theta_{c,t|t}^{(\widetilde{j},j)} = \theta_{c,t|t-1}^{(\widetilde{j},j)} \mathcal{P}_{c,t|t-1}^{(\widetilde{j},j)} H_j' \left[\mathcal{F}_{c,t|t-1}^{(\widetilde{j},j)} \right]^{-1} \eta_{c,t|t-1}^{(\widetilde{j},j)} \tag{27}$$

$$\mathcal{P}_{c,t|t}^{(\widetilde{j},j)} = \left(I - \mathcal{P}_{c,t|t-1}^{(\widetilde{j},j)} H_j' \left[\mathcal{F}_{c,t|t-1}^{(\widetilde{j},j)} \right]^{-1} H_j \right) \mathcal{P}_{c,t|t-1}^{(\widetilde{j},j)} \tag{28}$$

for $\widetilde{j}, j = 1, \ldots, N$. Steps 1-3 set the basic framework to compute the updated probabilities of each endogenous variable being in each state $s_{c,t}$. These probabilities will create the set of multivariate probabilities that span all the information about the states of nature necessary to estimate the parameters. Hence, the univariate models are only necessary to the extent that they allow the evaluation of the conditional density by the prediction- error-decomposition, which is necessary to update the univariate probabilities. From these steps, we save the one-step-ahead prediction errors (25) which will be used in the recursions for the probabilities.

Similar recursions are used for the multivariate model, namely:

- *Step 4*: Compute the projection of the state vector $E\left(\theta_t | \Omega_{t-1}, S_t^*\right) = \theta_{t|t-1}$ using information up to $t-1$ and the associated Mean-Squared-Error matrix $E\left[\left(\theta_t - \theta_{t|t-1}\right)\left(\theta_t - \theta_{t|t-1}\right)' | \Omega_{t-1}, S_t^*\right]$ $= \mathcal{P}_{t|t-1}$:

$$\theta_{t|t-1}^{(k,l)} = \widetilde{\mu}_{S_t} + F_l \theta_{t-1|t-1}^{k} \tag{29}$$

$$\mathcal{P}_{t|t-1}^{(k,l)} = F_l \mathcal{P}_{t-1|t-1}^{k} F_l' + Q \tag{30}$$

- *Step 5*: Evaluate the one-step-ahead prediction error $y_t - y_{t|t-1} = \eta_{t|t-1}$ and compute the associated MSE, $E\left[\left(y_t - y_{t|t-1}\right)\left(y_t - y_{t|t-1}\right)'\right] = \mathcal{F}_{t|t-1}$, using the projection of the state vector $\theta_{t|t-1}$:

$$\eta_{t|t-1}^{(k,l)} = y_t - H_l \theta_{t|t-1}^{(k,l)} - A_l Z_t \tag{31}$$

$$\mathcal{F}_{t|t-1}^{(k,l)} = H_l \mathcal{P}_{t|t-1}^{(k,l)} H_l' + \mathcal{G} \tag{32}$$

- *Step 6*: Update the state vector using information up to period t:

$$\theta_{t|t}^{(k,l)} = \theta_{t|t-1}^{(k,l)} \mathcal{P}_{t|t-1}^{(k,l)} H_l' \left[\mathcal{F}_{t|t-1}^{(k,l)}\right]^{-1} \eta_{t|t-1}^{(k,l)} \tag{33}$$

$$\mathcal{P}_{t|t}^{(k,l)} = \left(I - \mathcal{P}_{t|t-1}^{(k,l)} H_l' \left[\mathcal{F}_{t|t-1}^{(k,l)}\right]^{-1} H_l\right) \mathcal{P}_{t|t-1}^{(k,l)} \tag{34}$$

for $k, l = 1, ..., M^N$. Steps 4-6 are necessary to estimate the unobserved components present in θ_t namely, the cycles and the common component. Moreover, they are necessary to evaluate the conditional and marginal densities which will allow for the update of the multivariate probabilities, used to compute the log-likelihood and estimate the parameters. At each iteration of the filter, there are M^2 predictions, forecast errors and updates, along with the corresponding MSE matrices for each univariate model and $\left(M^N\right)^2$ predictions, forecast errors and updates, along with the corresponding MSE matrices for the multivariate model. A bivariate model ($N = 2$) is translated into three Kalman filters necessary to perform the algorithm, two univariate (one for each markov process)and one multivariate. At each iteration, with a two-state markov-chain, there will be 4 projections, forecast errors and updates of the state vector (plus MSE matrices) for each of the two univariate models and also, 16 projections, forecast errors and updates of the state vector (plus MSE matrices) for the multivariate model.

Assume that state k in $t-1$ and l in t, are associated with the following univariate states:

$$S_t = l \quad \text{if,} \quad s_{1,t} = i, s_{2,t} = g, ..., s_{N,t} = j$$
$$S_{t-1} = k \quad \text{if,} \quad s_{1,t-1} = \widetilde{i}, s_{2,t} = \widetilde{g}, ..., s_{N,t} = \widetilde{j}$$

This will help us in the definition of the forecast function for the multivariate probabilities. Also, we define:

Definition 1 $\delta_t \in [0,1]$ *defines a measure of synchronization between the elements of* y_t.

Remember that we have assumed in our model that $\delta_t = 1 - 0.5(1 - \vartheta_t/\sqrt{1+\vartheta_t^2})$, where ϑ_t is computed at each iteration of the multivariate Kalman filter. Again assuming that the hyperparameters of the model α are known at each point in time, we propose the following nonlinear recursive signal extraction method to obtain the probabilities, which is based in Hamilton (1989) and Camacho and Perez-Quiros (2001):

- *Step 7*: For a given country $c = 1, ..., N$, predict the univariate probabilities:

$$P\left(s_{c,t} = j, s_{c,t-1} = \widetilde{j}|\Omega_{c,t-1}\right) = P\left(s_{c,t} = j|s_{c,t-1} = \widetilde{j}\right) P\left(s_{c,t-1} = \widetilde{j}|\Omega_{c,t-1}\right) \tag{35}$$

and,

$$P\left(s_{c,t} = j|\Omega_{c,t-1}\right) = \sum_{\widetilde{j}=1}^{M} P\left(s_{c,t} = j, s_{c,t-1} = \widetilde{j}|\Omega_{c,t-1}\right) \tag{36}$$

- *Step 8*: Predict the multivariate probabilities using the function defined with possibly time-varying comovement between the elements of y_t:

$$P\left(S_t = l, S_{t-1} = k|\Omega_{t-1}\right) = \left(1 - \delta_{t|t-1}^{(k,l)}\right) P_{t|t-1}^{I} + \delta_{t|t-1}^{(k,l)} P_{t|t-1}^{D} \tag{37}$$

where $P_{t|t-1}^{I}$ and $P_{t|t-1}^{D}$ amounts to the probabilities of being in a given multivariate state when the univariate states of nature are statistically independent and dependent respectively, and $\delta^{(k,l)}$ defines the level of synchronization. This is equivalent to say that the multivariate probabilities depend on whether the unobserved components present in θ_t are completely independent or, at the other extreme, completely dependent. Note also that, in our theoretical definition of the filter, we assume state-dependence for δ_t, which is equivalent to assume that the level of comovement depends on the transitions between states. Additionally, since we have assumed in our empirical application that δ_t depends on ϑ_t, then the former will depend on the transitions (k, l), whenever ϑ_t does. In the case of full independence we can write the probability as:

$$P_{t|t-1}^{I} = P\left(s_{1,t} = i, s_{2,t} = g, ..., s_{N,t} = j, s_{1,t-1} = \widetilde{i}, s_{2,t-1} = \widetilde{g}, ..., s_{N,t-1} = \widetilde{j}|\Omega_{t-1}\right)^{I} = \tag{38}$$

$$= \mathrm{P}\left(s_{1,t} = i, s_{1,t-1} = \widetilde{i}|\Omega_{1,t-1}\right) \times ... \times P\left(s_{N,t} = j, s_{N,t-1} = \widetilde{j}|\Omega_{N,t-1}\right)$$

for $i \neq j$ and/or $\widetilde{i} \neq \widetilde{j}$, where c indexes the number of series in y_t, j indexes the states of nature at time t and $\widetilde{j}$ indexes the states of nature at time $t-1$. In the same way, in the case of full statistical dependence we can write the multivariate probability as:

$$P_{t|t-1}^{D} = P\left(s_{1,t} = i, s_{2,t} = g, ..., s_{N,t} = j, s_{1,t-1} = \widetilde{i}, s_{2,t-1} = \widetilde{g}, ..., s_{N,t-1} = \widetilde{j}|\Omega_{t-1}\right)^{D} = \tag{39}$$

$$= \begin{cases} P\left(s_{N,t} = j, s_{N,t-1} = \widetilde{j}|\Omega_{N,t-1}\right) & \text{if } i = g = \ldots = j \wedge \widetilde{i} = \widetilde{g} = \ldots = \widetilde{j} \\ 0 & \text{if } i \neq g, \ldots, j \vee \widetilde{i} \neq \widetilde{g}, \ldots, \widetilde{j} \end{cases}$$

where $P\left(s_{c,t} = j, s_{c,t-1} = \widetilde{j}|\Omega_{c,t-1}\right)$ is obtained with eq. (35) for $c = 1, \ldots, N$. When the series in y_t are fully synchronized, δ_t equals 1, attributing the full weight to the case in which all series are in the same state. For example, in the case of business cycle synchronization, when business cycles are fully synchronized, δ_t equals 1 and all the series in y_t are in the same state. In contrast, when cycles are completely independent and desynchronized, then δ_t equals zero and all the weight is attributed to the case in which cyclical states are statistically independent. This forecast function was proposed by Camacho and Perez-Quiros (2001) in the context of a multivariate autoregressive markov-switching model but assuming that δ_t is constant.

As can be seen from section 2.1 and in accordance with the theory of endogenous OCAs, several institutional, economic and policy events may have contributed to the increase in synchronization between countries in Europe: the European Monetary System in 1979; the Single Market with the liberalization in of the circulation of people, trade and capital flows; the ratification of the Maastricht treaty and of the Stability and Growth Pact; and lastly the creation of the EMU. If this is true, assuming constancy of δ_t would lead to biased estimates of the model's hyperparameters. Hence the worth of our approach.

After predicting the univariate and multivariate probabilities using information up to $t-1$, the following steps show how to update them, using information up to period t:

- *Step 9*: For a given country c, evaluate the univariate Joint and Marginal Densities with univariate probabilities:

$$f\left(y_{c,t}, s_{c,t} = j, s_{c,t-1} = \widetilde{j}|\Omega_{c,t-1}\right) = f\left(y_{c,t}|s_{c,t} = j, s_{c,t-1} = \widetilde{j}, \Omega_{c,t-1}\right) \times \tag{40}$$

$$\times P\left(s_{c,t}=j,s_{c,t-1}=\widetilde{j}|\Omega_{c,t-1}\right)$$

where $P\left(s_{c,t}=j,s_{c,t-1}=\widetilde{j}|\Omega_{c,t-1}\right)$ is drawn from eq. (35). Hence the marginal densities are obtained summing the joint densities over all states j and $\widetilde{j}$:

$$f\left(y_{c,t}|\Omega_{c,t-1}\right)=\sum_{j=1}^{M}\sum_{\widetilde{j}=1}^{M}f\left(y_{c,t},s_{c,t}=j,s_{c,t-1}=\widetilde{j}|\Omega_{c,t-1}\right) \tag{41}$$

Here, the conditional density for a univariate state-space representation, is obtained by the prediction error decomposition of the univariate Kalman filter defined as:

$$f\left(y_{c,t}|s_{c,t}=j,s_{c,t-1}=\widetilde{j},\Omega_{c,t-1}\right)=(2\pi)^{\frac{T}{2}}\left|\eta_{c,t|t-1}^{(\widetilde{j},j)}\right|^{-\frac{1}{2}}exp\left\{-\frac{1}{2}\eta_{c,t|t-1}^{(\widetilde{j},j)\prime}\mathcal{F}_{c,t|t-1}^{(\widetilde{j},j)-1}\eta_{c,t|t-1}^{(\widetilde{j},j)}\right\} \tag{42}$$

where $\eta_{c,t|t-1}^{(\widetilde{j},j)}$ and $\mathcal{F}_{c,t|t-1}^{(\widetilde{j},j)}$ come from the univariate Kalman-filter recursions defined in equations (25) and (26) respectively. This makes clear the need for univariate Kalman filters in this algorithm. Since the univariate probabilities are used to forecast the multivariate probabilities, we need at each point in time updates and forecasts of the univariate probabilities. Forecasts of the univariate probabilities are accomplished by knowing the updated probability in the previous period which in turn needs the prediction error from the Kalman filter to perform the update.

- *Step 10*: Evaluate Joint and Marginal Densities with multivariate probabilities:

$$f\left(y_t,S_t=l,S_{t-1}=k|\Omega_{t-1}\right)=f\left(y_t|S_t=l,S_{t-1}=k,\Omega_{t-1}\right)P\left(S_t=l,S_{t-1}=k|\Omega_{t-1}\right) \tag{43}$$

where $P\left(S_t=l,S_{t-1}=k|\Omega_{t-1}\right)$ comes from eq. (37). The marginal density is again obtained by summing the joint densities over all states:

$$f\left(y_t|\Omega_{t-1}\right)=\sum_{l=1}^{M^N}\sum_{k=1}^{M^N} f\left(y_t, S_t=l, S_{t-1}=k|\Omega_{t-1}\right) \tag{44}$$

The conditional density $f\left(y_t|S_t=l, S_{t-1}=k, \Omega_{t-1}\right)$ is obtained from the prediction error decomposition of the Kalman filter for the corresponding multivariate transition (k,l):

$$f\left(y_t|S_t=l, S_{t-1}=k, \Omega_{t-1}\right)=(2\pi)^{\frac{T}{2}}\left|\eta_{t|t-1}^{(k,l)}\right|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}\eta_{t|t-1}^{(k,l)'}\mathcal{F}_{t|t-1}^{(k,l)-1}\eta_{t|t-1}^{(k,l)}\right\} \tag{45}$$

where $\eta_{t|t-1}^{(k,l)}$ and $\mathcal{F}_{t|t-1}^{(k,l)-1}$ come from equations (31) and (32) respectively.

- *Step 11*: Update the univariate probabilities using Bayes rule:

$$P\left(s_{c,t}=j, s_{c,t-1}=\widetilde{j}|\Omega_{c,t}\right)=\frac{f\left(y_{c,t}, s_{c,t}=j, s_{c,t-1}=\widetilde{j}|\Omega_{c,t-1}\right)}{f\left(y_{c,t}|\Omega_{c,t-1}\right)} \tag{46}$$

where $f\left(y_{c,t}, s_{c,t}=j, s_{c,t-1}=\widetilde{j}|\Omega_{c,t-1}\right)$ and $f\left(y_{c,t}|\Omega_{c,t-1}\right)$ comes from eqs. (40) and (41) respectively. Summing over past states we obtain:

$$P\left(s_{c,t}=j|\Omega_{c,t}\right)=\sum_{\widetilde{j}=1}^{M} P\left(s_{c,t}=j, s_{c,t-1}=\widetilde{j}|\Omega_{c,t}\right) \tag{47}$$

- *Step 12*: Update the multivariate probabilities using Bayes rule:

$$P\left(S_t=l, S_{t-1}=k|\Omega_t\right)=\frac{f\left(y_t, S_t=l, S_{t-1}=k|\Omega_{t-1}\right)}{f\left(y_t|\Omega_{t-1}\right)} \tag{48}$$

where $f\left(y_t, S_t=l, S_{t-1}=k|\Omega_{t-1}\right)$ and $f\left(y_t|\Omega_{t-1}\right)$ comes from eqs. (43) and (44) respectively. Again summing over past states we obtain:

$$P\left(S_t = l|\Omega_t\right) = \sum_{k=1}^{M^N} P\left(S_t = l, S_{t-1} = k|\Omega_t\right) \tag{49}$$

Remember however that the Kalman filter produces $\left(M^N\right)^2$ posteriors which makes the filter virtually impossible to use. For example, with only two univariate states of nature and a sample of two endogenous variables, we would have 4 multivariate states and, thus, 4×4 posteriors at each iteration. As the number of states increases, the number of posteriors computed by the Kalman filter grows exponentially. Hence some method is needed to make the filter operational. As proposed by Harrison and Stevens (1976) and shown by Kim (1994), if we assume that $\theta_{t|t}^{(k,l)}$ equals $E\left[\theta_t|S_{t-1} = k, S_t = j, \Omega_t\right]$ and $P_{t|t}^{(k,l)}$ represents $E\left[\left(\theta_t - \theta_{t|t}^{(k,l)}\right)\left(\theta_t - \theta_{t|t}^{(k,l)}\right)^{\prime}|S_{t-1} = k, S_t = j, \Omega_t\right]$, the Kalman-filter posteriors can be reduced/collapsed by a factor M^N in the following way:

- *Step 13*: Using the multivariate updated probabilities, collapse the $\left(M^N\right)^2$ Kalman-filter posteriors:

$$\theta_{t|t}^{l} = \frac{\sum_{k=1}^{M^N} P\left(S_t = l, S_{t-1} = k|\Omega_t\right)\theta_{t|t}^{(k,l)}}{P\left(S_t = l|\Omega_t\right)} \tag{50}$$

$$\mathcal{P}_{t|t}^{l} = \frac{\sum_{k=1}^{M^N} P\left(S_t = l, S_{t-1} = k|\Omega_t\right)\left\{\mathcal{P}_{t|t}^{(k,l)} + \left(\theta_{t|t}^{l} - \theta_{t|t}^{(k,l)}\right)\left(\theta_{t|t}^{l} - \theta_{t|t}^{(k,l)}\right)^{\prime}\right\}}{P\left(S_t = l|\Omega_t\right)} \tag{51}$$

Note that the collapsing procedure is performed by computing weighted averages of the state vectors and MSE matrices with the probabilities of the states of nature at time t and $t-1$, integrating out the previous states. This considerably reduces the dimensionality of the procedure. As argued in Kim and Nelson (1999), these collapsed posteriors are only an approximation since $\theta_{t|t}^{(k,l)}$ does not compute exactly $E\left[\theta_t|S_{t-1} = k, S_t = j, \Omega_t\right]$ neither $P_{t|t}^{(k,l)}$ computes $E\left[\left(\theta_t - \theta_{t|t}^{(k,l)}\right)(\theta_t - \theta_{t|t}^{(k,l)})^{\prime}|S_{t-1} = k, S_t = j, \Omega_t\right]$ efficiently. Nevertheless, as already argued, this collapsing procedure turns the filter operational and makes the filter here proposed a version of the Generalized Pseudo Bayesian algorithm (Murphy (1998)). Recall that this collapsing procedure must be applied to the univariate Kalman filter posteriors for a theoretical transition $\left(\widetilde{j}, j\right)$, for $\widetilde{j}, j = 1, ..., M$ using for the effect the updated univariate probabilities obtained in step 11 above.

- *Step 14*: Using the univariate updated probabilities, collapse the M^2 Kalman-filter posteriors:

$$\theta^{j}_{c,t|t} = \frac{\sum_{\widetilde{j}=1}^{M} P\left(s_{c,t} = j, s_{c,t-1} = \widetilde{j} | \Omega_{c,t}\right) \theta^{(\widetilde{j},j)}_{c,t|t}}{P\left(s_{c,t} = j | \Omega_{c,t}\right)} \tag{52}$$

$$\mathcal{P}^{j}_{c,t|t} = \frac{\sum_{\widetilde{j}=1}^{M} P\left(s_{c,t} = j, s_{c,t-1} = \widetilde{j} | \Omega_{c,t}\right) \left\{\mathcal{P}^{(\widetilde{j},j)}_{c,t|t} + \left(\theta^{j}_{c,t|t} - \theta^{(\widetilde{j},j)}_{c,t|t}\right)\left(\theta^{j}_{c,t|t} - \theta^{(\widetilde{j},j)}_{c,t|t}\right)'\right\}}{P\left(s_{c,t} = j | \Omega_{c,t}\right)} \tag{53}$$

- *Step 15*: Obtain the final state vector θ_t by averaging over states:

$$\theta_{t|t} = \sum_{l=1}^{M^N} P\left(S_t = l | \Omega_t\right) \theta^{l}_{t|t} \tag{54}$$

With this we are able to predict the unobserved components in the state vector θ_t and the probabilities that the system is in each of the states, both univariate and multivariate.

To perform all these tasks we need estimates of the hyperparameters present in the state-space representation (14)-(17). As a by-product of the Kalman-filter recursions, we are able to evaluate the marginal density (44) through the prediction error decomposition for the multivariate model. After taking logarithms and summing over all observations that yields an approximate log-likelihood function that can be maximized over the allowable parameters' space,

$$\mathcal{L}\left(\widehat{\alpha}\right)=\max_{\alpha\in\Theta^{\kappa}}\ \sum_{t=1}^{T}ln\left[f\left(y_t|\Omega_{t-1}\right)\right] \tag{55}$$

where $f\left(y_t|\Omega_{t-1}\right)$ comes from equation (44). Hence, the estimation of the hyperparameters is performed resorting to the Quasi-Maximum Likelihood estimation analyzed in White (1982). Standard errors for the hyperparameters may be computed using the method proposed by White (1982) and described in detail in Hamilton (2001) and Canova (2007).

Remark 1 *The forecasting function for the multivariate probabilities (29) and the transition probabilities' matrices in case of statistical independence and dependence defined above imply that the multivariate transition probabilities' matrix is time varying and written as:*

$$\Psi_t=\left(1-\delta_t\right)\Psi^I+\delta_t\Psi^D \tag{56}$$

where δ_t is the measure of comovement between the elements of y_t.

In the context of the analysis of business cycles, this can be seen as a measure of synchronization between the cycles of y_t. If δ_t also depends on the states of nature as defined in above, i.e. if it is a component present in θ_t , then the multivariate transition probabilities may be written as:

$$\psi_{l,k,t}=\left(1-\delta_t^{(k,l)}\right)\psi_{l,k,t}^{I}+\delta_t^{(k,l)}\psi_{l,k,t}^{D} \tag{57}$$

which is equal to,

$$P\left(S_t=l|S_{t-1}=k\right)=\left(1-\delta_t^{(k,l)}\right)P\left(S_t=l|S_{t-1}=k\right)^I+\delta_t^{(k,l)}P\left(S_t=l|S_{t-1}=k\right)^D \tag{58}$$

To see the intuition behind this result it is useful to think of the markov-switching model presented in section 3.1. In such a model, even when the idiosyncratic structural characteristics (e.g. the duration) of each business cycles remain constant, the transition probabilities between different states of the business cycle will vary according to the level of synchronization displayed. To see this, consider the two extreme cases of statistical dependence and independence of markov-switching states. Imagine that we are interested in analyzing a bivariate relation and in period t, the cycles are completely independent such that δ_t equals zero. Hence the recession and expansion states are completely independent and the transition probabilities in the multivariate case are computed as a linear combination of the univariate ones. Assume now that in period $t+1$ synchronization improves and the cycle in country A lags the cycle in country B by one quarter. In this case, the transition probabilities in the multivariate case must necessarily change, or equivalently, the duration of common recessions (both countries in recession) and common expansions (both countries in expansion) must necessarily increase since business cycles have become more locked to each other. In the other extreme, imagine that one quarter latter business cycles have become completely synchronized in the sense that they precisely overlap. Hence, the probabilities of transition must be exactly the same across economies and moreover, the duration of common recessions and expansions i.e., the duration of the states in which both countries are in recession and in expansion must increase again since the countries experience exactly the same fluctuations. Thus, despite the idiosyncratic characteristics of the business cycles remain constant across time, the common characteristics will change as comovements change across economies.

This reasoning leads us further to the following:

Remark 2 *The time-varying nature of the multivariate transition probabilities implies that the expected duration* $E\left[\mathcal{D}|S_t = l\right]$ *of common states is also time varying and written as:*

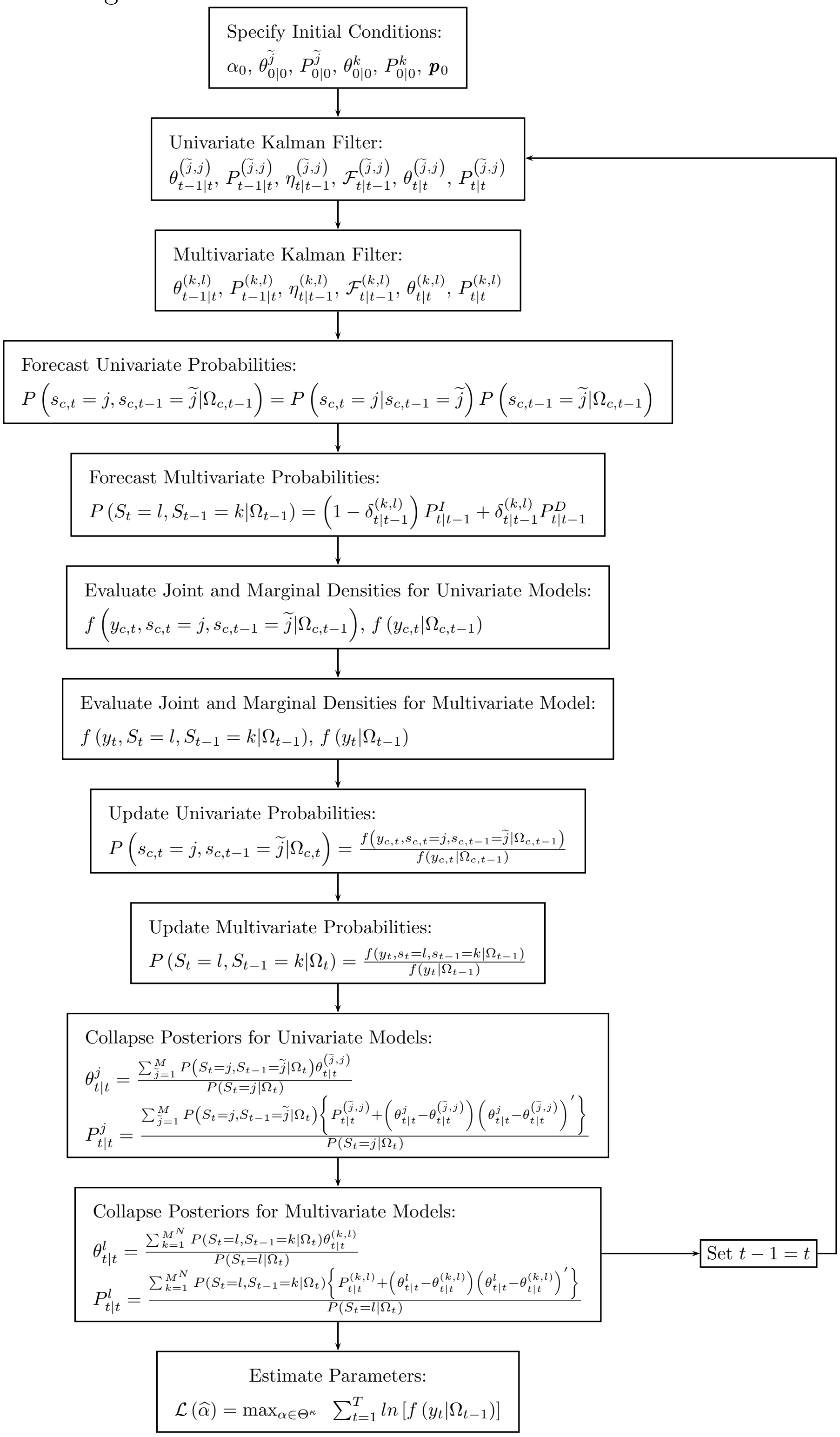
$$E\left[\mathcal{D}|S_t = l\right] = \left(1-\delta_t\right) E\left[\mathcal{D}|S_t = l\right]^I + \delta_t E\left[\mathcal{D}|S_t = l\right]^D \tag{59}$$

Filardo and Gordon (1998) have already studied the properties of markov-switching models with time-varying transition probabilities and time-varying durations of business-cycles. In their model, they assume that variation in the transition probabilities is conditional on a set of exogenous variables. Here, in contrast, all that matters for multivariate transition probabilities is the relation between business cycles. Our model of varying duration of common states has a different analytical formulation than that proposed by Filardo and Gordon (1998), since here the univariate transition probabilities are constant. Hence, the model provides us with another measure of synchronization between the endogenous variables in y_t, since it enables us to determine the expected time spent in common states. The computation of this measure is straightforward. After computing the multivariate transition probability matrix in the case of statistical dependence and independence we can simply apply the results from Hamilton (1989) and obtain:

$$E\left[\mathcal{D}|S_t = l\right] = (1-\delta_t)\left(1-\psi^{I}_{l,k,t}\right)^{-1} + \delta_t\left(1-\psi^{D}_{l,k,t}\right)^{-1} \tag{60}$$

Since the time-varying duration is a weighted average of the durations in case on statistical independence and dependence, it is now clear that a useful test for the null of common cycles would be $\delta_t = 1$. However, the time-varying nature of δ_t makes the computation of the test not so straightforward. Alternatively, one could adopt the strategy of testing certain restrictions on the multivariate transition probability matrix (20), namely, by testing whether at each point in time $\Psi_t = \Psi^D$. This amounts to test the nullity of some of the parameters of the matrix Ψ_t. Recall that when the markov-switching variables are fully dependent, the transition probability matrix assumes a particular parameterization in which the off-corner entries equal zero (see (22) for example). This procedure would however raise difficulties, since the transition probabilities are not identified under the null of no-significance as found by Hansen (1992, 1996a,1996b). We end this section presenting in figure 1 a flowchart that describes the proposed algorithm in detail.

Figure 1: Flowchart for Generalized Nonlinear Filter



3.4 Full-Sample Smoothing

This section describes an algorithm to produce estimates of the components of the state vector θ_t using all the information available in the data. Since the signal-extraction procedure defined above takes into account possible interdependencies between the univariate states of nature, the smoothing algorithm proposed by Kim (1994) becomes inoperable. A fixed-interval smoother can however be derived through the combination of Kim's (1994) smoother and the recursions defined above. Conditional on the estimated hyperparameters $\widehat{\alpha}$, for $t = T-1, T-2, ..., 1$,

- *Step 1*: Run through the filter defined above and store the sequences $\left\{\theta_{t|t-1}^{(k,l)}\right\}_{t=1}^{T}$, $\left\{P_{t|t-1}^{(k,l)}\right\}_{t=1}^{T}$, $\left\{\theta_{t|t-1}^{l}\right\}_{t=1}^{T}$, $\left\{P_{t|t-1}^{l}\right\}_{t=1}^{T}$, $\{P(S_t = l|\Omega_{t-1})\}_{t=1}^{T}$, $\{P(S_t = l|\Omega_t)\}_{t=1}^{T}$, $\{P(s_{c,t} = i|\Omega_{c,t})\}_{t=1}^{T}$ and $\{P(s_{c,t} = i|\Omega_{c,t-1})\}_{t=1}^{T}$;

- *Step 2*: Obtain the smoothed state vector and the corresponding MSE matrix for the multivariate model from the following equations:

$$\theta_{t|T}^{(l,k)} = \theta_{t|t}^{l} + P_{t|t}^{l} F_k' \left[P_{t+1|t}^{(l,k)}\right]^{-1} \left(\theta_{t+1|T}^{k} - \theta_{t+1|t}^{(l,k)}\right) \tag{61}$$

$$P_{t|T}^{(l,k)} = P_{t|t}^{l} + P_{t|t}^{l} F_k' \left[P_{t+1|t}^{(l,k)}\right]^{-1} \left(P_{t+1|T}^{k} - P_{t+1|t}^{(l,k)}\right) \left(P_{t|t}^{l} F_k' \left[P_{t+1|t}^{(l,k)}\right]^{-1}\right)' \tag{62}$$

- *Step 3*: Obtain the smoothed univariate probabilities using the approximation suggested by Kim (1994):

$$P\left(s_{c,t} = j, s_{c,t+1} = \widetilde{j}|\Omega_{c,T}\right) = \frac{P\left(s_{c,t+1} = \widetilde{j}|\Omega_{c,T}\right) P\left(s_{c,t+1} = \widetilde{j}|s_{c,t} = j\right) P\left(s_{c,t} = j|\Omega_{c,t}\right)}{P\left(s_{c,t+1} = \widetilde{j}|\Omega_{c,t}\right)} \tag{63}$$

sum over future states to obtain the smoothed probability of being in a given state:

$$P\left(s_{c,t}=j|\Omega_{c,T}\right)=\sum_{\widetilde{j}=1}^{M}P\left(s_{c,t}=j,s_{c,t+1}=\widetilde{j}|\Omega_{c,T}\right) \tag{64}$$

- *Step 4*: Obtain the smoothed multivariate probabilities from:

$$P\left(S_t=l,S_{t+1}=k|\Omega_T\right)=\left(1-\delta_{t|T}^{(l,k)}\right)P_{t|T}^{I}+\delta_{t|T}^{(l,k)}P_{t|T}^{D} \tag{65}$$

Again, we can compute each of the smoothed probabilities in case of statistical independence and dependence from the univariate smoothed probabilities as follows:

$$P_{t|T}^{I}=P\left(s_{1,t}=i,s_{2,t}=g,...,s_{N,t}=j,s_{1,t+1}=\widetilde{i},s_{2,t+1}=\widetilde{g},...,s_{N,t+1}=\widetilde{j}|\Omega_T\right)^{I}= \tag{66}$$

$$=\mathrm{P}\left(s_{1,t}=i,s_{1,t+1}=\widetilde{i}|\Omega_{1,T}\right)\times...\times P\left(s_{N,t}=j,s_{N,t+1}=\widetilde{j}|\Omega_{N,T}\right)$$

and,

$$P_{t|T}^{D}=P\left(s_{1,t}=i,s_{2,t}=g,...,s_{N,t}=j,s_{1,t+1}=\widetilde{i},s_{2,t+1}=\widetilde{g},...,s_{N,t+1}=\widetilde{j}|\Omega_T\right)^{D}= \tag{67}$$

$$=\begin{cases} P\left(s_{N,t}=j,s_{N,t+1}=\widetilde{j}|\Omega_{N,T}\right) & \text{if } i=g=\ldots=j\wedge\widetilde{i}=\widetilde{g}=\ldots=\widetilde{j} \\ 0 & \text{if } i\neq g,\ldots,j\vee\widetilde{i}\neq\widetilde{g},\ldots,\widetilde{j} \end{cases}$$

- *Step 5*: Collapse the Kalman-filter posteriors for the multivariate model:

$$\theta_{t|T}^{l}=\frac{\sum_{k=1}^{M^N}P\left(S_t=l,S_{t+1}=k|\Omega_T\right)\theta_{t|T}^{(l,k)}}{P\left(S_t=l|\Omega_T\right)} \tag{68}$$

$$P^l_{t|T} = \frac{\sum_{k=1}^{M^N} P\left(S_t = l, S_{t+1} = k|\Omega_T\right) \left\{ P^{(l,k)}_{t|T} + \left(\theta^l_{t|T} - \theta^{(l,k)}_{t|T}\right)\left(\theta^l_{t|T} - \theta^{(l,k)}_{t|T}\right)' \right\}}{P\left(S_t = l|\Omega_T\right)} \tag{69}$$

- *Step 6*: Compute the final value of the smoothed state vector:

$$\theta_{t|T} = \sum_{l=1}^{M^N} P\left(S_t = l|\Omega_T\right) \theta^l_{t|T} \tag{70}$$

With these steps, we are able to obtain predictions of the unobserved components at each point in time, taking into account all the information available in the sample.

3.5 The State-Space Representation of the Proposed Model

The filtering algorithm outlined in section 3.3 defines the guidelines for the estimation of the model proposed in section 3.1 to analyze business cycle synchronization. The application requires the decomposition of the main state-space representation for equations (11) to (13) together with (6) and (7) into simpler state-space representations for each trend-cycle models for each country under analysis, where the latter will serve as the basis to construct the probabilistic inference that will allow the estimation of the parameters of the multivariate representation.

We can cast the multivariate trend-cycle decomposition, in its stationary form, in the following state-space format:

$$\underbrace{\Delta Y_t}_{y_t} = \underbrace{\begin{pmatrix} I_N & -I_N & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}^* \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-W} \\ \vartheta_t \end{pmatrix}}_{\theta_t} + \underbrace{\Gamma_0 + \Gamma_1 S_t}_{A_{S_t}} \tag{71}$$

$$\underbrace{\begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-W} \\ \vartheta_t \end{pmatrix}}_{\theta_t} = \underbrace{\begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \cdots & \Phi_W & \mathbf{1} \\ I_N & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}^* \\ \mathbf{0} & I_N & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}^* \\ \vdots & \vdots & \vdots & \ddots & \vdots & \mathbf{0}^* \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}^* \\ 0 & 0 & 0 & \cdots & 0 & \rho \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ \vdots \\ x_{t-W-1} \\ \vartheta_{t-1} \end{pmatrix}}_{\theta_{t-1}} + \underbrace{\begin{pmatrix} \epsilon_t \\ \mathbf{0}^* \\ \mathbf{0}^* \\ \vdots \\ \mathbf{0}^* \\ \zeta_t \end{pmatrix}}_{\nu_t} \tag{72}$$

Where I_N is a $N \times N$ identity matrix, $\mathbf{0}$ is a $N \times N$ matrix of zeros, $\mathbf{0}^*$ is a $N \times 1$ vector of zeros and $\mathbf{1}$ is a $N \times 1$ vector of ones. Likewise, each univariate model can be defined as:

$$\underbrace{\Delta y_{c,t}}_{y_t} = \underbrace{\begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} x_{c,t} \\ x_{c,t-1} \\ x_{c,t-2} \\ \vdots \\ x_{c,t-W} \end{pmatrix}}_{\theta_t} + \underbrace{\gamma_{c,0} + \gamma_{c,1} s_{c,t}}_{A_{S_t}} \tag{73}$$

$$
\underbrace{\begin{pmatrix} x_{c,t} \\ x_{c,t-1} \\ x_{c,t-2} \\ \vdots \\ x_{c,t-W} \end{pmatrix}}_{\theta_t} = \underbrace{\begin{pmatrix} \phi_{c,1} & \phi_{c,2} & \phi_{c,3} & \cdots & \phi_{c,W} \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} x_{c,t-1} \\ x_{c,t-2} \\ x_{c,t-3} \\ \vdots \\ x_{c,t-W-1} \end{pmatrix}}_{\theta_{t-1}} + \underbrace{\begin{pmatrix} \epsilon_{c,t} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\nu_t} \tag{74}
$$

for $c = 1, ..., N$. It is straightforward now to note that the parameters present in the univariate state-space representation are the same as those present in the multivariate representation. The difference is that the former explicitly incorporates the effects of business cycle synchronization. Relating to the general state-space representation presented in the last section, here the $K \times 1$ vector Z_t is a vector of ones while ω_t and $\widetilde{\mu}_{S_t}$ are zero. Moreover, the univariate state-space representation (eq. (73)-(74)) will be used to obtain forecasts of the univariate probabilities for each state which will allow the computation of multivariate probabilities using eq. (37) giving the basis to evaluate the likelihood and estimate the parameters.

4 Data and Results

4.1 Data, Econometric Strategy and Model Restrictions

To estimate the model presented in section 3.1 we use times series of quarterly real GDP of 19 industrialized countries, covering, as a rule, the period 1970:1-2008:1. For some few exceptions, there were no available data for the entire period and smaller sample periods are considered. The 19 industrialized economies that we study are the following: aggregate Euro Area, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Norway, The Netherlands, Portugal, Spain, Sweden, Switzerland, UK, US.

For the aggregate Euro Area, the data come from the Euro Area Wide Model database for the period 1970:1-2002:3. The series was then updated using the growth rates of real GDP made available by the ECB for the aggregate Euro Area 11. Data for Austria, Canada, Finland, France, Germany, Japan, Norway, Spain, Sweden, Switzerland,UK and US were obtained from the IMF International Financial Statistics (IMF-IFS) dataset. For Greece, data for 1975:1-1991:1 and 2000:1-2008:1 were obtained from the IFS dataset, and were then completed using growth rates from the OECD Quarterly National Accounts (OECD-QNA) for the remaining periods. In the case of Italy, data are from the IMF-IFS for the period 1980:1-2008:1 and the series was then extended backwards using data from the OECD-QNA. In the case of Denmark, The Netherlands and Portugal, in which the data come from the IMF-IFS, it was only possible to cover the period 1977:1-2008:1. For Belgium, the sample covers the period 1980:1-2008:1 and comes also from the IMF-IFS. These last four countries make up the sole exceptions to our sample period of 1970:1-2008:4.

Seasonal adjustment has been performed with TRAMO-SEATS. A preliminary inspection of the data and their technical specifications led us to adjust for seasonality the time series of Austria, Belgium, Denmark, Finland, Greece, Japan, Sweden, Switzerland, Norway[22].

Our econometric strategy consists of estimating bivariate models. We first estimate the model for the aggregate Euro Area and each of the countries in the sample. Then, we estimate similar bivariate models replacing the Euro Area by the US.

We conducted the estimation under some restrictions which have been imposed in order to reduce the computational costs. First, for practical reasons the matrix of mean growth rates during the expansion phase is assumed to be diagonal i.e., $\Gamma_1 = diag\,(\gamma_{1,1}, \ldots, \gamma_{N,1})$. Second, we assume a simplified version of the VAR(W) process for the business cycles, imposing that the matrices of coefficients of the VAR are diagonal, i.e., $\Phi_w = diag\,(\phi_{11,w}, \ldots, \phi_{NN,w})$, for $w = 1, \ldots, W$ and, as usual in the unobserved-components literature, $W = 2$. The second restriction proves particularly valuable as it enables us to capture all the common variability between the business cycles through the common component ϑ_t and is consistent with the standard hypothesis in unobserved components models[23].

4.2 Results

This section presents the results for the estimations of the bivariate models formed by real GDP growth of each country and separately of the Euro Area and the US. The model corresponds to the one described in section 3.1, equations (eqs. (11)-(13)), (6) and (7). The estimation was carried out by maximum likelihood with the methods proposed in section 3.3.

[22]In fact, data for Greece, Japan, is available only partially adjusted. The adjustment made for Japan introduced an unusual observation in 1979:1 which was substituted by simple average over the next and previous quarters.

[23]As standard in the literature, we further impose several restrictions to ensure the correct identification of the model: standard deviations were constrained to be positive, transition probabilities constrained to lie in the closed interval $[0, 1]$, the coefficients of the autoregressive processes for the business cycles were restricted to avoid unit-roots and, the autoregressive coefficient on the for the common component was also restricted to to be $|\rho| < 1$. Estimation of the model was performed through an appropriate code written in GAUSS version 8, using *Optmum* to maximize the log-likelihood function and the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method to numerically search the maximum.

4.2.1 Bivariate Models with the Euro Area

Overview of the Results

Table 1 presents the quasi-maximum likelihood parameter estimates for each of the 18 bivariate models involving the Euro Area as reference. In the table, country 1 is always the Euro Area, while country 2 always represents the other country.

Table 1: Results for Bivariate Estimation with Euro Area

Param.	EA-AUS		EA-BGM		EA-CND		EA-DEN		EA-FIN	
$\gamma_{0,1}$	-0.10	(0.03)	-0.09	(0.05)	-0.21	(0.04)	-0.10	(0.01)	-0.44	(0.10)
$\gamma_{1,1}$	0.93	(0.03)	0.79	(0.01)	0.97	(0.08)	0.73	(0.03)	1.18	(0.05)
$\gamma_{0,2}$	-0.05	(0.03)	-0.19	(0.07)	-0.29	(0.07)	-1.18	(0.23)	-0.62	(0.22)
$\gamma_{1,2}$	0.94	(0.05)	0.95	(0.11)	1.34	(0.10)	1.86	(0.22)	1.56	(0.28)
$p_{0,1}$	0.77	(0.09)	0.80	(0.05)	0.32	(0.09)	0.57	(0.06)	0.80	(0.08)
$p_{1,1}$	0.89	(0.01)	0.93	(0.02)	0.91	(0.02)	0.93	(0.01)	0.93	(0.03)
$p_{0,2}$	0.76	(0.01)	0.80	(0.06)	0.50	(0.12)	0.71	(0.09)	0.67	(0.21)
$p_{1,2}$	0.90	(0.02)	0.92	(0.02)	0.85	(0.03)	0.92	(0.02)	0.92	(0.03)
$E\left[\mathcal{D}\|s_{1,t}=0\right]$	4.42	-	4.91	-	1.46	-	2.34	-	4.92	-
$E\left[\mathcal{D}\|s_{2,t}=0\right]$	4.10	-	4.99	-	1.98	-	3.47	-	3.02	-
ρ	0.48	(0.03)	0.80	(0.03)	0.42	(0.12)	0.75	(0.06)	0.43	(0.19)
$\phi_{1,1}$	0.81	(0.02)	0.56	(0.04)	1.21	(0.03)	0.74	(0.07)	1.15	(0.17)
$\phi_{2,1}$	0.04	(0.01)	0.14	(0.06)	-0.28	(0.06)	0.09	(0.04)	-0.21	(0.19)
$\phi_{1,2}$	0.48	(0.05)	0.43	(0.11)	1.21	(0.14)	0.66	(0.04)	1.15	(0.09)
$\phi_{2,2}$	0.45	(0.06)	0.16	(0.04)	-0.28	(0.15)	0.12	(0.04)	-0.24	(0.07)
σ_ζ	0.41	(0.04)	0.25	(0.07)	0.27	(0.09)	0.30	(0.06)	0.29	(0.03)
Λ_{ϵ_1}	0.27	(0.04)	0.26	(0.03)	0.34	(0.07)	0.25	(0.07)	0.33	(0.05)
Λ_{ϵ_2}	0.73	(0.05)	1.40	(0.13)	0.92	(0.04)	0.94	(0.01)	0.57	(0.05)
$-\mathcal{L}(\widehat{\alpha})$	-100050.21		-100081.93		-100065.82		-100049.04		-100015.52	

Note: Standard Errors in parentheses. $\gamma_{0,i}$ and $\gamma_{1,i}$ represent the mean growth rates of GDP during recessions and expansions; $p_{0,i}$ and $p_{1,i}$ the probability of transition from expansion to expansion and from recession to recession respectively; $\xi_{0,i}$ is the expected duration of a recession; $\phi_{w,i}$ are the parameters of the cycle equation; σ_ζ is the standard deviation of the common component while Λ_{ϵ_i} is the standard deviation of the business cycle, for country i.

In general, the algorithm for estimation seems successful in identifying the recessions and

Table 1 (Continued): Results for Bivariate Estimation with Euro Area

Param.	EA-FR		EA-GER		EA-GREE		EA-IT		EA-JP	
$\gamma_{0,1}$	-0.49	(0.09)	-0.19	(0.04)	-0.30	(0.10)	-0.15	(0.02)	-0.58	(0.10)
$\gamma_{1,1}$	1.23	(0.07)	1.10	(0.14)	1.09	(0.08)	0.98	(0.03)	1.34	(0.07)
$\gamma_{0,2}$	-0.37	(0.17)	-0.38	(0.03)	-0.30	(0.15)	-0.37	(0.09)	-0.48	(0.37)
$\gamma_{1,2}$	1.07	(0.21)	1.32	(0.06)	1.33	(0.23)	1.20	(0.09)	1.27	(0.41)
$p_{0,1}$	0.80	(0.04)	0.80	(0.03)	0.77	(0.04)	0.80	(0.26)	0.31	(0.23)
$p_{1,1}$	0.92	(0.03)	0.93	(0.04)	0.92	(0.03)	0.93	(0.17)	0.85	(0.02)
$p_{0,2}$	0.66	(0.09)	0.75	(0.01)	0.80	(0.08)	0.75	(0.08)	0.48	(0.21)
$p_{1,2}$	0.92	(0.04)	0.92	(0.03)	0.91	(0.02)	0.92	(0.04)	0.91	(0.03)
$E[\mathcal{D}\|s_{1,t}=0]$	4.90	-	4.91	-	4.42	-	4.91	-	1.44	-
$E[\mathcal{D}\|s_{2,t}=0]$	2.93	-	3.96	-	4.89	-	3.95	-	1.93	-
ρ	0.77	(0.10)	0.64	(0.09)	0.49	(0.12)	0.64	(0.17)	0.40	(0.05)
$\phi_{1,1}$	1.10	(0.08)	0.97	(0.08)	1.05	(0.06)	0.76	(0.01)	1.27	(0.06)
$\phi_{2,1}$	-0.30	(0.04)	-0.23	(0.04)	-0.16	(0.05)	-0.12	(0.10)	-0.39	(0.02)
$\phi_{1,2}$	0.94	(0.14)	0.58	(0.09)	0.92	(0.06)	1.12	(0.05)	1.49	(0.08)
$\phi_{2,2}$	-0.16	(0.10)	0.09	(0.02)	0.05	(0.05)	-0.31	(0.03)	-0.51	(0.08)
σ_ζ	0.39	(0.05)	0.41	(0.06)	0.26	(0.08)	0.37	(0.07)	0.25	(0.05)
Λ_{ϵ_1}	0.32	(0.03)	0.21	(0.04)	0.33	(0.07)	0.25	(0.04)	0.35	(0.04)
Λ_{ϵ_2}	0.25	(0.08)	0.63	(0.05)	1.75	(0.11)	0.58	(0.05)	0.92	(0.04)
$-\mathcal{L}(\widehat{\alpha})$	-99948.61		-100080.32		-100156.41		-100019.30		-100066.56	

Note: See above.

expansions phases in international fluctuationsas estimates of transition probabilities and mean growth rates are reasonable. It is remarkable that despite the high parameterization of the model, most parameter estimates are statistically significant[24].

The estimated mean growth rate during the expansion phase in the Euro Area ranges between 0.63% and 0.91%, while that for recessions ranges from -0.58% and -0.09%. Recessions tend to be more pronounced in Denmark, Finland, Japan, The Netherlands and Switzerland. As regards

[24] The table presents standard errors constructed as in White (1982). No t-ratios are computed since standard asymptotic theory cannot be applied due to the presence of nuisance parameters (the transition probabilities) under the null of no-significance, see Hansen (1992, 1996a,1996b). For an analysis of the finite-sample properties of the maximum-likelihood estimator for the markov switching model see Psaradakis, Z. and Sola, M. (1998). Such properties are however merely indicative, since the algorithm here presented is a two-step procedure, and thus very different from the baseline algorithm proposed by Hamilton (1989).

Table 1 (Continued): Results for Bivariate Estimation with Euro Area

Param.	EA-NRW		EA-NTH		EA-PT		EA-SP		EA-SWE	
$\gamma_{0,1}$	-0.17	(0.10)	-0.42	(0.20)	-0.40	(0.73)	-0.55	(0.07)	-0.29	(0.19)
$\gamma_{1,1}$	1.01	(0.09)	1.07	(0.56)	1.07	(0.71)	1.33	(0.10)	1.07	(0.24)
$\gamma_{0,2}$	-0.04	(0.01)	-0.61	(0.18)	-0.42	(0.42)	-0.40	(0.10)	-0.27	(0.22)
$\gamma_{1,2}$	1.10	(0.09)	1.40	(0.54)	1.27	(0.68)	1.40	(0.11)	0.99	(0.27)
$p_{0,1}$	0.77	(0.15)	0.80	(0.08)	0.52	(0.07)	0.79	(0.09)	0.77	(0.15)
$p_{1,1}$	0.90	(0.03)	0.93	(0.04)	0.91	(0.05)	0.91	(0.05)	0.90	(0.02)
$p_{0,2}$	0.76	(0.03)	0.49	(0.03)	0.80	(0.08)	0.46	(0.04)	0.66	(0.15)
$p_{1,2}$	0.91	(0.05)	0.91	(0.10)	0.92	(0.04)	0.90	(0.02)	0.91	(0.02)
$E[\mathcal{D}\|s_{1,t}=0]$	4.43	-	4.91	-	2.07	-	4.87	-	4.43	-
$E[\mathcal{D}\|s_{2,t}=0]$	4.13	-	1.95	-	4.96	-	1.86	-	2.98	-
ρ	0.45	(0.04)	0.67	(0.13)	0.83	(0.04)	0.38	(0.02)	0.43	(0.13)
$\phi_{1,1}$	1.04	(0.06)	0.82	(0.10)	0.72	(0.32)	0.86	(0.03)	1.07	(0.25)
$\phi_{2,1}$	-0.15	(0.03)	-0.05	(0.04)	-0.09	(0.13)	0.04	(0.02)	-0.18	(0.23)
$\phi_{1,2}$	0.61	(0.08)	0.81	(0.09)	0.40	(0.05)	1.00	(0.05)	0.66	(0.08)
$\phi_{2,2}$	0.35	(0.07)	-0.09	(0.08)	0.39	(0.06)	-0.10	(0.03)	0.30	(0.05)
σ_ζ	0.33	(0.07)	0.25	(0.10)	0.30	(0.06)	0.41	(0.04)	0.34	(0.07)
Λ_{ϵ_1}	0.26	(0.08)	0.27	(0.08)	0.24	(0.06)	0.24	(0.02)	0.27	(0.09)
Λ_{ϵ_2}	1.47	(0.09)	0.54	(0.10)	1.29	(0.06)	0.53	(0.03)	1.09	(0.12)
$-\mathcal{L}(\widehat{\alpha})$	-100134.52		-100001.54		-100084.33		-100034.65		-100096.79	

Note: See above.

expansions, higher mean growth rates are observed in Austria, Canada, Finland, Germany, Greece, Norway, Spain and the US.

The expected duration of recessions $E[\mathcal{D}|s_{c,t}=0]$ appears also in line with the estimates presented by the literature, ranging from 1.5 quarters to 5 quarters approximately for the Euro Area, which appears to be higher than in France and Germany. In fact, the expected duration of recessions in France is approximately 2 quarters lower and 1 quarter lower in Germany. In contrast, Austria, Belgium, Greece and Portugal have recessions with average durations between 4 to 5 quarters, roughly similar to the Euro Area.

Hence, our model and estimation algorithm seems to be able to identify the standard business cycles features. The parameter estimates for the cycles'autoregressive processes exhibit mean-

Table 1 (Continued): Results for Bivariate Estimation with Euro Area

Param.	EA-SWITZ		EA-UK		EA-US	
$\gamma_{0,1}$	-0.44	(0.17)	-0.33	(0.12)	-0.49	(0.21)
$\gamma_{1,1}$	1.16	(0.26)	1.11	(0.05)	1.25	(0.26)
$\gamma_{0,2}$	-0.69	(0.15)	-0.38	(0.00)	-0.13	(0.05)
$\gamma_{1,2}$	1.24	(0.05)	1.25	(0.09)	1.05	(0.06)
$p_{0,1}$	0.80	(0.08)	0.29	(0.11)	0.59	(0.24)
$p_{1,1}$	0.91	(0.02)	0.90	(0.04)	0.89	(0.16)
$p_{0,2}$	0.51	(0.10)	0.47	(0.05)	0.49	(0.06)
$p_{1,2}$	0.94	(0.03)	0.80	(0.03)	0.90	(0.09)
$E\left[\mathcal{D}\vert s_{1,t}=0\right]$	4.92	-	1.41	-	2.45	-
$E\left[\mathcal{D}\vert s_{2,t}=0\right]$	2.05	-	1.89	-	1.97	-
ρ	0.54	(0.09)	0.43	(0.06)	0.44	(0.17)
$\phi_{1,1}$	1.02	(0.03)	1.27	(0.15)	1.40	(0.05)
$\phi_{2,1}$	-0.11	(0.04)	-0.34	(0.14)	-0.45	(0.06)
$\phi_{1,2}$	0.78	(0.05)	0.79	(0.02)	1.04	(0.14)
$\phi_{2,2}$	0.02	(0.00)	0.11	(0.03)	-0.18	(0.03)
σ_ζ	0.33	(0.09)	0.31	(0.04)	0.27	(0.05)
Λ_{ϵ_1}	0.32	(0.04)	0.31	(0.04)	0.35	(0.04)
Λ_{ϵ_2}	0.93	(0.04)	0.65	(0.05)	0.68	(0.08)
$-\mathcal{L}(\widehat{\alpha})$	-100572.82		-100039.47		-100027.74	

Note: See above.

reversion as they should, which further lends credibility to our results.

The autoregressive parameter of the common component is significant in all the bivariate systems. However, its value varies considerably across estimations. In the models with Canada, Finland, Greece, japan, Norway, Spain and Sweden, the common component ϑ_t exhibits low persistence, and seems to be capturing high frequency information. Interestingly, these are countries found in the literature to be less synchronized with the Euro Area. Conversely, countries with a high persistence in the common component correspond to countries that are presented by the literature as having a business cycle highly correlated with the Euro Area cycle namely, Belgium, Denmark, France, Germany , Italy, The Netherlands and Portugal.

Figures 6-23 in Appendix A present the estimated business cycles for each bivariate estimation.

The vertical shaded lines indicate peaks and troughs according to the Bry-Boschan Quarterly (BBQ hereafter) algorithm proposed by Harding and Pagan (2002)[25].

In general, business cycles seem to be well identified, with periods of recession with markedly decreasing cyclical components, in line with the dates obtained with the BBQ. For the Euro Area, the 1970s and 1980s recessions, around the timing of the oil shocks, are apparent in the majority of the bivariate estimations. The similarity between the French and the Euro Area cycle is remarkable. Eyeballing the results, it is apparent that the recessions are well identified with periods of falling cyclical components as detached above. Moreover, the 2001 recession seems to have been rather mild. It is interesting to note that the German cycle doesn't share all these similarities, which is in line with the results in the literature that the French cycle is more synchronized with the Euro Area than the German.

The Japanese cycle is as usual the most idiosyncratic. The cyclical component is characterized by a significant period of expansion during most of the 70s and during the 80s. Afterwards, it is visible the economic slump that characterizes the Japanese economy. It is interesting however to note that our model estimates that the real output has been above the trend for most of the time, therefore suggesting that the slump has been one of weak trend growth. A similar pattern is identified for Spain at the beginning of the 1970s. Despite the well known oil shock, the business cycle is estimated above trend. This is consistent with the fact that the stagnation of Spain during the 70s and 80s has been very prolonged. The other countries' cycles seem regular, often crossing the zero line during classical (BBQ) recessions.

The estimated univariate smoothed probabilities presented in Figures 41-58, show that the periods of recession, for the vast majority of the estimations, are reasonably identified. For the Euro Area, recessions seem to be correctly identified, in view of the results elsewhere in the

[25] Artis et al. (2004) perform a dating exercise for the Euro Area business cycle, exploring the various definitions for business cycle, namely, the classical, growth and deviation cycle.

literature and the dating obtained from the BBQ algorithm. Interestingly, the 1993 and 2001 recessions in the Euro Area are timely detected, despite the non-existence of a negative cyclical component. Moreover, the 2001 slowdown is sometimes detected previously to the dates obtained from the BBQ algorithm.

For France and Germany, the similarities with the Euro Area timing of recessions are apparent. For the system with Germany, a recession starting around the beginning of 1995 is detected for both economies despite the inexistence of a downturn dated by the BBQ algorithm for the Euro Area. Moreover, Germany has a more protracted recession around 2001, which seems to affect the Euro Area estimation. This prolonged recession could well explain why France displays higher levels of synchronization with the Euro Area than Germany. A similar pattern is observed for Italy, which presents a longer recession starting in the first quarter of 2001. Synchronized recessions are also observed between the Euro Area and Austria, specially the ones during the first oil shock, the 1990s recession and part of the 2000s recession. For Greece and Norway, recession probabilities are not correctly identified.

As a preliminary study of the level of synchronization during recessions between the Euro Area and the core countries of the monetary union, we look now at figures 76 to 93, which graph the smoothed multivariate probabilities of being in each of the possible four multivariate states of each bivariate model. State one and four, are the probabilities of both economies being in expansion and recession respectively, and may be seen as an indicative measure of synchronization. This measure directly addresses the findings by Canova et al. (2007) and Girardin (2002) that synchronization is state dependent, i.e., the comovements tend to increase during recessions.

Recessions are highly synchronized for Austria, France, Germany and Italy as already argued. However, there is evidence of more prolonged recessions in Austria at the beginning of the 1970s and mid-1980s (see fig. 76). For France and Germany, we have similar pictures, since the recessions occurred at the timing of the first oil shocks were somewhat more protracted in both economies (see the lower left panel of figures 81 and 82 respectively). Our results suggest that Italy also spent more time in recession than the aggregate Euro Area during the 70s, judging from the increase in the probability in the lower left panel of figure 84. Hence, for the core countries of the Euro Area, the 1970s and 1980s recessions associated with the impact of the oil price shocks, were somewhat more prolonged. Our results also suggest a drop in synchronization during the 1980s in Portugal and in Finland during the 1990s, as shown in figures 88 and 80 respectively.

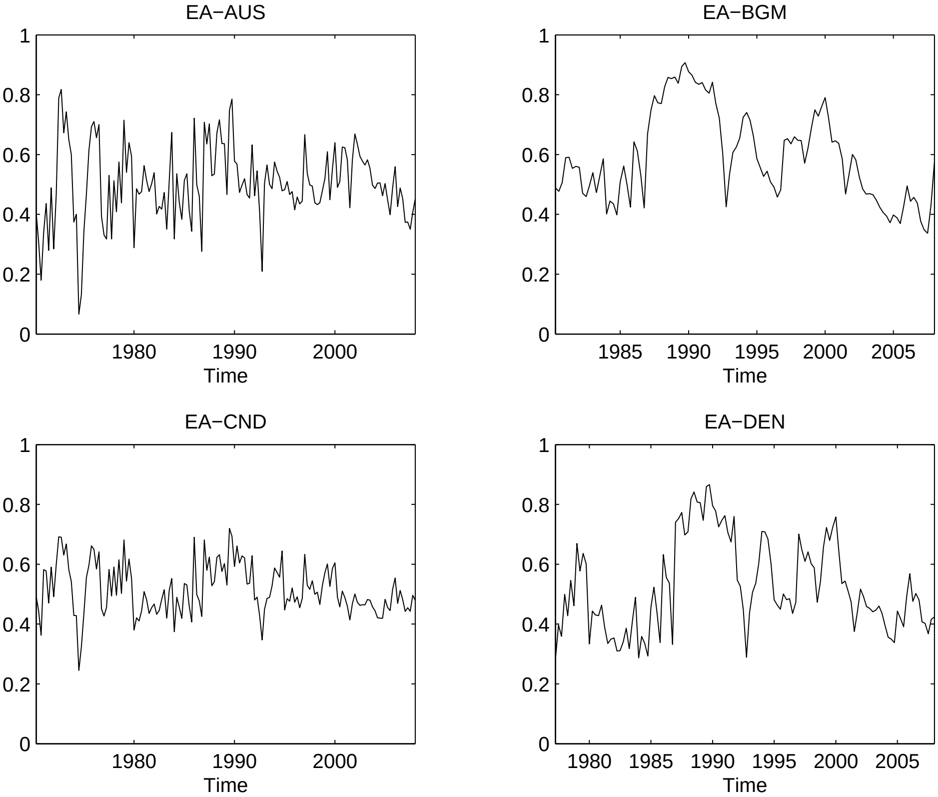
Some results are worth noting for the countries outside the Euro Area. Synchronization is particularly low in Canada, Denmark, Japan, and the UK (see figures 78, 79, 85 and 92 respectively). For Canada and Japan, this is the result of the more frequent recessions in the aggregate Euro Area. On the contrary, for the UK, the misalignments between the states of the business cycle emerge mainly due to the higher duration of recessions during the 70s and 80s in the UK, and during the 90s for the Euro Area. Our results further show that the low synchronization with the US, arise mainly due to the higher duration in its recession during the 1980s, but also, the late effects of the 1990s recession in the Euro Area.

Business Cycle Synchronization

We now turn to our main interest in this dissertation, which is to assess the estimated comovements between cyclical fluctuations of the studied countries. Albeit being somewhat informative, the smoothed multivariate probabilities analyzed in the previous section do not give a quantification neither picture of the temporal evolution of synchronization. As argued in section 3.1, we assume that the common component $\vartheta_{t|T}$ may capture the common variability between business cycles and hence, our suggestion of $\delta_{t|T}$ defined in equation (10) as an index of synchronization. Figure 2 shows the estimated indexes of synchronization $\delta_{t|T}$ for each system.

Overall, the results seem to indicate a rather low level of synchronization between the Euro Area cycle and most of the countries under analysis. Moreover, there seems to be an apparent

Figure 2: Estimated Synchronization Indexes with Euro Area

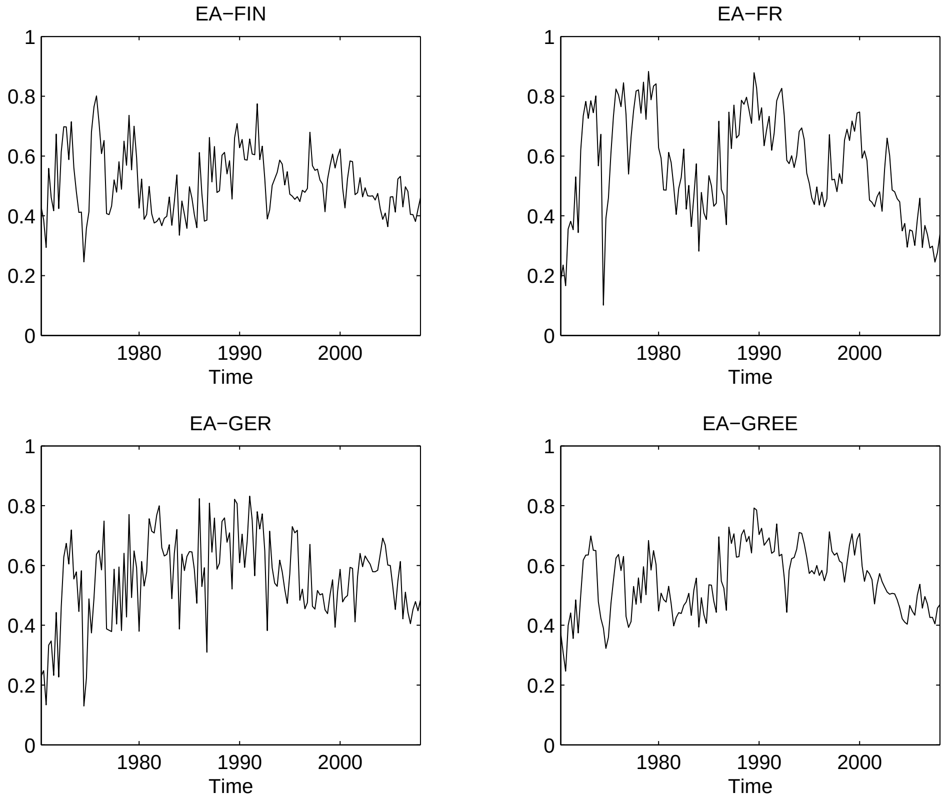


difference between the synchronization of the Euro Area members and the non-participants of the monetary union. We offer in the following paragraphs a more thorough analysis of the results.

First, the synchronization of the Euro Area countries is in average higher than that for the non-participants. We can state this fact by comparing the estimates of the synchronization index along the sample for Belgium, France, Germany, Italy, the Netherlands, Portugal and Spain, with the results for Canada, Japan, Switzerland, the UK and the US. This is a result which is in line with the literature on business cycle synchronization, for example, Belo (2001).

It is interesting to note, in particular for the Euro Area member countries, that the level of synchronization increases during the 1970s and the 1980s, peaking in the beginning of the 1990s and decreases onwards. This pattern can be observed in the estimated synchronization index of

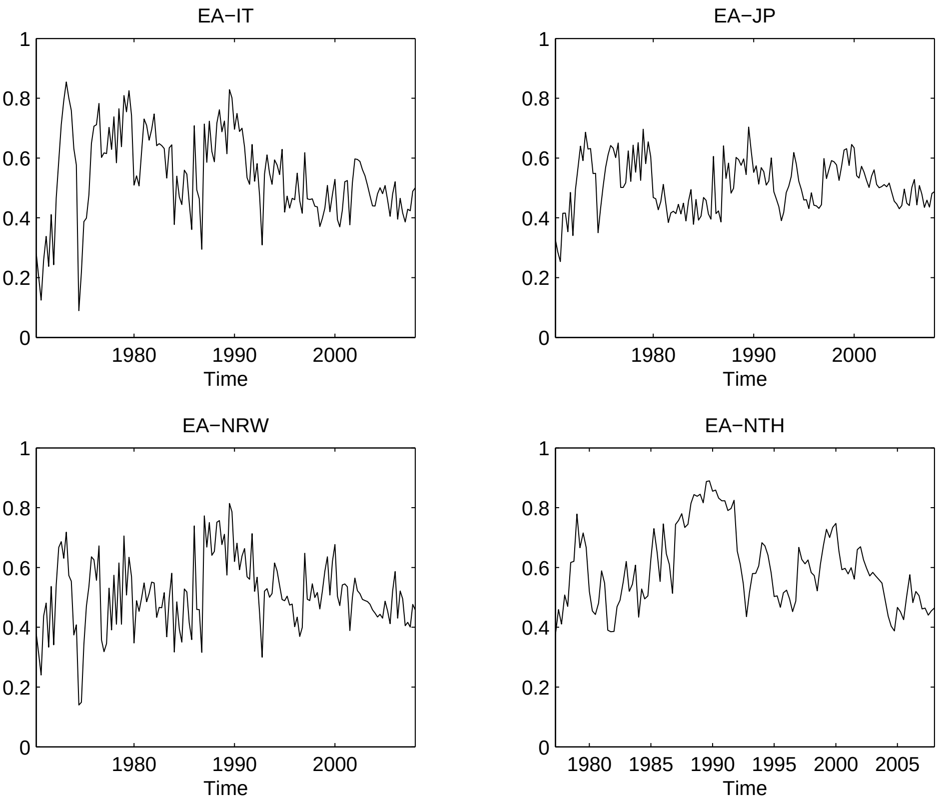
Figure 2 (Continued): Estimated Synchronization Indexes with Euro Area



Belgium, Denmark, Finland, France, Germany, Greece, Italy, Norway, the Netherlands, Portugal, Spain and Sweden. This fact is not easy to reconcile with the literature, mainly because the literature lacks models which assume time-varying synchronization or, when it recognizes this pitfall, doesn't provide an analysis for the Euro Area.

For some economies, we detect a slight increase in synchronization around the time of the introduction of the common currency in the Euro Area namely, in Austria, Belgium, Denmark, France, Germany, the Netherlands, Portugal, Spain. Nevertheless, we are not able to uncover a significant "*Euro effect*". Moreover, this increase in comovements is reversed in all the economies pointed, since synchronization decreases from the early 2000s onwards. The fact that the optimality of the currency areas is endogenous to the adoption of the common currencies and also, noting that the physical introduction of the common currency in the participant economies was made

Figure 2 (Continued): Estimated Synchronization Indexes with Euro Area

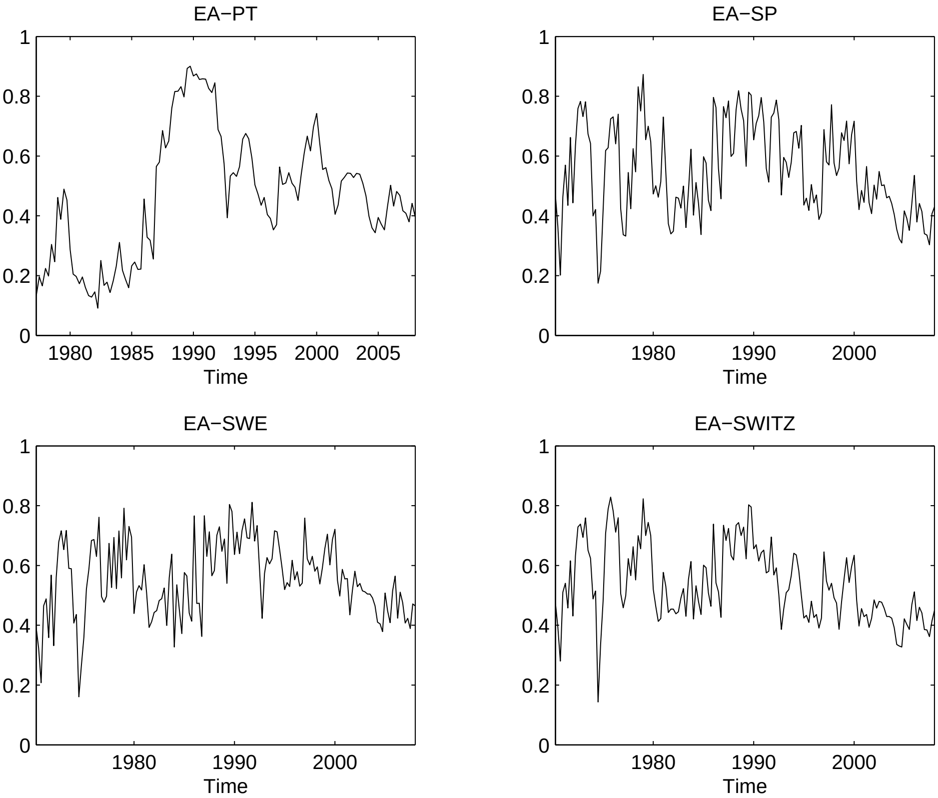


in 2001, leads us to believe that it can be to soon to detect a significant improvement in cyclical convergence.

Some non-participant economies display a particularly low level of synchronization with the aggregate Euro Area, namely, Canada, Japan, the UK and the US. This result was already pointed by the literature that analyses comovements within the G7, for example, Camacho and Perez-Quiros (2001), Stock and Watson (2005), Doyle and Faust (2005), partly in Canova et al. (2007) and Del Negro and Otrok (2008). On the contrary, other non-participant European countries, display levels of synchronization quite similar to those of some Euro Area economies, namely, Denmark, Sweden, and Switzerland.

Our results also suggest that France is the country with higher synchronization with the aggregate Euro Area, a result already found by the literature. Nevertheless, Germany and Italy also

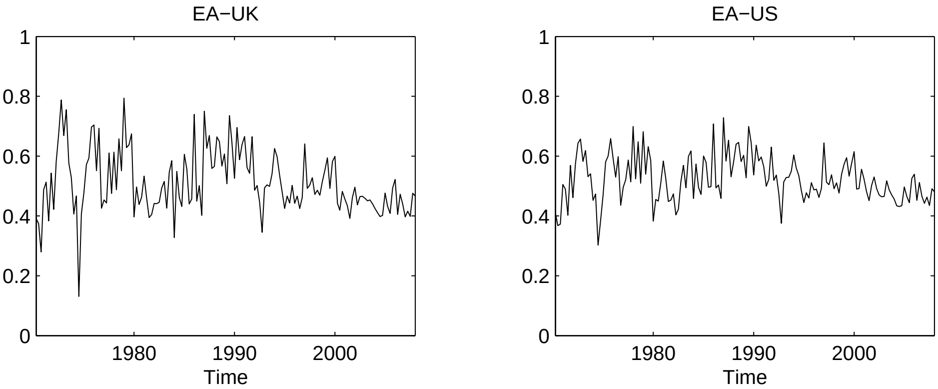
Figure 2 (Continued): Estimated Synchronization Indexes with Euro Area



attain relatively high levels of synchronization with the Euro Area. Commom to both economies, is also the break in the comovements during he 90s (despite the slight increase in comovements at the end of the decades, around the introduction of the Euro). For Germany, the drop in synchronization illustrates the effects of the reunification.

A puzzling result is that Germany's cyclical synchronization has roughly the same level than that of Greece. Despite that, this result has already been found elsewhere, showing that the level of synchronization of the two economies doesn't differ much, namely, Afonso and Furcery (2007), Mink et al. (2007) and Ferreira-Lopes and Pina (2008). The main difference between the two indexes emerges at the end of the sample, where the synchronization of Greece decreases considerably from the beginning of the 90s onwards (while that of Germany display a slight recovery in the 2000s).

Figure 2 (Continued): Estimated Synchronization Indexes for Models with Euro Area



One result that emerges from our estimations is that the level of synchronization decreased sharply in the mid-70s, around the recession of the first oil shock. This feature however has been detected in the previous section through the analysis conducted to the multivariate smoothed probabilities, which showed that for the majority of the economies, the probability of expansion in the Euro Area and recession in the other economies increased in this period, a consequence of a more prolonged recession in most of the economies.

The estimates for the Portuguese economy confirm why the economy has had so much difficulties in adapting to the Euro in the recent years. The results show a dramatic increase in the level of synchronization with the aggregate Euro Area previous to the entrance to the European Economic Community (EEC hereafter) in 1986. This increase was however reversed during the 90s, despite the participation in the Exchange Rate Mechanism (ERM hereafter). The start of this divergence coincides with the start of the disinflation period and the implementation of the nominal convergence policies. Despite a slight recovery at the end of the 90s, the cyclical synchronization continued to be rather low till the current period.

The results for Denmark show why this economy was placed for example by Bayoumi and Eichengreen (1993) in a group of countries which could represent the core of the currency union in Europe. During the second half of the eighties and the onset of the 90s, it featured a high degree of synchronicity with the aggregate Euro Area, and despite being lower than those of the 80s, the comovements recorded during the 90s were considerably higher than the ones of the other countries outside the currency union. Albeit not integrating the Euro Area, we estimate that Sweden has a comovement index similar to that of Finland.

This section presented the main results for the estimation of bivariate models with the aggregate Euro Area. We now end it summarizing the main conclusions obtained so far. These can be laid as follows:

First, the point estimates of the parameters of the model show that the algorithm proposed in this dissertation performs well in identifying the main features of international business cycles, namely, in identifying expansions and recessions, their expected durations and their dynamics (in terms of transition between states).

Second, our results show that business cycles across economies are reasonably well identified, with periods of recession being will flagged with decreasing cyclical components and increasing probabilities of recession. This good identification of states is also translated in a good identification of multivariate states, which allowed us to perform an analysis of the synchronicity of the states of the economy. With this analysis we dicover that: (i) for most of the countries, the states of the business cycle are well synchronized with the aggregate Euro Area but there are substantial differences; (ii) Euro Area countries tend to be more synchronized than non-participant countries; (iii) periods of divergence occur mainly during the 1970s and 1980s during the oil shocks, where most of the economies displayed prolonged periods of downturn when compared to the aggregate Euro Area.

Third, we've analyzed the estimates of our time-varying index of synchronization. In sum, our results suggest, firstly, that the Euro Area countries display higher levels of synchronization. Second, for most of the economies under analysis (in particular for the Euro Area countries) the estimated index of synchronization increases during the 70s and the 80s, peaks in the beginning of the 1990s and decreases from that period onwards. Third, notwithstanding the slight increase in the level of synchronization at the end of the 90s, we are not able to uncover a significant effect of the Euro (in particular in the Euro Area economies). Fourth, France is shown to be the country with higher synchronization with the aggregate Euro Area, despite the decrease from the end of the 80s onwards (and the good levels of comovements of Germany and Italy).

4.2.2 Bivariate Estimations with the US

In this section we present the results of the estimation of our bivariate model using as reference cycle the US, instead of the aggregate Euro Area. Even though the focus of this dissertation is the study of the Euro Area as a monetary union, this section has two purposes: first, to reassess the results obtained for the Euro Area using as a control the results that would be obtained for each country with a different, and important reference cycle; second, as a by-product, offer evidence that may be compared to the vast literature on the US economy.

Overview of the Results

Table 2 summarizes the estimates of the hyperparameters. In the table, country two always refers to the US, while country one represents each of the remaining economies.

In general the estimates are significant with the exception of some mean-growth rates. This happens in the estimations for Belgium, Denmark, Norway, the Netherlands and the UK. However, taken together with the results for the Euro Area, these results allow for a reasonable degree of confidence on the ability of the filter to estimate the hyparameters of the model[26].

Overall, we successfully identify the two phases of the business cycle, the recessions with a negative mean-growth rate, and the expansions with a positive growth rate. The mean-growth

[26] In general, as argued in Psaradakis and Sola (1998) the maximum likelihood estimator for markov- switching models behaves rather poorly in small samples. This could well be the case for some of our bivariate models in this section.

Table 2: Results for Bivariate Estimation with the US

Param.	AUS-US		BGM-US		CND-US		DEN-US		FIN-US	
$\gamma_{0,1}$	-0.52	(0.42)	-0.45	(0.24)	-0.35	(0.03)	-0.55	(0.22)	-0.22	(0.01)
$\gamma_{1,1}$	1.35	(0.40)	1.24	(0.31)	1.27	(0.09)	1.29	(0.19)	1.12	(0.04)
$\gamma_{0,2}$	-0.25	(0.45)	-0.37	(0.22)	-0.31	(0.01)	-0.14	(0.22)	-0.13	(0.05)
$\gamma_{1,2}$	1.22	(0.45)	1.29	(0.25)	1.18	(0.08)	1.07	(0.21)	1.09	(0.07)
$p_{0,1}$	0.78	(0.57)	0.80	(0.13)	0.77	(0.15)	0.52	(0.13)	0.41	(0.04)
$p_{1,1}$	0.91	(0.10)	0.93	(0.03)	0.91	(0.02)	0.91	(0.04)	0.90	(0.01)
$p_{0,2}$	0.74	(0.31)	0.51	(0.11)	0.74	(0.07)	0.52	(0.12)	0.43	(0.14)
$p_{1,2}$	0.92	(0.03)	0.92	(0.01)	0.92	(0.03)	0.91	(0.03)	0.83	(0.04)
$E\left[\mathcal{D}\vert s_{1,t}=0\right]$	4.45	-	4.92	-	4.43	-	2.07	-	1.69	-
$E\left[\mathcal{D}\vert s_{2,t}=0\right]$	3.91	-	2.03	-	3.89	-	2.09	-	1.74	-
ρ	0.49	(0.07)	0.80	(0.06)	0.80	(0.13)	0.67	(0.18)	0.64	(0.09)
$\phi_{1,1}$	0.50	(0.63)	0.30	(0.08)	0.95	(0.02)	0.56	(0.20)	1.30	(0.06)
$\phi_{2,1}$	0.45	(0.59)	0.10	(0.06)	-0.03	(0.04)	0.12	(0.07)	-0.36	(0.06)
$\phi_{1,2}$	1.13	(0.04)	0.84	(0.07)	0.79	(0.01)	0.93	(0.08)	1.10	(0.04)
$\phi_{2,2}$	-0.29	(0.02)	-0.16	(0.05)	0.10	(0.06)	-0.06	(0.02)	-0.22	(0.06)
σ_ζ	0.37	(0.03)	0.38	(0.07)	0.34	(0.07)	0.36	(0.16)	0.14	(0.01)
Λ_{ϵ_1}	0.88	(0.10)	1.39	(0.10)	0.79	(0.06)	0.86	(0.07)	0.64	(0.05)
Λ_{ϵ_2}	0.58	(0.08)	0.35	(0.06)	0.56	(0.05)	0.44	(0.08)	0.71	(0.05)
$-\mathcal{L}(\widehat{\alpha})$	-100129.50		-100125.50		-100106.09		-100096.08		-100083.10	

Note: Standard Errors in parentheses. $\gamma_{0,i}$ and $\gamma_{1,i}$ represent the mean growth rates of GDP during recessions and expansions; $p_{0,i}$ and $p_{1,i}$ the probability of transition from expansion to expansion and from recession to recession respectively; $\xi_{0,i}$ is the expected duration of a recession; $\phi_{w,i}$ are the parameters of the cycle equation; σ_ζ is the standard deviation of the common component while Λ_{ϵ_i} is the standard deviation of the business cycle, for country i.

rate of the US economy during recessions averages -0.18%, approximately -0.72% annually, while that during expansions equals 0.94% on average, which is translated into 3.76% approximately per year. Despite being relatively high, these growth rates are very similar across all the estimations. The other countries' estimates vary considerably, ranging from -1.13% for Japan to -0.20% for France during recessions and from 0.73% for Denmark and Switzerland to 1.13% for Japan during expansions. Japan's mean-growth rates are the highest in each of the two regimes (in absolute values). Recessions are on average more violent in Japan, the Netherlands, Sweden and Switzerland

Table 2 (Continued): Results for Bivariate Estimation with the US

Param.	FR-US		GER-US		GREE-US		IT-US		JP-US	
$\gamma_{0,1}$	-0.20	(0.05)	-0.51	(0.06)	-0.41	(0.84)	-0.51	(0.14)	-1.13	(0.37)
$\gamma_{1,1}$	0.95	(0.06)	1.27	(0.04)	1.37	(1.03)	1.25	(0.19)	2.26	(0.37)
$\gamma_{0,2}$	-0.36	(0.10)	-0.23	(0.07)	-0.03	(0.03)	-0.07	(0.01)	-0.20	(0.09)
$\gamma_{1,2}$	1.31	(0.07)	1.16	(0.10)	0.96	(0.06)	1.05	(0.06)	1.25	(0.14)
$p_{0,1}$	0.25	(0.09)	0.77	(0.06)	0.78	(0.31)	0.31	(0.18)	0.33	(0.09)
$p_{1,1}$	0.82	(0.01)	0.91	(0.01)	0.91	(0.03)	0.95	(0.01)	0.99	(0.00)
$p_{0,2}$	0.51	(0.27)	0.74	(0.06)	0.71	(0.32)	0.48	(0.11)	0.49	(0.13)
$p_{1,2}$	0.85	(0.03)	0.92	(0.01)	0.92	(0.11)	0.86	(0.03)	0.85	(0.05)
$E\left[\mathcal{D}\|s_{1,t}=0\right]$	1.33	-	4.44	-	4.47	-	1.44	-	1.48	-
$E\left[\mathcal{D}\|s_{2,t}=0\right]$	2.04	-	3.90	-	3.42	-	1.94	-	1.95	-
ρ	0.64	(0.08)	0.66	(0.03)	0.53	(0.29)	0.44	(0.07)	0.22	(0.19)
$\phi_{1,1}$	1.08	(0.04)	0.53	(0.10)	0.95	(0.15)	1.47	(0.07)	1.42	(0.09)
$\phi_{2,1}$	-0.15	(0.04)	0.42	(0.08)	-0.03	(0.03)	-0.54	(0.05)	-0.50	(0.06)
$\phi_{1,2}$	0.98	(0.06)	1.05	(0.05)	1.00	(0.06)	1.04	(0.10)	1.39	(0.16)
$\phi_{2,2}$	-0.06	(0.04)	-0.24	(0.04)	-0.20	(0.07)	-0.26	(0.04)	-0.44	(0.15)
σ_ζ	0.28	(0.04)	0.31	(0.06)	0.38	(0.08)	0.26	(0.04)	0.31	(0.11)
Λ_{ϵ_1}	0.38	(0.04)	0.79	(0.05)	1.69	(0.26)	0.74	(0.06)	0.76	(0.07)
Λ_{ϵ_2}	0.57	(0.04)	0.64	(0.05)	0.59	(0.07)	0.59	(0.05)	0.63	(0.07)
$-\mathcal{L}(\widehat{\alpha})$	-100027.35		-100115.94		-100211.33		-100086.68		-100124.84	

Note: See above.

while expansions are on average more pronounced in Canada, Finland, Greece, Japan, Norway and Spain.

The coefficients for the AR(2) process of the cyclical component show that these components are well identified and are mean-reverting stochastic processes. Some of the characteristics of the estimates shown in Table 2 are quite similar to those shown in Table 1 for the Euro Area. In the discussion in section 4.2.1, we found consistent evidence that countries with higher levels of persistence of the AR(1) process of the common component, were in general well synchronized with the reference cycle. We expect this pattern to be present in this panel as well. We note however that some of the countries that share this characteristic belong to the Euro Area, namely, Belgium, the Netherlands, and Spain (in addition to these, Canada, Norway and Sweden belong

Table 2 (Continued): Results for Bivariate Estimation with the US

Param.	NRW-US		NTH-US		PT-US		SP-US		SWE-US	
$\gamma_{0,1}$	-0.24	(0.23)	-0.67	(0.72)	-0.40	(0.45)	-0.31	(0.24)	-0.65	(0.34)
$\gamma_{1,1}$	1.30	(0.23)	1.46	(0.68)	1.30	(0.38)	1.40	(0.24)	1.53	(0.29)
$\gamma_{0,2}$	-0.18	(0.05)	-0.26	(0.19)	-0.24	(0.12)	-0.06	(0.12)	-0.01	(0.04)
$\gamma_{1,2}$	1.20	(0.12)	1.11	(0.09)	1.14	(0.09)	1.04	(0.13)	0.98	(0.10)
$p_{0,1}$	0.82	(0.05)	0.50	(0.69)	0.82	(0.12)	0.49	(0.44)	0.48	(0.11)
$p_{1,1}$	0.91	(0.07)	0.91	(0.12)	0.91	(0.03)	0.89	(0.14)	0.89	(0.04)
$p_{0,2}$	0.71	(0.08)	0.51	(0.37)	0.60	(0.22)	0.49	(0.94)	0.47	(0.32)
$p_{1,2}$	0.92	(0.04)	0.91	(0.02)	0.92	(0.03)	0.83	(0.35)	0.83	(0.07)
$E\left[\mathcal{D}\|s_{1,t}=0\right]$	5.43	-	2.00	-	5.45	-	1.97	-	1.94	-
$E\left[\mathcal{D}\|s_{2,t}=0\right]$	3.48	-	2.05	-	2.48	-	1.97	-	1.89	-
ρ	0.86	(0.05)	0.88	(0.17)	0.53	(0.08)	0.91	(0.03)	0.80	(0.06)
$\phi_{1,1}$	0.45	(0.08)	0.50	(0.21)	0.44	(0.05)	1.13	(0.29)	0.39	(0.15)
$\phi_{2,1}$	0.42	(0.10)	0.17	(0.22)	0.47	(0.09)	-0.30	(0.17)	0.15	(0.12)
$\phi_{1,2}$	1.04	(0.11)	0.96	(0.09)	1.06	(0.04)	1.26	(0.14)	1.05	(0.20)
$\phi_{2,2}$	-0.23	(0.06)	-0.19	(0.05)	-0.25	(0.04)	-0.40	(0.07)	-0.27	(0.10)
σ_ζ	0.27	(0.08)	0.22	(0.05)	0.34	(0.01)	0.12	(0.10)	0.21	(0.20)
Λ_{ϵ_1}	1.52	(0.10)	0.52	(0.04)	1.32	(0.10)	0.73	(0.13)	1.00	(0.04)
Λ_{ϵ_2}	0.55	(0.06)	0.53	(0.09)	0.51	(0.05)	0.67	(0.13)	0.65	(0.07)
$-\mathcal{L}(\widehat{\alpha})$	-100188.84		-100044.14		-100138.50		-100102.20		-100154.84	

Note: See above.

to this group).

The univariate transition probabilities seem to be reasonably estimated in general, with that of recessions higher than that of expansions, as usual. For the US, they imply estimates of the durations of recessions between 2 to 4 quarters approximately (sometimes slightly less), which is in line with the estimates from Hamilton (1989), Kim (1994) and many others. There are however some differences between the estimates for the remaining countries present in tables 1 and 2. Some of the estimates are considerably higher in bivariate models with the Euro Area reported in table 1, namely, for Finland, France, Italy, and Sweden.

Figures 24 to 40 in appendix A show the estimated cyclical components of real GDP for each bivariate system. Periods of recession seem to be well identified, with the cyclical components

Table 2 (Continued): Results for Bivariate Estimation with the US

Param.	SWITZ-US		UK-US	
$\gamma_{0,1}$	-0.81	(0.21)	-0.50	(0.37)
$\gamma_{1,1}$	1.54	(0.26)	1.28	(0.42)
$\gamma_{0,2}$	-0.04	(0.01)	-0.29	(0.20)
$\gamma_{1,2}$	0.98	(0.05)	1.16	(0.17)
$p_{0,1}$	0.35	(0.15)	0.71	(0.13)
$p_{1,1}$	0.78	(0.05)	0.91	(0.02)
$p_{0,2}$	0.44	(0.18)	0.35	(0.02)
$p_{1,2}$	0.83	(0.03)	0.92	(0.02)
$E\left[\mathcal{D}\|s_{1,t}=0\right]$	1.54	-	3.45	-
$E\left[\mathcal{D}\|s_{2,t}=0\right]$	1.79	-	1.54	-
ρ	0.65	(0.21)	0.50	(0.38)
$\phi_{1,1}$	0.71	(0.07)	0.70	(0.06)
$\phi_{2,1}$	0.03	(0.01)	0.24	(0.03)
$\phi_{1,2}$	1.17	(0.11)	1.02	(0.07)
$\phi_{2,2}$	-0.34	(0.06)	-0.21	(0.02)
σ_ζ	0.25	(0.07)	0.33	(0.13)
Λ_{ϵ_1}	0.87	(0.03)	0.65	(0.09)
Λ_{ϵ_2}	0.63	(0.04)	0.59	(0.06)
$-\mathcal{L}(\widehat{\alpha})$	-100135.12		-100092.37	

Note: See above.

decreasing and often crossing the zero line in periods identified as recessions by the BBQ dating algorithm for the identification of classical cycles (included in the graphs as dotted vertical lines).

The history of US business cycles that is told by the figures, is consistent with the literature. They clearly show three sharp recessions during the two oil price shocks' periods. A prolonged period of expansion till the third quarter of 1991, and two more recessions, namely in the beginning of 2001 and in 2007, can also be observed in the graphs.

For the remaining countries, the estimated business cycles are roughly in line with those estimated in models with the Euro Area. Some results are worth noting. In the case of Canada and Finland there is a violent drop of real output below its trend during the 1990s. The Japanese lost decade is also well documented here, by the significant fall in the output gap in the 1990s. It may be the case the there is an incorrect identification of our markov-switching model, which renders it unable to efficiently capture the cyclical dynamics of the Japanese economy, as the model estimates a considerable shock to the trend that makes the output gap systematically positive. A similar behavior is estimated for Spain, in which the 1970s recession is characterized by a positive (though falling) output gap (similarly to what was previously found in the estimation with the Euro Area).

Figures 59 to 75 present the estimated univariate smoothed probabilities of recession for each country. Recessions are identified with high success for the majority of the economies, as they are in accordance with the periods of downturn obtained from the BBQ algorithm. For the US economy, the model clearly detects six recessions, two during the 1970s, two in the beginning of the 80s, one in 1990, and one in 2001. One positive characteristic of our estimates is the detection of the recession beginning in the third quarter of 1990. Following McConnell and Perez-Quiros (2000), standard markov-switching models of US real output often fail to detect this recession if a break in the variance, occurred between 1983 and 1984, is not accounted for[27]. Our model does not imply any need to account for the *Great Moderation* in order to timely detect this period of downturn. The model seems however unable to detect the recent recession started in the fourth quarter of 2007.

There is a clear resemblance in the probabilities of recession of Canada and the US during the 80s, despite the slightly more protracted downturns in Canada. The similarity in the timing of recessions between the US and the other economies is less clear. When comparing the recession probabilities, one sees that no recession is detected in the beginning of 1970 for other countries, and the mid-1970s recessions in the US were more prolonged. Regarding this feature, Germany, Switzerland and the UK are exceptions, as they have spent more time in recession in the mid-1970s.

As an exercise for analyzing state-dependent synchronization between the US and our remaining sample of economies, we look at the smoothed multivariate probabilities in figures from 94 to 110.

[27] See also Kim and Murray (2002) and Lam (2004) for more on this subject.

Again, there is plenty of evidence of differences in the states of the economy across countries, which arise mainly during the 1970s and the 1980s.

Our results suggest the existence of significant desynchronization in the beginning of both decades, with the US being in recession while other countries were in the expansion phase. During the 1970s, this is observable through the significant increase in the probability in the lower left panel of the figures for Canada (figure 96), Greece (figure 101), Norway (figure 104) and the UK (figure 110). Note that with the exception of Greece, none of these countries integrates the Euro Area. During the 1980s, we observe desynchronized states between the US and Austria (figure 94), Belgium (figure 95), Denmark (figure 97), Italy (figure 102), Norway (figure 104) and the UK (figure 110).

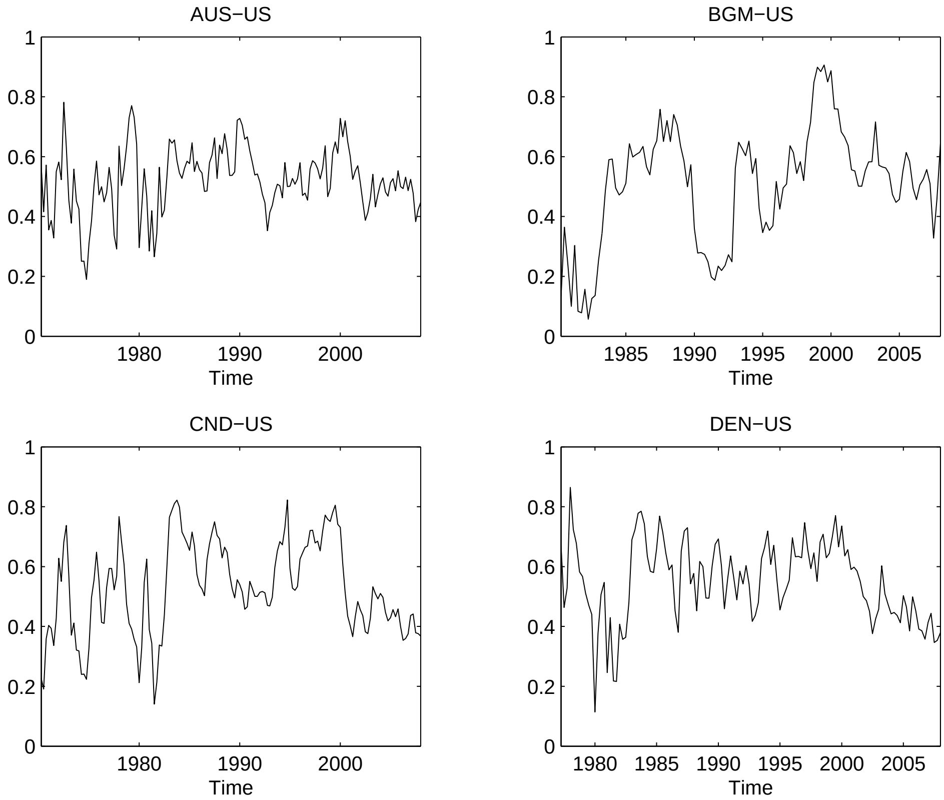
Additionally, differences in the states of the business cycle arise during the 1990s between the US and Belgium, France, Spain and Sweden. Hence, combined with the results for the previous two decades, it leads us to argue that divergence between the US and European countries may have increased in time.

Even when two economies are in the same phase of the business cycle (relating to the exercise we have just performed) comovements may be low simply due to differences in the characteristics of the recession/expansion namely, in terms of its deepness and steepness. A similar point was made by Harding and Pagan (2003) and Smith and Summers (2005) when using the concordance index. They state that it is often the case that two economies spend a high portion of time in the same state despite having a low correlation coefficient between the states of the business cycle. Hence, the previous exercise although informative, is incomplete.

Business Cycle Synchronization

We now turn to our comovement index $\delta_{t|T}$, computed from the time-varying estimates of the unobserved component $\vartheta_{t|T}$ according to (10). Figure 3 show the estimated indexes of comovements.

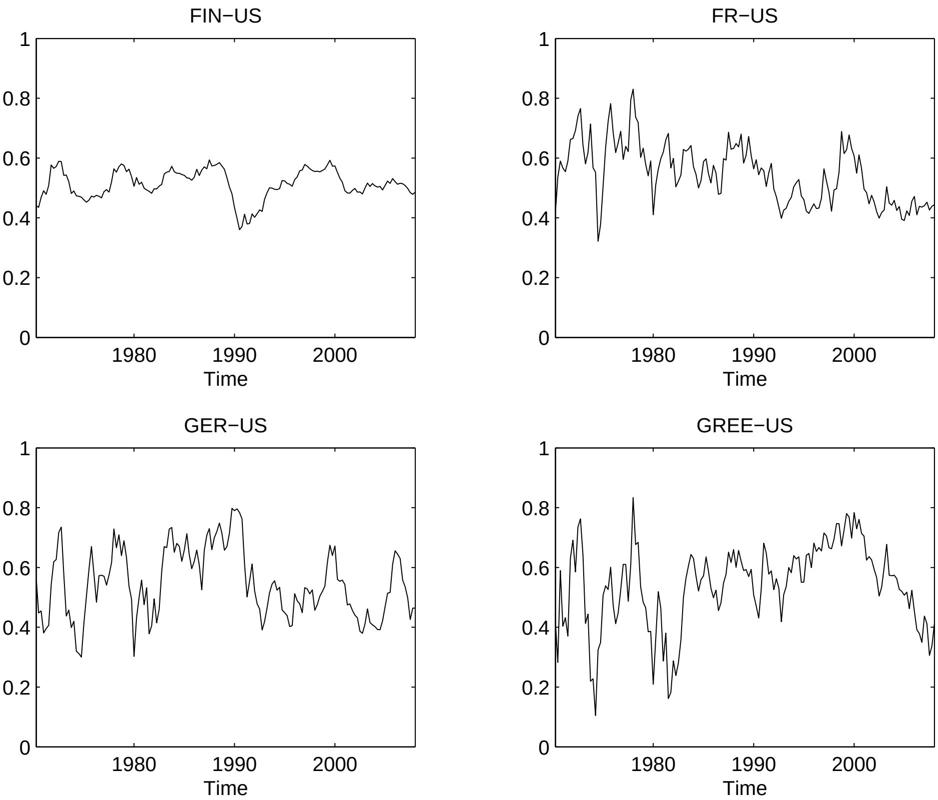
Figure 3: Estimated Synchronization Indexes with the US



The results suggest that for the majority of the Euro Area economies, the synchronization with the business cycle of the US either remained relatively stable across the 38 years of analysis (for example, in Austria, Finland, Germany and Spain), or decreased (for example, in France and Italy). In any case, however, the synchronization level with the US was not very high. The estimated index for France, in particular, remained stable during the 70s and the 80s, and decreased from the beginning of the 1990s onwards.

Our results further show that some Euro Area economies, namely, Belgium, Greece, the Netherlands and Portugal, increased their cyclical synchronization with the North American business cycle during the 70s, 80s and most of the 90s. In the end of the 1990s, the business cycles experienced a reduction in synchronization with that of the US.

Figure 3 (Continued): Estimated Synchronization Indexes with the US

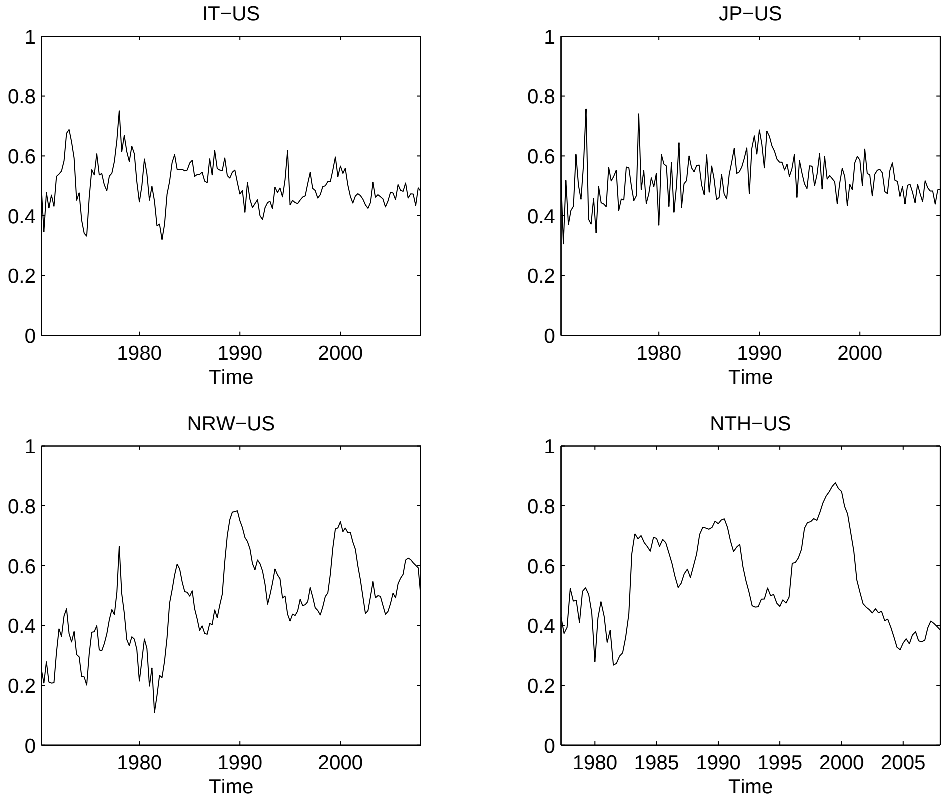


For the countries outside the Euro Area the results are rather mixed, with Canada, Denmark, Norway and the UK, increasing the synchronization during the three first decades, while Japan, Sweden and Switzerland revealing a quite stable level of synchronization throughout the period.

An interesting fact, common to the majority of the economies, is that the level of synchronization with the US business cycle increases during the 70s, the 80s and the 90s, peaking around the year 2000 and then decreases until the end of the sample. This characteristic is common to Belgium, Denmark, Greece, the Netherlands, Portugal and the UK. Since these countries are European (and with the exception of Denmark and the UK integrate the Euro Area), this result is particularly interesting, as the break in comovements coincides with the introduction of the Euro.

Additionally, our estimates reveal that Belgium, Canada, Greece and the Netherlands are the economies which display higher synchronization with the business cycle of the US economy, while

Figure 3 (Continued): Estimated Synchronization Indexes with the US

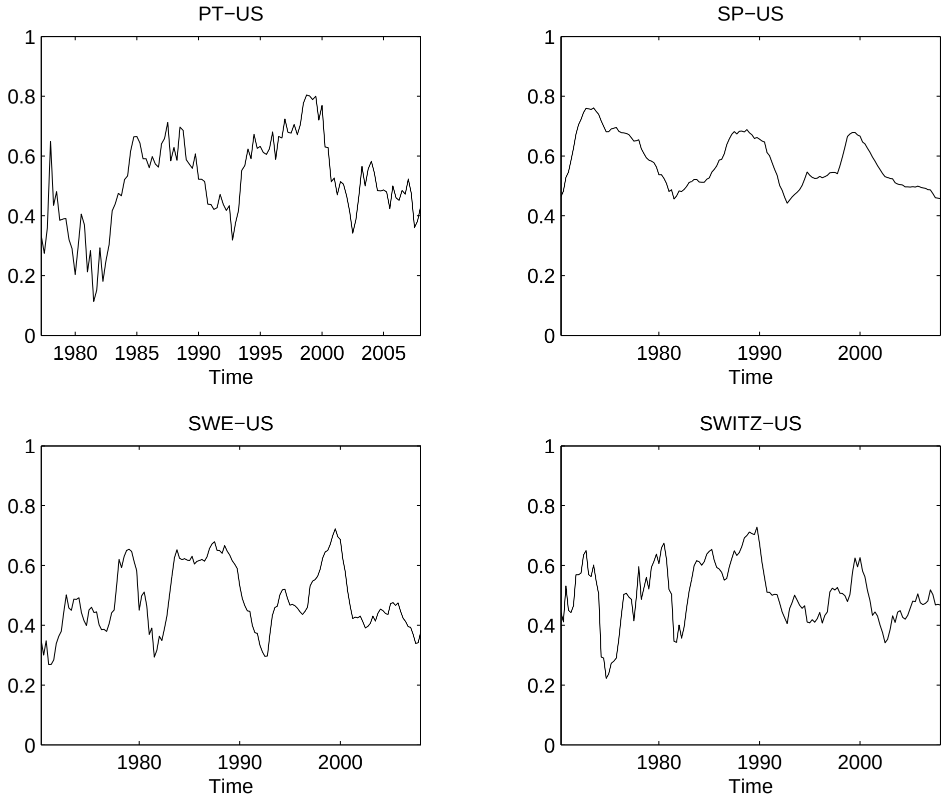


Finland, Italy, Japan and Spain are the countries with the lowest.

With respect to Canada, our estimates show that the cyclical synchronization increased during the 70s and remained stable during the 1989s and the 1990s. Nevertheless, our index uncovers a reduction in comovements starting in the end of the 90s, reaching a particularly low level.

The estimate for Japan suggests that its business cycle is loosely synchronized with that of the US. Moreover, the effects of the great depression of the 90s are also well depicted by the estimated index. The estimates show that the level of synchronization increased slightly till the end of the 1980s, but immediately reversed this effect, showing a decrease in comovements started in the beginning of the 1990s. it is interesting to note that this historical characteristic of the Japanese business cycle is more clearly obtained in the bivariate estimation with the US than with the aggregate Euro Area.

Figure 3 (Continued): Estimated Synchronization Indexes with the US



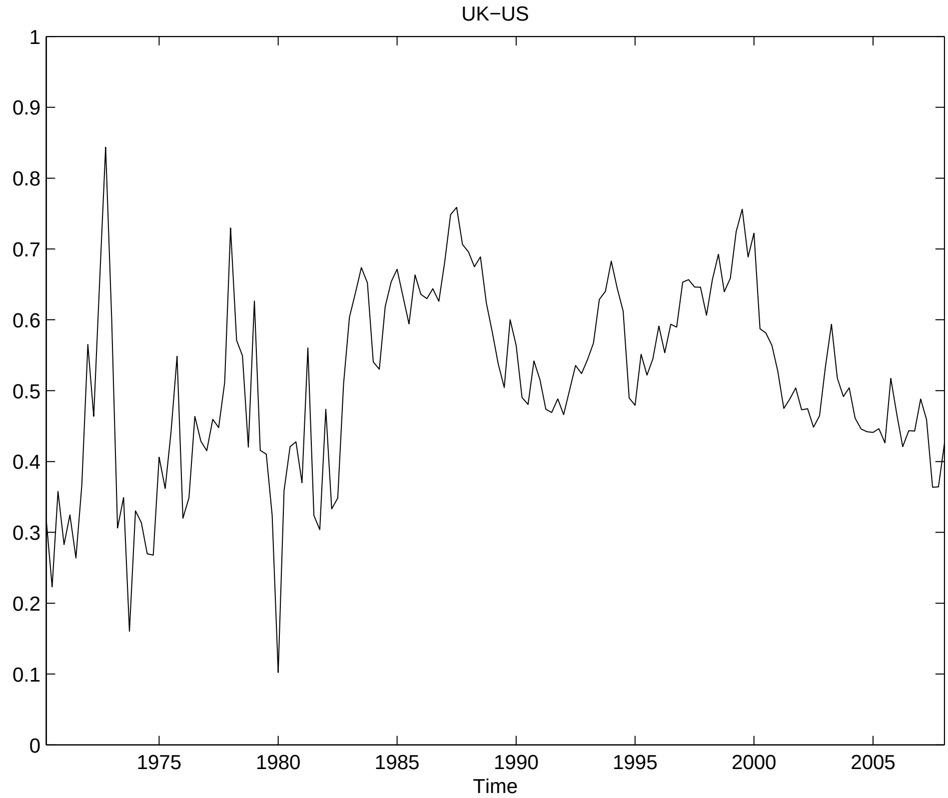
It is particularly interesting to see the substantial increase in synchronization with the US business cycle of those of Belgium, Greece, the Netherlands and Portugal during the 1990s. In bivariate models with the Euro Area, the estimated index of synchronization suggests a reduction in comovements started in the beginning of the decade. Notwithstanding this increase with the US, it is reassuring to observe that it is reversed at the end of the decade.

Summing up, we have reached the following main conclusions.

First, the point estimates of the model proposed confirm the success of our algorithm to identify the business cycles and the states of the economy, recessions and expansions.

Second, after confirming the good identification of recessions and expansions, through the estimated business cycles and also, the smoothed probabilities of recession and expansion, we've performed an analysis of synchronicity of the states of the economy using the multivariate proba-

Figure 3 (Continued): Estimated Synchronization Indexes with the US



bilities. Our results suggest that there is a substantial difference in the duration of the recessions during the 1970s and the 1980s, which is translated in a decrease in state-dependent synchronicity.

Recognizing the incompleteness in the analysis of the multivariate probabilities in detecting comovements between the business cycles, we analyzed the results from our time-varying index of synchronization with the US. In sum, our results suggest that, first, for the majority of the Euro Area economies, their levels of synchronization with the US business cycle either remained stable of show a tendency to decrease. Nevertheless, some economies (Belgium, Greece, the Netherlands and Portugal), increased their comovements till the end of the 1990s. Second, the level of synchronization of Belgium, Denmark, Greece, the Netherlands, Portugal and the UK, with the US business cycle increased during the 70s, the 80s and the 90s, peaked around the year 2000 and decreased from that period onwards. Third, for Canada, our estimates show that the increase in synchronization during the 70s, remained quite stable throughout the 80s and 90s but were reversed during the last 8 years of our sample.

4.3 Discussion of the Results

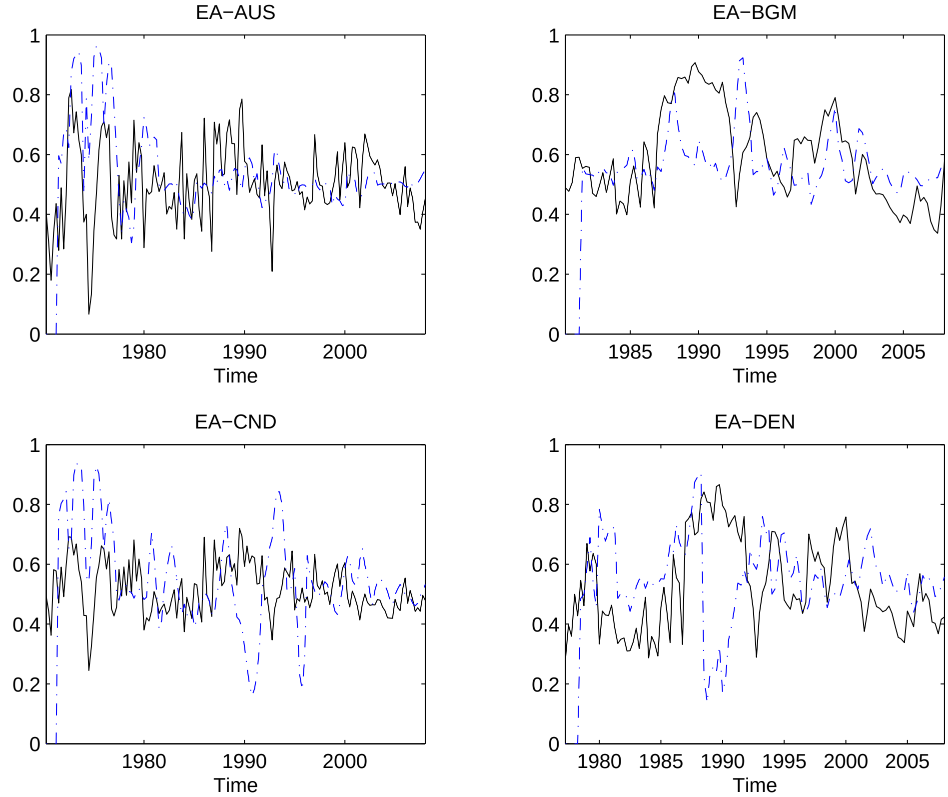
4.3.1 Rolling Covariances

In this section, we discuss our results with further detail, focusing on the estimates of the business cycle synchronization index. This is in fact main objective in this dissertation, i.e., to analyze the patterns of comovements between the business cycles of our sample of countries.

Our interpretation of the common component ϑ_t is that it captures the common variability between the estimated business cycles. Given the assumption of diagonality of the matrix of coefficients and of the orthogonality of the innovations of the VAR that drives the business cycles, this interpretation seems warranted since all the common variation will be captured by this component. Truly, under this interpretation, the component approaches the definition of a covariance. Our discussion of the results starts out exploring this parallelism, which is done comparing the estimates of the common component with a simple rolling covariance between the relevant pair of business cycles (over a 5-quarter rolling window). To ensure comparability, we transform the rolling covariance to lie within the closed interval $[0, 1]$ using equation (10). Figure 4 summarizes the results for the Euro Area.

The rolling covariances are in general remarkably similar to our synchronization indexes, which confirms our previous results. We are then able to reinforce the main conclusions outlined in section 4.2.1. One characteristic from the results, is that the computed rolling covariances tend to be more volatile than our estimated indexes of synchronization. The direct consequence of the increase in volatility is the tendency of the transformed covariance to approach one. Secondly, despite the substantial overlapping, we observe some lag in the variation of comovements when

Figure 4: Synchronization Indexes and Rolling Covariance with Euro Area



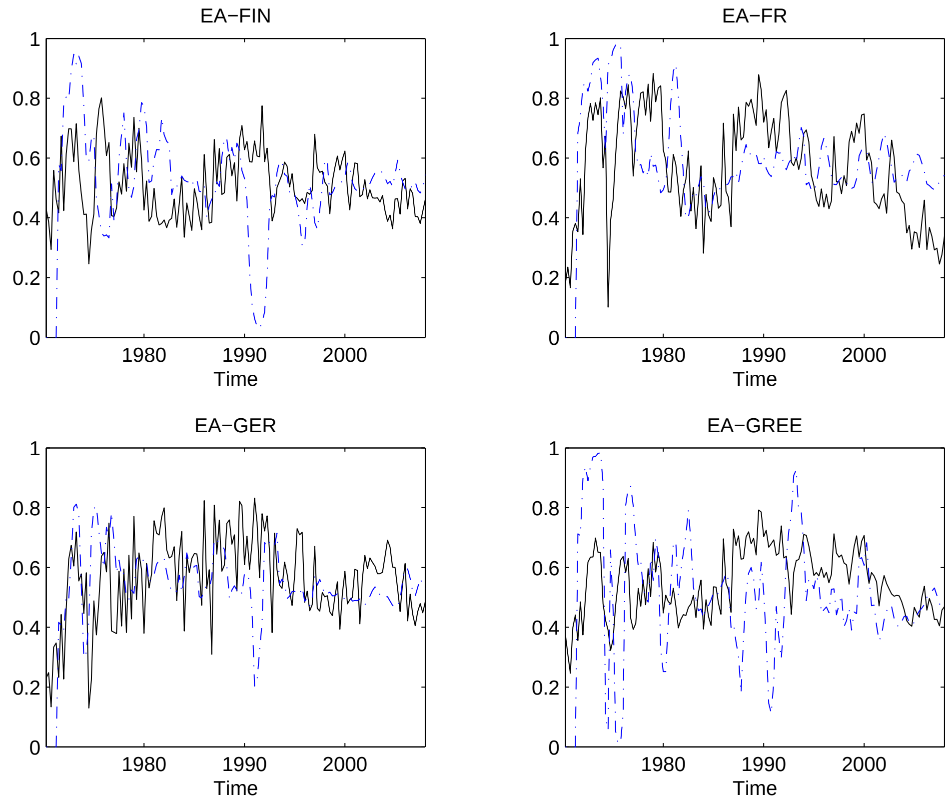
Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

compared to the synchronization index.

When we compare the rolling covariances computed between the aggregate Euro Area and the member countries of the currency area, we observe that their comovements are higher than for the countries outside the monetary union. When we compare the results for Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain, with those for Canada, Norway, Japan, the UK and the US, this result becomes clear.

Third, the rolling covariances also validate one characteristic already outlined in section 4.2.1, namely, the synchronization with the aggregate Euro Area increases during the 70s and 80s, but

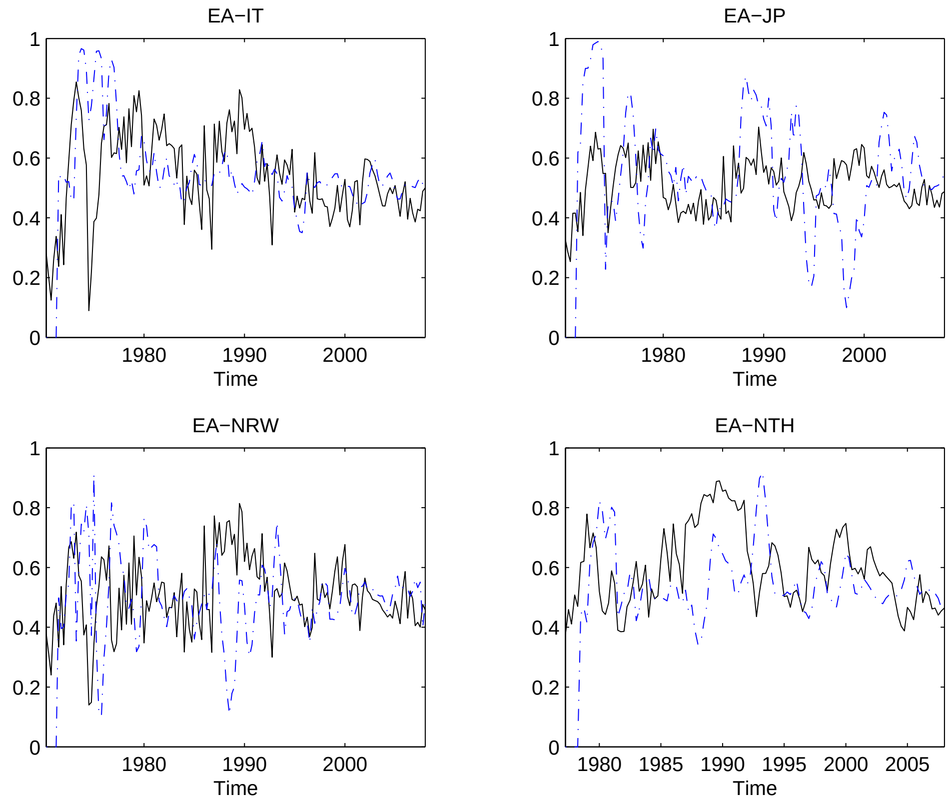
Figure 4 (Continued): Synchronization Indexes and Rolling Covariance with Euro Area



Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

from the beginning of the 1990s onwards exhibits a substantial decrease. This feature is observed for Belgium, Denmark, Finland, France, Germany, Greece, Italy, Norway, the Netherlands, Portugal, Spain and Sweden, ad now also for Switzerland. It is particularly interesting to see that the covariances confirm two already mentioned results regarding Portugal: (i) a substantial increase in the synchronization of its cycle with the Euro Area's during the 1980s, ahead of the entrance into the EEC, and (ii) the large reduction in synchronization with the business cycle of the Euro Area afterwards. It is thus possible that the policies pursued by the national policymakers, during the run-up for the common currency, have caused a reduction in the cyclical affiliation with the remaining economies of the monetary union.

Figure 4 (Continued): Synchronization Indexes and Rolling Covariance with Euro Area



Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

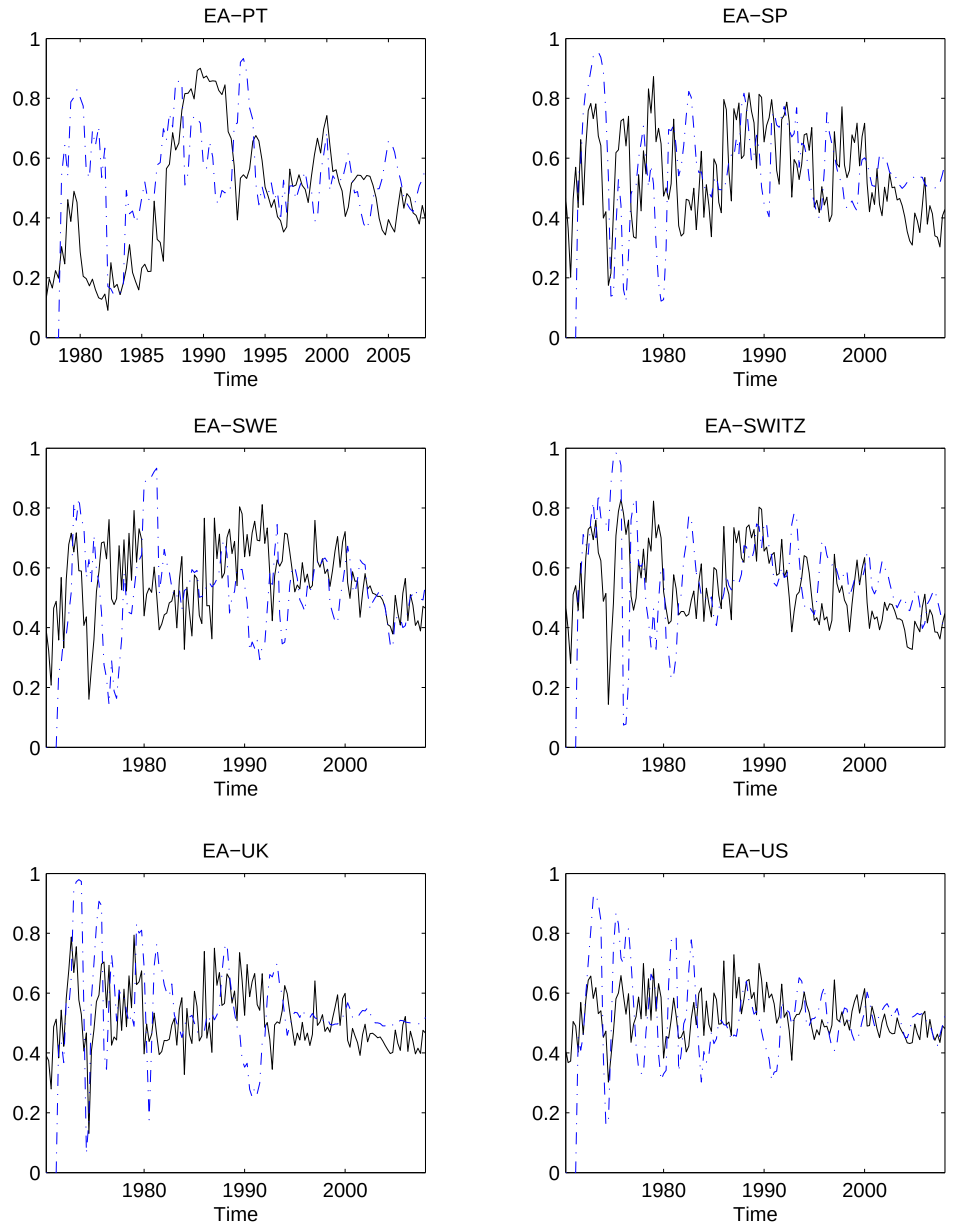
Fourth, the results confirm that in general, across the 38 years analyzed, France is the economy that displays higher cyclical synchronization with the business cycle of the aggregate Euro Area.

Fifth, the idiosyncrasy of the Japanese business cycle is once again confirmed, with a tendency to decrease in the last eighteen years, due to the well know depression, displaying the lowest synchronization with the Euro Area[28].

Sixth, the rolling covariances validate the substantial drop in synchronization in the mid-70s, during the first oil-shock recession. This characteristic was also previously found through the analysis of the multivariate probabilities (see the increase in the smoothed probability in the lower

[28] The Japanese depression is reviewed in Hayashi and Prescott (2002).

Figure 4 (Continued): Synchronization Indexes and Rolling Covariance with Euro Area

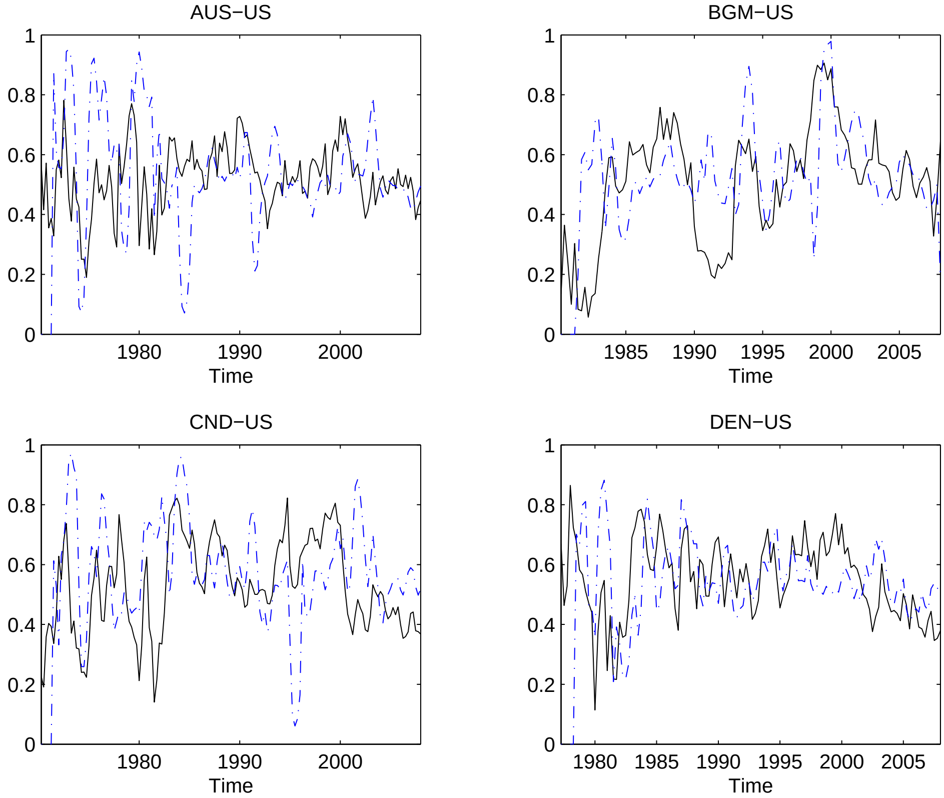


Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

left and right panels of figures from 76 to 93).

Figure 5 shows the comovement indexes $\delta_{t|T}$ and the rolling covariances of the output gaps

Figure 5: Synchronization Indexes and Rolling Covariance with the US



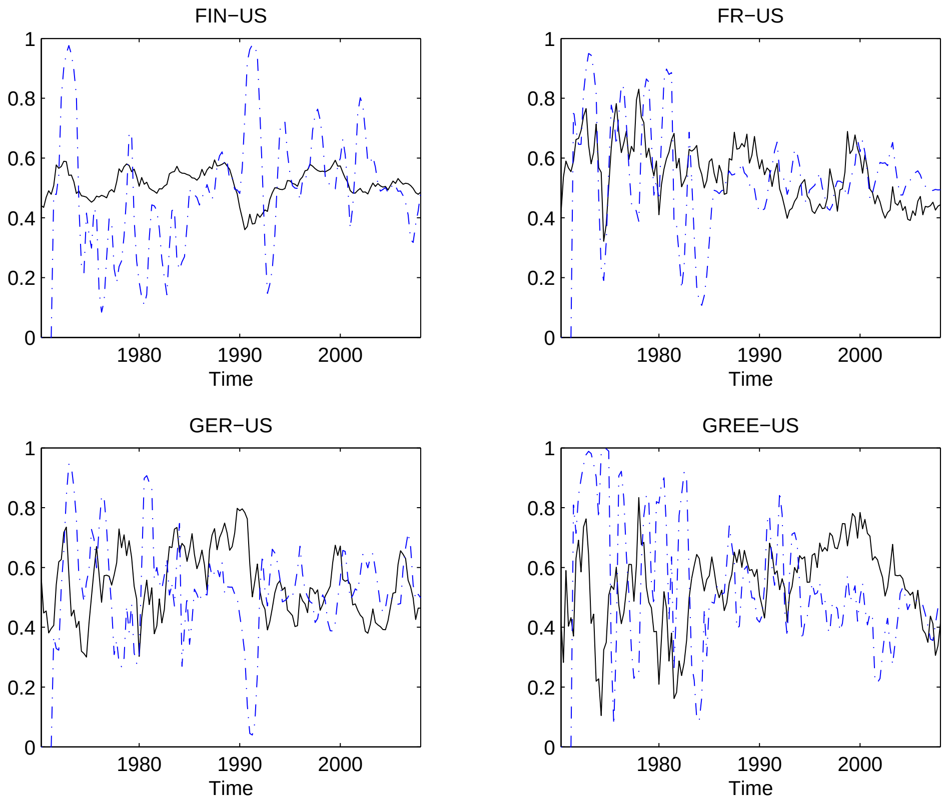
Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

foe the bivariate models with the US. Again, our results are confirmed, despite a somewhat less perfect overlap between the rolling covariances and our comovement indexes.

For the Euro Area economies, the results suggest that in fact, the majority of them either maintained a relatively stable level of cyclical synchronization with the business cycle of the US, namely, Austria, Finland, Germany, Italy and Spain, or decreased their comovements over time, namely, France.

When we compare the covariances with the smoothed index of synchronization, we confirm that for Belgium, Denmark, Greece, the Netherlands, Portugal, Sweden and the UK, the cyclical

Figure 5 (Continued): Synchronization Indexes and Rolling Covariance with the US

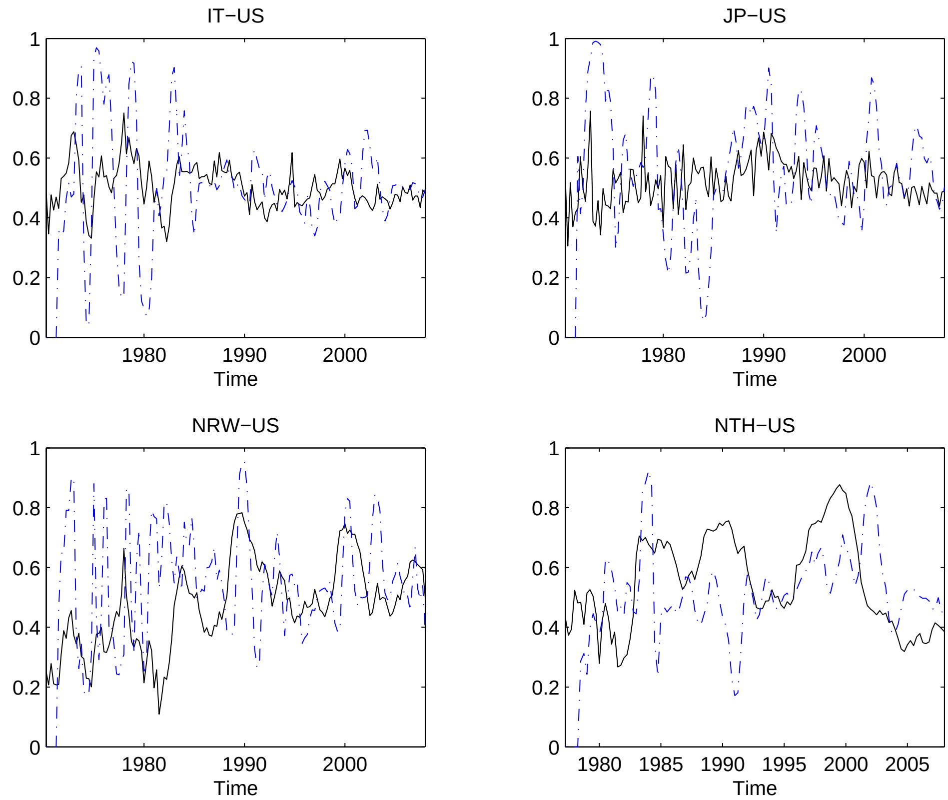


Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

synchronization with the US increased until the beginning of the 1990s, inverting this tendency at the onset of the 2000s. As these countries are European (and some of them participants in the Euro Area) and the break in the increase in comovements coincides with the introduction of the common currency, one may argue that this may constitute a possible effect of the Euro. Notwithstanding this possibility, one should note that the the US business cycle peaked in the first quarter of 2001, while these economies continued in expansion. This difference in the state of the business cycle contributed to the drop in comovements.

The results further confirm that Finland and Japan are the economies that exhibit the lowest comovements with the business cycle of the US, while Canada is the one that maintained consid-

Figure 5 (Continued): Synchronization Indexes and Rolling Covariance with the US

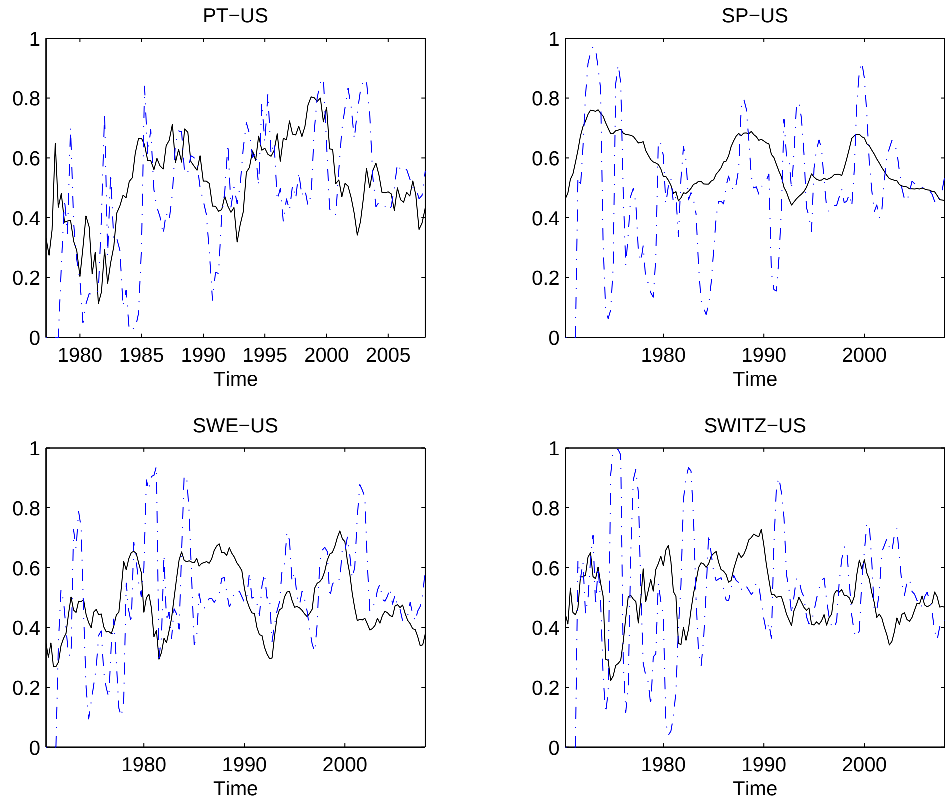


Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

erable cyclical comovements throughout the sample period (a result in line with the literature that analyzes business cycle synchronization within the G7).

Overall, for most of the cases, not only in models with the Euro Area but also in those with the US, we find that the business cycles are presently less synchronized than they've been during the previous thirty years. This result had already been noted by the IMF (2007, Chapter 4), which has argued that "*(...) while it is difficult to derive strong conclusions about the extent of synchronization, there is some evidence that national business cycles among industrial countries are now more synchronized than in the 1960s, although less so than during the 1970s and the first half of the 1980s."*. This result, which we have confirmed in this chapter, is of particular

Figure 5 (Continued): Synchronization Indexes and Rolling Covariance with the US

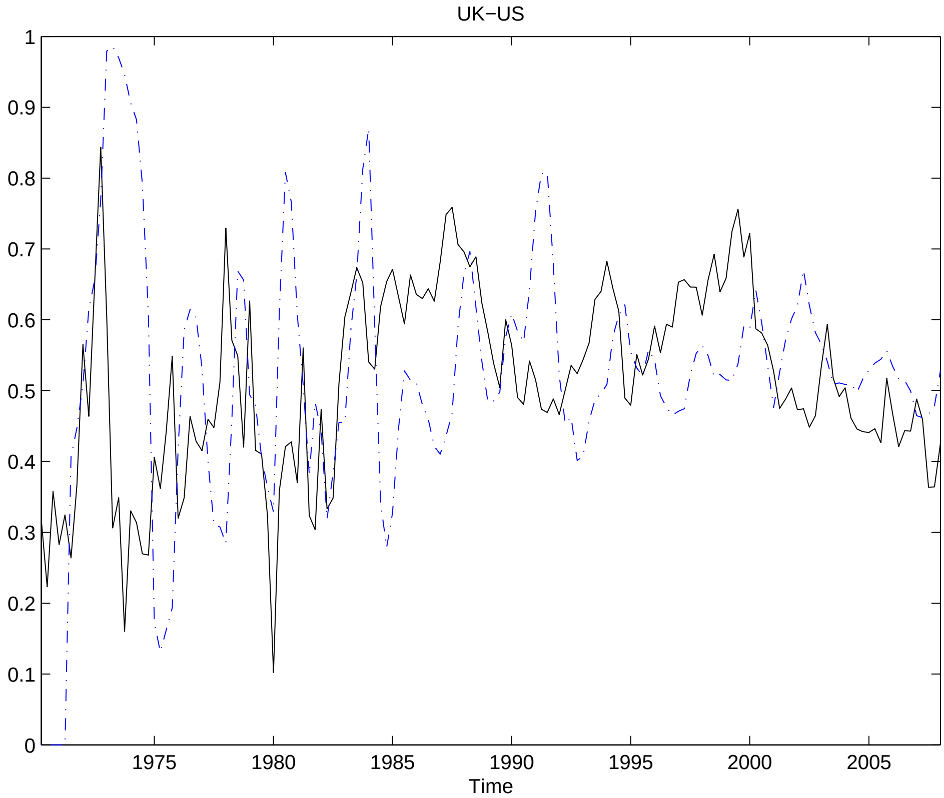


Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

importance in the Euro Area countries, since it means that the common monetary policy may be increasingly becoming ill suited for most of them.

Moreover, the IMF (2007) claims that the recent decrease in synchronization reflects the drop in global shocks and a rise in country-specific ones and recognizes the benefits of increased cross-border trade and financial linkages in fostering synchronization. Evidence on the prevalence of global and country-specific shocks had been previously provided by Stock and Watson (2005). They suggested that the reduction in the size of international shocks played an important role in the Great Moderation for Canada, France, Germany and the US, and that this moderation contributed to a large extent to the decrease in synchronization that they observed. As oil shocks

Figure 5 (Continued): Synchronization Indexes and Rolling Covariance with the US



Note: Solid line depicts the estimated index of synchronization and the dashed line the rolling covariance.

can be considered a truly global shock, they can explain in part, the moderation of international shocks. In fact, Blanchard and Gali (2008) show that the effects of oil price shocks in the most recent period are substantially different than during the 1970s, having smaller real effects today than in the past.

The cyclical divergence that we and others uncovered for the recent decade, may come from other sources as well. Specifically, the European integration process, namely, the liberalization of trade, may have increased the specialization in the production, as suggested by Krugman (1993), which contributed to foster idiosyncratic shocks. Additionally, the introduction of the common currency in Europe, may have induced an increase in idiosyncratic shocks as well, as countries lost their monetary policy that could be used to stabilize the economy. Nevertheless, the predominance of significant disparities in fiscal policy (measured by the government budget deficits), may have contributed to decrease business cycle synchronization, a result found in Silva (2009).

4.3.2 Variance Decomposition

Taking into account the arguments stated above, we perform an additional exercise, which intends to gauge the importance of common fluctuations in generating the real fluctuations observed across countries. For this, we suggest a simple variance decomposition of the total variance of the business cycle considering equation (12). It may be seen that there are two main sources of variance for business cycles, namely, the common component and the idiosyncratic innovations. Recall that in our framework, each business-cycle equation assumes the following form:

$$x_{c,t} = \phi_{c,11,1} x_{c,t-1} + \phi_{c,11,2} x_{c,t-2} + \vartheta_{t-1} + \epsilon_{c,t} \tag{75}$$

$$\vartheta_t = \rho \vartheta_{t-1} + \zeta_t \tag{76}$$

$$\epsilon_{c,t} \sim \mathcal{N}\left(0, \Lambda_{\epsilon_{c,t}}\right) \tag{77}$$

$$\zeta_t \sim \mathcal{N}\left(0, \sigma_\zeta^2\right) \tag{78}$$

Denoting by Var the variance operator and applying it to both sides of (76), we obtain:

$$Var\left(\vartheta_t\right) = \frac{\sigma_\zeta^2}{1-\rho^2} \tag{79}$$

Given that, from the definition above,

$$Var\left(\epsilon_{c,t}\right) = \Lambda_{\epsilon_c} \tag{80}$$

Then, a measure of the importance of common fluctuations relative to idiosyncratic fluctuations for country c is given by:

$$\mathcal{S} = \frac{Var\left(\vartheta_t\right)}{Var\left(\epsilon_{c,t}\right) + Var\left(\vartheta_t\right)} = \frac{\frac{\sigma_\zeta^2}{1-\rho^2}}{\Lambda_{\epsilon_c} + \frac{\sigma_\zeta^2}{1-\rho^2}} \tag{81}$$

By definition, this measure spans the interval $[0, 1]$. If this ratio equals zero, business cycles in country c are determined only by idiosyncratic fluctuations; when the ratio equals 0.5, idiosyncratic and common fluctuations have the same importance in generating the business cycle; when the ratio equals 1, common fluctuations are fully responsible for generating business cycles in country c.

As equation (81) shows, the measure $\mathcal{S}$ is a function of three hyperparameters of the model, namely, σ_ζ^2, ρ and Λ_{ϵ_c}; our estimation algorithm guarantees quasi-maximum likelihood estimates that may be used in the computation of $\mathcal{S}$ for each bivariate model.

Note that when the ratio is computed for a given country, it should also be computed for all the remaining countries in the model. The fact that common fluctuations are important to one country, doesn't necessarily mean that they will be equally important for the other countries, since these can have larger idiosyncratic components.

The motivation for this analysis is that highly synchronized countries must have an important portion of their business cycles generated by common fluctuations, and thus high values of $\mathcal{S}$. Note that we are not performing a complete variance decomposition of the business cycle equation since we do not take into account the effects of the autoregressive process, however, our intention here is to analyze the relative importance of the two sources of fluctuations (idiosyncratic and common), which justifies our approach. The results can be found in table 3.

For the majority of the Euro Area countries, the importance of the fluctuations that are common to the aggregate Euro Area is rather low. Yet, the importance of the common component in the

Table 3: Relative Importance of the Common Component in the Variance of Business Cycles

$\mathcal{S}$	EA	US
EA	-	0.42
AUS	0.29	0.19
BGM	0.08	0.18
CND	0.10	0.33
DEN	0.18	0.24
FIN	0.24	0.08
FR	0.86	0.47
GER	0.41	0.21
GREE	0.03	0.07
IT	0.42	0.13
JP	0.08	0.15
NRW	0.06	0.11
NTH	0.28	0.44
PT	0.15	0.08
SP	0.41	0.14
SWE	0.11	0.10
SWITZ	0.15	0.13
UK	0.22	0.26
US	0.16	-

Note: The table shows the estimates of $\mathcal{S}$ for each country in the first column for bivariate models with the aggregate Euro Area and the US.

overall cyclical variance is in general smaller in bivariate models of the Euro Area economies with the US, with an exception of Belgium, Greece an the Netherlands.

The results confirm that France is the country with higher synchronization with the aggregate Euro Area, as it exhibits the largest portion of variability attributed to common fluctuations in bivariate models with the Area. Germany's share of common fluctuations on its business cycle variability is also sizable and is identical to those of Italy and Spain. It seems that $\mathcal{S}$ is not larger for Germany because the variance of the idiosyncratic innovations in Germany is large, relative to the other countries. This is in line with our findings for the time-varying index of synchronization.

Overall, with only the exception of France, in all the economies of the Euro Area, the percentage of variability accounted by common fluctuations in bivariate models with the Area is less than 50%. Particularly noteworthy are the estimates for Belgium and Greece, which have the most idiosyncratic business cycles. Austria, Finland, the Netherlands and Portugal, have a share of variability caused by common fluctuations in the range from 15% to 29%.

For the countries outside the Euro Area, the importance of the common component for models with the Area is rather low, which is especially so for Japan and Norway. The percentage of the variance of the common component is in the case of Denmark, Switzerland, the UK and the US, quite similar to that of the more peripheric countries of the European currency union.

These results seem to suggest that there is no evidence of an Euro Area business cycle in our analysis. Not only the time-varying indexes have shown that for most of the countries, synchronization with the aggregate Euro Area decreased in the recent period, as the importance of common fluctuations in their business cycles seems to be limited. Our exercise has shown that for most of the countries we cannot identify an Euro Area group, that behaves significantly different from the remaining countries.

In the case of the bivariate estimations with the US, our results show that overall, the importance of common business cycle variability is less than 50%. France is, however, close to this mark. It is surprising that, some of the Eurozone countries have higher portion of business cycle variability explained by common fluctuations in bivariate models with the US than with the Euro Area (Belgium, Greece and the Netherlands). Nevertheless, in general, the business cycles of the Eurozone countries seem more affected by common fluctuations with the Euro Area, then by common fluctuations with the US.

It is also worth noting that the importance of common fluctuations with the US in the case of Canada is smaller than in the case of France, (33%). In contrast, Japan and the UK seem more affected by common fluctuations with the US than with Eurozone.

Summing up, we confirm that overall, synchronization is rather low. In general, countries belonging to the Euro Area tend to be more cyclically linked.

5 Conclusion

This dissertation focuses on the patterns of business cycle synchronization across a sample of 18 industrialized countries plus the aggregate Euro Area over the period 1970:1-2008:1. The study is thorough in the sense that, contrary to some literature, we do not restrict our attention to a subset of industrialized countries like the G7, but rather provide an analysis for a large sample including countries from North-America (US and Canada), Europe (including countries belonging to the Euro Area and also non-participants) and Japan.

In this study, we attempt to address a significant gap in the literature, that often doesn't recognize that synchronization can vary in time. We propose a novel approach to analyze business cycle synchronization, based on an unobserved-components model with markov-switching. Moreover, we assume the existence of time-varying interdependence of the states of the business cycle, an idea previously proposed by Camacho and Perez-Quiros (2001) in the context of a standard multivariate markov-switching model. Our approach is flexible since it models comovements as a time-varying process, identified inside the dynamic trend-cycle decomposition. We propose a new Kalman filter to recover the unobserved components and estimate the parameters. We also derive a full-sample smoother to re-compute the unobserved components of the model based on all in-sample information.

We show that the new filter can successfully identify the unobserved components namely, the business cycles and the common components of real fluctuations, and estimate the hyperparameters by quasi-maximum likelihood. Each state of the business cycle, expansion and recession, was shown to be correctly identified by their mean growth rates and the duration of each state; and the corresponding probabilities of expansion/recession were shown to be well estimated.

Our study shows contrasting results with most of the literature on business cycle synchronization within the Euro Area. We can group the main conclusions as follows.

First, we show that the cyclical synchronization of the Euro Area countries with the aggregate Area is higher than the synchronization for the remaining countries. Our estimates also suggest that synchronization increases during the 1970s till the beginning of the 1990s and decreases from that period onwards for Belgium, Denmark, Finland, France, Germany, Greece, Italy, Norway, the Netherlands, Portugal,Spain and Sweden. To our knowledge, this is a new finding in terms of comovements within the Euro Area. Interestingly, the drop in synchronization happens closely after the adoption of the Single Market rules within the EEC, but also, with the adoption of nominal convergence criteria by the member countries of the EMU, which would allow them to fulfill the rules for the adoption of the common currency.

Notwithstanding this drop in the level of comovements, we estimate a slight increase in synchronization around the timing of the introduction of the common currency, namely, in Austria, Belgium, Denmark, France, Germany, the Netherlands, Portugal, Spain. Nevertheless, this increase is so tenuous, that we believe we haven't uncovered a clear "*Euro Effect*", or a clear increase in synchronization, fostered by the common currency. The endogeneity of the currency areas however, leads us to believe that it may be relatively soon to observe a significant improvement in comovements within the currency union, especially due the persistent disparities in idiosyncratic policies inside the Union.

Second, our estimates suggest that France is the country with higher synchronization with the aggregate Euro Area (a finding already put forward by many others), while Japan exhibits the most idiosyncratic real fluctuations when contrasted with those of the European currency union.

Third, we find that the difficulties of the Portuguese economy in adapting to the Euro are related to the desynchronization of the country's business cycle with that of the Union aggregate. After a substantial increase in comovements during the 1980s, we estimate that real fluctuations in Portugal have become progressively less synchronized with those of the aggregate Euro Area since the beginning of the 90s.

Fourth, synchronization of the aggregate Euro Area business cycle with that of the US economy, was in general low throughout the last 38 years, and shows a tendency to decline during the 90s and the 2000s. Interestingly, the decline started in the late 1980s, around the ratification of the Single European Act and the introduction of the Common Market.

Fifth, cyclical comovements between Euro Area countries and the US are considerably lower that with the aggregate Euro Area. We show that the synchronization with the business cycle of the US, either remained relatively stable across the entire sample (in Austria, Finland, Germany and Spain), or dropped (in France and Italy). Some European countries, on the other hand (some of them participants of the Euro Area), increased their cyclical synchronization with the US business cycle during the 70s, 80s and most of the 90s, namely, Belgium, Greece, the Netherlands and Portugal. Nevertheless, in the end of the 1990s, this increase was reversed.

An interesting characteristic of our estimates, is that for Belgium, Denmark, Greece, the Netherlands, Portugal and the UK, the level of synchronization with the US business cycle increased till the end of the 1990s, and, around the years 1999-2000, decreased till the end of the sample. Since these countries are European (and with the exception of Denmark and the UK, integrate the Euro Area), this result is particularly interesting, as the break in comovements coincides with the introduction of the Euro.

In contrast with the results for the Euro Area, our estimates reveal that Canada has maintained a relatively high level of comovements throughout the entire period of analysis (a finding consistent with the literature on business cycle synchronization within the G7). Also in line with the literature, is the low comovements between Japan and the US, and the apparent drop in comovements initiated during the 1990s (due to the depression).

Finally, we provide a discussion of our results through two additional exercises in order to validate our findings. After contrasting our estimated indexes of synchronization with a 5-quarter rolling covariance, we confirm the essence of our conclusions. In most bivariate estimations the two measures of synchronization essentially overlap, which seems to validate our conclusions in this dissertation. We performed yet another exercise to evaluate the importance of common fluctuations with Euro Area and the US for each economy. For most of the countries, the importance of common fluctuations with the aggregate Area and the US is substantially lower than the importance of idiosyncratic fluctuations, as the later accounts for most of the variance in real fluctuations. This provides evidence that idiosyncrasies are not disappearing, or even that the specific national business cycles have not lost importance. Nevertheless, in France, common fluctuations with the Euro Area account for most of the business cycle variability.

These results directly answer our motivations for this research. Although the countries that participate in the Euro Area have their cycles apparently more synchronized with the aggregate Euro Area than with the US business cycle, we are not able to suggest that this difference is entirely due to the monetary integration. Despite the detection of an increase (even if slight) in synchronization around the years of the introduction of the Euro, we are not able to uncover a significant and permanent improvement in comovements, as this increase is immediately reversed in the more recent years.

Moreover, we do not find evidence of the emergence of an Euro-Area business cycle, since Area-wide fluctuations contribute little to the business cycles of the participant economies (a finding which contrast with the findings from recent literature). In sum, we find that integration in Europe, and in particular, monetary integration, hasn't made yet, a significant contribution to improve cyclical affiliations within Europe. Moreover, our findings are in accordance with those of the IMF (2007), as business cycles are now less synchronized than during the 1970s and the 1980s.

Our analysis also recognizes the need to bridge the analysis of business cycle comovements and the similarity of business cycles, since differences in the similarity of business cycles tend to directly affect its synchronization. Our assumption of the existence of nonlinearities in business cycles introduces an additional ingredient in our model, which is particularly important in detecting differences in the shape of business cycles.

In this thesis, we employ a simple modification of a standard multivariate trend-cycle decomposition of real output embodying regime shifts and use it to provide a rich set of information regarding the cyclical affiliations between countries. In spite of the success in estimating the model, some changes to our framework may be performed in order to account for recent developments in the literature on comovements between economic time series. A natural extension to our framework would be to model synchronization through time-varying stochastic correlation. Deterministic correlation was recently proposed by Luginbuhl and Koopman (2003) and Koopman and Azevedo (2008), but in their framework time variation is monotonic. A natural extension would then be the Dynamic Conditional Correlation proposed by Engel (2002). This would allow the utilization of a natural measure of synchronization, used widely in the literature.

Additionally, our algorithm suffers from a dimensionality problem, i.e., as the number of endogenous variables increases, the computational costs increase substantially, as more Kalman filters are needed to construct univariate probabilistic inference. Hence, we believe that a Bayesian approach could be pursued to estimate this class of models, exploring the benefits of Markov Chain Monte Carlo methods like the multi-move Gibbs sampler widely used in the markov-switching literature (see Kim and Nelson (1999a)).

The new algorithm for estimation of multivariate state-space models with markov-switching and the model presented and used in this dissertation has a wide range of applicability and may be used to study comovements in economic time series other than business cycles.

References

[1] Afonso, A. and Furceri, D. (2007) "Business Cycle Synchronization and Insurance Mechanisms in the EU.", ECB Working Paper, 844.

[2] Altissimo, F. and Violante, G. L. (2001) "The Non-Linear Dynamics of Output and Unemployment in the U.S..", *Journal of Applied Econometrics*, Vol. 16, 4, pp. 461-486.

[3] Artis, M. and Marcellino, M. and Proietti, T. (2004) "Dating the Euro Area Business Cycle.", pp. 7-33, *in* Reichlin, L. "The Euro Area Business Cycle: Stylized Facts and Measurement Issues.", CEPR, London.

[4] Azevedo, J. V. (2002) "Business Cycles: Cyclical Comovement Within the European Union in the Period 1960-1999. A frequency Domain Approach.", Banco de Portugal Working Paper, 5-02.

[5] Backus, D. K. and Kehoe, P. J. and Kydland, F. E. (1992) "International Real Business Cycles.", *Journal of Political Economy*, Vol. 100, 4, pp. 745-775.

[6] Backus, D. K. and Kehoe, P. J. and Kydland, F. E. (1995) "International Business Cycles: Theory and Evidence.", pp. 331-356, *in* Cooley, T. F. "Frontiers of Business Cycle Research.", Princeton University Press.

[7] Bayoumi, T. e Eichengreen, B. (1993) "Shocking Aspects of European Monetary Integration.", pp. 193-221, *in* Torres, F. and Giavazzi, F. "Adjustment and Growth in the European Monetary Union.", Cambrige University Press, Cambrige.

[8] Baxter, M. and King, R. (1999) "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series", *The Review of Economics and Statistics*, Vol. 81, 4, pp. 575-593.

[9] Baxter, M. and Kouparitsas, M. (2005) "Determinants of Business Cycle Comovement: a Robust Analysis.", *Journal of Monetary Economics*, Vol. 52, 1, pp. 113-157.

[10] Belo, F. (2001) "Some Facts about the Cyclical Convergence in the Euro Zone.", Banco de Portugal Working Paper, 7-01.

[11] Bengoechea, P. and Camacho, M. and Perez-Quiros, G. (2006) "A Useful Tool for Forecasting the Euro-Area Business Cycle Phases.", *International Journal of Forecasting*, Vol. 22, 4, pp. 735-749.

[12] Blanchard, O. J. and Gali, J. (2008) "The Macroeconomic Effects of Oil Price Shocks: Why are the 2000s so Different from the 1970s?", Universitat Pompeu Fabra Economics Working Paper Series, 1045.

[13] Bodman, P. and Crosby, M. (2005) "Are Business Cycles Independent in the G-7?", *International Economic Journal*, Vol. 19, 4, pp. 483-499.

[14] Boivin, J. and Giannoni, M. P. and Mojon, B. (2008) "How has the Euro Changed the Monetary Transmission?", NBER Working Paper, 14190.

[15] Böwer, U. and Guillemineau, C. (2006) "Determinants of Business Cycle Synchronization Across Euro Area Countries.", ECB Working Paper Series, 587.

[16] Camacho, M. and Perez-Quiros, G. (2001) "A New Framework to Analyze Business Cycle Synchronization.", Mimeo.

[17] Camacho, M. and Perez-Quiros, G. and Saiz, L. (2008) "Do European Business Cycles Look Like One.", *Journal of Economic Dynamics and Control*, Vol. 32, 7, 2165-2190.

[18] Canova, F. (2007) "Methods for Applied Macroeconomic Research.", Princeton University Press.

[19] Canova, F. and Ciccarelli, M. and Ortega, E. (2007) "Similarities and Convergence in G7 Cycles.", *Journal of Monetary Economics*, Vol. 54, 3, pp. 850-878.

[20] Canova, F. and Ciccarelli, M. and Ortega, E. (2008) "Did the Maastricht Treaty or the ECB creation alter the European Business Cycle?", Mimeo.

[21] Carlin, B. P. and Polson, N. G. and Stoffer, D. S. (1992) "A Monte Carlo Approach to Nonnormal and Nonlinear State-space Modeling.", *Journal of the American Statistical Association*, Vol. 87, 418, pp. 493-500.

[22] Carvalho, V. and Harvey, A. C. (2005) "Convergence in the Trends and Cycles of Euro-Zone Income.", *Journal of Applied Econometrics*, Vol. 20, 2, pp. 275-289.

[23] Chauvet, M. (1998) "An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching.", *International Economic Review*, Vol. 39, 4, pp. 969-996.

[24] Clark, T. E. van Wincoop, E. (2001) "Borders and Business Cycles.", *Journal of International Economics*, Vol. 55, 1, pp. 59-85.

[25] Corsetti, G. and Pesenti, P. (2002) "Self-Validating Optimum Currency Areas.", Federal Reserve Bank of New York Staff Reports, 152.

[26] Croux, C. and Forni, M. and Reichlin, L. (2001) "A Measure of Comovement for Economic Variables: Theory and Empirics.", *The Review of Economics and Statistics*, Vol. 83, 2, pp. 232-241.

[27] Darvas, Z. and Szapry, G. (2008) "Business Cycle Synchronization in the Enlarged EU.", *Open Economies Review*, Vol. 19, 1, pp. 1-19.

[28] de Haan, J. and Inklaar, R. and Jong-A-Pin, R. (2008) "Will Business Cycles in the Euro Area Converge? A Critical Survey of Empirical Research.", *Journal of Economic Surveys*, Vol. 22, 2, pp. 234-273.

[29] Del Negro, M. and Otrok, C. (2008) "Dynamic Factor Models with Time-Varying Parameters.", Federal Reserve Bank of New York Staff Reports, 326.

[30] Diebold, F. X. and Rudebusch, G. D. (1996) "Measuring Business Cycles: a Modern Perspective.", *The Review of Economics and Statistics*, Vol. 78, 1, pp. 67-77.

[31] Doyle, B. M. and Faust, J. (2002) "An Investigation of Co-movements Among the Growth Rates of the G-7 Countries.", *Federal Reserve Bulletin*, pp. 427-437.

[32] Doyle, B. M. and Faust, J. (2005) "Breaks in the Variability and Comovement of G-7 Economic Growth.", *The Review of Economics and Statistics*, Vol. 87, 4, pp. 721-740.

[33] Dueker, M. and Wesche, K. (2001) "European Business Cycles: New Indices and Analysis of their Synchronicity.", Federal Reserve Bank of St. Louis Working Paper Series, 1999-019B.

[34] Durland, J. M. and McCurdy, T. H. (1994) "Duration-Dependent Transitions in a Markov Model of U.S. GNP Growth.", *Journal of Business & Economic Statistics*, Vol. 12, 3, 279-288.

[35] Engel, R. (2002) "Dynamic Conditional Correlation: a Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models.", *Journal of Business & Economic Statistics*, Vol. 20, 3, pp. 339-350.

[36] Engel, J. and Haugh, D. and Pagan, A. (2005) "Some Methods for Assessing the Need for Non-Linear Models in Business Cycle Analysis.", *International Journal of Forecasting*, Vol. 21, 4, pp. 651-662.

[37] Eickmeier, S. and Breitung, J. (2006) "How Synchronized are New EU Member States with the Euro Area? Evidence from a Structural Factor Model.", *Journal of Comparative Economics*, Vol. 34, 3, pp. 538-563.

[38] Faia, E. (2007) "Financial Differences and Business Cycle Co-Movements in a Currency Area.", *Journal of Money, Credit and Banking*, Vol. 39, 1, pp. 151-185.

[39] Fatás, A. (1997) "EMU: Countries or Regions? Lessons from the EMS Experience.", *European Economic Review*, Vol. 41, 3-5, pp. 743-751.

[40] Ferreira-Lopes, A. and Pina, A. (2008) "Business Cycles, Core and Periphery in Monetary Unions: Comparing Europe and North America.", ISEG Documento de Trabalho, 21/2008.

[41] Filardo, A. J. and Gordon, S. F. (1998) "Business Cycle Durations.", *Journal of Econometrics*, Vol. 85, 1, pp. 99-123.

[42] Fonseca, R. and Patureau, L. and Sopraseuth, T. (2007) "Business Cycle Comovement and Labor Market Institutions: an Empirical Investigation.", RAND Labor and Population Working Paper Series, WR-511.

[43] Frankel, J. A. and Rose, A. K. (1998) "The Endogeneity of the Optimum Currency Area Criteria.", *Economic Journal*, Vol. 108, 449, pp. 1009-1025.

[44] Frankel, J. A. and Rose, A. K. (1997) "Is EMU More Justifiable ex post than ex ante?", *European Economic Review*, Vol. 41, 3-5, pp. 753-760.

[45] Friedman, M. (1993) "The "Plucking Model" of Business Fluctuations Revisited.", *Economic Inquiry*, Vol. 31, 2, pp. 171-177.

[46] Frühwirth-Schnatter, S. (2001) "Markov Chain Monte Carlo Estimation of Classical and Dynamic Switching and Mixture Models", *Journal of the American Statistical Association*, Vol. 96, 453, pp. 194-209.

[47] Furcery, D. and Karras, G. (2007) "Business-Cycle Synchronization in the EMU.", *Applied Economics*, Vol. 40, 12, pp. 1491-1501.

[48] Girardin, E. (2002) "Does Japan Share a Common Business Cycle with other East Asian Countries?", Mimeo.

[49] Hamilton, J. D. (1989) "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle.", *Econometrica*, Vol. 57, 2, pp. 357-384.

[50] Hamilton, J. D. (1994) "Time Series Analysis.", Princeton University Press.

[51] Hamilton, J. D. and Waggoner, D. F. and Zha, T. (2007) "Normalization in Econometrics.", *Econometric Reviews*, Vol 26, 2-4, pp. 221-252.

[52] Hansen, B. (1992) "The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP.", *Journal of Applied Econometrics*, Vol. 7, S, S61-S82.

[53] Hansen, B. (1996a) "Erratum: The Likelihood Ratio Test under Nonstandard Conditions: Testing the Markov Switching Model of GNP.", *Journal of Applied Econometrics*, Vol. 11, 2, 195-198.

[54] Hansen, B. (1996b) "Inference when a Nuisance Parameter is not Identified Under the Null Hypothesis.", *Econometrica*, Vol. 64, 2, pp. 413-430.

[55] Hansen, G. D. and Prescott, E. C. (2005) "Capacity Constraints, Asymmetries, and the Business Cycle", *Review of Economic Dynamics*, Vol. 8, 4, pp. 850-865.

[56] Harding, D. and Pagan, A. (2001) "Extracting, Analyzing and Using Cyclical Information.", Mimeo.

[57] Harding, D. and Pagan, A. (2002) "Dissecting the Cycle: a Methodological Investigation.", *Journal of Monetary Economics*, Vol. 49, 2, pp. 365-381.

[58] Harrison, P. J. and Stevens, C. F. (1976) "Bayesian Forecasting.", *Journal of the Royal Statistical Society*, B Vol. 38, 3, pp. 205-247.

[59] Hayashi, F. and Prescott, E. (2002) "The 1990's in Japan: a Lost Decade.", *Review of Economic Dynamics*, Vol. 5, 1, pp. 206-235.

[60] Imbs, J. (2004) "Trade, Finance, Specialization, and Synchronization.", *The Review of Economics and Statistics*, Vol. 86, 3, pp. 723-734.

[61] IMF (2007) "World Economic Outlook - Spillovers and Cycles in the Global Economy.", International Monetary Fund.

[62] Inklaar, R. and Jong-A-Pin, R. and de Haan, J. (2008) "Trade and Business Cycle Synchronization in OECD Countries - A Re-examination.", *European Economic Review*, Vol. 52, 4, pp. 646-666.

[63] Jansen, D. W. and Oh, W. (1999) "Modeling Nonlinearity of Business Cycles: Choosing between the CDR and STAR Models.", *The Review of Economics and Statistics*, Vol. 81, 2, pp. 344-349.

[64] Karras, G. and Stokes, H. H. (2001) "Time-Varying Criteria for Monetary Integration: Evidence from the EMU.", *International Review of Economics and Finance*, Vol. 10, 2, pp. 171-185.

[65] Kehoe, P. J. and Perri, F. (2002) "International Business Cycles with Endogenous Incomplete Markets.", *Econometrica*, Vol. 70, 3, 907-928.

[66] Keynes, J. M. (1936) "The General Theory of Employment, Interest and Money.", Macmillan, London.

[67] Kim, C. J. (1994) "Dynamic Linear Models with Markov-Switching.", *Journal of Econometrics*, Vol. 60, 1-2, pp. 1-22.

[68] Kim, C. J. and Murray, C. J. (2002) "Permanent and Transitory Components of Recessions.", *Empirical Economics*, Vol. 27, 2, pp. 163-183.

[69] Kim, C. J. and Nelson, C. R. (1999a) "State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications.", The MIT Press.

[70] Kim, C. J. and Nelson, C. R. (1999b) "Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components.", *Journal of Money, Credit, and Banking*, Vol. 31, 3, pp. 317-334.

[71] Kim, C. J. and Piger, J. (2002) "Common Stochastic Trends, Common Cycles, and Asymmetry in Economic Fluctuations.", *Journal of Monetary Economics*, Vol. 49, 6, pp. 1189-1211.

[72] Koopman, S. J. and Azevedo, J. V. (2008) "Measuring Synchronization and Convergence of Business Cycles for the Euro Area, UK and US.", *Oxford Bulletin of Economics and Statistics*, Vol. 70, 1, pp. 23-51.

[73] Kose, M. A. and Prasad, E. S. and Terrones, M. (2003) "How does Globalization Affect the Synchronization of Business Cycles.", *The American Economic Review*, Vol. 93, 2, pp. 57-62.

[74] Krugman, P. (1993) "Lessons of Massachusetts for EMU.", pp. 241-259, *in* Torres, F. and Giavazzi, F. "Adjustment and Growth in the European Monetary Union.", Cambrige University Press, Cambrige.

[75] Lam, Pok-Sang, (1990) "The Hamilton Model with a General Autoregressive Component: Estimation and Comparison with Other Models of Economic Time Series.", *Journal of Monetary Economics*, Vol. 26, 3, pp. 409-432.

[76] Lam, Pok-Sang, (2004) "A Markov-Switching Model of GNP Growth with Duration Dependence.", *International Economic Review*, Vol. 45, 1, pp. 175-204.

[77] Luginbuhl, R. and Koopman, S. J. (2003) "Convergence in European GDP Series: a Multivariate Common Converging Trend-Cycle Decomposition.", Tinbergen Institute Discussion Paper, 2003-031/4.

[78] Lumsdaine, R. L. and Prasad, E. S. (2003) "Identifying the Common Component of International Economic Fluctuations: a New Approach.", *The Economic Journal*, Vol. 113, 484, pp. 101-127.

[79] Marcellino, M. and Stock, J. and Watson, M. W. (2000) "A Dynamic Factor Analysis of the EMU.", Mimeo.

[80] McConnell, M. and Perez-Quiros, G. (2000) "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?", *The American Economic Review*, Vol. 90, 5, pp. 1464-1476.

[81] McKay, A. and Reis, R. (2008) "The Brevity and Violence of Contractions and Expansions.", *Journal of Monetary Economics*, Vol. 55, 4, pp. 738-751.

[82] McKinnon, R. (1963) "Optimum Currency Areas.", *The American Economic Review*, Vol. 53, 4, pp. 717-725.

[83] Mink, M. and Jacobs, J. P.A.M., de Haan, J. (2007) "Measuring Synchronicity and Co-Movement of Business Cycles with an Application to the Euro Area.", CESifo Working Paper, 2112.

[84] Morley, J. and Piger, J. (2008) "Trend/Cycle Decomposition of Regime-Switching Processes.", *Journal of Econometrics*, Vol. 146, 2, pp. 220-226.

[85] Mundell, R. (1961) "A Theory of Optimum Currency Areas.", *The American Economic Review*, Vol. 51, 4, pp. 657-665.

[86] Murphy, K. P. (1998) "Switching Kalman Filters.", Mimeo.

[87] Neftçi, S. N. (1984) "Are Economic Time Series Asymmetric Over the Business Cycle.", *Journal of Political Economy*, Vol. 92, 2, pp. 307-328.

[88] Obstfeld, M. and Rogoff, K. (1996) "Foundations of International Macroeconomics.", The MIT Press, Cambridge.

[89] Phillips, K. L. (1991) "A Two-Country Model of Stochastic Output with Changes in Regime.", *Journal of International Economics*, Vol. 31, 1-2, pp. 121-142.

[90] Psaradakis, Z. and Sola, M. (1998) "Finite-Sample Properties of the Maximum Likelihood Estimator in Autoregressive Models with Markov Switching.", *Journal of Econometrics*, Vol. 86, 2, pp. 369-386.

[91] Potter, S. M. (1995) "A Nonlinear Approach to US GNP.", *Journal of Applied Econometrics*, Vol. 10, 2, pp. 109-125.

[92] Potter, S. M. (1999) "Nonlinear Time Series Modeling: an Introduction.", *Journal of Economic Surveys*, Vol. 13, 5, pp. 505-528.

[93] Ricci, L. (1997) "A Model of an Optimum Currency Area.", IMF Working Paper, 76.

[94] Rünstler, G. (2004) "Modelling Phase Shifts Among Stochastic Cycles.", *Econometrics Journal*, Vol. 7, 1, pp. 232-248.

[95] Scheller, H. K. (2004) "The European Central Bank - History, Role and Functions.", European Central Bank.

[96] Shumway, R. H. and Stoffer, D. S. (1991) "Dynamic Linear Models with Switching.", *Journal of the American Statistical Association*, Vol. 86, 415, pp. 763-913.

[97] Sichel, D. (2009) "Business Cycle Asymmetry: a Deeper Look.", *Economic Inquiry*, Vol. 31, 2, pp. 224-236.

[98] Silva, R. (2009) "Business Cycle Association and Synchronization in Europe: a Descriptive Review.", *Issues in Political Economy*, Vol. 18, pp. 6-53.

[99] Smith, P. A. and Summers, P. M. (2005) "How Well do Markov Switching Models Describe Actual Business Cycles? The Case of Synchronization.", *Journal of Applied Econometrics*, Vol. 20, 2, pp. 253-274.

[100] Schiavo, S. (2008) "Financial Integration, GDP Correlation and the Endogeneity of Optimum Currency Areas.", *Economica*, Vol. 75, 297, pp. 168-189.

[101] Stock, J. H. and Watson, M. W. (1988) "A Probability Model of the Coincident Economic Indicators.", NBER Working Paper, 2772.

[102] Stock, J. H. and Watson, M. W. (2005) "Understanding Changes in International Business Cycle Dynamics.", *Journal of the European Economic Association*, Vol. 3, 5, pp. 968-1006.

[103] Teräsvirta, T. and Anderson, H. M. (1992)"Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models.", *Journal of Applied Econometrics*, Vol. 7, S, pp. S119-S136.

[104] Tsay, R. (1989) "Testing and Modeling Threshold Autoregressive Processes.", *Journal of the American Statistical Association*, Vol. 84, 405, pp. 231-240.

[105] van Dijk, D. and Teräsvirta, T. and Franses, P. H. (2002) "Smooth Transition Autoregressive Models - a Survey of Recent Developments.", *Econometric Reviews*, Vol. 21, 1, pp. 1-47.

[106] White, H. (1982) "Maximum Likelihood Estimation of Misspecified Models.", *Econometrica*, Vol. 50, 1, pp. 1-25.

A Appendix

This appendix comprises the figures not presented in the main body of the text for bivariate estimations with Euro Area and the US:

- Business Cycles for each country;
- Univariate Smoothed Probabilities of Recession for each country;
- Multivariate Smoothed Probabilities;

Figure 6: Estimated Cycles for Bivariate Model EA-AUS

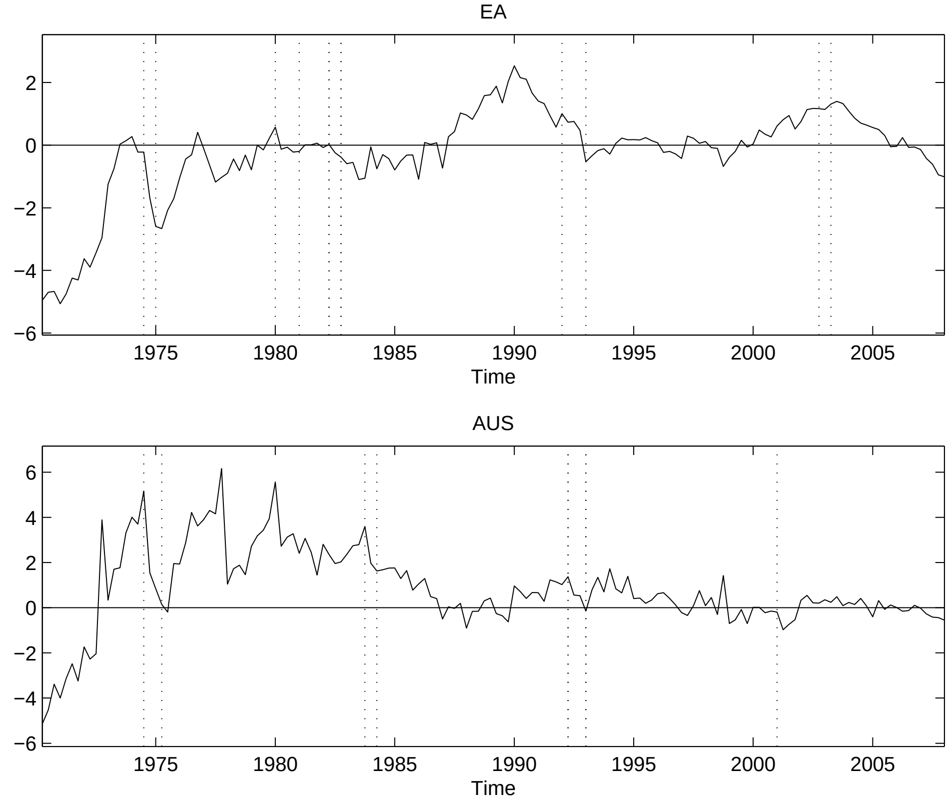


Figure 7: Estimated Cycles for Bivariate Model EA-BGM

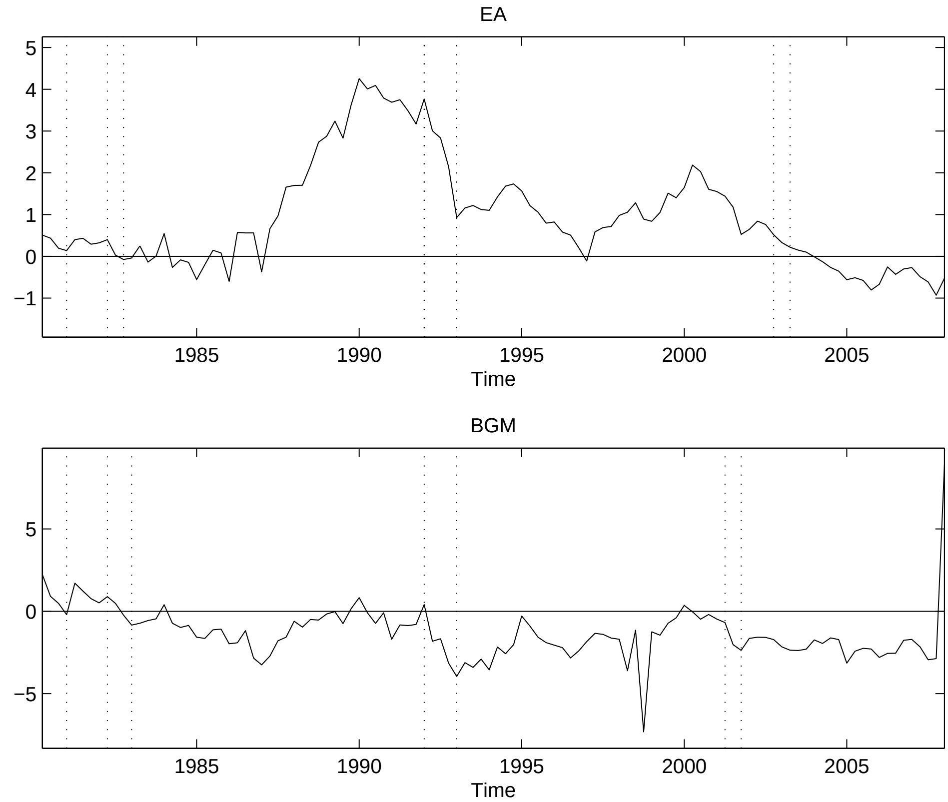


Figure 8: Estimated Cycles for Bivariate Model EA-CND

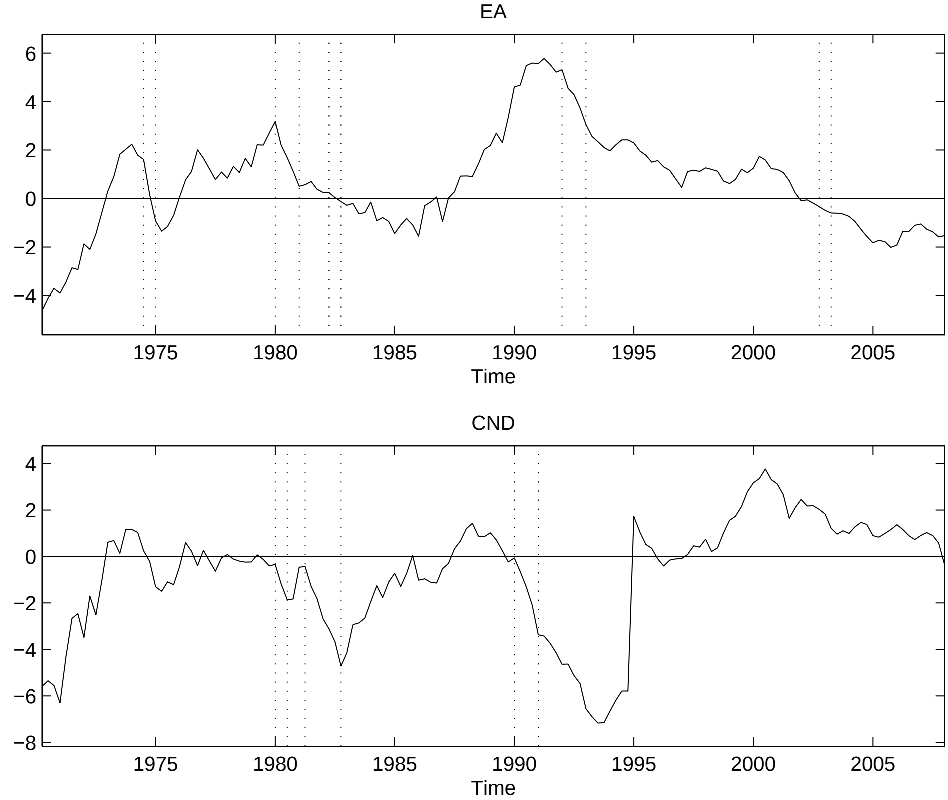


Figure 9: Estimated Cycles for Bivariate Model EA-DEN

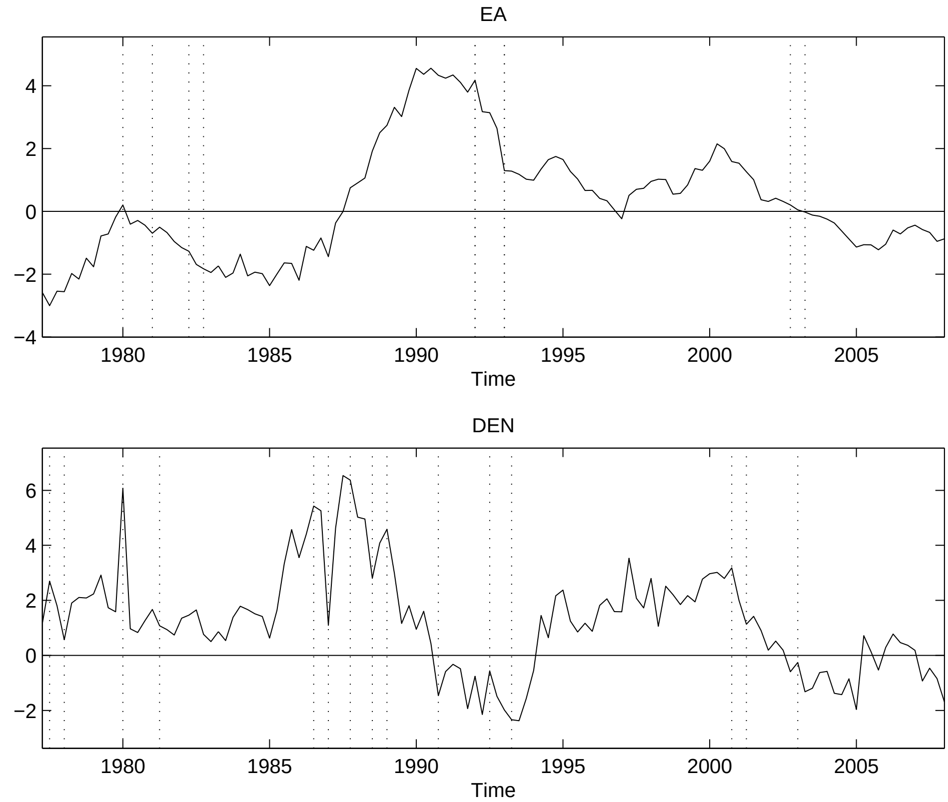


Figure 10: Estimated Cycles for Bivariate Model EA-FIN

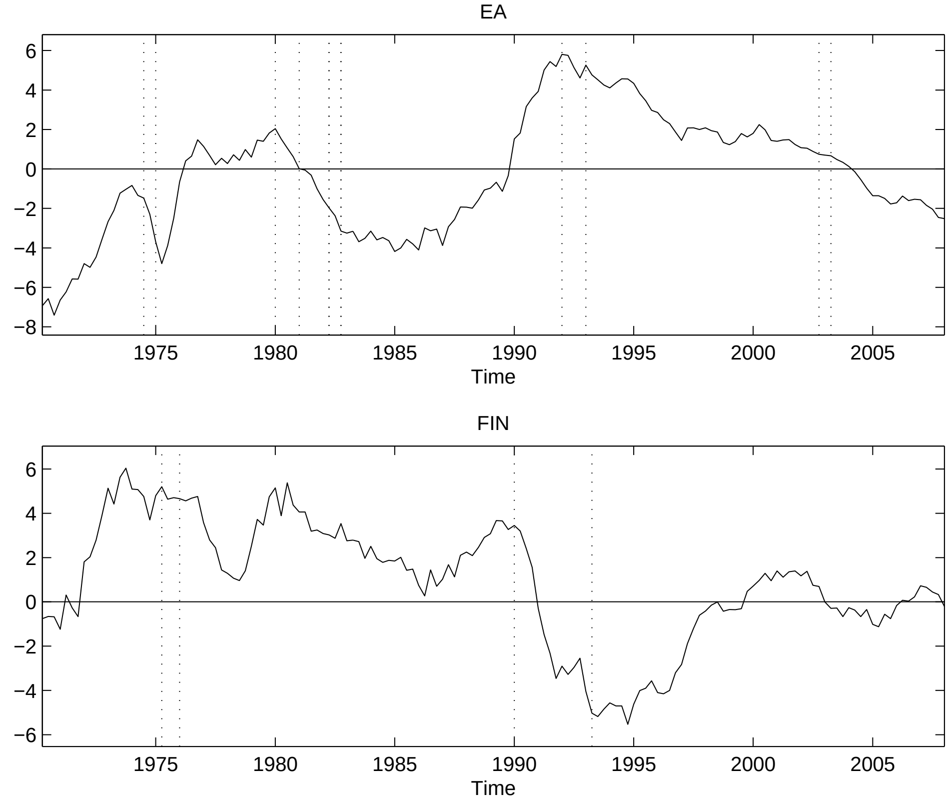


Figure 11: Estimated Cycles for Bivariate Model EA-FR

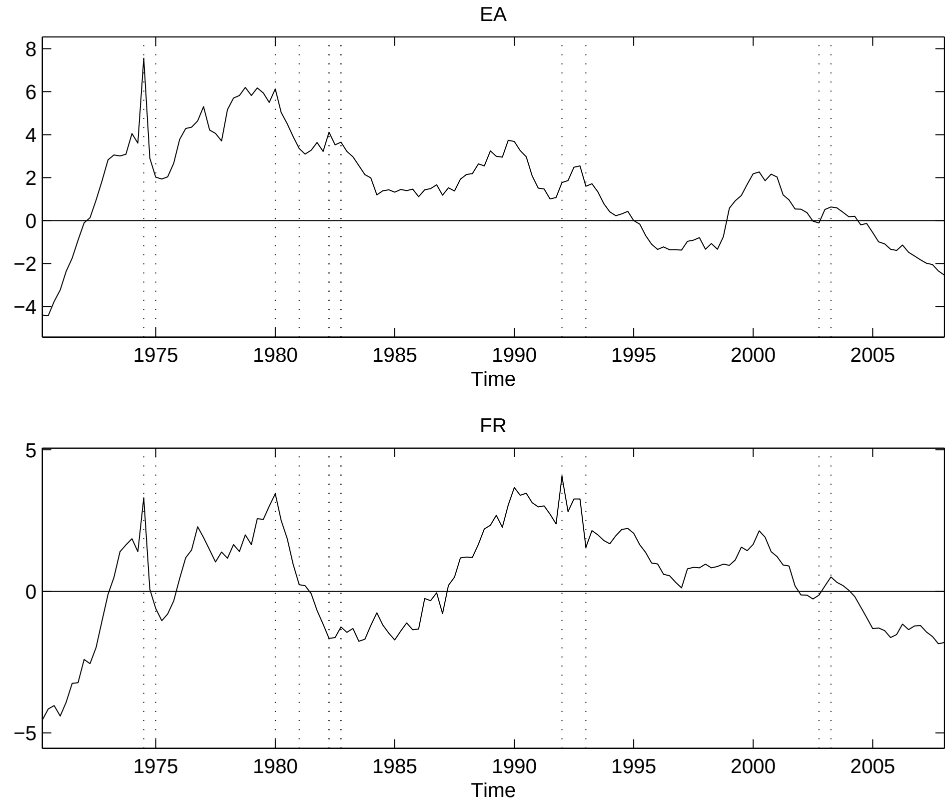


Figure 12: Estimated Cycles for Bivariate Model EA-GER

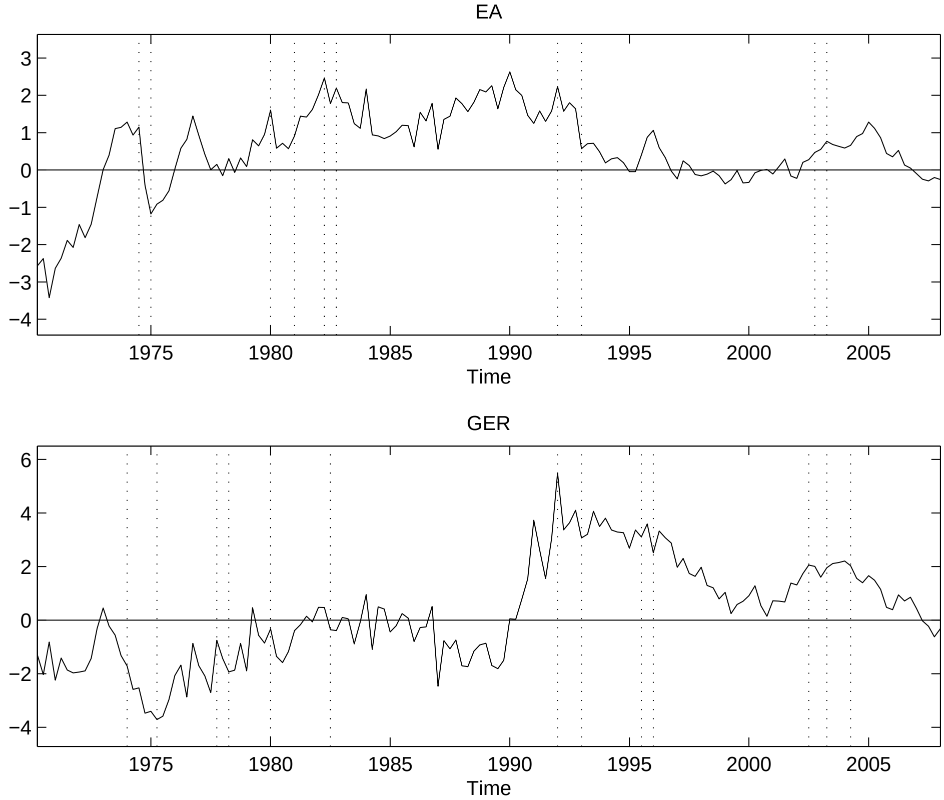


Figure 13: Estimated Cycles for Bivariate Model EA-GREE

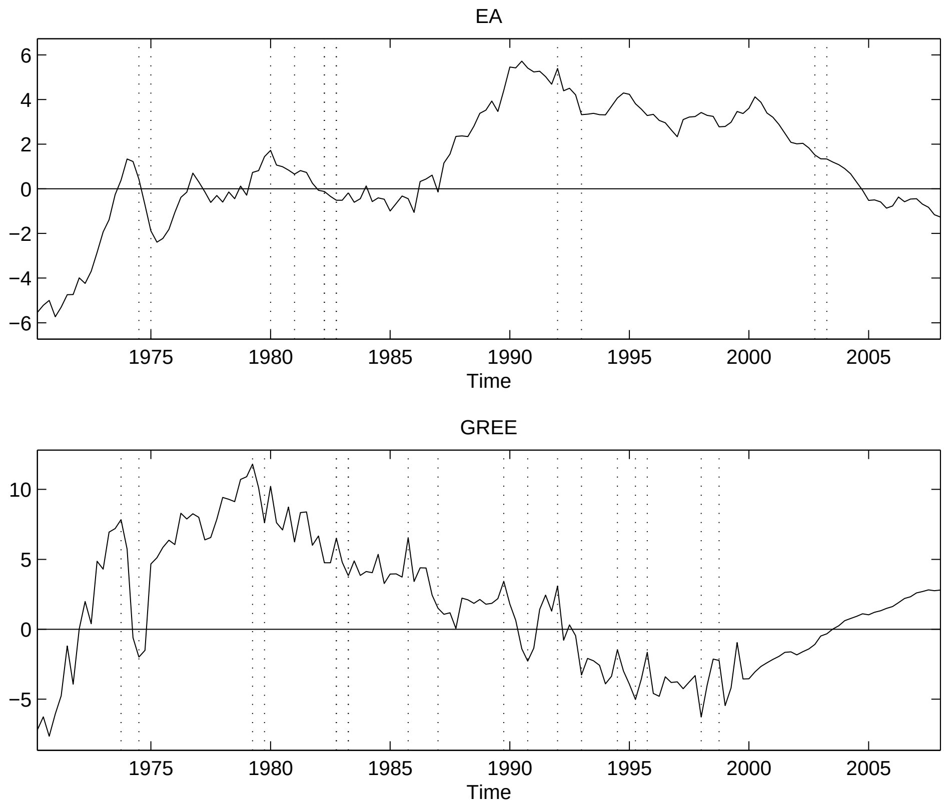
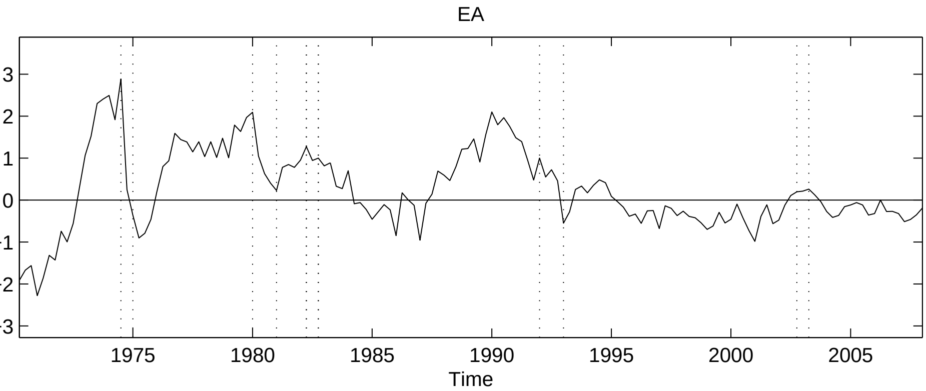


Figure 14: Estimated Cycles for Bivariate Model EA-IT



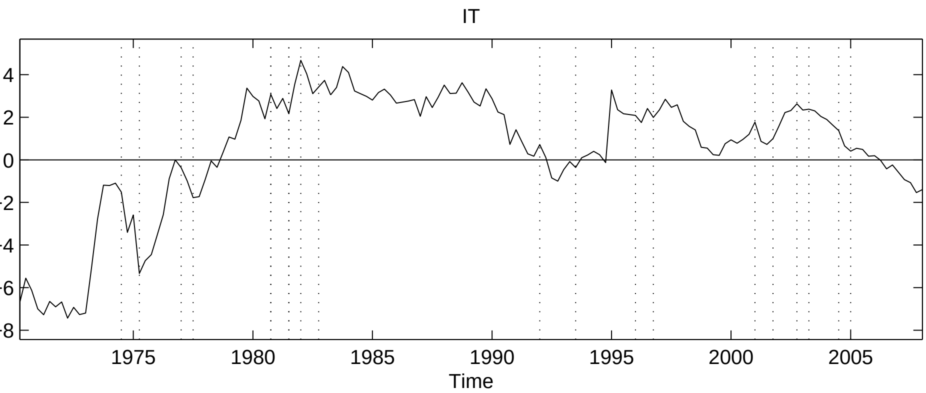
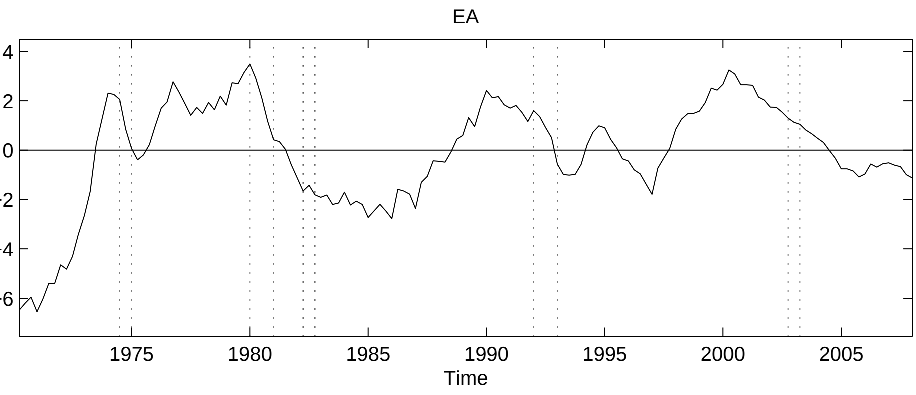


Figure 15: Estimated Cycles for Bivariate Model EA-JP



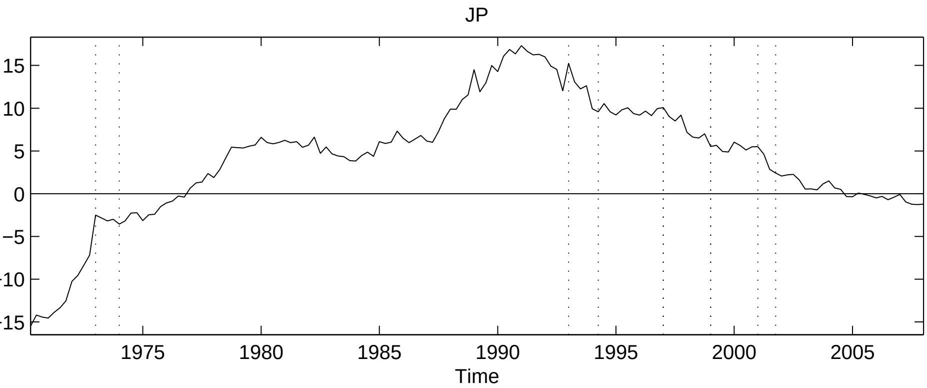


Figure 16: Estimated Cycles for Bivariate Model EA-NRW

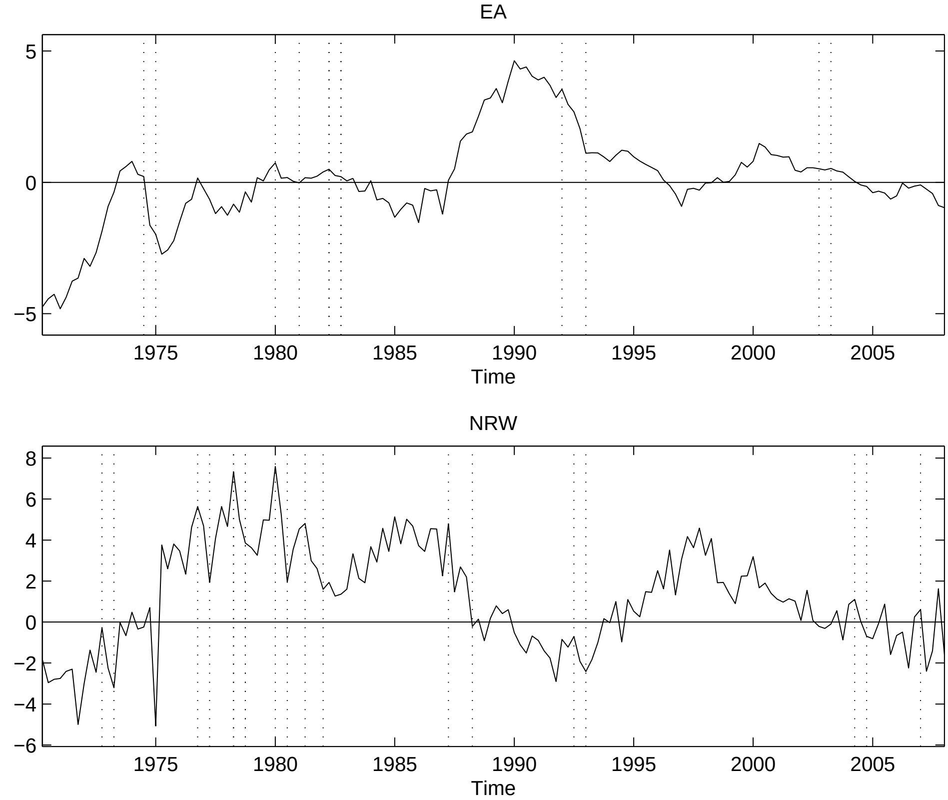


Figure 17: Estimated Cycles for Bivariate Model EA-NTH

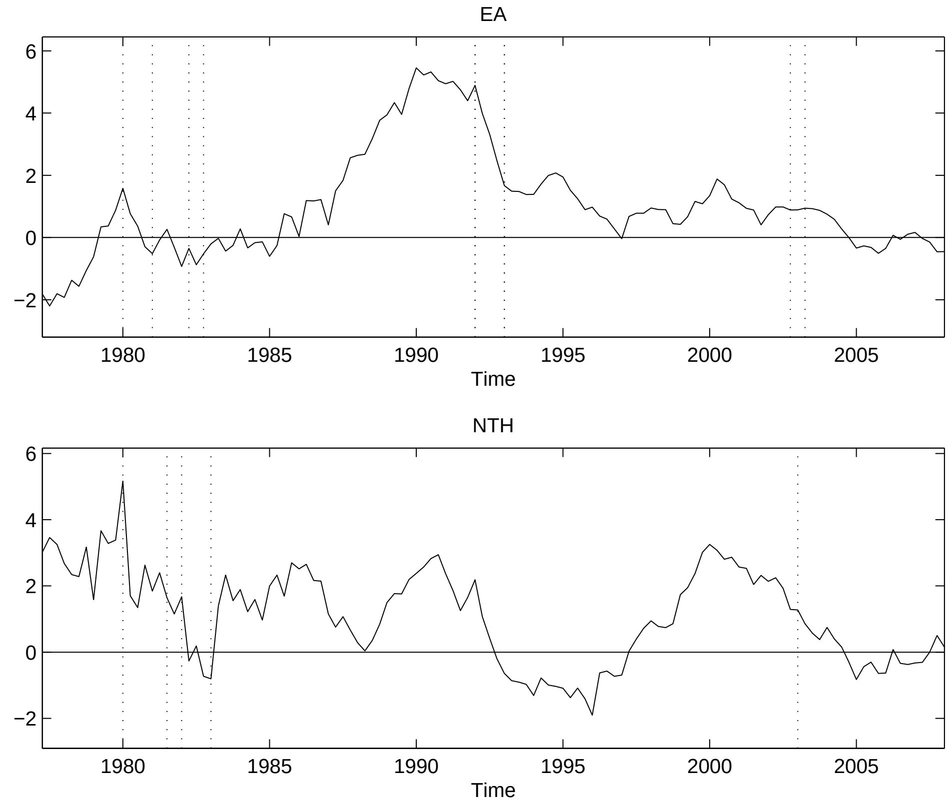


Figure 18: Estimated Cycles for Bivariate Model EA-PT

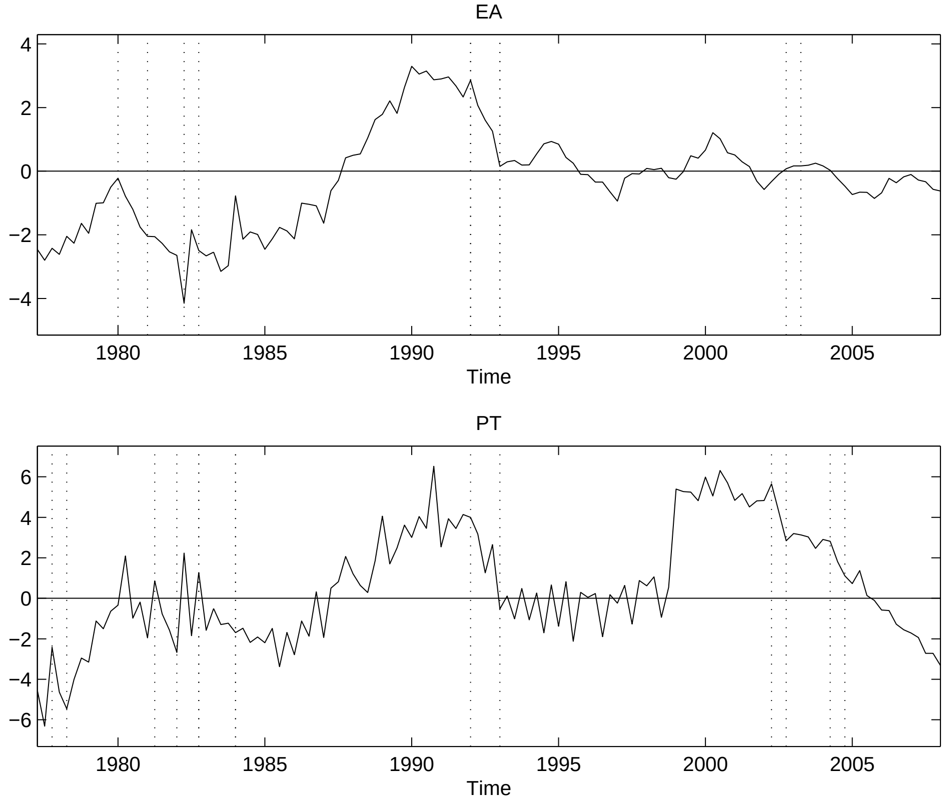


Figure 19: Estimated Cycles for Bivariate Model EA-SP

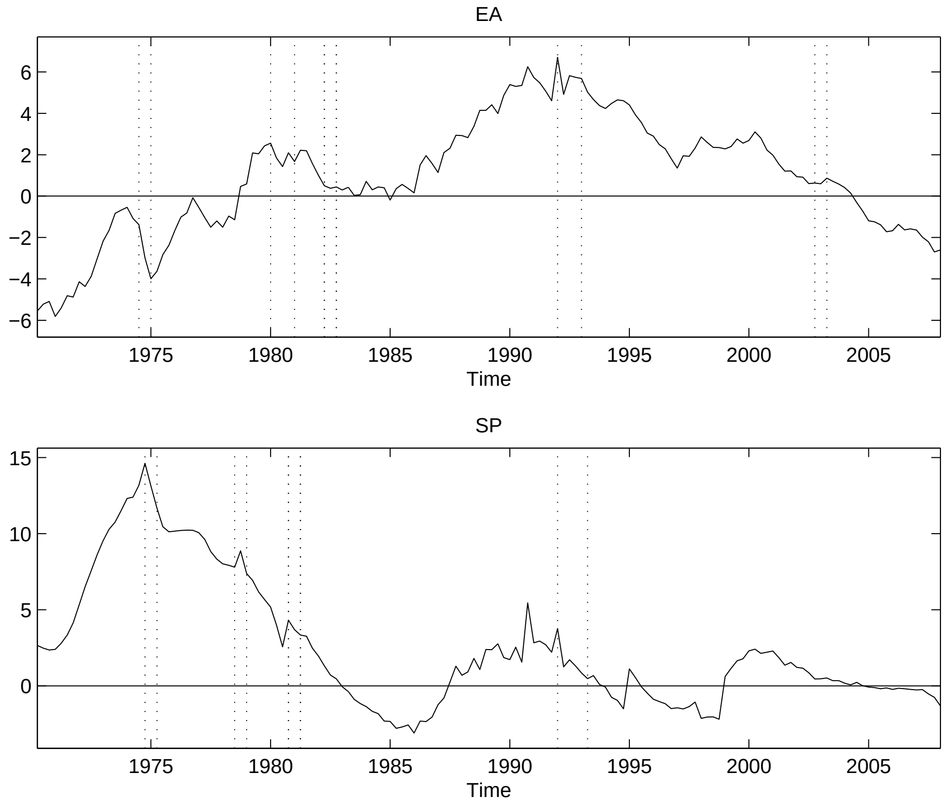


Figure 20: Estimated Cycles for Bivariate Model EA-SWE

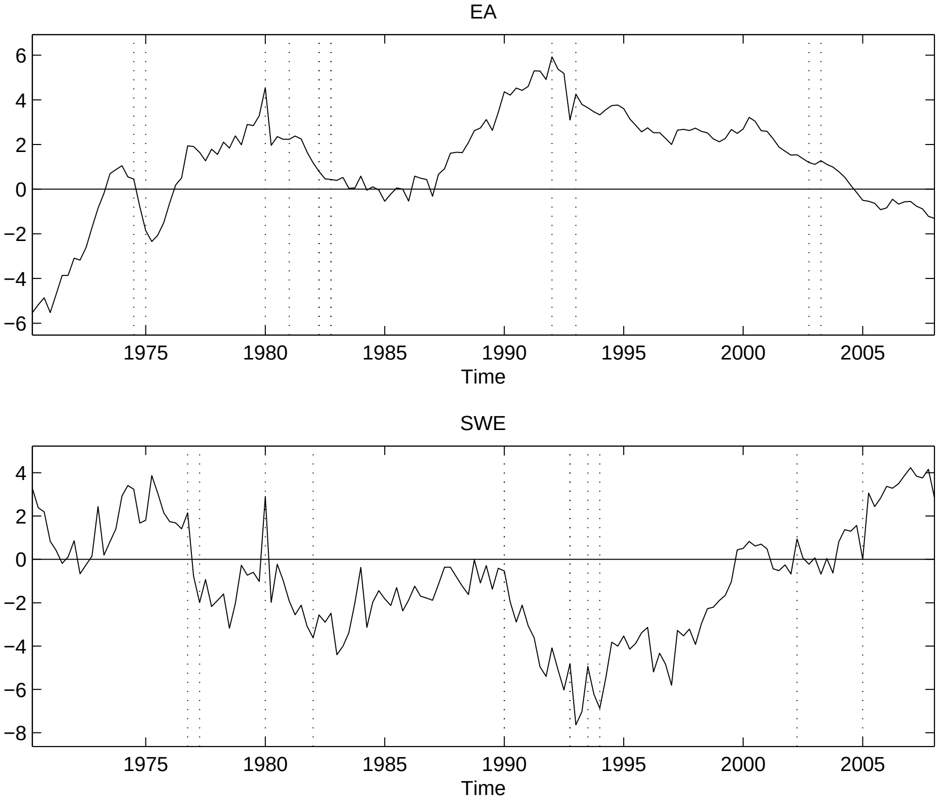


Figure 21: Estimated Cycles for Bivariate Model EA-SWITZ

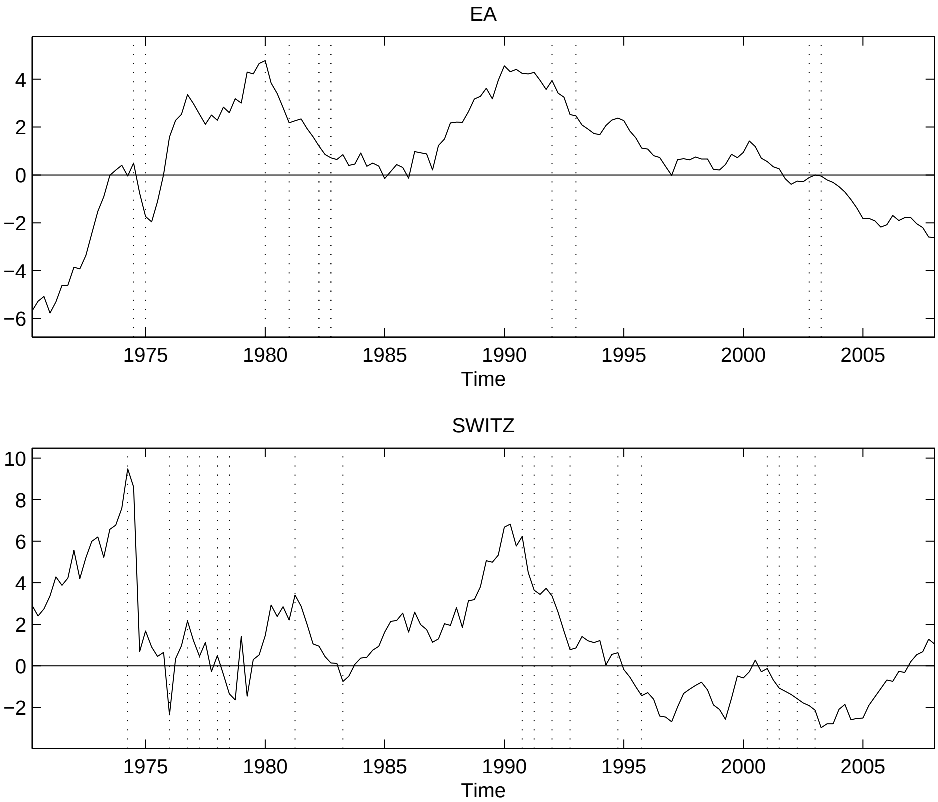


Figure 22: Estimated Cycles for Bivariate Model EA-UK

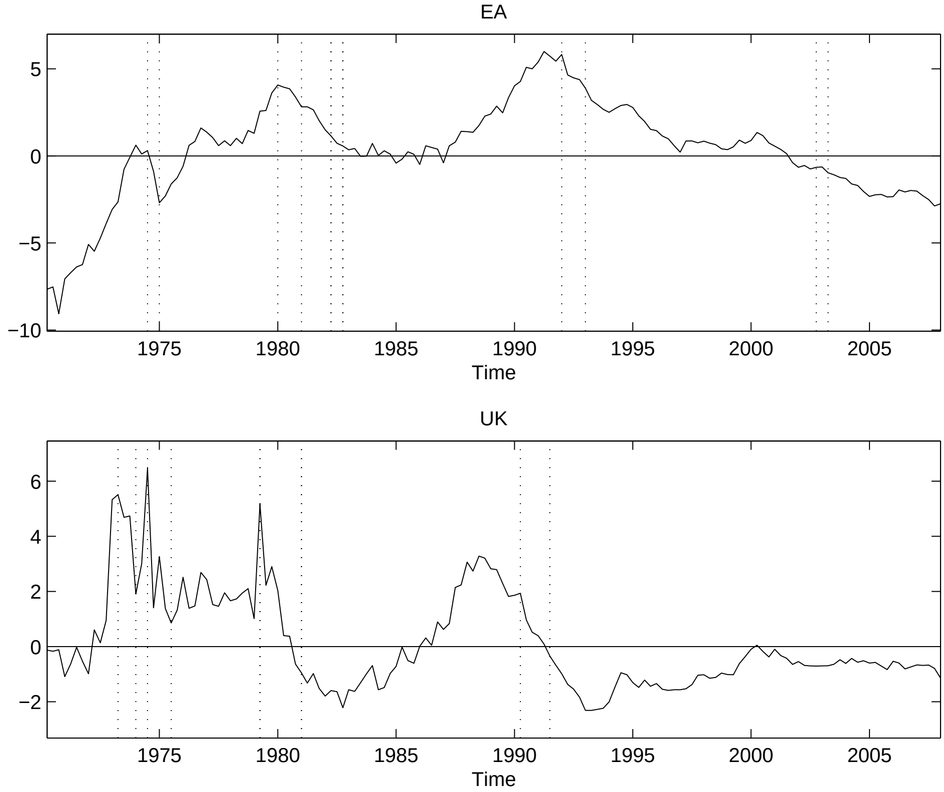


Figure 23: Estimated Cycles for Bivariate Model EA-US

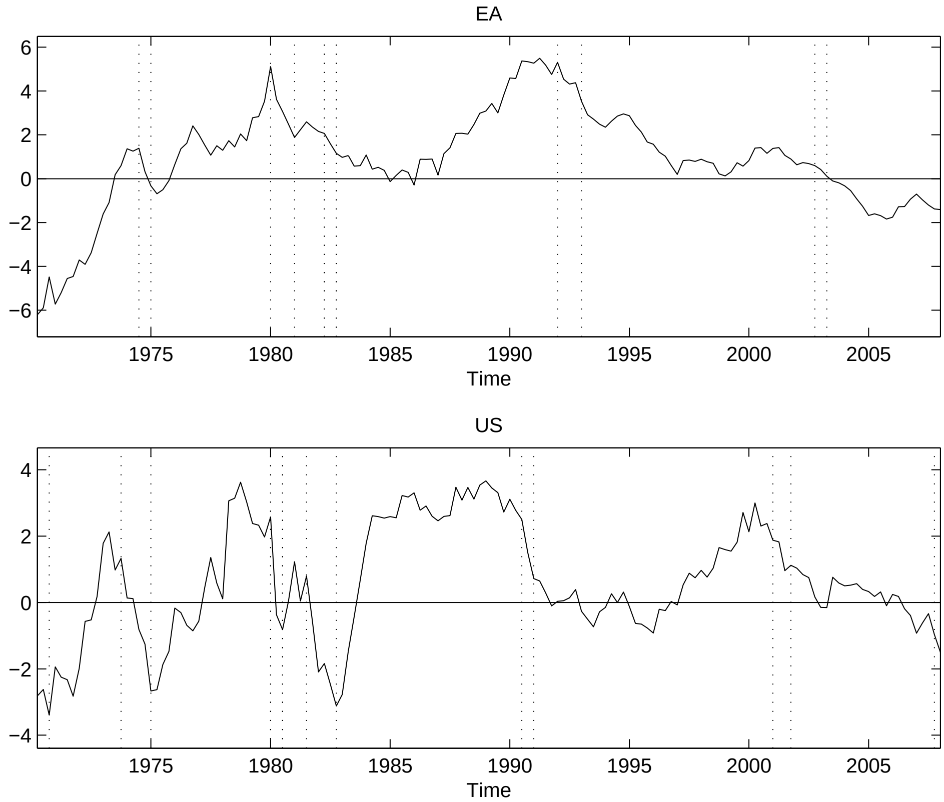


Figure 24: Estimated Cycles for Bivariate Model AUS-US

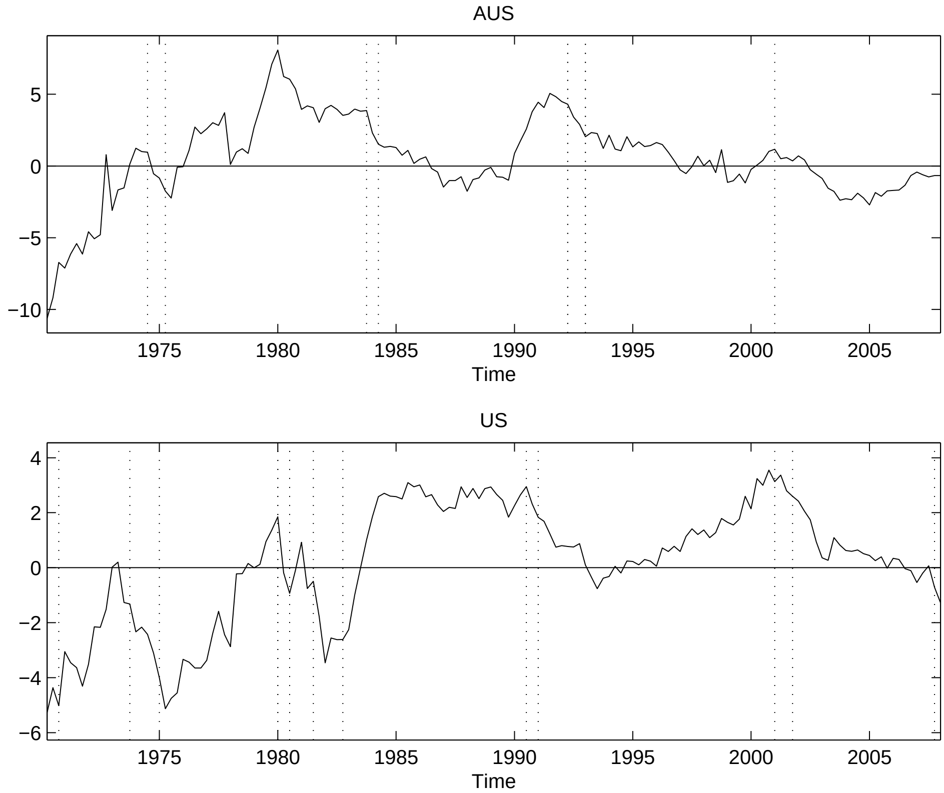


Figure 25: Estimated Cycles for Bivariate Model BGM-US

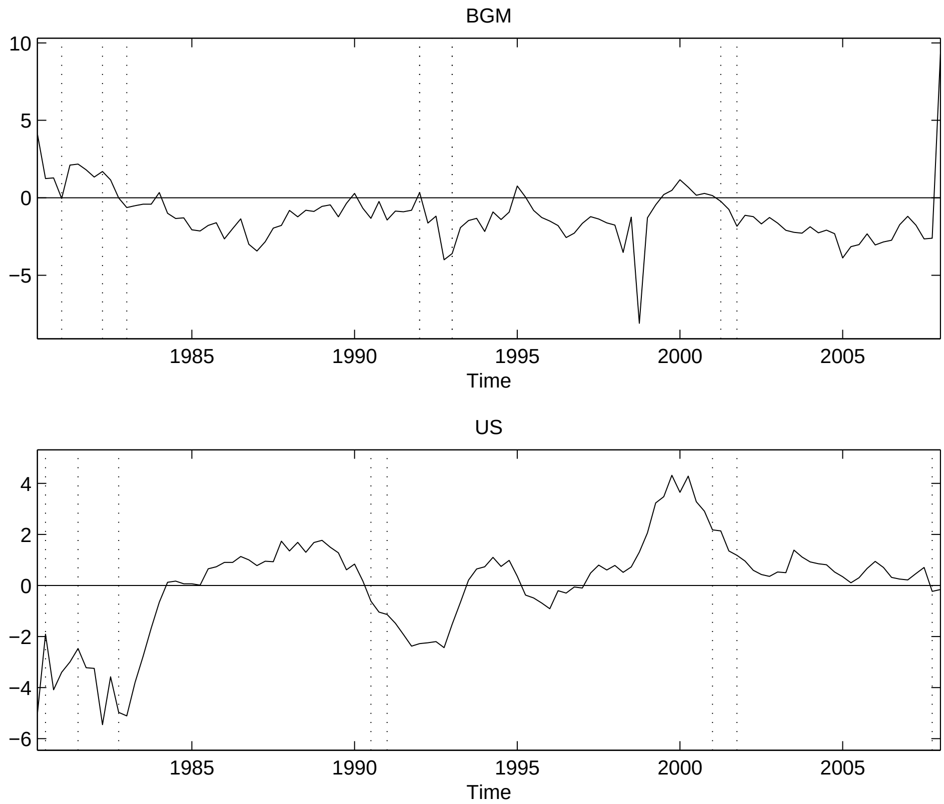


Figure 26: Estimated Cycles for Bivariate Model CND-US

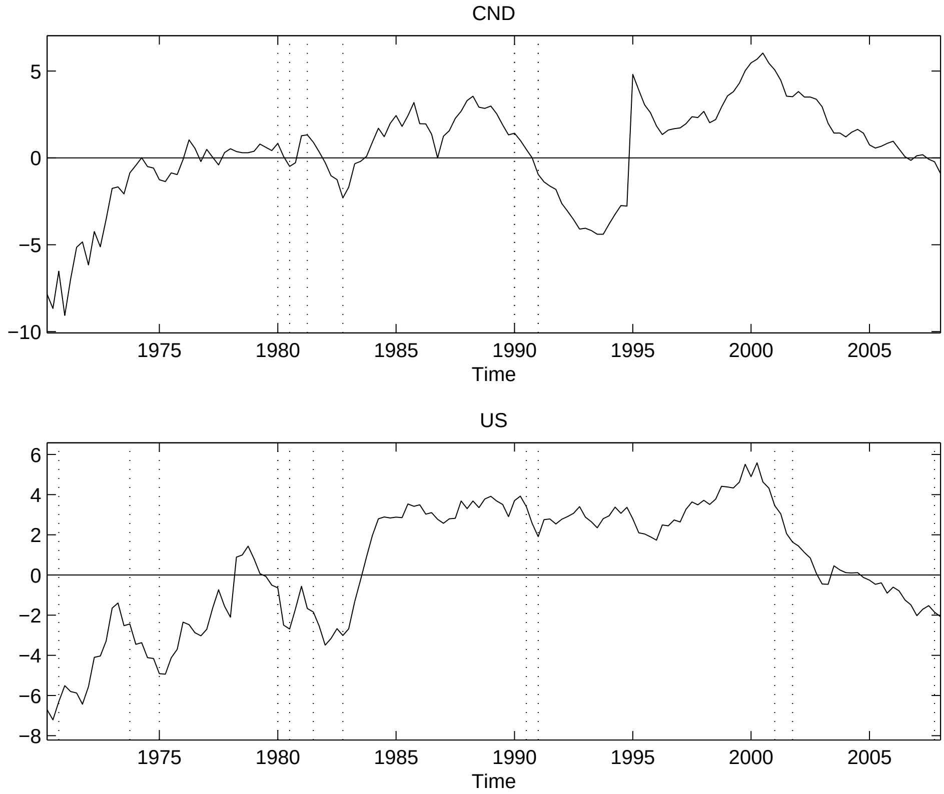


Figure 27: Estimated Cycles for Bivariate Model DEN-US

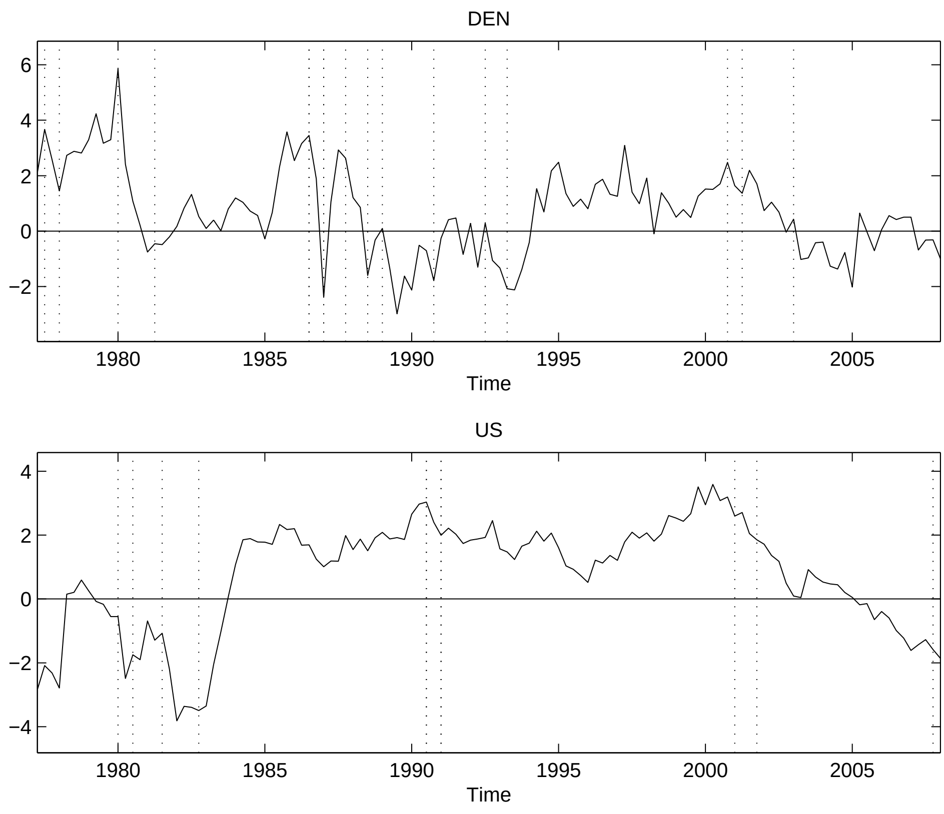


Figure 28: Estimated Cycles for Bivariate Model FIN-US

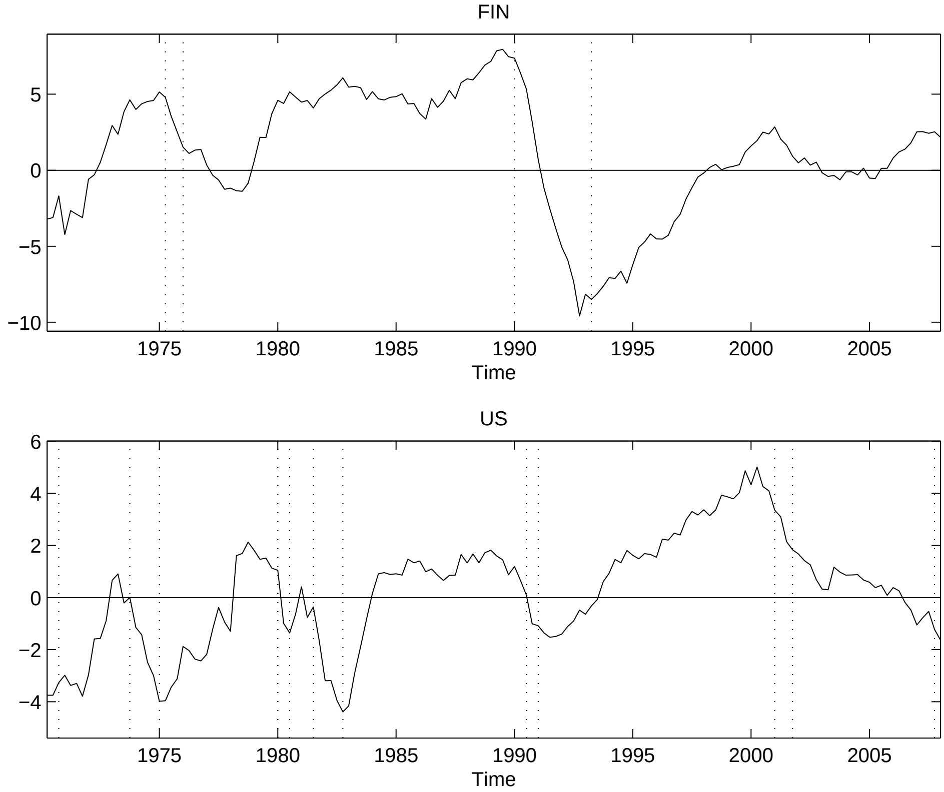


Figure 29: Estimated Cycles for Bivariate Model FR-US

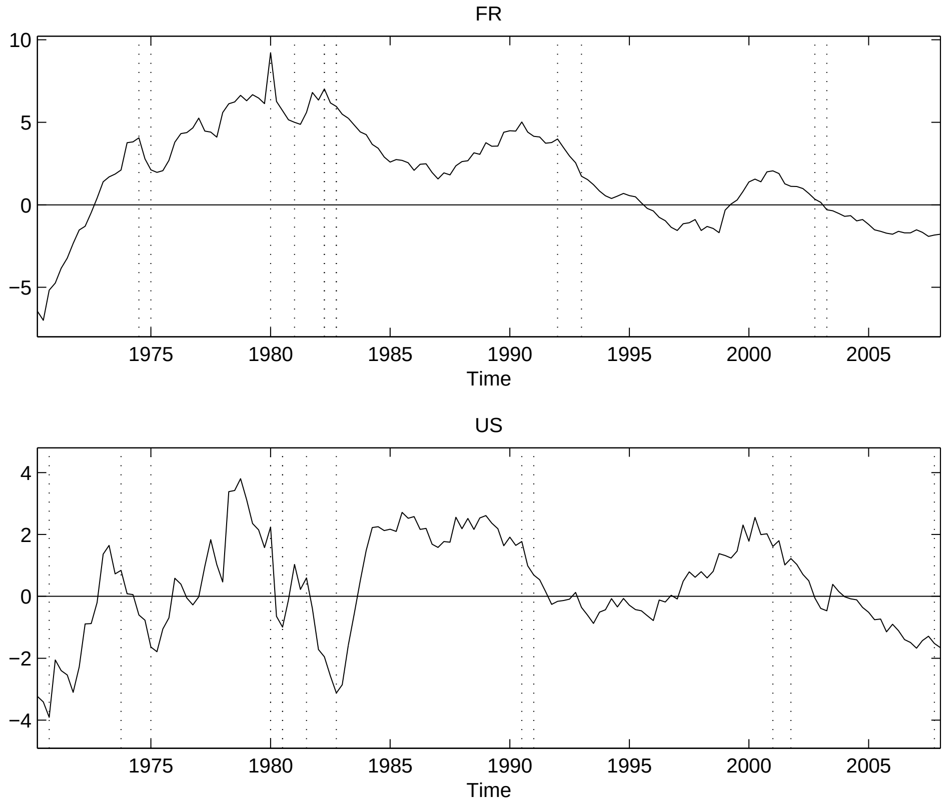


Figure 30: Estimated Cycles for Bivariate Model GER-US

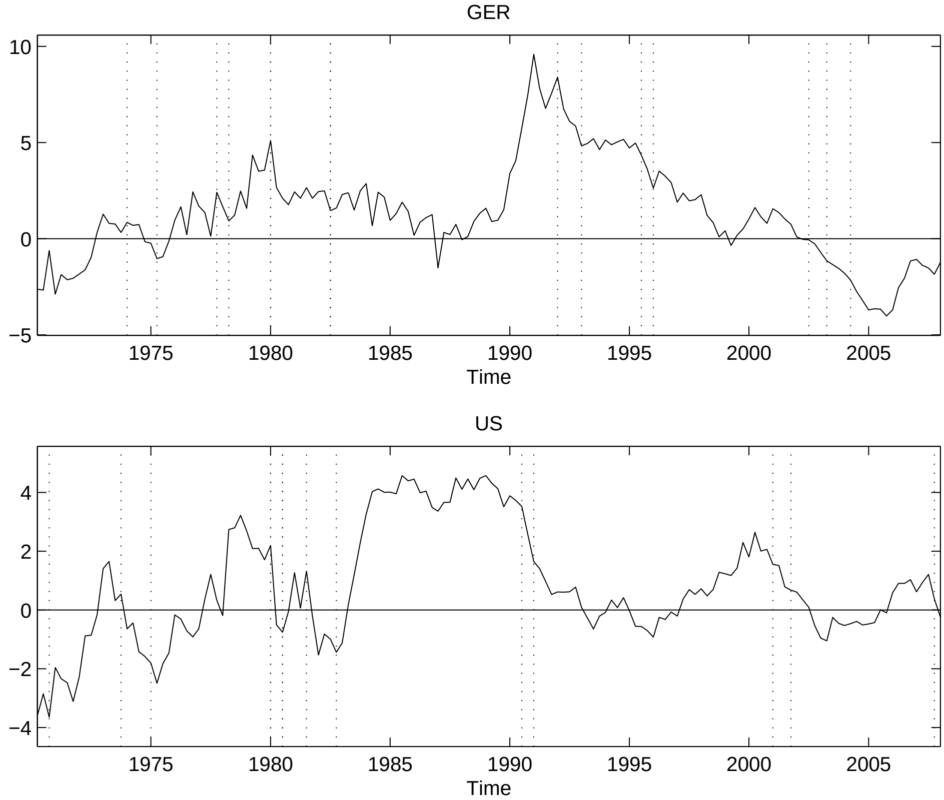


Figure 31: Estimated Cycles for Bivariate Model GREE-US

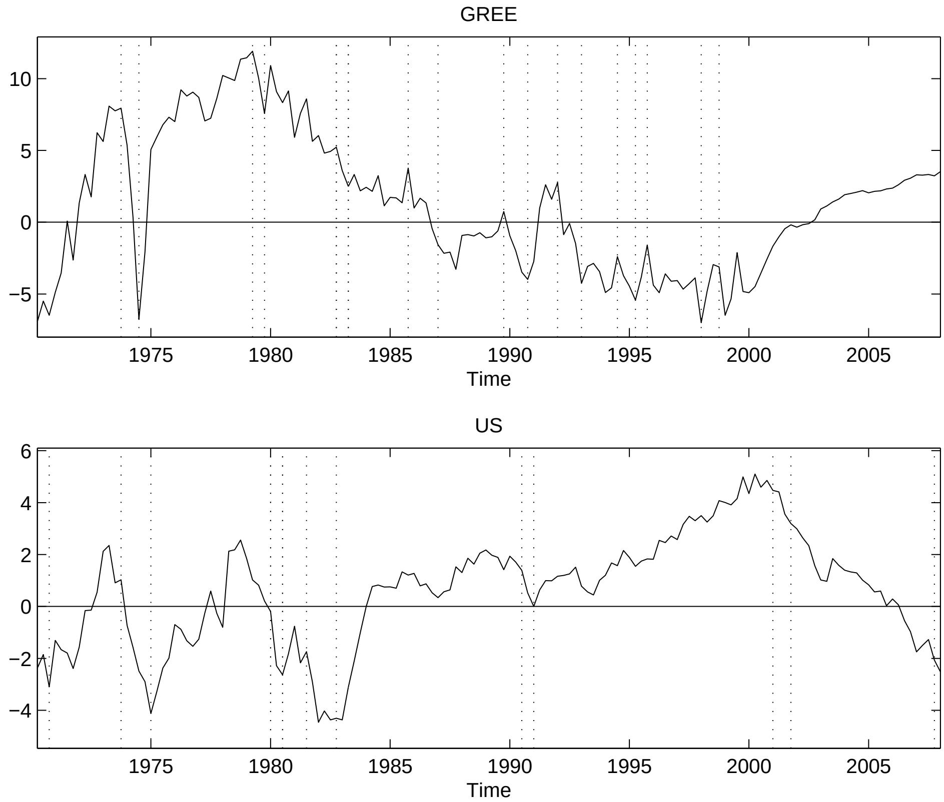


Figure 32: Estimated Cycles for Bivariate Model IT-US

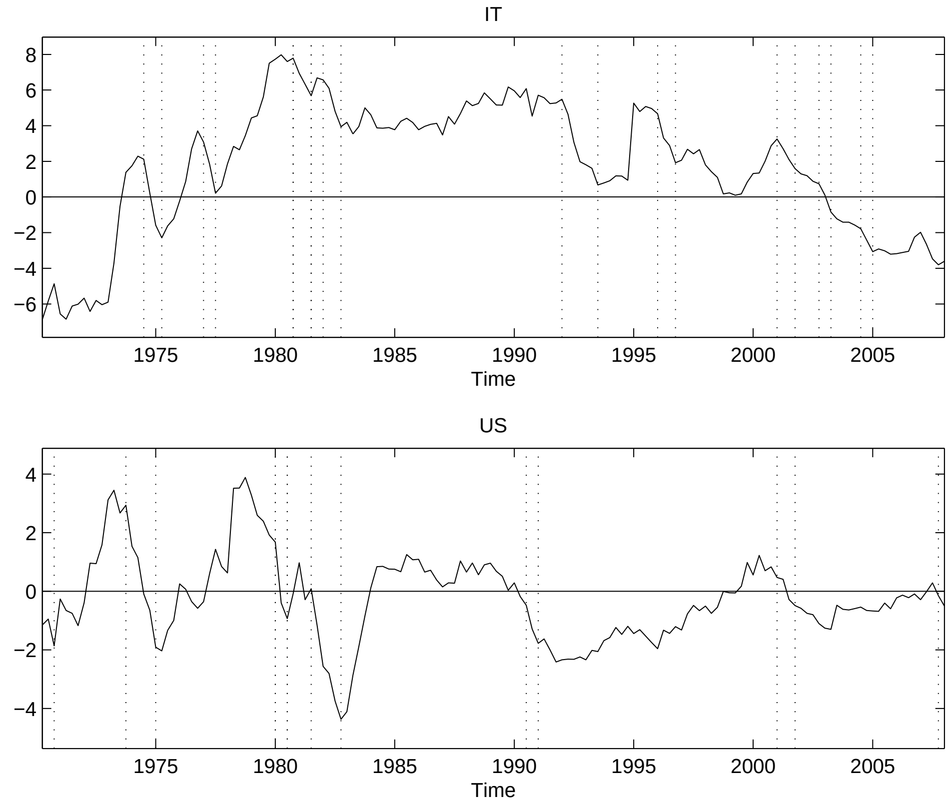


Figure 33: Estimated Cycles for Bivariate Model JP-US

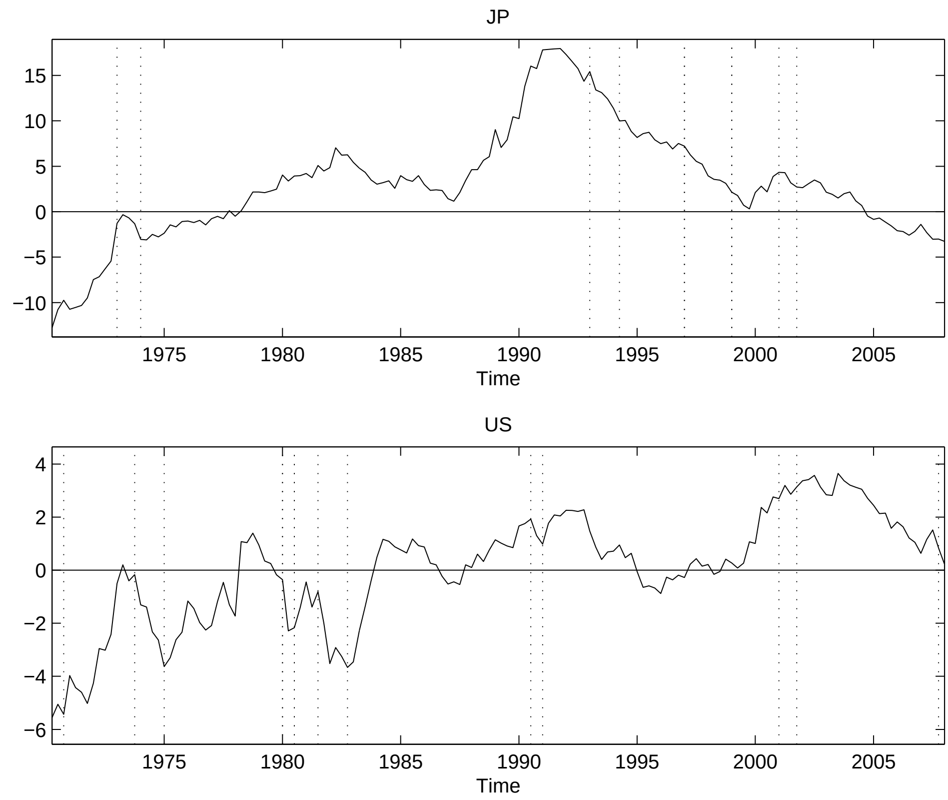


Figure 34: Estimated Cycles for Bivariate Model NRW-US

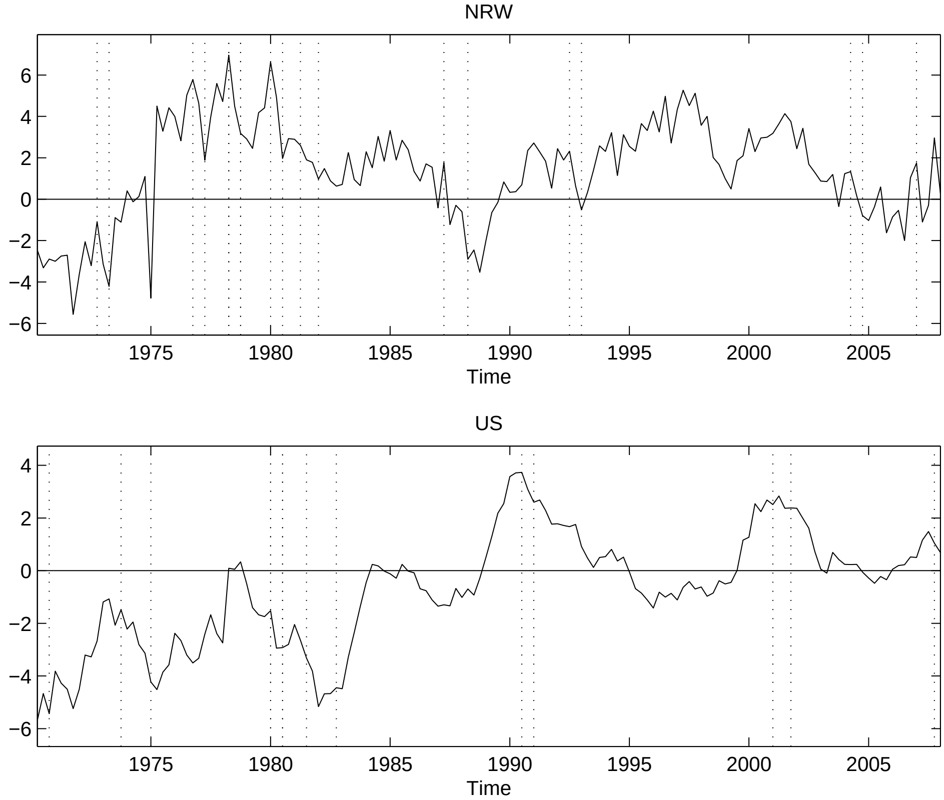


Figure 35: Estimated Cycles for Bivariate Model NTH-US

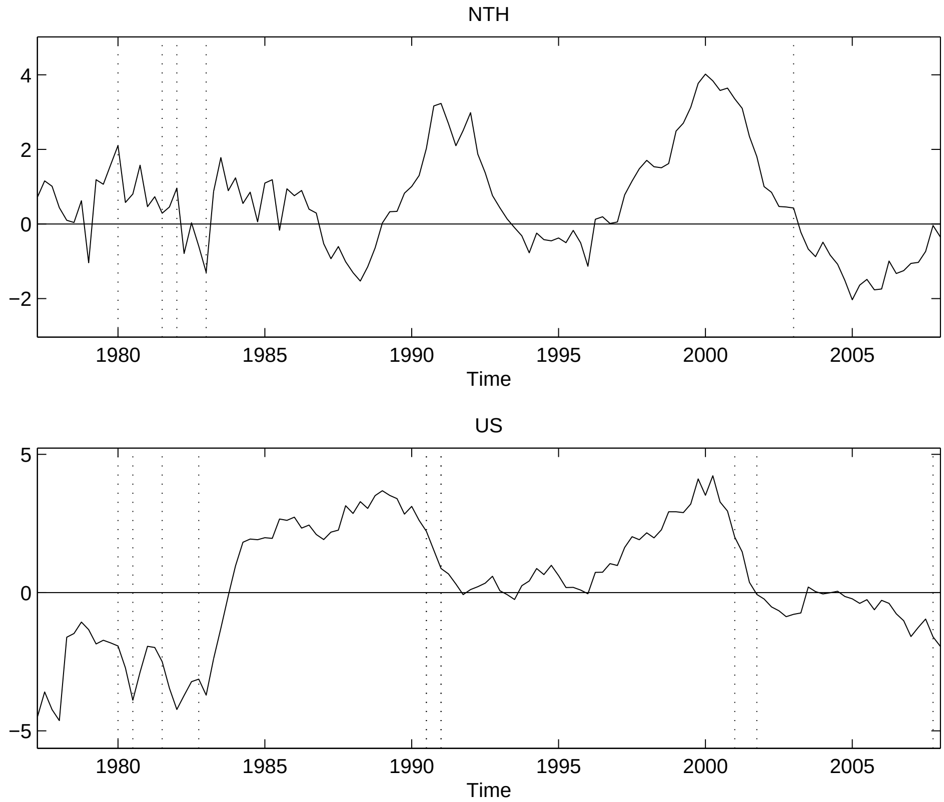


Figure 36: Estimated Cycles for Bivariate Model PT-US

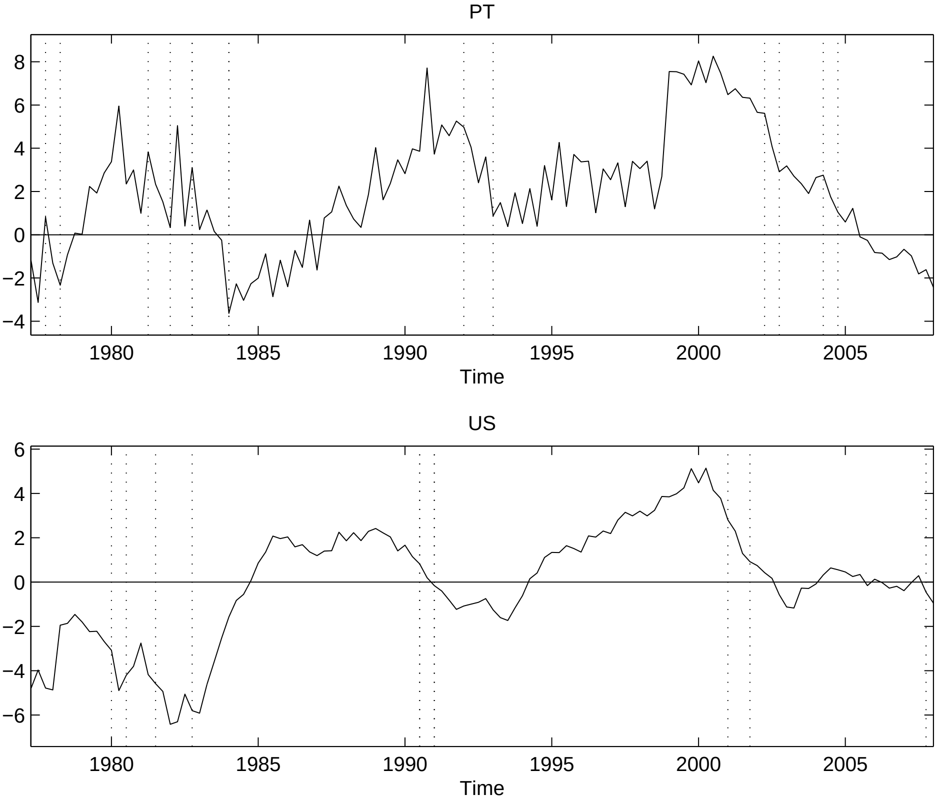


Figure 37: Estimated Cycles for Bivariate Model SP-US

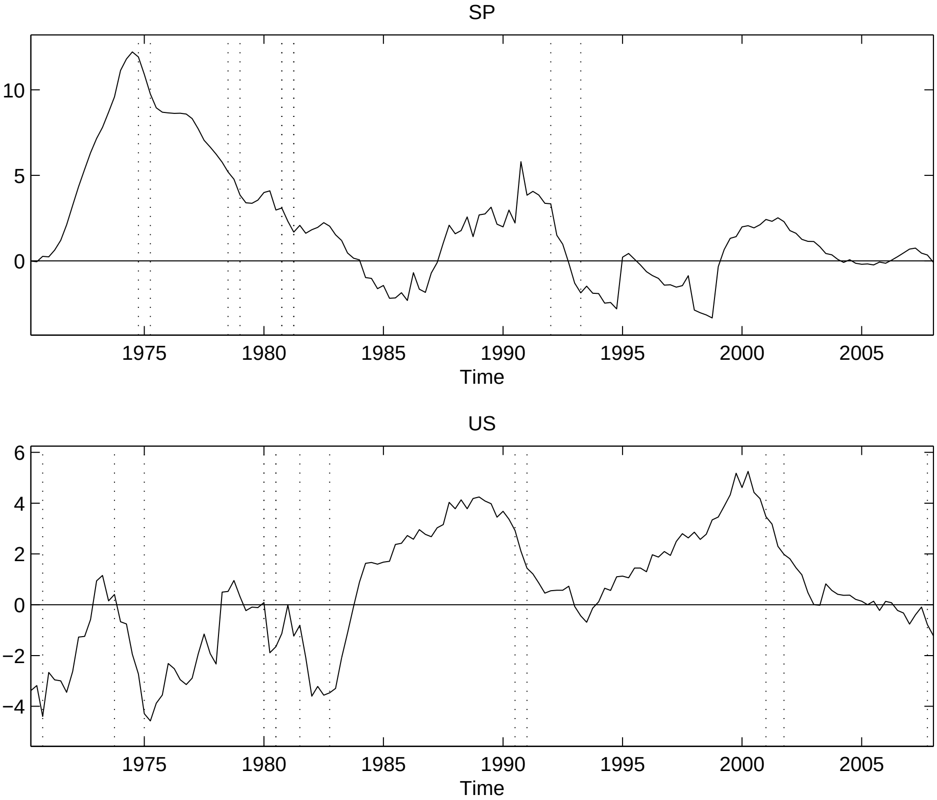


Figure 38: Estimated Cycles for Bivariate Model SWE-US

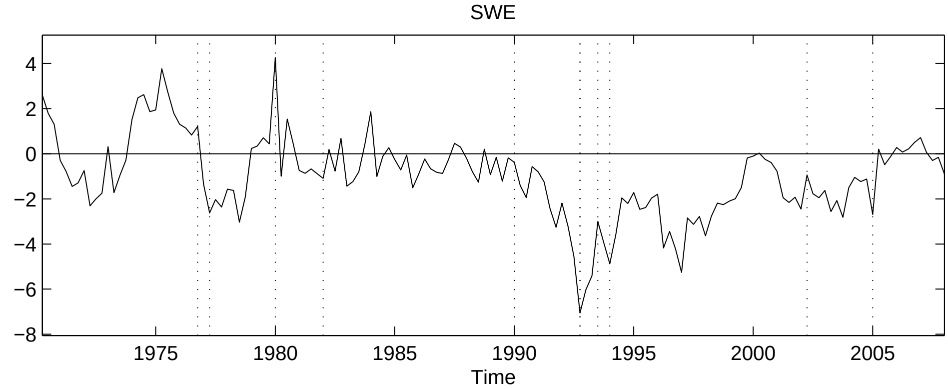


Figure 39: Estimated Cycles for Bivariate Model SWITZ-US

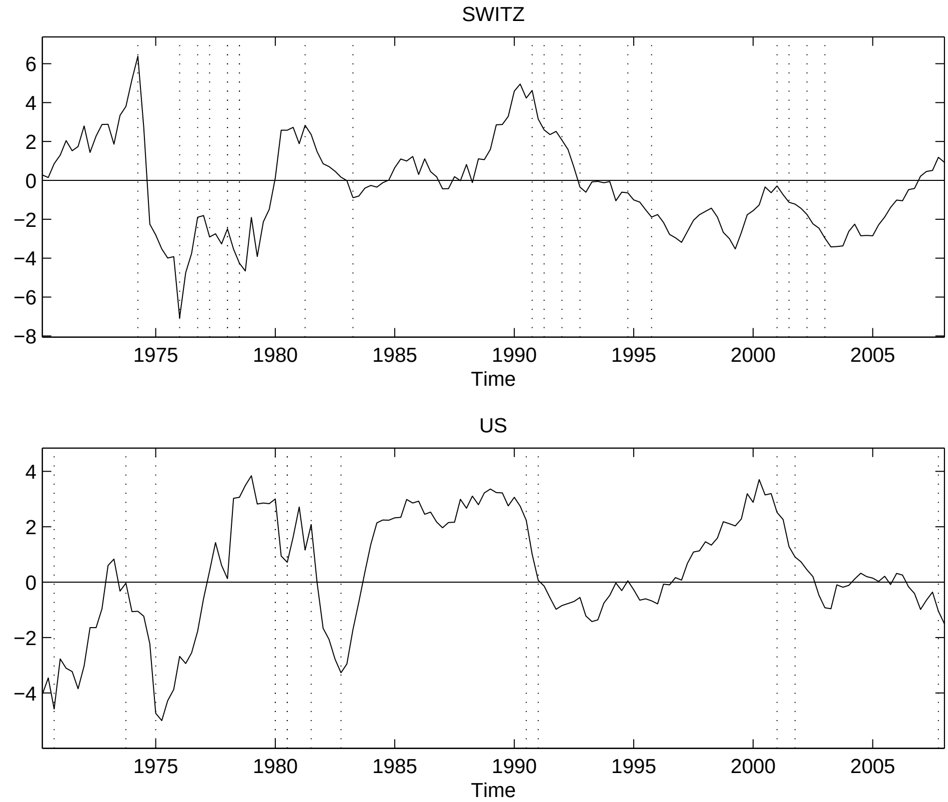


Figure 40: Estimated Cycles for Bivariate Model UK-US

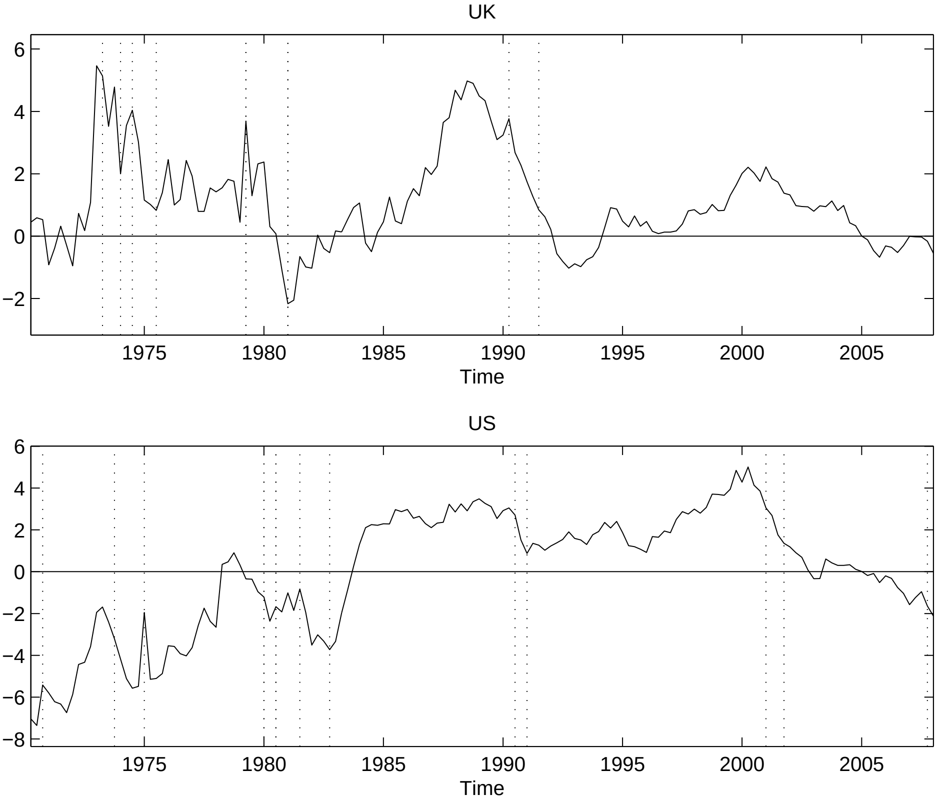


Figure 41: Smoothed Probabilities of Recession for Bivariate Model EA-AUS

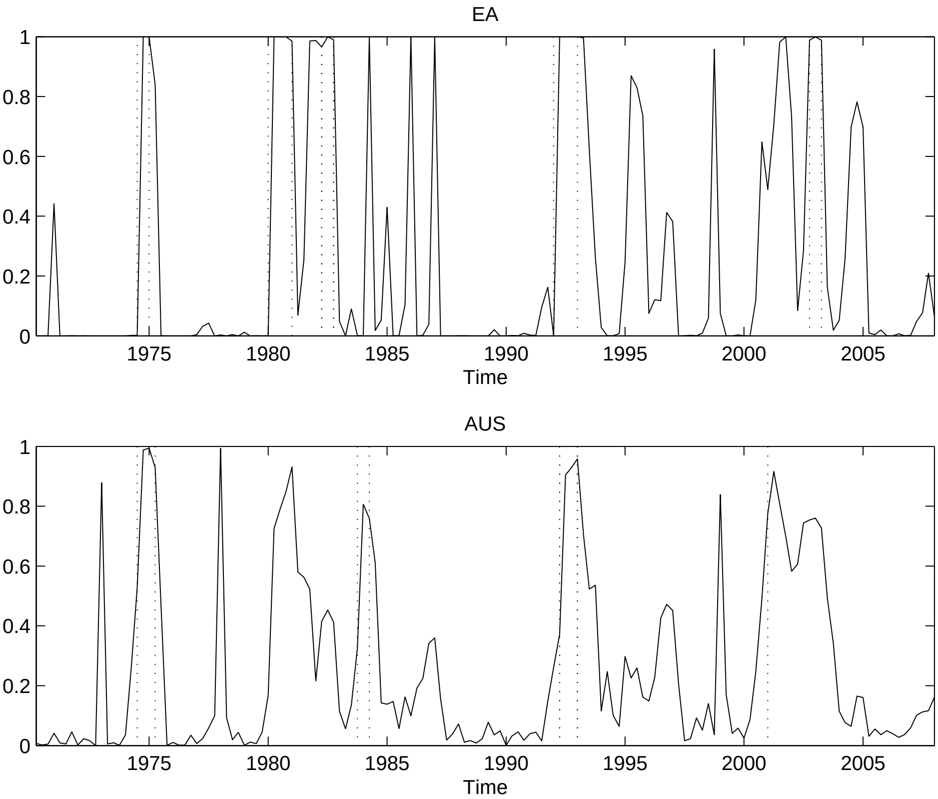


Figure 42: Smoothed Probabilities of Recession for Bivariate Model EA-BGM

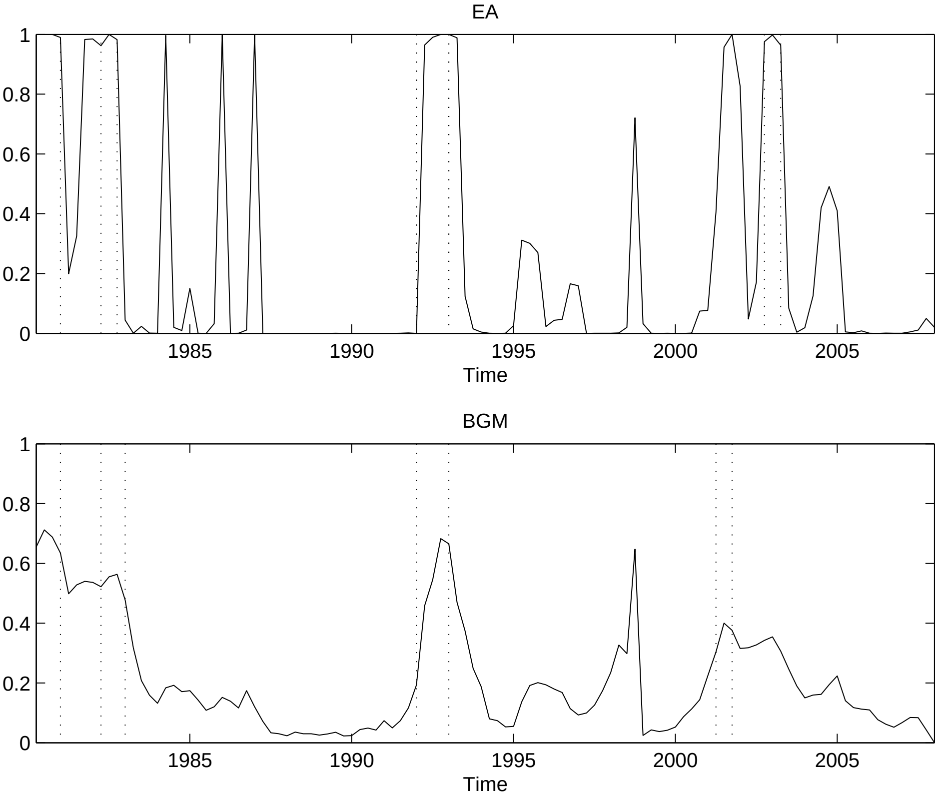


Figure 43: Smoothed Probabilities of Recession for Bivariate Model EA-CND

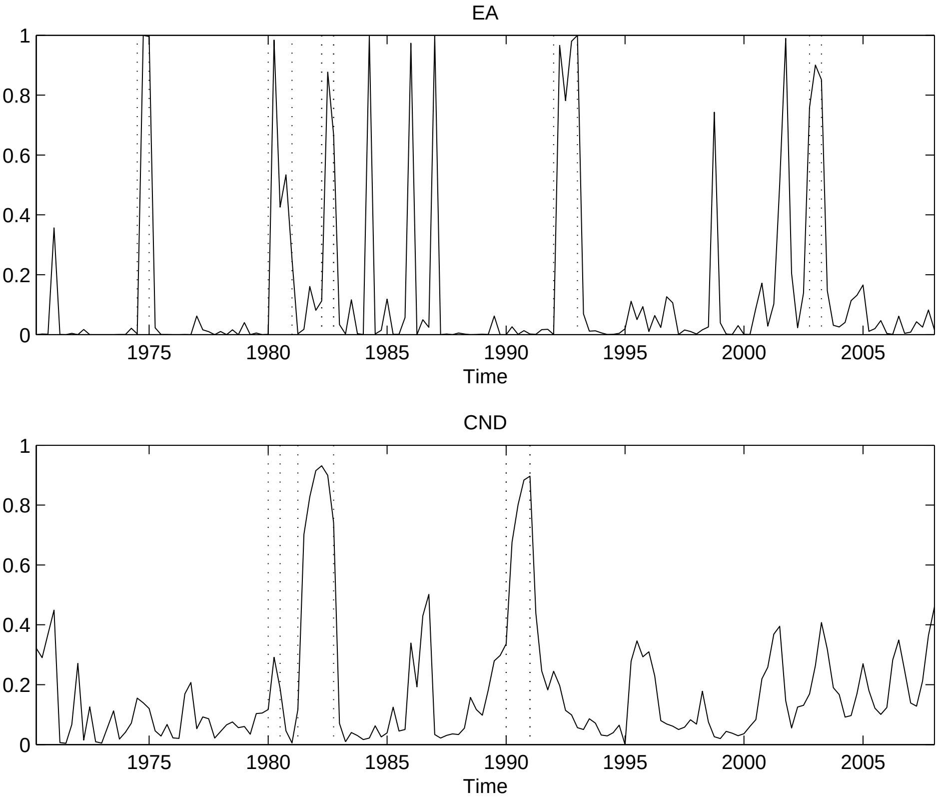


Figure 44: Smoothed Probabilities of Recession for Bivariate Model EA-DEN

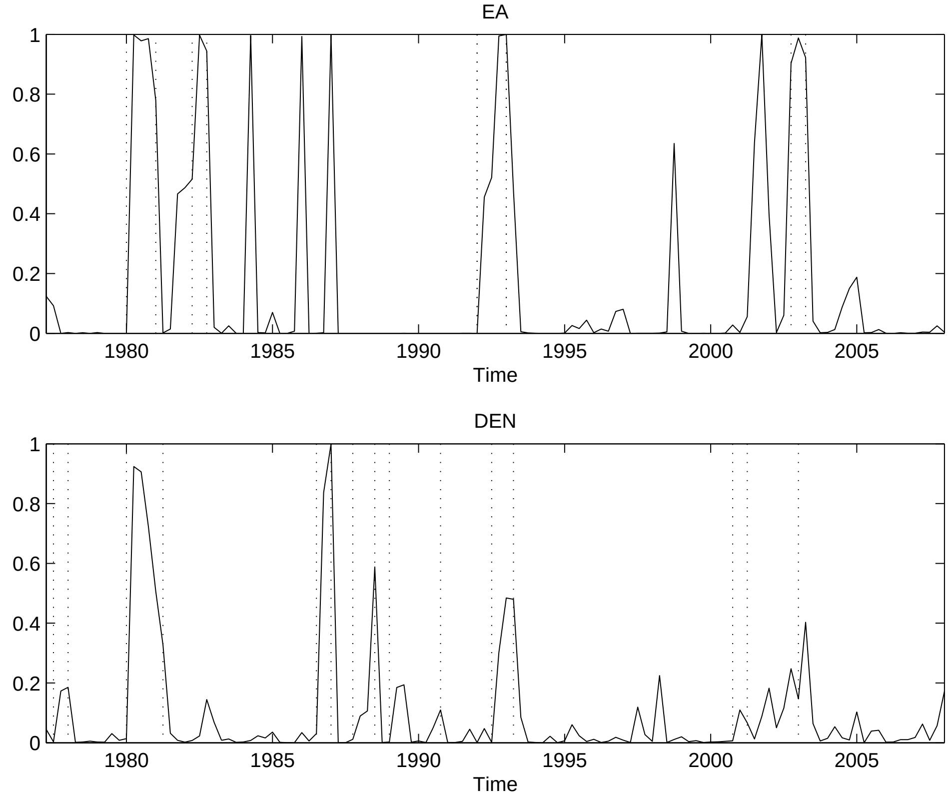


Figure 45: Smoothed Probabilities of Recession for Bivariate Model EA-FIN

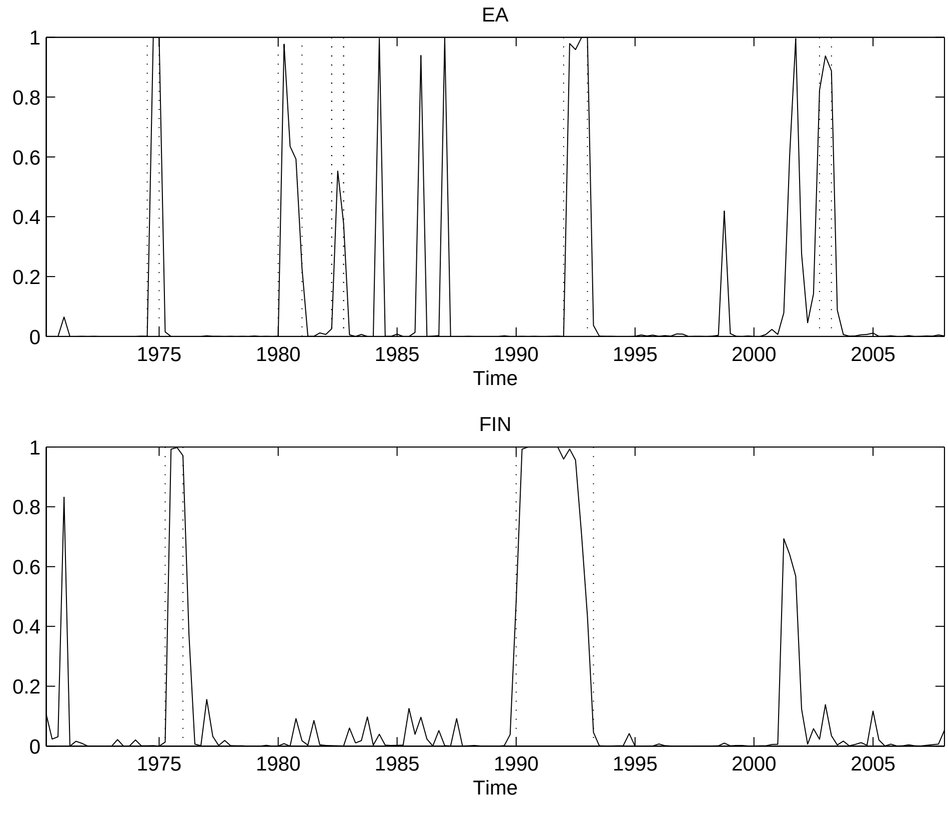


Figure 46: Smoothed Probabilities of Recession for Bivariate Model EA-FR

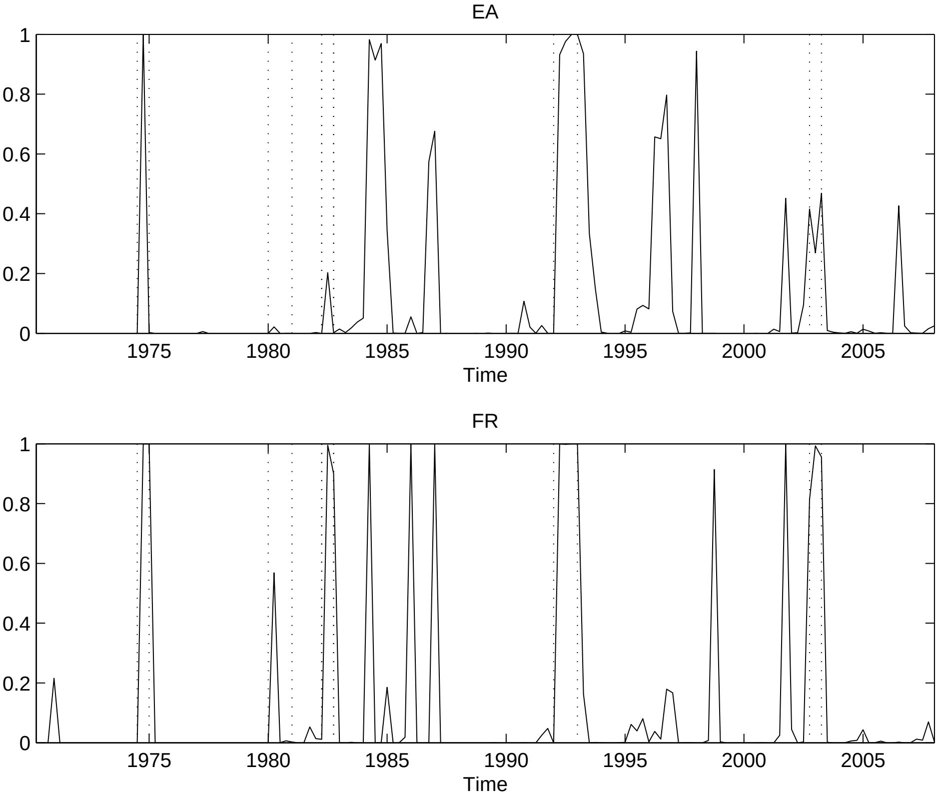


Figure 47: Smoothed Probabilities of Recession for Bivariate Model EA-GER

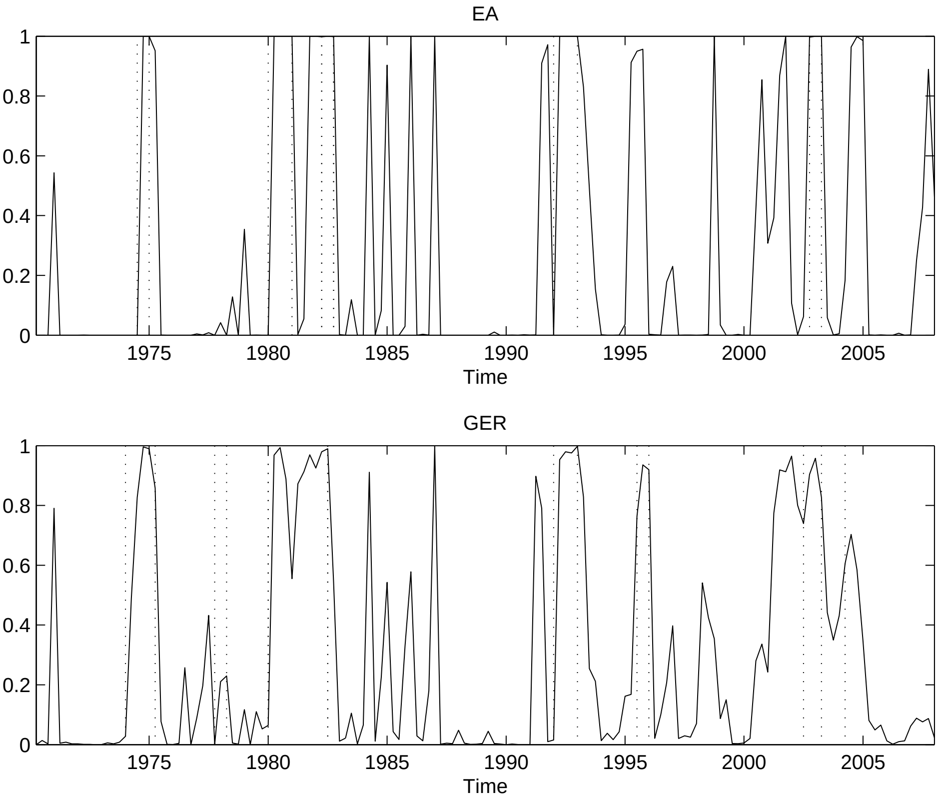


Figure 48: Smoothed Probabilities of Recession for Bivariate Model EA-GREE

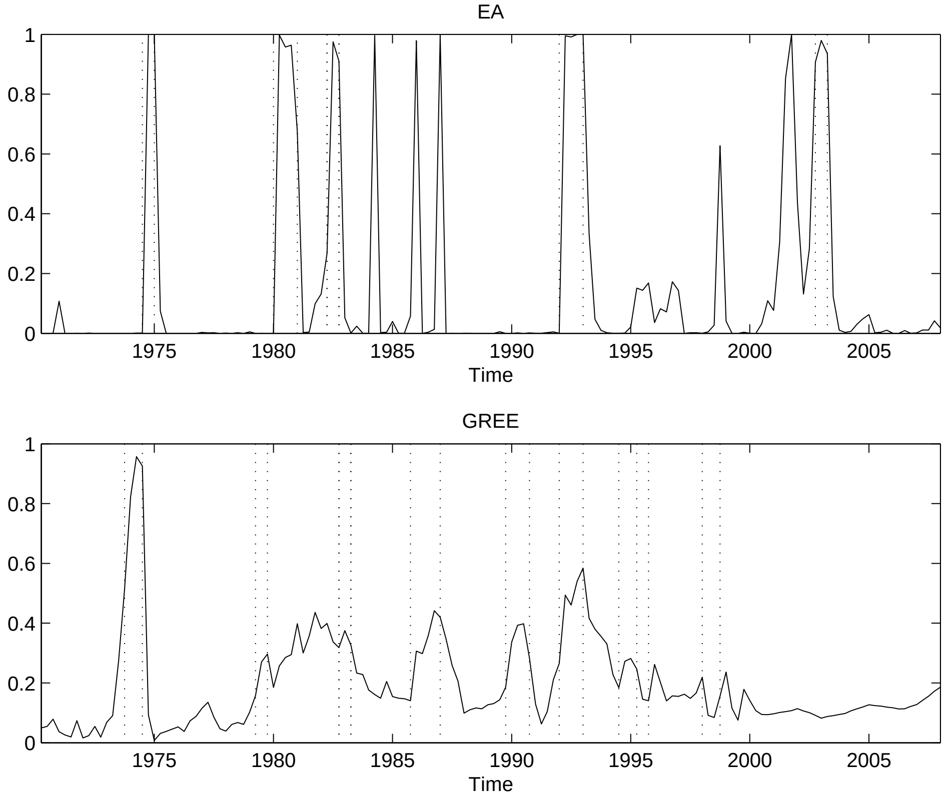


Figure 49: Smoothed Probabilities of Recession for Bivariate Model EA-IT

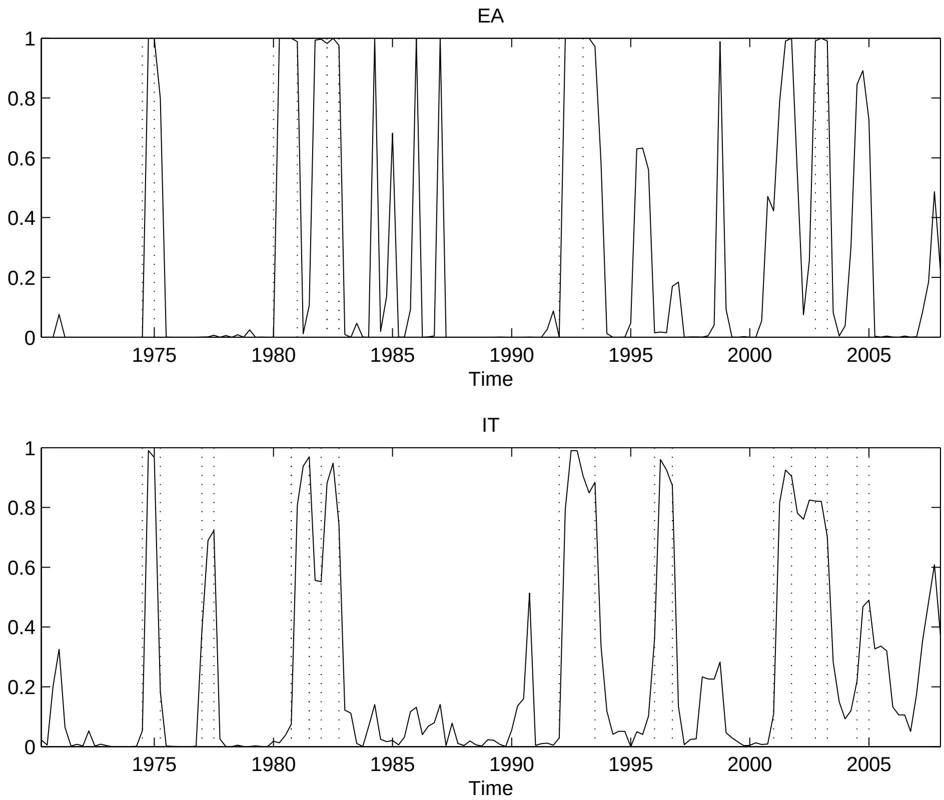


Figure 50: Smoothed Probabilities of Recession for Bivariate Model EA-JP

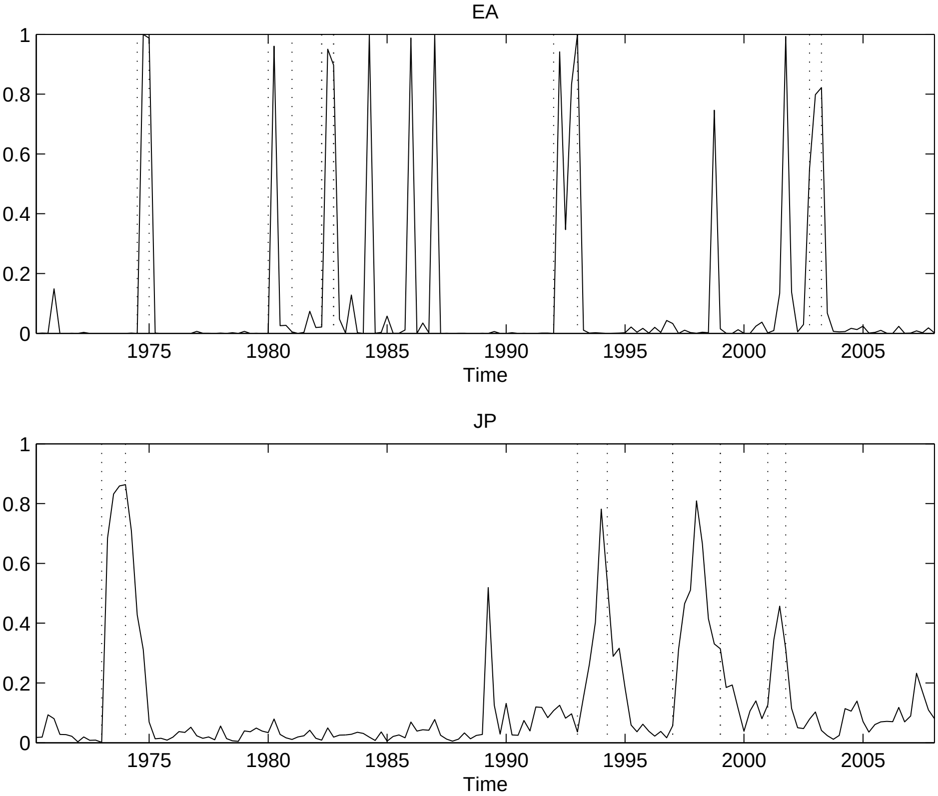


Figure 51: Smoothed Probabilities of Recession for Bivariate Model EA-NRW

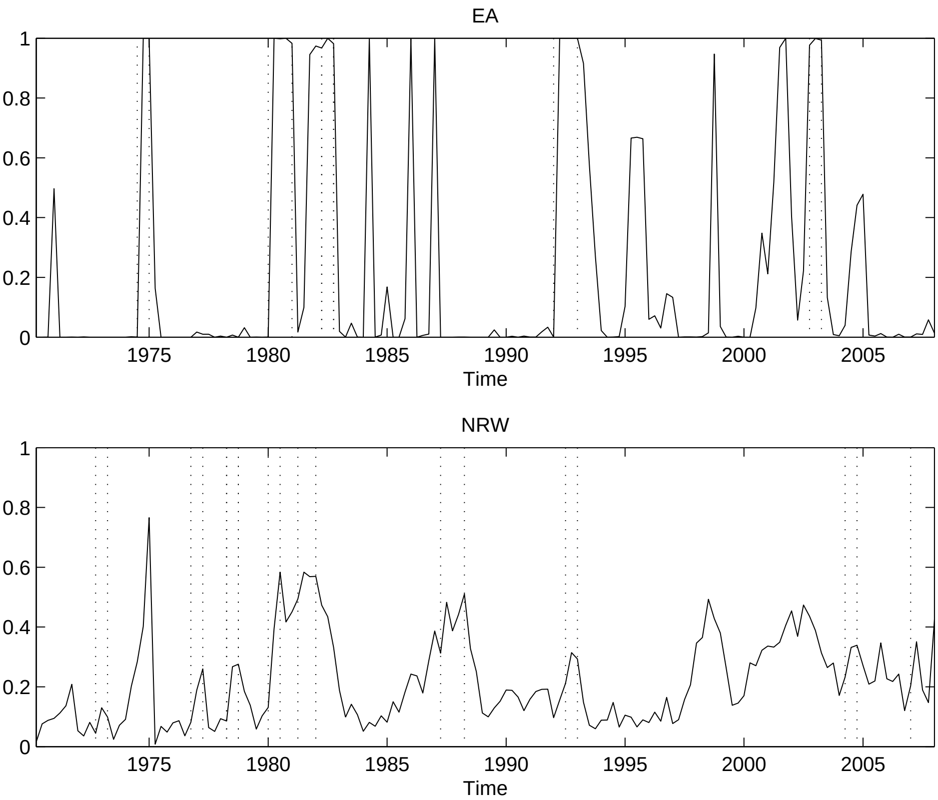


Figure 52: Smoothed Probabilities of Recession for Bivariate Model EA-NTH

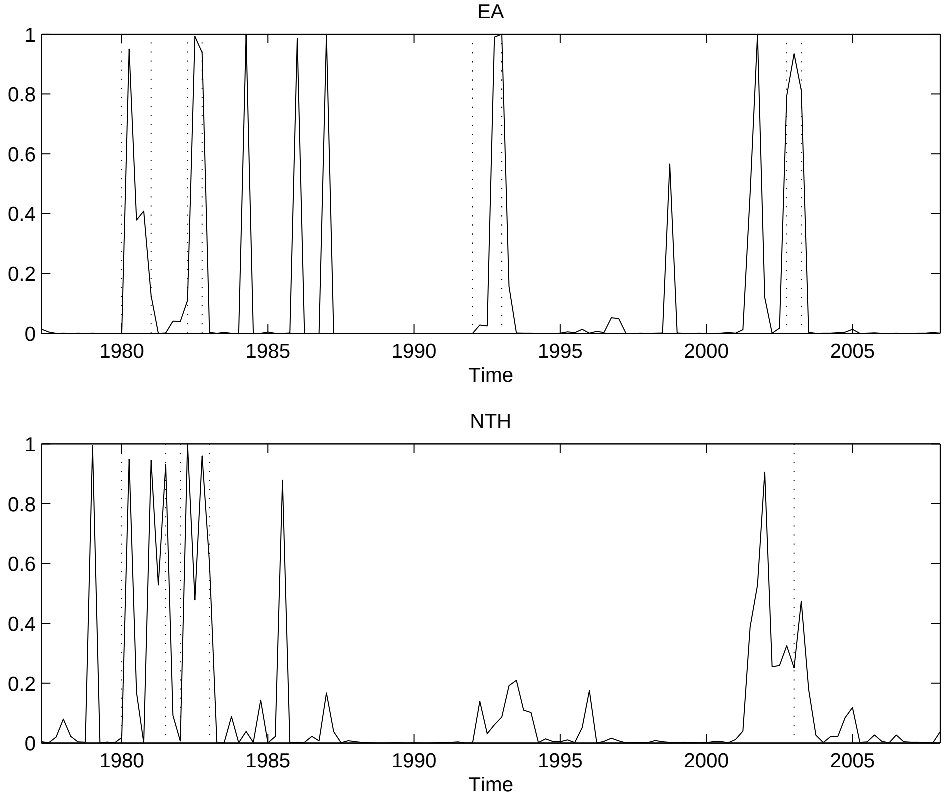


Figure 53: Smoothed Probabilities of Recession for Bivariate Model EA-PT

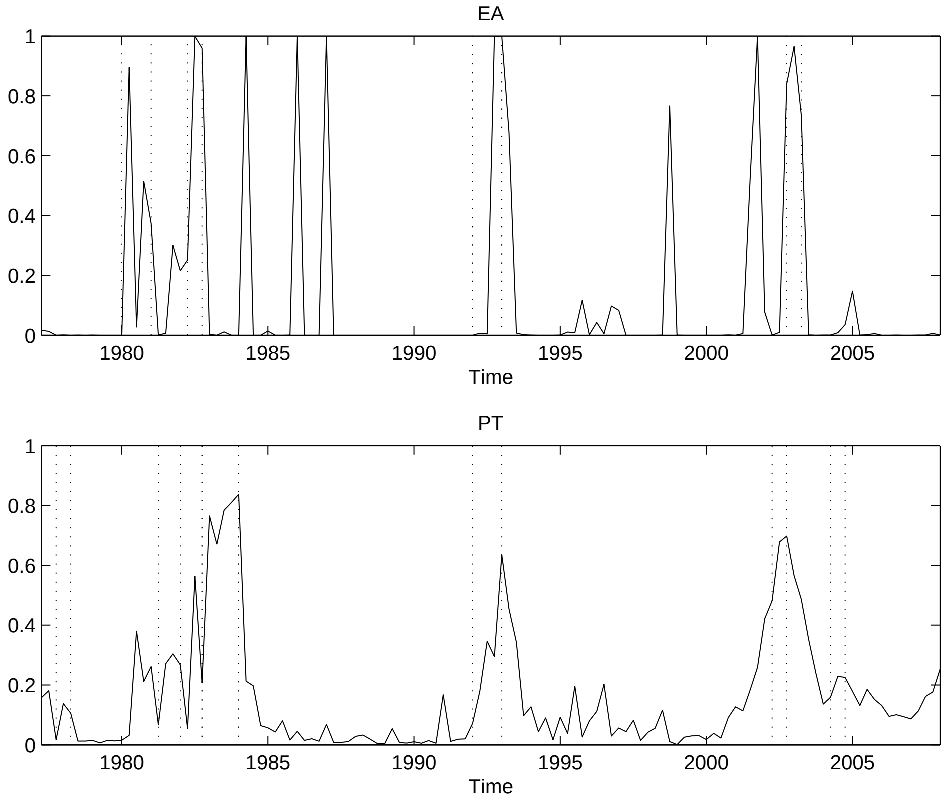


Figure 54: Smoothed Probabilities of Recession for Bivariate Model EA-SP

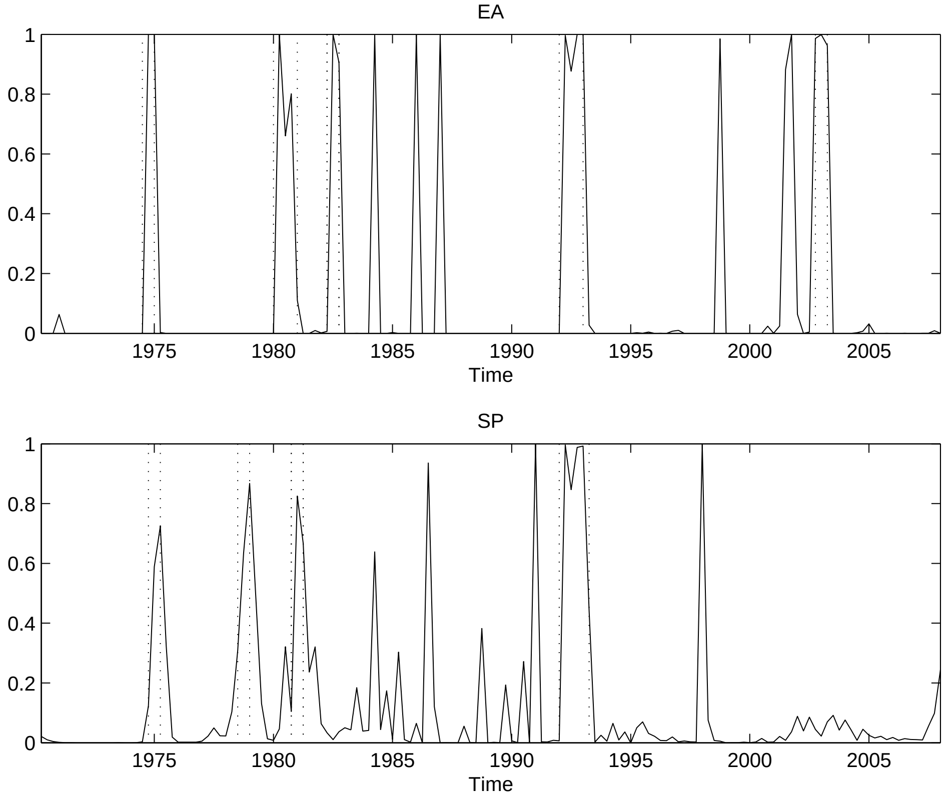


Figure 55: Smoothed Probabilities of Recession for Bivariate Model EA-SWE

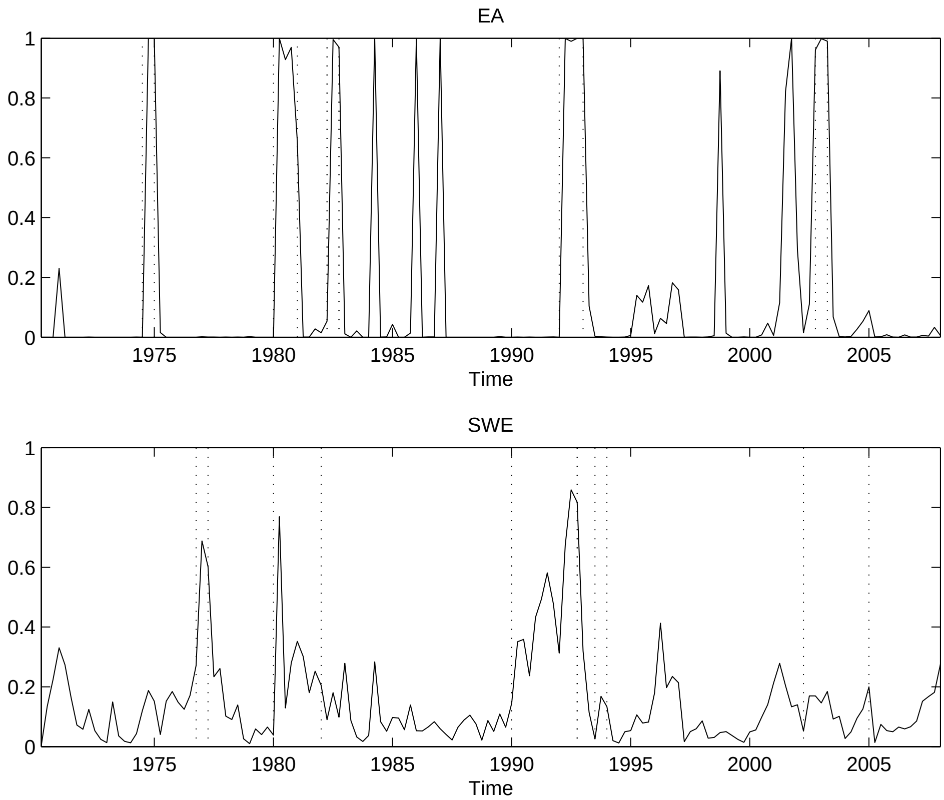


Figure 56: Smoothed Probabilities of Recession for Bivariate Model EA-SWITZ

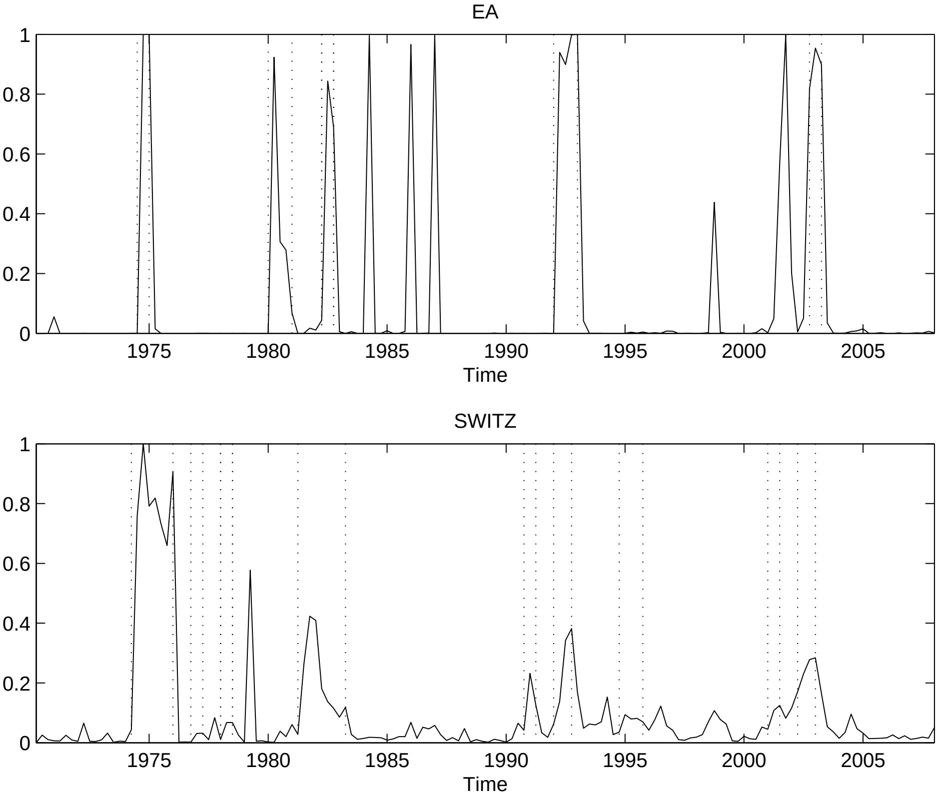


Figure 57: Smoothed Probabilities of Recession for Bivariate Model EA-UK

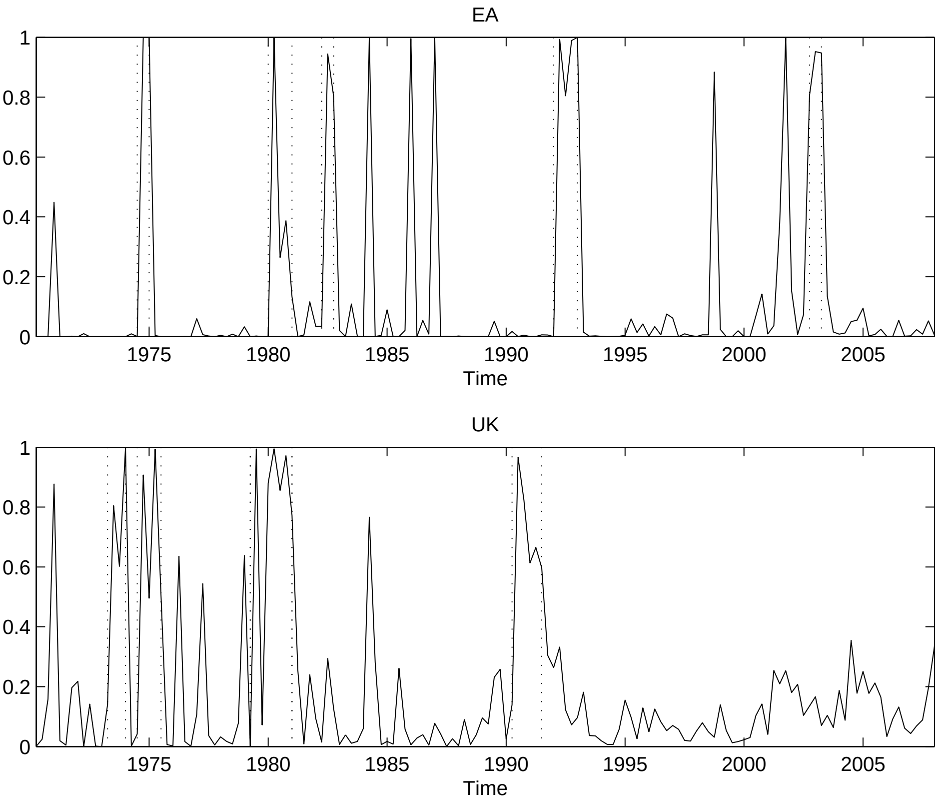


Figure 58: Smoothed Probabilities of Recession for Bivariate Model EA-US

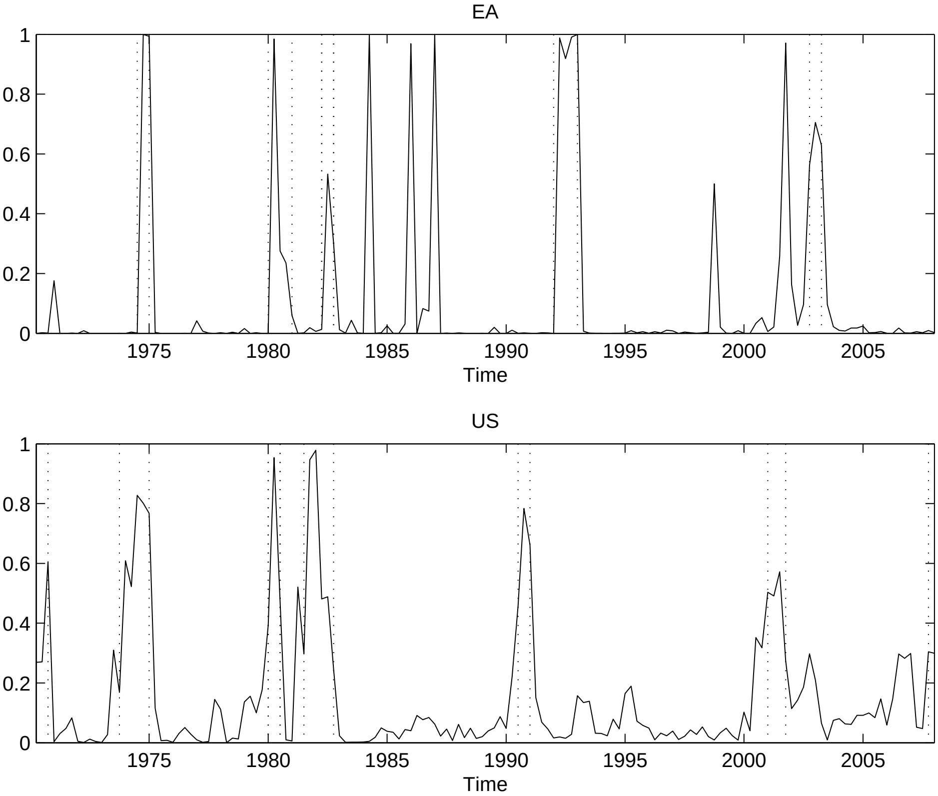


Figure 59: Smoothed Probabilities of Recession for Bivariate Model AUS-US

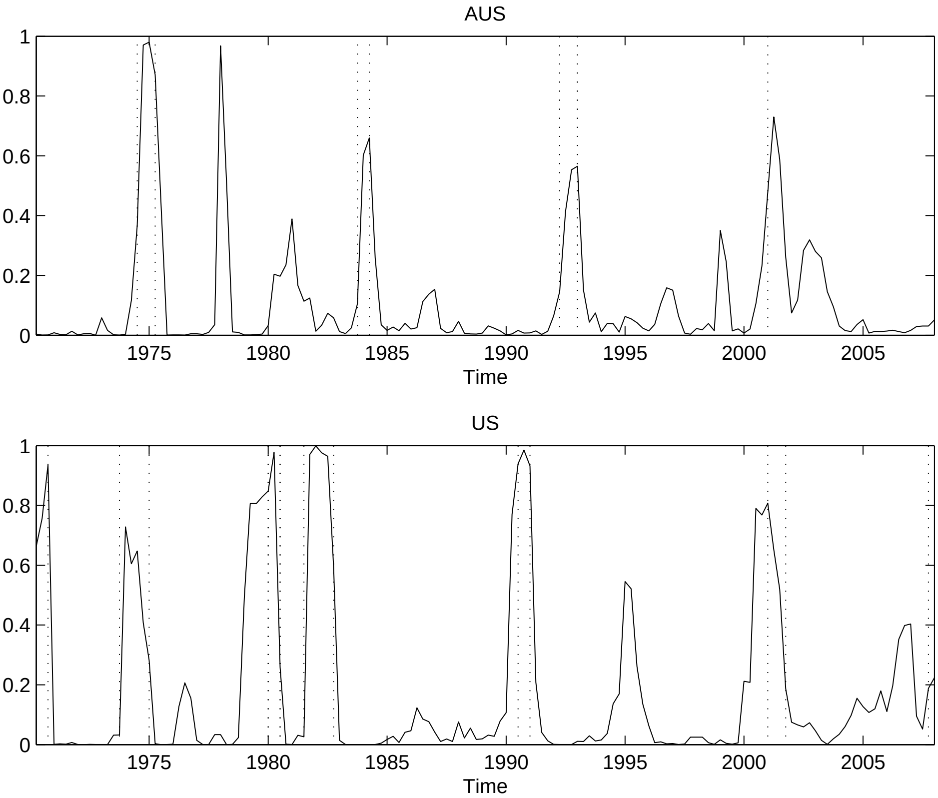


Figure 60: Smoothed Probabilities of Recession for Bivariate Model BGM-US

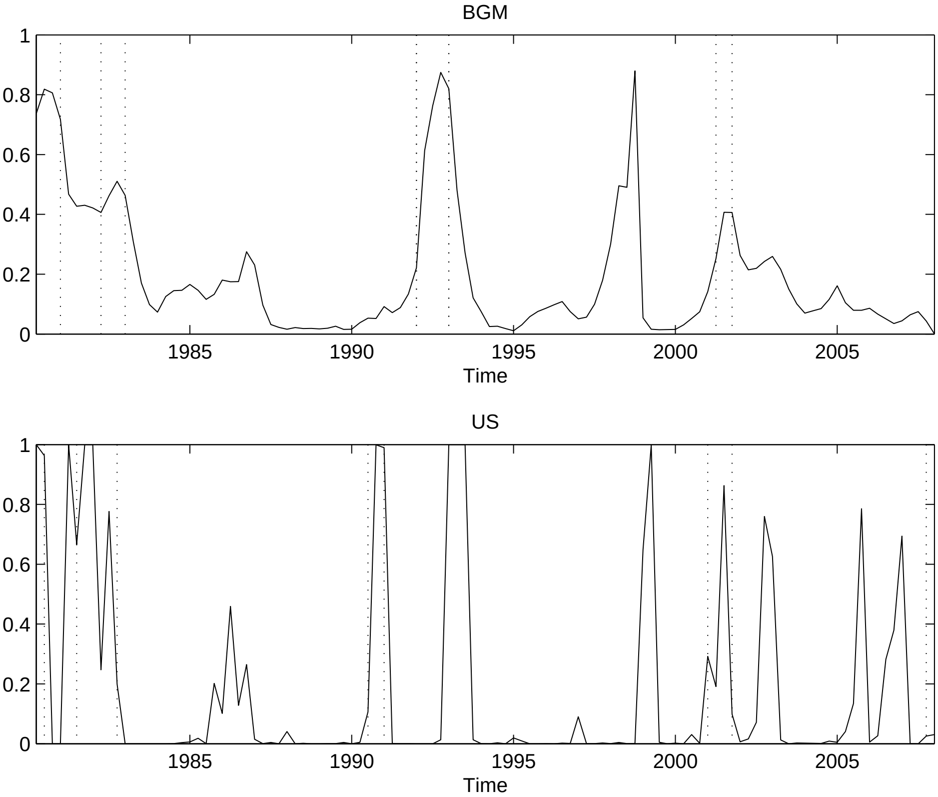


Figure 61: Smoothed Probabilities of Recession for Bivariate Model CND-US

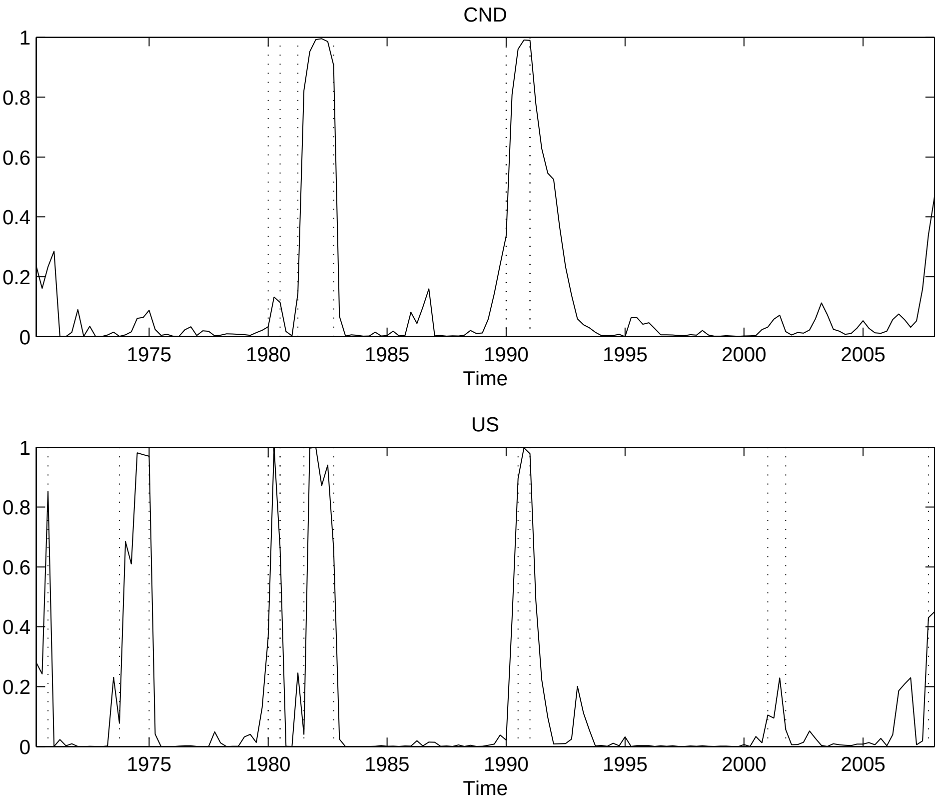


Figure 62: Smoothed Probabilities of Recession for Bivariate Model DEN-US

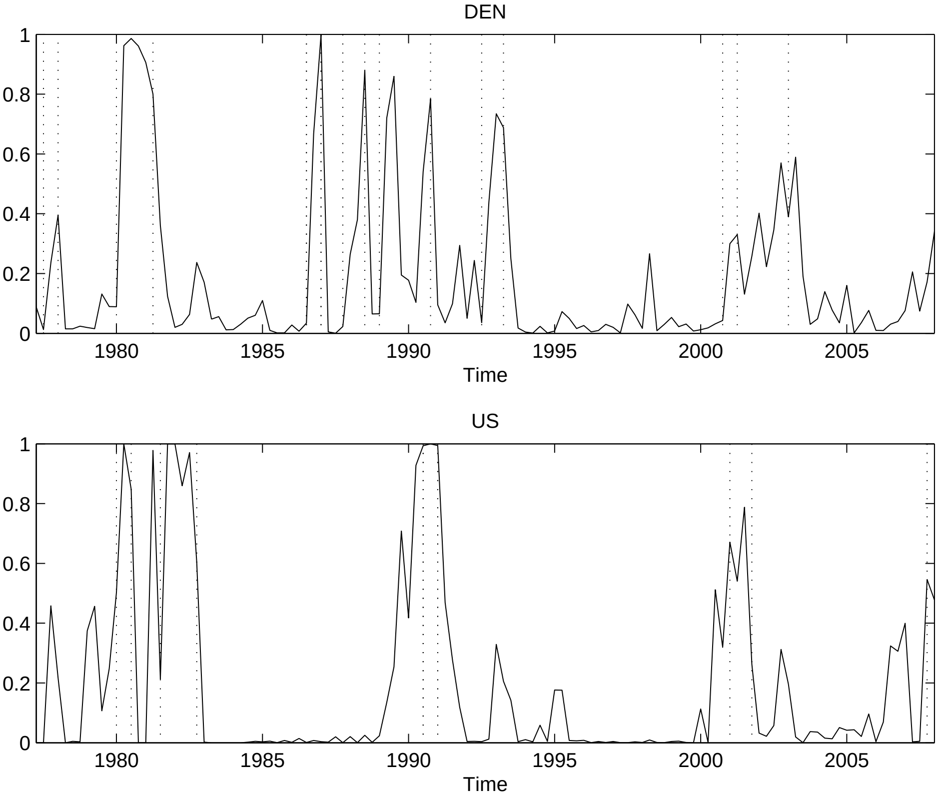


Figure 63: Smoothed Probabilities of Recession for Bivariate Model FIN-US

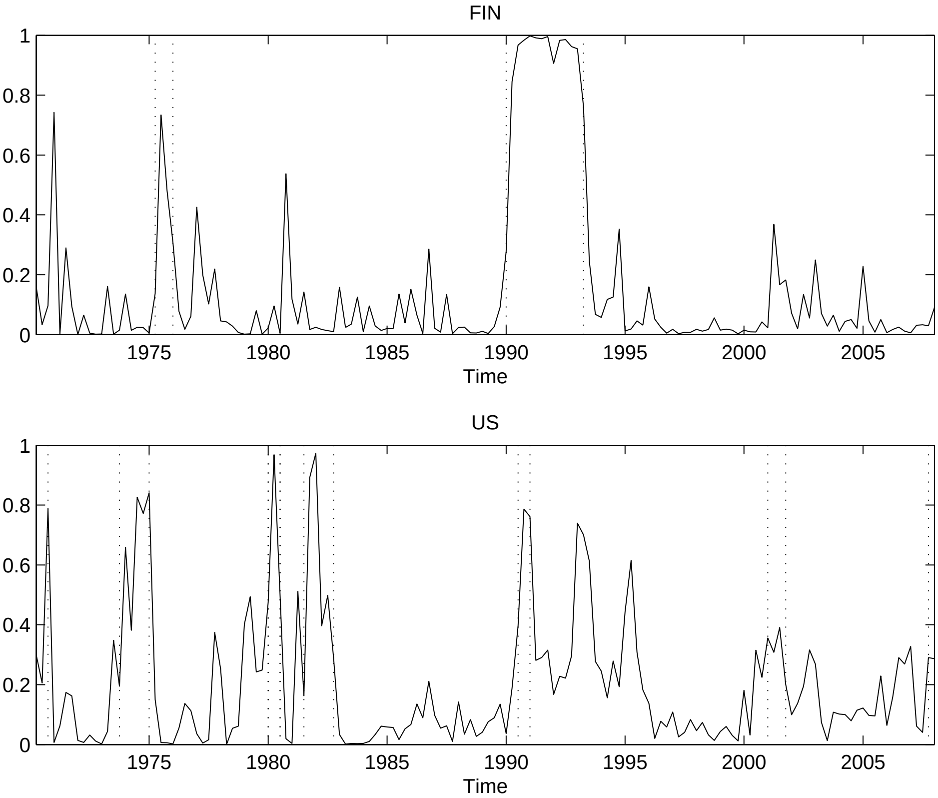


Figure 64: Smoothed Probabilities of Recession for Bivariate Model FR-US

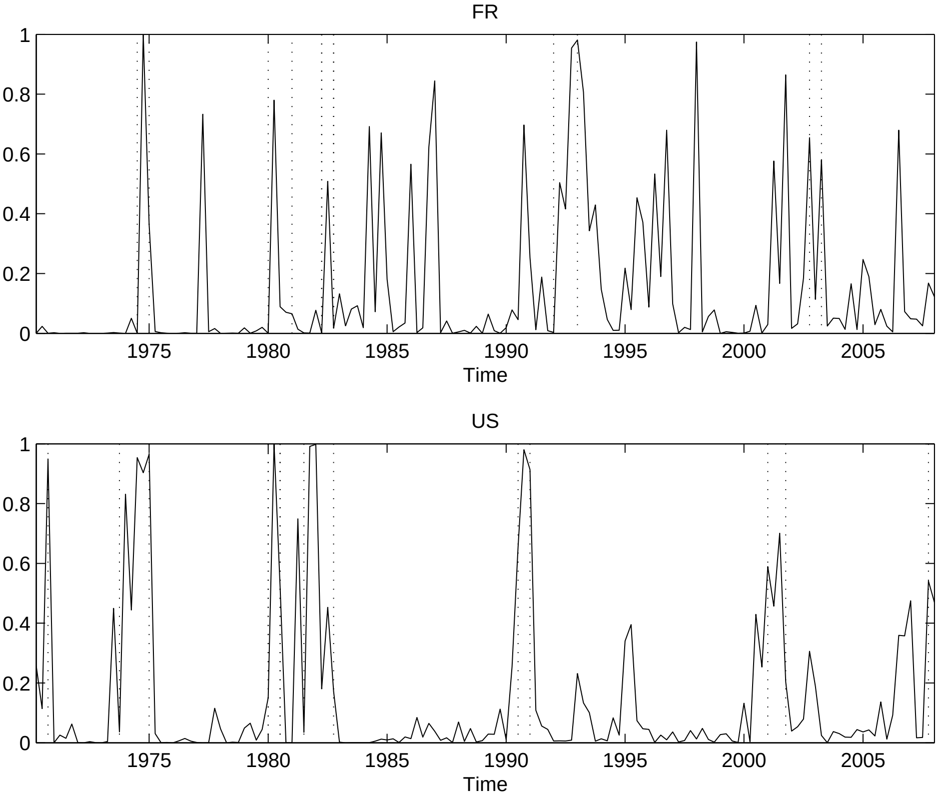


Figure 65: Smoothed Probabilities of Recession for Bivariate Model GER-US

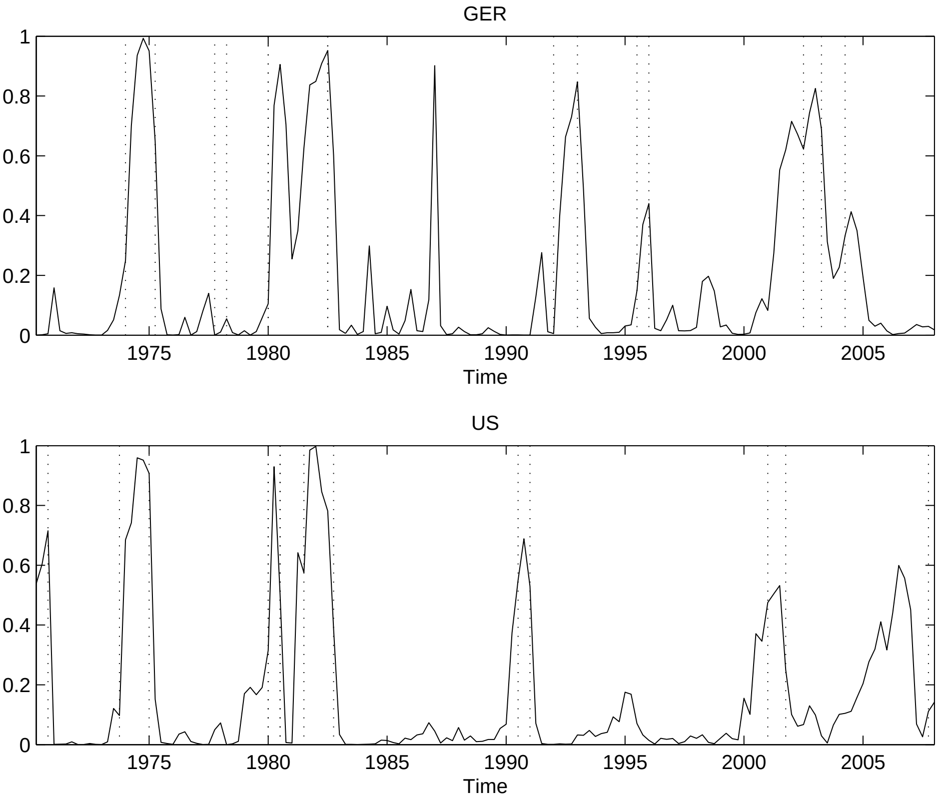


Figure 66: Smoothed Probabilities of Recession for Bivariate Model GREE-US

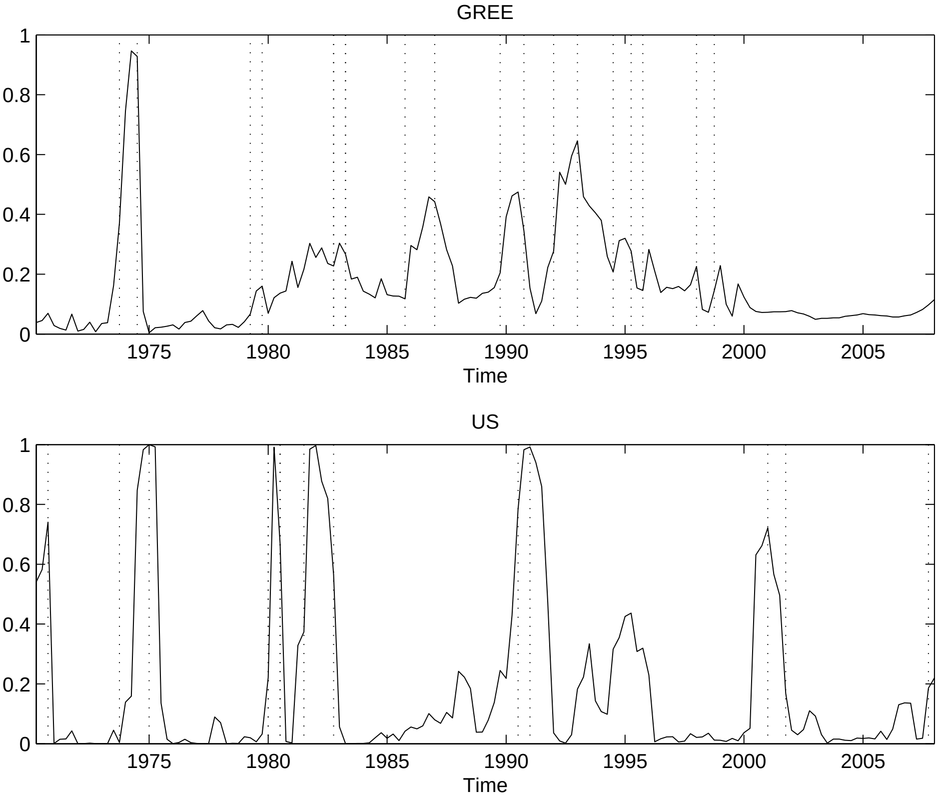


Figure 67: Smoothed Probabilities of Recession for Bivariate Model IT-US

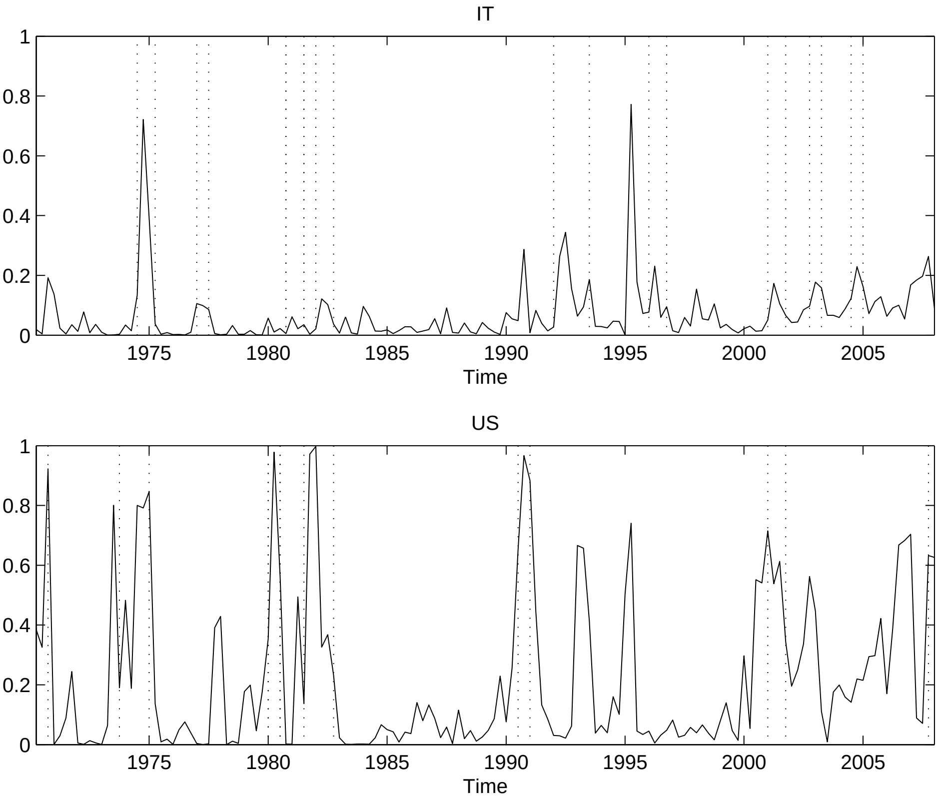


Figure 68: Smoothed Probabilities of Recession for Bivariate Model JP-US

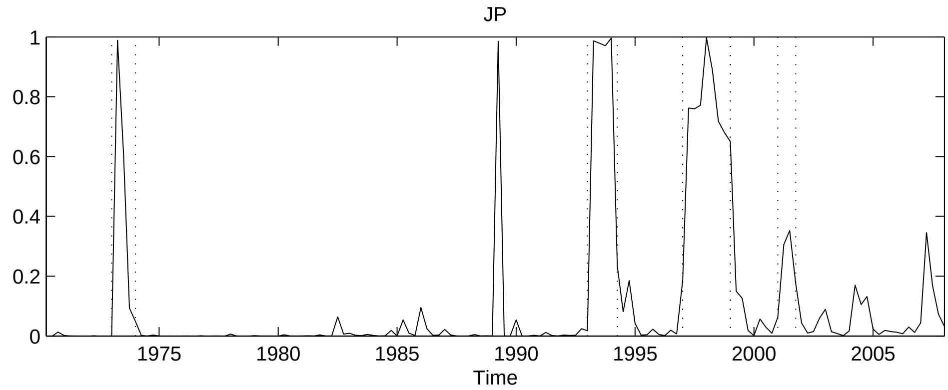


Figure 69: Smoothed Probabilities of Recession for Bivariate Model NRW-US

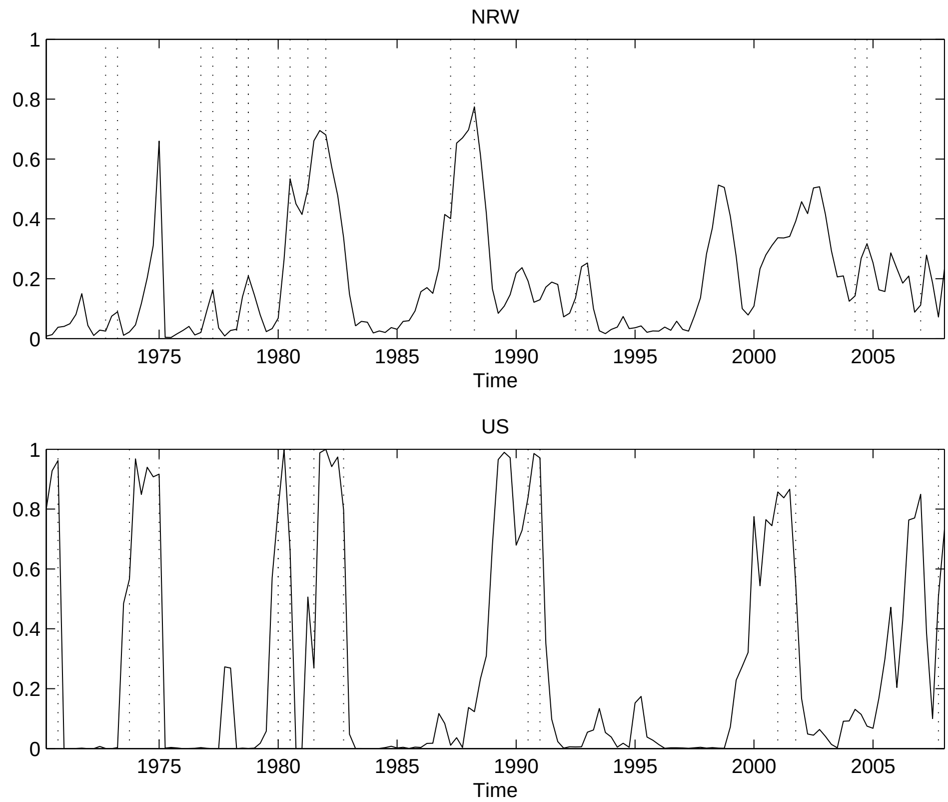


Figure 70: Smoothed Probabilities of Recession for Bivariate Model NTH-US

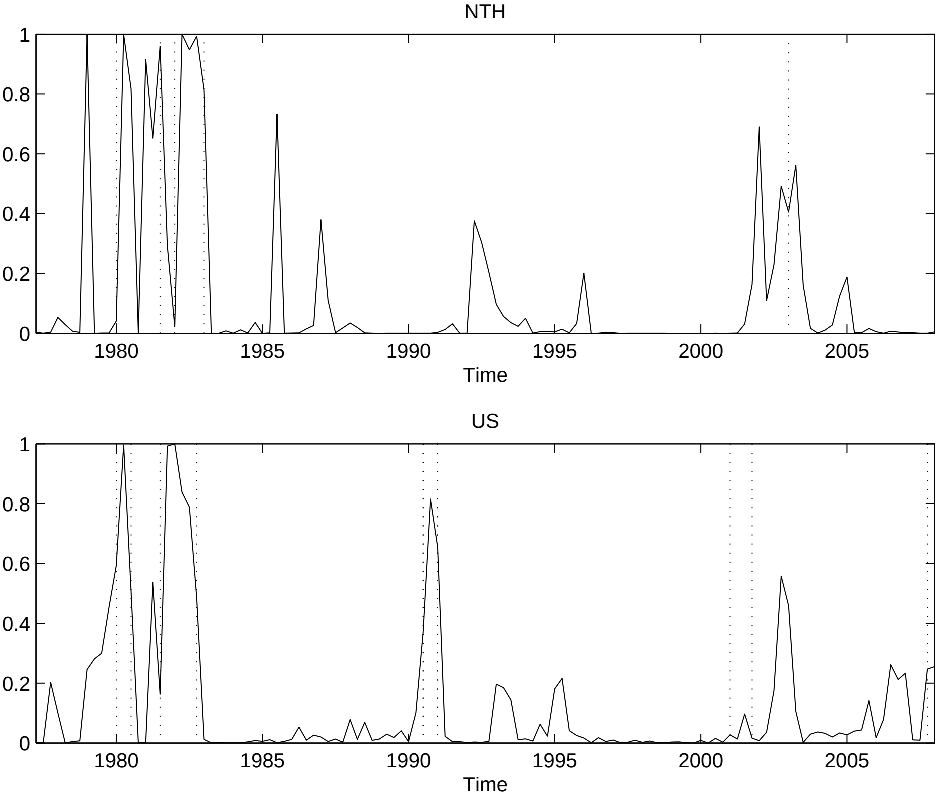


Figure 71: Smoothed Probabilities of Recession for Bivariate Model PT-US

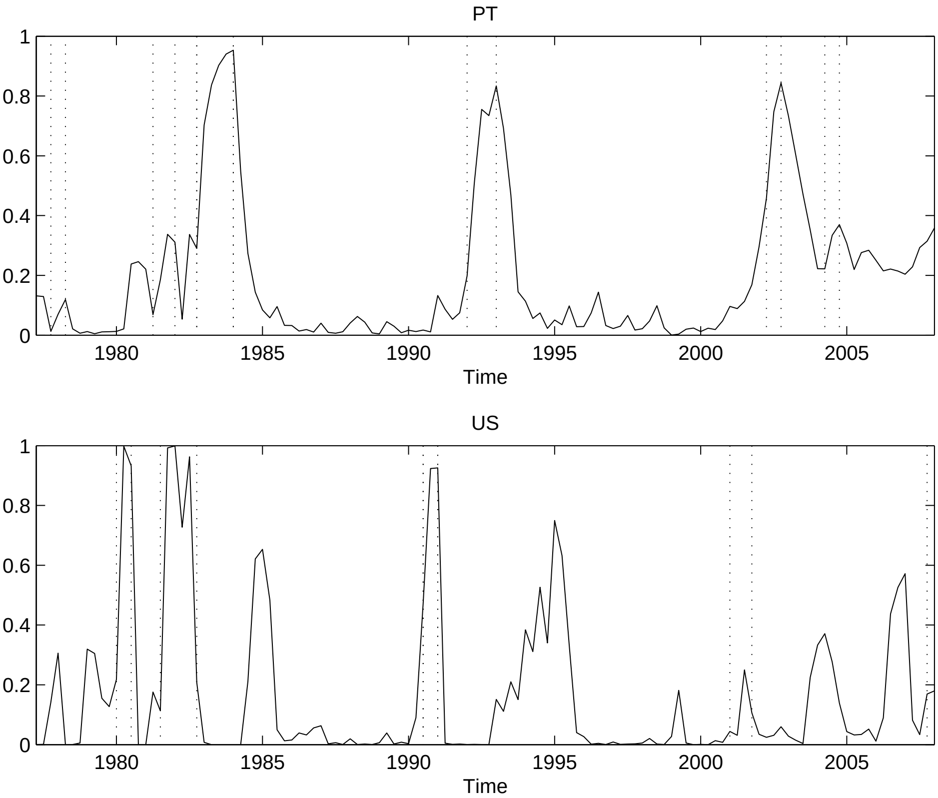


Figure 72: Smoothed Probabilities of Recession for Bivariate Model SP-US

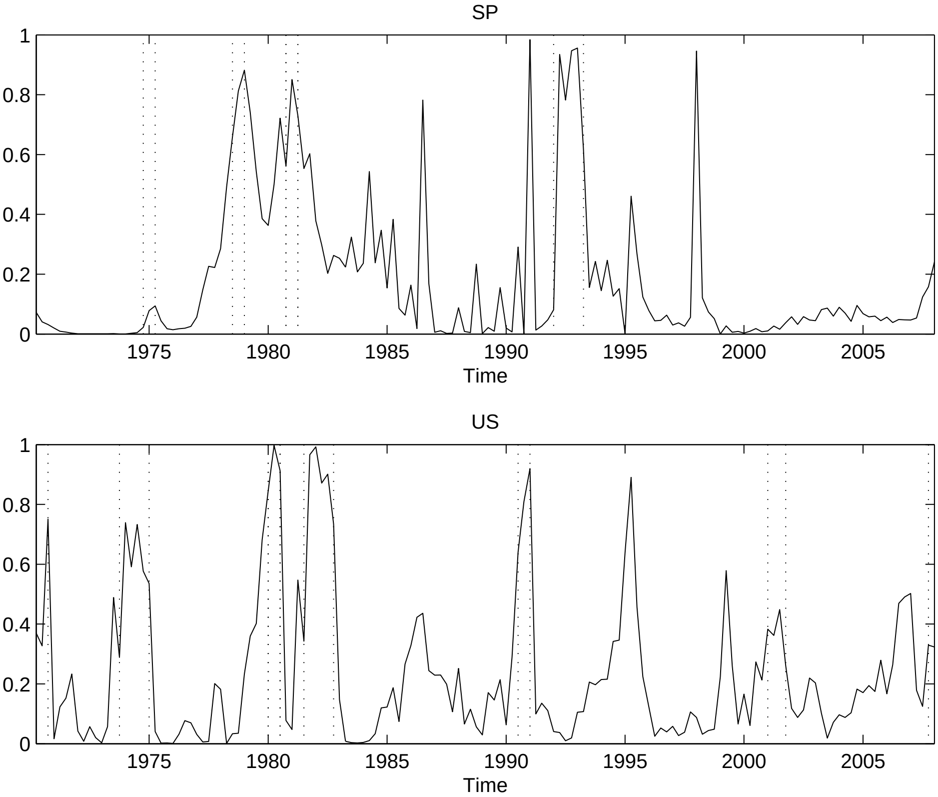


Figure 73: Smoothed Probabilities of Recession for Bivariate Model SWE-US

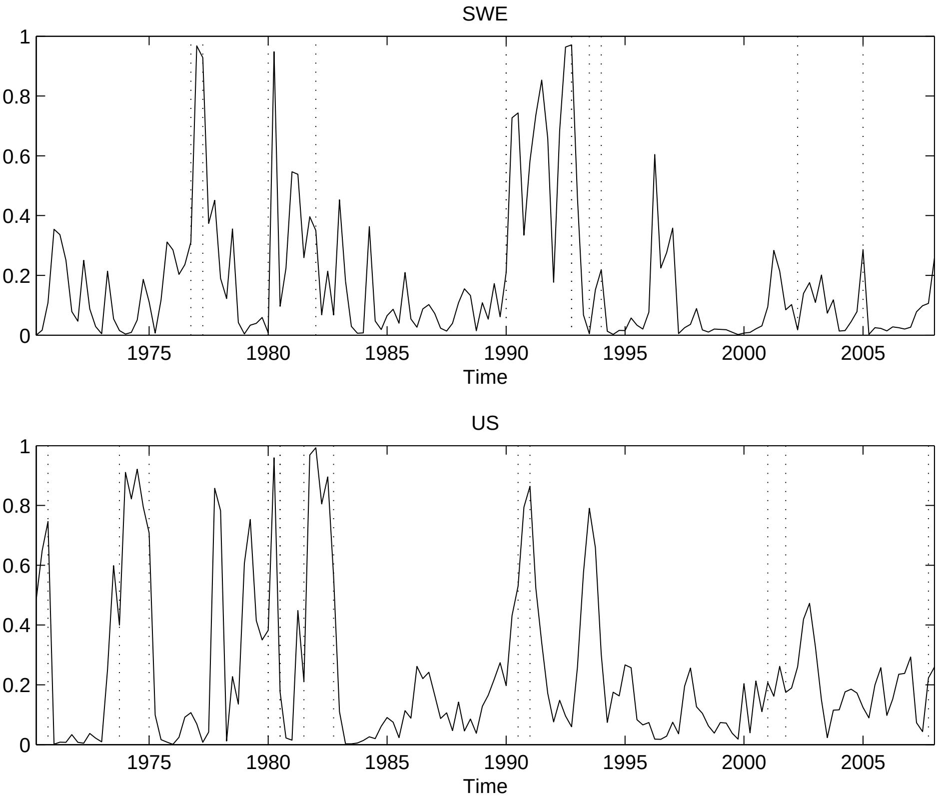


Figure 74: Smoothed Probabilities of Recession for Bivariate Model SWITZ-US

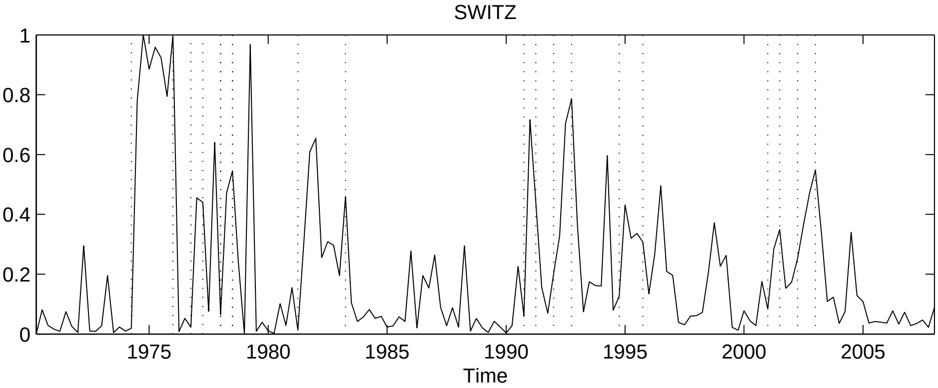


Figure 75: Smoothed Probabilities of Recession for Bivariate Model UK-US

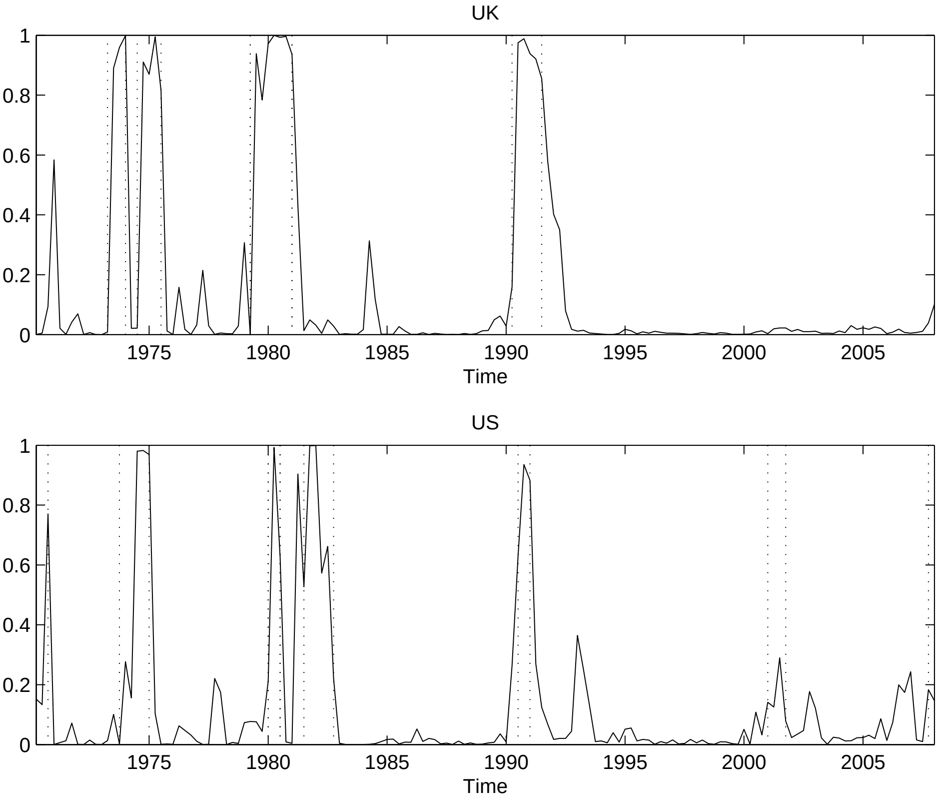


Figure 76: Smoothed Multivariate Probabilities for Bivariate Model EA-AUS

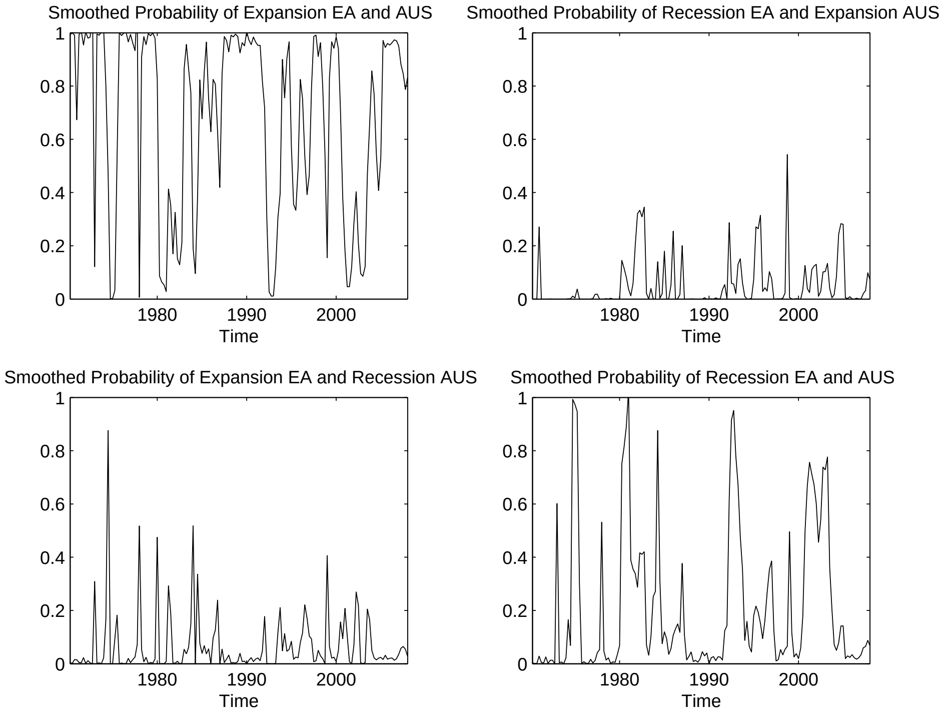


Figure 77: Smoothed Multivariate Probabilities for Bivariate Model EA-BGM

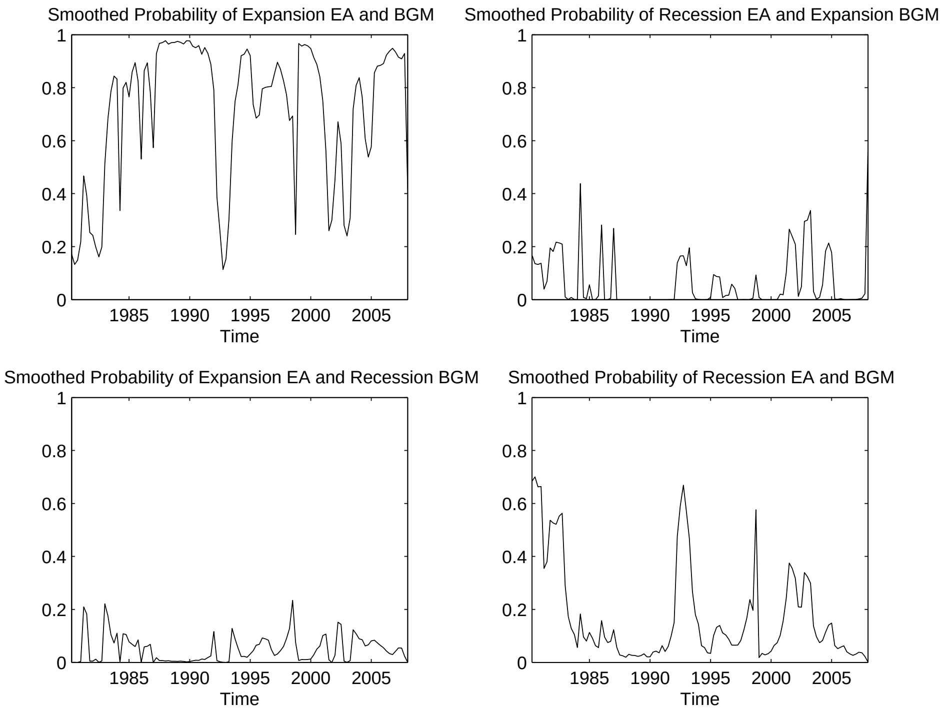


Figure 78: Smoothed Multivariate Probabilities for Bivariate Model EA-CND

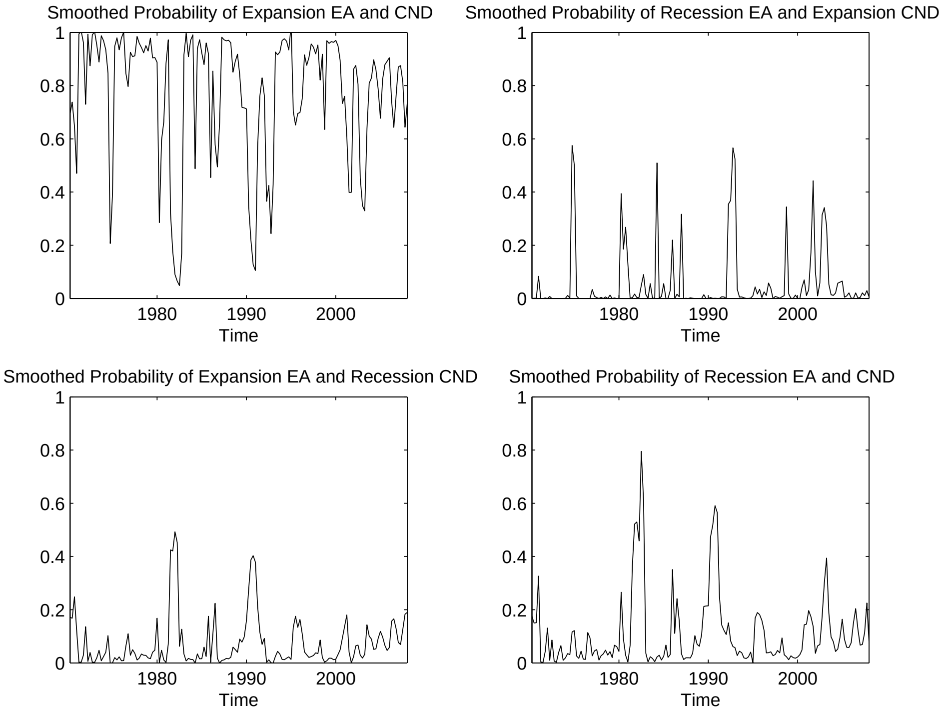


Figure 79: Smoothed Multivariate Probabilities for Bivariate Model EA-DEN

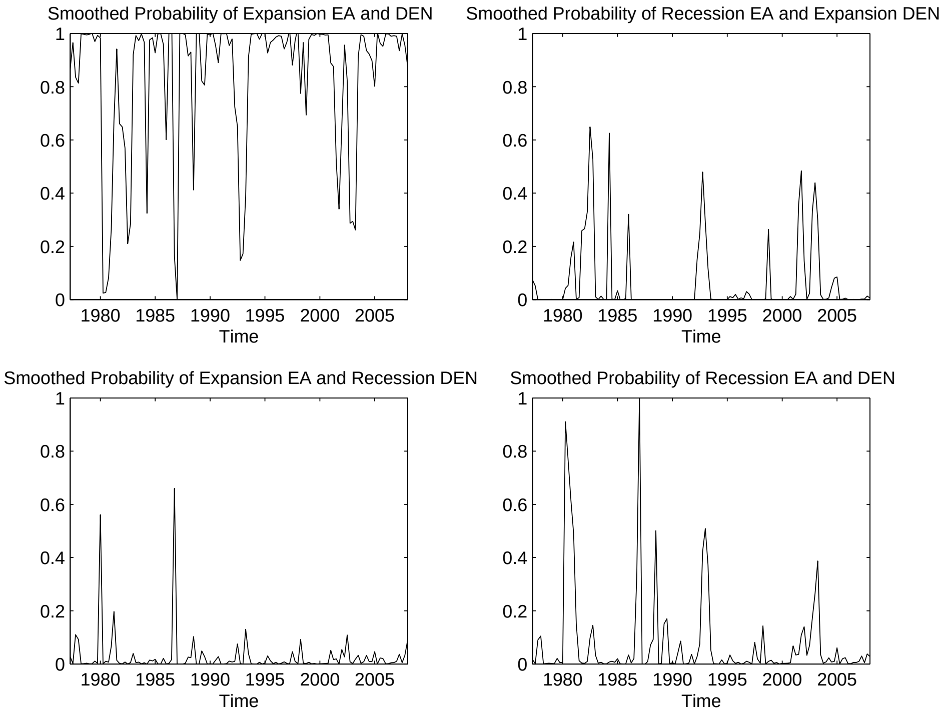


Figure 80: Smoothed Multivariate Probabilities for Bivariate Model EA-FIN

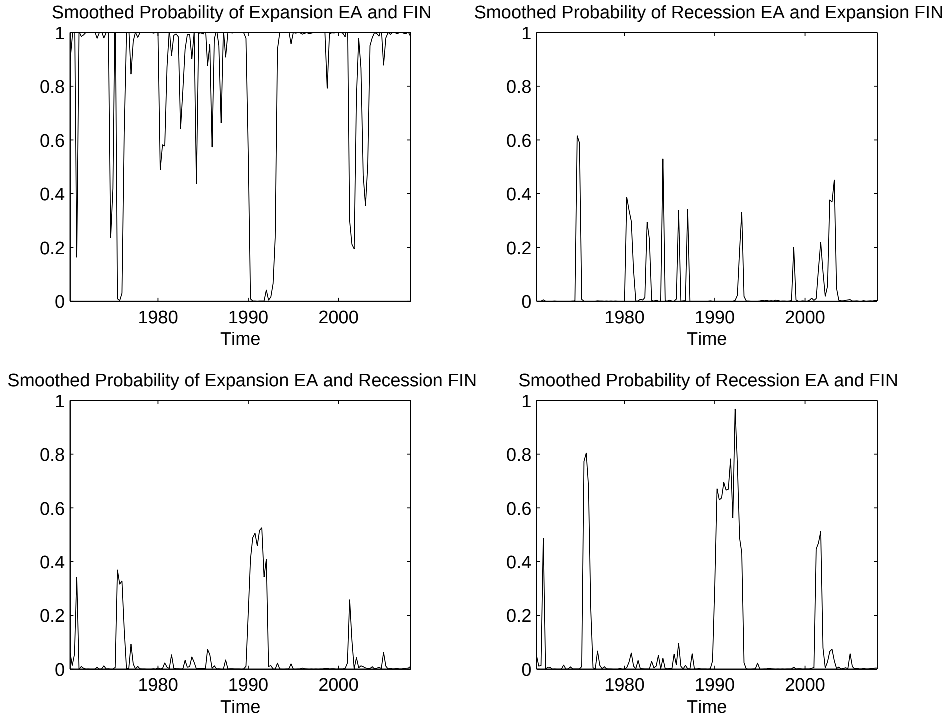


Figure 81: Smoothed Multivariate Probabilities for Bivariate Model EA-FR

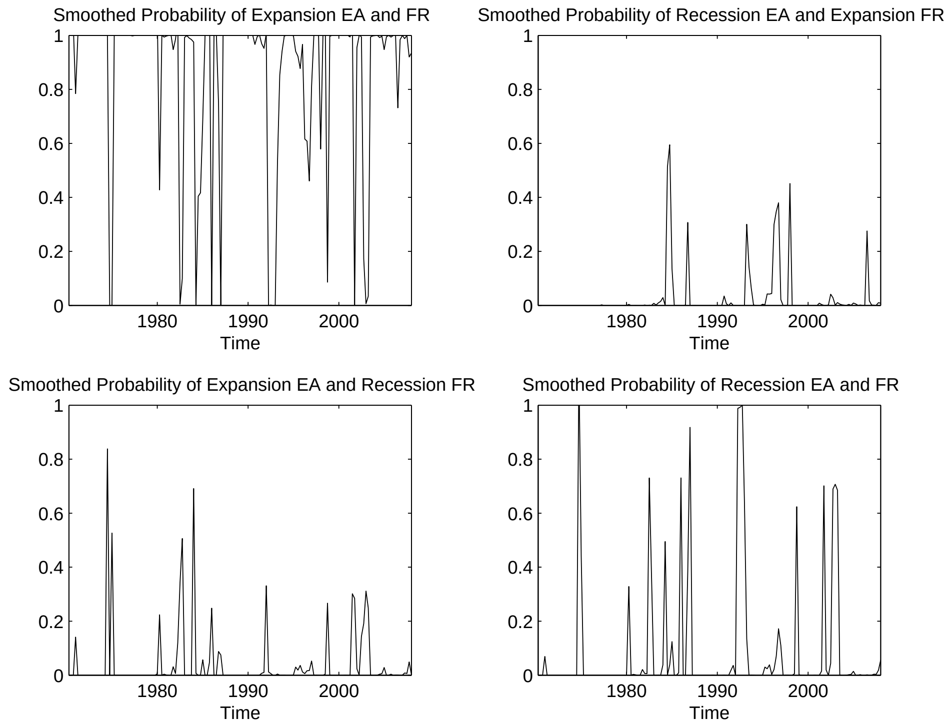


Figure 82: Smoothed Multivariate Probabilities for Bivariate Model EA-GER

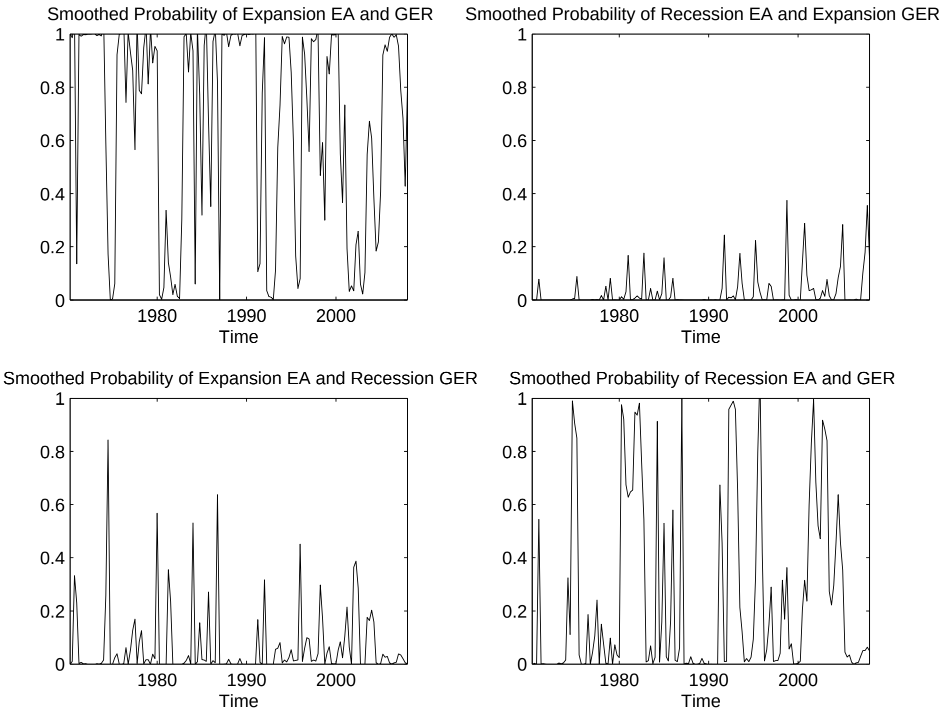


Figure 83: Smoothed Multivariate Probabilities for Bivariate Model EA-GREE

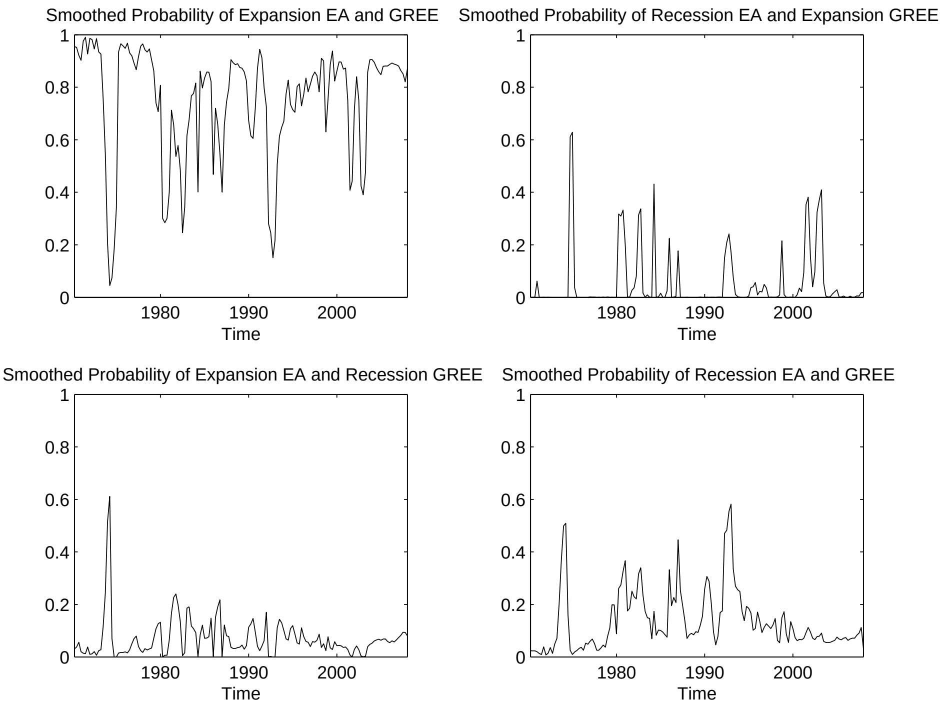


Figure 84: Smoothed Multivariate Probabilities for Bivariate Model EA-IT

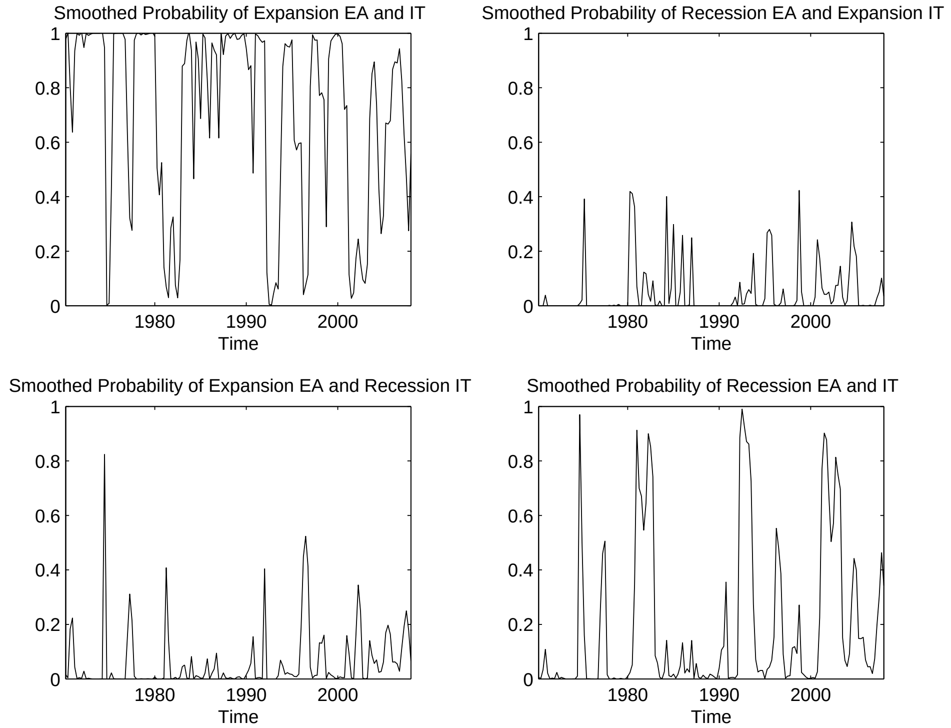


Figure 85: Smoothed Multivariate Probabilities for Bivariate Model EA-JP

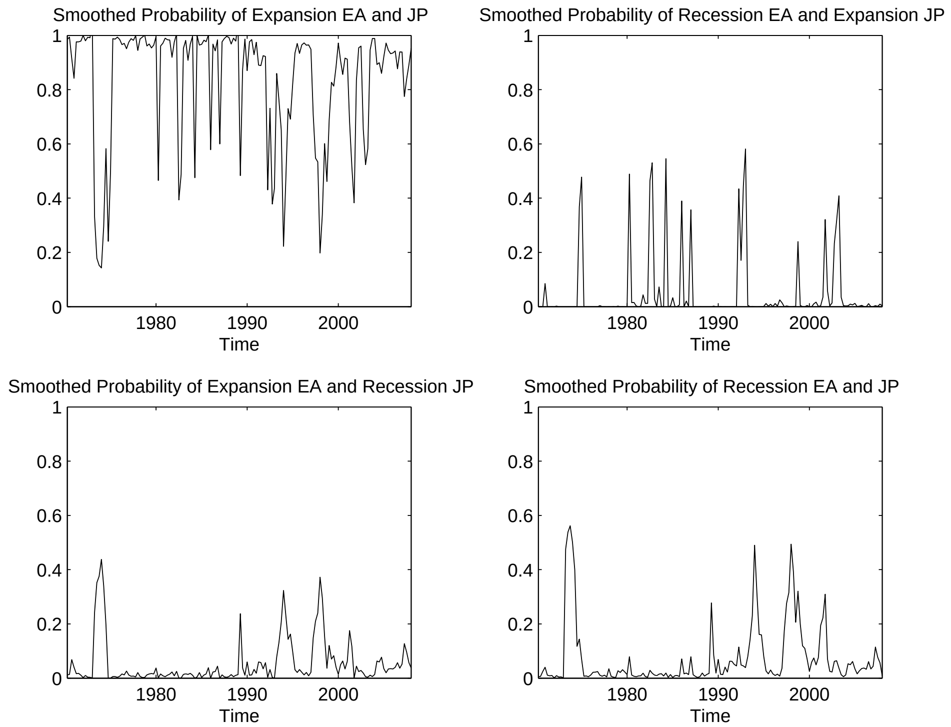


Figure 86: Smoothed Multivariate Probabilities for Bivariate Model EA-NRW

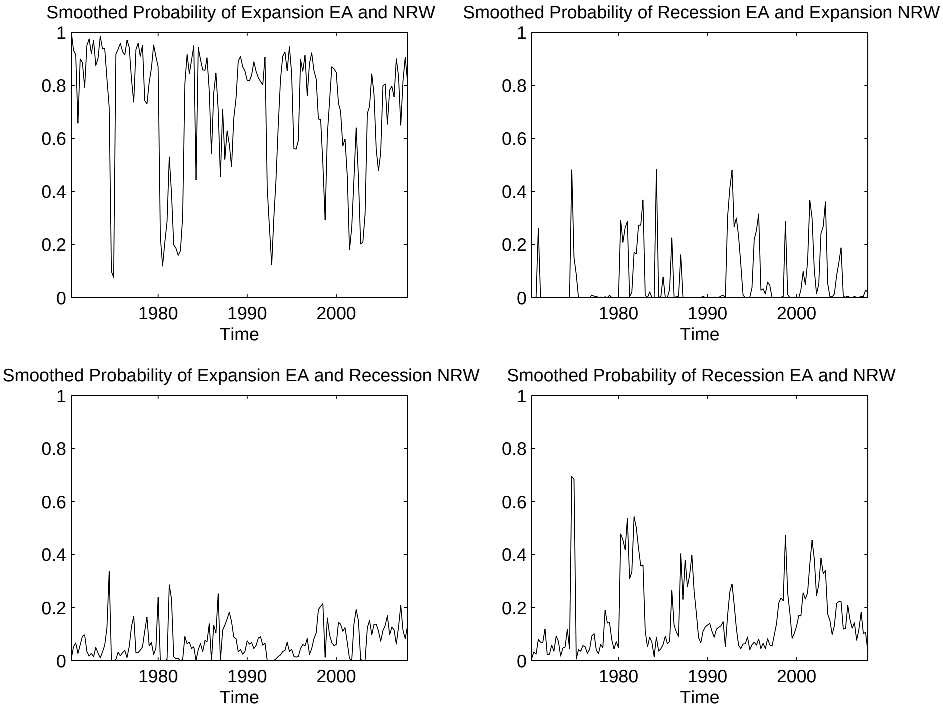


Figure 87: Smoothed Multivariate Probabilities for Bivariate Model EA-NTH

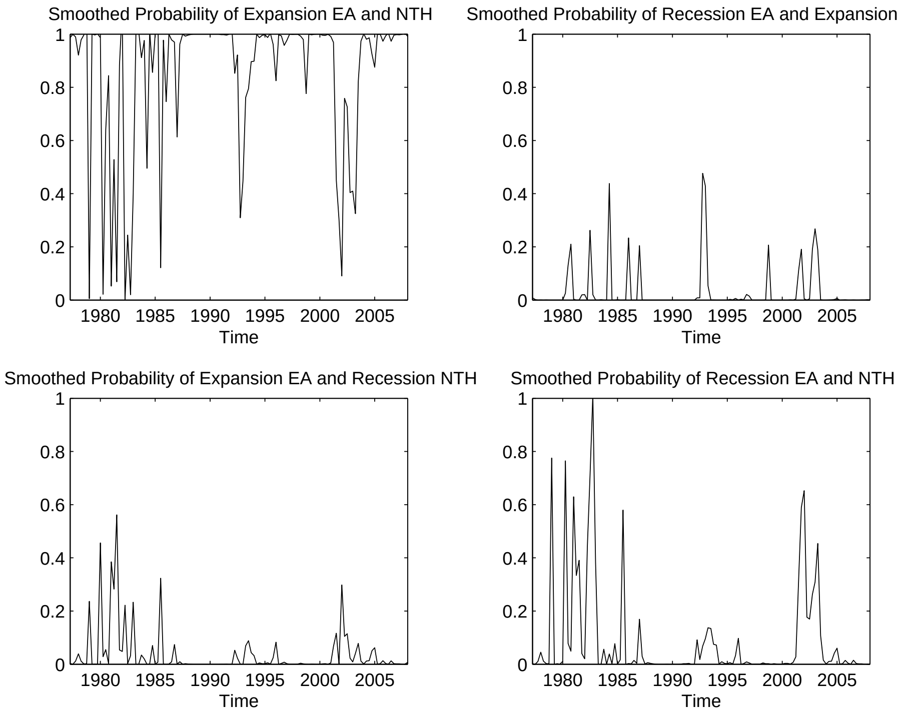


Figure 88: Smoothed Multivariate Probabilities for Bivariate Model EA-PT

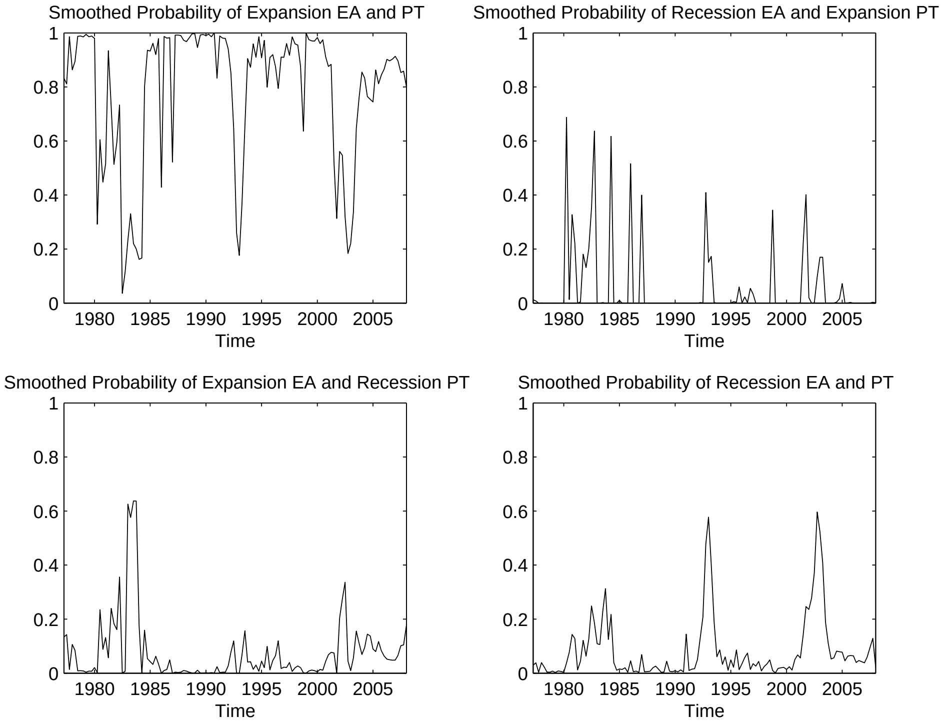


Figure 89: Smoothed Multivariate Probabilities for Bivariate Model EA-SP

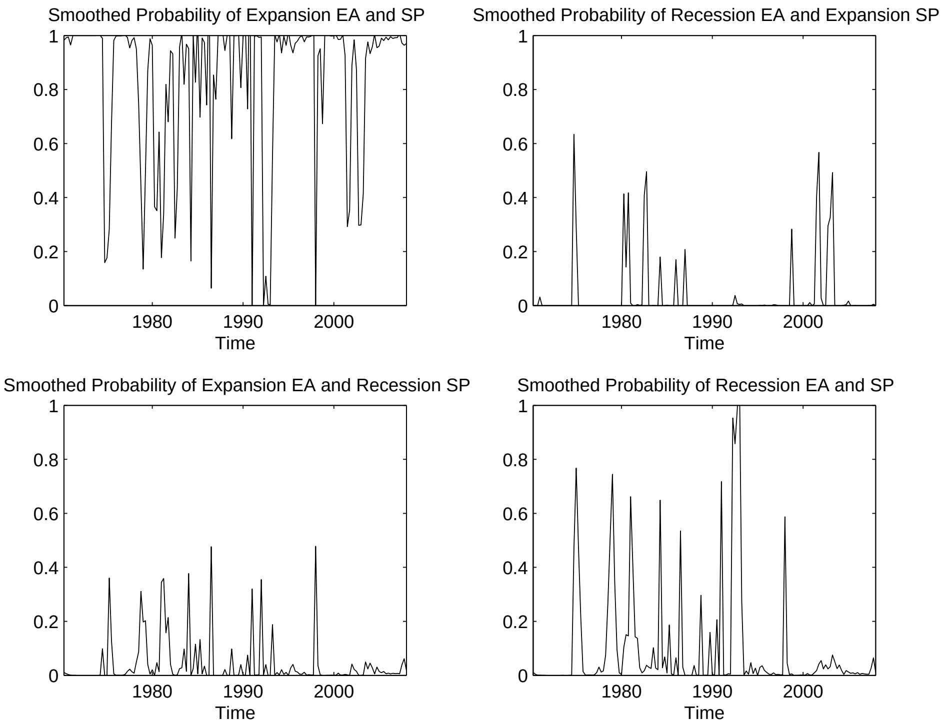


Figure 90: Smoothed Multivariate Probabilities for Bivariate Model EA-SWE

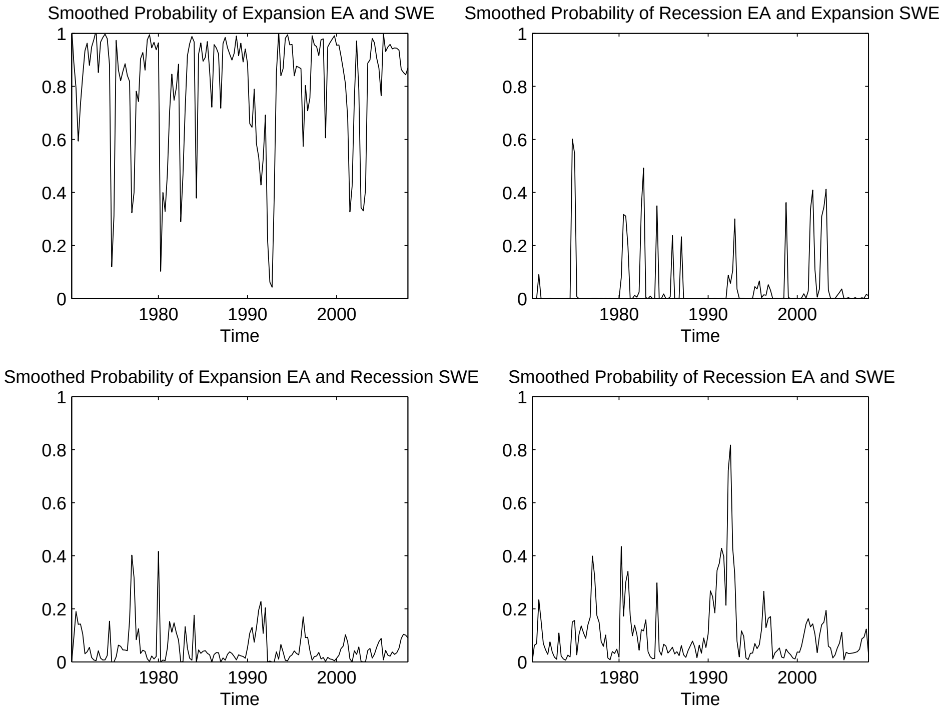


Figure 91: Smoothed Multivariate Probabilities for Bivariate Model EA-SWITZ

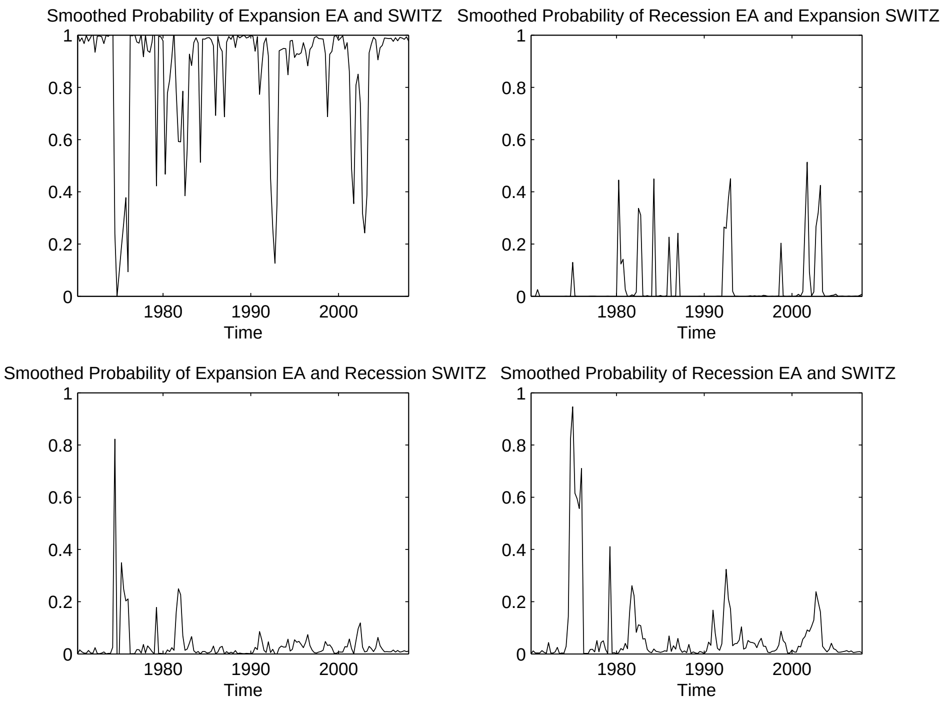


Figure 92: Smoothed Multivariate Probabilities for Bivariate Model EA-UK

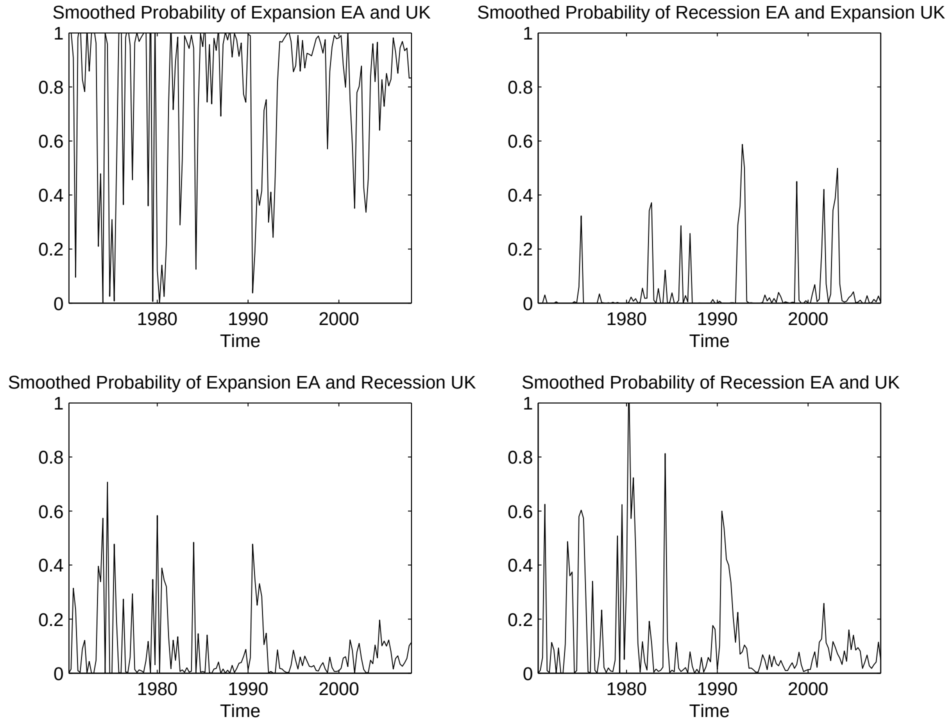


Figure 93: Smoothed Multivariate Probabilities for Bivariate Model EA-US

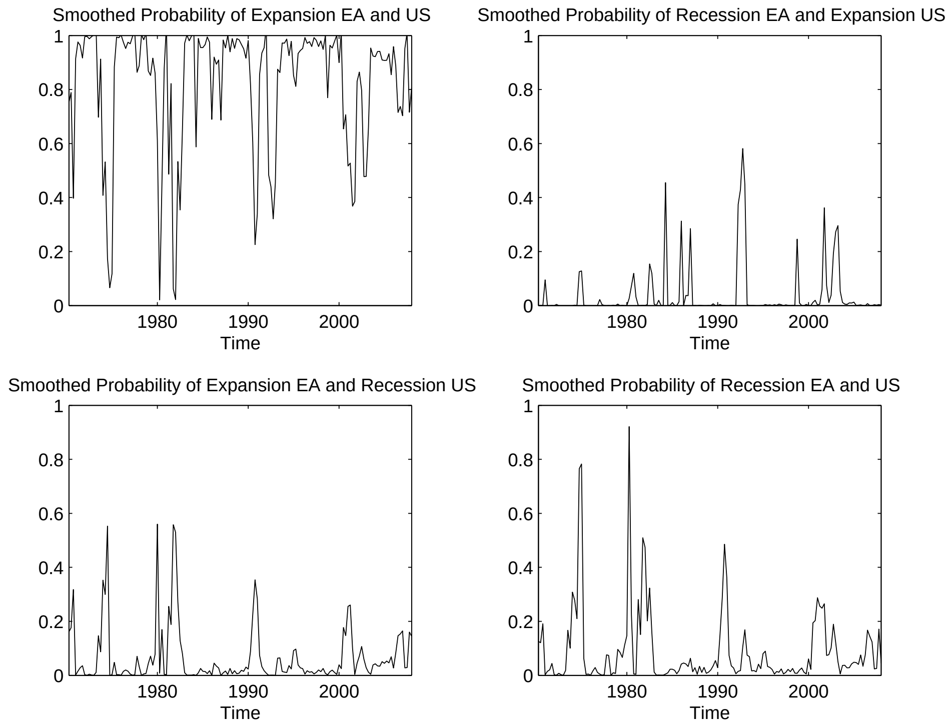


Figure 94: Smoothed Multivariate Probabilities for Bivariate Model AUS-US

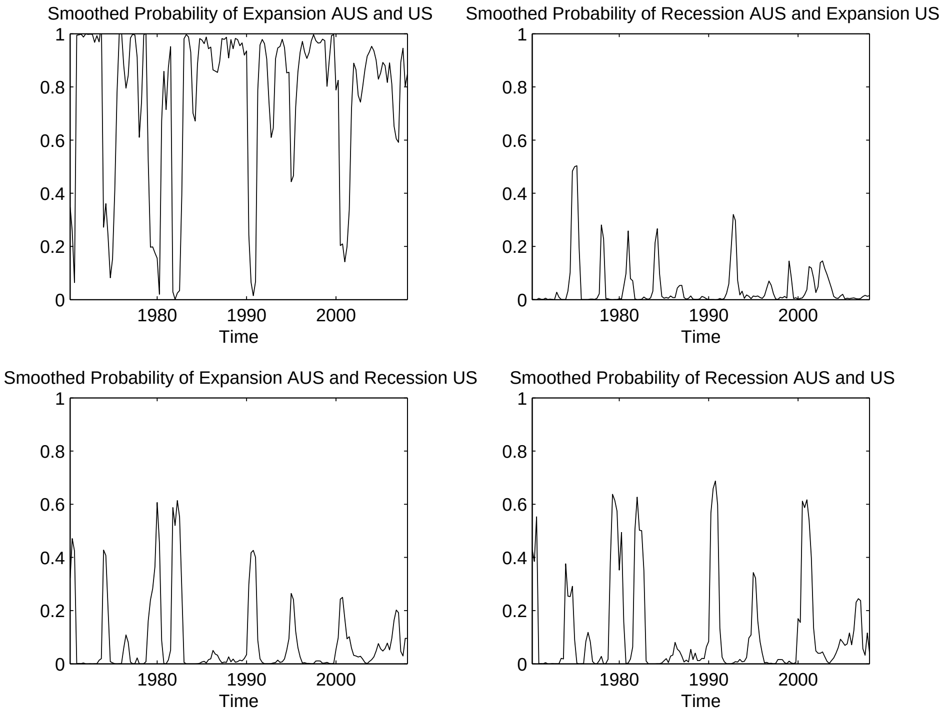


Figure 95: Smoothed Multivariate Probabilities for Bivariate Model BGM-US

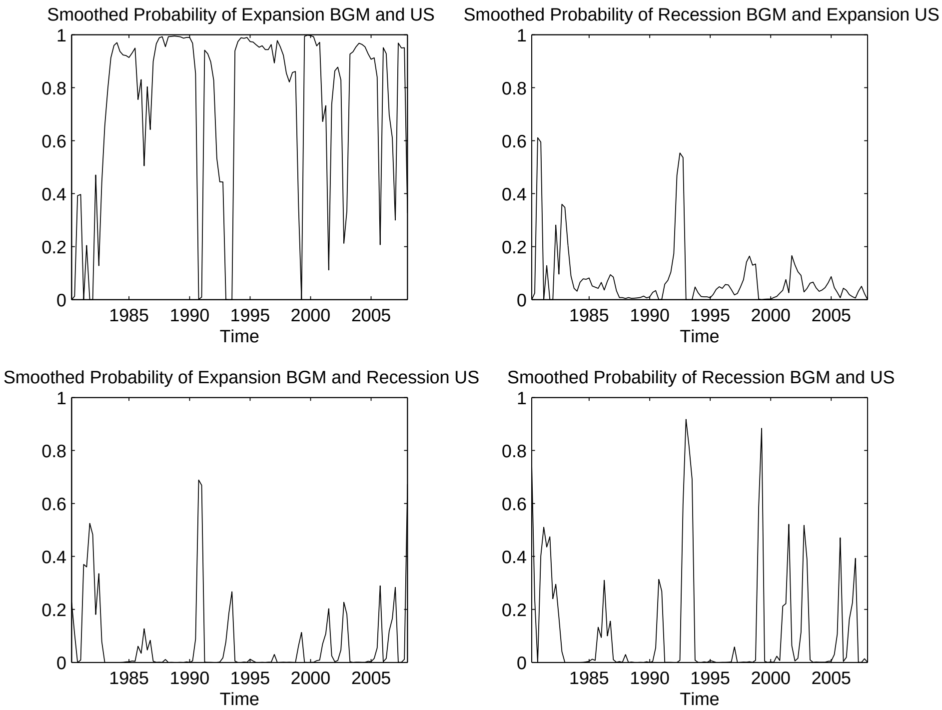


Figure 96: Smoothed Multivariate Probabilities for Bivariate Model CND-US

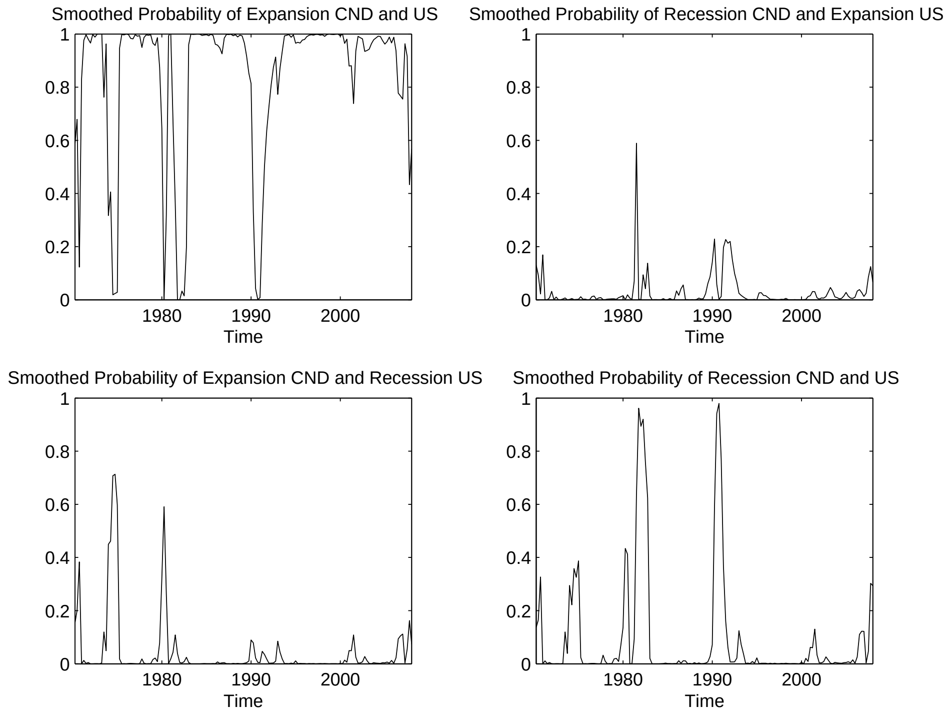


Figure 97: Smoothed Multivariate Probabilities for Bivariate Model DEN-US

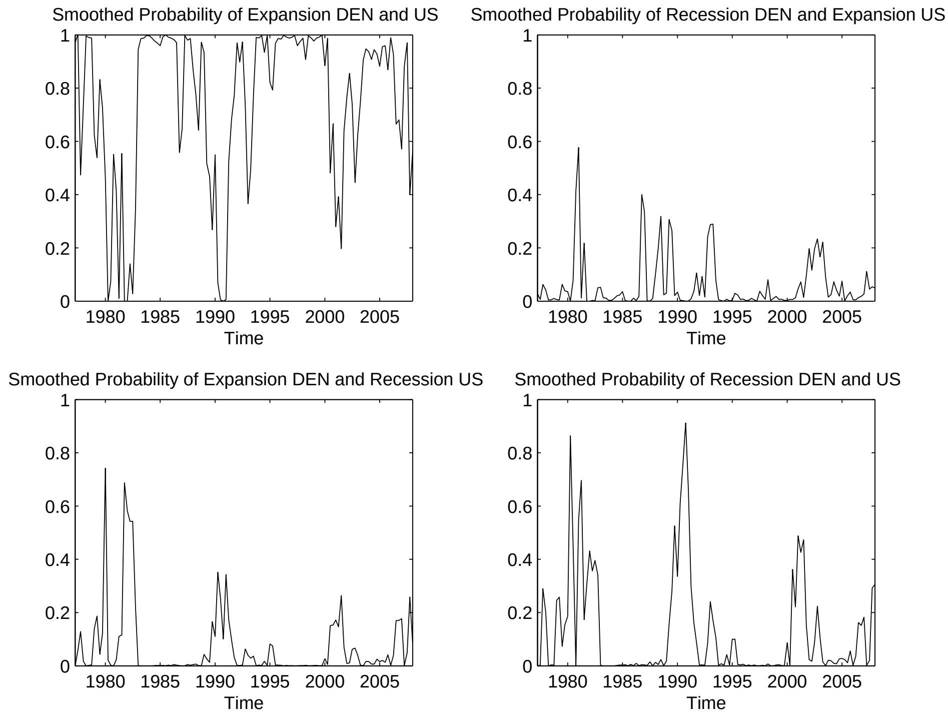


Figure 98: Smoothed Multivariate Probabilities for Bivariate Model FIN-US

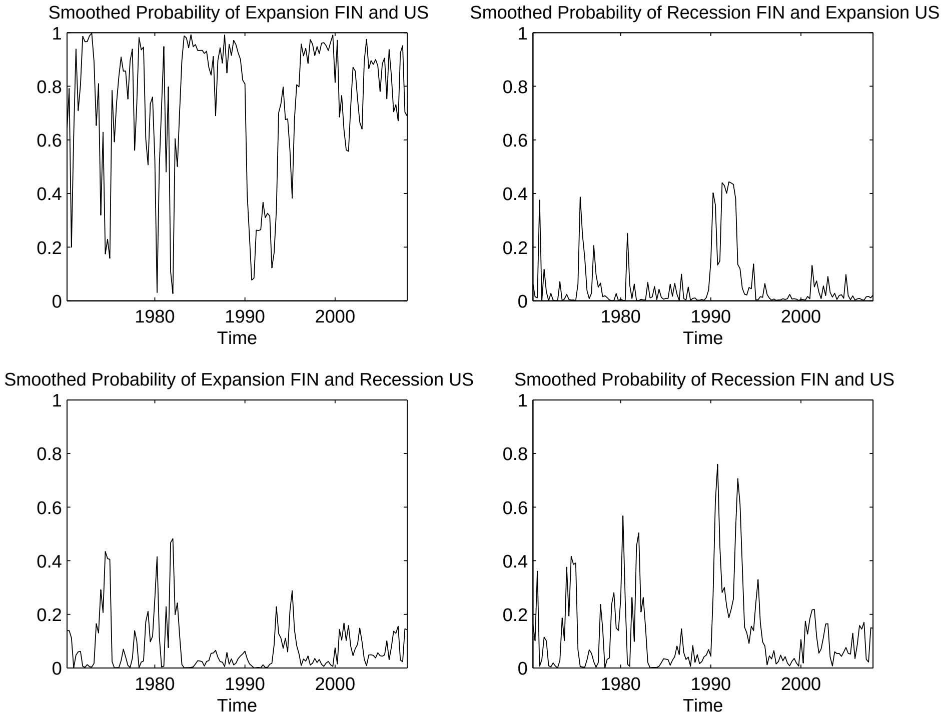


Figure 99: Smoothed Multivariate Probabilities for Bivariate Model FR-US

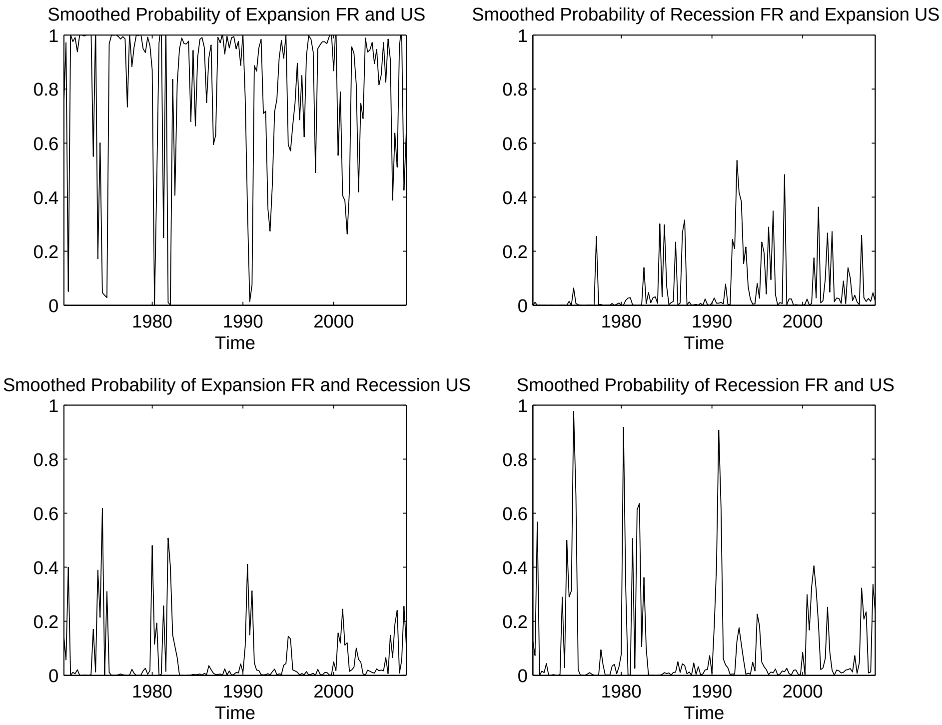


Figure 100: Smoothed Multivariate Probabilities for Bivariate Model GER-US

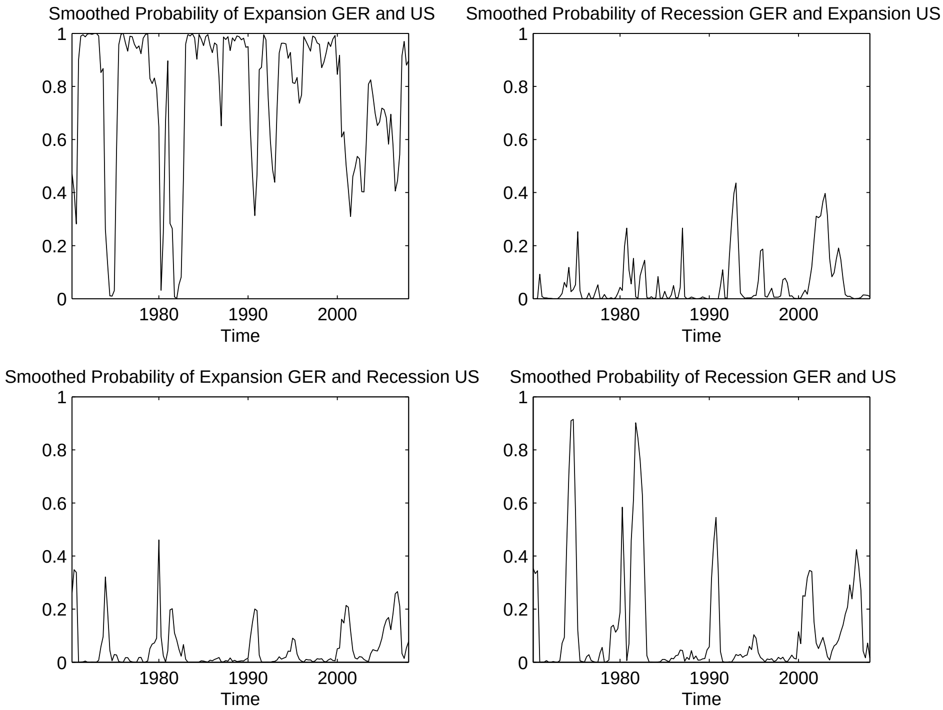


Figure 101: Smoothed Multivariate Probabilities for Bivariate Model GREE-US

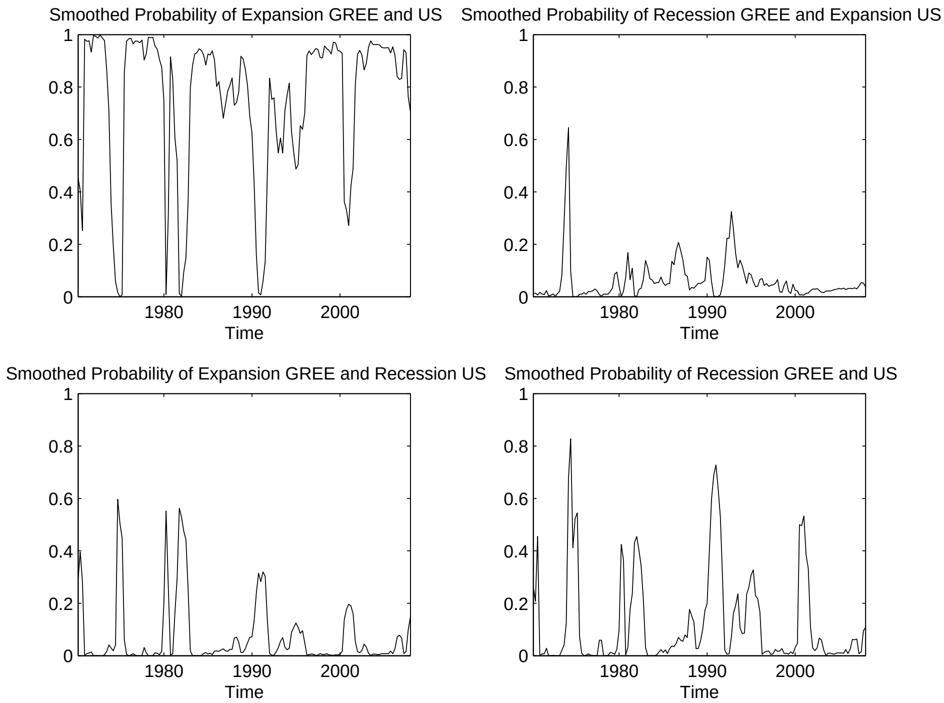


Figure 102: Smoothed Multivariate Probabilities for Bivariate Model IT-US

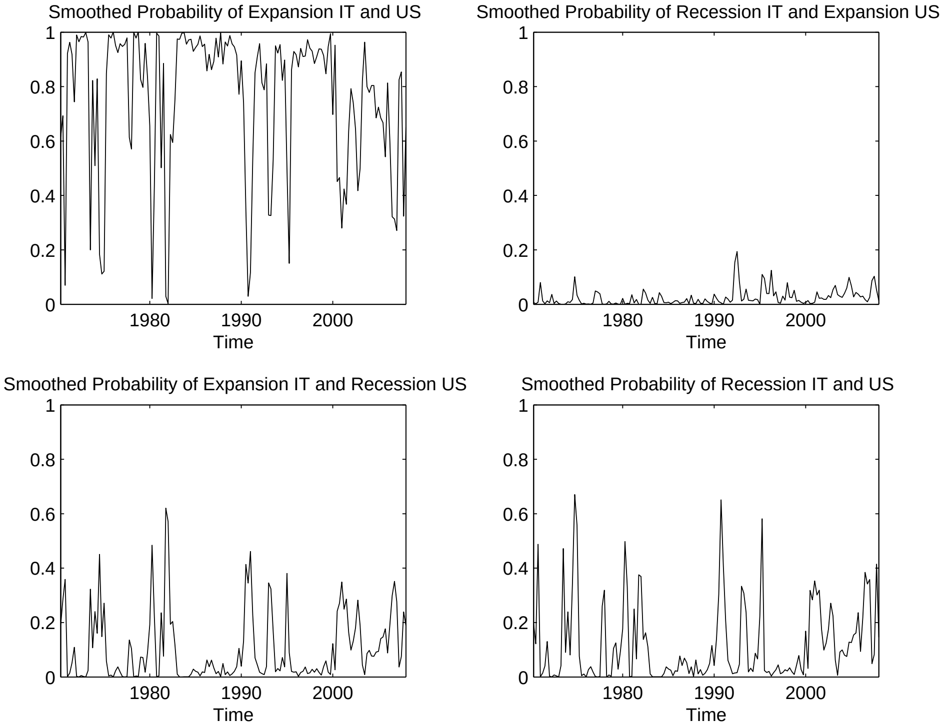


Figure 103: Smoothed Multivariate Probabilities for Bivariate Model JP-US

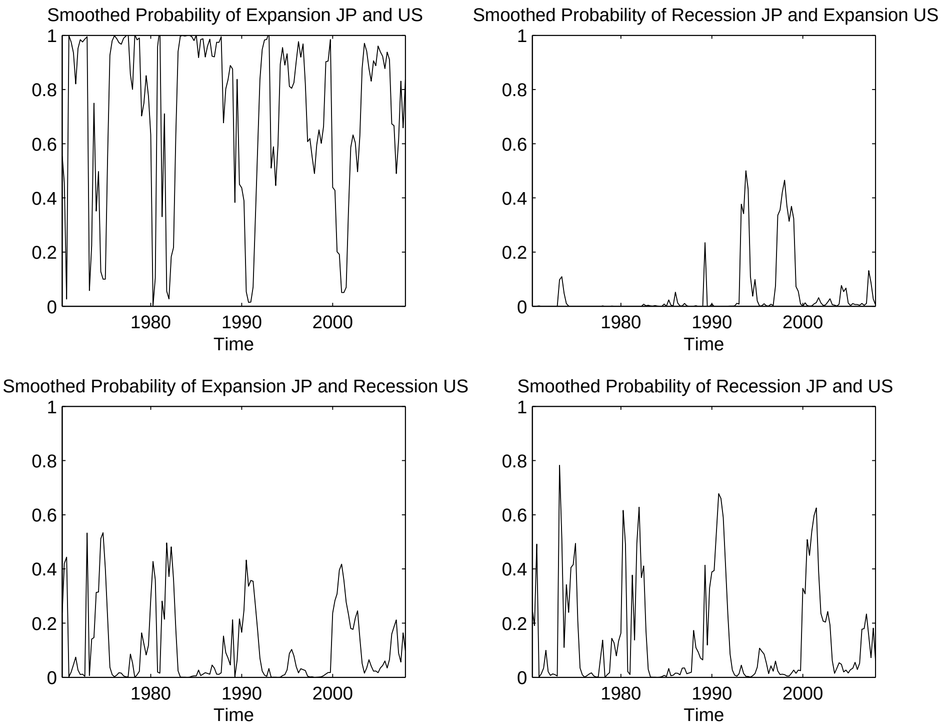


Figure 104: Smoothed Multivariate Probabilities for Bivariate Model NRW-US

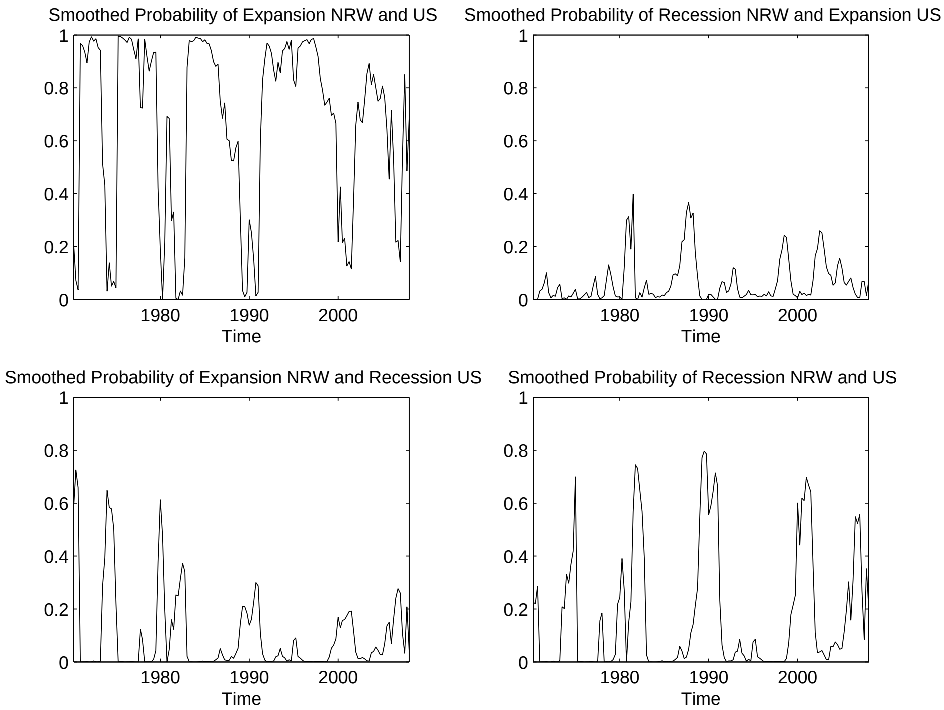


Figure 105: Smoothed Multivariate Probabilities for Bivariate Model NTH-US

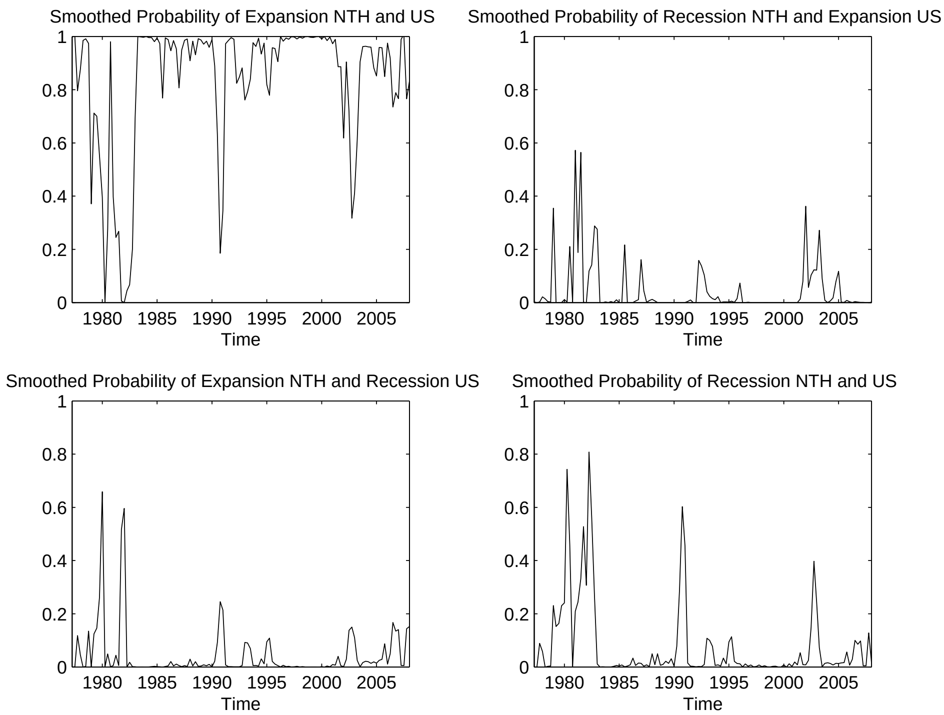


Figure 106: Smoothed Multivariate Probabilities for Bivariate Model PT-US

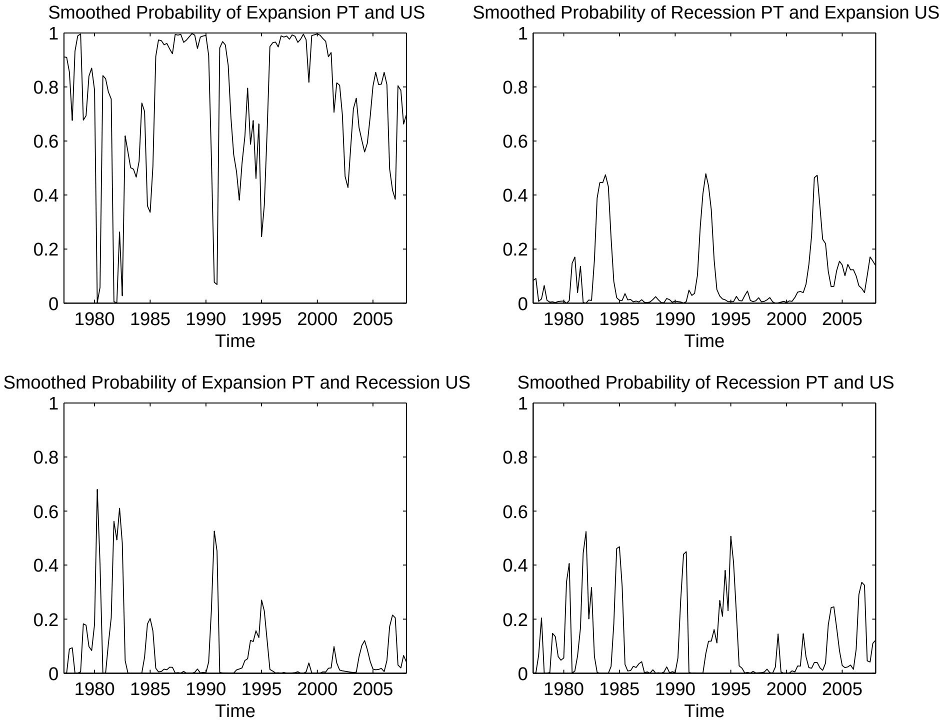


Figure 107: Smoothed Multivariate Probabilities for Bivariate Model SP-US

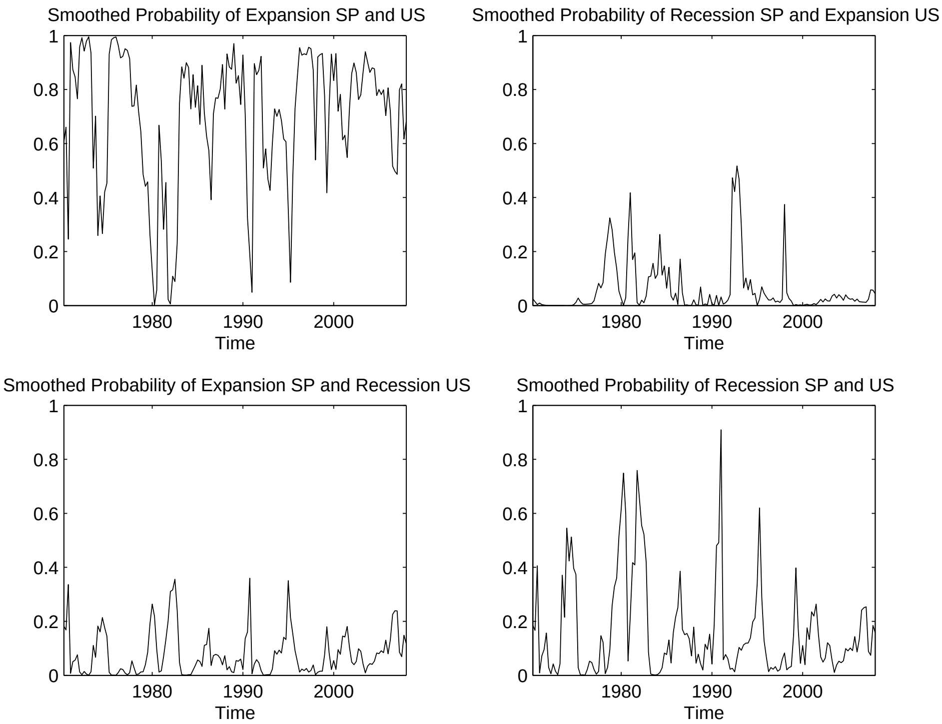


Figure 108: Smoothed Multivariate Probabilities for Bivariate Model SWE-US

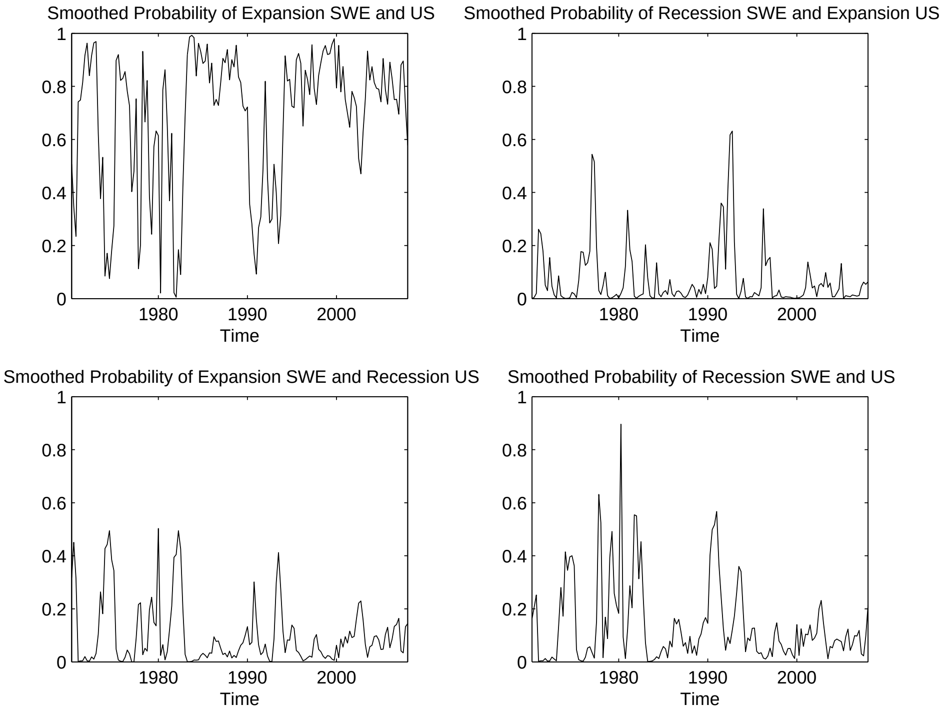


Figure 109: Smoothed Multivariate Probabilities for Bivariate Model SWITZ-US

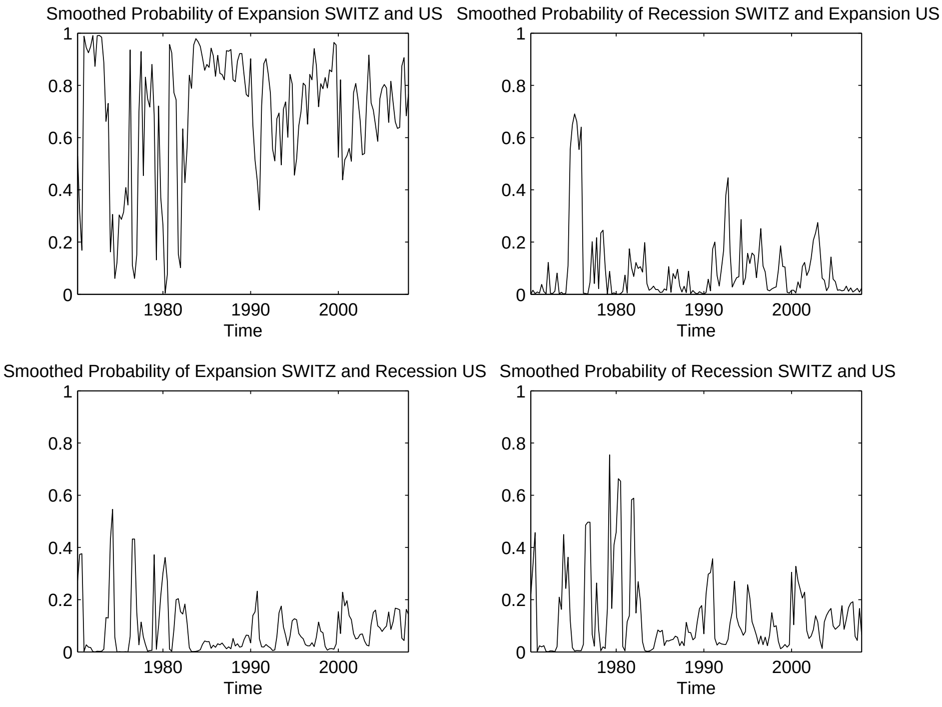


Figure 110: Smoothed Multivariate Probabilities for Bivariate Model UK-US