Lancaster University Management School
Working Paper
2006/048

Survival of the small firm and the entrepreneur under demand and efficiency uncertainty

Manuel Flores-Romero

Institute for Entrepreneurship and Enterprise Development
Lancaster University Management School
Lancaster LA1 4YX
UK

The LUMS Working Papers series can be accessed at http://www.lums.lancs.ac.uk/publications/
LUMS home page: http://www.lums.lancs.ac.uk/

Survival of the Small Firm and the Entrepreneur under Demand and Efficiency Uncertainty*

Manuel Flores†

September 2006

Abstract

The objective of this paper is to offer an answer to the question: why do some entrepreneurs own another firm after having closed an unsuccessful one? We present evidence that in the UK almost 44% of entrepreneurs own another firm, despite a previous unsuccessful experience. We develop an model to answer our question which predicts the existence of a mechanism of entrepreneurial selection in which entrepreneurs with talent above a individual-specific threshold are the only ones who own another firm in the future after having closed one, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs switch to waged work. We find that the lower the skills threshold is, the larger the probability of survival of the entrepreneur is. Moreover, low opportunities in the labour market, larger intrinsic tastes for entrepreneurship, longer periods in self-employment, over optimism, and a less risk averse attitude affect negatively the skills threshold and hence have a positive effect on the entrepreneur's survival.

Keywords: Small Firm, Entrepreneur, Survival, Entrepreneurial Skills

JEL Classification: L26, M13, J24, L25

*I want to thank Michael Waterson and Keith Cowling for their guidance and help, and to seminar participants at the University of Warwick, the 2004 Royal Economic Society conference, and the 2004 European Association for Research in Industrial Economics conference for their comments. I am also grateful to David Storey for his help. The data was kindly provided by Robert Blackburn from Kingston University to whom I am indebted. Funding from CONACYT Mexico is also acknowledged. Errors in this document are my exclusive responsibility.

†Institute for Entrepreneurship and Enterprise Development, Lancaster University Management School, LA1 4YW, Lancaster, UK. Tel: +44 1524 594860 E-mail: m.flores-romero@lancaster.ac.uk

1 Introduction

Why do some small firm owners start or buy a new firm after having closed one that was not successful? In this paper we try to answer this question. Why is this question relevant? Firstly, because straightforward intuition provides two conflicting hypotheses as answers. On one hand, we can argue that the small firm owner has learned from his mistakes and believes that in the following venture he can do better. This suggests the entrepreneur believes he has the potential talent to run successful firms, he was just unlucky in his previous venture. On the other hand, we can argue that the lack of success in his previous venture is a reflection on his lack skills to run a successful firm. If he does not have what it takes, why would he want to start a new firm? Secondly, the question is pertinent because we show that the decision to create a new firm following the closure of an unsuccessful one is a rather common occurrence amongst entrepreneurs. Utilising a survey of entrepreneurs in the United Kingdom who have recently experienced business closure, we show that almost 44% of these entrepreneurs start or buy a new firm, despite the failure of their previous venture. Do these entrepreneurs have the talent to run successful firms? Are they being irrational by hoping they will do better the next time? Neither questions cannot be answered affirmatively, but we believe answers one way or another will constitute findings which should be considered by policy makers interested in the encouragement of start-ups and promotion of entrepreneurial activity. Hence, the importance of tackling our initial question.

Our question is also relevant in the context of research on small firms. In the Economics literature, interest in research on small firms has been increasing in recent years. This can be regarded as a consequence of the recognition during the 80's and onwards that small firms account for a considerable share of economic activity (Acs and Audretsch 1990). In spite of the growing importance of small firms in the economy, it has been found that they consistently have a lower likelihood of survival than larger firms do, a finding which is now regarded as a stylised result in small firm research (Geroski 1995, Storey and Wynarczyk 1996). In the light of these results, efforts have been directed to unraveling the determinants of small firm survival (Bates 1990, Audretsch 1991, Kalleberg and Leicht 1991, Mata and Portugal 1994, Storey and Wynarczyk 1996). Despite the fact that in some of this research the role the entrepreneur plays as owner and manager has been acknowledged to some degree, the survival of the entrepreneur has been ignored. In other words, no research has examined the outcome of the decision between remaining in entrepreneurship by starting another firm after closure of the previous one, or starting work as an employee. Gladstone and Lee (1995, p. 27) on discussing the impact of the insolvency system in the UK on entrepreneurialism assert that '...there is still little or no concomitant recognition of, or accompanying policy, to rehabilitate entrepreneurs forced out of entrepreneurialism, even though they are a vital part of the competitive cycle. The impact of failure on the individual entrepreneur has been of minimal interest to most theorists...' Therefore, by trying to answer why some small firm owners wish to start a new firm after having closed an unsuccessful one, we also contribute to the efforts aimed at understanding the processes which determine the survival of small firms, and we also attempt to reduce the gap in the study of the forces which influence the survival of the entrepreneur.

The paper has the following structure. In the next section we present empirical evidence on entrepreneurial survival which shows the relevance of our initial question. Section 3 discusses some of the most influential theoretical models of firm survival and what they tell us about the survival of the entrepreneur. In Section 4 the model and assumptions we use to try to answer our initial question are introduced. In Section 5 we introduce an alternative model whose purpose is to provide a benchmark case for the predictions obtained by our general model. In this section we also present the predictions offered by our general model and by the alternative one, and we discuss the answers both models offer to our initial question. We found that our general model is successful in this task, whilst the alternative model fails. In Section 6 we extend our model to acknowledge the influence that opportunity costs in the labour market, intrinsic tastes for entrepreneurship, time-spent in self-employment, optimism, and risk aversion have on the survival of the entrepreneur. Finally, Section 7 summarises our findings.

2 Empirical evidence on the survival of the entrepreneur

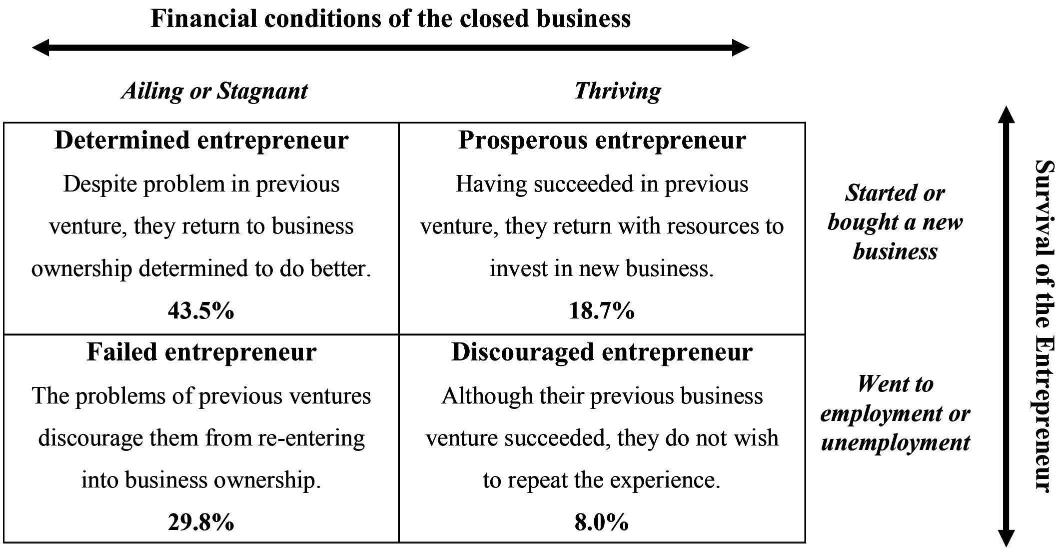
We employ the database Business Closures assembled by Kingston University to provide empirical evidence on the survival of the entrepreneur. This database is the result of a study carried out in the United Kingdom in 2000 with the objective of gathering information from a sample of owner-managers who had recently closed their businesses. The results of the study are impressive inasmuch as they provide evidence of the entrepreneurs' decision either to stick with entrepreneurship or adopt other alternatives, once the experience of business closure has occurred. Stokes and Blackburn (2001) explain that the research project comprised, as a first stage, the elaboration of the questionnaire to be used to interview the small firm owners. This questionnaire was built from interviews with 20 organisations of different types, including private organisations (e.g. banks, accountants), non-profit organisations, and government consultants, who had experience in advising small business. In a second stage, the questionnaire was sent to a sample of 2719 owner-managers who had recently closed their businesses, from which 380 usable observations were obtained. According to Stokes and Blackburn (2001), the sampling frame draws on a variety of sources which 'sought to be as representative as possible of the range of business closures and exit types, rather than for example, relying on official receivers of data' (p. 6). The usable responses were obtained from customers of the HSBC bank who closed a business account (36%), from Dun and Bradstreet lists of existing business (5%), and from closures identified during research on other projects by the Small Business Research Centre of Kingston University (59%).

An initial analysis of the data shows that 61% of the entrepreneurs in the sample remained in entrepreneurship after having experienced business closure, as opposed to 26% who opted for paid employment, 9% who retired, and 4% who joined other activities such as full-time education or travelling. Those who stayed in entrepreneurship are classified into those who started or bought a business again, 44%, and those who continued in an existing business, 17%. The latter category refers to the entrepreneurs, defined in the specialised literature as *portfolio entrepreneurs,* who were running several firms at the same time, and who after the closure of one business continued in entrepreneurial activities by running their other firms. However, given our initial question we are interested in the 44% share of entrepreneurs who started a new business after the closure of a previous one. In the specialised literature they are called *serial entrepreneurs*, and are defined as entrepreneur who run a succession of firms, one at a time. The relevance of this finding is that, to the best of my knowledge, serious research on the fate of the entrepreneur after a business closure had not been undertaken until the study on Business Closures by Kingston University. Similarly, these figures contribute to the growing body of evidence which rejects the idea that entrepreneurship is a *one time event*, with reference to the characteristic behaviour of serial and portfolio entrepreneurs who run several firms in succession or simultaneously (Macmillan 1986, Birley and Westhead 1993, Rosa 1998, Westhead and Wright 1998, Westhead et al 2003, Ucbasaran 2004).

However, this evidence does not offer any information on whether the closed business was successful or not. To amend this situation Figure 1 is adapted from Stokes and Blackburn's figure "A matrix of owners' responses to business closure" (2001, p. 17). It classifies the entrepreneurs into four quadrants, depending on the owner's view of whether the business was financially ailing or stagnant as opposed to thriving at the time of closure[1], and on whether the owner survived in entrepreneurship or not. Here we define *survival of the entrepreneur*,

[1] Information taken from the question: "How would you describe the financial situation of the business at the time you left or the business was closed?" where the option for answer was a scale from 1 (most ailing) to 5 (most thriving). We defined answers 4 and 5 as "Thriving".

Figure 1: A matrix of owners' responses to business closure



as the situation in which the entrepreneur, in the face of a business closure, remains in entrepreneurship by starting or buying a new firm instead of selecting waged employment[2]. Each quadrant also shows the percentage of owners who fit into each category. The top row of Figure 1 includes those entrepreneurs who started a new firm or bought an existing one, that is, those individuals who continued as serial entrepreneurs, as opposed to those in the bottom row who left entrepreneurship. The left column comprises those entrepreneurs who saw their firms as financially ailing or stagnant at the time of closure. Common sense would suggest that a firm's success is related to its financial performance. We can argue then that the left column shows those entrepreneurs who were running unsuccessful firms, contrary to those in the left column who regarded their firms as successful[3]. The top-left quadrant therefore represents those entrepreneurs who started a new firm, even though their previous

[2] Given our definition of *survival of the entrepreneur*, in this figure we leave out the share of entrepreneurs who continued other existing firms, went to retirement after the business closure, or engaged in other activities. The final sample comprises 262 observations.

[3] It has been widely documented that entrepreneurs are overly optimistic (Cooper et al 1988, Parker 2006). Furthermore, in the face of a business closure, the entrepreneur might be inclined to be even more upbeat about the performance of the closed firm. This would suggest that the share of entrepreneurs who regard their ventures as thriving in Figure **??** may be biased upwards. In any case, our interest, as explained later on, focuses on the *Determined entrepreneurs*, who had ailing or stagnant businesses. If anything, their share would be under represented, which would lend more support for the relevance of this study.

venture was unsuccessful. These are the *determined entrepreneurs* who represent 43.5% of the sample, and in fact they are the largest group of the four. Figure 1 also shows the other three possibilities: those who had successful firms and started or bought a new firm, the *prosperous entrepreneurs*; those who despite having successful firms did not want to remain in entrepreneurship, the *discouraged entrepreneurs*; those who were running unsuccessful firms, and changed to employment or unemployment afterwards, the *failed entrepreneurs*. Interestingly, the group of discouraged entrepreneurs represents the antagonistic case to the one we try to explain. Why did these individuals leave entrepreneurship if they had the experience of running a successful business? Is not this proof enough that they are skilful entrepreneurs? As will be shown, in our attempt to explain the reasons which drive the determined entrepreneurs to start a new firm despite not being successful in the previous one, we also try to explain the other three possibilities included in Figure 1.

3 Previous work

One of our interests is to present a case in favour of separating and differentiating between the small firm and the entrepreneur. In particular, we stress the fact that the survival of the small firm is different from the survival of the entrepreneur. The latter is defined as the outcome of starting a new firm after a business closure instead of switching to employment. Small firms are run by entrepreneurs. A small firm does not survive when it stops trading. However, this does not necessarily mean that the entrepreneur does not survive, as he can stay in entrepreneurship by starting or buying other firms. The issue is not trivial: as we have shown, evidence for the UK suggest that a significant amount of entrepreneurs return to entrepreneurship by starting a new firm or buying an existing one. Moreover, most of them engage in the new venture despite having an unsuccessful experience in the previous one. However, theoretical and empirical research in the Economics discipline has tended to focus on the one hand on investigating the small firm and the determinants of its survival, and on the other on the self-employed individual, i.e. the entrepreneur, and the factors associated with his decision to exit self-employment. Moreover, sometimes the firm and the entrepreneur have been used interchangeably, with the inaccurate implication that the survival of the small firm and the survival of the individual in self-employment are one and the same thing. No study has encompassed an integrated approach fully recognising the differences: to study the entrepreneur's decision to stay with or close his firm – the survival of the small firm – and if the firm closes down to investigate his decision to start a new firm or join employment – the survival of the entrepreneur.

With respect to theoretical studies on the survival of small firms, probably one of the earliest and most influential works is Jovanovic's (1982). He proposes a model in which firms start small and do not know their true level of efficiency, but learn about it as they age. As a result, efficient firms grow and survive while inefficient firms exit the market. Parker (2004, p. 211-212) enumerates eight implications which can be derived from this model for entrepreneurial ability, firm growth and survival, industry concentration, profits and output. One of the predictions of this model which has received some of the strongest empirical support is on survival rates with relation to firm size and firm age: young and small firms are less likely to survive compared to old and large firms, respectively. In Jovanovic (1982) the efficiency levels of the small firm can be thought of as representing the skills of the entrepreneur, which ultimately determine the survival and growth of the firm. This has had a significant influence on subsequent work which has modelled the entry and exit of small firms or entrepreneurs, as it became customary to assume that the profits of the small firm depended on the ability of the owner-manager, i.e., the entrepreneur (Frank 1988, Evans and Jovanovic 1989, Holmes and Schmitz 1990 1996, Cressy 1996).

Frank (1988) was the first one to explicitly model the profits of the small firm as a function of the entrepreneur's ability. In an analogous way to Jovanovic (1982), Frank assumes that entrepreneurs do not know their endowment of skills before entering with a firm, and as the firm spends more periods in the market, the entrepreneur learns about his skills. He also includes effort as a determinant of the firms profits, and as a determinant of the entrepreneur's own utility function: more effort leads to higher firm revenue thereby increasing the entrepreneur's utility, but it also increases dissatisfaction, affecting the entrepreneur's utility. One predictions of Frank's model stands out. The interplay between the entrepreneur's expected ability, the learning process of his true ability, and entry sunk costs provide an explanation for the *liability of adolescence* (Fichman and Levinthal 1991), in which exit rates by new firms are initially low in the few periods after entry, then are followed by a steep increase, and finally falter as time goes by. Given the level on the prior expectation of ability, larger sunk costs imply that the entrepreneur will wait longer for sufficiently bad results from the learning process before exiting. This can explain the low exit rates following entry. However, after this period is over exit rates peak drawing all inefficient entrepreneurs out of the market, which implies a subsequent reduction in the exit rates as only the skilful entrepreneurs remain.

Cressy (1996) proposes a model in which the human capital of the individual, that is his ability and skills, determines utility in self-employment and waged employment. He assumes that human capital is a positive concave function of the individuals age, and based on empirical evidence which suggests that self-employed individuals receive higher satisfaction from their jobs than wage workers do (Blanchflower and Oswald 1998), the model predicts that older entrepreneurs are more likely to stay in business compared to younger entrepreneurs. Despite the fact that in Cressy's model the subject of study is the entrepreneur not the firm, based on the latter prediction he builds and tests the hypothesis that there is a positive concave relationship between the entrepreneur's age and the survival of the small firm, overlooking the fact that even in the face of a firm's closure the individual can stay in business by starting or buying a new firm[4]. In fact, if in the models of Jovanovic (1982), Frank (1998), and Cressy (1996) firms are equated to entrepreneurs, one prediction that can be derived from them is that if the firm does not survive, the entrepreneur does not survive either, that is, he exits the market and never returns to entrepreneurship. By proposing a generalisation of such models in the Section 5 we formally demonstrate this[5]. This is completely at odds with the previously mentioned evidence that 62% of entrepreneurs who face a business closure return to entrepreneurship. Therefore, although models such as the ones in Jovanovic (1982) and Frank (1989) have shed light and improved our understanding on the survival of small firms, they are less effective when it comes to the survival of the entrepreneur.

4 The model

Individuals maximise their utility by choosing between working in a paid job or setting up and running a firm. The individual obtains utility $U = W$ when he is an employee and $U = \Pi + \xi$ when he is running his own firm, where W is the wage from the paid job and Π are the profits from running the firm. Following Blanchflower and Oswald (1998) $\xi \geq 0$ denotes the extra satisfaction to the individual for being his own boss. The individual knows his own ξ, which is constant over time, and has perfect foresight on future values of W. For the sake of simplicity we assume W remains constant. If the individual decides to run a firm, he will enter an industry characterised by monopolistic competition where the technological conditions of such industry and the phase of the product life cycle suggest incumbent firms are not affected by economies of scale, and as a result they do not need to grow to survive. The purpose of these assumptions is to simplify the analysis by allowing the firm to ignore

[4] Actually, the empirical evidence shows that age is positively related to both survival of the small firm (Bates 1990, Holmes and Schmitz 1996, Storey and Wynarczyck 1996, Gimeno et al 1997, Headd 2003) and survival in self-employment (Blanchflower and Meyer 1994, Holtz-Eakin et al 1994, Taylor 1999, van Praag 2003).

[5] Parker (2004) recognises this implication but only for the case of Jovanovic's model.

its competitors[6] and by eliminating the requirement to grow to survive found as an empirical regularity for small firms in the manufacturing sector[7].

In the small firm and entrepreneurship literature there is now a tradition to assume that the profits of the small firm depend on the talent or skills of its owner (see previous section). Hence, in our model we assume the profits of the small firm depend on the *efficiency* parameter, represented by θ^j. It denotes the skills of entrepreneur j to run a business, which are directly translated as the efficiency of the firm. A skilful entrepreneur would be represented by a high value in θ, which in turns means he is able to run his own firm more efficiently.

In contrast, several scholars in the entrepreneurship field assert that entrepreneurial activities are concerned with the process of discovery, evaluation, and exploitation of opportunities (Stevenson and Jarillo 1990, Shane and Venkataraman 2000). In fact, it has been suggested that the definition of entrepreneurship should encompass the creation of an organisation, for instance a small start-up, with the objective of pursuing an opportunity previously perceived (Bygrave and Hofer 1991). Accordingly, the entrepreneur's success and that of his organisation depend on the nature of the opportunity being exploited. However, the ability to successfully exploit an opportunity may or may not depend on the talent of the entrepreneur. Jennings and Beaver (1997) on discussing influences upon success potential for small firms point out that it is uncertain to what extent '...small business fail through lack of managerial skills...or through lack of competence in providing the product or service demanded by the customer, despite otherwise competent management in the organization' (p. 71). Several models have captured this notion and assumed that the profits of the small firm depend on the one hand on the skills of the entrepreneur to run the small business, and on the other on the nature of the opportunity being exploited which in some cases is contingent on entrepreneurial skills, and in others is an exogenous parameter. For instance, Holmes and Schmitz (1990) and Gifford (1993) assume in their models that there are two

[6] We use the definition of monopolistic competition proposed by Hart (1985). He points out, in the spirit of Chamberlin (1933), that monopolistic competition might be defined as one where there are many firms producing differentiated commodities, each firm is negligible so it can ignore its impact and reactions from other firms, and each firm faces a downward sloping demand curve.

[7] There is abundant literature which provides empirical support for the fact that small firms' presence and survival in the manufacturing sector depends on the technological conditions of the industry, suggesting that economies of scale play a role. See, for instance, White (1982), Acs and Audretsch (1989), Audretsch (1991), Mata and Portugal (1994), Geroski (1995). Interestingly, the very few studies that have looked at the service sector (e.g. Audretsch *et al* 1999), or at industries in the mature phase of the product life cycle (Agarwal and Audretsch 1999 and 2001, Agarwal and Gort 1996) have found that small firms in these circumstances do not need to grow to survive. This can be explained by accounting for the small size of the minimum efficient scale in these industries.

types of entrepreneurial abilities: talent to manage the business and talent to develop new products. Thus, as regards to the latter ability, the implication is that some entrepreneurs have a "natural talent" to perceive an opportunity and translate it into a product or service for which success is guaranteed in the form or high consumer demand. In contrast, Holmes and Schmitz (1995) as a result of their analysis on data on small business closures and the turnover of their owner-managers for the USA, come to the conclusion that there are characteristics of the business which influence its performance, that are separate from the talent of the entrepreneur. In their model they assume that in every start-up the business draws a "quality" from a distribution a business qualities, independent of the talent of the entrepreneur, in which the higher the quality, the higher the output of the small firm. Our model, therefore, follows the assumption that the profits of the small firm also depend on the nature of the opportunity being exploited, what we call the *demand* parameter, and it is represented by α. It denotes how successful the entrepreneur is in translating the opportunity into a product or service which is attracting customers or capturing a market niche. A high value in α^{ij} would mean that entrepreneur j is running the small firm i which is offering a product or service for which consumers' demand is high.

More specifically, we assume a profit function for the small firm of the following form

$$\Pi^{ij} = pQ\left(p, \alpha^{ij}\right) - \frac{1}{\theta^{j}} cQ\left(p, \alpha^{ij}\right) \ . \tag{1}$$

These are the profits for entrepreneur $j \in J$ running firm $i \in I$, where p is the price of the good/service offered or to be offered by the firm, and $Q(\cdot)$ represents the demand function. We also assume that the parameters α^{ij} and $\theta^{j}$are a source of uncertainty for the entrepreneur because they are unknown to him. That is, prior to owning a firm, the entrepreneur does not know his endowment of α^{ij} and θ^{j}. He only knows that the *true* demand for his firm's product and his *true* skills are random draws from some distributions. Once the entrepreneur runs a firm, nature determine these values, which are then revealed to the entrepreneur immediately. Specifically, we assume that

A.1 α is an i.i.d. stochastic variable with cumulative distribution function (CDF) $A(\cdot)$, with support $[0, \infty)$. $A(\cdot)$ is continuous and twice differentiable. $A(\cdot)$ is known by the individual.

A.2 θ is an i.i.d. stochastic variable. It has a CDF $G(\cdot)$ with support $(0, \infty)$. $G(\cdot)$ is continuous and twice differentiable with probability density function $g(\cdot)$. The individual knows the form of $G(\cdot)$.

We also adhere to the assumption that the opportunity being exploited is independent of the ability of the entrepreneur, i.e. the parameters α and θ are uncorrelated with each other. A consequence of this is that we restrict the nature of the entrepreneurial skills of the small firm owner. In other words, our framework defines that the entrepreneurial skills θ^j are limited to skills to manage the firm, and do not include the ability to develop and offer an attractive product. Without attempting an exhaustive list our definition of the parameter θ^j includes the entrepreneur's ability to organise and assume the responsibility of the activities of the small firm, to select the cheapest inputs given a desire level of quality, to deal with suppliers and perhaps employees, to ensure that the production of goods or services in the firm is done in the best possible way using the resources available, amongst others. In general, these represent the skills required to efficiently undertake the everyday activities of the firm and to tackle the problems that might arise. In this respect, our definition of entrepreneurial skills does not include the notion of *schumpeterian* innovators, but is closer to the definition of the entrepreneur's *general ability* put forward by Marshall[8]. Moreover, by further assuming that the parameter α is a random draw from a known distribution, we specify that whether or not the opportunity being exploited can be translated into a product or service for which there is high consumer demand is independent of the talent of the entrepreneur tu run a business, but depends more on "luck". That is, regardless of whether the entrepreneur believes he has identified a very attractive opportunity, we specify that when it comes down to running the business, the entrepreneur is not able to provide a better forecast for his product's demand apart from the mean of the distribution of the parameter α. In an alternative model we relax the assumption of independence between α^{ij} and θ^j and determine whether in this way we are able to offer an explanation to the empirical evidence contained in the previous section.

We further assume that

A.3 $Q\left(p, \alpha^{ij}\right) = D\left(\alpha^{ij}\right) - bp$ is the demand function the small firm faces, where $b > 0$ and p is the price of the good/service. Since the industry is subject to monopolistic competition, elasticities of substitution between any firm's good and the rest are assumed to be zero, and only the price of the firm's own good affects the firm's demand. The demand function has the following properties:

$$\partial D\left(\alpha^{ij}\right) / \partial \alpha > 0, \;\; \partial^2 D\left(\alpha^{ij}\right) / \partial \alpha^2 < 0 \; . \tag{2}$$

A.4 All operating firms have the same cost structure.

A.5 The cost function $\frac{1}{\theta^j} c Q\left(p, \alpha^{ij}\right)$ presents constant marginal costs, where $c > 0$. We also assume that there exists a $Q^{\max}$ such that for any $Q > Q^{\max}$, $c = \infty$.

[8] He defines the entrepreneur's general ability as the one which allows him 'to be able to bear in mind many things at a time, to have everything ready when wanted, to act promptly and show resource when anything goes wrong, to accommodate oneself quickly to changes, to be steady and trustworthy, to have always a reserve of force...' Marshall (1930, pp. 206 - 207).

Conditions A.4 and A.5 together imply all firms can potentially produce the same amount of output. Note that, due to the assumption of monopolistic competition, all firms are by definition small, which means that should the size of firms be measured by production levels, $Q^{\max}$ is small enough for all firms to fit in the small size category. Condition A.5 also tells us that marginal cost is constant over the feasible production range. Intuitively, this captures the assumption that firms are not affected by economies of scale, since the average cost is constant over the production range. Thus, growing to reach the minimum efficient scale is not a requisite for the firms to remain competitive.

Profit Maximisation

Once the entrepreneur is running a firm, his objective is profit maximisation. Since profits depend on the parameters α and θ, the maximised value of profits will depend on the specific parameters the individual is endowed with. The maximisation problem is $\max_p \Pi^{ij}$ where p represents the price the monopolistic firm chooses to apply. The first order conditions give

$$p \cdot \frac{\partial Q}{\partial p} + Q\left(p, \alpha^{ij}\right) - \frac{c}{\theta^j}\frac{\partial Q}{\partial p} = 0 \ , \tag{3}$$

which rearranged provides the standard mark-up rule

$$\frac{p - \frac{1}{\theta^j}c}{p} = \frac{1}{\varepsilon\left(\alpha^{ij}\right)} \ . \tag{4}$$

Equation (4) spells out the effect the demand and efficiency parameters have on the pricing strategy of the firm and, ultimately, on the profits of the firm. The more skilful the entrepreneur is, i.e., the higher θ^j, the higher the mark-up is. Likewise, the higher the demand parameter, the more inelastic the price elasticity $\varepsilon\left(\alpha^{ij}\right)$ is, and the higher the mark-up is.

Let the solution to the maximisation problem be expressed as

$$\max_p \Pi^{ij} \equiv \Pi^*\left(\alpha^{ij}, \theta^j\right) \tag{5}$$

where it can be shown that the second order conditions for a maximum are satisfied. The next lemmas are presented for mathematical rigour and establish the properties of the profits with respect to the demand and efficiency parameters. To save on notation we drop the superscripts i and j.

Lemma 1. *The profits are an increasing function of efficiency:* $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \theta} > 0$.

Proof. The first order conditions (3) are expressed as $-bp + D(\alpha) - bp + \frac{cb}{\theta} = 0$, thus $p^*(\alpha,\theta) = \frac{D(\alpha)}{2b} + \frac{c}{2\theta}$. The demand function is $Q(p,\alpha) = D(\alpha) + bp$. Hence, $Q^*(\alpha,\theta) =$

$\frac{D(\alpha)}{2} - \frac{bc}{2\theta}$, and $\Pi^*(\alpha, \theta) = \frac{(bc - \theta D(\alpha))^2}{4b\theta^2}$. From here it follows that $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \theta} = \frac{c}{2\theta^3}(\theta D(\alpha) - bc)$, which rearranged yields $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \theta} = \frac{c}{\theta^2}\left(\frac{D(\alpha)}{2} - \frac{bc}{2\theta}\right)$. Now, the terms inside the brackets in the right-hand-side are equivalent to $Q^*(\alpha, \theta)$. In this way we can write $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \theta} = \frac{c}{\theta^2} Q^*(\alpha, \theta)$ and thus $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \theta} > 0$. ■

Lemma 2. *The profits are a function of the demand parameter in the following way: whenever* $p^*(\alpha, \theta) > MC$ *we have* $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \alpha} > 0$*, and for* $p^*(\alpha, \theta) > MC + \frac{1}{2b}\left(\frac{\partial D(\alpha)}{\partial \alpha}\right)^2\left(-\frac{\partial^2 D(\alpha)}{\partial \alpha^2}\right)^{-1}$ *we have* $\frac{\partial^2 \Pi^*(\alpha,\theta)}{\partial \alpha^2} < 0$*, where* $MC = \frac{c}{\theta}$ is the marginal cost.

Proof. From the proof of Lemma 1 we obtain $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \alpha} = \frac{\partial D(\alpha)}{\partial \alpha}\left(\frac{D(\alpha)}{2b} - \frac{c}{2\theta}\right)$. From the value of $p^*(\alpha, \theta)$ we have $\frac{D(\alpha)}{2b} = p^*(\alpha, \theta) - \frac{c}{2\theta}$, which inserted in the term in brackets renders $\frac{\partial \Pi^*(\alpha,\theta)}{\partial \alpha} = \frac{\partial D(\alpha)}{\partial \alpha}\left(p^*(\alpha, \theta) - \frac{c}{\theta}\right)$. The sign of this expression is positive whenever $p(\alpha, \theta) > \frac{c}{\theta}$. Regarding the second part, we calculate $\frac{\partial^2 \Pi^*(\alpha,\theta)}{\partial \alpha^2} = \frac{1}{2b}\left(\frac{\partial D(\alpha)}{\partial \alpha}\right)^2 + \frac{\partial^2 D(\alpha)}{\partial \alpha^2}\left(p^*(\alpha, \theta) - \frac{c}{\theta}\right)$. We require $\frac{\partial^2 \Pi^*(\alpha,\theta)}{\partial \alpha^2} < 0$, so we evaluate under which conditions $\frac{1}{2b}\left(\frac{\partial D(\alpha)}{\partial \alpha}\right)^2 + \frac{\partial^2 D(\alpha)}{\partial \alpha^2}\left(p^*(\alpha, \theta) - \frac{c}{\theta}\right) < 0$. Rearranging for $p(\alpha, \theta)$ we find that $\frac{\partial^2 \Pi^*(\alpha,\theta)}{\partial \alpha^2} < 0$ takes place whenever $p^*(\alpha, \theta) > \frac{c}{\theta} + \frac{1}{2b}\left(\frac{\partial D(\alpha)}{\partial \alpha}\right)^2\left(-\frac{\partial^2 D(\alpha)}{\partial \alpha^2}\right)^{-1}$ where the second term on the right-hand-side is positive. ■

The decision to start a firm

The individual will choose to work in a paid job or to set up and run a firm depending on which state offers the largest utility. However, because the individual has no information on his true values of α and θ yet, the only information he has available to calculate the expected profits from running a firm is the distribution of these parameters. Under this situation, the maximisation problem is

$$V = \max\left\{W^j \ , \ E\left[\max_p \left(\Pi^{ij} + \xi^j\right)\right]\right\} \tag{6}$$

where V is the value function, Π^{ij} is specified by (1), and $E[\cdot]$ denotes the expectation operator. This expression is rewritten as

$$V = \max\left\{W^j \ , \ \underset{\alpha,\theta}{E}\left[\Pi^*(\alpha, \theta)\right] + \xi^j\right\} \tag{7}$$

where we define $\Pi^*(\alpha, \theta)$ as the value of maximised profits, and therefore

$$\underset{\alpha,\theta}{E}\left[\Pi^*(\alpha, \theta)\right] \equiv \int_0^\infty \int_0^\infty \Pi^*(\alpha, \theta)\, dA(\cdot)\, dG(\cdot) \tag{8}$$

as its expected value. Therefore the individual has an incentive to turn down a paid job and start a firm if

$$\underset{\alpha,\theta}{E}\left[\Pi^*\left(\alpha,\theta\right)\right] \geq W^j - \xi^j \ . \tag{9}$$

5 Results

Once the individual has decided to set up and run a firm at the end of a given period, it is assumed that the firm is formally in operation in the following one. The problem for the individual is to maximise his lifetime utility. Therefore, when he is running a firm he has to determine which state provides the largest utility. What are these states? It is straightforward to see that one possibility is to keep his current firm running. A second possibility is the waged worker state. This would imply closing his current business and moving to a paid job. A third possibility is the option of running a different firm, in other words, the possibility of closing the current business and starting a new one. Why is this last possibility relevant? In agreement with the suggestion that entrepreneurship is about the search and exploitation of profitable opportunities (Shane and Venkataraman 2000), the situation we are looking at is, for example, one where the entrepreneur was running a Chinese takeaway and decides to move to Mexican food, or to open a sports clothes retail outlet. In this sense, by new firm we refer to one where the opportunity recognised leads the entrepreneur to develop and offer a good or service differentiated from the one offered by the previous firm. Therefore, once the entrepreneur is running a firm, to maximise his utility he must decide whether keeping his current firm is the best choice, or if closing it and starting a new one provides the largest utility, or whether he is better off by closing it and changing to a paid job. We now define the rewards offered by each of these alternative states.

The utility from keeping the current firm running is calculated by

$$\max_{p_t}\Pi^{ij} \equiv \Pi^*\left(\alpha^{ij},\theta^j\right) \ . \tag{10}$$

This is the same expression as in (5). Once the individual has joined entrepreneurship, and given the nature of the parameters α^{ij} and θ^j, the procedure to calculate the profits the entrepreneur would obtain from starting a new firm is different from the one shown by the previous expression. θ^j represents the skills of the entrepreneur to manage the firm efficiently. These skills are time invariant, that is, talented entrepreneurs are always talented, whereas unskillful entrepreneurs are not able to enhance their skills[9]. Therefore, once the

[9]This assumption may seem very restrictive at first glance, since it eliminates any possibility for the

entrepreneur has started a firm for the first time and learned the *true* value of his efficiency, he is constrained to this value in the future. Thus every time he starts a new firm, his efficiency parameter remains the same. However, this is not true for the case of the demand parameter, as intuition suggests that different opportunities may face different levels of demand. In other words, every time the entrepreneur runs a new firm, he receives a different value of α. In the context of our previous example, it is easy to think that the new Mexican food outlet or the new sports clothes retail outlet the entrepreneur is planning to set up might not attract enough demand, regardless of whether the Chinese takeaway was a success or not. In terms of our model, this implies the expected profits of a new firm are calculated as follows :

$$\underset{\alpha}{E}\left[\underset{p_t}{\max}\Pi^{ij}\right] \equiv \int_0^{\infty} \Pi^*\left(\alpha, \theta^j\right) dA\left(\cdot\right) \ . \tag{11}$$

Note the difference between expression (8) and (11). Expression (8) represents the expected profits for a *de novo* entrepreneur, that is, for an entrepreneur who does not have any experience in running a firm. This means that a *de novo* entrepreneur does not know his endowment of both parameters α and θ, and as a consequence calculates expected profits by applying expectation over both parameters. In contrast, in our model an incumbent entrepreneur, i.e. one who has experience of running a firm, has already discovered his efficiency parameter. Therefore, if he is to start a new firm the unique uncertainty is the demand parameter. Hence, we find that the expected profits of a new firm as shown by (11) are calculated by applying expectation only over the demand parameter. To save on notation we define

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] \equiv \int_0^{\infty} \Pi^*\left(\alpha, \theta^j\right) dA\left(\cdot\right) \ . \tag{12}$$

Finally, the third alternative for the entrepreneur is to close his current firm and switch to employment. Given that expression (9) was the decision rule to enter the market, we set

$$\Pi_R^j = W^j - \xi^j \tag{13}$$

to indicate the reservation level of profits. Additionally, we assume that once the entrepreneur is running his firm, the value of the alternative of returning to waged work starts to depreciate. This is indicated by $\gamma^s \Pi_R^j$, where $0 < \gamma \leq 1$ is the rate of depreciation and s is the

improvement of management skills as a result of learning. However, in our model this is not the case because we assume the parameter θ^j embodies the endowment of potential skills, which can be regarded as time invariant. In this sense, entrepreneurs with low efficiency may improve the way they manage their firms over time, but they have a very limited range to extend their skills because by definition they have low potential.

number of periods the individual has been in self-employment. From expressions (10), (11) and (13) the problem for the entrepreneur is written as

$$V_E = \max \left\{ \Pi^* \left(\alpha^{ij}, \theta^j \right) , \ \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j \right) \right] , \ \gamma^s \Pi_R^j \ \right\} , \tag{14}$$

where V_E denotes the value function once the individual has joined entrepreneurship.

5.1 The alternative model

In the present study we specify that the efficiency parameter θ^j denotes the ability of the entrepreneur, which we restrict to skills to manage and deal efficiently with the everyday activities of the firm. In this context, the objective of this subsection is to build an alternative model where this assumption is relaxed, and to try to provide an answer to our initial question using an alternative model. In turn, the predictions of both the alternative model and the general one will be confronted. More specifically, we will allow the skills of the entrepreneur to include the ability to innovate. To this end consider the profit function

$$\Pi^j = \Pi \left(\varphi^j \right) \ . \tag{15}$$

Let φ^j represent some sort of *overall* entrepreneurial skills for individual j. Accordingly, we assume that the profits are a strictly increasing function of φ, i.e., $\partial \Pi / \partial \varphi > 0$. This function generalises the profit function that can be found in Jovanovic (1982) and Frank (1988), and in fact, in terms of the assumptions of our general model, it would imply that the parameters α^{ij} and θ^j are perfectly correlated. To appreciate this consider a profit function of the form revenue function minus cost function

$$\Pi = R(\cdot) - C(\cdot) \ ,$$

where the technology of production is included in the latter and the demand function in the former. This is the form of the profit function we consider in our model as shown by expression (1), where the demand parameter α^{ij} affects the revenue function directly, and the efficiency parameter θ^j influences the cost function. Jovanovic (1982) assumes the small firm produces in an industry with homogeneous product and perfect competition. Hence, he eliminates the need for the existence of a parameter like α^{ij} and concentrates on the cost function. In his model the efficiency parameter affects the cost function and is the only firm-specific determinant. Frank (1988) assumes that the talent of the entrepreneur is the only firm-specific determinant and that it affects the technology of production as well. He does not make any assumption on the type of competition the small firm faces. His model is equivalent to Jovanovic's in the sense that it discards the existence of a parameter like α^{ij} by implicitly assuming homogeneous product and concentrates on the technology of production. We believe the model presented in the previous section, henceforth our *general model*, offers a more realistic case by assuming monopolistic competition and hence the existence of differentiated products, than by simply assuming small firms compete in markets with homogeneous products. Under our general model, if we assume the profits of the small firm depend on a *single* parameter which embodies the *overall* entrepreneurial skills of the owner such as in (15), then it must be the case that these skills affect *both* the revenue function and the cost function. Hence, our argument that under such a scenario, it is as though the parameters α^{ij} and θ^{j} are perfectly correlated.

Note that the parameter φ^{j} therefore represents more than one characteristic of entrepreneurial skills, because under the form of a profit function like (15), the implication is that the entrepreneur has talent to manage the firm efficiently, as he does to identify attractive opportunities and develop products or services which are very appealing to customers, that is, to innovate or spot profitable opportunities. Under this assumption, a skilful entrepreneur running a small restaurant would be as successful as he would in running an small internet based company. Following a similar procedure as before, assume φ is an i.i.d. stochastic variable which follows a specific cumulative distribution function, and let

$$E\left[\Pi^{*}\left(\varphi\right)\right] \geq \Pi_{R}^{j} \tag{16}$$

represent the entry decision, where $\Pi_{R}^{j} = W^{j} - \xi^{j}$. Once the entrepreneur is running a firm, the entrepreneur has the option of keeping his current firm, or closing it down and moving to waged work, which is represented by the following expression

$$V_{A} = \max\left\{\max_{p_t}\Pi^{j}\left(\varphi\right),\ \gamma^{s}\Pi_{R}^{j}\right\}\ .$$

This expression is equivalent to

$$V_{A} = \max\left\{\ \Pi^{*}\left(\varphi^{j}\right), \gamma^{s}\Pi_{R}^{j}\right\}\ , \tag{17}$$

where $\Pi^{*}\left(\varphi^{j}\right)$ are the maximised profits. Note that the difference between this expression and expression (14) is that the profits of a possible new firm are not included in the former, because they would be the same as the profits of the current firm. In other words, given that the small firm profits depend only on the *overall* skills of the entrepreneur denoted by φ^{j}, and since these skills do not change from running one firm to another, the entrepreneur would expect the performance in the new firm to be the same as the one he experiences in the current firm: the same success in the small restaurant as in the video shop. This means that it would not make sense for the entrepreneur to shut the current firm and try to start a new one because he will not do any better in the new venture. As a result, expression (17) can be seen as a generalisation of models which assume that active entrepreneurs have only two options: to keep their current firm running or to close down and join employment.

For the firm to survive, the problem for the entrepreneur reduces to achieving profits satisfying the weak inequality

$$\Pi^* \left(\varphi^j\right) \geq \gamma^s \Pi_R^j \, . \tag{18}$$

From here we obtain the next proposition.

Proposition 1. *Let* $\Pi^j = \Pi\left(\varphi^j\right)$ *where* $j \in J$ *and* $\partial \Pi\left(\cdot\right)/\partial \varphi > 0$. *The maximisation problem is the one shown by (17). Define* φ_F *as the minimum level of expected entrepreneurial skills which would induce the entrepreneur to start a firm, and* $\varphi_{\min}$ *as the minimum level of entrepreneurial skills required to keep the firm running. Let* $H \subset J$ *be the subgroup for which expression (18) is not satisfied. Therefore,* $\forall j \in H$, $\varphi^j < \varphi_{\min} < \varphi_F$. *As a consequence, the waged work state will always provide the largest rewards to subgroup* H.

Proof. Let φ_F solve $\Pi_R^j = \Pi^*\left(\varphi_F\right)$. Since $\partial \Pi\left(\cdot\right)/\partial \varphi > 0$, any $\varphi^j > \varphi_F$ would satisfy (16) Let $\varphi_{\min}$ solve expression (18), $\Pi^*\left(\varphi_{\min}\right) = \gamma^s \Pi_R^j$. Thus, we rewrite this expression as $\Pi^*\left(\varphi_{\min}\right) = \gamma^s \Pi^*\left(\varphi_F\right)$. Since $\gamma < 1$, this implies $\varphi_{\min} < \varphi_F$. If for the subgroup H expression (18) is not satisfied and since $\partial \Pi\left(\cdot\right)/\partial \varphi > 0$, then it must be the case that $\forall j \in H$, $\varphi^j < \varphi_{\min} < \varphi_F$. Given that the value φ^j does not vary over time, the second argument of expression (17), which represents the reward in the waged work state, will always be the largest for subgroup H . ■

Proposition 1 states that when the only firm-specific determinant of the profits of the small firm are the overall entrepreneurial skills of the owner-manager, those entrepreneurs whose firms do not survive have skills lower than they expected and lower than the ones required to run a firm, therefore, it is optimal for them not to try to start another firm in the future. As a consequence, not only the firm will not survive, but also the entrepreneur will not attempt to start another firm again. As it stands, this alternative model does not seem to provide an intuitive answer to our initial question as to why some small firm owners start a new firm after having closed one that was not successful. According to Proposition 1 such entrepreneurs should not exist, since the fact that the small firm did not survive is simply because the entrepreneur does not have the necessary skills to run a successful firm. However, the empirical evidence provided before shows that a rather high share of entrepreneurs who close a firm own another one in the future, despite the lack of success in their previous venture. In the following subsection we present and evaluate the predictions of our general model, which provide a better explanation. Finally, another implication of the alternative model is that the terms *survival of the firm* and *survival of the entrepreneur* are equivalent, even though they have different meanings. According to the alternative model, failure by the entrepreneur to keep a small firm trading – the non survival of the small firm – indicates that he does not have the necessary entrepreneurial skills and therefore he is better off in a paid job – the non survival of the entrepreneur.

5.2 The general model

In our general model the problem for the entrepreneur is

$$V_E = \max \left\{ \Pi^* \left(\alpha^{ij}, \theta^j \right) , \ \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j \right) \right] , \ \gamma^s \Pi_R^j \ \right\} , \tag{19}$$

which is a repetition of expression (14). The entrepreneur's current firm will survive whenever the first argument is the largest. Therefore, sufficient conditions for the current firm to survive are

$$\Pi^* \left(\alpha^{ij}, \theta^j \right) > \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j \right) \right] \tag{20}$$

and

$$\underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j \right) \right] > \gamma^s \Pi_R^j \ . \tag{21}$$

However, our interest is to provide an explanation as to why some entrepreneurs start a new firm after having closed a previous one that was not successful. In terms of our model, it is as though the second term in (19) had the largest value. In other words, the situation is similar to one where inequality (20) is *not* satisfied, whilst inequality (21) *is* satisfied. This paves the way to our next proposition.

Proposition 2. *Let the profit function be the one shown by (1). The maximisation problem is the one in (19). Define $\theta_{\min}$ as the minimum level of entrepreneurial skills which satisfies the sufficient conditions for the current firm to survive (20) and (21). Assume all entrepreneurs share the same $\gamma^s \Pi_R^j$* and *let $D \subset J$ be the subgroup for which the second term in (19) is the largest. Therefore, $\forall j \in D, \ \ \theta^j > \theta_{\min}$. As a consequence, the waged worker state will always provide the smallest rewards to subgroup D.*

Proof. Let $\mathbb{A}$ represent a continuum of values of α^{ij} which solve expression (20) as an equality, and define $\overline{\alpha} \in \mathbb{A}$. Lemma 3 in the Appendix establishes the conditions under which $\mathbb{A}$ exist and the properties of $\overline{\alpha}$. Utilising Lemma 2 and assuming $p > MC$, expression (20) is satisfied whenever $\alpha^{ij} > \overline{\alpha}$, where $i \in I$, $j \in J$. From expression (9) let θ_F solve $\Pi_R^j = \underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta_F\right)\right]$, and from expression (21) let $\theta_{\min}$ solve $\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta_{\min}\right)\right] = \gamma^s \Pi_R^j$. Lemma 4 in the Appendix establishes under which conditions θ_F and $\theta_{\min}$ exist. Since all entrepreneurs share the same $\gamma^s \Pi_R^j$, then $\theta_{\min}$ does not vary across individuals. Therefore, $\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta_{\min}\right)\right] = \gamma^s \underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta_F\right)\right]$. Making use of Lemma 1, any $\theta^j > \theta_{\min}$ satisfies (21). If for subgroup D expression (20) is not satisfied, but expression (21) is, we must therefore have $\alpha^{ij} < \overline{\alpha}$ and $\theta^j > \theta_{\min}$, $\forall j \in D$, $i \in I$. Under these circumstances the second argument in (19) is the largest. Once the entrepreneur runs the new firm he receives a new draw for the demand parameter. Define the new draw as α^{lj}, where $i \neq l \in I$, $j \in D$. It can be the case that $\alpha^{lj} \gtreqless \overline{\alpha}$. Under any of these situations, either the first argument or the second one in (19) is the largest. This implies the third argument in expression (19), which represents the waged working state, has always the smallest value for subgroup D. ■

Proposition 2 argues that the reason why the *determined* entrepreneurs in Figure 1 continue in entrepreneurship after a business closure despite recently having an unsuccessful experience, is because they have entrepreneurial skills at least as large as the ones required to run a potentially successful firm, that is, skills above the threshold set by $\theta_{\min}$. Note that this is the opposite result to the one obtained from Proposition 1. A word of caution: the reader should not be confounded by the term successful. As implied by our general model, we use the term *successful firm* to refer to the case where running the small firm provides larger pecuniary and non-pecuniary rewards to the entrepreneur than working as an employee does, that is profits larger than $\gamma^s \Pi_R^j$. As a result, if this reservation level is very low, then the skills threshold $\theta_{\min}$ is also low. This implies that the threshold $\theta_{\min}$ is therefore idiosyncratic. We discuss this in more detail in a later section. In any case, this does not undermine the prediction of Proposition 2. that, *all other things being equal*, the determined entrepreneurs have high enough skills to run potentially successful firms. According to our model, the other condition to run a successful firm is to find a product or service which is appealing to customers or for which there is a market niche.

Two important questions to ask are how plausible and reliable this prediction is. We provide the following arguments to tackle such questions. First, such results are consistent with the view suggested by Audretsch (1995a) who asserts that entrepreneurs are always in the process of discovering whether they possess *the right stuff* in terms of the product they offer for which sufficient demand exists and whether they can produce it more efficiently than their rivals. More specifically he states that 'when a new firm is launched, its prospects are shrouded in uncertainty. If the new firm is built around a new idea, it is uncertain whether there is sufficient demand or if some competitor will have the same or even a superior idea. Even if the new firm is a clone, there is the question whether there is enough demand' (p. 67). Audretsch further asserts that 'an additional layer of uncertainty is how competent the new firm really is' (p. 67), which in terms of our model can be easily translated as referring to how skilful the owner-manager is. Storey (1990) acknowledges a possibility which is also in line with our assumptions. He declares that 'the creation and subsequent closure of a firm may merely be part of the small entrepreneur's way of searching out profitable opportunities...' (p. 47). Secondly, we believe the predictions of our model will gain strength if they account for all the possibilities as provided by the empirical evidence in Figure 1. That is, whilst Proposition 2 offers an explanation as to why the *determined entrepreneurs* in Figure 1 own another firm in the future, such explanation does not involve the other three cases. The following proposition presents the complete results of our general model.

Proposition 3. *Let the profit function be the one shown by (1). The maximisation problem is the one in (19). Assume all entrepreneurs share the same* $\gamma^s\Pi_R^j$ and *define* $\theta_{\min}$ *as the minimum level of entrepreneurial skills which satisfies the sufficient conditions for the current firm to survive (20) and (21), and define the group R as the one which contains the different combinations of* α^{ij} *and* θ^j *such that* $\Pi(\alpha^r,\theta^r) = \gamma^s\Pi_R^j$, $\forall(\alpha^r,\theta^r) \in R$.

(i) Let $M \subset J$ *be the subgroup for which the first term in (19) is the largest* and $M_1, M_2 \subset M$, $M_1 \cup M_2 = M$ *and* $M_1 \cap M_2 = \varnothing$. *Therefore* $\alpha^{ij} > \overline{\alpha}$ $\forall j \in M$, $\overline{\alpha} \in \mathbb{A}$, $\theta^j > \theta_{\min}$ $\forall j \in M_1$, $\theta_{\min} > \theta^j > \theta^r$ $\forall j \in M_2$, $i \in I$, $r \in R$.

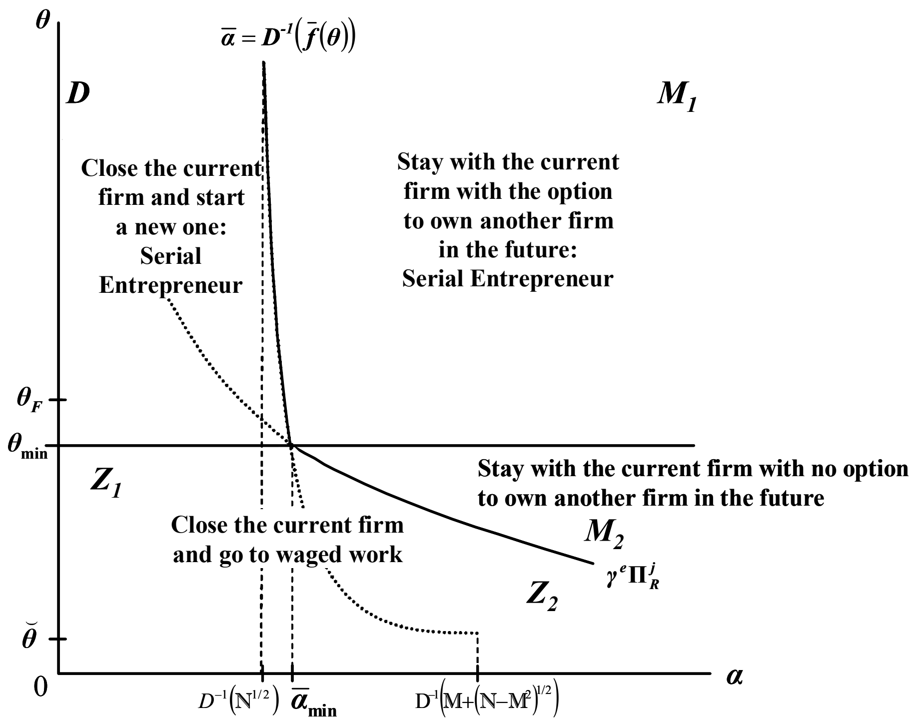
(ii) Let $D \subset J$ *be the subgroup for which the second term in (19) is the largest. Therefore* $\alpha^{ij} < \overline{\alpha}$, $\theta^j > \theta_{\min}$, $\forall j \in D$, $i \in I$, $\overline{\alpha} \in \mathbb{A}$.

(iii) Let $Z \subset J$ *be the subgroup for which the third term in (19) is the largest* and $Z_1 \cup Z_2 = Z$, *and* $Z_1 \cap Z_2 = \varnothing$. *Therefore* $\alpha^{ij} < \overline{\alpha}$ *and* $\theta^j < \theta_{\min}$, $\forall j \in Z_1$, $\overline{\alpha} \in \mathbb{A}$; $\alpha^{ij} > \overline{\alpha}$ *and* $\theta^j < \theta^r < \theta_{\min}$, $\overline{\alpha} \in \mathbb{A}$, $\forall j \in Z_2$, $i \in I$, $r \in R$.

(iv) For subgroups M_1 *and* M_2 *the sale of their firms or any shock which makes* $\alpha^{ij} < \overline{\alpha}$, $\overline{\alpha} \in \mathbb{A}$, $j \in M$, $i \in I$, *assuming* α^{ij} *is exposed to unforeseen shocks, has the effect of making the second argument in (19) the largest for subgroup* M_1, *and the third argument in (19) the largest for subgroup* M_2.

Proof. See the Appendix.

Figure 2: Survival of the small firm and the entrepreneur with the demand and efficiency parameters



Proposition 3 is represented graphically in Figure 2. It shows under which conditions, in terms of combinations of the parameters α and θ, it is optimal for the entrepreneur to keep his current firm, or shut it down and start a new one, or shift to waged work. More specifically, Figure 2 is an isoprofit map in the space (α, θ), where every point in its surface entails coordinates which form part of an isoprofit curve. In turn, every isoprofit curve is associated with a single value of profits for different combinations of α and θ. Lemma 5 in the Appendix establishes that such isoprofits curves have negative slope and are convex with respect to the origin. The farther to the north-east the isoprofit is placed, the higher the associated value of profits. In Figure 2 $\gamma^s \Pi_R^j$ is an isoprofit curve which corresponds to profits equivalent to the rewards in paid employment adjusted by the entrepreneurial spirit and by depreciation due to time spent in self-employment. Consistent with Proposition 3, $\gamma^s \Pi_R^j$ is assumed to be constant across individuals. Therefore, all the results obtained from Figure 2 should be interpreted with the suffix *all other things being equal.*

Figure 2 is divided into five areas determined by the intersection between the curve $\overline{\alpha} = D^{-1}\left(\overline{f}\left(\theta\right)\right)$ and $\gamma^s \Pi_R^j$, where the former represents a continuum of values of the demand parameter which makes the entrepreneur indifferent between the first and second arguments of expression (19*)*. Areas M_1 and M_2 contain the combinations of parameters α^{ij} and θ^j such that the best option for the entrepreneur is to keep his current firm, because it is providing profits larger than the reservation level $\gamma^s \Pi_R^j$. This explains why entrepreneurs placed in areas Z_1 and Z_2 find it optimal to close their business and switch to waged work. In contrast, area D comprises some combinations of α^{ij} and θ^j which are placed to the right of the reservation level, and some combinations located to the left of it. However, according to Proposition 3 the best course of action for entrepreneurs in area D is to shut their current firm and start a new one, because by doing so they have the option of offering a different product in the hope of drawing a higher α and hence reach a higher isoprofit curve. In terms of Figure 2 their anticipation is to reach a point on the curve represented by $\overline{\alpha} = D^{-1}\left(\overline{f}(\theta)\right)$ that crosses through the coordinates $(\overline{\alpha}_{\min}, \theta_{\min})$, and asymptotically converges to $\overline{\alpha} = D^{-1}\left(\mathbb{N}^{1/2}\right)$ [10]. The key point is that all entrepreneurs in area D have skills above the cut off point $\theta_{\min}$ so by starting a new firm and as a result becoming serial entrepreneurs, they expect to be placed in an isoprofit curve to the right of the reservation level. In this sense, what entrepreneurs in areas D and M_1 have in common is that all of them have skills above the threshold $\theta_{\min}$. Notice from Figure 2 that we can say that entrepreneurs in area M_1 are the ones who are running successful firms because of the high demand for their firm's product. Therefore, it is fair to say the only thing an entrepreneur in area D lacks is to offer a more appealing product, but his skills may be as good as the ones of those entrepreneurs who run successful firm. This is the case previously pointed out by Proposition 2.

Proposition 3 also establishes that the difference between entrepreneurs in areas M_1 and M_2 is that, if their current firms do not survive or should they sell them, the former have the option of remaining in entrepreneurship by starting a new firm, whereas the latter do not and have to move to employment. The intuition is as follows. Suppose an entrepreneur in area M_1 sells his firm. If he plans to start a new one and given that he has skills above $\theta_{\min}$, his expectation in terms of Figure 2 is to reach a point over the line represented by $\overline{\alpha} = D^{-1}\overline{f}(\theta)$, but only on the portion which starts from the coordinates $(\overline{\alpha}_{\min}, \theta_{\min})$ and goes upward. Any of these points would render profits higher than the reservation level.

[10] Lemma 3 in the appendix illustrates the properties of $\overline{\alpha} = D^{-1}\left(\overline{f}(\theta)\right)$. Amongst other things it demontsrates that $\overline{\alpha}$ is a decreasing function of θ. In our model this implies that even when the entrepreneur's skills to run a firm are not related to the success of the opportunity being exploited, more managerially able entrepreneurs are more measured in their expectations on the success of new opportunities. Accordingly, less skilful entrepreneurs are more optimistic. In a later section we discuss the scenario when entrepreneurs are overly optimistic regardless of the level of skills.

This is not the case for an entrepreneur in area M_2. Since he has skills lower than $\theta_{\min}$, by starting a new firm he expects to be placed in area Z, which means he would receive profits lower than the reservation level. This situation is equivalent to the one where there is a shock in demand and the firm does not survive. For instance, the shock can represent a downturn in demand because another firm entered the market and is offering a product similar enough to impinge on the demand of the original firm. If the shock is big enough, the small firm's demand may fall below $\overline{\alpha}$. Thus, it is as if entrepreneurs in area M_1 were *moved* to area D, where according to Proposition 3, these entrepreneurs are better off by closing their current firms and starting a new one. On the other hand, entrepreneurs in area M_2 would be placed in area Z after the shock, where they are better off by changing to paid work.

Proposition 3 therefore suggest the existence of a mechanism of entrepreneurial selection whereby only entrepreneurs with skills above the threshold set by $\theta_{\min}$ start new firms after having closed one, regardless of whether they were running business with high demand, area M_1, or low demand, area D. Also note that unlike the alternative model our general model does differentiate between survival of the small firm and survival of the entrepreneur. According to Figure 2, survival of the small firm takes place under the conditions described by areas M_1 and M_2, whereas survival of the entrepreneur occurs under the conditions described by areas D and M_1. More formally let P_E represent the probability that the entrepreneur survives, that is the likelihood of starting or buying a new firm after facing a business closure. Similarly, let P_F indicate the probability of survival of the entrepreneur's firm, that is when the individual decides that the best course of action is to stay with his current firm. From Proposition 3 and Figure 2 the probability that the entrepreneur survives is

$$P_E = P\left(\theta^j > \theta_{\min}\right) \ , \tag{22}$$

where $P(\cdot)$ designates a probability function. Likewise, the probability that the entrepreneur's firm survives is

$$P_F = P\left(\theta^j > \theta_{\min}, \alpha^{ij} > \overline{\alpha}\right) + P\left(\theta_{\min} > \theta^j > \theta^r, \alpha^{ij} > \overline{\alpha}\right) \ . \tag{23}$$

The key difference between these two expression is that the probability of survival of the entrepreneur shown by (22) does not depend on whether $\alpha^{ij} \gtreqless \overline{\alpha}$. Expression (22) is therefore equivalent to

$$P_E = \int_0^{\infty} g(\cdot)\, dg - \int_0^{\theta_{\min}} g(\cdot)\, dg \tag{24}$$

where $g(\cdot)$ is the probability density function of θ. However, for the case of P_F an expression equivalent to (24) cannot be obtained, as in agreement with proposition 3 we assume that α^{ij} is subject to unforeseen shocks once the entrepreneur has started a firm. This means that, despite knowing that every time the entrepreneur starts a new firm he draws a value of α from the *prior* cumulative distribution function $A(\cdot)$, we do not know the *posterior* distribution of α and therefore the probability that the entrepreneur's small firm survives cannot be calculated.

5.3 Explaining the Evidence

In the light of these results and considering the empirical evidence provided in Figure 1, a clearer picture starts to emerge. Figure 1 can be mapped onto Figure 2. The *failed entrepreneurs* from the former would correspond to area Z from the latter, the *discouraged entrepreneurs* would be placed in area M_2, while the *determined entrepreneurs* and *prosperous entrepreneurs* would be associated with areas D and M_1, respectively. Thus, our theory argues that the reason why the *determined entrepreneurs* and the *prosperous entrepreneurs* own another firm in the future is because they have skills large enough to run potentially successful small firms. In this sense, our theory suggests that the only difference between prosperous and determined entrepreneur might be that the latter were just *luckier* with their firms in the sense that they were running a firm with a very appealing product or service. Our theory also accounts for the cases of *failed entrepreneurs* and *discouraged entrepreneurs*. The former would be placed in area Z in Figure 2 and the latter in area M_2. According to our general model, the fact that a discouraged entrepreneur leaves entrepreneurship despite having owned a firm that was successful, can be explained by saying that he was *lucky* in running a firm with a very attractive product or service, whose success indeed compensated for his low skill in managing the firm. However, this unskilful entrepreneur does not expect to *hit the jackpot* again, so he prefers to switch to a paid job after having closed or sold his firm.

To offer these results our model relies on the assumption that the skills of the small firm owners are restricted to skills to manage the firm efficiently, and that such skills are not related with the ability to recognise and exploit attractive business opportunities. Let us assume the entrepreneur has both abilities. Thus, an entrepreneur who deals efficiently with the day-to-day activities of the small firm would also be good at recognising lucrative opportunities and translating them into appealing products. This is the case of the model discussed in Subsection 5.1, from which Proposition 1 was obtained. Such proposition establishes that those entrepreneurs whose firms do not survive have skills lower than required, and that it is optimal for them to not attempt to own another firm in the future. This means that for those entrepreneurs who run successful firms, they are better off starting new firms. In other words, under such a model only the prosperous and the failed entrepreneurs in Figure 1 exist. Therefore, the alternative model fails to explain the whole spectrum of empirical evidence, whereas our general model is successful. Accordingly, we can state that in general our assumption that the skills of the small firm owners are restricted to skills to manage the firm efficiently helps to explain the empirical evidence.

6 Determinant of the Skills Threshold

A key element in the mechanism of entrepreneurial selection is the threshold $\theta_{\min}$ as it determines which entrepreneurs survive, and which ones do not. As indicated by Figure 2, $\theta_{\min}$ is determined by the intersection between $\overline{\alpha} = D^{-1}\left(\overline{f}\left(\theta\right)\right)$ and $\gamma^{s}\Pi_{R}^{j}$. As defined by our model the reservation level $\gamma^{s}\Pi_{R}^{j}$ is a function of the opportunity cost in waged-employment i.e. the foregone wage W, the entrepreneurial spirit ξ, and time spent in self-employment. In a similar way the entrepreneur's expectation of success of exploiting new opportunities and his willingness to go in pursuit of such opportunities determine $\overline{\alpha} = D^{-1}\left(\overline{f}\left(\theta\right)\right)$. This suggests that on the one hand $\theta_{\min}$ is influenced by the labour market conditions the entrepreneur faces in paid employment, intrinsic tastes for entrepreneurship, and the length of spell in self-employment, and on the other it depends on how optimistic the entrepreneur is, and on his degree of risk aversion. Since all these elements are likely to vary across entrepreneurs, $\theta_{\min}$ is therefore expected to differ from one individual to the next. As a result, via $\theta_{\min}$, these elements also affect which individuals survive in entrepreneurship and which ones opt for the labour market. We analyse each of these factors in turn next which throws more light towards explaining the empirical evidence presented at the outset of this paper.

Low Opportunities in the Labour Market

In its simplest from the income choice model attributed to Knight (1921) illustrates the alternative individuals face between paid-employment and self-employment, in which the final decision rests on the expected earning from each activity. According to this model, when the labour market is depressed, such as in periods of high unemployment, we would expect the rates of entrepreneurship to be higher, as individuals would be *pushed* into this activity due to the lack of proper income from paid-employment. However, high levels of unemployment are usually accompanied by low levels of aggregate demand, meaning that income from self-employment activities can be low and highly uncertain, thus deterring people from joining entrepreneurship or even *pulling* entrepreneurs out from that activity. The final net effect is predicted to be ambiguous (Storey 1994, Parker 2004). Indeed, Parker (2004) reviews empirical evidence regarding the relationship between unemployment and self-employment which shows mixed results, thus highlighting the difficulty in isolating the push and pull factors mentioned above. However, the unemployment rate is only one way of measuring the level of – or lack of – labour market opportunities. In contrast to this, new evidence from a cross-country study from the Global Entrepreneurship Monitor (Acs et. al. 2004) has emerged which seems to suggest that scarce or unsatisfactory opportunities in waged employment indeed drives individuals to seek income from self-employment. The Global Entrepreneurship Monitor measures the entrepreneurial activity in several developed and developing countries, discriminating between opportunity entrepreneurship – the result of pursuing a perceived business opportunity in order to exploit it – and necessity entrepreneurship – individuals who enter entrepreneurship because all other options for work are either absent or unsatisfactory. The GEM shows that in low income countries necessity entrepreneurship is larger compared to higher income countries. According to Acs et. al. (2004) 'this is because people in richer countries tend to have access to more diversified labour markets...' therefore suggesting that individuals in poor countries face few or low quality labour market options which forces them to try self-employment out of necessity.

In the context of the present study and given the arguments presented above, we can expect entrepreneurs to be more likely to start or buy a new business after having closed one whenever they face poor prospects in paid-employment. From Proposition 2 we know that

$$\theta_{\min} = \frac{bc}{\mathbb{N} - 4b\gamma^s \Pi_R^j} \left(\mathbb{M} + \left(\mathbb{M}^2 - \mathbb{N} + 4b\gamma^s \Pi_R^j \right)^{1/2} \right) \tag{25}$$

where

$$\Pi_R^j = W^j - \xi^j \ .$$

and W is the wage the entrepreneurs would receive in paid-employment. We suppose that the poorer and more scarce the prospects of the self-employed in the labour market are, the lower the wage is. It can be shown that $\frac{\partial \theta_{\min}}{\partial W} > 0$. Define $H(\theta_{\min}) \equiv P_E$, and employing (24)

$$H(\theta_{\min}) \equiv \int_0^{\infty} g(\cdot)\, dg - \int_0^{\theta_{\min}} g(\cdot)\, dg \ . \tag{26}$$

Hence $H(\theta_{\min})$ represents the probability function that the entrepreneur survives. Clearly $\frac{\partial H}{\partial \theta_{\min}} < 0$ so

$$\frac{\partial H}{\partial W} = \frac{\partial H}{\partial \theta_{\min}} \cdot \frac{\partial \theta_{\min}}{\partial W} < 0 \ . \tag{27}$$

Figure 3: A lower skills threshold due to a) lower opportunities in the labour market or b) larger intrinsic tastes for entrepreneurship

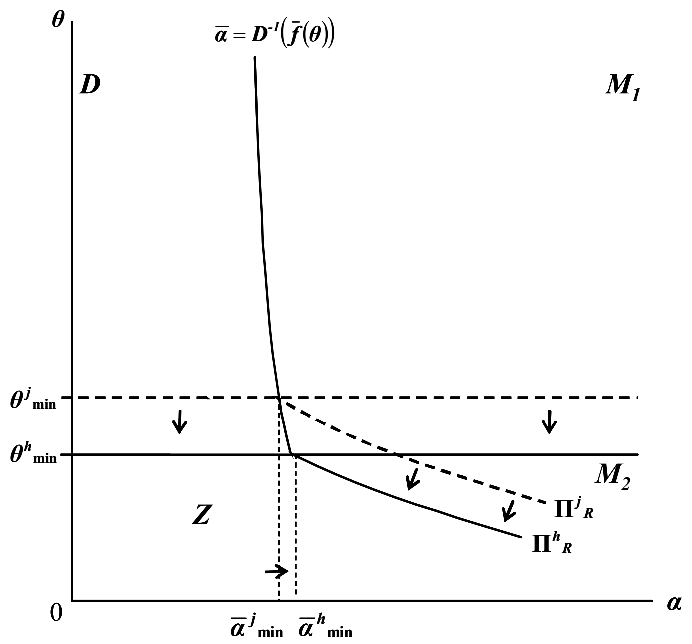


Figure 3 shows graphically the differences in the threshold $\theta_{\min}$ between entrepreneur j and h, where the latter has a lower value of W. A lack of opportunities in paid employment would result in a lower value in the wage for entrepreneur h, hence shifting the value of the entrepreneur's reservation level to the left. In turn, the threshold $\theta_{\min}$ is pushed down, hence increasing the probability that the entrepreneur h remains in entrepreneurship after a business closure. This insinuates that the *push* effect mentioned above not only applies for those facing self-employment for the first time, but also for those who are already in entrepreneurship and confront the closure of their business. If these entrepreneurs have a low opportunity cost in the labour market, they are also *pushed* into owning another firm and as a result staying in entrepreneurship.

Intrinsic Tastes for Entrepreneurship

Why does an individual become an entrepreneur? Due to the inter-disciplinary nature of entrepreneur research, this is a question that has been approached from different angles. The Psychology literature has dealt with the question extensively. One of its first attempts to provide an answer lies on the trait approach, that is, on trying to identify personal characteristics that lead some individuals to choose entrepreneurship, and others to select being employees. Amongst these traits we can find need for achievement, need for independence, and self-motivation (Burns 2001). Some of these traits can be labeled as *intrinsic tastes for entrepreneurship* since it can be said the individual will not receive any return from them unless he is undertaking entrepreneurial activities. However, the trait approach has been heavily criticised on several grounds, for instance a large number of traits have been identified which can amount to several dozens which in the end does not help to predict who turns into an entrepreneur and who does not (Delmar 2000, Parker 2004). However, Psychology research has not discarded the topic altogether, and its place new and more complex models of cognitive behaviour have been developed that '...take into account not only psychological characteristics of entrepreneurs, but also situational variables and personal background (e.g. age, sex)' (Delmar, 2000, p.145). Still the objective of these models is to explain the behaviour of individuals which can lead them to engage in entrepreneurship. And at the centre of some of these models lie key elements such as intrinsic motivation for entrepreneurship. Delmar (2000, p.151) mentions that 'intrinsically motivated behaviours are ones for which there is no apparent reward except for the activity itself'. Employing the richer and more elaborated framework of cognitive models of entrepreneurial behaviour, psychology researchers expect that individuals with more intrinsic motivation for entrepreneurship decide to join such an activity, as opposed to waged employment. In Economics research, the *extra satisfaction* that individuals obtain from self-employment, which can be equated to intrinsic tastes for entrepreneurship, has been recently included in models of occupational choice. The most notorious example is the one from Blanchflower and Oswald (1998, p. 26) where using British data they found that '...the self-employed report higher levels of job satisfaction and life satisfaction than employees'. Based on this, we can argue that larger intrinsic tastes for entrepreneurship, as captured by the entrepreneurial spirit variable ξ in our model, not only leads individual to pursue entrepreneurship, but amongst those already running a business, should they confront the closure of their firm, we can also anticipate that individuals with larger endowment of ξ are more likely to own a new firm, while those with a lower entrepreneurial spirit are more likely to choose employment.

Note that from (25) we have $\frac{\partial \theta_{\min}}{\partial \xi} < 0$, therefore using (26)

$$\frac{\partial H}{\partial \xi} = \frac{\partial H}{\partial \theta_{\min}} \cdot \frac{\partial \theta_{\min}}{\partial \xi} > 0 \, .$$

Similar to the case before, suppose two entrepreneurs, j and h, face different values of entrepreneurial spirit. In Figure 3 a larger ξ, as in the case of entrepreneur h, implies a lower reservation level, hence shifting the curve $\gamma^s \Pi_R$ to the left, dragging down the cut off point $\theta_{\min}$. The final result is that entrepreneurs with more intrinsic motivations are more likely to survive in entrepreneurship after facing the closure of their ventures.

Time Spent in Self-Employment

Our model is also consistent with the empirical evidence showing that the longer people remain in self-employment, the lower the probability of exit is. Evans and Leighton (1989) employing data for the USA found that hazard rates in self-employment decrease over time, that is the probability that the individual will exit entrepreneurship in a short period of time given that he has survived a specific period decreases the longer the period of survival is. In agreement, Blanchflower and Meyer (1994) utilising data for Australia show that the probability of leaving self-employment is lower for those who have spent longer periods in it. In terms of our model, from (25) we get $\frac{\partial \theta_{\min}}{\partial s} < 0$ where s denotes the number of periods in self-employment (e.g. years, quarters) and hence

$$\frac{\partial H}{\partial s} = \frac{\partial H}{\partial \theta_{\min}} \cdot \frac{\partial \theta_{\min}}{\partial s} > 0 \ \ .$$

Note that our model is not only consistent with the empirical evidence mentioned before, but it offers a more complete explanation. The studies mentioned above, and many others focusing on survival in self-employment, use data obtained by means of following samples or cohorts of individuals to identify their labour status – employment, self-employment, or unemployment. In the case of self-employment they do not distinguish if the entrepreneur has *moved* between different businesses. That is, they only recognise whether the entrepreneur is still is self-employment after a specific period, without mentioning whether in the process they have closed one or several business and started or bought new ones. In contrast, our model shows that those entrepreneurs with longer periods in self-employment, in case they close their business, have a higher likelihood in staying in entrepreneurship by means of owning a new business. On the other hand, we recognise a drawback of the present framework. The model assumes that entrepreneurial skills θ^j do not change over time, and that the individuals learn this value immediately after entry to entrepreneurship. Hence, only a single period would suffice to learn whether or not $\theta^j > \theta_{\min}$. In this sense, whether

$\theta_{\min}$ drops because of time spent in self-employment is irrelevant since θ^j does not change. A more complete model would therefore include changes in entrepreneurial skills through time, possibly due to learning and experience. Nevertheless, this does not invalidate the prediction of our model that amongst entrepreneurs who confront a business closure, those with more experience in self-employment are more likely to survive.

Optimism

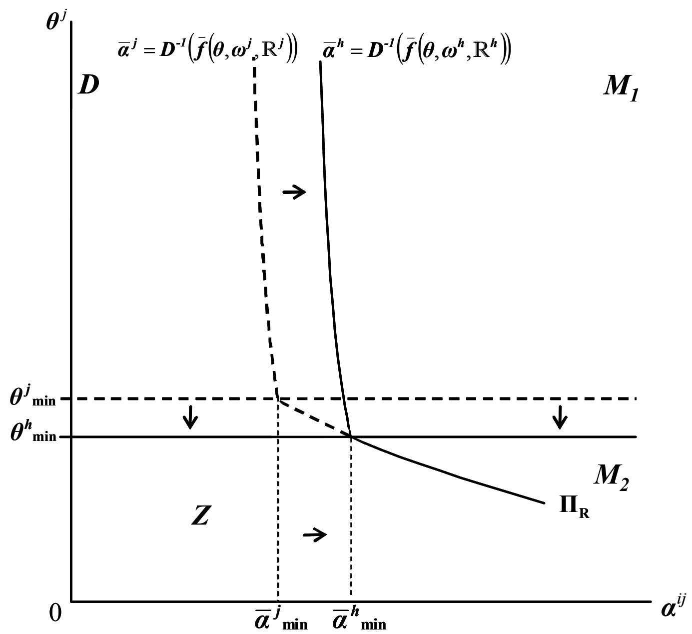
Entrepreneurs have been widely recognised to be overly optimistic in their chances of success in entrepreneurship (Cooper et. al. 1988, Parker 2006). A straightforward consequence of over-optimistic behaviour by entrepreneurs is that it leads to excess entry, with the concomitant implications for market efficiency. For instance, the larger number of entrepreneurs with unrealistically high expectations can lead to over-employment of external resources such as labour and capital, thus driving prices up and affecting realistic entrepreneurs in the process (Manove 2000). It can also explain the high small firm failure rates, as over-optimistic entrepreneurs would inadvertently enter the market with negative expected returns (de Meza and Southey 1996). Moreover, the presence of over-optimistic entrepreneur can be regarded as a negative externality for realistic ones, since one would expect equilibrium firm's profits to be lower than otherwise they would be (Parker 2006). Whatever the consequences of abnormal expectations, it is fair to assume a positive relationship between entry into entrepreneurship and optimism. However, in this study we are concerned with re-entry into entrepreneurship, and in particular whether over-optimism increases the probability of survival of the entrepreneur.

Let us define optimistic behaviour as one where the entrepreneur's expected profits from a new venture are larger than the *rational* level. Let the rational level be defined by (12) and the over-optimistic level as $\underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta^j\right)\right]+\omega^j$, where $\omega^j \geq 0$ represents the extent of over-optimism which can vary across entrepreneurs. The maximisation problem for an entrepreneur that is currently running a business is now

$$V_E = \max\left\{\Pi^*\left(\alpha^{ij},\theta^j\right),\ \underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta^j\right)\right]+\omega^j,\ \gamma^s\Pi_R^j\right\}. \tag{28}$$

Similar to proposition 3 and lemma 1 it can be shown that there exist a continuum of value of α that makes the entrepreneur indifferent between the first and second arguments of the above expression. Let these values be defined by $\overline{\alpha} = D^{-1}\left(\overline{f}'\left(\theta,\omega\right)\right)$ where it can be shown that $\frac{\partial\overline{\alpha}}{\partial\omega} > 0$. Therefore, for more optimistic entrepreneurs the curve which represents $\overline{\alpha}$ shifts to the left as depicted by Figure 4. As a result, the threshold $\theta_{\min}$ decreases. In fact,

Figure 4: A lower skills threshold due to a) more optimistic behaviour or b) lower degree of risk aversion



it can be demonstrated that $\frac{\partial \theta_{\min}}{\partial \omega} < 0$ and therefore

$$\frac{\partial H}{\partial \omega} = \frac{\partial H}{\partial \theta_{\min}} \cdot \frac{\partial \theta_{\min}}{\partial \omega} < 0 \ .$$

By believing that their chances in a new venture are higher, entrepreneurs which confront a business closure are more likely to continue in entrepreneurship by owning a new firm.

Risk Aversion

Most of the classical economists who reflected on entrepreneurship converged on the notion that a specific characteristic of entrepreneurs is that they bear the risks of the economic activities they undertake. On reviewing historical contributions to entrepreneurship research, van Praag (1999, pp. 327-328) declares that '[Richard] Cantillon's entrepreneur bears the risks as a consequence of selling (buying) at a certain price and buying (selling) at an uncertain price. [Jean-Baptiste] Say's entrepreneur bears the risk of losing capital and reputation due to experimenting and the chance of failure. The Marshallian entrepreneur is responsible for undertaking the business risk associated with the activities of his firm. ...[Frank] Knight...defines the entrepreneur as the decision maker whenever uncertainty is involved', where the square brackets are mine. In modern economic research this idea has been translated as the entrepreneur's choice between receiving a risk-free wage from employment, or engaging in self-employment, which includes owning and running a firm, with the inherent risks involved. In turn, the entrepreneurs' low risk aversion is a force which drives them to select self-employment (e.g. Kihlstrom and Laffont 1979, in Parker 2004 some models of occupational choice and how the individual's attitude towards risk affect this choice are presented). Moreover, taking advantage of the recent development of databases, some authors have been able to show empirically that entrepreneurs indeed experience a lower degree of risk aversion (Usitalo 2001, van Praag and Cramer 2001, although see Tucker 1988 where no significant effects between the individual's attitude toward risk and the decision to join self-employment is found).

The above deliberation reveals that the entrepreneur's degree of risk aversion is an important factor affecting the decision to switch to entrepreneurship. In terms of our model, the question to ask is whether risk aversion is also relevant when it comes to deciding between starting a new firm after having closed one, or switching to waged employment. To analyse this question, we employ the Arrow-Pratt measure of absolute risk aversion defined as

$$\mathbb{R}^j = -\frac{{U_{yy}}^j}{{U_y}^j} \ ,$$

where $U(y)^j$ is the individuals utility function from income y which originates from the wage W, or from the profits Π from entrepreneurship, and U_y and U_{yy} are the first and second derivative. We assume a concave utility function, hence $U_y > 0$ and $U_{yy} < 0$, which implies a risk averse individual, contrary to a risk neutral ($U_{yy} = 0$) or a risk lover person ($U_{yy} > 0$) (Jehle and Reny 2001). Now consider two risk averse entrepreneurs, j and h, where the latter is more risk-averse, that is

$$\mathbb{R}^j > \mathbb{R}^h \ .$$

If entrepreneur j is more risk averse compared to entrepreneur h, then the risk premium of j is larger than that of h, where the risk premium is defined as the amount of wealth the individual would be willing to sacrifice or the *price* he is willing to pay to avoid the uncertainty of a gamble (Varian 1992). Jehle and Reny (2001) define it as $\mathbb{P} \equiv E[y] - CE$ where $E[\cdot]$ is the expectation operator and CE is the *certainty equivalent*, such that $E[U(y)] \equiv U(CE)$. Our maximisation problem as stated by (19) implicitly assumes risk neutral entrepreneurs. Hence, to make things interesting we let

$$V_E^j = \max\left\{U\left(\Pi^*\left(\alpha^{ij},\theta^j\right)\right),\ \underset{\alpha}{E}\left[U\left(\Pi^*\left(\alpha,\theta^j\right)\right)\right],\ \gamma^s U\left(\Pi_R^j\right)\ \right\}. \tag{29}$$

and assume concavity in $U(\cdot)$. The certainty equivalent between running a new firm and staying with the current one is defined as

$$\underset{\alpha}{E}\left[U\left(\Pi^*\left(\alpha,\theta^j\right)\right)\right] \equiv U\left(\Pi^j_{CE}\right)\ , \tag{30}$$

where

$$\Pi^j_{CE} = \Pi^*\left(\alpha_{CE},\theta^j\right)\ .$$

Following a similar procedure as in Proposition 2 let $\overline{\alpha}^j$ define the continuum of values of the demand parameter which makes the entrepreneur indifferent between the first and second argument of expression (29)

$$\underset{\alpha}{E}\left[U\left(\Pi^*\left(\alpha,\theta^j\right)\right)\right] = U\left(\Pi^*\left(\overline{\alpha}^j,\theta^j\right)\right)\ . \tag{31}$$

Hence, from (30) and (31)

$$\alpha_{CE} = \overline{\alpha}^j = D^{-1}\left(\overline{f}\left(\theta,\mathbb{R}^j\right)\right)\ ,$$

where we included $\mathbb{R}$ as an argument to indicate that $\overline{\alpha}$ depends on the degree of *concavity* of $U(\cdot)$, which directly affects the risk attitude of the entrepreneur. The risk premium is therefore

$$\mathbb{P}^j \equiv \underset{\alpha}{E}\left[\Pi^*\left(\alpha^{ij},\theta^j\right)\right] - \Pi^*\left(\overline{\alpha}^j,\theta^j\right)\ , \tag{32}$$

or

$$\mathbb{P}^j \equiv \underset{\alpha}{E}\left[\Pi^*\left(\alpha^{ij},\theta^j\right)\right] - \Pi^*\left(D^{-1}\left(\overline{f}\left(\theta,\mathbb{R}^j\right)\right),\theta^j\right)\ .$$

If in the case of entrepreneurs j and h we have that $\mathbb{R}^j > \mathbb{R}^h$, then it must be the case that $\mathbb{P}^j > \mathbb{P}^h$. Therefore from (32) and since $\underset{\alpha}{E}\left[\Pi^*\left(\alpha^{ij},\theta^j\right)\right] = \underset{\alpha}{E}\left[\Pi^*\left(\alpha^{ih},\theta^h\right)\right]$ because the profit structure $\Pi^*(\cdot)$ and the uncertainty from α is the same for all entrepreneurs we must have $\Pi^*\left(\overline{\alpha}^j,\theta^j\right) < \Pi^*\left(\overline{\alpha}^h,\theta^h\right)$ or more specifically $\overline{\alpha}^j < \overline{\alpha}^h$. Figure 4 illustrates that a less risk averse attitude implies a shift to the left on the curve $\overline{\alpha}$, which in turn decreases the minimum required skills $\theta_{\min}$. The final effect is that less risk averse entrepreneurs, in the face of a business closure, are more likely to persist in entrepreneurship by starting or buying a firm, while more risk averse entrepreneur switch to employment[11].

[11] We are aware, however, that the properties of $\overline{\alpha}^j = D^{-1}\left(\overline{f}\left(\theta,\mathbb{R}^j\right)\right)$ change with higher or lower degrees of risk aversion. More specifically, $\frac{\partial\overline{\alpha}}{\partial\theta}$ will vary in accordance with the scale of concavity of the utility function. This means that, in strict sense, the shape of $\overline{\alpha}^h$ in Figure 4 will not be the same as the one from $\overline{\alpha}^j$. This, however, does not invalidate our prediction of a negative relationship between risk aversion and survival of the entrepreneur.

7 Conclusions

Why do some entrepreneurs wish to start new firms after having closed one that was not successful? This is the question which the present paper tries to tackle. Using information for the UK we showed the relevance of such a question: almost 44% of entrepreneurs who have closed a firm that was not successful start or buy another firm in the future. We also showed, amongst other cases, that some entrepreneurs despite having successful firms do not have the desire to own others firms again. In order to offer an explanation for this empirical evidence we proposed and developed a model whose principal assumption is that the profits of the small firm depend on two firm-specific parameters: the first one captures whether the opportunity being exploited is successful, what we call the demand parameter, and the second the skills of the owner to manage the firm efficiently. We assume such parameters are not correlated. On the contrary, we let the demand parameter to vary every time the entrepreneur starts a new firm.

To answer our initial question our model reveals a mechanism of entrepreneurial selection. Under this mechanism, entrepreneurs with skills above a specific threshold are the only ones who own another firm in the future after having closed one, regardless of the degree of success in their previous venture, whereas unskilful entrepreneurs prefer to switch to employment. For those entrepreneurs with skills large enough, this mechanism is consistent with the view that the closure of a business is only part of a process of searching and discovering profitable opportunities or the right stuff (Storey 1990, Audretsch 1995a). More crucially, we show that this mechanism also accounts for the empirical evidence on entrepreneurial survival provided at the outset of this paper. In particular, we find that the lower the specific skills threshold is, the higher the probability of survival of the entrepreneur is. Our theory also reveals that the skills threshold is idiosyncratic. That is, our theory predicts that low opportunities in the labour market, larger intrinsic tastes for entrepreneurship, longer periods in self-employment, over-optimism, and a lower degree of risk aversion negatively influence the skills threshold, and are therefore positively associated with the survival of the entrepreneur. These prediction are suitable to be tested by means of econometric analysis. This is a task that we leave for another study. On the other hand, since the general perception is that all these elements are positive associated with entrepreneurship, and since our definition of survival of the entrepreneur characterises the behaviour of serial entrepreneurs, i.e. those individual that after the closure of a business move to a new venture, our theory would suggest that amongst all entrepreneurs, serial entrepreneurs are the most entrepreneurial. Interesting research questions to emanate from here are whether the behaviour of serial entrepreneurs is associated with better firm performance on one the hand – e.g. innovation, profits, sales, job creation – and better (or worse) economic outcomes at regional, national or international levels on the other– for instance, *net* job creation , competition, efficiency gains, and economic growth .

Finally, this paper emphasises the need to differentiate between the survival of the small firm and the survival of the entrepreneur. Small firms are run by entrepreneurs. We say a small firm has not survived when it has stopped trading, though such an event does not necessarily mean the entrepreneur has not survived. The survival of the entrepreneur is the outcome of the choice between paid work or owning another firm, after having closed the previous one. Thus, future models of firms survival should explicitly differentiate between the firm and the entrepreneur, and acknowledge the fact that after the firm has closed the entrepreneur has the option of starting a new firm and therefore of staying in entrepreneurship.

Appendix. Proofs

Lemma 3. Let $\mathbb{M} = \int_0^\infty D(\alpha)\, dA(\alpha)$ and $\mathbb{N} = \int_0^\infty D(\alpha)^2\, dA(\alpha)$. Assume both integrals converge to positive numbers. Thus, there exist a continuum of value of α^{ij}, call it $\mathbb{A}$, where $\overline{\alpha} \in \mathbb{A}$ and supp$\mathbb{A} = \left(D^{-1}\left(\mathbb{N}^{1/2}\right), D^{-1}\left(\mathbb{M} + \left(\mathbb{N} - \mathbb{M}^2\right)^{1/2}\right)\right]$, such that $\Pi^*\left(\overline{\alpha}, \theta^j\right) = \underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] \; \forall \overline{\alpha}$.

Proof. From Lemma 1 $\Pi^*\left(\alpha^{ij}, \theta^j\right) = \frac{\left(bc - \theta^j D\left(\alpha^{ij}\right)\right)^2}{4b(\theta^j)^2}$, which we re-write as

$$\Pi^*\left(\alpha^{ij}, \theta^j\right) = \frac{bc^2}{4(\theta^j)^2} - \frac{cD\left(\alpha^{ij}\right)}{2\theta^j} + \frac{D\left(\alpha^{ij}\right)^2}{4b} \; .$$

Thus,

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] = \frac{bc^2}{4(\theta^j)^2} - \frac{c}{2\theta^j}\int_0^\infty D\left(\alpha^{ij}\right) dA(\alpha) + \frac{1}{4b}\int_0^\infty D\left(\alpha^{ij}\right)^2 dA(\alpha) \; ,$$

so

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] = \frac{bc^2}{4(\theta^j)^2} - \frac{c}{2\theta^j}\mathbb{M} + \frac{1}{4b}\mathbb{N} \; .$$

If any $\overline{\alpha} \in \mathbb{A}$ solves $\Pi^*\left(\overline{\alpha}, \theta^j\right) = \underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right]$, then it solves the equation

$$\frac{bc^2}{4(\theta^j)^2} - \frac{cD(\overline{\alpha})}{2\theta^j} + \frac{D(\overline{\alpha})^2}{4b} = \frac{bc^2}{4(\theta^j)^2} - \frac{c}{2\theta^j}\mathbb{M} + \frac{1}{4b}\mathbb{N} \; ,$$

or

$$\frac{D\left(\overline{\alpha}\right)^2}{4b}-\frac{cD\left(\overline{\alpha}\right)}{2\theta^j}+\frac{c}{2\theta^j}\mathbb{M}-\frac{1}{4b}\mathbb{N}=0\ .$$

The positive root which we obtain from here is

$$D\left(\overline{\alpha}\right)=\frac{bc+\left(\mathbb{N}\left(\theta^j\right)^2-2bc\mathbb{M}\theta^j+b^2c^2\right)^{1/2}}{\theta^j}\ , \tag{33}$$

where it can be shown that $\mathbb{N}\left(\theta^j\right)^2-2bc\mathbb{M}\theta^j+b^2c^2\geq 0$ for any $\theta^j\geq 0$. Therefore by virtue of the inverse-function theorem we get

$$\overline{\alpha}=D^{-1}\left(\frac{bc+\left(\mathbb{N}\left(\theta^j\right)^2-2bc\mathbb{M}\theta^j+b^2c^2\right)^{1/2}}{\theta^j}\right)\ . \tag{34}$$

Let $\overline{f}\left(\theta\right)\equiv\frac{bc+\left(\mathbb{N}\left(\theta^j\right)^2-2bc\mathbb{M}\theta^j+b^2c^2\right)^{1/2}}{\theta^j}$, then $\overline{\alpha}=D^{-1}\left(\overline{f}\left(\theta\right)\right)$. From expression (33) and employing the fact that $Var\left[D\left(\alpha\right)\right]=\mathbb{N}-\mathbb{M}^2>0$ it can be shown that $\frac{\partial D(\overline{\alpha})}{\partial\theta}<0$, and therefore $\frac{\partial\overline{\alpha}}{\partial\theta}=\frac{\partial\left(D^{-1}\left(\overline{f}(\theta)\right)\right)}{\partial\theta}<0$. Finally, by means of Lemma 1 note that $Q^*\left(\alpha^{ij},\theta^j\right)=\frac{D\left(\alpha^{ij}\right)}{2}-\frac{bc}{2\theta^j}\geq 0$, which is true for $\theta^j\geq\breve{\theta}=\frac{bc}{D(\alpha^{ij})}$, and taking expectation over α^{ij} we get $\underset{\alpha}{E}\left[\breve{\theta}\right]=\frac{bc}{\mathbb{M}}$, which represents a lower limit for the value of θ. Therefore, $D\left(\overline{\alpha};\theta=\frac{bc}{\mathbb{M}}\right)=\left(\mathbb{M}+\left(\mathbb{N}-\mathbb{M}^2\right)^2\right)$, and $\lim_{\theta\longrightarrow\infty}D\left(\overline{\alpha}\right)=\left(\mathbb{N}\right)^{1/2}$. Thus the support of $\mathbb{A}$ is $\left[D^{-1}\left(\mathbb{M}+\left(\mathbb{N}-\mathbb{M}^2\right)^2\right),D^{-1}\left(\mathbb{N}^{1/2}\right)\right)$.$\blacksquare$

Lemma 4. Let $\mathbb{M}=\int_0^\infty D\left(\alpha\right)dA\left(\alpha\right)$ and $\mathbb{N}=\int_0^\infty D\left(\alpha\right)^2dA\left(\alpha\right)$. Assume both integrals converge to positive numbers. Thus, under the conditions $\gamma^s\Pi_R^j\geq\frac{N-M^2}{4b}$ and $\Pi_R^j<\frac{N}{4b}$ there exist values of θ^j, which we name as θ_F and $\theta_{\min}$, which solve $\underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta_F\right)\right]=\Pi_R^j$ and $\underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta_{\min}\right)\right]=\gamma^s\Pi_R^j$, respectively.

Proof. Using $\Pi^*\left(\alpha,\theta_F\right)=\frac{(bc-\theta_FD(\alpha))^2}{4b(\theta_F)^2}$

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta_F\right)\right]=\frac{bc^2}{4(\theta_F)^2}-\frac{c}{2\theta_F}\mathbb{M}+\frac{1}{4b}\mathbb{N}\ .$$

If θ_F solves $\underset{\alpha}{E}\left[\Pi^*\left(\alpha,\theta_F\right)\right]=\Pi_R^j$, then it solves the equation

$$\frac{bc^2}{4(\theta_F)^2}-\frac{c}{2\theta_F}\mathbb{M}+\frac{1}{4b}\mathbb{N}-\Pi_R^j=0\ .$$

The positive root which we obtain from here is

$$\theta_F = \frac{bc}{\mathbb{N} - 4b\Pi_R^j}\left(\mathbb{M} + \left(\mathbb{M}^2 - \mathbb{N} + 4b\Pi_R^j\right)^{1/2}\right) \ ,$$

where we require $\Pi_R^j < \frac{\mathbb{N}}{4b}$, and $\mathbb{M}^2 - \mathbb{N} + 4b\Pi_R^j \geq 0$ or $\Pi_R^j \geq \frac{\mathbb{N}-\mathbb{M}^2}{4b}$ since $var\left[D\left(\alpha\right)\right] = \mathbb{N} - \mathbb{M}^2 > 0$. Similarly, $\theta_{\min}$ solves the equation

$$\frac{bc^2}{4(\theta_{\min})^2} - \frac{c}{2\theta_{\min}}\mathbb{M} + \frac{1}{4b}\mathbb{N} - \gamma^s\Pi_R^j = 0 \ .$$

The positive root which solve this equation is

$$\theta_{\min} = \frac{bc}{\mathbb{N} - 4b\gamma^s\Pi_R^j}\left(\mathbb{M} + \left(\mathbb{M}^2 - \mathbb{N} + 4b\gamma^s\Pi_R^j\right)^{1/2}\right) \ .$$

Following a similar procedure as above, we require $\gamma^s\Pi_R^j \geq \frac{\mathbb{N}-\mathbb{M}^2}{4b}$ and $\gamma^s\Pi_R^j < \frac{\mathbb{N}}{4b}$. Therefore since $\Pi_R^j > \gamma^s\Pi_R^j$ sufficient conditions for θ_F and $\theta_{\min}$ to exist are $\gamma^s\Pi_R^j \geq \frac{N-M^2}{4b}$ and $\Pi_R^j < \frac{N}{4b}$.

Proof of Proposition 3. Sufficient and necessary conditions which make the first term in (19) to be the largest are

$$\Pi^*\left(\alpha^{ij}, \theta^j\right) > \underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] \tag{35}$$

and

$$\Pi^*\left(\alpha^{ij}, \theta^j\right) > \gamma^s\Pi_R^j \ . \tag{36}$$

Whenever the following inequality is true

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] > \gamma^s\Pi_R^j \tag{37}$$

such conditions are satisfied. However we can have

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta^j\right)\right] < \gamma^s\Pi_R^j \tag{38}$$

and still satisfy them. From the last two expression and using Lemma 4 let $\theta_{\min}$ solve

$$\underset{\alpha}{E}\left[\Pi^*\left(\alpha, \theta_{\min}\right)\right] = \gamma^s\Pi_R^j \ . \tag{39}$$

If for subgroup M the first term in (19) is the largest, then from (35) and Lemma 1 and Lemma 3 we must have $\alpha^{ij} > \overline{\alpha}\ \forall j \in M$. From (39) and Lemma 2, any $\theta^j > \theta_{\min}$ satisfies

(37), which coupled with any $\alpha^{ij} > \overline{\alpha}$ also satisfies (36). We define $M_1 \subset M$ such that $\alpha^{ij} > \overline{\alpha}$, $\theta^j > \theta_{\min}$, $\forall j \in M_1$. Note that the left hand side of equation (39) contains the rewards in the wage worker state, i.e., it is the updated reservation level. Therefore, there must be an isoprofit curve representing this updated reservation level. As defined, let R represent the subgroup which contain the different combinations of α^{ij} and θ^j, respectively, which account for such isoprofit curve. That is they satisfy

$$\Pi^* \left(\alpha^r, \theta^r\right) = \gamma^s \Pi_R^j, \, \forall \left(\alpha^r, \theta^r\right) \in R \; . \tag{40}$$

Thus, for some $(\alpha^r, \theta^r) \in R$, say (α_-, θ_-) we have

$$\Pi^* \left(\alpha_-, \theta_-\right) = \gamma^s \Pi_R^j,$$

then from (39) we have that

$$\Pi^* \left(\alpha_-, \theta_-\right) = \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta_{\min}\right)\right] \; , \tag{41}$$

from Lemma 3 we know that

$$\Pi^* \left(\overline{\alpha}, \theta^j\right) = \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] \forall \overline{\alpha} \in \mathbb{A}, \theta^j \geq \breve{\theta} \; .$$

It can be shown that $\theta_{\min} \geq \breve{\theta}$, and since $\overline{\alpha}$ depends on the specific value of θ as shown by expression (34) we define $\overline{\alpha}_{\min} = D^{-1} \left(\overline{f} \left(\theta_{\min}\right)\right)$, which we call $\overline{\alpha}_{\min}$,

$$\Pi^* \left(\overline{\alpha}_{\min}, \theta_{\min}\right) = \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta_{\min}\right)\right]$$

and therefore

$$\Pi^* \left(\alpha_-, \theta_-\right) = \Pi^* \left(\overline{\alpha}_{\min}, \theta_{\min}\right) \; .$$

This means that the isoprofit curve represented by (40) crosses through the point $(\overline{\alpha}_{\min}, \theta_{\min})$. Lemma 5 below establishes that this isoprofit curve has negative slope and is convex to the origin. By means of (35), we have the restriction $\alpha^{ij} > \overline{\alpha}$ and given that the isoprofit curve (40) has negative slope, is convex to the origin, and crosses through $(\overline{\alpha}_{\min}, \theta_{\min})$, (38) and (36) are satisfied if and only if $\theta_{\min} > \theta^j > \theta^r$ and $\alpha^{ij} > \overline{\alpha}$. We define M_2 such that $\alpha^{ij} > \overline{\alpha}$, $\theta_{\min} > \theta^j > \theta^r$, $\forall j \in M_2$. Note that $M_1 \cap M_2 = \varnothing$ and $M_1 \cup M_2 = M$. This accounts for *(i)*.

Given that for subgroup D the second argument in (19) is the largest, sufficient and necessary conditions are $\underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] > \Pi^* \left(\alpha^{ij}, \theta^j\right)$ and $\underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] > \gamma^s \Pi_R^j$. By means of Lemma 1 and Lemma 3 the first inequality is satisfied whenever $\alpha^{ij} < \overline{\alpha}$, and making use of (39) and Lemma 2 the second inequality is satisfied whenever $\theta^j > \theta_{\min}$. Therefore, $\forall j \in D$, $\alpha^{ij} < \overline{\alpha}$ and $\theta^j > \theta_{\min}$. This accounts for *(ii)*.

For subgroup Z the third argument is the largest. Thus, the following are sufficient and necessary conditions for such a situation to take place

$$\Pi^* \left(\alpha^{ij}, \theta^j\right) < \gamma^s \Pi_R^j \tag{42}$$

and

$$\underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] < \gamma^s \Pi_R^j \tag{43}$$

where we can either have

$$\Pi^* \left(\alpha^{ij}, \theta^j\right) > \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] \tag{44}$$

or

$$\Pi^* \left(\alpha^{ij}, \theta^j\right) < \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] . \tag{45}$$

Using Lemma 2 and equation (39), condition (43) is satisfied whenever $\theta^j < \theta_{\min}$. By means of Lemma 1 and Lemma 3 (45) is met for $\alpha^{ij} < \overline{\alpha}$, and coupled with $\theta^j < \theta_{\min}$ implies that condition (42) is also satisfied. We define Z_1 such that $\forall j \in Z_1$, $\alpha^{ij} < \overline{\alpha}$ and $\theta^j < \theta_{\min}$. If both (42) and (44) are met, then condition (43) is satisfied. Thus, (44) is true whenever $\alpha^{ij} > \overline{\alpha}$. Given this restriction and since the left-hand-side term of (42) represent a level of profits equivalent to the isoprofit curve depicted by (40), which in turn is characterised by Lemma 5, condition (42) is satisfied if and only if $\theta^j < \theta^r < \theta_{\min}$. We define Z_2 such that $\forall j \in Z_2$, $\alpha^{ij} > \overline{\alpha}$ and $\theta^j < \theta^r < \theta_{\min}$. Note that $Z_1 \cap Z_2 = \varnothing$, and $Z_1 \cup Z_2 = Z$. This accounts for *(iii)*.

An entrepreneur in subgroup M will be willing to sell his current firm if and only if the offered price is higher than the profits of his current firm. That is, if the price is larger that the first term in expression (19).Once he has sold his business the problem for entrepreneurs in subgroup M becomes

$$V_M = \max \left\{ \underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right], \gamma^s \Pi_R^j \right\} , \tag{46}$$

where the first argument are the expected profits from starting a new firm, and the second the value of going to waged work. The entrepreneur will start another firm if $\underset{\alpha}{E} \left[\Pi^* \left(\alpha, \theta^j\right)\right] > \gamma^s \Pi_R^j$, from which we can obtain equation (39). Thus for $\theta^j > \theta_{\min}$ the first argument in (46) is the largest and for $\theta^j < \theta_{\min}$ the second argument in (46) is the largest. Note that

$\theta^j > \theta_{\min}\ \forall j \in M_1 \subset M$, $\theta_{\min} > \theta^j > \theta^r\ \forall j \in M_2 \subset M$, $M_1 \cap M_2 = \varnothing$ and $M_1 \cup M_2 = M$, therefore the second argument in expression (19) is now the largest for subgroup M_1, and the third argument the largest for subgroup M_2. This accounts for the first part of *(iv)*. Now, assume that α is subject to unforeseen shocks, that is

$$\tilde{\alpha}^{ij} = \alpha^{ij} + \epsilon$$

where the expected value of ϵ before starting a new firm is assumed to be zero, but it changes once the entrepreneur's owns the new firm and the entrepreneurs does not know the new expected value nor the distribution of ϵ. Substituting $\tilde{\alpha}^{ij}$ for α^{ij} in expression (19) and assume there is a shock which makes $\tilde{\alpha}^{ij} < \overline{\alpha}$, then condition (35) is no longer met. For subgroup M_1 this situation is equivalent to the one for subgroup D for which the second argument in (19) is the largest, because $\theta^j > \theta_{\min}\ \forall j \in (M_1 \cup D)$, and $\tilde{\alpha}^{ij} < \overline{\alpha}\ \forall j \in D$. However, for subgroup M_2 the situation is equivalent to the one for subgroup Z_1 for which the third argument in (19) is the largest, because $\theta^j < \theta_{\min}\ \forall j \in (M_1 \cup Z_1)$ and $\tilde{\alpha}^{ij} < \overline{\alpha}$ $\forall j \in Z_1$. This completes the proof. ■

Lemma 5. The isoprofits curves of the (maximum expected) profits function in the space (α, θ) have negative slope and are convex to the origin.

Proof. For simplicity all superscripts and subscripts are dropped. The slope of the isoprofits is determined by

$$\frac{\partial \theta}{\partial \alpha} = -\frac{\partial \Pi\left(\alpha, \theta\right) / \partial \alpha}{\partial \Pi\left(\alpha, \theta\right) / \partial \theta} .$$

From the proof of Lemma 1 and Lemma 2 we obtain,

$$\frac{\partial \theta}{\partial \alpha} = -\frac{\theta^2}{bc} \cdot \frac{\partial D\left(\alpha\right)}{\partial \alpha} .$$

From assumption A.3 we have that $\frac{\partial D(\alpha)}{\partial \alpha} > 0$, therefore $\frac{\partial \theta}{\partial \alpha} < 0$. The convexity of the curves is established by

$$\frac{\partial^2 \theta}{\partial \alpha^2} = -\frac{\theta^2}{bc} \cdot \frac{\partial^2 D\left(\alpha\right)}{\partial \alpha^2} .$$

From assumption A.3 we have that $\frac{\partial^2 D(\alpha)}{\partial \alpha^2} < 0$. Therefore, $\frac{\partial^2 \theta}{\partial \alpha^2} > 0$ and the isoprofits are convex to the origin. ■

References

[1] ACS, Z. and D. B. AUDRETSCH (1989) "Small Firms in U.S. Manufacturing: A First Report", *Economics Letters*, vol. 31 (4), pp. 399-402.

[2] ACS, Z. and D. B. AUDRETSCH (1990) "Small Firms in the 1990s", in Zoltan Acs and David Audretsch (eds.), *The Economics of Small Firms: A European Challenge*, Dordrecht: Kluwer Academic Publishers.

[3] ACS, Z. J., P. ARENIUS, M. HAY, and M. MINNITTI (2004) *Global Entrepreneurship Monitor: 2004 Executive Report*, Babson College and London Business School.

[4] AGARWAL, R. and D. B. AUDRETSCH (1999) "The Two Views of Small Firms in Industry Dynamics: A Reconciliation", *Economics Letters*, vol. 62 (2), pp. 245-251.

[5] AGARWAL, R. and D. B. AUDRETSCH (2001) "Does Entry Size Matters? The Impact of the Life Cycle and Technology on Firm Survival", *Journal of Industrial Economics*, vol. 49 (1), pp. 21-43.

[6] AGARWAL, R. and M. GORT (1996) "The Evolution of Markets and Entry, Exit and Survival of Firms", *Review of Economics and Statistics*, vol. 78 (3), pp. 489-498.

[7] AUDRETSCH, D. B. (1991) "New-Firm Survival and the Technological Regime", *Review of Economics and Statistics*, vol. 73 (3), pp. 441-450.

[8] AUDRETSCH, D. B. (1995) *Innovation and Industry Evolution*. Cambridge, MA.: The MIT Press.

[9] AUDRETSCH, D. B., L. KLOMP and R. THURIK (1999) "Does Service Differ from Manufacturing? The Post-entry Performance of Firms in Dutch Services" in Audretsch and Thurik (eds.) *Innovation, Industry Evolution and Employment*, Cambridge, UK: Cambridge University Press.

[10] BATES, T. (1990) "Entrepreneur Human Capital Inputs and Small Business Longevity", *Review of Economics and Statistics*, vol. 72 (4), pp. 551-559.

[11] BIRLEY, S. and P. WESTHEAD (1993) "A Comparison of New Businesses Established by 'Novice' and 'Habitual' Founders in Great Britain", *International Small Business Journal*, vol. 12 (1), pp. 38-60.

[12] BLANCHFLOWER, D. G. and A. J. OSWALD (1998) "What Makes an Entrepreneur?", *Journal of Labour Economics*, vol. 16 (1), pp. 26-60.

[13] BLANCHFLOWER, D. G. and MEYER, B. D. (1994) "A Longitudinal Analysis of the Young Self-Employed in Australia and the United States", *Small Business Economics*, vol. 6 (1), pp. 1-19.

[14] BURNS, P. (2001) *Entrepreneurship and Small Business*, Palgrave.

[15] CHAMBERLIN, E. H. (1933) *The Theory of Monopolistic Competition*. Cambridge, MA: Harvard University Press.

[16] COOPER, A. C., C. Y. WOO, and W. C. DUNKELBERG (1988) "Entrepreneurs' Perceived Chances for Success", *Journal of Business Venturing*, vol.3 (2), pp. 97-108.

[17] CRESSY, R. (1996) "Are Business Startups Debt-Rationed?", *Economic Journal*, vol. 106 (438), pp. 1253-1270.

[18] DELMAR, F. (2001) "The Phsycology of the Entrepreneur", in Sara Carter and Dylan Jones- Evans (eds), *Enterprise and Small Business*, FT/ Prentice Hall.

[19] DE MEZA, D. and C. SOUTHEY (1996) "The Borrower's Curse: Optimism, Finance and Entrepreneurship", *Economic Journal*, vol. 106 (435),. pp. 375-386.

[20] EVANS, D. S. and B. JOVANOVIC (1989) "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints", *Journal of Political Economy*, vol. 97 (4), pp. 808-827.

[21] EVANS, D. S. and L. S. LEIGHTON (1989) "Some Empirical Aspects of Entrepreneurship", *American Economic Review*, vol. 79 (3), pp. 519-535.

[22] FICHMAN, M. and D. LEVINTHAL (1991) "Honeymoons and the Liability of Adolescence: A New Perspective on Duration Dependence in Social and Organizational Relationships", *Academy of Management Review*, vol.16 (2), pp. 442-468.

[23] FRANK, M. Z. (1988) "An Intertemporal Model of Industrial Exit", *Quarterly Journal of Economics*, vol. 103 (2), pp. 333-344.

[24] GEROSKI, P.A. (1995) "What Do We Know About Entry?", *International Journal of Industrial Organization*, vol. 13 (4), pp. 421-40.

[25] GIFFORD, S. (1993) "Heterogeneous Ability, Career Choice and Firm Size", *Small Business Economics*, vol. 5 (4), pp. 249-259.

[26] GIMENO, J., T. B. FOLTA, A. C. COOPER, and C. Y. WOO (1997) "Survival of the Fittest? Entrepreneurial Human Capital and the Persistence of Underperforming Firms", *Administrative Science Quarterly*, vol. 42 (4), pp. 750-783.

[27] GLADSTONE , B. and J. L. LEE (1995) "The Operation of the Insolvency System in the U.K.: Some Implications for Entrepreneurialism", *Small Business Economics*, vol. 7 (1), pp. 55-66.

[28] HEADD, B. (2003) "Redefining Business Success: Distinguishing Between Closure and Failure", *Small Business Economics*, vol. 21 (1), pp. 51-61.

[29] HART, O. D. (1985) "Monopolistic Competition in the Spirit of Chamberlin: A General Model", *Review of Economic Studies*, vol. 52 (4), pp. 529-546.

[30] HOLMES T. J. and J. A. SCHMITZ (1990) "A Theory of Entrepreneurship and its Application to the Study of Business Transfers", *Journal of Political Economy*, vol. 98 (2), pp. 265-294.

[31] HOLMES T. J. and J. A. SCHMITZ (1995) "On the Turnover of Business Firms and Business Managers", *Journal of Political Economy*, vol. 103 (5), pp. 1005-1038.

[32] HOLMES T. J. and J. A. SCHMITZ (1996) "Managerial Tenure, Business Age, and Small Business Turnover", *Journal of Labour Economics*, vol. 14 (1), pp. 79-99.

[33] HOLTZ-EAKIN, D., D. JOULFAIAN, and H. S. ROSEN (1994) "Sticking it Out: Entrepreneurial Survival and Liquidity Constraints", *Journal of Political Economy*, vol. 102 (1), pp. 53-75.

[34] JEHLE, G. A. and P. J. RENY (2001) *Advanced Microeconomic Theory*. Boston: Addison Wesley.

[35] JENNINGS, P. and G. BEAVER (1997) "The Performance and Competitive Advantage of Small Firms: A Management Perspective", *International Small Business Journal*, vol. 15 (2), pp. 63-75.

[36] JOVANOVIC, B. (1982) "Selection and the Evolution of Industry", *Econometrica*, vol. 50 (3), pp. 649-670.

[37] KALLEBERG, A. L. and K. T. LEICHT (1991) "Gender Organizational Performance: Determinants of Small Business Survival and Success", *Academy of Management Journal*, vol. 34 (1), pp. 136-161.

[38] KIHLSTROM R. E. and J. J. LAFFONT (1979) "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion", *Journal of Political Economy*, vol. 87 (4), pp. 719-748.

[39] KNIGHT, F. H. (1921) *Risk, Uncertainty and Profits*. Boston: Houghton Mifflin.

[40] MACMILLAN, I. C. (1986) "To Really Learn about Entrepreneurship, Let's Study Habitual Entrepreneurs", *Journal of Business Venturing*, vol. 1 (3), pp. 241-243.

[41] MANOVE, M. (2000) "Entrepreneurs, Optimism and the Competitive Edge", mimeo, Boston University and CEMFI.

[42] MARSHALL, A. (1930) *Principles of Economics*. London: Macmillan and Co.

[43] MATA, J. and J. A. P. PORTUGAL (1994) "Life Duration of New Firms", *Journal of Industrial Economics*, vol. 42 (3), pp. 227-245.

[44] PARKER, S. C. (2004) *The Economics of Self-Employment and Entrepreneurship*. Cambridge, UK: Cambridge University Press.

[45] PARKER, S. C. (2006) "New Agendas in the Economics of Entrepreneurship: Optimism, Education, Wealth and Entrepreneurship", American Economics Association Conference.

[46] ROSA, P. (1998) "Entrepreneurial Process of Business Cluster Formation and Growth by 'Habitual' Entrepreneurs", *Entrepreneurship Theory and Practice*, vol. 22 (4). pp. 43-61.

[47] SHANE, S. and S. VENKATARAM (2000) "The promise of Entrepreneurship as a Field of Research", *Academy of Management Review*, vol. 25 (1), pp. 217-226.

[48] STEVENSON, H. H. and J. C. JARILLO (1990) "A Paradigm of Entrepreneurship: Entrepreneurial Management", *Strategic Management Journal*, vol. 11 (5), pp. 17-27.

[49] STOKES, D. and R. BLACKBURN (2001) *Opening up Business Closures: a Study of Businesses that Close and Owners' Exit Routes*. A Research Report for HSBC, Small Business Research Centre, Kingston University.

[50] STOREY, D. J. (1994) *Understanding the Small Business Sector.* London: Routledge.

[51] STOREY, D. J. (1990) "Firm Performance and Size", in Zoltan Acs and David Audretsch (eds.), *The Economics of Small Firms: A European Challenge*, Dordrecht: Kluwer Academic Publishers.

[52] STOREY, D. J. and P. WYNARCZYK (1996) "The Survival and Non Survival of Micro Firms in the UK", *Review of Industrial Organization*, vol. 11 (2), pp.211-229.

[53] TAYLOR, M. P. (1999) "Survival of the Fittest? An Analysis of Self-Employment Duration in Britain", *Economic Journal*, vol. 109 (454), pp. 140-155.

[54] TUCKER, I. B. (1988) "Entrepreneurs and Public-sector Employees: The Role of Achievement Motivation and Risk in Occupational Choice", *Journal of Economic Education*, vol. 19 (3), pp. 259-268.

[55] UCBASARAN, A. D. (2004) *Business Ownership Experience, Entrepreneurial Behaviour and Performance: Novice, Habitual, Serial and Portfolio Entrepreneurs.* Ph.D. Dissertation, University of Nottingham.

[56] USITALO, R. (2001) "Homo Entreprenaurus?", *Applied Economics*, vol. 33 (13), pp. 1631-1638.

[57] van PRAAG, C. M. (1999) "Some Classics Views on Entrepreneurship", *De Economist*, vol. 147 (3), pp. 311-335.

[58] van PRAAG, C. M. (2003) "Business Survival and Success of Young Small Business Owners", *Small Business Economics*, vol. 21 (1), pp. 1-17.

[59] van PRAAG, C. M. and J. S. CRAMER (2001) "The Roots of Entrepreneurship and Labour Demand: Individual Ability and Low Risk Aversion", *Economica*, vol. 68 (269), pp. 45-62.

[60] VARIAN, H. R. (1992) *Microeconomic Analysis.* New York: W. W. Norton & Company.

[61] WESTHEAD, P. and M. WRIGHT (1998) "Novice, Portfolio, and Serial Founders: Are They Different?", *Journal of Business Venturing*, vol. 13 (3), pp. 173-204.

[62] WESTHEAD, P., D. UCBASARAN, and M. WRIGHT (2003) "Differences between Private Firms Owned by Novice, Serial, and Portfolio Entrepreneurs: Implications for Policy Makers and Practitioners", *Regional Studies*, vol. 37 (2), pp. 187-200.

[63] WESTHEAD, P., D. UCBASARAN, M. WRIGHT, and M. BINKS (2005) "Novice, Serial, and Portfolio Entrepreneurs Behaviour and Contributions", *Small Business Economics*, vol. 25 (1), pp. 109-132.

[64] WHITE, L. J. (1982) "The Determinants of the Relative Importance of Small Business", *Review of Economics and Statistics*, vol. 64 (1), pp. 42-49.