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**The effects of forecasting errors and mis-specification**

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This paper represents a long-standing collaboration between Robert Fildes and Brian Kingsman who sadly died before it could be completed.

# **DEMAND UNCERTAINTY and LOT SIZING IN MANUFACTURING SYSTEMS:**

## **The effects of forecasting and mis-specification**

**Robert Fildes and Brian Kingsman**

This paper proposes a methodology for examining the effect of demand uncertainty and forecast error on lot sizing methods, unit costs and customer service levels in MRP type manufacturing systems. A number of cost structures were considered which depend on the expected time between orders. A simple two-level MRP system where the product is manufactured for stock was then simulated. Stochastic demand for the final product was generated by two commonly occurring processes and with different variances. Various lot sizing rules were then used to determine the amount of product made and the amount of materials bought in.

The results confirm earlier research that the behaviour of lot sizing rules is quite different when there is uncertainty in demand compared to the situation of perfect foresight of demand. The best lot sizing rules for the deterministic situation are the worst whenever there is uncertainty in demand. In addition the choice of lot sizing rule between 'good' rules such as the EOQ turns out to be relatively less important in reducing unit cost compared to improving forecasting accuracy whatever the cost structure. The effect of demand uncertainty on unit cost for a given service level increases exponentially as the uncertainty in the demand data increases. The paper also shows how the value of improved forecasting can be analysed by examining the effects of different sizes of forecast error in addition to demand uncertainty. In those manufacturing problems with high forecast error variance, improved forecast accuracy should lead to substantial percentage improvements in unit costs.

Key Words: forecast error / MRP/ lot sizing/ buffer stocks and service levels/ demand uncertainty/ value of forecasting/ manufacturing systems/ forecast mis-specification

## 1. INTRODUCTION

MRP systems are in widespread use for co-ordinated production and operations planning in manufacturing systems. These typically assume a deterministic environment with the emphasis on co-ordinating production at all levels and the purchasing of components and materials so as to meet the planned Master Production Schedule (MPS). The MPS provides the linkage between the demands of the outside customers and the detailed planning and scheduling of internal production supply facilities. The MPS, once determined, is passed onto the material requirements planning system. Uncertainty can affect the performance of the MRP system in a variety of ways. Random variations can occur in internal manufacturing lead times, in purchasing lead times or stochastic yield variations at production stages. Since these latter activities mainly fall within the span of control of the company management, they can be managed in order to reduce significantly the amount and impact of uncertainty. External demand from customers is however normally outside the company's control, except for gross influences like promotions, and is stochastically varying over time in most practical situations. Thus the MPS has to be developed on the basis of forecasts of demand.

The scheduled production will either be insufficient to meet the realised actual demand, thus producing shortages, or exceed the realised demand resulting in unanticipated stocks. As several authors have pointed out, for example Vargas and Dear (1991), the effect of demand uncertainty on simple deterministic lot-sizing techniques, and their corresponding effectiveness, such as are commonly used in MRP applications, is an important problem. While all researchers seem to agree with the conclusions of De Bodt and Van Wassenhove (1983a) and others that demand uncertainty dramatically increases lot-sizing costs, the simple questions, of how those costs increase with forecasting error and the relationship to demand uncertainty, how different lot sizing rules perform and whether the form of demand pattern and the forecasting model used are important, remain unresolved. The contribution made in this paper is thus to integrate two distinct research areas, manufacturing systems and forecasting.

In order to investigate this problem researchers must make a number of choices, (i) on the form of the MRP system, including the number of levels and the interdependence within the production process, production/ delivery lead times, and back-ordering rules, (ii) the lot sizing rules to be considered and their application at differing levels of the MRP system, and, (iii) the costing of the various activities, finished inventory, work in progress, ordering, and service level. Particularly

important issues for research in this area, as explored further in the next section, are the form of the demand and forecast functions. These issues include the levels of forecast error to be included in the experimental design, and how possible bias is to be treated. It is questionable whether previous published research has treated these issues correctly.

Researchers have tackled the problem of uncertainty in MRP systems in two distinct ways: firstly, through case studies with changes in forecast errors arising from the use of different forecasting methods (Gardner, 1990, Lee et al., 1993, Enns, 2002 ) and, secondly, through developing a simulation of the whole MRP system, including the demand distribution and forecast errors. This paper adopts the second approach with the aim of developing a satisfactory methodology for quantifying the effects of forecast error in manufacturing systems.

The production system simulated here is composed of two stages, a raw material (or a component) having a purchasing and delivery lead time and a manufacturing process (or assembly) to make the final product from the material ( or component), requiring one or more periods to be completed. The major areas of interest in the simulation experiment are the costs of producing the final product and the customer service level achieved, together with the effect of demand forecast errors. The research presented follows most earlier work in assuming the only stochastic variation is in the demand for the final products. It is assumed that internal manufacturing lead times and purchasing lead times are fixed and also that internal yields are always 100%. The components are pushed up to the final level as scheduled. They are not delayed because the demand for the final product has changed. All variation is absorbed into the stock of finished items. The methodology we describe here extends straightforwardly to more complex production systems with manufacturing uncertainty, interdependent demand and capacity constraints. This paper is in six further sections. We next consider demand uncertainty and appropriate ways of specifying the forecast error. Section three describes the simulated manufacturing system including costs and lot sizing rules while section four describes the simulation experiment. The substantive results of the simulation are presented in section 5 for differing levels of demand uncertainty. Section 6 considers the effects of different forms of forecast error and forecasting model misspecification (in addition to the demand uncertainty). The paper concludes with a discussion of the results, both substantive and methodological.

The results provide conclusive evidence that the forms of the demand and forecast functions impact on the relative performance of the plausible lot sizing rules, and secondly, the absolute size of

the impact is both economically significant for typical values of the demand parameters and increases substantially as forecast error increases. Estimates of the effects of forecast error are included - improvements in forecasting accuracy are worth achieving only if the absolute level of accuracy improves markedly. In addition, the form of the demand function and any specification error is shown to be significant. Equally important, the paper demonstrates an effective methodology for investigating the effects of forecast error on manufacturing systems.

## **2. DEMAND UNCERTAINTY AND FORECAST ERRORS IN MANUFACTURING SYSTEMS**

Many studies have evaluated the performance of simple deterministic lot-sizing rules in manufacturing systems and the effects of uncertainty on the various rules have also generated considerable interest, with Wemmerlov (1989) offering an early summary. The problems with the earlier research include a limited range of lot sizing rules, (Vargas and Dear, 1991), and a lack of clarity in distinguishing between demand uncertainty and forecast error, (De Bodt and Van Wassenhove, 1983a, Lee and Adam, 1986). The lot sizing rules examined by these researchers included those commonly adopted in organisations - more complex stochastic rules that take into account the demand distribution were excluded. While much of the research analysed a simple production system (e.g. De Bodt and Wassenhove, 1983b, Lee and Adam, 1986, Ritzman and King, 1993, Chen et al, 2000a) a few researchers have considered MRP systems closer to the complexity observed in practice e.g. Vargas and Dear, 1991, Enns, 2002. The problem with the more complex studies is that the quantitative results are likely to be specific to the distinctive features of the chosen production system. More particularly, unless due care is taken in choosing the criteria by which performance is evaluated, the results will follow automatically from the choice of cost parameters. For example, if service level is not taken into account (Lee and Adams, 1986) then lot-sizing rules which lead to low inventories and correspondingly low service levels will automatically be preferred.

While not discounting the need to examine complex systems we will follow the approach adopted by Wemmerlov and Whybark (1984) and Wemmerlov (1989). These studies examined a single level, single product production system in a rolling horizon environment. The conclusion of Wemmerlov's research (1989) gave further support to De Bodt and Van Wassenhove (1983a) - that an inventory system facing uncertain demand is fundamentally different to one with a known demand and

that results from the latter case will not apply in most practical situations. He also found no significant difference in performance between the six best lot-sizing rules when demand was uncertain, concluding that this diminished the value of lot-sizing research. (However some other common rules performed poorly; nor was economic impact considered).

The inadequate incorporation of service level has led to further confusion in past research: failing to distinguish between the (unknown) bias in the forecast, and a buffering strategy of forecast inflation that aims to moderate the effects of forecast errors by increasing the forecast (Lee and Adam, 1986). Such a buffering strategy has long been criticised as inadequate in simple production systems when compared to the alternative of using safety stocks, e.g. Vargas and Dear (1991).

In an attempt to overcome the major problem of service level associated with most of the previous work Wemmerlov (1989) partially controlled for service level by using a buffer to ensure that the service level was 100%. In the research described here we go further by considering a two level production system including service level as a continuous variable. This permits us to evaluate the effects of realistic improvements in forecasting accuracy on differing levels of service and unit costs.

## **2.1 Demand Models Used In MRP Research**

Earlier authors when examining the effects of uncertainty on MRP systems have considered only limited models of the demand generation process and forecast function. De Bodt and Van Wassenhove (1983a, 1983b) compared various lot sizing rules when demand was generated by  $d_t = \mu_d + \varepsilon_t$  where  $\varepsilon_t$  is  $N(0, \sigma^2)$  and  $\mu_d$  is constant. The demand forecasts were derived using exponential smoothing. They concluded that with any level of demand uncertainty greater than zero total costs were increased about 10%; the impact was highest when ordering costs were low compared to inventory costs (and the time between orders was therefore short). Also, the choice of lot-sizing rule was unimportant with the exception of Lot-for-Lot which performed uniformly poorly. Mean demand was set at 200 and  $\sigma$  varied from 0 to 50 giving corresponding coefficients of variation (CV) of 0 to 0.25. The MRP system was such that current demand was known and could be immediately supplied. There were therefore no stockouts. This key assumption limits the usefulness of their work. They also fail to match their generation process for demand (random about a constant level) to their forecasting method (exponential smoothing which is non-stationary). This is a common mistake (see for example, Newbold and Bos (1989) and Chatfield and Koehler (1991)), compounding forecasting error with mis-

specification error which makes interpretation of their results difficult.

Lee and Adam (1986) and Lee, Adam and Ebert (1987) examined more complex MRP systems. Demand was generated by  $d_t = \mu_d$ , a constant while the forecasts were seen as randomly distributed around  $\mu_d$ , as  $N(\delta, \sigma^2)$  where  $\delta$  is the bias in the forecast. Lot-for-Lot again performed poorly. A wide range for the coefficient of variation was considered. Despite the wide range, the error standard deviation had little effect on performance while the bias was evaluated as much more important. However the more extreme values within Lee and Adam's simulation and the specification of the demand generating function can be dismissed as unrealistic, both in form and for their choice of error distribution parameters, see Fildes and Beard (1992) and Fildes et al (2004). The conclusion they drew that the bias was the more important factor again arises from the cost parameters considered. More recently, research has focussed on bullwhip effects and supply chain coordination, (see for example, Chen et al, 2000a, 2000b, Xu et al, 2001 Zhao et al, 2001). While more care has been taken in specifying both the DGP and the forecasting model one of their fundamental and unanalysed assumptions is that the forecasting model used by the manufacturer is mis-specified.

## 2.2 Independent Demand and Corresponding Forecasting Methods

Although a plethora of different forecasting methods have been proposed as suitable for forecasting independent demand, recent large scale empirical studies have concluded that a limited range of simple methods perform as well as their more complex alternatives, (Fildes and Makridakis, 1995, Makridakis et al, 2000, Fildes et al, 2004). Particularly for monthly or weekly data on item demand, simple autoregressive statistical models should adequately characterise most of the situations encountered in practice. Two simplifications from the typical series observed in practice have been made here: extreme (or outlying) observations have been neglected and the data are assumed to be non-seasonal. The effects of these assumptions require further research. We therefore propose to analyse the following demand model, where  $D_t$  is the demand in period  $t$  and  $e_t$  is random white noise:

$$D_t = \delta + \rho D_{t-1} + e_t - \theta e_{t-1}$$

This model, which is a realistic generalisation of earlier work, includes a wide range of data types that are commonly observed in practice as the forecasting literature has established, see for example, Fildes et al (2004). For  $\theta=\rho=0$  the model is just the 'constant mean demand model' (contrast the Lee and Adam's formulation described earlier) while for  $\rho=1$  and  $\theta < 1$  the model is the ARIMA (0,1,1) model for which exponential smoothing is the optimal forecasting method. For  $\rho < 1$  and  $\theta=0$



the model is a stationary AR(1) process. The above model is the basis of the most common forecasting methods used in production systems.

The optimal one-step ahead forecast of demand for period  $t$ , made in period  $t-1$  after that demand is known, for the above data generation process is given by:

$$opt F_{t-1,t} = \delta + \rho D_{t-1} - \theta e_{t-1}$$

The error term,  $e_{t-1}$ , is the forecast error observed in the previous period, the difference between the realised demand and the forecast and, at time  $t$ , is non-stochastic. In general, of course, such an optimal forecast would be unavailable to the scheduler, not least because the parameters would have to be estimated from the data. In this study we assume the parameters are known exactly; the only difference between the optimal forecast and the data generation process arises because of the random error,  $e_t$ .

Two particular demand models are considered in the research as follows:

- (i)  $\delta = 1000, \rho = 0, \theta = 0$ : the constant demand model, with mean level, 1000, hence

$$opt F_{t-1,t} = 1000 \quad \text{and} \quad D_t = 1000 + e_t$$

- (ii)  $\delta = 100, \rho = 0.9, \theta = 0$ : an almost non-stationary process, with mean level 1000, hence

$$opt F_{t-1,t} = 100 + 0.9D_{t-1} \quad \text{and} \quad D_t = 100 + 0.9D_{t-1} + e_t$$

The distribution assumed for  $e_t$  includes a range of values for the noise variance but the mean is assumed to be zero. The reason for this choice is that a constant bias model (such as that used by Lee and Adam) would be forecastable over time. The noise distribution is assumed normal (although demand and the forecasts of demand are constrained to be positive). Additional research is needed to examine the effects of non-normal errors and outlying observations which are often observed in demand data. The general form for the forecast error variance for an AR(1) model for  $k$  steps ahead is :

$$\text{var } e_t(k) = \frac{(1 - \rho^{2k})}{(1 - \rho^2)} \sigma^2$$

For the constant demand model, with  $\theta = 0$ , the forecast error variance is constant for all steps ahead. For the AR( $\rho=0.9$ ) model the standard deviation is increased to  $1.345\sigma$  for 2 steps ahead,  $1.943$  for 6

steps ahead and 2.201 for 12 steps ahead.

The complete specification of this data generation process depends on the values chosen for the noise variance. The values used in the experiment represent a plausible range of values for fast moving items according to surveys of forecasting accuracy reported in the literature (Fildes and Beard, 1992; Fildes et al, 2004). For one-step ahead (3 months ahead or less) forecasts, the average mean absolute percentage error (MAPE) was found to be 16%. For a normal distribution this roughly translates into a coefficient of variation of 0.20. In the simulation experiment we will therefore use values of the coefficient of variation ranging from 0 to 0.4. For  $e_t$  assumed Normal the range of values of the noise standard deviation included in the simulation experiment are therefore: 0, 50, 100, 200, 400 (around a mean of 1000).

### **3. THE PRODUCTION AND LOT SIZING ENVIRONMENT**

#### ***3.1 Planning Horizon and Action Horizon***

In uncertain environments, MRP is usually developed using a rolling schedule. When using a rolling schedule, the production schedule is replanned at regular intervals. Decisions are made taking into account the forecast demands over some future fixed planning horizon. In each successive period, lot-sizing decisions are made to minimise costs over the planning horizon ahead from that period. Orders for raw material then are placed and production occurs. An ordering decision by the manufacturer now leads to the scheduled arrival of that order some lead time into the future. Here it is assumed that once orders have been placed, they cannot be cancelled or altered in any way. Thus a portion of the production schedule determined for the planning horizon is frozen each planning cycle. This will include all those periods with the scheduled arrival of an outstanding order placed in previous periods. This frozen portion will be at least the maximum lead time,  $L$ , to produce the item through all of the possible stages of purchasing and manufacturing. It may also include a further action horizon, a length of time, of say  $A$  periods, over which any recommended orders are made firm orders, which cannot then be changed in later periods. Given the assumptions of constant processing and lead times, only the total length of the frozen period matters, not how the frozen period is decomposed into the maximum manufacturing lead time and the action horizon.

The planner can only make changes to ordering decisions for that part of the planning horizon beyond the frozen period, the free planning horizon. The length of the free horizon dictates the

number of periods over which demands can be batched into different lot sizes. The longer it is the greater the possibility of reducing costs by allowing bigger orders where advantageous. The total planning horizon is the frozen plus free periods. The simulation model allows different values for both the frozen and free horizons to be examined. The experiments covered values of 2, 5 and 7 periods for the frozen period and 12, 24 and 36 for the free horizon.

### **3.2 Costs**

The costs included in this experiment consist of the cost of placing orders and the cost of holding stock. There are two major measures of performance, (i) the average ordering and stockholding costs per unit of demand and (ii) the service level achieved, defined as the proportion of demand satisfied immediately either from the opening stock or the current period's output.

Since most of the deterministic lot-sizing rules aggregate the net requirements over future time periods to give the lot sizes, the expected time between successive orders is an appropriate method of ranking alternative sets of cost parameters, the ordering cost and (end of period) stockholding cost. This was determined from the order cycle given for the classical Economic Order Quantity (EOQ), based on the average demand per period implied by the demand generation processes used. It is often known as the Time Between Orders measure, TBO. The stockholding cost was set at 1 and the expected average period demand at 1000. Ordering cost values were chosen to give TBO values, measured in periods, in the middle of the ranges 1 - 2, 2 - 3, 3 - 4 and 4 - 5. These values imply the order would be expected to cover 1, 2, 3, 4 and 5 periods' demands respectively. The corresponding values for the ordering cost will then be 1750, 3500, 7000, 11500. (A TBO of 1 was considered but has quite different performance characteristics with EOQ incurring extra stockholding costs – this has been omitted from the analysis.)

### **3.3 Service Level and Safety Stocks**

A service level performance measure has been used rather than including stockout costs as part of the total cost, since the latter are almost impossible to estimate in any meaningful way in practice. We adopted the approach of letting the service level, here defined as the fraction of total demand satisfied immediately from stock and current output, be a factor in the experiments. This allowed us to examine the trade-offs between costs and service level. Clearly, under the same cost parameters and

forecast error conditions, different lot sizing rules result in different performance costs and different service levels. In order to render the results comparable, the models were run with different levels of target end of period stocks. If this target level is a positive quantity then this is equivalent to holding safety stock. Lower service levels are only possible by deliberately forcing the system on average to have stockouts. This can be achieved by allowing backorders or lost sales up to some pre-set level, i.e. by making the safety margin negative.

### 3.4 Lot Sizing Model for Backorders and Lost Sales Case.

As discussed above, the existence of an action horizon,  $A$ , means that at any time,  $t$ , recommended orders to be placed within the next  $A$  periods are made up of firm orders which cannot be changed at a later time. Thus at time  $t$ , orders already placed for periods  $t+1$  to  $t+A-1$  are outstanding orders that were placed at an earlier time and cannot be altered in any way. These orders will arrive a lead time  $L$  after they were placed, in periods  $t+1+L$  to  $t+A-1+L$ . Let us denote the frozen interval as discussed above, the sum of the action horizon and the lead time in periods, as  $F = A-1+L$ . Thus, lot sizing decisions taken at time  $t$  cannot affect the backorders or lost sales arising over the frozen part of the schedule, periods  $t+1$  to  $t+F$  but start only for orders that will arrive in periods  $t+F+1$  onwards. The problem is therefore to find the orders from this period  $t+F+1$  onwards that minimise the costs of meeting the forecast demand over the next  $F+P$  periods where  $P$  is the planning horizon. Let  $F_{t,t+k}$  be the forecast of demand (plus any buffer) for period  $t+k$  made at the end of period  $t$ , after the demand for period  $t$  is known, for  $k = 1$  to  $F+P$ . There are outstanding orders from earlier periods,  $OO_{t+k}$ , scheduled to arrive for period  $t+k$  for  $k = 1$  to  $F$ . Let  $I_{t,t}$  be the known stock at the end of period  $t$ .  $I_{t,t+k}$  is defined as the stock projected for the end of period  $t+k$  on the basis of the forecast period demands, outstanding orders and the ordering decisions made in period  $t$ .

Different service levels are generated by specifying the target stock levels to be achieved at the end of every period including buffers. Unlike the standard lot sizing procedure for deterministic demands, the aim here is to ensure that the projected start of period stock plus the order scheduled to arrive at the start of that period less the forecast demand is at least some specified stock level,  $SS$ . If  $SS$  is positive then it is a safety stock to guard against demand variations. If  $SS$  is negative, then it is the maximum level of backorders that can be planned to be carried from one period to the next or the maximum shortfall in meeting the predicted demand for a period in the lost sales case. So in lot size planning in period  $t$ , new orders should be scheduled to arrive in period  $t+k$ ,  $O^*_{t,t+k}$ , to ensure that

$I_{t,t+k} = I_{t,t+k-1} + OO_{t+k} + O^*_{t,t+k} - F_{t,t+k} \geq SS \quad \text{for } k = F+1 \text{ to } F+P$	$(1)$
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treating the forecasts as realised demands. The lot sizing problem at the end of period  $t$  can thus be expressed as to determine the orders scheduled for arrival in period  $k$ ,  $O^*_{t,t+k}$ , that minimise the total costs of ordering and stockholding over periods  $t+F+1$  to  $t+F+P$  whilst satisfying the above constraint for all periods.

$\text{Minimise } \sum_{k=F+1}^{F+P} [C_o \delta_{t,t+k} + C_h \text{Max}(0, I_{t,t+k})]$	$(2)$
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where  $C_o$  is the ordering cost and  $C_h$  the inventory holding charge per unit, and  $\delta_{t,t+k}$  is an indicator representing whether an order is placed in period  $t+k$  to arrive in period  $t+k+L$ .

For the backorders case, it can be shown, see Fildes and Kingsman (1997), that independently of whether SS is positive or negative, the introduction of the target end of period stock levels, SS, requires the determination of the best lot sizes to meet the net requirements ( $NR_{t,k}$ ) for each period over the  $P$  period planning horizon from  $k = t+F+1$  to  $t+F+P$ , which are given for  $j = 1$  to  $P$  by

$NR_{t,t+F+j} = \text{Min} \left\{ F_{t,t+F+j}, \text{Max} \left\{ 0, \sum_{k=F+1}^{F+j} F_{t,t+k} + SS - I_{t,t+F} \right\} \right\}$	$(3)$
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where  $I_{t,t+F}$  is given by continuously applying the stock transition equation (2) above with  $O^*_{t,t+k} = 0$  over the frozen part of the schedule, the periods  $k = 1$  to  $k = F$ . It can be seen that any backorders occurring over the frozen part of the schedule are cumulated until period  $t+F+1$  and then added to the forecast demand for period  $t+F+1$ . Adding in  $NR_{t,t+k} = 0$  for  $k = 1$  to  $F$ , this provides the forecast net requirements that must be satisfied over the planning horizon  $F + P$  for lot sizing planning at each time period  $t$ , using various lot sizing algorithms. The problem remains deterministic for lot size planning at the start of each period even when planned safety stocks or planned backorders are allowed. Since it has been expressed as determining the lot sizes to minimise the ordering and stockholding costs of meeting a given set of demands, it satisfies the conditions for which the standard lot sizing heuristics were devised.

### 3.5 Lot sizing rules

Six alternative lot-sizing methods were evaluated in the simulation, as given below. Fuller descriptions are given in texts such as Orlicky (1975) and Silver, Pyke and Peterson (1998). In all

cases, the currently uncovered net requirements over the free part of the planning horizon, taking account of any backorders that have accumulated, are firstly determined to give the demands to be satisfied. The average period demand is the average of the uncovered net requirements divided by the total number of periods in the free part of the planning horizon. However, if the net requirement in the first free period (or first few free periods) is zero then this is ignored in the averaging. In this particular case, no order will be placed and the simulation moves on to the following period updating the system with the actual period demand.

**EOQ (Economic Order Quantity):** The lot size is set equal to the standard Wilson Economic Order Quantity calculated on the basis of the average period demand, as above. If the EOQ is insufficient to meet the first period's net requirement, the lot size is made the smallest multiple of the EOQ that achieves this.

**POQ (Period Order Quantity):** This converts the EOQ lot size to the nearest integer number of periods demands, (again based on the average period demand over the free part of the planning horizon as above).

**LUC (Least Unit Cost):** The lot size is increased to cover succeeding periods until the item unit cost is at a minimum. The item unit cost is the ordering cost plus the stockholding costs for the lot divided by the lot size. If there is a tie, then the smaller number of periods is taken.

**LTC (Least Total Cost or Part Period Balancing):** This is based on the concept that the least total cost occurs where the ordering and stockholding costs are most equal. The lot size is increased to cover succeeding periods until the stockholding cost is closest to the ordering cost. Again, if there is a tie, the smaller number of periods is taken.

**Silver Meal Heuristic:** The objective is to minimise the costs per period, so the total costs as in the LUC method are divided by the number of periods. If a period within the total covered by the lot has zero net requirement then it counts as a period for the division.

**Wagner Whitin algorithm:** This procedure gives the optimal solution for minimising the costs of meeting a given set of deterministic requirements over a given planning horizon.

In addition, the simulation included the Lot -for-Lot method and the incremental heuristic, mainly as a check of the accuracy of the simulation programme. However, the results for these

methods are excluded from the analysis due to their consistently poor performance.

#### 4. THE SIMULATION EXPERIMENT

Within the production system described in the previous section a number of arbitrary decisions have been made that could affect the results we report. Demand has been generated over 20,000 periods but the variability of demand itself suggests it is wise to repeat the simulation experiment in order to establish the effect on demand, service level and the various costs. Five replications were carried out and detailed examination analysis of these runs showed that there was very little difference between the outcomes of each of the replications. The safety stock was set as a buffering factor times the forecast error deviation. The service level recorded was the average value over the 5 replications for that buffering factor, and similarly for the unit costs. The buffering values were chosen so as to cover service levels in the range 90% to 99.5%.

The random numbers used were generated using the Turbo Pascal function 'random' based on fixed seeds so that each experimental run used common random numbers to reduce variance when evaluating those factors determining unit cost (Law and Kelton, 1991). The normal distribution generator was that proposed by Box-Muller (1958).

In addition the effects of the following parameters on the results have been examined:

- (1) **frozen interval** - The experiment was run with three different frozen periods, 2 (the minimum possible given the two period nature of the production process), 5 and 7. The results reported in this paper are for a frozen period of 5. Unsurprisingly the shorter the frozen interval, the lower the unit cost curve. The ranking of the LSRs remained essentially unchanged (average correlation=.97). The change in the relative performance<sup>1</sup> (when comparing results across frozen intervals) is small, averaging around 1%.

Because the frozen interval determines the first period in which the schedule can be changed it also affects the actual error standard deviation under which the plans are set. The error standard deviation for the first period in which the schedule can be changed, denoted by  $\sigma(F+1)$  where  $F$  is the length of the frozen interval in periods, for both DGPs has been set at the values 50, 100 etc using equation (3). (Only in the case of the autoregressive DGP does this lead to a rebasing of the

error standard deviation; for the constant demand process the error deviation remains constant whatever the lead time.)

- (2) **planning horizon** - the simulations were run with three different free planning horizons, 12 , 24 and 36 periods, for some options. Note that the full planning horizon is the relevant frozen interval plus this free horizon. The results were virtually identical with an average correlation of .99 approximately. This was as expected because even the smaller free planning horizon is more than twice the maximum TBO of between 4 and 5 periods. All the experiments reported are for a free planning horizon of 24 periods except where noted for the Wagner-Whitin algorithm.
- (3) **simulation run length and warm-up period** - the experiment starts with zero stocks and zero demand over the frozen interval. The actual simulation period was for 20,000 periods, having first ‘run-in’ the simulation for a warm-up period of 72 periods. The warm-up period outcomes are not included in the analysis of the results
- (4) **validation** – the programme was broken into a number of test sub-programs covering the lot sizing rules, the demand generation and the forecast generation processes and the rolling schedule calculations. Each were verified against manual calculations.

## 5. THE EFFECTS OF DEMAND UNCERTAINTY WITH OPTIMAL FORECASTING

The results of the various simulations are presented in terms of two key measures of performance as discussed earlier. The Total Cost(*i,j*) is the sum of the ordering costs and holding costs incurred for the *i*th combination of experimental factors and the *j*th simulation replication. The mean unit cost for the *i*th combination of experimental factors is the average over the *J* replications, i.e.,

$$Mean Unit Cost(i) = \frac{1}{J} \sum_{j=1}^J Unit Cost(i,j) = \frac{1}{J} \sum_j \frac{Total Cost(i,j)}{Total Demand(i,j)}$$

The total satisfied demand each period is that met immediately from stock or the current period’s output, so the mean service level for the *i*th combination of experimental factors is

$$Mean Service Level(i) = \frac{1}{J} \sum_j Service Level(i,j) = \frac{1}{J} \sum_j \frac{Total Satisfied Demand(i,j)}{Total Demand(i,j)}$$

Experiments were performed on two demand processes. The first is the Constant Demand Data

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<sup>1</sup> Measured as the average unit cost of (EOQ and LTC) vs (POQ and LUC).



Generation Process, where the period demands are essentially random variates from a normal distribution with a mean of 1000 and standard deviations ranging from 50 to 400. The second is a more typical demand pattern in manufacturing, which is when there is a strong autoregressive pattern in the data. The case examined is Autoregressive parameter = 0.9. The overall variation in demand is larger than for the constant mean demand case and there are apparent cycles in the data.

Figure 1a presents some examples of the trade off curves obtained for the various lot sizing rules with a Constant demand DGP which show the mean unit cost(i) graphed against the mean service level(i)for each of the buffer values used in the experiments when there is perfect foreknowledge of the future demands. This is the deterministic demand case for which the lot-sizing rules were originally designed. The results shown are for a TBO value of 5 and for two values for the error standard deviation, 100 and 400. (It should be remembered that a TBO = 5 means that the average time between orders is 4 periods, which is given by an ordering cost of 7000 relative to a period holding cost of 1). An immediate tentative finding from Figure 1 is that there is little difference between the unit costs values of the rules for the lower error standard deviation, apart from the EOQ which costs significantly more, except for the lower service levels. The relative ordering of the unit costs of the lot sizing rules remains the same over the whole range of service levels shown. The differences between

Figure 1: Unit Cost vs Service Level  
Various Lot Sizing Rules: Time Between Orders 4 Periods

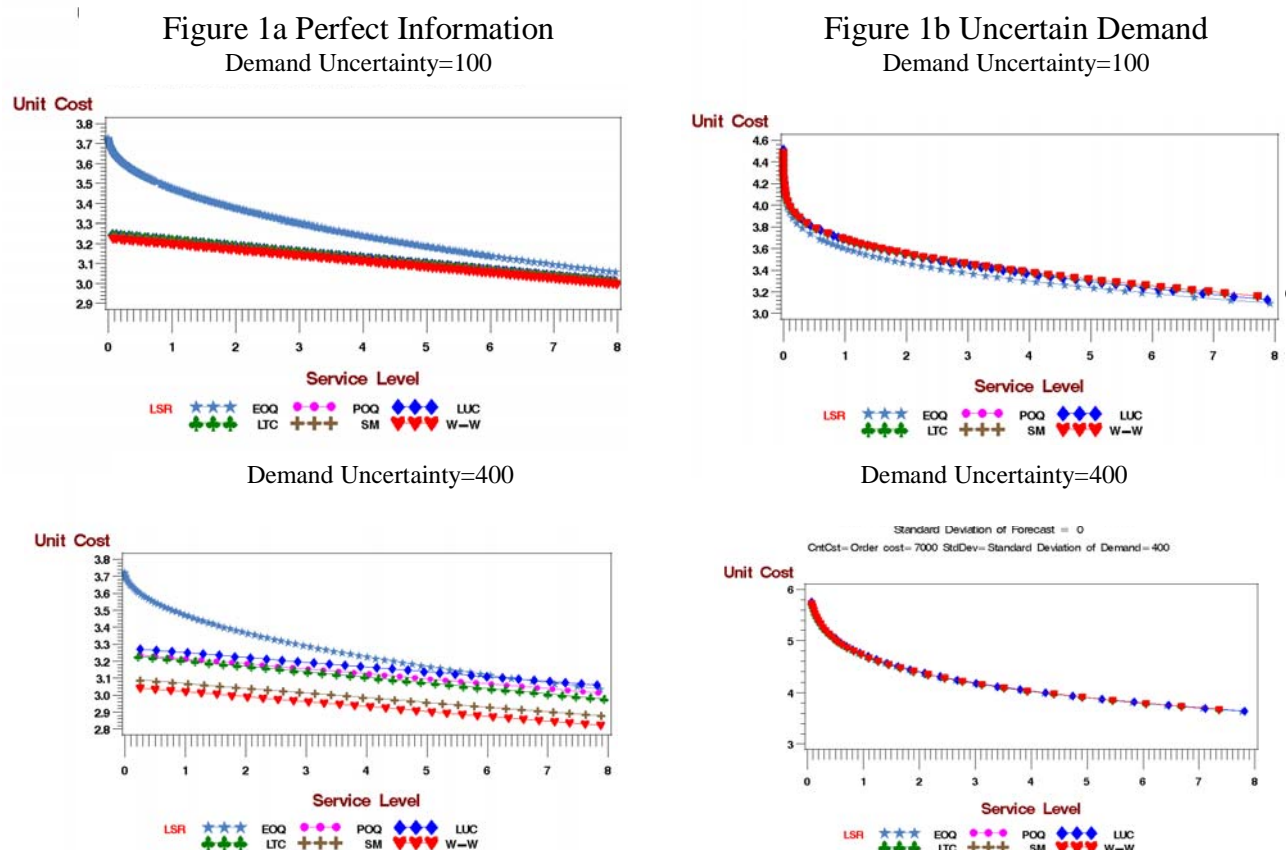


Figure 1b shows the trade off curves obtained for the various lot sizing rules for the same case as in Figure 1a but now based on forecasts of the future demands each period. The ordering decisions are based on the forecast of demands over the planning horizon at each period in turn, not the actual demands as for Figure 1a. Note that in this case, the unit costs become asymptotic with the vertical axis as the service level tends to 100%. An immediate finding from Figure 1b is that there is again a consistent difference between the different lot-sizing rules over the whole range of service levels shown, but that the differences are much smaller than those of Figure 1a, the deterministic demand situation with perfect foresight. The relative ordering of the unit costs of the lot sizing rules is the same for both levels of demand uncertainty but the differences becomes smaller as the noise standard deviation increases. The other tentative finding is that ordering of the rules in terms of unit costs is the reverse of what was found for the perfect foresight case. The EOQ is always the best, whilst the WW ‘optimal’ algorithm is almost always the worse. However, there is not a lot of difference between the rules other than for the EOQ.

Graphs such as Figure 1 are an illuminating way of presenting complex results, but, unfortunately, the service levels resulting for each lot sizing rule and level of demand uncertainty for the same buffering factor (and hence safety stock) differ. Although one can draw general conclusions from such figures, they do not allow the direct derivation of a quantitative summary of the relative performance for all costs structures and the different levels of the error variance for a particular Data Generating Processes. The solution to this difficulty was to model the trade off curves quantitatively to give the relationship between the unit cost and the service level. They were modelled individually for each lot sizing rule, error deviation, data generation process and cost structure (TBO) by

$$\log_e(\text{Unit Cost}_i) = \mu + \beta_1(\text{Service Level}_i\%) + \beta_2(\text{Service Level}_i\%)^2 + \beta_3(\text{Service level})^5 + \beta_4(\text{Service level})^3 + \lg(\text{Service level}) + \varepsilon_i.$$

where  $i$  is the experimental setting. Various simpler alternative specifications were considered such as a log-log function of mean service level but this flexible form was preferred. The curves were estimated for the service level less greater than 90%. The maximum percentage error in predicted unit cost is less than .1% with  $R^2=1$  in all cases. Different plausible functional led to the same rankings of the various LSRs.

Using these fitted curves, it is then possible to estimate the unit costs for each lot sizing rule, TBO and error deviation for any particular service level and so perform a quantitative analysis of

relative performances. Two target service levels were selected, 99% and 96%, representing high and a more modest but acceptable level of service achievement. The detailed values by all TBOs from 2 to 5 and for all four error deviations are available from the author for both the Constant Demand and Autoregressive Demand DGPs.

An analysis of ranking the lot-sizing rules performance by unit cost for each of the TBO and error deviation combinations confirmed the earlier result on their relative ordering, see Table 1. The ordering is as expected when there is perfect foreknowledge of demand for both demand DGPs. The optimal WW rule is first followed quite closely by SM. The EOQ is always worst

**Table 1 – Ranking the Rules by Unit Cost**

(measured by ranking each method for each TBO and level of noise uncertainty and summing the ranks)

Constant Demand GDP		Autoregressive Demand DGP	
Perfect Foreknowledge of Demand	Forecasting Demand	Perfect Foreknowledge of Demand	Forecasting Demand
WW	EOQ	WW	LTC
SM	LUC	SM	EOQ
POQ, LTC	LTC, POQ, SM	LUC	POQ ,
LUC		LTC, POQ	
EOQ	WW	EOQ	SM, WW, LUC

by far. The order of the other three rules varies between the two DGPs. The ordering is reversed when you have to forecast the demand using the ‘optimal’ method for the particular process. The WW rule, the optimal for the deterministic situation is now very much the worst rule. The EOQ, the worst by far in the deterministic case, is best for the Constant demand DGP and second best for the Autoregressive DGP. The LTC rule, which was poor in the deterministic case, is now the best for the Autoregressive DGP. Thus having to forecast demand and plan on the basis of a rolling horizon in a stochastic demand situation gives very different results to those found under the deterministic demand situation for which the lot-sizing rules were devised.

To assess the implications of these results we need firstly to check if there is a significant difference between the performance of the rules. In order to avoid any assumption of normality a

non-parametric test, the Friedman Test for the two way analysis of variance by ranks, was performed to test if there are significant differences in performance between the six lot sizing rules and the effect of varying TBOs and error deviations<sup>3</sup>. The Friedman statistic,  $FR = \frac{12}{bk(k+1)} \sum_{i=1}^k T_i^2 - 3b(k+1)$

approximately follows a chi-squared distribution with k-1 degrees of freedom, where  $T_i$  is the sum of ranks for the  $i^{th}$  lot size, so  $k = 6$ , and  $b$  is the number of factor levels, e.g., 16 if all TBOs 2 to 5 and error deviation values are considered, or 4 if only TBO or error deviation is considered. The values of the statistic are given in Table 2.

The  $\chi^2$  values for 5 degrees of freedom with  $\alpha = 0.1, 0.05$  and  $0.01$  are 9.23, 11.07 and 15.08 respectively. The values of the Friedman statistic in the first line of Table 2 indicate that overall there are significant differences in performance between the different lot sizing rules for the Constant Demand Process but for the Autoregressive Process, the p value is between 5% and 10%. An examination of the test statistics within TBOs (showing the variation in performance across error deviations) and within error deviations (showing the variation in performance across TBOs) shows somewhat more variation in performance across TBOs though the differences are not pronounced.

	Constant Demand DGP		AutoRegressive Demand DGP	
	96% Service level	99% Service Level	96% Service level	99% Service Level
ALL DATA	31.4	25.3	10.7	10.6
<b>Within TBOs</b>				
TBO = 2	14.3	8.86	6.0	4.11
TBO = 3	10	5.43	6.50	9.18
TBO = 4	10	7.43	1.32	2.07
TBO = 5	17.7	9.33	10.6	11.5
<b>Within Error Deviations</b>				
Error Deviation = 50	10.2	13.4	10.3	10.2
Error Deviation = 100	10.2	10.2	11.1	8.54
Error Deviation = 200	10.2	10.2	6.7	9.14
Error Deviation = 400	3.4	1.39	14.2	11.7

**Table 2 - Dependency of Mean Unit Costs on Lot Sizing rule for Different TBOs and Error Deviations: (Friedman's  $\chi^2$  Test Statistics)**

<sup>3</sup> The use of common random numbers, while introducing dependence between unit costs for different treatments, does not undermine either Friedman's test or the ANOVA tests considered later as the comparisons are blocked (as in a paired t-test).

Another way to measure the relative performances is to calculate the ‘regret’ of using a particular lot sizing rule, the increase in unit cost compared to the best method for that TBO, error deviation and service level. Let  $C_{i,j,k,l}$  be the average unit cost for lot sizing rule  $i$ , using TBO  $j$  and error deviation  $k$  for service level  $l$ . The lowest unit cost is  $Mn_{j,k,l} = \min_i \{C_{i,j,k,l}\}$ . Then the ratio

$$RC_{i,j,k,l} = \frac{(C_{i,j,k,l} - Mn_{j,k,l})}{Mn_{j,k,l}}$$

expressed as a percentage, is the cost regret of using the  $i^{\text{th}}$  lot sizing rule for the TBO, error deviation and service level compared to the lowest cost rule for that situation. The overall average regret values for each of the lot sizing rules for each service level for the Constant and Autoregressive Demand DGPs are displayed in Table 3. This confirms the earlier deductions above.

It can be seen for the Constant Demand DGP that the EOQ rule always gives the best performance by far for both service levels, with a regret of very close to 0%. The LUC rule is slightly better than POQ, LTC and SM rules, which are again slightly better than the WW rule. The size of the regret falls as the error deviation increases and also as the TBO takes higher values. The potential saving in choosing the EOQ rule compared to any other is around 8% for a TBO of 2 but falls to only 1.4% for the highest TBO of 5, for the 96% service level. For the lowest error deviation the regret is 6.8% falling to less than 1% for the highest value of 400. The impact becomes smaller as the service level increases from 96% to 99%.

**Table 3 - The Overall Average Percentage Regret by Error Deviation by Lot Sizing Rule.**  
**3a) Constant Demand DGP**

Lot Sizing Rules	Service Level							
	96%				99%			
	Error Deviation				Error Deviation			
	50	100	200	400	50	100	200	400
EOQ	0	0	0	0.075	0	0	0	0.05
POQ,	6.75	4.85	2.35	0.875	4.525	3.9	2	0.525
LUC	6.55	4.025	1.25	0.3	5.175	3.875	1.475	0.275
LTC	6.75	4.85	2.35	0.875	4.525	3.9	2	0.525
SM	6.75	4.85	2.35	0.875	4.525	3.9	2	0.525
WW	6.8	4.9	2.4	0.9	4.55	3.95	2.025	0.55
	TBO				TBO			
	2	3	4	5	2	3	4	5
EOQ	0.00	0.08	0.00	0.00	0.00	0.03	0.00	0.03
POQ,	8.43	2.88	2.08	1.45	5.85	2.05	1.73	1.33
LUC	5.90	3.33	1.80	1.10	4.80	2.93	1.80	1.28
LTC	8.43	2.88	2.08	1.45	5.85	2.05	1.73	1.33
SM	8.43	2.88	2.08	1.45	5.85	2.05	1.73	1.33
WW	8.43	2.88	2.08	1.63	5.85	2.05	1.73	1.45

**3b) Autoregressive (0.9) Demand DGP**

Lot Sizing Rules	Service Level							
	96%				99%			
	Error Deviation				Error Deviation			
	50	100	200	400	50	100	200	400
EOQ	0.00	0.00	0.68	2.18	0.00	0.00	0.75	1.53
POQ,	3.98	1.75	0.45	0.70	3.13	1.45	0.20	0.48
LUC	3.33	0.93	0.65	2.18	3.15	1.13	0.70	2.05
LTC	3.98	1.73	0.18	0.00	3.13	1.45	0.03	0.05
SM	3.98	1.75	0.73	1.23	3.13	1.45	0.45	0.70
WW	4.78	2.53	1.30	1.43	3.78	2.05	0.88	0.90
	TBO				TBO			
	2	3	4	5	2	3	4	5
EOQ	0.35	1.23	0.65	0.63	0.83	0.63	0.23	0.50
POQ	4.48	1.20	0.68	0.53	0.70	0.63	0.40	0.48
LUC	3.53	1.95	0.95	0.65	1.00	0.90	0.53	0.78
LTC	4.30	0.85	0.50	0.23	0.33	0.33	0.33	0.25
SM	4.48	1.70	0.90	0.60	0.80	0.70	0.47	0.50
WW	4.48	1.58	0.70	3.28	3.13	2.87	2.83	2.60

Comparing Table 3b with 3a shows that the regrets from using the best rule in each case is smaller for the Autoregressive DGP than from the Constant demand DGP. The EOQ is best for the lower error deviations whilst the LTC rule is best for the two higher error deviations. The EOQ rule is best for a TBO of 2 but the LTC takes over for higher TBOs of 3, 4 or 5. The autoregressive results show that generally the percentage regret from using the wrong rule is less than 2 %. Thus, unlike the Constant demand DGP, the choice of lot-sizing rule to use for the Autoregressive DGP is unimportant except for very low error deviations and TBOs.

To summarise the results, for both processes, the choice of lot-sizing rule becomes less critical as the demand uncertainty increases, as the average time between orders increases and as the service level increases and for higher TBOs and service levels is close to inconsequential.

It is also valuable to assess the increase in cost arising when forecasting uncertain demand compared with the costs if future demand was known. This is illustrated in Table 4 where costs are compared to those calculated with perfect foresight of the (stochastically) generated demand pattern for both DGPs. This shows the percentage increase in unit cost for the best lot sizing rule above the unit cost for the corresponding WW cost based for the deterministic case of perfect foresight (disaggregated for the different TBOs and error deviations). The bottom row of the table shows the average increases over TBO values from 2 to 5. The increases are much higher for the 99% service level than for the 96% service level. The values derived indicate a complex pattern of behaviour, with the standardised unit cost increase rising as the time between orders lengthens, the error deviation increases and the service level increases. There is again a significant difference between the two DGPs. The cost increases due to the uncertainty in the demand are much greater for the Autoregressive DGP than for the Constant Demand DGP. The average of the 96% and 99% service levels can be used as an overall indicator of the effect of changes in the error deviation. The rate of increase in unit cost as the error deviation increase is greater than a linear rate, but over the range given is an increase of around 74% and 93% for every 100 units increase in the error deviation for the Constant Demand DGP and the Autoregressive DGP respectively.

These results contradict De Bodt and Wassenhove's often repeated conclusion (1983a) that with any level of demand uncertainty greater than zero, costs were increased by about 10%. In fact, this result is an artefact arising from the attempt to eliminate service level as a factor in the experiments they carried out.

**Table 4 - The Percentage Increase in the Unit Cost of the Best LSR Rule Above the WW**  
**Solution based on Perfect Demand Information**

**4a) Constant Demand DGP**

TBO	Service Level							
	96%				99%			
	Error Deviation				Error Deviation			
	50	100	200	400	50	100	200	400
2	15.8	20.7	40.3	102.9	26.5	35.6	64.2	146.3
3	7.9	10.9	22.8	60.3	15.1	20.6	38.5	88.6
4	4.1	6.0	13.2	37.2	9.1	12.6	23.8	56.1
5	2.3	3.6	9.0	26.3	6.3	8.6	16.9	40.6
Average	7.5	10.3	21.3	56.7	14.2	19.3	35.8	82.9

**4b) Autoregressive (0.9) Demand DGP**

TBO	Service Level							
	96%				99%			
	Error Deviation				Error Deviation			
	50	100	200	400	50	100	200	400
2	18.3	31.1	71.4	180.0	31.7	51.6	106.6	243.2
3	9.1	15.9	38.0	108.1	17.6	29.1	60.6	150.6
4	4.6	8.5	22.7	67.7	10.6	17.4	37.8	97.1
5	2.7	5.5	15.4	48.1	7.2	12.4	27.1	69.9
Average	8.7	15.3	36.9	101.0	16.8	27.6	58.0	140.2

## 6. THE VALUE OF FORECASTING and THE EFFECTS OF MISSPECIFICATION

Now the above results are indicative of the value of forecasting. Transparently, changes in the demand uncertainty leads to significant changes in the unit cost when service level is kept fixed. However, this does not directly answer the question posed in the introduction: what is the value of forecasting. To most practitioners, the forecast error is the difference between the actual and the forecast value. However, this error combines the randomness in the process generating the demands and the errors arising from not using the optimal forecast and most earlier researchers in this area have conflated the two. It is only the latter that represents the potential value of improving forecasting accuracy. Our analyses to date have assumed the use of the optimal forecasting method. The non-optimal forecasting system can now be defined as:

$$F_{t-1,t+k} =_{opt} F_{t-1,t+k} + v_t$$

for each of the DGPs so that the actual forecast equals the optimal forecast plus random noise



(assuming the same error is made across lead times). It follows that the overall forecast error is

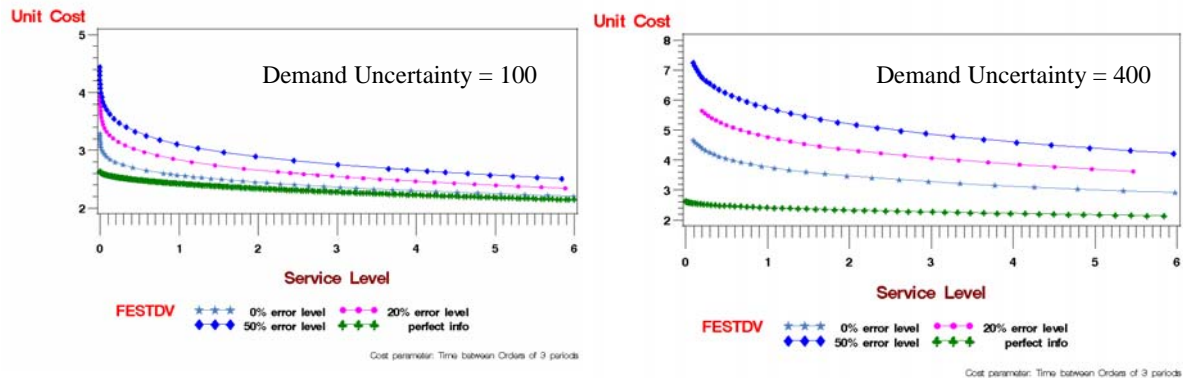
$$D_{t+k} - F_{t-1,t+k} = e_{t+k} - v_t$$

This distinction between randomness in the data generation process and the consequential forecasting errors permits the introduction of a wide range of forecasting error characteristics, including bias and forecast model specification error. However, in this paper we only examine the case where  $v_t$  has error standard deviation  $\kappa\sigma$  where  $\kappa$  is chosen so that the overall error deviation,  $\sigma\sqrt{(1+\kappa^2)}$ , takes values of 110%, 120% and 150% of the minimum attainable, given by  $\sigma$ . The results from introducing forecast error should look similar to those just presented for different levels of randomness in the data generation process alone despite the two sources of error being independently simulated. (This will not in general be true in the common case where  $e_t$  and  $v_t$  are correlated.)

There are thus two sources of uncertainty in the demand prediction process. One, the forecasting error, can be reduced by getting closer to the optimal forecasting method for the underlying demand generation process (DGP). Clearly, as there will be inevitable errors in identifying and estimating the parameters of the DGP, it will never be possible to eliminate such forecasting errors altogether. The second source, the process error deviation, is the random variation in the DGP itself. This can be reduced only by attempting to manage the demand process, for example by attempting to change customers' behaviour or through collaborative forecasting. The use of specific ordering days in a week or month for different groups of customers in inventory/distribution systems offers an illustration. It is important in guiding management effort towards process improvements to distinguish as much as possible between the impact of these two sources of uncertainty on unit costs and service level.

Figure 2 shows the effects on the unit cost / service level trade-off curves of increasing the forecasting error by 20% and 50% compared to when the forecast is optimal (i.e.  $v_t \equiv 0$ ). This is further contrasted with the curve based on perfect foresight information. The illustrative graph of the results shown is derived using the EOQ lot sizing rule and for a TBO of 3 periods for different standard deviations in the data generation process.

Figure 2: Unit Cost vs Service Level as it is affected by Forecast Error Levels  
Based on the EOQ: Time Between Orders 3 Periods



In order to quantify the value of forecasting, the logarithm of unit costs has been modelled as a function of possible explanatory factors as given in Table 5. As before, the model was of  $\log(\text{average unit cost})$ . The data set modelled was for mean service levels lying between 93% and 99.9% to avoid distortions at the extremes of the data matrix. This range includes those values of practical interest.

Qualitative Factors	Quantitative Factors
Lot Sizing rules (LSR),	Standard deviation of the DGP (STD) and $\text{STD}^2$
Cost Structure (TBO)	Standard deviation of the forecast uncertainty - measured as $\text{FORDEV} = \text{STD} * (\text{error}\%)$ and transformed as $\text{FORDEV}^2$ $\text{PROD} = \text{STD} * \text{FORDEV}$
Forecast Error level (FORDEV)	Overall error deviation, $\text{OVLLLDV}$
Interaction of (LSR and TBO) and (LSR and STD)	The out-of-stock level, $\text{SERV}$ - measured as: $(100\% - \text{service})$ , included in the same form as eq. 3. i.e. $\text{LgSERV}$ , $\text{SERV}$ , $\text{SERV}^2$ , $\text{SERV}^{1/2}$ , $\text{SERV}^{3/2}$ .

Table 5 Possible Explanatory Factors in a Model of  $\text{Lg}_e(\text{Cost})$

In the model specification the service level has been included in the same form as was used to

estimate the 99% and 96% service levels in section 5. The results show that the factors LSR and Cost (as measured by TBO) are significant. Perhaps unsurprisingly, the largest reduction in variance is due to the Cost (TBO) parameter. The standard deviation of the data generation process, STD, and the forecast error deviation, FORDEV, were included linearly. The inclusion of the variances,  $STD^2$  and  $FORDEV^2$ , as well as the standard deviations led to improved predictions for the larger values of the overall deviation (OVLLLDV). The RMSE is low suggesting a satisfactory model (with interquartile ranges of approximately  $\pm 5\%$  for the error) with  $R^2$  of 99%. Details of the associated analysis of variance summary table are omitted as the inevitable consequence of using such a large data set is almost all factors were found to be ‘significant’.

Graphical analysis plus the inclusion of additional cross-product terms such as the often insignificant interactions between OVLLLDV and LSR and TBO suggested that the model was broadly adequate and the error distribution was approximately normal but it failed to capture all the non-linearities induced by the interaction of STD, the standard deviation of the DGP, and the cost structure, TBO. However, including such interaction terms had no effect on the qualitative conclusions we draw and little effect on the quantitative estimates we make since the coefficients are small. The final specification adopted included a dummy variable to distinguish between the situation with zero forecast error deviation ( $FORDEV=0$ ) and non-zero forecast error. Its interaction with LSR was also included.

The key interaction to include is that between the cost parameter and the overall uncertainty (OVLLLDV), preferred to both the process standard deviation (STD) or the forecast error deviation (FORDEV). This shows that the impact of the overall deviation lessening as the time between orders (TBO) increases. Essentially the shorter the time between orders the more sensitive the system is to uncertainty and forecast error. The impact of the forecast error deviation is larger, compared to the process error though there is a qualitative reduction in cost in moving from the zero forecast error deviation to the non-zero case, i.e small amounts of forecast error lead to overall improvements in performance.

The above model was used to evaluate the performance of the different lot sizing rules, see Table 6. Overall EOQ, POQ and LUC performed best (although the estimated cost differences are small, typically less than 1%) as shown in the column ‘Estimated Effect’ in Table 6. However, this overall

results disguises a more complex picture that becomes apparent if the analysis is carried out only for those points with zero forecast error deviation on the one hand and positive deviation on the other. While overall EOQ is not the uniform best performer, for zero forecast error deviation it is significantly better than its competitors (confirming table 1 and 3) whilst the best performers with forecast error were EOQ, POQ and LUC which differed significantly from the rest. Whatever the error deviation, there was no significant difference found between Silver-Meal and Wagner-Whitin.

Contrast	Estimated Effect	Estimated Effect	Estimated Effect
	All data	No forecast error	With forecast error
EOQ vs POQ	-0.741**	-2.80**	0.09
EOQ vs LUC	-0.800**	-2.65**	-0.06
POQ vs LUC	-0.060	0.15	-0.15
EOQ vs LTC	-1.276**	-2.80**	-0.61**
EOQ vs SM	-0.803**	-2.80**	0
EOQ vs WW	-0.838**	-2.85**	-0.03
EOQ/LUC vs rest	-0.516**	-1.50**	-0.11
EOQ/LUC/ POQ vs rest	-0.690**	-1.51**	-0.33**
SM vs WW	-0.035	-0.06	-0.03

**Table 6: The % difference in unit cost:**

#### **Testing the difference between Lot Sizing Rules - the Contrasts**

(\*\* represents a contrast significant at less than 1%)

Estimating the value of forecasting requires some assumptions about the size of the overall uncertainty that arises in practice and also how it is divided between the noise in the process generating demand and the forecast errors incurred from using a non-optimal forecasting method. Only the second source of error is lessened by improved forecasting. It was stated in section 2 that, in practice for short term forecasting, weekly or monthly up to three steps ahead, the standard error of the forecast for faster moving items would usually not typically exceed 40% of the mean demand (i.e. a value of 400 if the mean demand was 1000). Improvements in forecasting of around 20-50% in the overall forecast error are possible by implementing appropriate forecasting techniques to match the particular characteristics of the demand data used in the MRP system (Fildes et al, 1998, Fildes et al, 2004). Thus, it would be indicative of possible savings in unit cost to consider the improvements

expected for values of 200 and 400 in the overall uncertainty, derived from different mixes in process noise and forecast error. Using the above response model specification, the estimated % change in cost is independent of cost, LSR and service level. Using a first order approximation for the response function the percentage improvement in unit cost for an h% improvement in forecasting accuracy is given by:

$$\beta_{TBO, OVLLDV} \frac{FORDEV}{OVLLDV} + \beta_{fordev} + 2\beta_{fordevsq} FORDEV + \beta_{prod} STD ) * FORDEV * h$$

where FORDEV is the forecast error, and OVLLLDV is the overall uncertainty. The  $\beta$  coefficients have the obvious meaning with  $\beta_{TBO, OVLLDV}$  representing the interaction term. In quantitative terms  $\beta_{fordev}$  dominates the improvement from improved accuracy but the formula shows the size of the forecast error is also critical. Illustrative exact results are shown in Table 7 for EOQ. The approximation underlines the importance of the forecast error deviation relative to the process noise. In effect major improvements in cost can be achieved where the overall uncertainty is high and a major component of this is forecast error. However, the table also highlights the difficult of making such gains.

**Table 7a. The Value of Forecasting for Different % Improvements in Forecasting Error**

**– the % reduction in Unit Costs for the AR(0) process (averaged over TBO)**

						<b>Forecast Improvement</b>							
		-10%	-10%	-10%	-20%	-20%	-20%	-50%	-50%	-50%	-100%	-100%	-100%
<b>Standard deviation of DGP</b>		100	200	400	100	200	400	100	200	400	100	200	400
<b>Forecast Error Deviation as % of DGP</b>	10%	0.2	0.3	0.4	0.4	0.6	0.8	0.9	1.6	2.1	1.9	3.2	4.2
	20%	0.4	0.6	0.8	0.7	1.2	1.6	1.8	3.0	4.0	3.6	6.1	8.0
	50%	0.8	1.4	1.9	1.6	2.7	3.7	4.0	6.9	9.2	8.3	13.8	18.2

The final stage in this analysis of the value of forecasting is to analyse the results obtained for the second autoregressive generating process through the analysis of variance. The model had the same characteristics as that for constant demand (although some slight improvement could have been obtained by using the interaction between TBO and the forecast uncertainty). The estimated co-variates are similar with a similar cost sensitivity to forecast error improvements as shown in Table 7b.

Because of the greater interdependence of LSR and cost structure, unlike the simple constant demand model, there is no simple equivalence between the results of the analysis of contrasts and the ranking results in Table 1.

**Table 7b. The Value of Forecasting for Different % Improvements in Forecasting Error**  
– the % reduction in Unit Costs for the AR(.9) process (averaged over TBO)

						<b>Forecast Improvement</b>							
		-10%	-10%	-10%	-20%	-20%	-20%	-50%	-50%	-50%	-100%	-100%	-100%
<b>Standard deviation of DGP</b>		100	200	400	100	200	400	100	200	400	100	200	400
<b>Forecast Error Deviation as % of DGP</b>	10%	0.2	0.3	0.4	0.3	0.6	0.8	0.8	1.4	2.0	1.5	2.7	3.9
	20%	0.3	0.6	0.8	0.7	1.2	1.7	1.6	2.8	4.1	3.1	5.4	7.8
	50%	0.9	1.6	2.3	1.9	3.2	4.5	4.5	7.7	10.7	8.2	14.0	19.4

An alternative approach to the problem of estimating the value of improved forecasting accuracy is by using the raw simulation results and comparing the unit cost of dropping from 50% forecast error to 20% to 0% for fixed service (at 96% and 99%). This approach gives a higher estimates of the forecast sensitivity, with median forecast sensitivity of 10% at the 96% service level (17% at 99%) and 11% at 96% service (20% at 99%) at process standard deviations of 200 and 400 process uncertainty respectively. Interestingly a small amount of forecast error and low process uncertainty resulted in a small cost improvement. For the AR(.9) process the effects of forecasting improvement are uniformly larger with approximate unit cost improvements of 5% and 8% above the AR(0) benchmark values given above.

### *Misspecification*

In applications, simple forecasting methods are usually applied without any prior analysis of the appropriate underlying mode of the DGP. The effect is to introduce both forecast error and misspecification error. As noted in the introduction such misspecification is a common feature of various supply chain models discussed in the literature including most recently Chen et al, 2000a and Zhao et al, 2001. A full discussion of its effects of misspecification would ensure the paper was overly long. However, we have considered two cases, where the DGP is the constant demand model and the forecasting model is the AR(.9) and vice versa. Table 8 gives the results averaged over the two service

levels and lot-sizing rules EOQ, POQ and LTC which ensured the ‘better’ LSRs have been used.

Forecasting Generating Process		Forecast Noise:	Data Generating Process	
			AR(0)	AR(.9)
			Mean	Mean
	AR(0)	00%	15.4%	46.1%
		20%	26.4%	50.0%
		50%	39.3%	54.8%
	AR(.9)	0	25.6%	25.1%
		20%	67.6%	46.7%
		50%	107%	67.4%

**Table 8 The effects of forecast model misspecification**

(averaged over the ‘better’ lot sizing rules, EOQ, POQ and LTC, averaged over TBOs 3 &4, for process uncertainty of 100 and 200 and service levels).

The table illustrates the effects of using a mis-specified model (typically the case in previous research) compared with using an optimal model for the particular case. The table is split into four quadrants, the first under the heading AR(0) repeats the unit cost where the DGP is an AR(0) process and the optimal forecasting model specification has been used to model the DGP (but with three levels of forecast error, 0% 20% and 50%, of the demand uncertainty). The fourth quadrant shows the results where the DGP is again the AR(0) but the forecasting model that is used is AR(.9) (again with the three levels of noise). Quadrants two and three are similar but the DGP is AR(.9). The results strongly confirm the lack of symmetry in the two specification errors – a greater percentage error is incurred by assuming the process is AR(0) if in fact it is AR(.9) than vice versa but only when the forecast error noise is around 20% or lower – i.e. no dominant strategy is available for selecting a forecasting model. In addition, an analysis of the more detailed results show the specification error effects on costs increases with TBO, process uncertainty and service level. No differences in performance were observed for these ‘better’ performing lot sizing rules. Thus, earlier research (which makes assumptions equivalent to mis-specifying the forecasting model adopted in the simulation) is likely to have misstated the effects of demand uncertainty, potentially substantially, and their conclusions will be dependent on the nature of the misspecification assumed in the research design.

## 7. DISCUSSION and CONCLUSIONS

In this research we have demonstrated a methodology for integrating aspects of manufacturing research with forecasting. The methodological issues we have explored aimed to highlight deficiencies in previous research designs: the conflation of demand uncertainty with forecasting error uncertainty and model mis-specification, the use of 100% service levels to draw conclusions and the pervasive use of Analysis of Variance when the issue of significance can easily be dealt with through increased sample size, thus neglecting the much more fundamental issue of the practical importance of the results. Size matters! (Zilian and McCloskey, 2004). The substantive aim of the research has therefore been to show how to estimate the value of improved forecasting and to flesh out the claims made by software suppliers and forecasters alike that accurate forecasting is critical to many manufacturing operations

Various stylised facts spring from the substantive analysis we have presented.

- Unit cost (expressed as a percentage of the unit cost based on perfect foresight) increases exponentially with demand uncertainty in contrast to de Bodt's and Van Wassenhove's (1983a) assertion. The discrepancy with this study apparently arises from their focus on service levels of 100% only.
- In the more general framework established here, the best lot sizing rules when demand uncertainty exists remain very different to those based on deterministic demand which assumes perfect foresight. This strengthens, for example, Wemmerlov and Whybark, (1984),
  - A possible explanation of the relative performance of the various lot sizing rules is that for a given set of forecast demands, the standard lot sizing heuristics cumulate demands over an integer number of periods. This avoids the deliberate creation of carryover stock as occurs with the EOQ rule. The differences between the forecasts and the actual demands will create unplanned carry over stock and also periods with small net requirements. The existence of demand uncertainty thus negates the advantage of the rounding to match period demands of the lot sizing rules in the deterministic demand situation. The EOQ rule carries an implicit safety stock also. Thus when the final product demand has to be forecast, using the EOQ for lot sizing works. However, in the AR(9) case with higher demand uncertainty, the LTC rule performed best. The substantially poorer performance of the Silver-Meal heuristic and Wagner Whitin was unexpected. Once forecast errors is acknowledged, there will always be carryover stock from period to period, so any method that concentrates on minimising costs per period is



not perhaps as appropriate as examining costs per unit acquired. For lot sizing research to make a contribution to improved manufacturing, uncertain demand must be explicitly considered in a realistic context that includes service level. Applying the results from deterministic lot sizing research is clearly inadequate (Wemmerlov, 1989).

- The choice of lot sizing rule can make up to a 5% difference in unit cost for the constant demand DGP. The choice is less important for the autoregressive DGP (around 2%).
- Sensitivity to the choice of lot sizing rule diminishes with increasing demand uncertainty, increasing TBO and increasing service.
  - For a TBO of 2 periods there is a benefit of around 12 % for low levels of randomness at the 96% level, lower at 8% for 99% service
  - Focusing on 100% service gives misleading results
- The benefits of improved forecasting increases, depending on the relative sizes of the stochastic variation in the demand generation process and the forecasting errors. These are substantially greater than the effects arising from the choice of lot sizing rule.
  - Potential savings are around 8% for higher levels of demand and forecast uncertainty.
- Sensitivity to forecast error is higher for the autoregressive process ( by around 5 percentage points on a base of 15-20% for high process uncertainty) .and for lot sizing choice is lower.
- There is no best method of forecasting. Misspecification in choosing the forecasting model to use leads to increased forecast error and increased costs.
  - There is no dominant strategy of forecasting model selection as measured by regret, despite the common practice in both research and organisational forecasting practice of using a generic model such as exponential smoothing in all applications.

Taking the results as a whole, they show that while it is sometimes important to consider the choice of lot sizing rule, attempts to reduce the forecast errors are likely to prove more valuable, as the lot sizing choice is relatively straightforward depending on TBO. This reduction can be achieved both by moving closer to the optimal forecasting method for the problem and also by trying to reduce the variation in the data generation process itself.

The value of improvements in forecasting depends quite critically on the cost structure of the manufacturing process. The emphasis on recent developments in production and operations management, whether called achieving world class manufacturing, lean production methods or JIT production, has been on the benefits of reducing lot sizes, so the values for lower TBOs may be better

indicators of the impact of demand uncertainty. If manufacturing is improved to make lower lot sizes more economic for production, then this results in the costs of satisfying customer demands being even more sensitive to uncertainties in demand, whether due to inherent process variation or to inadequate forecasting. Thus, it is important to manage the demand generation process as well as to take actions to make lower lot sizes more economic in order to improve the overall performance.

A number of substantive research questions remain, in particular whether the results generalise to other (more complex and realistic) production processes, in particular through the inclusion of capacity constraints, correlated demand and JIT procedures. Case research is needed here. Further lot sizing rules could be usefully added in to the comparisons we have made. In addition, little of the research done on forecasting has addressed the types of data generation processes observed in manufacturing systems which may well be severely non-normal, perhaps with intermittent demand, outliers and changes in trend (Gardner, 1990, Fildes et al, 2004). The importance of this issue, totally neglected in earlier lot sizing research, has been demonstrated in the results for the more realistic autoregressive process, which show a higher sensitivity to forecast error. We would expect that the common types of specification error common arising when automatic forecasting systems are used, would lead to increased advantages to more accurate, properly specified forecasting models. The key question remaining is what improvements in forecasting accuracy are realistically achievable. As this paper has shown, the benefits to success in achieving such improvements are worth the effort.

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