Investigating the use of stochastic forecast for RTC of urban drainage systems

Investigation sur l'usage de la prevision stochastique pour la gestion en temps réel d'un système d'assainissement

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RÉSUMÉ

Le contrôle en temps réel est la base d'une gestion efficace des systèmes de drainage urbain. Une utilisation optimale des volumes de rétention des bassins par exemple, peut être achevée en considérant les prévisions de ruissellement lors de la prise de décision. Toutefois, ces prévisions sont sujettes à des incertitudes significatives. Cette incertitude devrait être considérée lors de la prise de décision. Une approche stochastique incluant les prévisions de ruissèlement, appelée modèle « greybox » de contrôle en temps réel est présentée. Ces modèles permettent une description dynamique des incertitudes des prévisions. De plus, ils permettent une adaptation continue face aux ruissellements observés. Les méthodes permettant de générer des prévisions stochastiques considérées dans le cadre de la prise de décision sont décrites. La qualité des prévisions est comparée à l'aide de deux événements sur les modèles de prévision déterministes et le volume au déversoir. Nous sommes donc en mesure de démontrer le potentiel des modèles stochastiques, mais l'amélioration de ce travail se poursuit.

ABSTRACT

Real time control is considered a mean to efficiently improve the performance of urban drainage systems. A globally optimal utilisation of e.g. storage volume in basins can best be achieved by considering runoff forecasts in the decision setup. These forecasts, however, are subject to significant uncertainty. This uncertainty should be considered in the decision making. An approach that incorporates stochastic multistep runoff predictions from so-called greybox models into a real time control setup is presented. These models provide a dynamic description of forecast uncertainties and they simultaneously allow a continuous adaption of the model states to observed runoff. Methods for generating stochastic forecasts and incorporating these into the decision making framework are described. Using two sample events, the forecast quality is compared to state-of-the-art deterministic forecasting models and the effect on control decisions and the resulting overflow volume is evaluated. We can demonstrate potential of the stochastic models but identify a need for model adaptivity and modified model structures that allow for a more general modelling of forecast uncertainties.

KEYWORDS

Greybox model, Real time control, Runoff forecast, Uncertainty, Urban drainage system

1   INTRODUCTION

Integrated real time control (RTC) of urban drainage system is increasingly seen as an efficient approach to improve the performance of these infrastructures and reduce the impact to the natural water environment (see the discussion in Rauch et al., 2005). While the cost-efficiency of RTC is increasingly demonstrated by both simulation studies (e.g. Dirckx et al., 2011) and full scale applications (e.g. Fradet et al., 2011), recent developments in radar nowcasting (Thorndahl et al., 2009;2010), on-line measurements and available computational capacity have boosted the potential field of application of these tools. These include also integration of field measurements with mathematical models, which provide information regarding the future evolution of the controlled system (defined as Model Predictive Control – MPC).

The Storm- and Wastewater Informatics project (SWI) is currently investigating the development of integrated modelling tools for improving the performance of urban drainage systems, both in term of reducing flooding risk and in terms of quality of receiving waters (i.e. reducing combined sewer overflows (CSO) and improving the performance of wastewater treatment plants – WWTP). The SWI project is investigating, among others, tools for nowcasting precipitation based on radar measurements, modelling of bottlenecks in WWTP (i.e. secondary clarifier), and strategies for integrated MPC of the entire wastewater system.

In the urban drainage field, MPC is mainly focusing on estimating the evolution of the collected runoff in the near future. While radar-based nowcasting tools provide an estimate of the future rainfall across the catchment, this needs to be translated into water volumes by rainfall-runoff models. As the reaction time (and thus the controls) in urban drainage systems is usually short (below hours), simple and fast conceptual hydrological models (based on, for example, time-area method, linear reservoir cascade) are commonly used. These models are however affected by a great number of sources of uncertainty (e.g. Deletic et al., 2012), which makes uncertainty estimation essential to apply their results in a reliable manner. Rainfall-runoff models need also to be updated each time new field measurements are available, in order to reduce the discrepancy between the modelled and the measured environmental variable and thus increase the confidence in the MPC results. Information regarding the estimated level of uncertainty also needs to be included in the control strategy, in order to find the optimal solution based on the estimated confidence in the available measurements and/or model predictions. Examples of these approaches for control of drainage systems in an uncertain context are presented in Raso et al. (2012) and Vezzaro and Grum (2012).

Alternative approaches for estimation of runoff flows are based on stochastic greybox models, where uncertainty is explicitly addressed by including a stochastic term in the model structure (Breinholt et al., 2011). Greybox models provide a dynamic estimation of the model output uncertainty, information that can directly be used by the control strategy. Further, through an extended Kalman filter setup, the models can adapt to new runoff observations in an online setting. This state updating usually ensures that forecasts are generated from a correct starting point but also provides flexibility for handling erroneous or missing observations. By using a lumped model structure, which is computationally fast and thus suitable for MPC applications, it is thus possible to control the drainage system based on the dynamically estimated level of uncertainty.

The aim of this paper is to compare the performance of two approaches for on-line estimation of runoff volumes and their effects when used in an integrated control strategy. The study focused on the Lynetten catchment (located in Copenhagen, Denmark), which is controlled by an integrated RTC approach (the Dynamic Overflow Risk Analysis (DORA) approach, described in Vezzaro and Grum (2012)). The first approach that was considered was a deterministic conceptual rainfall-runoff model, which utilizes a fixed probability distribution to describe the uncertainty in the estimated runoff volume. This approach was compared with a greybox model, which provides a dynamic estimation of uncertainty. As the main focus of DORA is the reduction of CSO, the comparison aimed at investigating the differences in the estimated CSO risk calculated by using the output from the two approaches. A simple hydrological was used to compare the effects of the two forecast model types on the control strategy and the resulting overflow volumes.

2 MATERIAL AND METHODS

2.1 Case study – The Lynetten catchment

2.1.1 Catchment and data description

The Lynetten catchment covers the central area of Copenhagen (Denmark) and it has an area of approximately 76 km². The catchment discharges to the Lynetten wastewater treatment plant (WWTP), which is regulated by a STAR® system for advanced control of wastewater processes (Thomsen and Ônnerth, 2009). This platform is also used to control flows and storage capacity across the catchment, allowing an integrated control: WWTP operations can be defined according to the forecasted inflow to the plant, and the storage capacity can be optimized by looking at the actual capacity of the WWTP. A detailed description of the study area can be found in Breinholt and Sharma (2010).

The integrated control of the Lynetten catchment considers eight overflow structures (connected to basins and pumping stations – see Figure 1), discharging to recipients with different sensitivity to CSO. The total storage capacity is about 114200 m³. Rainfall falling on the catchment is quantified by radar measurements, which are dynamically calibrated against data provided by the gauges belonging to network of the Danish Water Pollution Committee, operated by the Danish Meteorological Institute (Jørgensen et al.(1998)). Further details on the radar nowcasting methods can be found in Thorndahl et al. (2009;2010).

Available measurements are the basin outflow and water levels, i.e. inflow to each basin is not directly measured, and is calculated from the basin water balance. Therefore, no direct measurements of overflow volumes are available for all the overflow structures considered in the study. These are estimate by running an off-line detailed hydrodynamic model (built in the MIKE URBAN software, DHI (2008)), which uses radar measurements as input. The same model is used by Copenhagen's water utility to elaborate statistics on unmeasured overflows, which are also used to evaluate bathing water criteria (Clauson-Kaas et al., 2008).
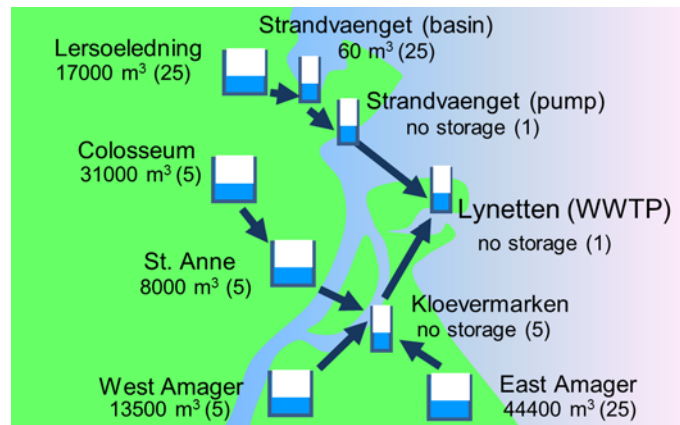

Figure 1. Scheme of the Lynetten catchment. Storage volumes and overflow prices (in brackets) are listed for each node of the network.

2.1.2 Integrated control strategy (DORA)

The storage capacity in the Lynetten catchment is controlled by the Dynamic Overflow Risk Assessment (DORA) strategy (Vezzaro and Grum, 2012). This strategy aims to reduce the overflow risk in the different nodes of the drainage network by minimizing a global cost function, which is defined as follows:

$$Cost = \sum_{i=1}^{N_{ba\sin s}} \left(C_{cr,i} + C_{F,i} - C_{hor,i} \right) \qquad (1)$$

The first term ($C_{cr,i}$) describes the cost due to overflows generated by the runoff volume that already entered the drainage system; the second term ($C_{F,i}$) expresses the cost due to expected overflow events in the time interval defined by the forecast horizon (in this study set to 2 hr – based on the reliability of radar nowcasting); the third term ($C_{hor,i}$) is a factor which optimizes the system when no rainfall is expected (i.e. in dry weather periods and during the emptying phase of the basins).

Overflow costs are defined as linearly proportional to the overflow volume, with a cost c_i (listed in brackets in Figure 1) which reflects the different sensitivity of the receiving water body. For example, discharge from the most sensitive overflow structures, such as bathing areas (e.g. Lersoeledning, East Amager), is defined as 25 times more expensive as overflow at the inlet of Lynetten WWTP.

The cost of expected overflow at the i-th discharge point ($C_{F,i}$) is calculated by considering the expected runoff volume ($V_{F,i}$) and the related uncertainty level $p(V_{F,i})$, which is estimated by rainfall-runoff forecast models (see section 2.2), and the available storage volume (V_{cr} – critical volume, which takes into account the available volume in the basin and the difference between inflow and outflow). By combining the estimated overflow probability (showed in red in Figure 2) with the linear cost relationship, it is possible to calculate the overflow risk ($C_{F,i}$):

$$C_{F,i} = \int_{V_{CR,i}}^{\infty} c_i \cdot V_{F,i} \cdot p(V_{F,i}) \, dV_{F,i} \qquad (2)$$

A genetic algorithm is used to minimize the total cost (eq. 1) by varying the outflow from each basin ($Q_{out,i}$).
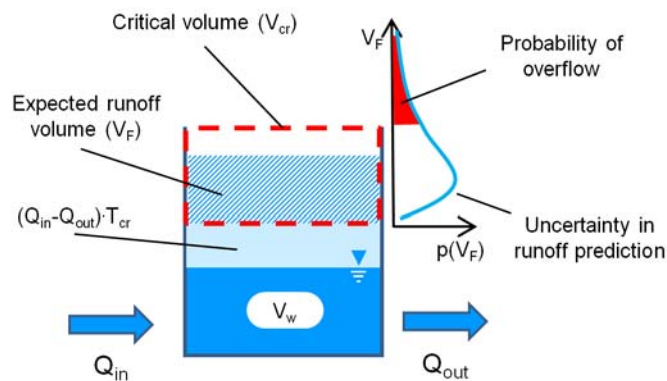

Figure 2. Scheme of a basin as schematized in DORA. Probability of overflow (in red) is calculated by considering the actual water volume (V_w), the available storage volume (V_{cr}), and the estimated level of uncertainty in runoff predictions $p(V_{F,i})$.

2.2   Runoff forecast models

2.2.1   Deterministic model

Currently, runoff for each of the eight subcatchments in Lynetten is estimated by using a conceptual hydrological rainfall-runoff model. Rainfall predictions, based on the radar nowcasting for an interval of two hours (see section 2.1.1), are used as input to a linear cascade model, which provides the inlet hydrograph for the i-th basin. The model is coded in Wateraspects[TM] (Grum et al., 2004) and model parameters are estimated trough auto-calibration based on a Baysian approach every 10 minutes (i.e. each time a new radar nowcast is available). The calibration procedure re-estimates the model main parameters, which are the catchment impervious area A and the discharge constants of the linear reservoirs. A similar approach, based on an informal Bayesian methodology, has also been applied by Leonhardt et al. (2012).

The expected runoff volume ($V_{F,i}$) is subsequently calculated by integrating the hydrograph until the time critical T_{cr} (hatched area in Figure 2). The uncertainty in this estimation is assumed to follow a pre-defined gamma distribution. This assumption has been made for simplicity, as it is possible to analytically calculate the integral listed in eq. 2 (Vezzaro and Grum, 2012).

2.2.2   Stochastic model

Similar to the deterministic case, we apply conceptual hydrological rainfall-runoff models for generating stochastic runoff predictions with radar nowcasts as input. Breinholt et al. (2011) give a detailed description of the model structure. The open-source software CTSM (Kristensen and Madsen, 2003) is applied for the modelling process and we obtain a state space model layout with the following system equations:

$$d\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} A \cdot P_t + a_0 - \dfrac{1}{K} S_1 \\ \dfrac{1}{K} S_1 - \dfrac{1}{K} S_2 \\ \dfrac{1}{K} S_2 - \dfrac{1}{K} S_3 \end{bmatrix} dt + \begin{bmatrix} \sigma_1 \cdot S_1 & & \\ & \sigma_2 \cdot S_2 & \\ & & \sigma_3 \cdot S_3 \end{bmatrix} d\omega_t \qquad (3)$$

In eq. 3 (S_1, S_2, S_3) correspond to the reservoir states in the lumped model, (A) to the effective area, (P_t) to the rainfall input, (a_0) to the mean dry weather flow and (K) to the time lag constant. In addition to the physical model part, the system equations also include a stochastic term consisting of a random process ($d\omega_t$) with state dependent variance ($\sigma \cdot S$). This term is used to model uncertainties resulting from uncertain (rainfall) inputs and an incomplete description of reality by the model.

The so-called observation equation relates flow predictions from the model to basin inflow observations (Q_i) at time step (i). We describe variations in dry weather flow by a harmonic function (D_i) and consider the flow observations subject to a random normal error (e_i).

$$Q_i = \frac{1}{K} S_{3,i} + D_i + e_i \qquad (4)$$

Depending on the variance of states and observations, this model layout allows for an adjustment of the states at every time step to match the observations. This state updating is performed through an extended Kalman filtering routine. It is also possible to include the model parameters (A) and (K) as additional states corresponding to e.g. purely random variables in the system equations 4. The parameters are then estimated as part of the state updating and the model can adapt to different behaviour of the system e.g. in summer and winter or for different rainfall characteristics. We intend to implement this kind of behaviour in the next version of forecasting models and hence do not perform a constant re-calibration as for the deterministic models.

Parameters for the stochastic flow forecasting models used in this work are derived by minimizing the difference between the distributions of multistep flow predictions and the empirical distribution of the corresponding flow observations using the continuous ranked probability score (CRPS) as criterion (Gneiting and Raftery, 2007;Löwe et al., submitted). A 4-week-period of observations in spring 2012 was used for estimation of the model parameters.

Probabilistic predictions of runoff volumes are obtained through a scenario approach. 1000 possible scenarios of flow forecasts up to the maximal considered horizon of 60 time steps or 2 hours are generated from the greybox models. For each scenario, the predicted runoff volume up to a given horizon is determined and subsequently a distribution of runoff volumes is derived from the different scenarios.

2.2.3 Comparison setting

The comparison between the two approaches for runoff estimation in an integrated RTC context was carried out by using DORA to control a simplified model of the Lynetten catchment, which is implemented in Wateraspects™. Flows between the different basins are simulated by a simple routing function, i.e. the full hydrodynamic description of the system (including e.g. back-water effects) was neglected in this study. The inputs to the model were eight flow time series, which were generated by running a calibrated MikeUrban model of the Lynetten catchment. The inputs of the MikeUrban model were the rainfall intensities which were recorded by the radar: this allowed for considering rainfall spatial variability in the study.

Two sample events were considered here to demonstrate the effect of different forecast models on real time control. These events were selected according to data availability and to the response of the system (i.e. overflow volumes), which was estimated based on the detailed model simulations. Both the rain events had similar magnitude, with a total runoff volume of about 13 mm and a duration of about 8-9 hours. Due to the bad quality of the measurements available for the St. Annae basin, no forecasts were used in the simulations for this point (i.e. only the actual volume of stored water and the flows from the upstream basins were used by DORA).

3   RESULTS AND DISCUSSION

3.1   Runoff predictions

As an example, Figure 3b,d shows predicted inflows to two of the basins considered in the control setup for the Lynetten catchment for a forecast horizon of 120 minutes. We see that the stochastic forecast for the Colloseum basin (left, event from 24/09/2012) does not capture the system behaviour very well. The deterministic forecast underestimates the runoff volumes, but generally describes the behaviour of the system better. This is a result of the stochastic model parameters being estimated only to a 4 week period containing 3 rain events whereas the deterministic model is recalibrated every 10 minutes.   In the Strandvænget catchment (right, event from 12/10/2012) we obtain a better prediction of the true runoff volumes as the rain events considered in the estimation period for this model were more representative for forecasting this event. For the deterministic model we see a similar behaviour of capturing the general system behaviour correctly but underestimating the runoff volume.
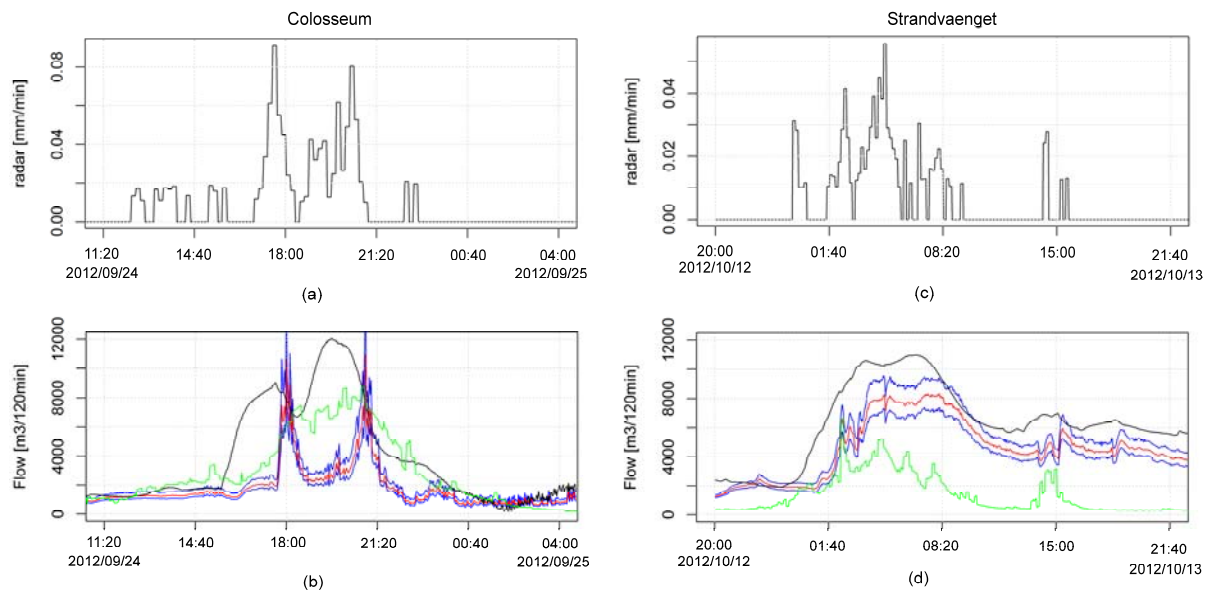

Figure 3. Top: Radar rainfall observations. Bottom: measured basin inflow (black), stochastic predicted flows (red with 95% confidence bounds shown in blue) with a forecast horizon of 120 minutes; and deterministic predictions (green). Left: Colloseum (event from 2012/09/24). Right: Strandvænget (event from 2012/10/12).
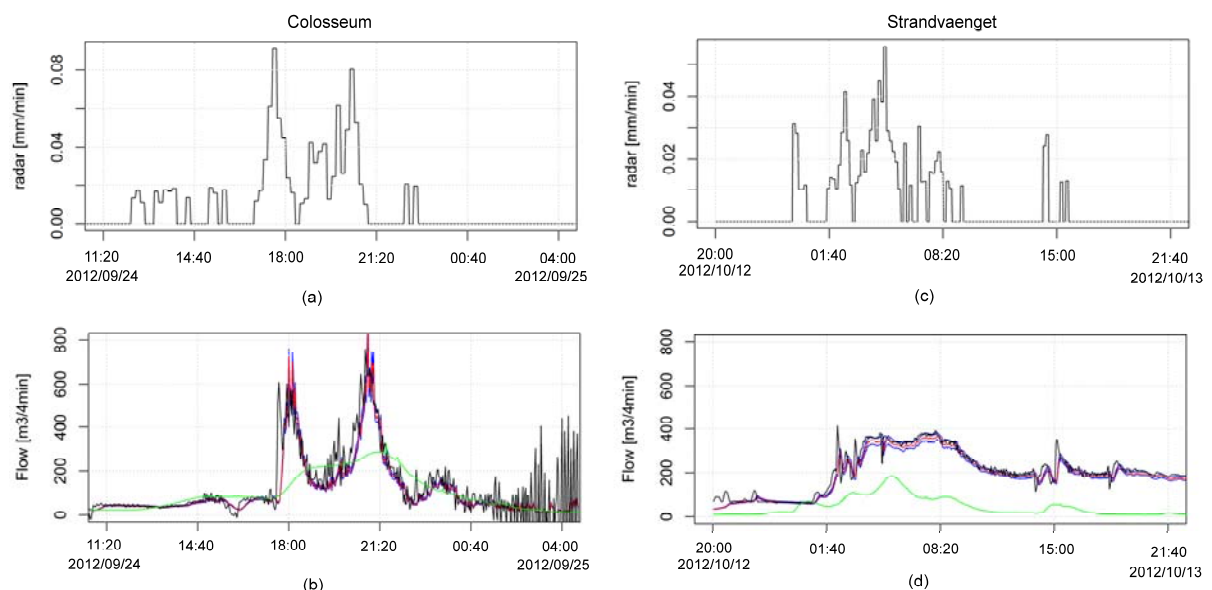

Figure 4. Top: Radar rainfall observations. Bottom: measured basin inflow (black), stochastic predicted flows (red with 95% confidence bounds shown in blue) with a forecast horizon of 4 minutes; and deterministic predictions (green). Left: Colloseum (event from 2012/09/24). Right: Strandvænget (event from 2012/10/12).

Table 1. Overflow volumes and cost simulated for the controlled drainage system with deterministic and stochastic forecasts, event from 24/09/2012

Control Point	Overflow Volume [m^3]			Overflow Cost		
	deterministic	stochastic	variation	deterministic	stochastic	variation
Lersoeledning	38759	23259	-40%	968972	581473	-40%
Strandvænget Basin	8584	0	-100%	214595	0	-100%
Strandvænget Pump	7417	0	-100%	7417	0	-100%
Colloseum	0	0	-	0	0	-
St. Anne	8392	0	-100%	50348	0	-100%
West Amager	0	0	-	0	0	-
East Amager	0	0	-	0	0	-
Kloevermarken	1156	13785	+1093%	5780	68926	+1093%
Lynetten	0	0	-	0	0	-
Total	64307	37044	-42%	1247111	650398	-48%

Looking at a shorter forecast of 4 min horizon in Figure 4b,d, we see that the state updating inherent in the greybox models almost automatically leads to good forecast results as the initial state values of the models for generating forecasts are always set to a good value. The deterministic models, although auto-calibrated every 10 minutes, do not include this property and are outperformed by the stochastic models.

Finally, we can see that the uncertainty bounds provided by the stochastic models appear reasonable on the short horizons (Figure 4b,d), but too narrow on the long horizons (Figure 3b,d). In the model estimation we optimize the model for generating multistep predictions. This means, that the derived uncertainty description is a compromise between longer and shorter forecast horizons and will typically be too narrow for the longer horizons. Further, the model structure describing forecast uncertainties depending on the predicted state value is not optimal for modelling the actually observed uncertainties of the predictions as the predicted states can be quite far from the truth. There appears to be a general tendency that estimating the model parameters using the CRPS as objective function then results in rather small predicted uncertainties, that on the other hand maximize the information content (or resolution) of the forecasts.

3.2 Overflow risk

Table 1 and Table 2 show the simulated overflow volumes and costs for the two considered events. In the first event, the stochastic predictions produce a clear reduction of the overflow volumes by improving the use of the available storage and by increasing the overflow volume in less sensitive points of the drainage systems (Kloevermarken). This resulted in an overall reduction of the CSO cost of 50%. Figure 5 shows how the stochastic forecasts resulted in a greater water storage in the southern part of the catchment (Figure 5b,d) and in a lower filling of the basins in the northern part of the catchment (Figure 5a,c). Interestingly, a better use of the storage in the Colosseum catchment (not shown), resulted in better performance of the downstream St. Anne basin, where no forecasts were available.

In the second event, utilisation of the stochastic predictions results in an increase of overflow cost of approximately 15%. The main reason for this increase is a strong underestimation of expected runoff volumes at Lersoeledning (one of the most sensitive points in the system) by the forecast model (see Figure 6a). Based on this information, DORA defined lower outflows from the basin (Figure 6b), resulting in an increased overflow when the storage capacity was exceeded. Figure 6c shows how the Lersoeledning basin is filled at the same rate with both the deterministic and the stochastic forecast. The increased COS volume is thus due to the lower optimal flow, which is defined according to the information provided by the forecasts.

Table 2. Overflow volumes and cost simulated for the controlled drainage system with deterministic and stochastic forecasts, event from 12/10/2012

Control Point	Overflow Volume [m^3]			Overflow Cost [DKK]		
	deterministic	stochastic	variation	deterministic	stochastic	variation
Lersoeledning	16991	21293	+25%	424744	532317	+25%
Strandvænget Basin	1718	287	-83%	42944	7172	-83%
Strandvænget Pump	0	0	-	0	0	-
Colloseum	0	0	-	0	0	-
St. Anne	5120	477	-91%	30720	2863	-91%
West Amager	0	0	-	0	0	-
East Amager	0	0	-	0	0	-
Kloevermarken	104	6373	+6030%	520	31866	+6030%
Lynetten	0	0	-	0	0	-
Total	23933	28430	+19%	498960	574218	+15%
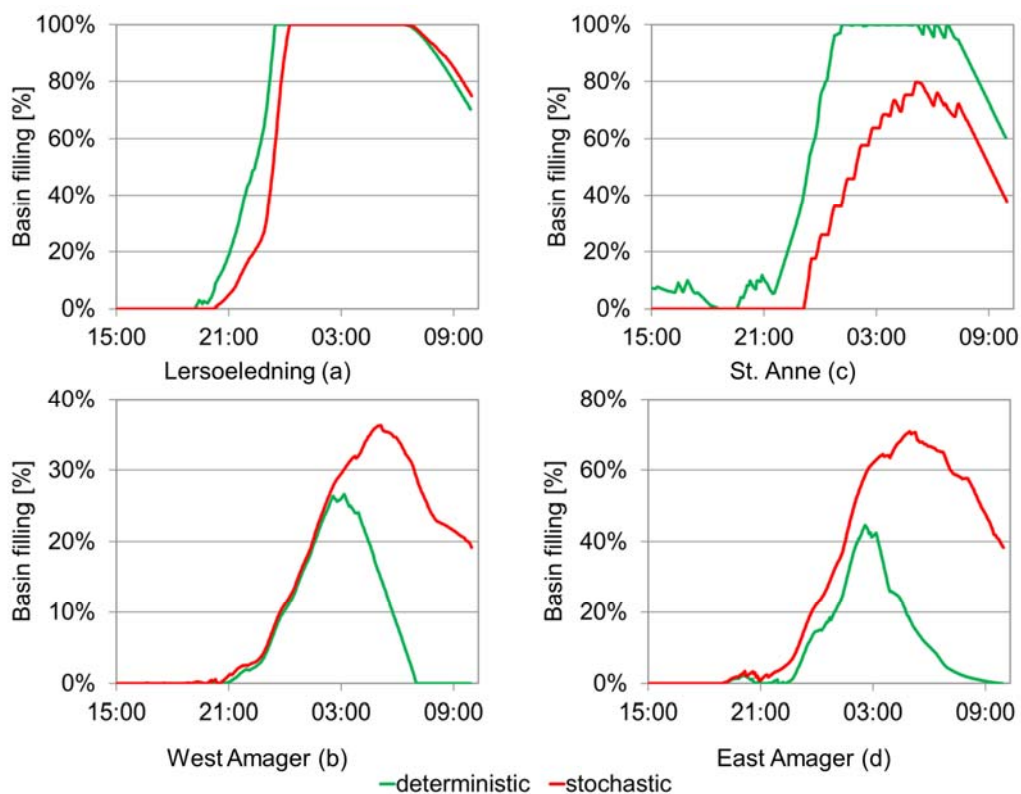
Figure 5. Simulated filling degree in four major basins (event from 2012/09/24)

However, also in this event we can see how overflows are reduced at the more critical points Strandvænget and St. Anne and moved to Kloevermarken. As suggested in the previous section, an adaptive model should provide improved control results for this event. As observed for the first event, the control strategy utilizing the stochastic forecasts stored greater volumes in the southern part of the catchment (Amager West and East).
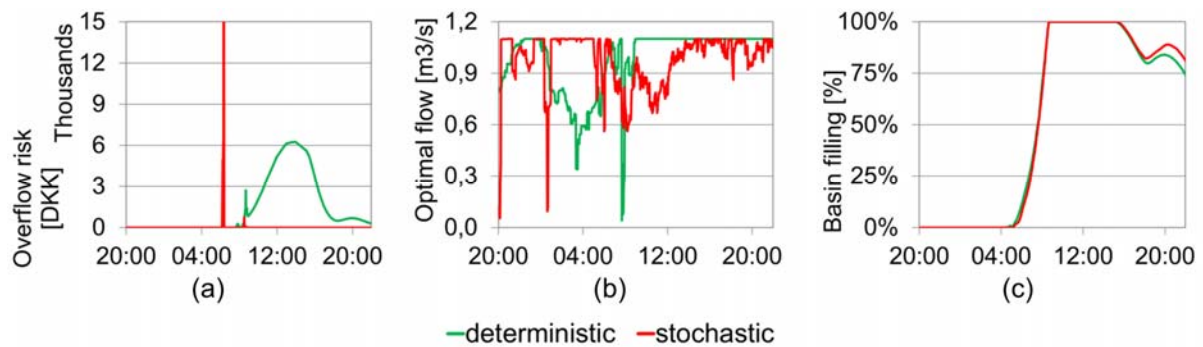
Figure 6. Simulated overflow risk (a), optimal outflow, and filling degree (c) for Lersoeledning for the event starting on 2012/10/12.

Finally, we need to point out that the here performed analysis based on two events can only give an impression of the effects obtained by different forecasting systems. The behaviour of a real time control system depends very much on the event characteristics. As shown for example in Vezzaro and Grum (2012), conclusions on the performance of integrated control approaches can only be drawn on the basis of a great number of events. Given the limited amount of available data, this analysis is postponed to a further stage of the SWI project.

4   CONCLUSION

The implementation of stochastic runoff forecasting models into a real time control system has been shown. Decision making based on the stochastic forecasts can in general provide reduced overflow volumes from sewers. Based on two sample events, very oppositional results are obtained. Applying the stochastic forecasts models for decision making leads to a clear reduction of overall overflow volumes in the first event and a shift of overflow volume to a less critical location.

In the second event, an underestimation of runoff volumes at a critical basin by the stochastic forecasting models results in an increase of overflow at this basin and also the overall amount of overflow. Conclusions on the general performance of the stochastic forecasting system can off course not be drawn based on two events but require the simulation of extended periods.

Analysing the model predictions, however, a need for adaptivity of the prediction models can be identified. The models partly failed to properly reproduce the system behaviour as they only include a simplified physical description of the system and the data used for model calibration were not representative for the rain events considered when testing the control system. Allowing model parameters to vary in time is a way to obtain improved forecasts and can easily obtained in the stochastic models through extended Kalman filtering.

Further, the currently applied state dependent uncertainty description in the forecasting models is identified as insufficient. With the applied structure, wrong physical forecasts will also lead to wrong uncertainty forecasts and the models fail to capture in particular the high forecast uncertainty at the beginning of rain events. Making the models uncertainty description subject to the rainfall input rather than the predicted states should be one step towards proper uncertainty predictions.

AKNOWLEDGMENTS

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