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ESSAYS ON TECHNICAL ANALYSIS AND MARKOV CHAINS

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A thesis submitted in fulfillment of the requirement for the award of the Degree of Doctor of Economics

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Declaration

I certify that except where due acknowledgment is made, this thesis presented to the Lisbon School of Economics and Management (ISEG) in fulfillment of a PhD degree is solely my work. The copyright of this thesis belongs to the author. Quotation from it is permitted, provided that full acknowledgment is made. This thesis may not be reproduced without the prior written consent of the author. I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party. I confirm that in Chapter Four there is a jointly co-authored paper with João Nicolau.

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To God

To my mother and my beloved Raquel, Valeria and Violeta.

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Abstract

The efficient market hypothesis (Fama, 1970) has been one of the most fundamental pillars of modern financial theory. According to the weak-form of the efficient market hypothesis, prices should reflect all available information. Consequently, it should not be possible to earn excess returns consistently from any investment strategy that attempts to predict asset price movements based on historical data (Fama, 1965; and Fama & Miler, 1972).

Nevertheless, in recent decades, empirical studies have provided evidence that models used for forecasting stock markets, such as technical analysis (TA), which are based on past stock price and volume, can lead to sustainable profitability. Indeed, the TA methodology, which is one of the most widely-used financial market forecasting tools, has been classified as a high-performing method, capable of predicting the stock market.

TA is classified as a price forecasting and market timing methodology, based on the assumptions that markets move in trends, and that these trends persist, suggesting some sort of serial dependency of the behavior of past prices series. In the TA jargon, market action discounts everything.

In this dissertation, we empirically study the predictive power of technical analysis indicators and propose a new theoretical framework, based on a well-defined statistical and mathematical platform. Accordingly, we introduce a new TA methodology, based on multivariate Markov chains. Using as a source the MTD-Probit model proposed by Nicolau (2014), we explore the use of the Markov chain to explain the departure from the martingale property when data snooping is statistically controlled.

Resumo

A hipótese do mercado eficiente (Fama, 1970) tem sido um dos mais fundamentais pilares da teoria financeira moderna. De acordo com a forma fraca da hipótese, os preços dos ativos financeiros devem refletir todas as informações disponíveis. Consequentemente, não é possível obter consistentemente retornos superiores à média do mercado com qualquer estratégia de investimento destinada a prever oscilações dos preços das ações com base em dados históricos (Fama, 1965; e Fama & Miller, 1972).

No entanto, nas últimas décadas, estudos empíricos têm fornecido indícios de que os modelos utilizados para a previsão do mercado de ações com base em informações históricas, como a análise técnica (AT), podem conduzir a uma rentabilidade sustentável. Efetivamente, a metodologia da AT, uma das ferramentas de previsão de mercado financeiro mais amplamente utilizada, tem vindo a ser classificada como um método de alta performance, capaz de prever os mercados de ações.

A AT é uma metodologia de previsão de preços e "timing" de mercado que se baseia nas premissas de que os mercados oscilam por tendências, e de que essas tendências persistem, sugerindo algum tipo de dependência em série com base no seu comportamento passado. No jargão da AT, o mercado desconta tudo.

Nesta dissertação, estudamos empiricamente a capacidade de previsão de indicadores de análise técnica e propomos um novo quadro teórico, baseado numa metodologia estatística e matemática bem definida. Neste sentido, apresentamos uma nova metodologia de AT, com base em cadeias de Markov multivariadas. Utilizando como fonte o modelo MTD-Probit proposto por Nicolau (2014), exploramos o uso da cadeia de Markov para explicar o desvio em relação à propriedade de Martingale quando o "data-snooping" é estatisticamente controlado.

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Chapter 1

Introduction and Research Overview

1.1 Introduction

The widespread use of technical analysis (TA) as a leading stock market forecasting tool (see, e.g. Skynkevich, 2012) is still challenging the concept of market efficiency. Since the study of Fama (1970), the efficient market hypothesis (EMH) has been one of the most fundamental pillars in modern finance theory. According to the EMH (Fama, 1965, 1966 and Fama & Miller, 1972), prices should reflect all available information, and it should therefore not be possible to earn excess returns consistently from any investment strategy based on historical data. Consequently, the best conditional choice for future prices should be the current price. That is to say, buying and holding the security is the best investment strategy.

Nevertheless, in recent decades, new empirical evidence has suggested that the stock market can be inefficient, and that it is possible to obtain abnormal stock returns that are not fully explained by common risk measures. In particular, some authors have addressed the possibility that TA could result in sustainable profitability (Murphy, 1986; Sweeney,1986,1988; Brown and Jennings, 1989; Brock et al., 1992; Blume et al.,1994; Neely et al., 1997; Gencay, 1998; Hsu et al., 1999; Lo et al., 2000; Griffioen, 2003; Park and Irwin, 2004 and 2007; Hsu et al., 2010; Neely and Weller, 2011; and Hsu et al., 2013).

The main objective of this dissertation is to study the effectiveness of the technical analysis methodology, and to propose a new multivariate Markov chain (MMC) model to forecast financial market behavior. We believe that the use of this methodology is of special interest in finance, as it is theoretically robust, well defined, and parsimonious. Indeed, the MMC estimation process does not require any extensive set of assumptions, such as the normality distribution, or the existence of homoscedasticity in the series under analysis.

In this context, our key hypothesis is that financial markets are in some way inefficient and that the use of a robust forecasting technique can lead to a substantial profit opportunity. Additionally, we believe that financial time series display a non-random behavior which depends on some independent explanatory variables, and therefore, for forecasting purposes, we have to consider these intrinsic features. This concept is relatively similar to that of the econometrics models which are used to characterize and model financial time series.

In the MMC framework, nevertheless, when the number of categorical data (say s) and the number of states that each financial data can assume (say m) becomes large, any model estimation becomes rapidly intractable, even with moderate values of s and m (e.g. Raftery, 1985 and 1994; Raftery and Tavare, 1994; Berchtold, 1985; Ching et al., 2002 and 2008; and Zhu and Ching, 2010). Nonetheless, a new MMC estimation procedure, called "MTD-

Probit" (Nicolau, 2014), has led to a simpler approach which facilitates model parameter estimation and its statistical inference.

As a result, in this dissertation we forecast the Financial Time-Stock Exchange 100 Share Index (FTSE 100), using a simple trading strategy based on the MTD-Probit estimation method. We call this procedure the "markovian MMC indicator" (MMCI). As far as we know, this methodology has never been used to forecast the stock markets.

Furthermore, we carry out inference and model selection, and apply the White (2000) Bootstrap Reality Check (RC) and the Hansen (2005) Superior Predictive Ability (SPA) data-snooping bias tests. These tests allow us to forecast the Index, based on a large set of parameters and co-variables, without any data mining spurious results.

1.2 Research Overview

This dissertation is structured in five interconnected essays, and is divided into two main sections. We follow a "basic focus rationale" and produce a concise study, which could be of innovative interest for the academic and business community alike.

The first section is based on the study of the standard technical analysis framework. Its main focus is to expand our knowledge of the profitability of technical analysis in an unexplored empirical area. In Chapter Two, we therefore propose a study of TA profitability in the Euronext Lisbon stock exchange index, PSI-20.

We analyse the performance of a total of 152,071 trading rules, checking, after considering the costs involved, for the existence of superior returns, compared to adopting the reference strategy, which is buying and retaining the asset (buy-and-hold).

To the best of our knowledge, this is the first paper in which the data snooping controlled methodology has been applied in an extensive one-country study of the Portuguese financial market, which is a relatively "young and less-capitalized" market in a well-developed region.

In the second section, we analyse the previous understanding of the technical analysis methodology and propose a new forecasting instrument, which is dependent on MMC mathematical framework. The main objective is to provide a sound theoretical structure, based on a well-defined statistical and mathematical platform, which is feasible to be applied in the "real" world. This section covers four chapters.

In Chapter Three, we explore the standard Markov chains test methodology applied in financial market studies, and propose some reconfiguration to expand its main findings.

In Chapter Four, co-authored with João Nicolau, we closely study the theoretical formulation of the MMC methodology, and thus propose a more structured formulation of the tools used.

Then, in Chapter Five, we introduce a new multivariate Markov chain forecasting methodology based on the MTD-Probit model proposed by Nicolau (2014). We call this procedure the "Markovian MMC indicator", and use it to forecast the FTSE 100 index.

In Chapter Six, we consolidated all previously discussed concepts, and examine a new research prospect which combines both the Markov chain framework and the standard technical analysis methodology. As a result, we present a new methodology to maximize the performance of any technical analysis trading strategy.

It is well known by financial market professionals that one major difficulty with the use of TA methodology is how to correctly forecast stock price movement signals without being confused by false signals. These trading noises can be seen even in the best-behaved stock price series, and are one of the most challenging problems, as late entries and exit points are responsible for lower investment return.

In this context, we use the MTD-Probit model as a noise control method for the TA strategies, and apply it to forecast the FTSE 100 Index. Our main objective is to provide evidence that this methodology not only potentially controls and filters out false trading signals, but that it is also an important step for the study of TA predictive power.

To the best of our knowledge, this is the first time that Markov chain methodology has been used in conjunction with technical analysis, as part of a stock market investment strategy.

Chapter 2

Testing the Profitability of Technical Analysis in the PSI-20 Index

Abstract

In this paper, we present a new evidence of the profitability of the technical analysis trading rules in the Portuguese Stock Exchange PSI-20 Index. We apply a total of 152,071 simple and complex trading strategies and test for superior performance compared to the buy-and-hold trading strategy. It has been found that economically significant excess returns over the buy-and-hold trading benchmark strategy are generated, before take in account the effects of data-snooping bias. Nevertheless, the data-snooping tests suggest that the best rule performance across sub-samples is not significant at any significant conventional test level. Indeed, in spite of the wide number of rules tested in this study, our superior profitability could be due to chance rather than to the existence of high-performance strategies. Under such circumstances, the possibility of spurious results is a reasonable assumption.

Keywords: technical analysis, efficient market hypothesis, data-snooping, PSI-20 Index.

2.1 Introduction

The efficient market hypothesis (Fama, 1970) has been one of the most fundamental pillars in modern financial theory. According to the weak-form of the efficient market hypothesis, prices should reflect all available information; therefore, it should not be possible to earn excess returns consistently with any investment strategy that attempts to predict asset price movements based on historical data (Fama, 1965; and Fama & Miler, 1972).

Nevertheless, in recent decades, empirical studies have provided evidence that models used for forecasting stock markets, based on past stock price and volume, such as technical analysis (TA), can produce sustainable profitability. Indeed, the TA methodology, which is one of the most widely-used financial market forecasting tools (e.g. Shynkevich, 2012), has been considered a high-performance method, capable of predicting the stock market (see among others Sweeney, 1988, and Brock. et al., 1992, Hudson et al., 1996, Taylor, 2000, Skouras, 2001, Park and Irwin, 2004 and 2007, Marshall et al., 2010, Neely and Weller, 2011, and Hsu et al., 2013).

Formally, TA is a price forecasting and market timing methodology, based on the assumptions that markets move in trends, and that these trends persist, suggesting some sort of serial dependency about the behavior of past prices series. In the TA jargon, market action discounts everything. Amongst the possible explanations for the superior performance of TA, is the possibility of a nonlinear stochastic dynamic in stock returns (Berchold and Raftery, 2002), as well as some sort of short-run time inefficiency (Timmermann and Granger, 2004). From this perspective, the TA strategies' superior profitability could be the result of exploiting those intrinsic characteristics, despite the lack of strong formal mathematical and statistical structures.

However, it has been also suggested that the use of TA to forecast financial markets can be profitable only if data-snooping is not statistically controlled. Indeed, it is well known that data-snooping is a typical problem in financial time-series analysis (Lo, 1990, Brock, 1992, White, 2000, Hansen, 2005, Romano and Wolf, 2005, Hsu and Kuan, 2005, Park and Irwin, 2007, Wang, 2007, Romano et al., 2008, Hsu, Hsu, & Kuan, 2010, Park and Irwin, 2010, Day and Lee, 2011, Neuhierl and Schlusche, 2011, Chen et al., 2011, and Yu, 2013).

In this paper, we study the TA profitability in the Euronext Lisbon stock exchange index, the PSI-20. Its main objective is to expand our knowledge of the profitability of technical analysis in an unexplored empirical area. Although there is voluminous literature on the study of the performance of TA, little research has been undertaken to study the Portuguese stock market. Indeed, to the best of our knowledge, this is the first paper in which the data snooping controlled methodology is applied in an extensive one-country study of the Portuguese financial market, which is a relatively "young and less capitalized" market in a well-developed region.

We contribute to the literature in two ways. Firstly, we produce a novel study of the TA rules' profitability, using a unique broad sample of trading rules. We analyse the performance of a total of 152,071 trading rules, based on well-known mathematically-defined trading rules, checking for the existence of superior returns, after considering the costs involved, compared to adopting the reference strategy, which is buying and retaining the asset (buy-and-hold). Secondly, we test the TA profitability adjusting accordingly for data-snooping bias, by applying the White (2000) "Bootstrap Reality Check" (RC) and the Hansen (2005) (SPA) tests.

The study shows that before adjusting for data-snooping and transaction costs there is some evidence that TAI rules are capable of consistently producing superior performance over the buy-and-hold benchmark. It was seen that the benchmark is outperformed with an excess return that lies between 43.45% (2011 to 2014) and 14.63% (1993 to 2002). Nevertheless, the data-snooping tests suggest that the best rule performance across sub-samples is not significant at any conventional test level. Indeed, in spite of the high number of rules tested in this study, our superior profitability could be due to chance rather than to the existence of high-performance strategies. Thus, we conclude that there is non-significant evidence of abnormal profitability of the TAI strategies applied to forecast the PSI-20, and therefore we cannot rejected the weak-form of the efficient market hypothesis (Fama, 1965 and 1970).

The remainder of this study is organized as follows. The theoretical consideration related to the efficient market hypothesis and technical analysis is presented in Section 2.2. In section 2.3, we present an overview of the PSI-20 Index and define the study sample. In sections 2.4 and 2.5, the trading rules modelling framework and data-snooping test that are used in our study are presented, respectively. In Section 2.6, we present the empirical evaluation of the TA profitability, using controlled data-snooping tests. Finally, Section 2.7 concludes the paper.

2.2 A Brief Literature Review

The use of TA is probably one of the most popular and oldest [1] investment tools among practitioners, which is used mainly as a complement for fundamental analysis. Indeed, as is acknowledged by Menkhoff (2010) in a survey with 692 fund managers in five countries, TA is a highly used methodology and "is obviously in widespread and of relevant use among fund managers" (p.2573).

However, despite its widespread use, the empirical studies in the area are ambiguous and controversial. On the one hand, there is a body of research that validates the market efficiency and presents contrary evidence for the use of TA as a method that could generate abnormal returns, based on publicly available market information (Fama, 1966; Bessembinder and Chan, 1995 and 1998; Allen and Karjalainen, 1999; Ready, 2002; Li and Wang, 2007; and Hoffmann and Shefrin, 2014).

On the other hand, several other studies have shown that TA could be a high-performance method capable of analyzing any fundamental stochastic structures presented in financial data series (Sweeney, 1986; Neftci, 1991; Brock et al., 1992; Blume et al., 1994; Sullivan et al., 1999; Lebaron, 1999; Lo et al., 2000; Qi and Wu, 2006; Cheung et al., 2011; Mitra, 2011; Metghalchi et al., 2012; and Shynkevich, 2012).

[1] The TA principles were established as far back as the late 1800´s with the Japanese candlestick charting techniques.

There are some explanations for these controversial results. We would like to point out two of them. First, the research in this area has proven to be difficult to model, because it requires specially-designed forecasting models, as they often exhibit a near-random behavior, which is characterized by non-stationarity, dependence on higher moments, heteroscedasticity, and specially, nonlinear behavior (Bollerslev, Chou and Kroner, 1992, Hamilton, 1994, Berchtold and Raftery, 2002, and Gonzalez and Rivera, 2009).

Furthermore, as TA methodology is a highly diverse group of techniques and methods, empirical studies in this area were formulated using several different approaches, ranging from simple trading rules as moving averages, through a complex graphic pattern of analysis recognition [2].

Second, it is considered that the existence of any TA profitable trading rule is just the result of data-snooping. Indeed, it is well known that any empirical result in the financial time-series analysis could produce controversial results, because of the data-snooping bias (e.g. Lo and MacKinlay, 1990, Brock et al., 1992, White, 2000, Hansen, 2005, Lin et al., 2010, Bajgrowicz and Scaillet, 2012, and Kuang et al., 2014).

In this context, the study of technical trading rules' performance on the Portuguese Stock Exchange is an unexplored empirical area. Indeed, there is only one published global empirical study that examines the predictive-ability of moving average trading rules for 16 European stock markets (Metghalchi et al., 2012), over the 1990 to 2006 period, that included the Portuguese market. Applying the White "Reality Check" (2000) test, the authors presented empirical results that support the superior profitability hypothesis from the technical analysis strategies in the PSI-20 Index.

2.3 The Index, data, and sample selection

In this section, we describe the PSI-20, data and sample selection methodology used in the empirical study of the profitability of TAI rules applied to the PSI-20.

2.3.1 The PSI-20

The PSI-20 Index is the Portuguese stock market index and the benchmark for structured products, funds, exchange traded funds and futures. The Index that was created on 31/12/1992, with a 3,000 points base level, and is a composite of the twenty largest companies in terms of a free float market capitalization. Nowadays, it has a market capitalization of €41.69 billion (December 31, 2014), and is part of the pan-European stock exchange group, Euronext, alongside Brussels's BEL20, Paris's CAC 40, and Amsterdam's AEX.

Formally, the PSI-20 is a market value-weighted index with a selection principle based

[2] An extensive review of the use of TA in different scenarios can be found in Park and Irwin (2004) and Menkhoff and Taylor (2007).

on the free float adjusted market capitalization. Companies are selected based on their "velocity threshold" and a minimum "free float" market capitalization of €100 million. The free float is the total of listed shares available for trading, and velocity is the daily ratio of the number of traded and listed shares. The Index composition is annually review in March by an independent PSI Steering Committee. For the annual review, the constituent companies must have an annual trading free float velocity of at least 25% to avoid replacement that can occur quarterly in June, September and December. On January 1, 2014, the Index rules changed. In the new context, the PSI-20 reduced its constituents to a minimum of 18 companies, and lowered the free float market capitalization minimal requirement. Table 2.1 summarizes the PSI-20 composition on December 31, 2014.

Table 2.1: PSI-20 Composition

Ticker Symbol	Company Name	Sector (ICB)	Index Weighting (%)	Index Cap. (%)	Float
EDP	EDP	Utilities	19.72	2.98	0.75
GALP	GALP ENERGIA-NOM	Oil & Gas	13.98	2.11	0.45
JMT	J.MARTINS, SGPS	Retail	13.89	2.10	0.40
NOS	NOS, SGPS	Media	8.93	1.35	0.50
EDPR	EDP RENOVAVEIS	Utilities	7.80	1.18	0.25
BCP	B.COM.PORTUGUES	Banks	5.89	0.89	0.85
CTT	CTT CORREIOS PORT	Industrial Goods & Services	5.57	0.84	0.70
SON	SONAE	Retail	5.42	0.82	0.40
PTC	P.TELECOM	Telecommunications	3.59	0.54	0.70
SEM	SEMAPA	Basic Resources	3.14	0.47	0.40
PTI	PORTUCEL	Basic Resources	3.13	0.47	0.20
BPI	BANCO BPI	Banks	2.36	0.36	0.25
RENE	REN	Utilities	1.67	0.25	0.40
EGL	MOTA ENGIL	Construction & Materials	1.62	0.25	0.45
ALTR	ALTRI SGPS	Industrial Goods & Services	1.52	0.23	0.45
BANIF	BANIF SA	Banks	0.84	0.13	0.70
TDSA	TEIXEIRA DUARTE	Construction & Materials	0.49	0.07	0.25
IPR	IMPRESA,SGPS	Media	0.44	0.07	0.50

Data Source: Euronext Lisbon, December 31, 2014

2.3.2 Data Sample Selection and Descriptive Statistics Results

In this study, we consider a large Index data sample from January 01, 1993 to December 31, 2014, obtained from the Datastream database. In Table 2.2, we report a summary of the descriptive statistics for the daily log returns on the indices considered in the paper.

We conduct our study on four subsequent sub-samples, in order to provide a dynamic analysis of trading rules performance (e.g. Sullivan et al., 1999, and Park and Heaton, 2014). We consider these sub-samples based on two combined criteria: a time-frame large enough to produce consistent parameter estimation, and some important historical facts that might generate some PSI-20 structural breaks.

In this framework, we selected the first sub-sample based on Portugal's entrance to the Euro zone in 2002[3]. We believe that the Euro provoked a permanent effect on the

[3] On January 1, 2002 twelve of the countries in the European Union issued their new euro banknotes and coins.

Index, since it reduced its exchange rate exposure. The second sub-sample corresponds to the sub-prime crisis period in 2008, when major worldwide financial institutions collapsed. The third sub-sample is defined by the European Union financial assistance package signed in May, 2011. Finally, the last period ends in 2014.

Table 2.2: PSI-20 Index Returns Descriptive Statistics

Period	01/01/1993 31/12/2014	01/01/1993 31/12/2001	01/01/2002 30/08/2008	01/09/2008 31/04/2011	01/05/2011 31/12/2014
N(Obs.)	5552	2229	1699	682	939
Mean Daily (%)	8.46E-03	0.043	5.79E-03	-0.0171	-0.0506
Max. (%)	14.6161	12.0992	4.8244	10.1959	5.4612
Min. (%)	-12.0992	-14.6161	-6.0125	-10.3792	-4.2669
SD	1.1789	1.1635	0.8777	1.5971	1.3252
Skewness	-0.4539	-0.9048	-0.5545	0.1825	-0.3211
Kurtosis	15.8468	24.6197	7.7458	11.5300	3.8445
ρ(1)	0.089*	0.112*	0.054**	0.045	0.121*
ρ(2)	-0.001*	-0.009*	0.044**	-0.056	0.030*
ρ(3)	0.009*	0.036*	0.059*	-0.043	-0.033*
ρ(4)	0.037*	0.079*	0.020*	0.054	-0.049*
ρ(5)	-0.015*	0.023*	0.002*	-0.072*	-0.041*
ρ(6)	-0.024*	-0.012*	-0.034*	-0.078**	0.020*
Q(6)	56.81*	46.26*	17.03*	14.58**	19.88*
JB	38370.12*	43714.83*	1505.22*	2076.24*	44.04*

Notes: Mean Daily (%) is the mean sample log-return, SD is the standard deviation, JB are the Jarque-Bera test statistics, ρ(n) is the estimated auto-correlation at lag n for each series and Q(n) are the Ljung-Box-Pierce test statistics for the nth lag. * **Statistical Significance at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

Figure 2.1 shows the Index behavior for the entire sample period and sub-samples.

Figure 2.1: PSI - 20 Sample

In general terms, our data sample series can be considered consistent for most financial series distributions. Indeed, as can be seen in Table 2.2, the highest mean daily buy-and-hold return for the PSI 20 Index is 0.043%, which equates to 250 trading days per year, a yearly average of 10.75%. Additionally, the table also shows that the Index is skewed to the left, which indicates that extreme negative returns are more probable than extreme positive ones. The sample excess kurtosis level reveals that the return series has fatter tails than the normal distribution, i.e. low positive and negative returns are more probable. The results show that the first sub-sample (1993-2001) is the highest leptokurtic (24.61) and skewed (-0.9048) period, and that the last sub-sample (2011-2014) has the lowest kurtosis (3.84) and negative asymmetry (-0.3211). There is also evidence of some significant autoregressive process in the PSI-20 return dynamics for some sub-samples, which is a common phenomenon in stock indices returns series. Nevertheless, this linear time dependence can also reflect the existence of a certain level of linear predictability in the index return.

Finally, the Ljung-Box Q statistics rejects, at the 1% level, the null hypothesis of no autocorrelations in the first six lags for the Index and, based on the Jaque-bera test results, the null hypothesis of normality is also rejected.

2.4 TA Rules Modeling Framework

The main goal of this paper is to evaluate TA profitability performance to predict stock market behavior. In this context, it is crucial to select an appropriate set of technical rules since this is an essential step to ensure properly tested procedures. Therefore, in this paper we adopt three basic rules selection criteria: (1) relevance of the instrument; we chose the most widely tools used in the financial market and in the academic literature; (2) replication capacity; we considered only mathematically well-formulated rules, and (3) analytical appropriateness; we selected the rules that are by construction "Markovian times", as proposed by Neftci (1991). In this scenario, we choose to study technical indicators trading (TAI) rules.

2.4.1 Technical Indicators Trading Rules

In the TA methodology there are special kinds of rules based not on the subjective judgment of figures or chart patterns analysis. Instead, they are focused on market variables data transformation such as trade price, volume and volatility, which can easily be quantified and tested (Murphy, 1986). These strategies can be seen as mathematically well-defined methods for foreseeing securities, based only on the past behavior. Indeed, in the case of these rules, study of historical data is enough to identify some aspects of price dynamics that can produce buy or sell signals, which can be used not only to foresee future changes in prices, but also to provide the information needed to create or adjust any taken market strategy adopted.

In this paper we consider an extensive set of TAI rules, drawn from a wide variety of parametrization specifications that are presented in previous academic studies and also the technical analysis manuals (see e.g. Edwards and Magee, 2012, and Pring, 2012). As acknowledged by Sullivan et al.(1999), the list of trading rules should be “vastly larger than those compiled in previous studies, and we include the most important types of trading rules that can be parsimonious parametrized and that do not rely on "subjective" judgments” (p.1655).

In this context, we choose a broad set of starting parameters that are presented in the financial literature, such as the number of days of the different horizons time measures, the size of the increase or decrease necessary to generate a buy or sell signal, the number of days’ rate of change in price or volume and overbought/oversold levels. We selected a parameter set that is diversified enough to avoid the type of “survivorship bias” problem related to the best performing historical rules (Sullivan et al., 1999).

Furthermore, since one of the trickiest aspects in technical analysis is the inaccuracy created by short-run false signals we combine TAI strategies, using some complex strategies to confirm an initial trading signal. We want to study multi-indicator trading rules that could help minimize the trading of signal-to-noise and increase profitability (Hsu et al., 2010). We provide an analysis of four complex trading rules. We test the MFI&RSI (Yen and Hsu, 2010), PPO&PVO, PMA&VMA and BBS&RSI. The list of trading rules is presented in Table 2.3 and in Appendices 1 and 2, we comprehensively detail how the rules and parameter values used in our analysis were defined. As a result, we select a total of 152,071 TAI trading rules parametrization, based on 36 different sets of simple and complex double-rules, provided by the practitioners and academic mainstream literature in the area (see e.g. Brock et al., 1992, and White, 2000).

Table 2.3: TAI Strategies

TAI Rules	Abbreviation	Number of Rules
Bollinger Bands	BBS-EMA and BBS-SMA	1,890
Commodity Channel Index	CCI	4,080
Chaikin Oscillator	CHO	173
Chaikin Money Flow	CMF	210
Moving Average Convergence Divergence	MACD	9,660
Moving Average Filters based on Price	PEMA and PSMA	75,918
Moving Average Filters based on Volume	VEMA and VSMA	
Money Flow Index	MFI and MFI - Divergence	7,920
Percentage Price Oscillator	PPO	3,479
Percentage Volume Oscillator	PVO and PVO-Divergence	6,958
Rate-of-Change	ROC and ROC Divergence	168
Relative Strength Index	RSI, RSI-Divergence	5,652
Stochastic Oscillator	STO, Fast and Slow STO	1,372
William R%	WRI	280
Complex Rule	BBS&RSI	8,820
Complex Rule	MFI&RSI	7,560
Complex Rule	PEMA&VEMA and PSMA&VSMA	14,452
Complex Rule	PPO&PVO	3,479
	Total Simple and Complex Trading Rules	152,071

2.4.2 TAI Financial Strategy

This section presents our trading rule methodology used to forecast the PSI-20 Index. We assume the existence of some sort of serial dependency on prices that can be seen as a generalization of McQueen and Thorley´s (1991) approach for analyzing stock returns predictability.

We propose evaluating a market strategy using a very simple approach. We assume that the investor buys or sells the PSI-20 Index, according to the trading signal based on the TAI estimation model.

We study the proposed strategy under two different investor behavior assumptions: the one-day strategy (ODS) and the trend reversal strategy (TRS). In the first strategy, we assume the naive and costly hypothesis that any signal lasts for a one-day period only. In the second strategy, we consider that the investor liquidates the position, only if it has a trend reversal signal, for example, from a buy signal to a sell signal.

To sum up, the procedure can be described through the following algorithm:

Step 1: For each sub-sample, we split our dataset into two segments. Then, we use the first t observations to determine the first TAI signals for buy, sell or no action, where the sell signal implies short selling. The size of t is given by the minimum size that is needed to calculate all the TAI trading rules.

Although it is not possible to sell short owing to legal or market restrictions, we follow the approach that it is essential to accurately calculate a total trading rule profitability. Additionally, if our investment rule indicates a no-change market (no action) we account for no return [4].

Step 2: We use the $t + 1$ observations to re-value the next TAI signals, and so on, sequentially, until we reach a time horizon of n predictions trading signals.

Step 3: We record all the returns that our trading rules are generating and measure total net returns. Mathematically, the returns are determined based on the signal function for the kth TAI rule, $k = 1, ..., M$, given by:

$$R^*_{k,t} = R_{k,t} - R^0_t, \tag{2.1}$$

$$R_{k,t} = I_{k,t}R_t - abs(I_{k,t} - I_{k,t-1})Tc, \tag{2.2}$$

$$R_t = ln(p_t/p_{t-1}), \tag{2.3}$$

[4] We could equally account the overnight cash rate, calculated on the basis, for example, of the "3-month Treasury Bill Yield".

where $R^*_{k,t}$ is the one-day excess return of the kth TAI strategy discounting the market benchmark strategy R^0_t, which in our case is the buy-and-hold trading strategy, after accounting for the one-way transaction cost Tc. Furthermore, p_t is the daily closing quote of the index at time t and $I_{k,t}$ is a variable indicator for the kth TAI rule, which takes the values 1,0 or -1, respectively if we take a long, no action or short position in t, respectively.

Step 4: For each model set up, we calculate the percentage success rate (PSR), based on the predictive accuracy of the trading signals generated in the previous steps, as follows:

$$PSR_k = V_k/n, \tag{2.4}$$

where V_k is the number of times that our kth TAI trading model estimated signal matches the real market movement in our forecasting horizon.

Step 5: We evaluate the performance of our forecast methodology, using the White (2000) "Bootstrap Reality Check" and the Hansen (2005) data-snooping test. More details of the bootstrap method and tests applied in this study are presented in Section 5.

2.4.3 Transaction Costs

In this study, we do not consider transaction costs directly, but make a simple assumption that $Tc = 0$. There is no doubt that an investment rule is profitable only when its profit is greater than any trading costs. However, the recent introduction of a new computational trading floor process and online trading systems have lowered the overall "transactional costs" (see e.g. Bessembinder and Chan, 1995, Mitra, 2010, Bajgrowicz and Scaillet, 2012, and Kuang et al., 2014). Therefore, it is very difficult to choose any previous or recommended one-way transaction costs level.

To minimize the effects of this "somewhat unrealistic assumption" (Bajgrowicz and Scaillet, 2012), we present a break-even transaction costs analysis based on the methodologies of Hsu et al. (2010) and Mitra (2010). Then, we calculate the "potential margins for profitability" (PMP) that is the level of Tc which could offset any foreseen profitability. As proposed the PMP is the break-even transaction cost, which measures the trading rule capacity to absorb any transaction costs (see e.g. Hsu et al., 2016). It is estimated as follows:

$$PMP = \frac{R_{Tk}}{N_k}, \tag{2.5}$$

where R_{Tk} and N_k are respectively, the total return and the number changing signals generated along the investment period horizon for the kth TAI rule. In our investment methodology, the transaction cost depends of the type of market strategy adopted. In the case of ODS, it is payable twice in each investment decision (round-trip cost), that is:

$$N_k = \sum_{t=1}^{n} 2 * abs(I_{k,t}). \tag{2.6}$$

However, in the TRS case, the transaction cost should be considered initially when a buy/sell signal generates an investment position, and secondly, when a new signal is generated; requiring a change in the previous investment decision as follows:

$$N_k = \sum_{t=1}^{n} abs(I_{k,t} - I_{k,t-1}). \tag{2.7}$$

2.5 Data-Snooping Bias

Many authors have raised concerns about reusing the same data set to test model forecasting accuracy, as this could generate a data-snooping bias (Lo, 1990, Brock, 1992, Hsu and Kuan, 1999, White, 2000, Hansen, 2005, Hsu and Kuan, 2005, Romano et al., 2005, Park and Irwin, 2010, Day and Lee, 2011, Neuhierl and Schlusche, 2011, Chen et al., 2011, and Yu, 2013). Indeed, the possibility of spurious results is a reasonable assumption since superior profitability could be due to chance rather than to the existence of high-performance strategies.

Two different approaches are described in the literature to overcome such biases. The first approach is to validate the forecasting results based on an available comparable data set or in out-of-sample testing (see, e.g. Lo and MacKinlay, 1990). However, such a procedure is not only dependent on existence of a comparable data set, but it is also highly sensitive to the arbitrary sample splitting choice.

A second approach is to test forecasting performance comparing the weighted distance between two alternative competing strategies. If this pairwise comparisons shows any statistically significant divergence, then we cannot consistently reject the null hypothesis that there is no profitable trading rule.

Nevertheless, the use of this methodology has an important pitfall, since using the same data set for a large number of competing strategies, can generate a sequential testing bias. In this case, a null hypothesis is a composite hypothesis of several individual hypotheses and, as a consequence, if we are testing each of the models separately (at some level α), then the overall test size increases whenever we test a new hypothesis.

To overcome the sequential test problem, some studies proposed new tests to provide a solution to the data-snooping problem. The methodology is based on the “best rule” (Sullivan et al., 1999, White, 2000, Hansen, 2005, and Shynkevich, 2012), verifying whether there is a superior rule within a “universe” of rules that could outperform some benchmark models, for example the buy-and-hold trading strategy or mean zero criterion.

2.5.1 The RC and SPA Tests

In this study, we use the White RC and Hansen SPA tests to provide accurate analysis of the profitability for our TAI trading rules taking into account data-snooping effects.

On the one hand, White (2000) proposes to test the predictive superiority of a trading rule (model) based on the performance measure relative to the benchmark trading strategy.

Formally, following the literature (see e.g. Lai and Xing, 2008, Hsu et al., 2010, and Metghalchi et al., 2012), let f_k $(k = 1, ..., M)$ denote the excess return of the kth trading rule to the benchmark model or performance measure (White, 2000) and $\varphi_k = \mathrm{E}(f_k)$. The null hypothesis is that there is no superior trading rule in the universe of the M trading rules:

$$H_0 : \max_{1 \leq k \leq M} \varphi_k \leq 0. \tag{2.8}$$

The rejection of (2.8) implies that at least one of the models has superior performance over the benchmark and is evidence against the EMH. In this context, White (2000) proposes a statistic to test this null hypothesis based on the maximum of the normalized sample average:

$$\overline{V}_n = \max_{1 \leq k \leq M} \sqrt{n} \bar{f}_k, \tag{2.9}$$

where $\bar{f}_k = \sum_{i=1}^{n} f_{k,i}/n$ with $f_{k,i}$ being the ith observation of f_k and $f_{k,1}, ..., f_{k,n}$ are the computed returns in a sample of n past prices for the kth trading rule. Additionally, the author approximates the sampling distribution of $\overline{V}_n$ [5] by:

$$\overline{V}_n^* = \max_{1 \leq k \leq M} \sqrt{n}(\bar{f}_k - \varphi_k). \tag{2.10}$$

In this set-up, White (2000) suggests using the Politis and Romano (1994) stationary bootstrap method (SB)[6] to compute the p-values of (2.10), based on the empirical distribution of $\overline{V}_n$, which is obtained with realizations of B bootstrapped samples, $b = 1, ..., B$, of the following statistic:

$$\overline{V}_n^*(b) = \max_{1 \leq k \leq M} \sqrt{n}(\bar{f}_k^{\,*}(b) - \bar{f}_k), \tag{2.11}$$

where $\bar{f}_k^{\,*}(b) = \sum_{i=1}^{n} f_{k,i}^*(b)/n$ denote the sample average of the bth bootstrapped sample $\{f_{k,1}^*(b), ..., f_{k,n}^*(b)\}$. White´s reality check test p-value is then obtained comparing $\overline{V}_n$ with the quantiles of the empirical distribution of $\overline{V}_n^*(b)$, computing:

[5] White (2000) shows in the corollary 2.4 that, under a suitable regularity condition, the distribution of $\overline{V}_n$ and $\overline{V}_n^*$ are asymptotically equivalent.

[6] In Appendix 3 we provide an explanation of the SB method. For a more detailed explanation see, e.g. Romano and Wolf (2005).

$$\hat{p}_{RC} = \sum_{b=1}^{B} \frac{I_{RC}}{B}, \tag{2.12}$$

where I_{RC} is an indicator function that takes the value one if $\overline{V}^*_n(b)$ is higher than $\overline{V}_n$. The null hypothesis is rejected whenever $\hat{p}_{RC} < \alpha$, where α is a given significance level.

On the other hand, Hansen (2005) points out that the RC test has two major limitations as the null distribution is obtained under the "least favorable configuration" [7] and the statistic is not studentized. As a result, the author proposes two improvements to produce a more powerful and less conservative test. First, Hansen (2005) proposed the studentization of White´s RC test statistic on Eq. (2.9):

$$\tilde{V}_n = max[\max_{1 \leq k \leq M} \frac{\sqrt{n}\bar{f}_k}{\hat{\sigma}_k}, 0], \tag{2.13}$$

where $\hat{\sigma}_k^2$ is a consistent estimate of $\sigma_k^2 = var(\sqrt{n}\bar{f}_k)$. In this paper, we estimate $\hat{\sigma}_k$ based on the stationary bootstrapped resamples of $\sqrt{n}\bar{f}_k$ (see, e.g. Hansen, 2005 and Hsu and Kuan, 2005).

Secondly, the author suggests that under the null, when there are some $\varphi_k < 0$ and at least one $\varphi_k = 0$, the limiting distribution of (2.10) depends only on the trading rules with zero or higher mean returns. As a result, Hansen´s "superior predictive ability" data-snooping test discards the irrelevant or poor performance models re-centering the null distribution based on a preset threshold rate given by $-\sqrt{2loglogn}$[8]:

$$\tilde{V}_n{}^*(b) = max[\max_{1 \leq k \leq M} \frac{\sqrt{n}\bar{Z}_k{}^*(b)}{\hat{\sigma}_k}, 0], \tag{2.14}$$

$$\bar{Z}_k{}^*(b) = \sum_{i=1}^{n} \frac{Z^*_{k,i}(b)}{n}, \tag{2.15}$$

$$Z^*_{k,i}(b) = f^*_{k,i}(b) - \bar{f}_k.\mathrm{I}_{\{\sqrt{n}\frac{\bar{f}_k}{\hat{\sigma}_k} \leq -\sqrt{2loglogn}\}}, \tag{2.16}$$

where $\bar{Z}_k{}^*(b)$[9] is the sample average of the bootstrapped re-centered performance measure $Z^*_{k,i}(b)$, and $\mathrm{I}_{\{.\}}$ is an indicator function taking on the value of one if the condition is satisfied and zero otherwise. In this scenario, the consistent p-values of $\tilde{V}_n$ are determined by the empirical distribution of $\tilde{V}_n{}^*(b)$, $b = 1, .., B$, and is computed by:

[7] White (2000) obtain the null distribution based on irrelevant models, i.e. $\varphi_1 = \varphi_2 = ... = \varphi_M = 0$, artificially enhancing the p-values of the RC test (see, e.g. Hsu et al., 2010).

[8] Hansen´s threshold is motivated by the law of the iterated logarithm. Nonetheless, as pointed out by Hansen (2005), other threshold values can also produce valid results with different p-values in finite samples, for example, Hsu and Kuan (2005) used $n^{\frac{1}{4}}/4$. The log is the natural logarithm.

[9] In this paper, we use the same $\hat{\sigma}_k$ in $\tilde{V}_n$ and $\tilde{V}_n{}^*(b)$ (see e.g. Shynkevich, 2012).

$$\hat{p}_{SPA} = \sum_{b=1}^{B} \frac{I_{SPA}}{B}, \tag{2.17}$$

where I_{SPA} is an indicator function takes value one if $\tilde{V_n}^*(b)$ is higher than $\tilde{V_n}$. In a similar fashion to the RC test, the null hypothesis is rejected whenever $\hat{p}_{SPA} < \alpha$.

Hansen (2005) also proposes two additional estimators in order to provide a lower and upper boundary to the consistent p-value of the conventional former test. On the one hand, the lower boundary is based on stricter configuration that eliminates any negative performance model and is given by:

$$Z^{l*}_{k,i}(b) = f^*_{k,i}(b) - max(\bar{f}_k, 0). \tag{2.18}$$

On the other hand, the upper bound considers the inclusion of the poor and least favorable alternatives as suggested in the RC test:

$$Z^{u*}_{k,i}(b) = f^*_{k,i}(b) - \bar{f}_k, \tag{2.19}$$

where $Z^{l*}_{k,i}(b) \leq Z^{c*}_{k,i}(b) \leq Z^{u*}_{k,i}(b)$. In the literature, the SPA test given by (2.16) is called the SPA_c and the lower and the upper bounds are referred to as the SPA_l and SPA_u, respectively.

2.6 Empirical Evaluation

In this section, we provide the empirical evaluation of the best rule performance and analyze our data-snooping bias controlled results for a total of 152,071 trading strategies.

2.6.1 Best Performing TAI Trading Rules

The results for the TAI models for the PSI-20 are presented in Tables 2.4 to 2.7 for each of the four sub-samples[10]. In the Tables, the first column highlights the top 10 performing TAI strategies, based on the log return criteria, while the second column reports the mean return for these strategies.

Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The numbers in parentheses are the standard t-ratios testing the significance of the returns and the difference of the mean buy and the mean sell returns[11]. In columns 6 to 8, we report the PSR which is the, number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The

[10] For the first period we do not use strategies based on volume, since this variable is not provided in the main financial databases.

[11] The t-statistics for the buy-sell mean return difference is computed according to Brock et al.(1992).

All(% , *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample.

Additionally, the number of trades are reported in columns 9 to 11, where $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. In our study, the buy and sell returns were computed without considering the possibility of an additionally risk-free overnight return when the trading rule indicates the no position (out of the market).

Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any superior out-performance.

2.6.1.1 Detailed Technical Analysis Empirical Evidence

Table 2.4 presents the profitability of our TAI trading rules for the first sub-sample data from January 01, 1993 to December 31, 2001, where the mean buy-and-hold return is 0.0632%. In the Table, we observe that all the mean daily returns are significant and the t-test for the difference between buy and sell mean returns are not significant.

In the case of the ODS, the best 10 rules mean daily return are based on two types of TAI trading strategies. The first is a centered oscillator: the Rate-of-change indicator (ROC). The ROC is also referred to as Momentum. It is an oscillator that measures the percentage change in price from one period to the next, comparing the current price to the price "t" periods ago, and fluctuates above and below the zero line.

The second TAI is a trend-following indicator, based on the exponential moving averages indicator (EMA) of the PSI 20 price Index. The EMA is a TA trading strategy that uses two exponential moving averages to generate crossover signals. These crossovers involve the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average.

In the second part of Table 2.4, we present the best-performance rules under the TRS. In the TRS case, as expected, we have a higher mean daily return, 10.92 basis points (bps), and a lower number of trades than in the ODS case. In this sample, we observe that the best 10 trading rules are based on a mix of simple and complex TAI strategies. On the one hand, the best performance rule is based on the Moving Average Convergence-Divergence (MACD) indicator. The MACD is a trend-following strategy that uses the difference between long and short moving averages to identify market opportunities. The indicator fluctuates above and below the zero line as the moving averages converge, cross and diverge. Convergence occurs when the moving averages move towards each other. Divergence occurs when the moving averages move away from each other.

We also observed the presence of simple and complex EMA strategies. The complex strategy is based on the Bollinger Bands indicator (BBS). The BBS is a trading strategy that uses standard deviations and stock price moving averages to generate buying and selling signal bands. The signal is given by the band´s crossover. A bullish crossover occurs when the middle band crosses below the lower standard deviation band. A bearish crossover occurs when the middle band crosses above the higher standard deviation band. In this set-up, the indicator combines the Index standard deviations with its exponential moving average to generate buy and sell signals.

Table 2.5 shows the profitability of our TAI trading rules for the second sub-sample data from January 01, 2002 to August 08, 2008, where the mean buy-and-hold return is 0.0651%. In the Table, we observe for the ODS case that the *t*-test for the mean daily return and the difference between buy and sell mean daily returns are not significant for most of the trading rules. On the other hand, in the TRS case, only the mean daily returns are all significant.

Additionally, we also observe that the best 10 strategies in the ODS and TRS are based on a single type of TAI strategy, the ROC and the EMA, respectively. In the ODS case, the three best rules presented a significantly higher daily mean return (7.4 bps). On the other hand, in the TRS case, the best trading rules have the same performance (8.64 bps). Furthermore, we also observed a lower expected trading activity.

Table 2.6 shows the profitability of our TAI trading rules for the third sub-sample data from September 01, 2008 to April 31, 2011, where the mean buy-and-hold return is -0.0596%. The results for the mean daily return and the difference between buy and sell mean daily returns are non-significant for most of the trading rules in the ODS case. Furthermore, for the TRS case the trading rules mean return and their buy-sell differences are highly non-significant.

We observe for the ODS, that the best 10 rules presented approximately the same daily mean return, from 11.17 bps to 10.07 bps, based only on the Percentage Volume Oscillator indicator (PVO). The PVO is a momentum oscillator based on volume that measures the difference between two volume-based EMA strategies as a percentage of a larger moving average.

In the second part of Table 2.6, the best 10 TAI which are also based on a variety of TAI strategies have a mean daily return from 16.64 bps to 13.05 bps. On one the hand, we have a complex trading rule strategy based on the BBS and the Relative Strength Index (RSI). The RSI is a momentum oscillator that measures the speed and change of price movements.

On the other hand, the Percentage Price Oscillator (PPO) is a momentum oscillator that measures the difference between two moving averages as a percentage of the larger moving average. The oscillator moves into positive and negative terrain as a function of the difference between the shorter moving average and the longer moving average. Note, that contrary to previous sub-sample findings, in both types of market strategies, ODS and TRS, we verify that there is non-significant difference in the number of trades.

Table 2.7 shows the profitability of our TAI trading rules for the last sub-sample data from May 01, 2011 to December 31, 2014, where the mean buy-and-hold return is -0.0123%. In this sub-sample, we observe that not only all mean daily returns *t*-test are significant,

Table 2.4: The Best 10 TAI Trading Strategies Return for the PSI-20

One-day ahead Strategy - 01/01/1993 to 31/12/2001											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
ROC(10,0.01)	0.0897(3.45)*	0.0327(0.71)	0.1596(4.72)*	-0.1268(1.51)	48.49	50,00	59,72	881	1100	7	0.045
ROC(10,0)	0.0895(3.44)*	0.0334(0.73)	0.1582(4.69)*	-0.1248(1.49)	48.44	50.25	59.72	886	1102	0	0.045
ROC(12,0)	0.0891(3.43)*	0.0328(0.71)	0.1583(4.69)*	-0.1255(1.50)	48.79	50.87	59.94	885	1103	0	0.045
PEMA(24,26,25,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,50,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,48,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,35,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,40,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,45,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
PEMA(24,26,30,0,0)	0.0883(3.40)*	0.0354(0.72)	0.1451(4.45)*	-0.1097(1.23)	48.84	45.90	64.32	798	1190	0	0.044
Trend Reversal Strategy											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
MACD(5,10,16,0.1)	0.1092(4.22)*	0.0485(0.44)	0.2192(1.98)***	-0.1707(0.82)	50.81	64.06	52.89	137	109	1742	0.619
MACD(5,10,16,0.15)	0.1042(4.02)*	0.0428(0.41)	0.2155(2.06)***	-0.1728(0.88)	50.20	63.93	51.71	155	124	1709	0.588
BBS-SMA&RSI(3,0.5,0,10,75)	0.1032(3.98)*	0.0660(0.41)	0.1472(0.42)	-0.0811(0.03)	48.44	40.42	68.16	1	8	1979	11.394
BBS-EMA&RSI(3,0.5,0,10,75)	0.1032(3.98)*	0.0660(0.41)	0.1472(0.37)	-0.811(0.03)	48.44	40.42	68.16	1	6	1981	14.649
BBS-SMA&RSI(3,0.5,0,10,70)	0.0981(3.78)*	0.1955(0.14)	0.1025(0.34)	0.930(0.04)	47.94	12.69	90.92	1	16	1971	5.733
BBS-EMA&RSI(3,0.5,0,10,70)	0.0981(3.78)*	0.1955(0.14)	0.1025(0.29)	0.930(0.04)	47.94	12.69	90.92	1	12	1975	7.497
BBS-EMA&RSI(3,0.5,0,15,75)	0.0964(3.66)*	0.0405(0.06)	0.1620(0.34)	-0.1216(0.10)	47.33	71.27	39.32	4	6	1978	9.582
PEMA(5,10,0,0)	0.0949(3.61)*	0.0533(0.30)	0.1342(1.09)	-0.0809(0.25)	49.35	49.75	61.97	73	73	1842	0.689
PEMA(5,10,35,0.015,0.015)	0.0939(3.61)*	0.0533(0.37)	0.1342(1.74)	-0.0809(0.33)	48.59	37.93	70.62	114	188	1686	0.740
PEMA(5,10,30,0.015,0.015)	0.0939(3.13)*	0.0212(0.17)	0.1717(2.17)***	-0.1505(0.66)	48.59	37.93	70.62	114	188	1686	0.740

Notes: The first column highlights the top 10 performing TAI strategies, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily returns from the buy and sell trading signals, respectively. The $Buy - Sell(\%)$ is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns. In columns 6 to 8, we report the PSR which is the number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *** Statistical Significance at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

Table 2.5: The Best 10 TAI Trading Strategies Return for the PSI-20

One-day ahead Strategy - 01/01/2002 to 30/08/2008

	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
ROC(10,0)	0.0740(3.63)*	0.01674(0.39)	0.1230(4.84)*	-0.1063(1.56)	46.72	36.52	69.67	324	667	0	0.037
ROC(10,0.01)	0.0737(3.62)*	0.0159(0.37)	0.1231(4.84)*	-0.1072(1.56)	46.72	36.24	69.67	322	667	2	0.037
ROC(10,0.2)	0.0733(3.68)*	0.0205(0.45)	0.1281(4.86)*	-0.1076(1.50)	45.31	33.70	66.11	300	625	66	0.039
ROC(5,0.2)	0.0396(2.02)***	-0.0250(0.54)	0.0961(3.45)*	-0.1207(1.64)	42.68	33.10	57.95	300	572	119	0.022
ROC(5,0)	0.0388(1.90)	-0.0410(1.02)	0.0977(3.73)*	-0.1390(2.09)***	44.60	38.20	64.02	356	635	0	0.019
ROC(5,0.025)	0.0377(1.85)	-0.0430(1.05)	0.0988(3.71)*	-0.1418(2.10)***	44.40	37.64	62.76	349	619	23	0.019
ROC(5,0.01)	0.0375(1.83)	-0.0420(1.04)	0.0968(3.67)*	-0.1391(2.08)***	44.40	37.92	63.39	353	628	10	0.019
ROC(5,0.15)	0.0373(1.87)	-0.0250(0.56)	0.0900(3.25)*	-0.1151(1.60)	43.09	35.39	58.79	314	588	89	0.021
ROC(5,0.1)	0.0370(1.84)	-0.0280(0.65)	0.0896(3.29)*	-0.1176(1.66)	43.59	36.00	60.25	323	600	68	0.020
ROC(5,0.05)	0.0358(1.76)	-0.0410(1.00)	0.0948(3.52)*	-0.1362(1.99)***	43.79	37.08	61.51	343	611	37	0.019

Trend Reversal Strategy

	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
PSMA(5,12,25,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,25,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,120,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,12,30,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,80,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,20,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,70,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,55,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,60,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599
PSMA(5,10,90,0.01,0.01)	0.0864(4.25)*	0.0356(0.38)	0.1410(1.95)	-0.1053(0.64)	48.23	42.42	68.41	68	80	843	0.599

Notes: The first column highlights the top 10 performing TAI strategies, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The $Buy - Sell(\%)$ is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns. In columns 6 to 8, we report the PSR which is the number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *** Statistical Significance at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

Table 2.6: The Best 10 TAI Trading Strategies Return for the PSI-20

One-day ahead Strategy - 01/09/2008 to 31/04/2011											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
PVO(5,10,5,0.01,0.01)	0.1117(2.19)***	0.2664(1.46)	0.4031(1.72)	-0.1366(0.33)	24.49	19.49	21.03	76	78	287	0.160
PVO(5,10,5,0,0)	0.1071(2.09)***	0.2590(1.41)	0.3935(1.64)	-0.1345(0.32)	24.04	19.49	20.09	76	76	289	0.155
PVO(5,10,3,0.01,0.01)	0.1069(2.16)***	0.2542(1.38)	0.4177(1.77)	-0.1636(0.39)	23.36	18.98	19.62	75	74	292	0.158
PVO(5,10,3,0,0)	0.1063(2.15)***	0.2634(1.41)	0.4028(1.73)	-0.1394(0.33)	23.36	18.98	19.62	74	75	292	0.157
PVO(5,10,14,0.15,0.15)	0.1024(1.93)	0.1954(1.41)	0.3036(1.42)	-0.1082(0.32)	27.21	24.10	23.37	104	89	248	0.117
PVO(5,10,3,0.05,0.05)	0.1010(2.08)***	0.2374(1.36)	0.3897(1.68)	-0.1523(0.38)	23.36	19.49	19.62	80	73	288	0.146
PVO(5,20,9,0,0)	0.1008(1.96)***	0.2582(1.46)	0.3269(1.44)	-0.0688(0.17)	24.26	20.51	21.03	81	81	279	0.137
PVO(5,20,9,0.01,0.01)	0.1008(1.96)***	0.2582(1.46)	0.3269(1.44)	-0.0688(0.17)	24.26	20.51	21.03	81	81	279	0.137
PVO(5,10,16,0.15,0.15)	0.1008(1.90)	0.1783(1.29)	0.3097(1.46)	-0.1313(0.39)	27.44	24.10	23.83	105	90	246	0.114
PVO(5,10,20,0.01,0.01)	0.1007(2.09)***	0.4543(2.39)**	0.2088(0.87)	0.2455(0.57)	22.45	19.49	17.29	70	72	299	0.156
Trend Reversal Strategy											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
PVO(5,20,3,0.2,0.2)	0.1664(1.93)	0.1661(0.79)	0.2197(1.49)	-0.0536(0.16)	47.39	71.80	31.78	102	63	276	0.382
BBS-SMA&RSI(3,0.5,0,25,75)	0.1491(1.72)	0.1538(0.12)	0.1815(0.18)	-0.0276(0.02)	49.21	75.90	31.78	3	1	437	8.221
PPO(5,10,12,0.2)	0.1476(1.70)	0.0159(0.81)	0.1595(0.69)	-0.0005(0.00)	50.57	75.90	35.05	94	67	280	0.423
PPO(5,10,9,0.2)	0.1418(1.63)	0.1551(0.80)	0.1477(0.61)	0.0073(0.02)	50.79	75.90	35.51	94	66	281	0.403
PPO(5,10,20,0.15)	0.1353(1.56)	0.1538(0.74)	0.1314(0.62)	0.0224(0.06)	49.89	70.77	38.32	87	72	282	0.337
PPO&PVO(5,20,9,0.05,0.05)	0.0133(1.70)	0.1717(0.38)	0.1483(0.38)	0.0233(0.03)	46.03	52.82	45.80	18	18	405	0.994
BBS-SMA&RSI(3,0.5,0,25,70)	0.1322(1.52)	0.1476(0.12)	0.1335(0.14)	0.0142(0.00)	48.53	71.80	34.11	3	2	436	5.829
PPO(5,10,14,0.2)	0.1321(1.52)	0.1436(0.70)	0.1395(0.71)	0.0041(0.01)	50.79	78.00	33.65	94	68	279	0.371
PPO(5,10,20,0.1)	0.1321(1.52)	0.1752(0.85)	0.0996(0.41)	0.0756(0.17)	49.66	61.03	46.73	75	71	295	0.302
VSMA(5,12,0.2)	0.1305(1.82)	0.4269(0.19)	0.1475(0.17)	0.2794(0.09)	35.60	20.51	46.26	2	2	437	7.192

Notes: The first column highlights the top 10 performing TAI strategies, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The $Buy - Sell(\%)$ is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns. In columns 6 to 8, we report the PSR which is the number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *** Statistical Significance at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

but also it is the most diversified set of TAI best trading rules. Additionally, contrary to previous findings, we verify that there are non-significant differences in and the number of trades between the ODS and TRS strategies. However, the results for the buy and sell mean daily returns are very similar to previous periods. Indeed, the *t-statistics* for these differences are highly non-significant, therefore we cannot reject the null hypothesis of similar results for the mean return on buy days and sell days.

In the ODS case, the best TAI trading rules are based on the PPO, the MACD and a new indicator, the Money Flow Index (MFI). The MFI is an oscillator that uses both price and volume to measure buying and selling pressures. The MFI is positive when the price rises (buying pressure) and negative when the price declines (selling pressure). A ratio of positive and negative money flow is then calculated to create an oscillator that moves between zero and one hundred. As a momentum oscillator it is used to identify reversals and price extremes with a diversity of signals.

In the TRS case, we observed that the best 10 trading strategies are based on the MACD and the PPO. Additionally, in the case of the best return strategy, we also have a new complex TAI based on the EMA. This indicator uses the PSI 20 price and volume moving averages to generate crossover buy and sell signal bands. A bullish crossover occurs when the shorter price and volume exponential moving averages cross above the longer moving averages.

Finally, there is no evidence of any tendency of over-time market efficiency. As pointed out by Timmermann and Granger (2004), any trading method that is publicly available and profitable can be incorporated into prices and therefore provide the necessary “force” to re-establish the market efficiency.

2.6.2 Robustness Check

The standard statistical t-tests presented in this section have a major weakness since they are formulated based on the stationary, time independent and normally distributed mean returns hypothesis. Nevertheless, asset return distributions are known to be non-normal, auto-correlated and they have time-varying moments. Furthermore, as pointed out by White (2000), the standard statistical inference based on individual testing understate the possibility of a Type I error when we are choosing the best trading rule. Indeed, in this case the mean return statistical distribution will be affected and the test will be biased towards the rejection of the null hypothesis because of data-snooping (see, e.g. Hsu et al., 2016). In this context, any superior significant performance may be the spurious result of test bias.

In this section, we take into account this issue and provide some robustness results to examine if the trading rules presented achieve good economic and statistical performance.

Table 2.7: The Best 10 TAI Trading Strategies Return for the PSI-20

One-day ahead Strategy - 01/05/2011 to 31/12/2014											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
PPO(5,20,3,0.2)	0.0939(3.64)*	0.2418(2.30)**	0.4592(4.02)*	-0.2174(1.16)	22.92	25.54	13.84	153	79	466	0.141
PPO(5,20,3,0.1)	0.0936(3.99)*	0.2961(2.56)**	0.5445(4.40)*	-0.2483(1.19)	20.92	21.54	12.89	124	68	506	0.170
PPO(5,20,3,0.15)	0.0854(3.42)*	0.2148(1.91)	0.5101(4.35)*	-0.2953(1.49)	21.49	22.77	13.52	139	74	485	0.140
PPO(5,10,3,0.2)	0.0795(3.03)**	0.2863(2.32)**	0.3018(2.37)**	-0.0155(0.07)	21.06	21.23	13.21	128	81	489	0.133
PPO(5,20,5,0.2)	0.0738(2.88)**	0.2264(1.86)	0.3015(2.67)**	-0.0752(0.34)	21.06	19.08	16.04	121	99	478	0.117
MACD(5,12,3,0.2)	0.0720(2.67)**	0.3453(2.59)**	0.1786(1.33)	-0.1666(0.65)	20.49	18.77	14.78	111	95	492	0.122
PPO(5,20,3,0.05)	0.0718(3.45)*	0.3638(3.00)*	0.3234(2.20)***	-0.0404(0.17)	19.05	18.15	10.69	94	65	539	0.157
MFI(4,3,20,70)	0.0702(2.16)***	0.1060(1.03)	0.2115(2.04)***	-0.1055(0.48)	28.22	21.23	30.19	139	179	380	0.077
PPO(5,10,5,0.2)	0.0699(2.69)**	0.3057(2.27)***	0.2400(1.74)	-0.0658(0.26)	19.48	18.46	12.58	111	83	504	0.126
MFI(4,3,5,70)	0.0698(2.08)***	0.1060(1.03)	0.1946(1.91)	-0.0886(0.40)	29.08	21.23	32.39	139	193	366	0.073
Trend Reversal Strategy											
	Mean Daily Log Return				PSR			Number of Trades			PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	No. Neutral	(%)
PSMA&VSMA(5,12,0.2)	0.1545(3.25)*	0.2253(0.26)	0.1510(0.21)	0.0744(0.05)	46.42	51.39	46.86	3	3	692	8.988
VSMA(5,12,0.2)	0.1545(3.25)*	0.2253(0.26)	0.1510(0.21)	0.0744(0.05)	46.42	51.39	46.86	3	3	692	8.988
MACD(5,12,3,0.2)	0.1384(2.84)**	0.1267(0.16)	0.2188(0.30)	-0.0921(0.06)	51.00	75.69	34.28	111	95	492	0.608
PPO(5,10,5,0.2)	0.1243(2.55)**	0.1149(0.89)	0.1952(1.43)	-0.0803(0.31)	49.43	74.46	32.08	111	83	504	0.436
PSMA&VSMA(5,12,0.25)	0.1147(2.40)**	0.1960(1.38)	0.0919(0.64)	0.1041(0.39)	45.99	46.77	50.63	2	3	693	8.007
VSMA(5,12,0.25)	0.1147(2.40)**	0.1960(1.38)	0.0919(0.64)	0.1041(0.39)	45.99	46.77	50.63	2	3	693	8.007
PPO(5,20,3,0.1)	0.1100(2.26)**	0.0949(0.10)	0.2124(0.28)	-0.1174(0.06)	50.57	80.92	27.99	124	68	506	0.446
PPO(5,20,9,0.15)	0.1098(2.25)**	0.1050(0.11)	0.1603(0.20)	-0.0552(0.03)	50.72	73.85	35.54	117	96	485	0.355
PPO(5,20,5,0.15)	0.1065(2.19)***	0.1051(0.86)	0.1461(0.89)	-0.0410(0.17)	49.43	70.77	35.85	111	92	495	0.408
PPO(5,10,3,0.2)	0.1062(2.18)***	0.0990(0.78)	0.1676(1.23)	-0.0687(0.28)	47.99	74.15	29.25	128	81	489	0.414

Notes: The first column highlights the top 10 performing TAI strategies, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The $Buy - Sell(\%)$ is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns. In columns 6 to 8, we report the PSR which is the number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, *No.Buy* and *No.Sell* are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *** Statistical Significance at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

2.6.2.1 Transaction Costs

It is well known that one of the most common problems in correctly defining the economic performance of any trading rules is related to the size of the transaction costs involved. Indeed, the transaction costs charged to an investor are unknown since these costs depends on many different aspects, such as the type of investor, investment size, and the technological level of the trading floor systems. Indeed, as presented by Shynkevich (2012), the investor may be trading from a relatively low cost of 5 bps (Hsu et al., 2010), for a single trip transaction, to a less conservative assumption of 20 basis points (Shynkevich, 2012).

In our case, the results show that the number of trades and the break-even cost ($PMP\%$) across the sub-samples varies substantially. We observed that the PMP range from a high of 14.65 % (1993 to 2001) to a low of 4.4 bps (1993 to 2001). These are the boundaries where the evidence of abnormal profitability of the TAI and the EMH rejection should be analyzed. As a result, in relation to transaction costs, there is some evidence of abnormal profitability in the use of the technical analysis methodology.

2.6.2.2 Results of Data-snooping Tests

Table 2.8 summarizes the daily and annualized mean return, based on 250 trading days. It equally gives a summary of White and Hansen´s p-values of the best rules in our sample. The RC and SPA test results are presented based on the stationary bootstrapped with $B = 500$ interactions and a the geometric distribution parameter set as $q = 0.1$ (see, e.g. Politis and Romano, 1994 and Hansen, 2005).

Table 2.8 provides a summary of results for the best explanatory variables set-up for the PSI 20 Index, with the columns described as follows: $BestTradingRule$ represents the best performance strategies; $Trading\ Days$ is the number of forecasted trading days; $B\&H\%$ is the annualized buy-and-hold benchmark return for the forecast period, based on 250 trading days; $TAIAn.Ret\%$ gives the annualized mean return performance without discount the benchmark return for the period; $TAIAn.Perf\%$ is the performance on an annual basis, considering the benchmark return; $TAIM.Ret\%$ is the mean log return of the best rule; finally, PMP is the break-even one day transactions cost. Additionally, P_{RC} is the RC p-value test results and SPA_l, SPA_c and SPA_u are the lower, consistent and upper SPA p-values, respectively.

As observed, there is some evidence that TAI rules are capable of consistently producing superior performance over the buy-and-hold benchmark for the PSI-20 across sub-samples. Before adjusting for data-snooping and transaction costs, it was observed that the benchmark is outperformed with an excess return that lies between 43.45% (2011 to 2014) to 14.63% (1993 to 2002). More specifically, the TAI best rules results show the highest annualized performance (28.30% and 43.45%) in the last sub-sample (2011-2014), during the European Union financial assistance package period. Alternatively, in the first (1993 to 2002) and the second (2002 to 2008) sub-samples, we have the lowest result (14.63% and 17.87%).

Table 2.8: The Best Investment Strategy Data-Snooping Results for the PSI-20 Index

Period	$BestTradingRule$	$Trading$ $Days$	$B\&H$ %	$TAIAn.Ret$ %	$TAIAn.Perf$ %	$TAIM.Ret$ %	PMP %	P_{RC}	SPA_l	SPA_c	SPA_u
One-day ahead Strategy											
sub-sample I (1993-2001)	ROC(10,0.01)	1988	7.79	22.42	14.63	0.0897	0.045	1.00	1.00	1.00	1.00
sub-sample II (2002-2008)	ROC(10,0)	991	3.74	18.49	14.75	0.0864	0.037	1.00	1.00	1.00	1.00
sub-sample III (2008-2011)	PVO(5,10,5,0.01,0.01)	441	1.46	27.94	26.47	0.1117	0.160	0.9145	0.8923	0.8923	0.8923
sub-sample IV (2011-2014)	PPO(5,20,3,0.2)	698	-4.52	23.48	28.30	0.0939	0.141	0.9435	0.9327	0.9327	0.9327
Trend Reversal Strategy											
Period	$BestTradingRule$	$Trading$ $Days$	$B\&H$ %	$TAIAn.Ret$ %	$TAIAn.Perf$ %	$TAIM.Ret$ %	PMP %	P_{RC}	SPA_l	SPA_c	SPA_u
sub-sample I (1993-2001)	MACD(5,10,16,0.1)	1988	7.79	27.31	19.51	0.1092	0.619	1.00	0.9652	0.9652	0.9652
sub-sample II (2002-2008)	PSMA(5,12,25,0.01,0.01)	991	3.74	21.61	17.87	0.0864	0.599	1.00	1.00	1.00	1.00
sub-sample III (2008-2011)	PVO(5,20,3,0.2,0.2)	441	1.46	41.59	40.13	0.1664	0.382	0.8973	0.8453	0.8453	0.8453
sub-sample IV (2011-2014)	PSMA&VSMA(5,12,0.2)	698	-4.52	38.63	43.45	0.1545	8.988	0.9134	0.8752	0.8752	0.8752

Notes: The $BestTradingRule$ represents the best performance strategies; $Trading\ Days$ is the number of forecasted trading days; $B\&H\%$ is the annualized buy-and-hold benchmark return for the forecast period, based on 250 trading days; $TAIAn.Ret\%$ gives the annualized mean return performance without discount the benchmark return for the period; $TAIAn.Perf\%$ is the performance on an annual basis, considering the benchmark return; $TAIM.Ret\%$ is the mean log return of the best rule; finally, PMP is the break-even one day transactions cost. Additionally, P_{RC} is the RC p-value test results and SPA_l, SPA_c and SPA_u are the lower, consistent and upper SPA p-values, respectively.

Nevertheless, the data-snooping tests suggest that the best rule performance across sub-samples is not significant at any significant conventional test level. Indeed, in spite of the high number of rules tested in this study, our superior profitability could be due to chance rather than to the existence of high-performance strategies. Under such circumstances, the possibility of spurious results is a reasonable assumption.

Hence, we conclude that there is non-significant evidence of abnormal profitability of the TAI strategies applied to forecast the PSI-20, and therefore we cannot reject the weak-form of the efficient market hypothesis (Fama, 1965 and 1970).

2.7 Conclusion

Reproducing the words of Fama (1970) : "In short, the evidence in support of the efficient markets model is extensive, and (somewhat unique in economics) contradictory evidence is sparse." Nonetheless, the widespread use of technical analysis as a leading stock market forecasting instrument is still challenging the idea of market efficiency.

Indeed, over the years, academic research has studied TA use as a high-performance method capable of predicting financial market securities. In this sense, the financial market could experience time inefficiencies that raise a major question: are there some forecast models, based only on the past price movements, which could be used as forecasting methods?

This paper makes two main contributions to answer this question. Firstly, we produce a novel study of the profitability of TA rules, using a unique broad sample of 152,071 trading rules in an unexplored empirical area. Indeed, to the best of our knowledge, this is the first paper that applies data-snooping controlled methodology to broadly study the Portuguese financial market, which is a relatively "young and less capitalized" market in a well-developed region.

Secondly, we test the technical analysis profitability adjusted for data-snooping bias by applying the White (2000) "Bootstrap Reality Check" and the Hansen (2005) tests. Although, there is some "reasonable" evidence that the TA methodology is capable of consistently producing superior profitability, our test results discard the existence of high-performance strategies. Under these conditions, we conclude that we cannot reject the EMH in the PSI 20 Euronext Lisbon stock exchange index.

This is a very important result which draws attention to the importance of controlling data-snooping to avoid the possibility of spurious results. Undoubtedly, as suggested by Hsu et al.(2016), when we are searching through a huge number of trading rules, a skeptic might say that they are surprised no over-performing strategy has been found since "if you torture the data long enough, it'll confess to anything"[12].

[12] In Hsu et al. (2016), this citation is attributed to Economics Nobel Laureate Ronald Coase.

Appendix 1

In this appendix we have summarized the Technical Analysis Indicators used in our study, based on the notations taken from Edwards and Magee (2012) and Pring (2014) and the initial scenario table.

Table 2.9: Technical Analysis Indicators

TAI Rules	Trading Rule Definition
Bollinger Bands - BBS	This is an indicator that uses standard deviations and stock price moving averages to generate buying and selling signal bands. The signal is given by the band's crossover. A bullish crossover occurs when the middle band crosses below the lower standard deviation band. A bearish crossover occurs when the middle band crosses above the higher standard deviation band.
Commodity Channel Index - CCI	The CCI measures the current price level relative to an average price level over a given period of time to generate overbought and oversold signals. The indicator measures the difference between a security's price change and its average price change. As such, high positive readings indicate that prices are well above their average, which is a show of strength. Low negative readings indicate that prices are well below their average, which is a show of weakness. Readings above +100 reflect strong price action that can signal the start of an uptrend. If they fall below -100 it reflects weak price action that can signal the start of a downtrend.
Chaikin Oscillator - CHO	The CHO is an indicator designed to anticipate directional changes in prices by measuring the momentum behind the movements. This oscillator generates signals with crosses above/below the zero line or with bullish/bearish divergences.
Chaikin Money Flow - CMF	This indicator measures the amount of money flow volume over a specific look-back period, typically 20 or 21 days. The resulting oscillator fluctuates above/below the zero line weighing the balance of buying or selling pressure. The CMF usually fluctuates between -.50 and +.50 with zero as the center-line.
Moving Average Convergence-Divergence -MACD	The MACD is a trend-following indicator. It uses the difference between long and short moving averages to measure a momentum. The indicator fluctuates above and below the zero line as the moving averages converge, cross and diverge. Convergence occurs when the moving averages move towards each other. Divergence occurs when the moving averages move away from each other. A 9-day EMA of the MACD line is used as a performance indicator as a signal line to identify market opportunities.
Money Flow Index - MFI	An indicator that uses both price and volume to measure buying and selling pressures. The MFI is positive when the price rises (buying pressure) and negative when the price declines (selling pressure). A ratio of positive and negative money flow is then calculated to create an oscillator that moves between zero and one hundred. As a momentum oscillator, it is used to identify reversals and price extremes with a diversity of signals. There is another version of this indicator, called MFI - Divergence, which compares the cross-over signal generated to buy or sell with its maximum or minimum level and with the price level.
Price Exponential and Simple Moving Average Indicators - PEMA\PSMA	The PEMA\PSMA investment strategy uses two exponential\simple moving averages to generate price crossover signals. These crossovers make the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average. In this paper we use the PEMA\PSMA indicator not only to generate buy and sell signals based on price and volume, but we also use its average, as an indicator of performance and a signal line to identify market opportunities.
Percentage Price Oscillator - PPO	A momentum oscillator that measures the difference between two moving averages as a percentage of the larger moving average. The value of the PPO becomes increasingly positive as the shorter moving average distances itself from the longer moving average reflecting a strong upside momentum. For negative values of the PPO, this indicates that the shorter moving average is below the longer moving average. Increasing negative values indicate that the shorter moving average is distancing itself from the longer moving average, reflecting strong downside momentum.
Percentage Volume Oscillator - PVO	A momentum oscillator for volume. The PVO measures the difference between two volume-based EMA as a percentage of a larger moving average. The PVO is positive when the shorter volume EMA is above the longer volume EMA and negative when the shorter volume EMA is below the longer volume EMA. There is also another type of this indicator called PVO - Divergence, which compares the generated cross-over signal to buy or sell with its maximum or minimum level for a price level.
Rate-of-Change - ROC	This indicator is referred to as Momentum. It is an oscillator that measures the percentage change in stock price from one period to the next. The ROC compares the current price to the price "t" periods ago, and fluctuates above and below the zero line. Moreover, the ROC is used by combining its signal with the divergence in stock price, called ROC - Divergence. In this case a buy(sell) signal is produced if the current ROC value is higher than its previous value, for a lower price.
Relative Strength Index -RSI	A momentum oscillator which measures the speed and change of stock price movements. The RSI oscillates between zero and 100. The indicator is considered overbought when above 70 and oversold when below 30. There is a modification of this indicator called RSI - Divergence, which compares the generated cross-over signal to buy or sell with its maximum or minimum level for some price level.
Stochastic Oscillator - STO	The STO measures the level of the closing stock price relative to the high-low range over a given period of time. When the STO is above 50 the indicator signals that the closing price is in the upper half of the range. In contrast, when it is below 50, this indicates the closing price is in the lower half. A STO reading below 20 signals that the price is near its lowest level for the given time period. However, for high readings (above 80) the rule indicates that the price is near its highest level. There are two other versions of Stochastic Oscillator which use an EMA of the STO to generate cross-over signals to buy or sell. These are the fast and slow STO.
Volume Exponential and Simple Moving Average Indicators -VEMA\VSMA	The VEMA\VSMA investment strategy uses two exponential\simple moving averages to generate volume crossover signals. These crossovers involve the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average. In this paper we use the VEMA\VSMA indicator not only to generate buy and sell signals based on price and volume, but we also use its average, as an indicator of performance and a signal line to identify market opportunities.
Williams %R Indicator - WRI	Technical indicator which reflects the level of the closing stock price relative to the highest high´ for a look-back period. The WRI oscillates from 0 to -100. Readings from 0 to -20 are considered overbought. Readings from -80 to -100 are considered oversold.

Appendix 2

In this appendix we summarize the parameters used in our TAI strategies.

Table 2.10: TAI Parameter Definition

Parameter Definitions

n= number of days used to calculate the rule
up = upper thresholds to initiate a position
low= lower thresholds to initiate a position
b = band to initiate a position
s=number of days of the short moving average
l = number of days of the long moving average
d=number of days of the second short moving average
sd= standard deviation multiplier

Trading Rule	Abbreviation	Parameters	
	BBL-PEMA (n,sd,b)	n	3,7,10,12,14,16,20,25,30,35,40,45,50,55,60
Bollinger Bands		sd	0.5,1,1.25,1.5,1.75,2,2.25,2.5,3
	BBL-VEMA (n,sd,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	4,6,8,10,15,20,22,24,26,28,30,35,40,45,50
Commodity Channel Index	CCI (n,up,low)	up	70,75,80,85,90,95,100,110,120,130,140
		low	-70,-75,-80,-85,-90,-95,-100,-110,-120,-130,140
BBS-EMA&RSI	BBL-EMA(n,sd,b,vp) & RSI(n,s,up,low)	s	3,7,10,20,30,40,50
		sd	0.5,1,1.25,1.5,1.75,2,2.25,2.5,3
		b	0,0.01,0.025,0.05,0.10
BBS-SMA&RSI	BBL-SMA(n,sd,b,vp) & RSI(n,s,up,low)	up	70,75,80,85,90,95
		low	5,10,15,20,25,30
Chaikin Oscillator	CHO (s,l)	s	3,7,10,12,14,16,18,20,22,24,26,28,30,35 40,45,50,60,70
		l	5,7,10,12,14,16,18,20,24,26,28,30,35,40,50,60,70
Chaikin Money Flow	CMF (n,b)	n	3,5,7,10,12,14,16,20,22,24,26,28,30,35,40,45,50,55,60,70,75,80,85,90,95,100,120
		b	0,0.01,0.025,0.05,0.10,0.15,0.20,0.25,0.30
		s	3,5,7,10,12,14,16,20,25,30
Moving Average Convergence- Divergence	MACD (s,l,n,b)	l	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50,60,70
		n	3,5,9,12,14,16,20
		b	0,0.005,0.01,0.015,0.02,0.025,0.05,0.10,0.15,0.20
		n	4,6,8,10,12,14,16,20,25,30,35
Money Flow Index	MFI (n,s,up,low)	s	3,5,9,12,14,16,20
		up	60,65,70,75,80,85,90,95
		low	5,10,15,20,25,30,35,40
		n	3,5,10,12,14,16,20,26
MFI&RSI	MFI(n,s,up,low) & RSI(n,s,up,low)	s	3,5,7,9,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50,55,60
		up	60,65,70,75,80,85,90,95
		low	5,10,15,20,25,30,35,40
	PEMA\PSMA (n,l,b)	s	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50
Moving Average Filters		l	10,12,14,16,18,20,22,24,26,28,30,35,40,50,60,70
	VEMA\VSMA (n,l,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,10,12,14,16,20,25,30,35,40,45,50
Percentage Price Oscillator	PPO (n,s,d,b)	s	10,20,24,28,32,40,50,60,70
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,10,12,14,16,20,25,30,35,40,45,50
Percentage Volume Oscillator	PVO (n,s,d,b)	s	10,20,24,28,32,40,50,60,70
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	3,5,7,10,12,14,16,20,25,30
PPO&PVO	PPO(n,s,d,b) & PVO(n,s,d,b)	s	5,10,12,14,16,20,25,30,35,40,45,50
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
PEMA&VEMA	PEMA&VEMA (n,l,b)	s	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50
		l	10,12,14,16,18,20,2224,26,28,30,35,40,45,50,60,70
PSMA&VSMA	PEMA&VEMA (n,l,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
Rate-of-Change	ROC (n,b)	n	5, 10, 12,14,16, 20, 25,30,35,40,45,50
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,7,9,10,12,14,16,20,22,24,25,30,45,52
Relative Strength Index	RSI (n,s,up,low)	s	2,4,6,10,12,14,16,20
		up	60,65,70,75,80,85,90
		low	5,10,15,20,25,30,35,40
		n	5, 10, 12, 14,16, 15,20,25
Stochastic Oscillator	STO (n,up,low)	up	5,10,15,20,25,30,35
		low	65,70,75,80, 85,90,95
		n	5, 10,12, 14,16,20,25,30,35
William R%	WRI (n, up, low)	up	-5, -10,-15,-20,-25,-30
		low	-70,-75,-80,-85,-90,-95

Appendix 3

Stationary Block Bootstrap method

The basic idea of the stationary bootstrap method is to construct random data blocks that are independent, yet preserve the time dependence inside each block. The unknown population distribution structure is approximated by block sampling distributions based on a statistical model. As such, the stationary bootstrap methodology provides a re-sampling method which is applicable for weakly-dependent time series, where the pseudo-time series are stationary time series.

The method is based on two basic steps that provide proper consistency and weak convergence properties. Firstly, the original series is re-sampled into a set of b random length overlapping blocks of observations, determined by the realization of a geometric distribution with parameter $q \in (0, 1)$. In this case, the average block size is the inverse of q. Secondly, the stationary bootstrap method "wraps" the data around in a "circle" to avoid the block end effects (Politis and Romano, 1994, p.1304). The idea is to choose a large enough block length, preferably based on the sample size, so that observations greater than $1/q$ time units apart will be nearly independent.

However, the major difficulty of this method lies in choosing the size of q. Indeed, the size of the block is a controversial topic in the literature (e.g. Sullivan et al., 1999; Hsu and Kuan, 2005; Metghalchi et al., 2012, and Hsu et al., 2010), as a small size will not reproduce the data dependence, and a large value will reduce the statistical efficiency. In this study we adopt what is usually presented in the previous research in this area, and set $q = 0.1$.

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Chapter 3

The Efficient Market Hypothesis of Stock Prices in the Markov Chain Framework

Abstract

The paper presents a new Markov chain framework to test the efficient market hypothesis (EMH) in the top 20 most capitalized worldwide stock markets. Our approach consists of testing the EMH, based on two different methodologies. Firstly, we use the standard Anderson and Goodman (1957) time-homogeneity and time-dependence Markov chain tests. Secondly, we apply a new framework, based on the Polansky (2007) methodology for detecting multiple change-points and the MTD-Probit estimation model (Nicolau, 2014).The MTD-Probit model is a new approach for estimating high-order and multivariate Markov chains.

Keywords: Markov chains, efficient markets hypothesis, time-homogeneity and time-dependence tests.

3.1 Introduction

The efficient market hypothesis (Fama, 1970) is one of the most fundamental pillars in modern financial theory. According to the weak-form of the efficient market hypothesis (EMH), prices should reflect all available information. Consequently, it would not be possible to earn excess returns consistently with any investment strategy that tries to predict asset price movements based on historical data (Fama, 1965; and Fama & Miler, 1972).

Nevertheless, in recent decades, new empirical evidence has suggested that the stock market is not efficient, thereby admitting the possibility of obtaining abnormal stock returns that are not fully explained by common risk measures (e.g. Brock et al., 1992; Hsu et al., 1999; Lo et al., 2000; Park and Irwin, 2004 and 2007; Hsu et al., 2010; Hsu et al., 2013; Neely et al, 2014). Among the possible arguments against the EMH is the possibility of a nonlinear stochastic dynamic in stock returns (Berchold and Raftery, 2002) and seemingly short-run time inefficiencies (Timmermann and Granger, 2004). From this perspective, past information should be helpful and informative to explain future price movements, and therefore challenge the concept of market efficiency.

In this scenario, many different approaches have been used to test the EMH, in particular, the Markov chain test methodology. The use of the Markov chain framework is of special interest in finance, not only because it is applied in a wide range of fields, from genetics to economics, but also because it is theoretically robust, well-defined and parsimonious.

There are three main limitations for the use of this method to test the EMH. Firstly, the Markov chain test allows one to consider the nonlinear temporal dependence of stock returns, as long as the time-homogeneity of the transition probability matrix (TPM) is not rejected. However, the use of the Markov chain to test random walk behavior implicitly assumes (ad hoc) time-homogeneity, and therefore fails to fully account for the interdependence between time-homogeneity and time-dependence properties (Fielitz and Bhargava, 1973; Bickenbach and Bode, 2001;Tan and Yilmaz, 2002).

Secondly, the standard Anderson and Goodman (1957) Markov chain time-homogeneity test methodology may be un-informative, as it may fail to identify the true break date. Indeed, the test is likely to falsely indicate a break, when one does not exist. This is either because the break dates are known in advance, or/and they are chosen arbitrarily (Tan and Yilmaz, 2002). Consequently, the results can be highly sensitive to these arbitrary choices and different researchers can easily reach very distinct conclusions.

Finally, the Markov chain time-dependence tests are mostly formulated on the use of maximum likelihood estimation procedures. However, when we study higher-order Markov chains, even with moderate time-dependence and a number of states, the estimation procedure becomes impracticable, on account of the large number of parameters and constrains (e.g. Raftery, 1985; Raftery and Tavare, 1994; Berchtold, 2001; Ching et al., 2002 and 2008; Zhu and Ching, 2010; Nicolau, 2014). Nonetheless, recently, a new high-order Markov chain model (HOMC) estimation procedure, called MTD-Probit (Nicolau, 2014), has introduced a simplifying approach that facilitates the model parameter estimation and statistical inference.

The main objective of this study is to propose a new method to test the EMH of stock prices using time-dependence and time-homogeneity Markov chain test procedures. We contribute to the literature in three ways. Firstly, we present a new methodology for detecting and estimating change-points for a discrete-time Markov chain based on the MTD-Probit model. Our research is based on the methodology of Tan and Yilmaz (2002) [1] to evaluate the predictability of stock returns, and on the Polansky (2007) Markov chain time-homogeneity test for an unknown number of change-points. Secondly, we analyse the EMH using a unique broad sample of 4,474 stocks and indices from the top 20 most capitalized worldwide stock markets. Finally, we apply the standard [2]Anderson and Goodman (1957) test, based on non-parametric, contingency table type mathematical procedures, as this allows us to statistically compare the empirical results of previous EMH tests.

The study shows that there is some evidence that the stock market can be efficient in a wide variety of stock exchanges around the globe. Nonetheless, in the Anderson and Goodman (1957) methodology, the American (DJIA and NASDAQ Indices) and the UK (FTSE 100 Index) markets represents a first, or higher-order time-homogeneous Markov chain process.

The paper proceeds along the following lines. Previous studies on Markov chain based statistical tests and applications to stock markets are reviewed briefly in Section 3.2. In Section 3.3, we explore the basic Markov chain theory and the Markov chain test methodology is described in Section 3.4. In section 3.5, detailed uses of the test procedures are presented. The empirical evaluation of the Markov chain tests is reported in Section 3.6. Finally, Section 3.7 presents findings and conclusions.

3.2 A Brief Literature Review

Initially, Niederhoffer and Osborne (1966) applied the Markov chain methodology to test some non-random behavior of ticker transactions price changes from one transaction to another for seven stocks included in the Dow Jones Index for a 22-day trading period in October 1964. Considering the “ad hoc” stationarity of the TPM, their results suggested that stock market ticker prices display non-random properties for price reversal. Dryden (1969) used Markov chains to study the deviations from random walk in the U.K. stock market. He studied the behavior of the number of shares whose quoted prices for different categories were rising, falling, or remained unchanged from the previous day in the London Stock Exchange, from January 1963 to April 1967. Although, the studied process proved not to be stationary, he concluded that there is some evidence of dependence among successive daily price changes.

Later, Fielitz and Bhargava (1973) and Fielitz (1975) tested the order of dependence of a three-state Markov chain of the daily returns of 200 individual stocks between 1963 and 1968. As a result, they rejected random walk in favour of first or higher-order dependence in the daily returns of those individual stocks, although they observed some structural breaks in the series. In Ryan’s (1973) paper, he explores the relevance of the theory of

[1]Tan and Yilmaz presented a detailed description of the Markov chain technique and the evaluation of the small and large sample properties of the time-dependence and time-homogeneity tests.

[2]We use the term “standard” for the Anderson and Goodman (1957) time-homogeneity test.

Markov processes in the analysis of stock price movements based on the study of Dryden (1969), without the test of time-homogeneity. He showed that price movements which do not display random walk behavior could be followed by a Markovian stochastic process. It is also relevant to point out the work of Gregory and Sampson (1987), who tested the independence of forecast errors in the forward foreign exchange market, using Markov chains, and concluded that the available cross-country information was useful for predicting the future forward exchange forecast errors for the data of six foreign exchanges.

Finally, McQueen and Thorley (1991) applied two state (up-down) Markov chain tests to the annual real and excess returns of the equally-weighted and value-weighted portfolios of all stocks in the New York Stock Exchange, between 1947 and 1987. The authors tested random walk against second-order dependence. As result, they rejected the random walk hypothesis in favour of possible long-horizon mean-reversion in stock returns. Although they considered a period of 40 years of the New York Stock Exchange (NYSE), it can be implicitly assumed that the TPM is time-homogeneous throughout the period of analysis, and that it tested for the EMH.

3.3 The Basic Markov Chain Theory

In this section, we present the Markov chain framework used in this study[3].

3.3.1 First-order Markov Chain

A sequence of discrete-time random variables $X = \{X_t, t \geq 0\}$ taking values on countable or finite set of states $\mathscr{M} := \{1, ..., m\}$, is a first-order discrete-state, discrete-time Markov chain (FOMC), if the present state at time t is conditionally independent of those up to the $t-1$ immediate past state. That is:

$$P_t(X_t = i_0 | X_{t-1} = i_1, X_{t-2} = i_2, ...X_0 = i_t) = P_t(X_t = i_0 | X_{t-1} = i_1) := p_{i_1 i_0}(t), \quad (3.1)$$

where at time t, X_t is called the state of the process and $p_{i_1 i_0}(t)$ is the conditional probability that the Markov chain process jumps from state i_1 to i_0, from time $t-1$ to t, for all sequences of constants $\{i_t, ..., i_0\} \in \mathscr{M}$ [4].

In particular, if the probability $p_{ij}(t)$ is time-invariant, that is $p_{ij}(t) = p_{ij}$, for ∇t and $\nabla i, j \in \mathscr{M}$, then the process X_t is called as first-order time-homogeneous Markov chain. In this case, the Markov chain is completely determined by the one-step transition probability matrix $P = \{p_{ij}\}$:

[3] See e.g. Berchold and Raftery (2002).
[4] The condition presented in Eq.(3.1) is so-called the Markov or memory-less property.

$$
P = X_{t-1} \quad \overset{X_t}{\begin{vmatrix} p_{11} & p_{12} & \cdots & \cdots & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & \cdots & \cdots & p_{2m} \\ . & . & \cdots & \cdots & \cdots & . \\ . & . & \cdots & \cdots & \cdots & . \\ . & . & \cdots & \cdots & \cdots & . \\ p_{i1} & p_{i2} & \cdots & p_{ij} & \cdots & p_{im} \\ . & . & \cdots & \cdots & \cdots & . \\ . & . & \cdots & \cdots & \cdots & . \\ . & . & \cdots & \cdots & \cdots & . \\ p_{m1} & p_{m2} & \cdots & \cdots & \cdots & p_{mm} \end{vmatrix}} \tag{3.2}
$$

$$
0 \leq p_{ij} \leq 1, \tag{3.3}
$$

$$
\sum_{j=1}^{m} p_{ij} = 1. \tag{3.4}
$$

which summarizes all m^2 transition probabilities p_{ij}, $i, j \in \mathscr{M}$, and an initial distribution $P_0 = (p_{01}, p_{02}, \ldots, p_{0m})$, $\sum_{j=1}^{m} p_{0j} = 1$, describing the starting probabilities of the various states. In this set-up, the $m(m-1)$[5] transition probabilities can be estimated through the maximum likelihood estimates (MLE) (see, e.g. Anderson and Goodman, 1957: 92; and Basawa and Rao, 1980: 54 f.), as follows:

$$
\widehat{p}_{ij} = n_{ij} / \sum_{j=1}^{m} n_{ij}, \tag{3.5}
$$

where n_{ij} denotes the observed absolute number of one-step transitions from state i to j, and $\sum_{j=1}^{m} n_{ij}$ accounts for all the transitions count from state i.

3.3.2 High-order Markov Chains

A sequence of discrete-time random variables $X = \{X_t, t \geq 0\}$ taking values on countable or finite set of states $\mathscr{M} = \{1, ..., m\}$, is the discrete-state, discrete-time homogeneous Markov chain process[6] of order k, if the time-invariant transition probability of the next state, conditional to the past and present states, depends only on the previous $t - k$ time states. That is:

[5] Given the condition on Eq.(3.4), there are (m − 1) independent probabilities in each row of the matrix P.

[6] Throughout this section, to simplify the nomenclature, we study the high-order time-homogeneous Markov chain.

$$P_t(X_t = i_0|\, X_{t-1} = i_1, X_{t-2} = i_2, ...X_0 = i_t) =$$

$$P\left(i_0|\, i_1, ..., i_k\right) := P(X_t = i_0|\, X_{t-1} = i_1, .., X_{t-k} = i_k), \tag{3.6}$$

for all sequences of constants $\{i_t, ..., i_0\} \in \mathscr{M}$ and $t \in \{0, 1, 2....\}$ and can be estimated through the following MLE expression:

$$\hat{P}_j\left(i_0|\, i_1, ..., i_k\right) = \frac{n_{i_1 i_2 \ldots i_k i_0}}{\sum\limits_{i_0=1}^{m} n_{i_1 i_2 \ldots i_k i_0}}, \tag{3.7}$$

where $n_{i_1 i_2 \ldots i_k i_0}$ is the number of times the sequence $i_k \to i_{k-1} \to ....i_0$ or the number of transitions of type $X_{t-1} = i_1, .., X_{t-k} = i_k, X_t = i_0$, where the sum is over all values $i_1, ..., i_k, i_0$ with $n_{i_1 i_2 \ldots i_k i_0} > 0$.

3.3.3 The MTD-Probit Estimation Method

The use of the MLE for modelling high-order chains can be problematic. Indeed, when the number of state m is relatively large and the sample size is small, or even moderate, the total number of parameters to be estimated is $m^k(m-1)$. In practical terms, this means that the numerator as well as the denominator on Eq.(3.7) may be zero in most cases, or very close to zero. As a consequence, the parameters can be neither efficiently estimated nor identified with a finite sample size (Nicolau, 2014).

To overcome this problem, Ching et al. (2002) considered a simplifying hypothesis, which is, in fact, an extension of Raftery (1985), for modeling high-order Markov chains. It involves assuming that a natural model to estimate the transition probability matrix (TPM) for a kth-order Markov chain on Eq.(3.6) is through a linear combination of $\{P_1\left(i_0|\, i_1\right), ..., P_k\left(i_0|\, i_k\right)\}$, where $P_k\left(i_0|\, i_k\right) := P\left(X_t = i_0|\, X_{t-k} = i_k\right)$, as follows:

$$P^{MTD}\left(i_0|\, i_1, ..., i_k\right) := P\left(i_0|\, i_1, ..., i_k\right) =$$

$$\sum_{g=1}^{k} \lambda_g P_g\left(i_0|\, i_g\right) = \lambda_1 P_1\left(i_0|\, i_1\right) + ... + \lambda_k P_k\left(i_0|\, i_k\right). \tag{3.8}$$

To ensure that the results of the model are probabilities, is impose that:

$$\sum_{g=1}^{k} \lambda_g = 1,$$

and

$$0 \leq \sum_{g=1}^{k} \lambda_g P_g\left(i_0|\, i_g\right) \leq 1. \tag{3.9}$$

Where $P_g(i_0|\, i_g)$ are elements of an $m \times m$ transition probability matrix[7] and λ_g is the weight parameter associated with the lag g (Berchtold and Raftery, 2002).

The expression on Eq.(3.8) is called the mixture transition distribution (MTD) model (Raftery, 1985). In this model, with the condition that the $0 \leq \lambda_{ji} \leq 1$, the inequality (3.9) is automatically satisfied. In this case, the λ-parameters may be interpreted as probabilities, and the estimation procedure is easier to implement. Indeed, the number of parameters to be estimated is substantially reduced to $m(m-1) + (k-1)$ and each additional lag adds only one additional parameter.

Nonetheless, although the MTD model tries to overcome the difficulties for estimated HOMC with parsimony and is easier to implement, one of the main challenges in applying this model is linked to the estimation process, the way the nonlinear constraints deal with the numerical optimization and the range of dependence patterns that the model can capture, especially negative partial effects (e.g. Berchtold, 2001, Lèbre and Bourguignon, 2008, Chen and Lio, 2009, and Nicolau, 2014).

However, recently a new MTD estimation process called MTD-Probit (Nicolau, 2014) was proposed. The MTD-Probit model is based on a specification which is completely free from constraints, facilitating the estimation procedure. Additionally, it has a more accurate specification for $P(i_0|\, i_1, ..., i_k)$ which does not alter the consistency of the MLE. More specifically, the MTD-Probit model suggests modeling HOMC, as follows:

$$P(i_0|\, i_1, ..., i_k) = P^{\Phi}(i_0|\, i_1, ..., i_k) := \frac{\Phi(\eta_0 + \eta_1 P_1(i_0|\, i_1) + ... + \eta_k P_k(i_0|\, i_k))}{\sum\limits_{i_0=1}^{m} \Phi(\eta_0 + \eta_1 P_1(i_0|\, i_1) + ... + \eta_k P_k(i_0|\, i_k))}, \quad (3.10)$$

where η_i, $\eta_i \in \mathbb{R}$ and $i \in \mathscr{M}$, are parameters to be estimated, and Φ is the (cumulative) standard normal distribution function. In this scenario, the log-likelihood for the kth-order Markov chain is expressed as:

$$logL(k) = \sum_{i_1 i_2, ... i_k i_0} n_{i_1 i_2 ... i_k i_0} log(P^{\Phi}(i_0|\, i_1, ..., i_k)), \quad (3.11)$$

and the MLE η_j can be expressed[8] as:

$$\hat{\eta}_j = argmax_{\eta_1, \eta_2, ..., \eta_k, \eta_0} logL \quad (3.12)$$

In addition, the parameters $P_\kappa(i_0|\, i_\kappa)$, $1 < \kappa < k$ can be consistently estimated as usual through $\hat{P}_\kappa(i_0|\, i_\kappa) = \frac{n_{i_\kappa i_0}}{\sum_{i_0=1}^{m} n_{i_\kappa i_0}}$ where $n_{i_\kappa i_0}$ is the number of transitions of type

[7] It should be observed that the $P_1(i_0|\, i_1)$ is not the same as the first-order Markov chain probability $p_{i_1 i_0} := P(X_t = i_0|\, X_{t-1} = i_1)$.

[8] As suggested by Nicolau (2014), we have used the constrained maximum likelihood module in GAUSS software (Aptech Systems, Chandler, Arizona, United States) that allows switching between several algorithms (BFGS, Broyden-Fletcher-Goldfarb-Shanno, DFP, Davidon-Fletcher-Powell, Newton, BHHH, Berndt-Hall-Hall-Hausman, scaled BFGS and scaled DFP) depending on either of three methods of progress: change in function value, number of iterations or change in line search step length.

$X_{t-\kappa} = i_\kappa$ to $X_t = i_0$.

3.4 The Markov Chain Tests Methodology

In this section, we briefly consider the test methodology that is applied in this study, based on first-order Markov chains. We opted to follow this approach as it not only simple, but is also the statistical and probabilistic framework that is truly most applicable for high-order Markov chains.

3.4.1 Introduction

The use of the Markov chain test methodology can be an important alternative test procedure for testing EMH in the presence of a nonlinear stochastic dynamics in stock returns. Indeed, the Markov chain test allows one to consider the nonlinear temporal dependence of stock return.

However, it is well-referenced in the literature that the Markov chain methodology can only be used to test EMH when there are no structural breaks. If the time-homogeneity of the sample TPM is not tested, the use of the total sample for forecasting events may be misleading as the evolution of the stock market stochastic process prior to a structural break may not be informative for the subsequent period's process developments.

In summary, for more realistic inference and a statically meaningful interpretation of the EMH validity, it is necessary to apply a testing procedure that contemplates not only the exam of the time-dependence structure, but also the time-homogeneity properties of the observed TPM.

Nonetheless, the use of the Markov chain framework to test the random walk behavior has not been addressed in its entirety, given that to date, the underlying statistical assumptions, namely the time-homogeneity of stock price series, have not been properly tested (Fielitz, 1975; Tan and Yilmaz, 2002).

Indeed, as referenced by Tan and Yilmaz (2002), the Monte Carlo results of the finite-sample properties of the Anderson and Goodman (1957) Markov chain test methodology show that structural breaks are difficult to detect. Furthermore, the test results are also affected by undesirable power and size characteristics, whereby the longer the time period under consideration, the higher the risk of structural breaks, and consequently some empirical results become misleading (see, e.g. Bickenbach and Bode, 2001).

Additionally, the use of the standard Markov chain test methodology, based on χ^2-square and Likelihood-Ratio tests (e.g. Anderson and Goodman, 1957, Goodman, 1958 and 1959, Billingsley, 1957 and 1961; Basawa and Rao, 1980) also demostrates some major problems related with the MLE parameter estimation.

These facts motivated us to design and develop a new method to test the EMH on stock prices using time-dependence and time-homogeneity Markov chain test procedures. Our study is based on the Tan and Yilmaz (2002) [9] methodology for evaluating the predictability of stock returns and Polansky's (2007) Markov chain time-homogeneity test for an unknown number of change-points.

In addition, we also applied the new MTD-Probit (Nicolau, 2014) model, which facilitates the parameter estimation procedure and its statistical inference for high-order Markov chains.

3.4.2 The Polansky (2007) Markov chain time-homogeneity Test

Polansky (2007) developed a new testing method for detecting and estimating change-points in Markov chains, when the number of break dates and their locations are unknown. The methodology is based on a single observed realization of a discrete-time Markov chain stochastic process, with fixed sample size and deterministic parameters change.

3.4.2.1 The Polansky Test Method

In Polansky (2007), three cases of change-points are addressed: (1) the number and locations are known; (2) the number is known, but locations are unknown, and; (3) the number and locations are unknown. We briefly review below the propose method using the Polansky's usage and notation.

Suppose $X_0, ..., X_n$ is a realization of length $n+1$ from a discrete first order Markov chain, with state space $S_c = \{1, 2, ..., c\}$, where c is a positive finite integer, and with transition probability matrix P. Let P change during the observed realization. Therefore, in this model there is a sequence of transition probability matrix $T_0, T_1, ..., T_\tau$, and positive integers $0 = \psi_0 < \psi_1 < ... < \psi_\tau < \psi_{\tau+1} = n$ such that $P_j = T_i$ for $j = \psi_i, ..., (\psi_{i+1} - 1)$ and $i = 1, ..., \tau$. The points $\psi_1, ..., \psi_\tau$ are defined as change-points.

Next, if the number and locations of change-points are known, we can then compute the maximum likelihood estimation of $T_0, T_1, ..., T_\tau$ given by $\widehat{T}_0, ..., \widehat{T}_\tau$, by applying the estimator of Eq.(3.5) to the sequence of observations $\{X_0, ..., X_{\psi_1}; X_{\psi_1}, ..., X_{\psi_2}; ....; X_{\psi_\tau}, ..., X_n\}$, respectively. In this case, a likelihood ratio test of the equality of the TPM is developed for testing $H_0 : T_0 = T_1 = ... = T_\tau$ versus $H_1 : T_i \neq T_j$ for some $i \neq j$, which is given by:

$$\Gamma = -2(\sum_{r=0}^{\tau} L(\psi_r, \psi_{r+1}) - L(0, n)), \tag{3.13}$$

where

[9]Tan and Yilmaz (2002) presented a detailed description of the Markov chain technique and the evaluation of the small and large sample properties of the time-dependence and time-homogeneity tests.

$$L(a,b) = \sum_{i,,j \in \xi(a,b)} n_{ij}^{a,b} log(n_{ij}^{a,b} / \sum_j n_{ij}^{a,b}) = \sum_{i,j} n_{ij}^{a,b} log(\widehat{p}(i,j)), \tag{3.14}$$

with $n_{ij}^{a,b}$ denoting the observed absolute number of transitions from state i to j in the time segment (a,b), $\sum_j n_{ij}$ accounts for all the transitions count from state i , and ξ(a,b) is understood to contains all indices (i,j) such that $n_{ij}^{a,b} > 0$. The standard asymptotic test theory applies and the test statistic Γ has an asymptotic χ^2-square distribution with $c(c-1)\tau$ degrees of freedom. Hence, a level α test of $H_0 : T_0 = T_1$ rejects the null hypothesis when $\Gamma > \chi^2_{1-\alpha, c(c-1)\tau}$.

Alternatively, when the location of change-points is unknown, it is proposed that the change-points $\psi_1, ..., \psi_\tau$ are added to the likelihood function as unknown parameters. In this case, the maximum likelihood estimators for $\psi_1, ..., \psi_\tau$, as indicate by the author, will generally not exist in closed form, and can be found algorithmically as:

$$(\tilde{\psi}_1, ..., \tilde{\psi}_\tau)' = argmax \left\{ \psi_1 < \psi_2 < .... < \psi_\tau \in \{1, ..., n-1\} : \sum_{r=0}^{\tau} L(\psi_r, \psi_{r+1}) \right\}, \tag{3.15}$$

where

$$\sum_{r=0}^{\tau} L(\psi_r, \psi_{r+1}), \tag{3.16}$$

is the maximum observed likelihood, conditional on $\psi_1, ..., \psi_\tau$. Thus, a size α test of $H_0 : T_0 = T_1 = ... = T_\tau$ versus $H_1 : T_i \neq T_j$ for some $i \neq j$ is developed in the same manner as on Eq. (3.13), where the likelihood ratio test statistic is given by:

$$\tilde{\Gamma} = -2(\sum_{r=0}^{\tau} L(\hat{\psi}_r, \hat{\psi}_{r+1}) - L(0,n)), \tag{3.17}$$

where an α size test will reject the null hypothesis when $\tilde{\Gamma} > \tilde{\gamma}_\alpha$. However, when the locations of the change-points are unknown, the standard asymptotic test theory is no longer valid (Polansky, 2007). In this case, the value of $\tilde{\gamma}_\alpha$ is estimated using the bootstrap. It is proposed to simulate B realization of length n from a Markov chain with TPM $\widehat{T}$ and initial state X_0, from the observed sample realizations[10].

For each simulated realization, the test statistic is computed by Eq.(3.17). Then $\tilde{\gamma}_\alpha$ is approximated by the $1-\alpha$ sample percentile of $\tilde{\Gamma}_1^*, ..., \tilde{\Gamma}_B^*$. The p-value for the test is approximated with:

$$\tilde{p} = \frac{1}{B+1} \left[1 + \sum_{i=1}^{B} \delta(\tilde{\Gamma}_i^* \geq \tilde{\Gamma}) \right], \tag{3.18}$$

[10]In the appendix 2, we provided an explanation of the Markov chain estimation method.

where δ is an indicator function which takes value one if $\tilde{\Gamma}_i^*$ is higher or equal than $\tilde{\Gamma}$. The null hypothesis is rejected whenever $\tilde{p} < \alpha$, where α is a given significance level.

Finally, when the chain dynamics is unknown, i.e. the number and location of the change-points are unknown, it is proposed to adopt some criteria that penalize models that have more parameters, instead of simply introducing parameter r in the likelihood function. Indeed, since r controls the number of parameters required to fit the observed data, and hence the dimension of the parameter space, by the principle of parsimony, it is reasonable to prevent over-fitting the data (Polansky, 2007, p. 6018). Accordingly, the author suggests using the AIC (Akaike, 1974) and the BIC (Schwarz, 1978) in the formulation on Eq.(3.17).

$$AIC(r) = -2\sum_{r=0}^{\tau} L(\hat{\psi}_r, \hat{\psi}_{r+1}) + 2c(c-1)(\tau+1), \tag{3.19}$$

$$BIC(r) = -2\sum_{r=0}^{\tau} L(\hat{\psi}_r, \hat{\psi}_{r+1}) + ln(n)c(c-1)(\tau+1), \tag{3.20}$$

where the AIC and BIC estimates are given by $\widehat{r}_{AIC} = argmin_{0\leq r\leq n} AIC(r)$ and $\widehat{r}_{BIC} = argmin_{0\leq r\leq n} BIC(r)$, respectively. Therefore, once the value of r is estimated using either measure, the time-homogeneity likelihood test is computed using $\widehat{r}$ on Eq.(3.17).

3.4.3 The Anderson and Goodman´s Standard Markov Chain Tests

The Anderson and Goodman (1957)[11] time-homogeneity and time-dependence tests are based on the standard non-parametric χ^2-square methods[12] applied in testing contingency tables. Their utilization is the equivalent of checking whether there is no statistically-significant difference between the observed frequency and the corresponding expected frequency.

3.4.3.1 The Anderson and Goodman's time-homogeneity Test

The Anderson and Goodman's time-homogeneity test involves dividing the entire sample T into D, $d = \{1, 2, ..., D\}$, with mutually independent and equi-length periods of observations, and testing whether or not the transition probabilities estimated for each of the equal sub-samples are significantly statistically different for those estimated for the entire sample [13](see e.g. and Tan and Yilmaz, 2002). The criteria according to which the sub-samples are

[11]For a more comprehensive analysis, see also Goodman (1958) and Billingsley (1961).

[12]The Anderson and Goodman (1957) proposed test is asymptotically equivalent to the likelihood ratio test.

[13]As pointed by Bickenbach and Bode (2001), taken literally, the tests just compare multinomial distributions (rows of transition matrices) rather than Markov processes. A test of, e.g. whether two sub-samples (r = 1, 2) follow the same Markov process does not take into account whether or not the initial distributions are likely to emerge from that Markov (Bickenbach and Bode, 2001, p.8).

defined are *ad-hoc* and depend on the hypothesis to be tested against. More specifically, if p_{ij}^d is the first-order transition probability corresponding to period d, that is:

$$p_{ij}^d = P_t(X_t = i_0 | X_{t-1} = i_1), t \in [(d-1)\Delta, d\Delta]. \tag{3.21}$$

where $\Delta = [(T+1)/D]$. In this context, we test for $i, j \in \mathscr{M}$ and $d = \{1, 2, ..., D\}$the following null hypothesis:

$$H_o : p_{ij}^d = p_{ij}, \tag{3.22}$$

against the alternative of transition probabilities differing between periods:

$$H_a : \exists p_{ij}^d \neq p_{ij}, \tag{3.23}$$

The proposed testing method is a χ^2-square asymptotically distributed statistics, given by:

$$\chi_i^2 = \sum_{d,j} n_{ij}^d (\widehat{p}_{ij}^d - \widehat{p}_{ij})^2 / (\widehat{p}_{ij}) \sim \chi_{df}^2, \tag{3.24}$$

where n_{ij}^d is the total number of observed transitions from state$i$ in the period d. Under the null hypothesis χ_i^2 has an asymptotically χ^2-square distribution with $(m-1)(D-1)$ degrees of freedom. Additionally, since under the null hypothesis $\widehat{p}_{ij}^d$ are mutually independent across sub-samples, then the *dth* transition probabilities can be estimated similar to Eq.(3.5).

We observe that the preceding result on Eq.(3.24) shows similarity to the usual procedures for contingency tables. Indeed, it is the equivalent to checking whether there is no statistically significant difference in the observed frequency of the sub-intervals from its corresponding expected frequency for the entire interval (Anderson and Goodman, 1957). That is:

$$\chi_i^2 \longleftrightarrow \sum_{t,j} (obs.freq - exp.freq)^2 / (exp.freq). \tag{3.25}$$

Additionally, if we consider the joint hypothesis that $p_{ij}^d = p_{ij}$, for all $i, j \in \mathscr{M}$ and $d = \{1, 2, ..., D\}$, the χ^2-square test is set as:

$$\chi^2 = \sum_i \chi_i^2 = \sum_i \sum_{d,j} n_{ij}^d (\widehat{p}_{ij}^d - \widehat{p}_{ij})^2 / (\widehat{p}_{ij}). \tag{3.26}$$

The null hypothesis is rejected if the computed χ^2 statistic is greater than the $(1-\alpha)-$*quartile* of the limiting χ^2-square distribution with $m(m-1)(D-1)$ degrees of freedom.

3.4.3.2 The Anderson and Goodman's time-dependency Test

As proposed by Anderson and Goodman (1957), the time-dependence structure of a stochastic process can also be tested using a standard non-parametric χ^2-square test. The procedure tests the null hypothesis of zero-order (statistical independence) against the alternative hypothesis that it is first or higher-order, which is equivalent to testing the EMH. More specifically, the following null hypothesis is tested:

$$H_o : p_{ij} = p_i p_j, \nabla i, j \in \mathscr{M}, \tag{3.27}$$

where p_i is the marginal probabilities. The proposed χ^2-test statistic is:

$$\chi_i^2 = \sum_j n_{ij}(\widehat{p}_{ij} - \widehat{p}_i\widehat{p}_j)^2/(\widehat{p}_i\widehat{p}_j), \tag{3.28}$$

where $\widehat{p}_i = n_i./n$ and $\widehat{p}_j = n._j/n$ are the estimated marginal probabilities, $n_i. = \sum_j n_{ij}$ and $n._j = \sum_i n_{ij}$ are the total number of observed transitions from state$i$ and j, respectively, and $n = \sum_i n_i . = \sum_j n._j$is the total number of observed transitions in all rows and columns, which is equal to the sample size. Under the null hypothesis χ_i^2 has an asymptotically χ^2-square distribution with $(m-1)^2$ degrees of freedom[14].

Additionally, if we consider the joint hypothesis that $p_{ij} = p_i p_j$, for all $i, j \in \mathscr{M}$, the χ^2-square test is set as:

$$\chi^2 = \sum_i \chi_i^2 = \sum_i \sum_j n_{ij}(\widehat{p}_{ij} - \widehat{p}_i\widehat{p}_j)^2/(\widehat{p}_i\widehat{p}_j), \tag{3.29}$$

the null hypothesis is rejected if the computed χ^2 statistic is greater than the $(1-\alpha)-$ *quartile* of the limiting χ^2-square distribution with $m(m-1)^2$ degrees of freedom.

The standard Anderson and Goodman's time-dependence test can be generalized to test if the chain is of order $k-1$ against the alternative hypothesis of order k. For example, the null hypothesis that the chain is first-order against the alternative that is second-order:

$$H_o : p_{1ij} = ... = p_{kij} = ... = p_{mij} = p_{ij}, \nabla i, j \in \mathscr{M}, \tag{3.30}$$

is based on similar χ^2-square test procedure (see, e.g. Anderson and Goodman, 1957).

There is, however, a major limitation regarding the generalization of the test procedure. In this case, modelling these probabilities when m and k is relatively large and the sample size is small or even moderate, is unfeasible, as the total number of parameters is $m^k(m-1)$.

[14]In terms of frequencies, the χ^2-square test statistic for all $i, j \in \mathscr{M}$ can be computed by:

$$\chi_i^2 = \sum_{i,j}[n_{ij} - n(n_i./n)(n._j/n)]^2/n(n_i./n)(n._j/n).$$

For example, as can be seen in Table 3.1, to test for the $4th$-order time-dependence in a five states Markov chain, we need to calculate 2500 parameters.

Table 3.1: Number of Parameters for Testing Different Order Markov Chains

	Markov Chain Order			
States	1	2	3	4
2	2	4	8	16
3	6	18	54	162
4	12	48	192	768
5	20	100	500	2500

3.5 The EMH Test Procedure

3.5.1 Introduction

Our EMH test procedure follows the proposed Tan and Yilmaz (2002) test structure using both the Anderson and Goodman (1957) and the Polansky (2007) time-homogeneity tests. However, our methodology is different in two aspects. Firstly, we developed a new approach to test for higher-order dependency of our data series using the MTD-Probit model and the Bayes Information Criterion (BIC) [15].

Secondly, we proposed a slightly modified Polansky time-homogeneity test procedure that is computationally simplified. Although the Polansky (2007) methodology can be successfully applied to detect and estimate change-points, a major limitation exist which is related with time computational restrictions in implementing the proposed method. For example, for a series of lengthy n=500 and five possible unknown break-dates, the method would have to perform over 252 billion estimations (Polansky, 2007, p.6025).

We address this problem of estimation of break-dates by proposing an efficient algorithm that is not only computationally reasonable in our time horizon of approximately five trading years (n≈1250 trading days), but is also suitable to test the EMH. We believe that our method is both parsimonious and theoretically acceptable.

Given this, our tests procedures can generate one of two conclusions. The first conclusion relates to the chain order, when we do not reject its time-homogeneity[16]. In this case, if the series is not of zero order, we do not support the EMH. Alternatively, if the time-homogeneity is rejected, we conclude that the time-dependence and hence the EMH, cannot be tested in the considered sample, as the series is not stationary.

[15] We choose the BIC test because it penalizes the likelihood for the number of independent parameters being tested. For more results see e.g. Schwarz (1978) and Katz (1981).

[16] Based on the Anderson and Goodman (1957) and the Polansky (2007) time-homogeneity tests

3.5.2 The HOMC time-dependence Test Procedure

Our study proposes a new method to test time-dependence. Firstly, we apply the standard Anderson and Goodman's time-dependence methodology to test the null hypothesis of statistical independence (zero-order) against the alternative hypothesis that the process can be characterized by a first-order or second-order time-dependence Markov chain.

Next, we use the BIC criterion to test from the zero-order through to the $5th$-order time-dependence, with $0 \leq k \leq 5$, based on the MTD-Probit maximum log-likelihood estimated $logL(\hat{k})$. In this study, when $k = 5$, the result is classified as $k \geq 5$. We follow this approach to avoid the problems imposed by unsuitable large number of independent parameters being estimated in high-order Markov chains, even for relatively small m and k. More concretely, in the BIC measure we use the maximum MTD-Probit log-likelihood estimate $logL(\hat{k})$, corrected for the number of independent parameters $q = m(m-1) + k$[17] and the sample size n [18] :

$$BIC(k) = -2logL(\hat{k}) + qln(n), \tag{3.31}$$

where the chain order $k = (0, 1, ..., 5)$ is estimate by $\hat{k}_{BIC}$:

$$\hat{k}_{BIC} = argmin_{0 \leq k \leq 5} BIC(k). \tag{3.32}$$

In this set-up, before we test the time-homogeneity of the considered sample, our time-dependence test procedures can generate one of three conclusions *a priori*. The first relates to the support of the EMH. If the process is statistically independent (zero-order) based on the standard Anderson and Goodman´s test result, and if the BIC is minimized for the zero-order chain, we then conclude that the EMH cannot be rejected.

However, if the standard test and the BIC test rejected the zero-order against first or higher order, we have to reject the EMH and conclude the time-dependence following the BIC test results. Finally, if there are opposite results between the Anderson and Goodman's and the BIC test, we then consider this case to be inconclusive and cannot proceed with the time-homogeneity test. For example, we can have the BIC test supporting the EMH and the standard test not rejecting a first or higher-order time-dependence, and therefore it is inconclusive.

3.5.3 The Polansky time-homogeneity Test Procedure

We propose a simplified test procedure that is not only computational reasonable in our time horizon, but is also suitable to test the EMH. In order to allow some computational parsimony for the process, we apply the change-point search process in steps of 126 trading

[17]In the MTD-Probit model, a constant term is introduced in the estimation process. In this way, the estimation process involves one additional parameter.

[18]The sample size depend of the chain order, that is $n = T - k$.

days (half trading year) and a TPM composed of a minimal of 252 daily observations (one trading year). The following example illustrates our proposed method.

Step 1: We test the simplest case of one change-point, i.e $r = 1$. That is to say, we compute the likelihood function to the sequences of observations $X_0, ..., X_{\psi_1}$ and $X_{\psi_1}, ..., X_n$. In this set-up, the first two segments likelihood functions, are given by:

$$\tilde{\Gamma}_{\psi_1} = L(0, X_{252}) + L(X_{252}, n), \tag{3.33}$$

$$\tilde{\Gamma}_{\psi_1} = L(0, X_{252+126}) + L(X_{252+126}, n). \tag{3.34}$$

Step 2: The method continues iteratively until we reached the last search step:

$$\tilde{\Gamma}_{\psi_1} = L(0, X_{n-252}) + L(X_{n-252}, n), \tag{3.35}$$

and identified the single change-point that maximize the sequence of likelihood functions.

Step 3: We repeat the last step sequentially for $r = 2, 3, 4, ...\tau$. Next, we apply the BIC test to the maximum likelihood estimations for each change-points r and identify the number and locations of the break dates that minimized the $\hat{r}_{BIC}$[19].

Step 4: We apply the Polansky likelihood homogeneity-test procedure on Eq.(3.17), based on the last step estimated change-point.

Step 5: Finally, we simulate the test statistic p-value, based on the bootstrap as suggested by Polansky (2007). In the Polansky study, the value of $\tilde{\gamma}_\alpha$ is estimated using the Efron (1979) bootstrap methodology. In this paper, however, in order to preserve a possible correlation dynamics in the stock prices series, we adopt the Politis and Romano (1994) stationary bootstrap method (SB)[20].We use the SB with $B = 500$ interactions and with the parameter of the geometric distribution set as $q = 0.1$ (see, e.g. Politis and Romano, 1994 and Hansen, 2005).

In this last step, we compute the test p-value based on the empirical distribution of $\tilde{\Gamma}_\tau$, which is obtained with realizations of B bootstrapped samples $\tilde{\Gamma}^*_1, ..., \tilde{\Gamma}^*_B$, based on the observed sample realizations of length n from a Markov chain with the transition probability matrix T , the initial state p_0 and the $\hat{\tau}$ estimate change-points. The Polansky test p-value is approximated using the Eq.(3.18).

[19] It is important to notice that given the characteristic of the maximum likelihood methodology, it is not surprising that the change-point estimated in this step are equal to maximum number of possible change-points.

[20] In the Appendix we provide an explanation of the SB method.

3.5.4 The Anderson and Goodman's time-homogeneity Test Procedure

To apply the standard test, we established some sample and sub-sample conditions. Initially, we set a minimum sub-sample size, which is established based on two technical restrictions. Firstly, we need to choose enough d sub-samples to allow us to have a reliable test for homogeneity. Secondly, for reliable inferences, any sub-sample size should have a minimum number of observed transitions to avoid the rejection of the null hypothesis against the alternative hypothesis, due to insufficient data size.

Indeed, the reliability of TPM estimations is subjected not only to the data generating process which should be Markovian, but also to a trade-off between the size of the sample data needed for reliable estimates and the likelihood of violating the time-invariant initial hypothesis due to the existence of structural breaks (Bickenbach and Bode, 2001). We also take into consideration that the minimum size is at least four years of daily price returns observation, or approximately one thousand trading market days.

In this set-up, we also propose to evaluate the time-homogeneity based on a standard approach. Firstly, we verify the time homogeneity of the entire sample considered. Next, as a second step, if the security price return is not time-invariant during the period, we then proceed to generate new sub-samples over the most recent time sub-interval until the chain time-homogeneity is accepted. For example, if our sub-sample is divided into four and five subgroups, $d = \{4, 5\}$, we will have approximately 250 and 200 daily observations[21], respectively.

3.5.5 The State aggregation Method

The use of a Markov chain methodology to study the EMH requires the aggregation of a continuous time series process into a discrete state space sequence of finite states. In practice, although any continuous time series can be aggregated around a discrete-valued regularly-spaced process, there is no optimal aggregation method that could preserve all the statistical properties of the original time series.

In this study, we adopt the methodology proposed by Fielitz and Bhargava (1973) and arbitrarily categorize the random sequence of asset log returns, $Y = \{Y_t, t \geq 0\}$, into a discrete state Markov chain stochastic process $X = \{X_t, t \geq 0\}$ based on its mean absolute deviation (MAD), as a measure of dispersion, as follows:

$X_t = 1$ if $Y_t \leq \bar{Y} - \upsilon * MAD$, corresponding to the bear market;

$X_t = 2$ if $\bar{Y} - \upsilon * MAD < Y_t < \bar{Y} + \upsilon * MAD$, for a neutral market ; and

$X_t = 3$ if $Y_t \geq \bar{Y} + \upsilon * MAD$, corresponding to the bull market.

[21]We believe that this sample data size is adequate not only for the application of reliable tests, but also as a representative to describing the behavior of the markets under analysis.

Where $\bar{Y}$ is the observed mean of Y, υ is an appropriate constant, that is set up to 0.5 [22], and:

$$MAD = \sum_{t=1}^{T} \|Y_t - \bar{Y}\|/T. \tag{3.36}$$

As such, the log returns continuous state space is mapped into state space$\{1, 2, 3\}$, which allows us to incorporate the direction of change in the Index returns into the analysis, and its magnitude as a function of parameter υ (see, e.g. Niederhoffer and Osborne, 1966, and Fielitz and Bhargava, 1973).

3.6 Empirical Examination

In this section, we provide the empirical evaluation of the EMH of stock prices using our time-dependence and time-homogeneity Markov chain test procedures.

3.6.1 Data Sample Selection and Statistics Results

The empirical analysis for this study is based on a broad worldwide sample of market indices. We adopt the Metghalchi et al.(2012) methodology for market selection. We use stock market capitalization distribution percentiles as a main variable to define our stock market indices' sample of the top 20 stock exchanges.

In Table 3.2, we present our list of stock market indices which are ranked and classified for January, 2015, according to the financial market capitalization criterion. Our minimum market size (baseline) is the Brazilian Stock Exchange ($20th$ in market capitalization in 2015). The small market indices group is constituted from this point up until the median of the total market capitalization distribution (NYSE Euronext (Europe)).

The next category is the medium markets, which are selected if the Index capitalization is between the median and the 75th percentile. Finally, large-sized markets are defined for markets where the Index capitalization is greater than the $75th$ percentiles (NYSE Euronext (US)).

3.6.2 Main Sample Index Statistics Results

Table 3.3 presents descriptive statistics of daily log returns for the indices considered in the paper. The data consist of the daily closing prices of 21 worldwide stock exchange indices, obtained from the Datastream database. The sample used comprises approximately four

[22] In Fielitz and Bhargava (1973) and Fielitz (1975) it is proposed that $\upsilon = 0.5$ makes the three states approximately equiprobable.

Table 3.2: Data Sample and Market Capitalization - January, 2015

Financial Markets	Name	Country	Rank Market Cap (USA bn)	Market Cap (USA bn)	Stock Market Indices	Stock Symbol	Ranked
New York Stock Exchange (NYSE) Euronext(US)	DJIA	US	1	19,223	S&P 500 and DJIA -Dow Jones Industrial Average	^GSPC and ‘DJI	Large
NASDAQ	NASDAQ	US	2	6,831	NASDAQ	^IXIC	Upper-median
London Stock Exchange	FTSE-100	UK	3	6,187	FTSE 100	^FTSE	Upper-median
Japan Stock Exchange	Nikkei 225	Japan	4	4,485	Nikkei 225	^N225	Upper-median
Shanghai Stock Exchange	SSE	China	5	3,986	SSE - Shangay Composite Index	^SSEC	Upper-median
Hong Kong Stock Exchange	HSI	Hong Kong	6	3,325	HSI - Hang Seng Index	^HSI	Upper-median
NYSE Euronext (Europe)	AEX	Netherlands	7	3,321	AEX Index	^AEX	Lower-median
NYSE Euronext (Europe)	CAC	France	7	3,321	CAC Index	^FCHI	Lower-median
NYSE Euronext (Europe)	BEL 20	Belgium	7	3,321	BEL 20 Index	^BFX	Lower-median
NYSE Euronext (Europe)	PSI 20	Portugal	7	3,321	PSI 20 Index	^PSI20.NX	Lower-median
Shenzhen Stock Exchange	SZSE	China	8	2,285	SZSE Component Index	SHE:399106	Lower-median
TMX Group	S&P/TSX	Canada	9	1,939	S&P/TSX Composite Index	^GSPTSE	Lower-median
Deutsche Börse	DAX	Germany	10	1,762	DAX	^GDAXI	Lower-median
Bombay Stock Exchange	BSE 30	India	11	1,682	BSE 30 - S&P BSE SENSEX lndex	^BSESN	Lower-median
National Stock Exchange of India	S&P Nifty 50	India	12	1,642	S&P Nifty 50 Index	^NSEI	Small
SIX Swiss Exchange	SMI	Switzerland	13	1,516	SMI - Swiss Market lndex	^SSMI	Small
Australian Securities Exchange	AOI	Australia	14	1,272	AOI - All Ordinaries Index	^AORD	Small
Korea Exchange	KOSPI	South Korea	15	1,251	KOSPI Composite Index	^KS11	Small
NASDAQ OMX Nordic Exchange	OMX N. 40	Northern Europe	16	1,212	OMX NORDIC 40	^OMXN40	Small
JSE Limited	FTSE/JSE	South Africa	17	951	FTSE/JSE All-Share	J203:L	Small
Spanish Stock Exchange	Ibex-35	Spain	18	942	Ibex-35	^IBEX	Small
Taiwan Stock Exchange	TAIEX	Taiwan	19	861	TAIEX - Taiwan Cap.Weighted Stock Index	^TWII	Small
BM&F Bovespa	Ibovespa	Brazil	20	824	Ibovespa - Bovespa Index	^BVSP	Small

Data Source: World Federation of Exchanges, January 2015. * Multiple indices.

years of log returns of the indices daily closing price from the period of January 01, 2010 to December 31, 2014.

From this table it can be inferred that the highest mean daily return is the NASDAQ Index (5.72 basis point), and the lowest is the OMXN.40 Index (-3.79 basis points), which equates to 250 trading days per year, with an approximate average of -9.48% and 14,30 % per year, respectively. The mean daily return volatility is highest in the Spanish market (IBEX 35), with a standard deviation of 1.58%, and lowest in the Canadian market (S&P/TSX) with a standard deviation of 0.82%.

Additionally, the table also shows that most of the indices are skewed to the left, which indicates that extreme negative returns are more probable than extreme positive ones.The sample excess kurtosis level reveals that the indices return series has fatter tails than the normal distribution, i.e. the low positive and negative returns are more probable. Indeed, the Jarque-Bera portmanteau test (JB) supports the non-normal nature of the sample distribution, as it strongly rejects the null hypothesis of normality at the one percent level, for all individual indices.

Regarding the linear time dependence properties, we observe that there is significant evidence of first-order autocorrelation across the sample, at 5% level or better, accounting for a total of nine indices, with a varying coefficient ranging from 0.106 in the Portuguese market (PSI 20) to -0.058 in the American market (DOW JONES). Finally, based on the Ljung-Box Q statistics, there is also significant autocorrelation, of up to six lags, for some of the Index returns. The null hypothesis of no autocorrelation for all six lags tested is rejected for 13 indices, at 5% level or better.

Although, there is no evidence of autocorrelation in some indices, this does not mean that these indices' returns are independent over time. Indeed, there is the possibility of nonlinear time dependence in the observed data sample. Hence, the use of the Markov chain test methodology can be an important procedure for testing EMH.

3.6.3 Results of the time-dependence Test

Tables 3.4 and 3.5 present a summary of our time-dependence test results for the study of the EMH, using the standard Anderson and Goodman (1957) and the Polansky (2007) methodology.

In Table 3.4, we present the summary results of the standard testing of the Markov chain order for 1% of statistical significance. The zero-order time-dependence is observed for a total of 12 indices in financial markets with a very mixed type of attributes. Indeed, under this conditions, we see both the young and small PSI 20 Index market, as well as the more developed and capitalized markets, such as, for example, the Nikkei 225 and HSI indices.

Table 3.3: Indices Descriptive Statistics

Indices	N(Obs.)	Mean (%)	Max. (%)	Min. (%)	S.D. (%)	Skewness	Kurtosis	ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(5)	ρ(6)	Q(6)	JB
S&P 500	1257	0.0475	4.6317	-6.8958	1.0092	-0.4771	7.6493	-0.064**	0.051**	-0.076*	0.023*	-0.111*	0.001*	32.03	1179.84*
DOW JONES	1257	0.0415	4.1533	-5.7061	0.9188	-0.4300	7.1254	-0.058**	0.056**	-0.059*	0.012**	-0.108*	-0.000*	27.40	930.09*
NASDAQ	1257	0.0572	5.1592	-7.1489	1.1276	-0.4211	6.5613	-0.033	0.035	-0.083*	0.009**	-0.071*	-0.014*	18.39	701.43*
FTSE 100	1296	0.0149	5.0323	-4.7792	0.9739	-01776	5.5442	0.032	-0.028	-0.012	-0.052	-0.008	-0.004	6.13	356.36*
Nikkei 225	1237	0.0399	5.5223	-11.1534	1.3788	-0.7330	8.0449	-0.033	0.025	0.012	-0.094*	0.032**	0.023**	15.22	1422.52*
SSE	1210	-2.32E-04	6.0399	-5.4968	1.1980	-0.2086	5.4650	0.003	0.001	0.031	0.012	0.002	-0.024	2.09	312.13*
HSI	1254	6.26E-03	5.5187	-5.8270	1.1547	-0.2712	5.4013	0.011	0.015	-0.024	-0.032	0.003	-0.042	4.72	316.65*
AEX	1280	0.0166	7.0722	-4.5689	1.1012	-0.0755	5.9903	0.084***	-0.017	-0.019	-0.049	-0.042	0.011	9.37	478.12*
CAC 40	1280	4.88E-03	9.2208	-5.6346	1.3543	0.0175	6.539	0.000	-0.039	-0.035	-0.061***	-0.034*	0.025	10.59	666.88*
BEL 20	1280	0.0194	8.9550	-5.4925	1.1503	0.1616	7.6530	0.022	-0.056***	-0.037***	-0.097*	-0.061*	0.015*	23.71	1160.25*
PSI 20	1280	0.0456	10.1959	-5.5071	1.3385	-0.0219	6.4932	0.106*	-0.010*	-0.039*	-0.066*	-0.021*	0.023*	23.20	650.91*
S&P/TSX	1269	0.0165	3.9409	-4.1226	0.8154	-0.3676	5.5201	0.061**	0.011***	-0.091*	-0.030*	-0.098*	-0.019*	29.41	364.39*
DAX	1278	0.0378	5.2104	-5.9947	1.2612	-0.1883	5.6179	0.063**	-0.042**	-0.035**	-0.048**	-0.045**	-0.009**	14.51	372.48*
BSE 30	1232	0.0364	3.7035	-4.2129	1.0489	-0.0128	3.8277	0.066**	0.005***	-0.053**	-0.022**	0.021***	0.034***	11.49	35.20*
SMI	1275	0.0238	4.9029	-4.2428	0.9264	-0.2917	6.5603	0.097*	-0.027*	-0.020**	-0.036*	-0.056*	0.039*	21.03	691.51*
AOI	1262	7.70E-03	3.4368	-4.2998	0.8859	-0.2567	4.4631	0.021	0.006	-0.014	-0.046	-0.058	0.003	7.80	126.44*
KOSPI	1236	9.84E-03	4.900	-6.4202	1.0607	-0.4031	6.8546	0.022	-0.030	-0.010	-0.074***	0.001	-0.079**	16.32	798.67*
OMXN.40	1275	-0.0379	6.8268	-7.1697	1.2664	0.2669	6.6877	0.013	-0.000	-0.059	-0.061***	-0.053**	-0.023**	13.68	737.58*
Ibex-35	1278	-0.0130	13.4836	-6.8739	1.5778	0.3269	8.3705	0.067**	-0.064*	-0.040*	-0.090*	-0.0018*	0.052*	27.35	1558.59*
TAIEX	1235	0.0102	4.4594	-5.7422	0.9979	-0.4783	5.6591	0.074*	-0.044*	-0.028**	-0.028**	0.002**	-0.064**	16.17	410.95*
Ibovespa	1250	-0.0270	4.9760	-8.4307	1.4129	-0.1463	4.6732	-0.021	0.026	-0.029	-0.001	-0.001	-0.063	7.45	150.27*

Notes: (1)The mean sample log-return (Mean (%)) and the standard deviation.(S.D. (%)) are reported in percentage . (2) JB are the Jarque-Bera test statistics, ρ(n) is the estimated autocorrelation at lag n, and Q(n) are the Ljung-Box-Pierce test statistics for the nth lag. The Ljung-Box Q -statistics p -values are reported with the estimated autocorrelation. ***Statistical Significance of the two tail test at the 10% level for a two-tailed test. **Statistical Significance at the 5% level.*Statistical Significance at the 1% level.

Table 3.4: Results of the Anderson and Goodman's Time-dependence Test

Indices	Chain Order Test Result
S&P 500	Second or Higher Order*
DJIA	Second or Higher Order*
NASDAQ	Second or Higher Order*
FTSE 100	First-order*
Nikkei 225	Zero-order*
SSE	Zero-order*
HSI	Zero-order*
AEX	Second or Higher Order*
CAC 40	First-order*
BEL 20	Zero-order*
PSI 20	Zero-order*
S&P/TSX	Zero-order*
DAX	Second or Higher Order*
BSE 30	Zero-order*
SMI	Second or Higher Order*
AOI	Zero-order*
KOSPI	Zero-order*
OMX N. 40	Zero-order*
Ibex-35	First-order*
TAIEX	Zero-order*
Ibovespa	Zero-order*

Note: *The results are for 1 % of statistical significance.

In Table 3.5, as a follow up the test procedures, we apply the BIC test for higher orders of dependency, based on MTD-Probit log-likelihood estimation. From Table 3.5, we can observe that all indices are classified with a zero-order time-dependence.

Table 3.5: Results of the BIC Test for HOMC

	Chain Order					
Indices	0th	1th	2th	3th	4th	5th
S&P 500	2386.47*	4112.94	4108.67	4108.91	4109.76	4101.85
DJIA	2386.47*	4114.49	4110.10	4097.47	4081.07	4082.08
NASDAQ	2386.47*	4119.93	4099.13	4094.78	4088.61	4086.89
FTSE 100	2457.78*	4341.74	4339.26	4338.98	4329.95	4327.78
Nikkei 225	2348.52*	3985.49	3972.33	3973.46	3973.52	3974.65
SSE	2296.37*	3802.37	3809.63	3802.76	3799.16	3799.05
HSI	2378.67*	4089.59	4087.19	4080.84	4091.37	4092.42
AEX	2429.22*	4266.45	4267.14	4268.24	4264.28	4259.86
CAC 40	2429.22*	4256.08	4250.24	4248.15	4238.31	4234.20
BEL 20	2429.22*	4264.82	4944.22	4261.36	4249.12	4241.74
PSI 20	2429.22*	4252.70	4246.79	4245.43	4244.59	4242.64
S&P/TSX	2414.61*	4209.46	4204.51	4200.94	4195.43	4195.00
DAX	2423.02*	4246.54	4241.94	4236.19	4222.51	4217.88
BSE 30	2337.53*	3939.45	3929.09	3930.24	3930.89	3932.00
SMI	2421.24*	4226.41	4225.91	4222.39	4223.36	4210.48
AOI	2397.46*	4154.31	4148.55	4134.24	4119.35	4116.87
KOSPI	2343.91*	3968.56	3969.29	3970.50	3971.10	4610.05
OMX N. 40	2421.24*	4240.41	4236.00	4234.96	4209.16	4202.31
Ibex-35	2423.02*	4252.05	4235.58	4234.69	4216.74	4225.68
TAIEX	2345.32*	3965.98	3962.56	3962.49	3948.10	3963.94
Ibovespa	2372.29*	4072.38	4067.23	4061.46	4061.96	4069.30

Notes: BIC, Bayesian information criterion.* Lowest BIC value.

Based on the results in Table 3.4 and 3.5, and in line with the previous discussion regarding the EMH and the Markov chain framework test procedures, our indices sample can be tested for time-homogeneity.

3.6.4 Results of the time-homogeneity Test

In Table 3.6, we provide the Polansky time-homogeneity test results, based on the stationary bootstrap with $B = 500$ interactions and the parameter of the geometric distribution parameter set as $q = 0.1$ (see, e.g. Politis and Romano, 1994 and Hansen, 2005). As can be seen, we cannot statistically reject the null hypothesis of time-homogeneity for the considered indices in our time horizon for any conventional significance test level.

Table 3.6: Results of the Polanski's Time-homogeneity Test

Indices	P-Value		Chain Order Test
	BIC	AIC	Result
S&P 500	0.5968	0.5768	Time-homogeneous
DJIA	0.4691	0.4671	Time-homogeneous
NASDAQ	0.5110	0.5309	Time-homogeneous
FTSE 100	0.5509	0.6267	Time-homogeneous
Nikkei 225	0.5629	0.5230	Time-homogeneous
SSE	0.6068	0.5788	Time-homogeneous
HSI	0.1397	0.1197	Time-homogeneous
AEX	0.5689	0.6188	Time-homogeneous
CAC 40	0.4970	0.5709	Time-homogeneous
BEL 20	0.5729	0.5788	Time-homogeneous
PSI 20	0.5788	0.5589	Time-homogeneous
S&P/TSX	0.5988	0.6367	Time-homogeneous
DAX	0.5629	0.5309	Time-homogeneous
BSE 30	0.5549	0.5469	Time-homogeneous
SMI	0.5629	0.5808	Time-homogeneous
AOI	0.5629	0.5389	Time-homogeneous
KOSPI	0.5489	0.5349	Time-homogeneous
OMX N. 40	0.5329	0.5868	Time-homogeneous
Ibex-35	0.4751	0.5050	Time-homogeneous
TAIEX	0.5469	0.5589	Time-homogeneous
Ibovespa	0.4830	0.4930	Time-homogeneous

In Table 3.7, we report the standard Anderson and Goodman's time-homogeneity test results. For this purpose, we split our sample data n into d sub-samples, $d = \{4, 5\}$, which has approximately n/d observations, that are equivalent to approximately 300 and 250 trading days respectively. We believe that our sample data size is adequate, not only for the application of reliable tests, but also to be representative to describe the behavior of the markets under analysis.

The standard test suggests that the S&P 500, the CAC 40, and also the BSE 30 indices are not statistically time-homogeneous at 5% significant level. That is to say, there is some evidence of a structural break in the sample time series. In this case, the use of the total sample TPM estimations may be misleading for forecasting the evolution of the stock market stochastic process.

Table 3.7: Results of the Anderson and Goodman´s Time-homogeneity Test

	$d = 4$	$d = 5$
Indices	χ^2-square	χ^2-square
S&P 500	25.34	39.25**
DJIA	15.35	24.29
NASDAQ	16.75	29.95
FTSE 100	16.03	26.71
Nikkei 225	22.76	25.88
SSE	18.13	26.66
HSI	18.21	19.39
AEX	26.41	31.69
CAC 40	33.64**	31.27
BEL 20	21.96	26.01
PSI 20	15.54	17.76
S&P/TSX	15.52	21.35
DAX	12.80	15.52
BSE 30	37.02*	39.95**
SMI	11.92	28.14
AOI	18.55	19.67
KOSPI	27.95	39.88**
OMX N. 40	18.70	21.52
Ibex-35	11.49	22.94
TAIEX	16.03	25.71
Ibovespa	13.52	20.22

Notes:.**Statistical Significance at the 5% level.
*Statistical Significance at the 1% level.

Based on the results in Table 3.6 and 3.7, our indices sample is suitable for testing the EMH.

3.6.5 The Efficient Market Hypothesis

In Table 3.8, we present the summary results of the standard testing of the EMH for $d = 5$, for 1% of statistical significance, and compare this with our Polansky time-homogeneity and time-dependence tests results. As can be seen, there are ten indices where we cannot reach a conclusion regarding the EMH. Nevertheless, when we fully account for the interdependence between time-homogeneity and time-dependence properties, we cannot reject the EMH in eleven financial markets. These results are very important.

Nonetheless, is interesting to note that in the Anderson and Goodman (1957) methodology, the American (DJIA and NASDAQ Indices) and the UK (FTSE 100 Index) markets represents a first, or higher-order time-homogeneous Markov chain process. This is a challenging result. Indeed, given the worldwide importance of these markets, we suggest that in future work an alternative testing methodology should be used to test the EMH on these indices.

Table 3.8: Results of the Anderson and Goodman's and Polansky Tests

Indices	Anderson and Goodman' Test		Polansky' Test		Order Result
	Time-Homogeneity	Chain Order	Time-Homogeneity	Chain Order	
S&P 500	Time-inhomogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
DJIA	Time-homogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
NASDAQ	Time-homogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
FTSE 100	Time-homogeneous	First-order	Time-homogeneous	Zero-order	Inconclusive
Nikkei 225	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
SSE	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
HSI	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
AEX	Time-homogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
CAC 40	Time-inhomogeneous	First-order	Time-homogeneous	Zero-order	Inconclusive
BEL 20	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
PSI 20	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
S&P/TSX	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
DAX	Time-homogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
BSE 30	Time-inhomogeneous	Zero-order	Time-homogeneous	Zero-order	Inconclusive
SMI	Time-homogeneous	Second or Higher Order	Time-homogeneous	Zero-order	Inconclusive
AOI	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
KOSPI	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
OMX N. 40	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
Ibex-35	Time-homogeneous	First-order	Time-homogeneous	Zero-order	Inconclusive
TAIEX	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order
Ibovespa	Time-homogeneous	Zero-order	Time-homogeneous	Zero-order	Zero-order

Table 3.9: Results of the Time-Homogeneity and Time-Dependence Tests

Market Exchange Index	Country	Number of Assets	Time-Homogeneity Assets (%)	First or Higher Chain Order (%)	Time-Homogeneous First or Higher Chain Order
CAC 40	France	18	61.11	72.22	44.44
DAX	Germany	29	86.20	31.03	20.69
FTSE 100	England	94	80.85	45.74	39.36
HSI	Hong Kong	46	82.60	39.13	30.43
Ibovespa	Brazil	51	72.54	50.98	31.37
NASDAQ	US	1986	73.81	72.35	49.29
NYSE	US	1486	80.21	47.64	32.70
Dow	US	29	89.65	37.93	31.03
PSI 20	Portugal	18	50.00	77.77	66.66
Ibex-35	Spain	31	87.09	41.93	41.93
India Market	India	686	59.47	76.67	43.00
	All Markets Total	4474	74.18	62.98	41.86

3.6.6 A Robustness Results

In this section we applied the standard test methodology to a broad universe of stocks from worldwide security markets, verifying the proportion of stocks that are time-homogeneous [23] and which can be characterized as a first, or higher order Markov chain. As far we know, this is the first time that Markov properties were tested in a very representative sample of globally-listed securities, considering not only time-dependence, but also time-homogeneity properties.

The data sample consists of a universe of 4,474 stocks[24] from worldwide stock exchanges, obtained from the Datastream database. The sample used comprises approximately four years of log returns of the indices daily closing price from the period of January 01, 2010 to December 31, 2014. The main standard tests results are presented in Table 3.9, for a 5% significance level, using the MAD dispersion measure and a test time interval of $d = 4$.

In Table 3.9, the fourth column reports the proportion of time-homogeneous assets using the Anderson's test methodology. The fifth column highlights the proportion of first-order or higher order computed without considering the sample time-homogeneity. Finally, in the last column, we present the results which fully apply the test methodology. That is to say, we fully account for the interdependence between time-homogeneity and time-dependence properties, using the standard test methodology.

From this Table, it can be inferred that an incorrect consideration of the standard Markov chain methodology leads to the misleading conclusion that a stock market has a higher predictive power, than when the time-homogeneity is tested. Indeed, when we compare the results in Columns 5 and 6, we observe that approximately 50% more stocks are either first or higher order Markov chains.

Furthermore, we observe that approximately 42% of the stocks in this study are char-

[23] We use Tan and Yilmaz (2002) nomenclature and time-homogeneity test which is significantly equivalent to testing the stationarity of the process as used by Fielitz and Bhargava (1973).

[24] For some markets, we have excluded a few stocks because the data were corrupted or non-existent.

acterized by first or higher order Markov chains when the time-homogeneity property of the data series is considered. For example, almost half of the stocks listed in the NASDAQ market have a short-time memory and the EMH is rejected.

This is a very important novel result, and it undoubtedly indicates that a significant number of stocks in the main financial markets can be used for predictive purposes, based on the efficient behavior of their respective market Index, especially in some of the less developed and/or younger markets.

3.7 Conclusions

This paper makes two main contributions to the literature. With regard to the methodology, we propose a new Markov chain test procedure. We apply the MTD-Probit model (Nicolau, 2014) to estimate and test the time-dependence for HOMC. Furthermore, we propose a simplify time-homogeneity test procedure that is not only computationally reasonable to detect and estimate the true change-points for a discrete-time Markov chain, but is also suitable to test the EMH.

Regarding its application, we explored the proposed procedure to obtain new evidence for the EMH hypothesis in the main worldwide stock market indices. Our empirical results for time-invariant Markov chains suggest that the stock market can be efficient. Nonetheless, the results are inconclusive for the American and the UK financial markets. Indeed, using the Anderson and Goodman (1957) methodology, these markets represent a first, or higher-order time-homogeneous Markov chain process. This is an issue that may be worth studying in future research.

We also perform the Markov chain tests on a broad sample of 4,474 stocks and interesting robustness result emerges. The study showed that a lack of full accountability of the interdependence between the time-homogeneity and time-dependence properties can lead to a conclusion that a stock market has a higher predictive power than when only the time-homogeneity is tested.

Appendix 1

Stationary Block Bootstrap method

The basic idea of the stationary bootstrap method is to construct random data blocks that are independent, yet preserve the time dependence inside each block. The unknown population distribution structure is approximated by block sampling distributions based on a statistical model. As such, the stationary bootstrap methodology provides a re-sampling method which is applicable for weakly-dependent time series, where the pseudo-time series are stationary time series.

The method is based on two basic steps that provide proper consistency and weak convergence properties. Firstly, the original series is re-sampled into a set of b random length overlapping blocks of observations, determined by the realization of a geometric distribution with parameter $q \in (0, 1)$. In this case, the average block size is the inverse of q. Secondly, the stationary bootstrap method "wraps" the data around in a "circle" to avoid the block end effects (Politis and Romano, 1994, p.1304). The idea is to choose a large enough block length, preferably based on the sample size, so that observations greater than $1/q$ time units apart will be nearly independent.

However, the major difficulty of this method lies in choosing the size of q. Indeed, the size of the block is a controversial topic in the literature (e.g. Sullivan et al., 1999; Hsu and Kuan, 2005; Metghalchi et al., 2012, and Hsu et al., 2010), as a small size will not reproduce the data dependence, and a large value will reduce the statistical efficiency. In this study we adopt what is usually presented in the previous research in this area, and set $q = 0.1$.

Appendix 2

Markov Chain Simulation Method

In this appendix we describe a bootstrap implementation of the Polansky tests in detail for a FOMC stochastic process. As was presented, the Markov chain process for the observed sample is fully determined once is known the estimated TPM matrix $\hat{P}$ and the initial probability vector p_0, describing the starting probabilities of the various states.

Our objective is to generate a sequence of random variables $X = \{X_t, t = 1, 2, ..., T\}$ with three states $\{1, 2, 3\}$ and the data generating process given by the MLE expression on Eq.(3.5) in our sample data. We use the following algorithm:

Step 1: Estimate $\hat{P}$ and assigning arbitrarily values for the initial term of the sequence, $X_0 = i$, $i \in \{1, 2, 3\}$. Thus, we assume in the estimation process, that the distribution of the initial state of the Markov chain has all of its mass concentrated at $X_0 = i$.

Step 2: Simulate a random variable $u \sim U(0, 1)$.

Step 3: Generate X_t given X_{t-1}, as follows:

$$X_t = \left\{ \begin{array}{lll} 1 & if & u \in [0, \hat{p}_{i1}) \\ 2 & if & u \in [\hat{p}_{i1}, \hat{p}_{i1} + \hat{p}_{i2}) \\ 3 & if & u \in [\hat{p}_{i1} + \hat{p}_{i2}, 1] \end{array} \right\}$$

where $\hat{p}_{ij} = P(X_t = j | X_{t-1} = i)$, for $i, j \in \{1, 2, 3\}$. For example, assuming that $X_0 = 1$, then $X_1 = j$, can be generated as follows:

$$X_1 = \left\{ \begin{array}{ccc} 1 & if & u \in [0, \hat{p}_{11}) \\ 2 & if & u \in [\hat{p}_{11}, \hat{p}_{11} + \hat{p}_{12}) \\ 3 & if & u \in [\hat{p}_{11} + \hat{p}_{12}, 1] \end{array} \right\}$$

where $\hat{p}_{11} = P(X_1 = 1|X_0 = 1)$ and $\hat{p}_{21} = P(X_1 = 2|X_0 = 1)$.

Step 4: Return to steps 2 and 3, until $t = T$.

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Chapter 4

Estimation and Inference in Multivariate Markov Chains

Abstract

The literature of Markov chains has recently focused on modeling multiple categorical data sequences. The usual procedure for handling these multivariate Markov chains (MMC), with "m" categorical data and "s" states, consists of expanding the state space by considering m^s new states. This model rapidly becomes intractable even with moderate values of "m" and "s" due to the excessive number of parameters to estimate. Ching et al. (2002) found a way to cope with the intractability of the conventional MMC. They also suggested a method of estimation that proved to be inefficient. Zhu and Ching (2010) proposed another method of estimation based on minimizing the prediction error with equality and inequality restrictions. However, both these procedures treat the estimation problem as a mechanic method, without addressing the statistical inference problem. In this article we try to overcome this shortcoming and, at the same time, we propose a new approach to estimate MMC (under Ching et al. hypothesis) which avoids imposing equality and inequality restrictions on the parameters. We illustrate the model and the estimation method with two applications on financial time series data.

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REGULAR ARTICLE

Estimation and inference in multivariate Markov chains

João Nicolau · Flavio Ivo Riedlinger

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Keywords Multivariate Markov chains · Nonlinear least squares · Predictability of investment recommendations · Statistical inference

Mathematics Subject Classification 62M02 · 62M05 · 62M10

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1 Introduction

Markov chains are applied in a number of fields such as physics, chemistry, information sciences, queueing theory, internet, economics and finance, social

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sciences, biology, etc (more recent applications can be found, for example, in Tsai and Yen, 2011 and Faraz and Saniga, 2011). Recently the literature has focused on modeling multiple categorical data sequences. When the number of categorical data, say s, and the number of states each data can take on, say m, are low, one can expand the state space by considering a first-order Markov chain with m^s states. However, this model rapidly becomes intractable even with moderate values of m and s due to the excessive number of parameters to estimate.

In this context, Ching et al. (2002) found a way to cope with the intractability of the conventional multivariate Markov chain (MMC) by developing a model with far fewer parameters based on a mixture transition distribution model. This hypothesis was already considered by Raftery (1985) for modeling high-order Markov chains, as an extension of Pegram (1980). This model allows both the intra and inter-transition probabilities among the categorical data. They also propose a method to estimate the parameters based on linear programming. The MMC model has been applied to Markov chain Monte Carlo (2000), demand predictions (Ching et al. 2002), credit risk (Kijima et al. 2002), reproductive biology (McDonnell et al. 2002), stock markets (Maskawa, 2003), DNA sequences and Genetic Networks (Ching et al. 2006), weather simulation (Yang et al. 2011), credit rating (Siu et al. 2005, Fung et al. 2012).

Recently Zhu and Ching (2010) have proposed a method of estimation based on minimizing the prediction error involving equality and inequality restrictions. They do not address the statistical inference problem (Ching et al. 2002, do not focus this issue either). Our article has two goals: first we propose a new approach to estimate MMC which avoids imposing equality and inequality restrictions on the parameters, which facilitate the model estimation and the statistical inference. Furthermore, we address the statistical inference of MMC models as proposed by Ching et al. (2002). We illustrate the model and the estimation method with two applications on financial time series data.

This article is organized as follows: in the next section we present the main results concerning the estimation and inference of MMC. In the last section we illustrate the model and the estimation method with two applications on financial time series data.

2 Estimation Statistical Inference of Multivariate Markov Chain

Consider the multivariate stochastic process $\{(S_{1t}, ..., S_{st})\,; t = 1, 2, ...\}$ where S_{jt} $(j = 1, ..., s)$ can take on values on the set $\{1, 2, ..., m\}$. We may rewritten this process as $\left\{\left(\mathbf{x}_t^{(1)}, ..., \mathbf{x}_t^{(s)}\right); t = 1, 2, ...\right\}$ where

$$\mathbf{x}_t^{(j)} = \begin{cases} (1, 0, 0, ..., 0)' & \text{if } S_{jt} = 1 \\ (0, 1, 0, ..., 0)' & \text{if } S_{jt} = 2 \\ \vdots & \vdots \\ (0, 0, 0, ..., 1)' & \text{if } S_{jt} = m. \end{cases}$$

Given $S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s$, the kth element of $\mathbf{x}_t^{(j)}$ is a random variable that takes on the value one with probability

$$P\left(S_{jt} = k \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right).$$

Modeling these probabilities using the conventional Markov chain is impracticable since the total number of states of the process increases exponentially with s (there are m^s states). A simplifying hypothesis is considered in Ching et al. (2002). It involves assuming that the probability $P\left(S_{jt} = k \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right)$ can be written as a convex linear combination of $P\left(S_{jt} = k \middle| S_{1,t-1} = i_1\right), ..., P\left(S_{jt} = k \middle| S_{s,t-1} = i_s\right)$, i.e.

$$\begin{aligned} P\left(S_{jt} = k \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right) &= \lambda_{j1} P\left(S_{jt} = k \middle| S_{1,t-1} = i_1\right) \\ &+ ... + \lambda_{js} P\left(S_{jt} = k \middle| S_{s,t-1} = i_s\right) \end{aligned} \quad (1)$$

where $0 \leq \lambda_{ji} \leq 1$ and $\sum_{i=1}^{s} \lambda_{ji} = 1$. The first approach to estimate the parameters λ_{jk} is described in Ching et al. (2002). This method solves a minimization problem involving the stationary vector. As referred to in Zhu and Ching (2010), this may imply a large error when the data sequence period is not long enough. This method is certainly not optimal in the mean square error sense (since it does not involve the conditional mean).

We notice that a probability like $P\left(S_{jt} = k \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right)$ is formally identical to the conditional moment

$$\mathrm{E}\left(\mathcal{I}\left(S_{jt} = k\right) \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right),$$

where $\mathcal{I}(A)$ is an indicator function that takes on the value one if A is true. Let $\mathbf{P}^{(jk)}$ be a $m \times m$ matrix with elements $P_{ab}^{(jk)} := P\left(S_{jt} = a \middle| S_{k,t-1} = b\right)$. Therefore

$$\underbrace{\begin{bmatrix} \mathrm{E}\left(\mathcal{I}\left(S_{jt} = 1\right) \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right) \\ \vdots \\ \mathrm{E}\left(\mathcal{I}\left(S_{jt} = m\right) \middle| S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right) \end{bmatrix}}_{\mathrm{E}\left(\mathbf{x}_t^{(j)} \middle| S_{1,t-1}=i_1,...,S_{s,t-1}=i_s\right)} =$$

$$= \lambda_{j1} \underbrace{\begin{bmatrix} P_{1i_1}^{(j1)} \\ \vdots \\ P_{mi_1}^{(j1)} \end{bmatrix}}_{i_1 th \text{ column of } \mathbf{P}^{(j1)}} + ... + \lambda_{js} \underbrace{\begin{bmatrix} P_{1i_s}^{(js)} \\ \vdots \\ P_{mi_s}^{(js)} \end{bmatrix}}_{i_s th \text{ column of } \mathbf{P}^{(js)}}.$$

Now define $\mathcal{F}_{t-1}$ as all the available information of the system at time $t-1$, i.e. let $\mathcal{F}_{t-1}$ be the σ-algebra generated by $\left\{\left(S_{1,t-1}, ..., S_{s,t-1}\right), \left(S_{1,t-2}, ..., S_{s,t-2}\right), ...\right\}$. This means that the hypothesis (1) can be put in the following terms:

$$\mathrm{E}\left(\mathbf{x}_t^{(j)} \middle| \mathcal{F}_{t-1}\right) = \lambda_{j1} \mathbf{P}^{(j1)} \mathbf{x}_{t-1}^{(1)} + ... + \lambda_{js} \mathbf{P}^{(js)} \mathbf{x}_{t-1}^{(s)} \quad (2)$$

where

$$\mathrm{E}\left(\mathbf{x}_t^{(j)}\middle|\mathcal{F}_{t-1}\right) := \begin{bmatrix} \mathrm{E}\left(\mathcal{I}\left(S_{jt}=1\right)|\mathcal{F}_{t-1}\right) \\ \vdots \\ \mathrm{E}\left(\mathcal{I}\left(S_{jt}=m\right)|\mathcal{F}_{t-1}\right) \end{bmatrix}$$

(obviously, $\mathrm{E}\left(\mathcal{I}\left(S_{jt}=k\right)|\mathcal{F}_{t-1}\right) = \mathrm{E}\left(\mathcal{I}\left(S_{jt}=k\right)|S_{1,t-1},...,S_{s,t-1}\right)$, given the Markovian nature of the process). To illustrate this, assume that $s=3$ (three categorical sequences) and $m=2$. Suppose that at time $t-1$ one observes $S_{1t-1}=1$, $S_{2t-1}=1$ and $S_{3t-1}=2$, i.e.

$$\mathbf{x}_{t-1}^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \mathbf{x}_{t-1}^{(2)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \mathbf{x}_{t-1}^{(3)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then the conditional mean of $\mathbf{x}_t^{(1)}$ is

$$\mathrm{E}\left(\mathbf{x}_t^{(1)}\middle|\mathcal{F}_{t-1}\right) = \lambda_{11}\underbrace{\mathbf{P}^{(11)}\mathbf{x}_{t-1}^{(1)}}_{1st\text{ col. of }\mathbf{P}^{(11)}} + \lambda_{12}\underbrace{\mathbf{P}^{(12)}\mathbf{x}_{t-1}^{(2)}}_{1st\text{ col. of }\mathbf{P}^{(12)}} + \lambda_{13}\underbrace{\mathbf{P}^{(13)}\mathbf{x}_{t-1}^{(3)}}_{2nd\text{ col. of }\mathbf{P}^{(13)}}.$$

In practise, the probabilities $\mathbf{P}_{ab}^{(jk)}$ have to be estimated. Consistent estimates can be obtained as follows (see Ching et al. 2002):

$$\hat{P}\left(S_{jt}=a|S_{k,t-1}=b\right) = \frac{\sum_{t=1}^{n}\mathcal{I}\left(S_{jt}=a,S_{k,t-1}=b\right)}{\sum_{t=1}^{n}\mathcal{I}\left(S_{k,t-1}=b\right)}.$$

We now address the estimation of λ_{jk}. An important step in the estimation procedure consists of representing the MMC through an equation with a martingale difference error term, where the standard nonlinear least squares can be readily applied. Using the fact that any process $\mathbf{x}_t^{(j)}$ can be always written as $\mathbf{x}_t^{(j)} = \mathrm{E}\left(\mathbf{x}_t^{(j)}\middle|\mathcal{F}_{t-1}\right) + \mathbf{u}_t^{(j)}$ with $\mathbf{u}_t^{(j)} := \mathbf{x}_t^{(j)} - \mathrm{E}\left(\mathbf{x}_t^{(j)}\middle|\mathcal{F}_{t-1}\right)$, we represent the MMC as

$$\mathbf{x}_t^{(j)} = \sum_{k=1}^{s}\lambda_{jk}\mathbf{P}^{(jk)}\mathbf{x}_{t-1}^{(k)} + \mathbf{u}_t^{(j)}, \qquad j=1,2,...,s \tag{3}$$

where $\mathbf{u}_t^{(j)} := \mathbf{x}_t^{(j)} - \mathrm{E}\left(\mathbf{x}_t^{(j)}\middle|\mathcal{F}_{t-1}\right)$ is the prediction error, which by construction is a martingale difference. To impose $\sum_{i=1}^{s}\lambda_{ji}=1$ we replace λ_{js} by $\lambda_{js} = 1-\lambda_{j1}-...-\lambda_{j,s-1}$ in equation (3). Rearranging the terms leads to

$$\begin{aligned}\mathbf{x}_t^{(j)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)} = {} & \lambda_{j1}\left(\mathbf{P}^{(j1)}\mathbf{x}_{t-1}^{(1)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)}\right) + \lambda_{j2}\left(\mathbf{P}^{(j2)}\mathbf{x}_{t-1}^{(2)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)}\right) \\ & + ... + \lambda_{j,s-1}\left(\mathbf{P}^{(j,s-1)}\mathbf{x}_{t-1}^{(s-1)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)}\right) + \mathbf{u}_t^{(j)}.\end{aligned}$$

To simplify the notation let us rewrite the previous equation as:

$$\begin{aligned}\mathbf{y}_t^{(j)} &= \lambda_{j1}\mathbf{z}_{t-1,1}^{(j)} + \lambda_{j2}\mathbf{z}_{t-1,2}^{(j)} + ... + \lambda_{j,s-1}\mathbf{z}_{t-1,s-1}^{(j)} + \mathbf{u}_t^{(j)} \\ &= \sum_{k=1}^{s-1}\lambda_{jk}\mathbf{z}_{t-1,k}^{(j)} + \mathbf{u}_t^{(j)}\end{aligned}$$

where $\mathbf{y}_t^{(j)} := \mathbf{x}_t^{(j)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)}$ and $\mathbf{z}_{t-1,k}^{(j)} = \left(\mathbf{P}^{(jk)}\mathbf{x}_{t-1}^{(k)} - \mathbf{P}^{(js)}\mathbf{x}_{t-1}^{(s)}\right)$.

We have yet to deal with the restrictions $0 \leq \lambda_{jk} \leq 1$. We consider an approach that consists of replacing the parameters of interest with an auxiliary function that guarantees the validity of the restrictions. Hence, let λ_{jk} be defined as $\lambda_{jk} = h\left(\theta_{jk}\right) = e^{\theta_{jk}} / \left(1 + e^{\theta_{jk}}\right)$, $k = 1, 2, ..., s-1$. By construction $0 \leq \lambda_{jk} \leq 1$, for any value θ_{jk}.

Finally, the model with the restrictions $0 \leq \lambda_{ji} \leq 1$ and $\sum_{i=1}^{s} \lambda_{ji} = 1$ is

$$\mathbf{y}_t^{(j)} = \sum_{k=1}^{s-1} h\left(\theta_{jk}\right) \mathbf{z}_{t-1,k}^{(j)} + \mathbf{u}_t^{(j)}.$$

The idea is simple: we first obtain an estimate for θ_{jk} and then recover the original parameter λ_{jk}, through the equation $\lambda_{jk} = e^{\theta_{jk}} / \left(1 + e^{\theta_{jk}}\right)$. We emphasize that the estimation, as we will see below, is carried out without imposing any kind of restrictions on the parameters θ_{jk}, although the restrictions on the original parameters λ_{jk} are maintained. The estimation of θ_{jk} is clearly a nonlinear least squares (NLS) problem. Define the NLS estimator as

$$\hat{\boldsymbol{\theta}}_j = \arg\min_{\boldsymbol{\theta}_j} \frac{1}{n} \sum_{t=2}^{n} q_t\left(\boldsymbol{\theta}_j\right),$$

where $q_t\left(\boldsymbol{\theta}_j\right) = \sum_{i=1}^{m} \left(y_{it}^{(j)} - \sum_{k=1}^{s-1} h\left(\theta_{jk}\right) z_{i,t-1,k}^{(j)}\right)^2$ and $\boldsymbol{\theta}_j = \left(\theta_{j1}, ..., \theta_{j,s-1}\right)'$. We recall that $\mathbf{y}_t^{(j)}$ is a $m \times 1$ vector. A generic element in this vector is identified as $y_{it}^{(j)}$, $i = 1, ..., m$. In the same way, a generic element of the $m \times 1$ vector $\mathbf{z}_{t,k}^{(j)}$ is $z_{i,t,k}^{(j)}$. Under some mild regularity conditions, including $\left\{\left(\mathbf{y}_t^{(j)}, \mathbf{z}_{t,k}^{(j)}\right)\right\}$ is a stationary and weakly dependent process, we have

$$\hat{\boldsymbol{\theta}}_j \xrightarrow{p} \boldsymbol{\theta}_j \text{ and } \sqrt{n}\left(\hat{\boldsymbol{\theta}}_j - \boldsymbol{\theta}_j\right) \xrightarrow{d} N\left(\mathbf{0}, \boldsymbol{\Sigma}\right)$$

where $\boldsymbol{\Sigma} = \mathbf{A}^{-1}\mathbf{B}\mathbf{A}$ and

$$\mathbf{A} = \mathrm{E}\left(\frac{\partial^2 q_t\left(\boldsymbol{\theta}_j\right)}{\partial \boldsymbol{\theta}_j \boldsymbol{\theta}_j'}\right), \qquad \mathbf{B} = \lim_{n\to\infty} \frac{1}{n} \operatorname{Var}\left(\sum_{t=1}^{n} \frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial \boldsymbol{\theta}_j}\right)$$

(see, for example, Hayashi 2000, chap. 7).

Remark 1 The assumption that $\left\{\left(\mathbf{y}_t^{(j)}, \mathbf{z}_{t,k}^{(j)}\right)\right\}$ is a stationary and weakly dependent process is a weak condition. Suppose that $P^{(jk)}$ for $j, k = 1, ..., s$ are irreducible and aperiodic. Then $\mathbf{x}_t = \left(\mathbf{x}_t^{(1)}, ..., \mathbf{x}_t^{(s)}\right)'$ has a unique stationary distribution (see Billingsley, 1999). It follows that a function of stationary process is also a stationary process, thus $\left\{\left(\mathbf{y}_t^{(j)}, \mathbf{z}_{t,k}^{(j)}\right)\right\}$ remains a stationary process. Moreover, all moments of $\mathbf{x}_t$ are bounded in view of the finite state assumption. Due to the fact that all moments are finite, to stationarity and the fact that $\mathbf{x}_t$ is α-mixing with a geometric rate of decay (see Billingsley, 1999), any sample mean follows the law of large numbers and any sum properly standardized has an asymptotic normal distribution by the central limit theorem. These results may be used to easily verify the hypotheses defined in propositions 7.1 and 7.8 of Hayashi (2000) concerning the consistency and asymptotic normality of the NLS estimator.

Remark 2 By the law of large numbers (see previous remark) the matrix $\mathbf{A}$ can be consistently estimated by

$$\hat{\mathbf{A}} = \frac{1}{n}\sum_{t=1}^{n}\frac{\partial^2 q_t\left(\hat{\boldsymbol{\theta}}_j\right)}{\partial\boldsymbol{\theta}_j\boldsymbol{\theta}_j'}.$$

To estimate $\mathbf{B}$ we use the fact that

$$\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j} = -\sum_{i=1}^{m}\left(y_{it}^{(j)} - \sum_{k=1}^{s-1}h\left(\theta_{jk}\right)z_{i,t-1,k}^{(j)}\right)\sum_{k=1}^{s-1}\frac{h\left(\theta_{jk}\right)z_{i,t-1,k}^{(j)}}{\partial\boldsymbol{\theta}_j}$$

is a martingale difference (hence its conditional and marginal moment is zero). In effect

$$\mathrm{E}\left(\left.\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j}\right|\mathcal{F}_{t-1}\right) = -\sum_{i=1}^{m}\mathrm{E}\left(y_{it}^{(j)} - \left.\sum_{k=1}^{s-1}h\left(\theta_{jk}\right)z_{i,t-1,k}^{(j)}\right|\mathcal{F}_{t-1}\right)\sum_{k=1}^{s-1}\frac{h\left(\theta_{jk}\right)z_{i,t-1,k}^{(j)}}{\partial\boldsymbol{\theta}_j}$$

and $\mathrm{E}\left(y_{it}^{(j)} - \left.\sum_{k=1}^{s-1}h\left(\theta_{jk}\right)z_{i,t-1,k}^{(j)}\right|\mathcal{F}_{t-1}\right) = 0$ by construction. Moreover, by stationarity we have

$$\frac{1}{n}\,\mathrm{Var}\left(\sum_{t=1}^{n}\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j}\right) = \frac{1}{n}\sum_{t=1}^{n}\mathrm{E}\left(\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j}\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j'}\right) = \mathrm{E}\left(\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j}\frac{\partial q_t\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j'}\right).$$

Therefore, a consistent estimator of $\mathbf{B}$ is

$$\hat{\mathbf{B}} = \frac{1}{n}\sum_{t=1}^{n}\frac{\partial q_t\left(\hat{\boldsymbol{\theta}}_j\right)}{\partial\boldsymbol{\theta}_j}\frac{\partial q_t\left(\hat{\boldsymbol{\theta}}_j\right)}{\partial\boldsymbol{\theta}_j'}.$$

Given $\hat{\boldsymbol{\theta}}_j$ we recover the parameters of interest:

$$\hat{\lambda}_{jk} = \begin{cases} h\left(\hat{\theta}_{jk}\right) & k = 1, 2, ..., s-1 \\ 1 - \sum_{i=1}^{s-1}h\left(\hat{\theta}_{ji}\right) & k = s \end{cases},$$

Obviously $\hat{\lambda}_{jk} \xrightarrow{p} \lambda_{jk}$ by Slutsky's theorem (given that h is a continuous function). The asymptotic distribution of $\hat{\boldsymbol{\lambda}}_j = \left(\hat{\lambda}_{j1}, ..., \hat{\lambda}_{j,s-1}\right)'$ is given by the delta theorem

$$\sqrt{n}\left(\hat{\boldsymbol{\lambda}}_j - \boldsymbol{\lambda}_j\right) \xrightarrow{d} N\left(\mathbf{0}, \frac{\partial\mathbf{h}\left(\boldsymbol{\theta}_j\right)'}{\partial\boldsymbol{\theta}_j}\boldsymbol{\Sigma}\frac{\partial\mathbf{h}\left(\boldsymbol{\theta}_j\right)}{\partial\boldsymbol{\theta}_j}\right),$$

where $\mathbf{h}(\boldsymbol{\theta}_j) = (h(\theta_{j1}), ..., h(\theta_{j,s-1}))'$. In particular, for a scalar estimate $\hat{\lambda}_{ji}$ we have

$$\sqrt{n}\frac{\left(\hat{\lambda}_{ji} - \lambda_{ji}\right)}{\operatorname{Var}\left(\hat{\lambda}_{ji}\right)^{1/2}} \xrightarrow{d} N(0,1), \qquad \operatorname{Var}\left(\hat{\lambda}_{ji}\right) = \left(\frac{\partial h\left(\hat{\boldsymbol{\theta}}_{ji}\right)}{\partial \theta_{ji}}\right)^2 \operatorname{Var}\left(\hat{\theta}_{ji}\right).$$

The delta theorem can be applied again to obtain the asymptotic distribution of $\hat{\lambda}_{j,s} = 1 - \sum_{k=1}^{s-1} h\left(\hat{\theta}_{jk}\right)$.

This procedure has to be repeated for other values $j \in \{1, ..., m\}$.

3 Examples

In this section we illustrate the model and the estimation method with two examples from the financial time series area.

MMC is very promising in modeling ratings over time, since ratings have the level of measurement required by MMC models. In the first example we illustrate how MMC may be used to study the predictability of stock or investment recommendations. For illustration purposes we consider Citigroup's investment recommendation produced by the Bank of America/Merrill Lynch (BofA/ML) research department during the period January 1994-December 2009. Let S_{1t} be the analysts' recommendations, with state space $\{1, 2, 3\}$ defined according to BofA/ML research: $S_{1t} = 1$ (buy) if at time t the annualized return expectation is higher than 10%; $S_{1t} = 2$ (hold) if at time t the annualized return expectation lays in the interval $(0, 10\%)$ and $S_{1t} = 3$ (sell) otherwise (BofA/ML also considers the "rating dispersion guidelines for coverage cluster"). These recommendations express the particular analyst's opinion about the company's future prospects. To compare the ability of the analysts to correctly predict future returns we also collect Citigroup's monthly returns (since analysts' recommendations are disclosed at the beginning of each month, we consider monthly returns at the beginning of the month too). Monthly returns are split into three categories $\{1, 2, 3\}$ leading to the second stochastic process $\{S_{2t}\}$ to be defined as follows: $S_{2t} = 1$ if at time t the annualized return is higher than 10%; $S_{2t} = 2$ if at time t the annualized return lays in the interval $(0, 10\%)$ and $S_{2t} = 3$ if otherwise. Let $\hat{\mathbf{x}}_t^{(j)}$ be defined as $\hat{\mathbf{x}}_t^{(j)} := \sum_{k=1}^{s} \hat{\lambda}_{jk}\hat{\mathbf{P}}^{(jk)}\mathbf{x}_{t-1}^{(k)}$ (see equation (3)). Implementing the method described in previous section we obtained:

$$\hat{\mathbf{x}}_t^{(1)} = \underset{(0.058)}{0.9907}\hat{\mathbf{P}}^{(11)}\mathbf{x}_{t-1}^{(1)} + \underset{(0.058)}{0.009}\hat{\mathbf{P}}^{(12)}\mathbf{x}_{t-1}^{(2)} \tag{4}$$

$$\hat{\mathbf{x}}_t^{(2)} = \underset{(0.214)}{0.530}\hat{\mathbf{P}}^{(21)}\mathbf{x}_{t-1}^{(1)} + \underset{(0.214)}{0.470}\hat{\mathbf{P}}^{(22)}\mathbf{x}_{t-1}^{(2)} \tag{5}$$

(standard error in parentheses) where $P_{ab}^{(jk)} := P\left(S_{jt} = a \middle| S_{kt} = b \right)$ and

$$\hat{\mathbf{p}}^{(11)} = \begin{bmatrix} 0.86 & 0.06 & 0.09 \\ 0.14 & 0.91 & 0.00 \\ 0.00 & 0.03 & 0.91 \end{bmatrix}, \quad \hat{\mathbf{p}}^{(12)} = \begin{bmatrix} 0.35 & 0.35 & 0.27 \\ 0.46 & 0.65 & 0.46 \\ 0.19 & 0 & 0.27 \end{bmatrix}$$

$$\hat{\mathbf{p}}^{(21)} = \begin{bmatrix} 0.33 & 0.43 & 0.45 \\ 0.29 & 0.33 & 0 \\ 0.38 & 0.24 & 0.55 \end{bmatrix}, \quad \hat{\mathbf{p}}^{(22)} = \begin{bmatrix} 0.42 & 0.35 & 0.41 \\ 0.38 & 0.18 & 0.18 \\ 0.19 & 0.47 & 0.41 \end{bmatrix}.$$

Equation (4) suggests that past returns have little impact on the level of future ratings. Ratings exhibit strong persistence, i.e. BofA/ML research tends to maintain previous ratings (notice that the elements on the diagonal of $\hat{\mathbf{p}}^{(11)}$ are relatively high). This is especially true for the ratings $S_{1t} = 2$ (hold) and $S_{t1} = 3$ (sell). On the other hand, equation (5) suggests that the analyst stock recommendations may have value for investors (the 0.530 estimate is statistically significant), although the accuracy of those recommendations may not be very high as the following example illustrates. Suppose that in the period $t-1$ one has $S_{1t-1} = 1$ and $S_{2t-1} = 1$ (there is a recommendation to buy and the annualized return is above 10%), so $\mathbf{x}_{t-1}^{(1)} = (1,0,0)'$ and $\mathbf{x}_{t-1}^{(2)} = (1,0,0)'$. Using the estimates of equation (5) one obtains $\hat{\mathbf{x}}_t^{(2)} = (0.38, 0.33, 0.29)'$. The first entry of $\hat{\mathbf{x}}_t^{(2)}$ (value 0.38) represents the probability that the annualized returns are above 10% at time t (given $S_{1t-1} = 1$ and $S_{2t-1} = 1$). This probability is relatively low given that there was a recommendation to buy in the previous period. Another interesting scenario is when $S_{1t-1} = 3$ and $S_{2t-1} = 1$ (there was a recommendation to sell and the annualized return was above 10%), so $\mathbf{x}_{t-1}^{(1)} = (0,0,1)'$ and $\mathbf{x}_{t-1}^{(2)} = (1,0,0)'$. This scenario involves information with contrarian signs. Using equation (5) again we obtain $\hat{\mathbf{x}}_t^{(2)} = (0.44, 0.18, 0.38)'$. Despite a sell recommendation, the most likely scenario is a bull market in the following period. This exercise allows us to conclude that an informative recommendation (in the sense that there is a correlation between rating and future returns) may not be accurate enough to present valuable information to investors.

In the second example we consider a multivariate Markov chain to model the SP500, Nikkei 225 and DAX stock indices (we analyze weekly data from January 6, 1965 to March 30, 2011). This example can be seen as a generalization of McQueen and Thorley's (1991) approach to analyzing the predictability of stock returns . They consider a Markov chain model to test the random walk hypothesis of stock prices. Their Markov chain is defined by two states: one to represent high returns and the other to represent low returns. Our generalization consists in expanding the number of categorical data (one to three) and the number of states or regimes that each process can take on (we will consider 10 states). The main purpose of our application is only to illustrate how MMC can be used in practice, but several interesting conclusions can be drawn from the data.

Let r_{1t}, r_{2t} and r_{3t} be the returns associated with the SP500, Nikkei 225 and DAX respectively. We split the returns into 10 categories data as follows.

Let $q_\alpha^{(i)}$ be the α-quantile of the marginal distribution of r_{it}, i.e. $q_\alpha^{(i)}$ is such that $P\left(r_{it} \leq q_\alpha^{(i)}\right) = \alpha$, and $\hat{q}_\alpha^{(i)}$ the corresponding sample quantile (for simplicity we will refer to the $\hat{q}_{0.10}$ as the $10th$ percentile, the $\hat{q}_{0.20}$ as the $20th$ percentile, and so on). We have

$$\begin{aligned}
S_{it} &= 1 \text{ if } r_{it} \leq \hat{q}_{0.10}^{(i)}, \\
S_{it} &= 2 \text{ if } \hat{q}_{0.10}^{(i)} < r_{it} \leq \hat{q}_{0.20}^{(i)} \\
&\;\;.... \\
S_{it} &= 10 \text{ if } r_{it} \geq \hat{q}_{0.90}^{(i)}
\end{aligned}$$

(the higher the value S_{it} takes on the higher the associated return; for example $S_{1t} = 10$ means that at time t the return of the SP500 index is above the $90th$ percentile). This conversion causes some loss of information. However, Markov chains with more than two states can capture nonlinear dynamics. This is valid for univariate Markov chain models as well as for multivariate model. This feature turns out to be of fundamental importance in our model as we see below. Implementing the method described in the previous section we obtained.

$$\begin{aligned}
\hat{\mathbf{x}}_t^{(1)} &= \underset{(0.072)}{0.351}\hat{\mathbf{P}}^{(11)}\mathbf{x}_{t-1}^{(1)} + \underset{(0.070)}{0.265}\hat{\mathbf{P}}^{(12)}\mathbf{x}_{t-1}^{(2)} + \underset{(0.072)}{0.384}\hat{\mathbf{P}}^{(13)}\mathbf{x}_{t-1}^{(3)} \\
\hat{\mathbf{x}}_t^{(2)} &= \underset{(0.075)}{0.182}\hat{\mathbf{P}}^{(21)}\mathbf{x}_{t-1}^{(1)} + \underset{(0.067)}{0.575}\hat{\mathbf{P}}^{(22)}\mathbf{x}_{t-1}^{(2)} + \underset{(0.075)}{0.243}\hat{\mathbf{P}}^{(23)}\mathbf{x}_{t-1}^{(3)} \\
\hat{\mathbf{x}}_t^{(3)} &= \underset{(0.072)}{0.241}\hat{\mathbf{P}}^{(31)}\mathbf{x}_{t-1}^{(1)} + \underset{(0.069)}{0.344}\hat{\mathbf{P}}^{(32)}\mathbf{x}_{t-1}^{(2)} + \underset{(0.072)}{0.415}\hat{\mathbf{P}}^{(33)}\mathbf{x}_{t-1}^{(3)}
\end{aligned}$$

(the $\hat{\mathbf{P}}^{(ij)}$ matrices of 10×10 dimension, are too large to be presented here. They are available upon request). A striking feature of these results is that all estimates are statistically significant. It is clear from our simulations that the model can anticipate strong increases or decreases in returns. For example, we may ask what the conditional probability function of S_{1t} is given that $S_{1t-1} = 1, S_{2t-1} = 1$ and $S_{3t-1} = 1$ (in other words, what are the probabilities associated with S_{1t}, given that all the three returns were below the 10th percentile in the last period). The answer is given by the vector $\hat{\mathbf{x}}_t^{(1)}$ (of 10×1 dimension) when all vectors $\mathbf{x}_{t-1}^{(1)}$, $\mathbf{x}_{t-1}^{(2)}$ and $\mathbf{x}_{t-1}^{(3)}$ take on the value 1 at the first entry (and zero in other entries) - see Table 1.

Table 1: Cond. prob. of S_{1t} given $S_{1t-1} = 1, S_{2t-1} = 1, S_{3t-1} = 1$

1	2	3	4	5	6	7	8	9	10
0.145	0.086	0.103	0.078	0.075	0.067	0.040	0.09	0.102	0.214

Table 1 shows that the probability of the SP500 being in a bull market (i.e. $S_{1t} = 10$) after the three indices were below the $10th$ percentile in the previous week is relatively high (the probability is 0.214) and higher than the probability of the SP500 continuing below the $10th$ percentile. We may also express this conclusion in terms of the original variables as follows,

$$P\left(r_{1t} > q_{0.90}^{(1)} \middle| r_{1t-1} < q_{0.10}^{(1)}, r_{2t-1} < q_{0.10}^{(2)}, r_{3t-1} < q_{0.10}^{(3)}\right) = 0.214$$
$$> P\left(r_{1t} < q_{0.10}^{(1)} \middle| r_{1t-1} < q_{0.10}^{(1)}, r_{2t-1} < q_{0.10}^{(2)}, r_{3t-1} < q_{0.10}^{(3)}\right) = 0.145.$$

Another similar exercise can be done, using as conditioning set $S_{1t-1} = 10, S_{2t-1} = 10$ and $S_{3t-1} = 10$. The conditional probabilities of S_{1t} are given in table 2.

Table 2: Cond. prob. of S_{1t} given $S_{1t-1} = 10, S_{2t-1} = 10, S_{3t-1} = 10$

1	2	3	4	5	6	7	8	9	10
0.123	0.113	0.090	0.092	0.090	0.100	0.084	0.100	0.108	0.100

Table 2 shows that the probability of the SP500 being in a bear market after the three indices were above the $90th$ percentile in the previous week is relatively higher than the probability of the SP500 continuing above the $90th$ percentile. Both these extreme cases may be related to the famous quotation by Mandelbrot who stated that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". Our results go a bit further: not only do they confirm Mandelbrot's idea (that low values of S_{it-1} tend to be followed by low or high values of values of S_{it}, but not by moderate values) but also enables us to conclude that a bull (bear) market is more likely to be followed by a bear (bull) market.

4 Concluding Remarks

We merely illustrate the potential use of the model, but there are several other issues that can be exploited. In fact, since it is quite easy to obtain conditional moments (such as means, variance, skewness and kurtosis) as well as Markov times and marginal moments, many interesting finance applications can be devised in the context of the model. The results also suggest that the model may be able to generate trading rules. This is an issue that may be worth analyzing in a future work.

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Chapter 5

The Profitability in the FTSE 100 Index: a New Markov Chain Approach

Abstract

In this paper, we propose a new method to predict stock market trends based on the multivariate Markov chain (MMC) methodology. Our approach consists of forecasting the one-period ahead FTSE 100 Index behavior, using the MTD-Probit model. The MTD-Probit model is a new approach for estimating MMC, based on multiple categorical data sequences that can be used to forecast financial markets. In this context, we propose a simple trading strategy and analyze its profitability using the White "Reality Check" and the Hansen SPA data snooping bias tests. Our empirical results suggest that the MTD-Probit model applied to the FTSE 100 Index cannot significantly out-perform the buy-and-hold benchmark after data-snooping is controlled.

Keywords: multivariate Markov chains, data-snooping test, MTD-Probit.

5.1 Introduction

One of the most intricate tasks in the field of finance is to forecast stock market behavior. The main challenge lies in choosing a model that is theoretically consistent and feasible when applied in the "real" world, in the presence of market inefficiencies. Usually, the best choice should be suggested by the underlying theory; however, the model is often selected through its capacity to reproduce the key characteristics of the given time series data.

The main objective of this paper is to study the effectiveness of the Multivariate Markov chain (MMC) methodology in forecasting the behavior of financial market. We believe that the MMC approach is of special interest in finance, as it is not only a theoretically robust, well-defined and parsimonious method, but also can capture nonlinear dynamics. For the practitioner, the Markov approach is also an appealing model, as it can be used to derive the security price behavior and the nature of the successive movements of such prices, based on documented price information.

Most articles in this area which applied the Markov chain theory are based on the first-order univariate discrete-time chain (e.g. Mills and Jordanov, 2003, Svoboda and Lukas, 2005, Doubleday and Esungei, 2011, Vasanthi et al., 2011, and Onwukwe and Samson, 2014). However, financial series can also depend on some explanatory variables lagged over more than one period.

In this case, when we expand the probabilistic structure and consider an MMC model that depends on multiple categorical data sequences the estimation process can be problematic. Indeed, with a finite sample size, when the number of categorical data, say s, and the number of states that each security financial data can take on, say m, are relatively large and the sample size is small or even moderate, the parameters cannot be efficiently estimated, even with moderate values of s and m (e.g. Nicolau, 2014). Nonetheless, a new MMC model estimation procedure, called MTD-Probit (Nicolau, 2014), has led to a simpler approach which facilitates the model parameter estimation and its statistical inference.

In this paper, we study the Financial Time-Stock Exchange 100 Share Index (FTSE 100) forecast ability, using a simple trading strategy based on the MTD-Probit estimation method. We call this procedure the markovian MMC indicator (MMCI). Also, to carry out inference and model selection, we apply the White (2000) Bootstrap Reality Check (RC) and the Hansen (2005) Superior Predictive Ability (SPA) data-snooping bias tests. These tests allow us to forecast the Index based on a large set of parameters and covariates statistically controlling the possibility of data mining spurious results.

Although, in the literature, the MMC methodology has been used for a long time in several fields, ranging from genetics to economics (see, e.g Berchtold and Raftery, 2002, and Ching et al., 2004 and 2008), as far as we know, this methodology has never been study to forecast stock markets.

The study shows that there is some evidence that the MTD-Probit trading rules applied to the FTSE 100 Index can consistently out-perform the buy-and-hold benchmark after transaction costs. Nevertheless, the data-snooping tests suggest that the best rule performance is not significant at any conventional test level. Under such circumstance, we conclude that there is non-significant evidence of abnormal profitability of the MTD-Probit model and therefore we cannot reject the weak-form of the efficient market hypothesis (Fama, 1965 and 1970).

We organize the rest of this study as follows. In Section 5.2, we present the MMC methodology. Detailed descriptions of the trading rules modeling framework we use in our study are presented in Section 5.3. In Section 5.4, we present the data-snooping test methodology. In Section 5.5, we report the empirical evaluation of the profitability based on the MMC framework and data snooping tests. Finally, in Section 5.6 we close with some concluding remarks.

5.2 Methodology

In this section, we present our forecasting methodology based on the new MMC method: the MTD-Probit (Nicolau, 2014). We explore a very simple multivariate markovian investment rule and test its capacity to forecast stock behavior, using the buy-and-hold trading strategy as a market benchmark. Our main hypothesis is that the behavior of security prices and the nature of the successive movements of these prices can be forecast based on past information available and multiple correlated categorical data sequences.

5.2.1 Multivariate Markov Chain Model and MTD-Probit Estimation Method

Throughout this study, we apply a first-order MMC methodology to model the stock market behavior. Formally, we consider a multivariate stochastic Markov process $\{S_{1,t},...,S_{s,t};t=1,2,...\}$ where the present state $S_{j,t}$, $j\in\{1,2,...s\}$, can take values in the finite set $\{1,2,...,m\}$. Furthermore, $S_{j,t}$ depends on the previous values and/or some explanatory variables of $S_{1,t-1},...,S_{j,t-1},...,S_{s,t-1}$, which are used to explain $S_{j,t}$. In this context, a natural model to predict $S_{j,t}$ is based on its transition probabilities:

$$P_j\left(i_0|\,i_1,...,i_s\right):=P\left(S_{j,t}=i_0|\,S_{1,t-1}=i_1,...,S_{s,t-1}=i_s\right), \tag{5.1}$$

that can be easily estimated through the maximum likelihood estimates (MLE):

$$\hat{P}_j\left(i_0|\,i_1,...,i_s\right)=\frac{n_{i_1i_2...i_si_0}}{\sum_{i_0=1}^{m}n_{i_1i_2...i_si_0}}, \tag{5.2}$$

where $n_{i_1i_2...i_si_0}$ is the number of transitions of type $S_{1,t-1}=i_1,...,S_{s,t-1}=i_s,S_{j,t}=i_0$.

However, modeling these probabilities when s and m are relatively large and the sample size is small or even moderate is impracticable, as the total number of parameters is $m^s\left(m-1\right)$. In practical terms, this means that the numerator as well as the denominator of (5.2) may be, in most of cases, zero or very close to zero. As a consequence, the parameters can be neither efficiently estimated nor identified with a finite sample size.

To overcome this problem, Ching et al. (2002) considered a simplifying hypothesis, which is, in fact, an extension of Raftery (1985), for modeling high-order Markov chains. It involves assuming that the probability (5.1) can be written as a linear combination of $\{P_{j1}(i_0|\, i_1), ..., P_{js}(i_0|\, i_s)\}$, where $P_{jk}(i_0|\, i_k) := P(S_{j,t} = i_0|\, S_{k,t-1} = i_k)$, that is:

$$P(S_{j,t} = i_0|\, S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s) = P_j^{MTD}(i_0|\, i_1, ..., i_s) :=$$

$$\lambda_{j1} P_{j1}(i_0|\, i_1) + ... + \lambda_{js} P_{js}(i_0|\, i_s), \tag{5.3}$$

where $\sum_{i=1}^{s} \lambda_{ji} = 1$ and

$$0 \leq \sum_{k=1}^{s} \lambda_{jk} P_{jk}(i_0|\, i_k) \leq 1. \tag{5.4}$$

The expression on Eq.(5.5) is called the mixture transition distribution model (MTD) (Raftery, 1985). In this model, we can impose the condition that the $0 \leq \lambda_{ji} \leq 1$, and can satisfy the inequality (5.4). In this case, the λ-parameters may be interpreted as probabilities, and the estimation procedure is easier to implement. Indeed, the number of parameters to be estimated is substantially reduced to $m(m-1) + (s-1)$ and each additional explanatory variable adds only one additional parameter.

Nonetheless, although the MTD model tries to overcome the difficulties for estimated MMC with parsimony and is easier to implement, one of the main challenges in applying this model is linked to the estimation process, the way the nonlinear constraints are deal with the numerical optimization and the range of dependence patterns that the model can capture, especially negative partial effects (e.g. Berchtold, 2001, Lèbre and Bourguignon, 2008, Chen and Lio, 2009, and Nicolau, 2014).

However, recently a new MTD estimation process called MTD-Probit (Nicolau, 2014) was proposed. The MTD-Probit model is based on a specification which is completely free from constraints, facilitating the estimation process. Additionally, it has a more accurate specification for $P_j(i_0|\, i_1, ..., i_s)$ which does not alter the consistency of the MLE. More specifically, the MTD-Probit model suggests modeling MMC, as follows:

$$P_j(i_0|\, i_1, ..., i_s) = P_j^{\Phi}(i_0|\, i_1, ..., i_s) := \frac{\Phi(\eta_{j0} + \eta_{j1} P_{j1}(i_0|\, i_1) + ... + \eta_{js} P_{js}(i_0|\, i_s))}{\sum_{i_0=1}^{m} \Phi(\eta_{j0} + \eta_{j1} P_{j1}(i_0|\, i_1) + ... + \eta_{js} P_{js}(i_0|\, i_s))}, \tag{5.5}$$

where $\eta_{ji} \in \mathrm{R}(j = 1, ..., s; i = 1, ..., s)$ are parameters to be estimated, and Φ is the (cumulative) standard normal distribution function. In this scenario, when $S_{j,t}$ is the dependent variable the likelihood is:

$$logL = \sum_{i_1 i_2, ... i_s i_0} \eta_{i_1 i_2 ... i_s i_0} log(P_j^{\Phi}(i_0|\, i_1, ..., i_s)), \tag{5.6}$$

and the MLE can be expressed[1] as:

$$\hat{\eta}_j = argmax_{\eta_{j_1},\eta_{j_2},\ldots\eta_{j_s}} logL \tag{5.7}$$

In addition, the parameters $P_{jk}\left(i_0|\, i_k\right)$, $k = 1, \ldots .s$ can be consistently estimated in advance through $\hat{P}_{jk}\left(i_0|\, i_k\right) = \frac{n_{i_k i_0}}{\sum_{i_0=1}^{m} n_{i_k i_0}}$ where $n_{i_k i_0}$is the number of transitions of type $S_{k,t-1} = i_k$ to $S_{j,t} = i_0$.

5.3 Markovian Financial Strategy

In this section, we present a trading rule methodology used to forecast the FTSE 100 Index. We assume the existence of some sort of serial dependency on prices, which can be seen as a generalization of McQueen and Thorley´s (1991) approach for analyzing the predictability of stock returns.

5.3.1 Modeling Framework

The integration and globalization of financial markets in the last few decades has increased the interdependence among world stock markets and increased the possibility of mean and volatility spillovers[2]. Indeed, as per recent studies[3], the liberalization of capital movements and advanced computer technology has boosted the co-movements of stock prices among markets (see, e.g. Hamao et al., 1990, Kanas, 1998, Forbes and Ricobon, 2002, Baele, 2005, Christiansen, 2007, Chan et al. 2008, Abou-zaid et al. 2011, Natarajan et al., 2014, and Akca and Ozturk, 2016).

Therefore, it is of fundamental importance in any asset-pricing model to incorporate the impact of the correlation between stock mean and volatility among financial markets. Indeed, the spillover effects have strong implications for investors´ optimal asset allocation, especially in the more capitalized financial markets (e.g. Natarajan et al., 2014).

In this study, we investigate if the MTD-Probit model can predict the FTSE 100 Index behavior. The FTSE 100 Index, which represents 70 percent of the equity capitalization of all United Kingdom equities, is the most important index in Europe. Indeed, in December 2015, the London Stock Exchange (LSE) was rated as the most capitalized stock exchange

[1] As suggested by Nicolau (2014), we have used the constrained maximum likelihood module in GAUSS software (Aptech Systems, Chandler, Arizona, United States) that allows switching between several algorithms (BFGS, Broyden-Fletcher-Goldfarb-Shanno, DFP, Davidon-Fletcher-Powell, Newton, BHHH, Berndt-Hall-Hall-Hausman, scaled BFGS and scaled DFP) depending on either of three methods of progress: change in function value, number of iterations or change in line search step length.

[2] The co-movements in return and volatility among markets have been commonly termed as mean and volatility spillover, respectively.

[3] For an in-depth review of the literature in the area see Singh et al, 2015.

in Europe, followed by Frankfurt and Paris and the third largest in the world [4].

We propose evaluating a market strategy using a very simple approach. We assume that the investor buys or sells the FTSE 100 Index according to trading signal based on the MTD-Probit estimation model, and liquidates the position only if it has a trend reversal signal, for example, from a buy signal to neutral or sell signal.

Additionally, as a result of the possibility of information transmission among markets, we include the mean and volatility spillover impact from the regional players (Europe aggregated) and global markets as covariates in the estimation process of the FTSE 100 Index. Then, we use the log return and log return volatility of the Frankfurt and Paris financial markets, represented by the DAX and the CAC 40 indices respectively, to study the regional interdependence of the financial market; and we use the American market based on the S&P 500 and the NASDAQ indices to proxy global market spillover effects.

5.3.2 The MTD-Probit Forecast Strategy

The MTD-Probit forecast procedure can be summarized in the following algorithm:

Step 1: We categorize our data sample. Firstly, we map our indices log return to a first-order three state Markov chain based on an interval around their observed sample median [5]. We consider the following example. Let $r_{1,t}$ be the log return associated with the FTSE 100 and $q^{(1)}_{0.5}$ be the median of the marginal distribution of $r_{1,t}$, i.e. $q^{(1)}_{0.5}$ is such that $P(r_{1,t} \leq q^{(1)}_{0.5}) = 0.5$, and $\hat{q}^{(1)}_{0.5}$ the corresponding sample median. Then, we can map the FTSE 100 as follows:

$S_{1,t} = 1$ if $r_{1,t} \leq \hat{q}^{(1)}_{0.5} - k_1$, corresponding to the bear market;

$S_{1,t} = 2$ if $\hat{q}^{(1)}_{0.5} - k_1 < r_{1,t} < q^{(1)}_{0.5} + k_1$, for a neutral market; and

$S_{1,t} = 3$ if $r_{1,t} \geq \hat{q}^{(1)}_{0.5} + k_1$, corresponding to the bull market.

As such, the FTSE 100 continuous state space is mapped into state space $\{1, 2, 3\}$, which allows us to incorporate the direction of change in the Index returns into the analysis, and its magnitude as a function of parameter k_1(see, e.g. Niederhoffer and Osborne, 1966, and Fielitz and Bhargava, 1973).

Secondly, we map our indices return to short-term volatilities. In this case, we first determine the returns volatilities over a specific time-horizon of k_2 days and compute the median of their marginal distribution. Then, we consider an interval map using the median as a neutral benchmark. This study adopts the sample variance to proxy the return volatility $\hat{v}_{i,z} = \sum_{z=t}^{t+(k_2-1)} (r_{i,z} - \mu_{i,h})^2 / k_2$, where $\mu_{i,h}$ is the estimated ith index sample mean for the k_2 time-horizon, that is $\mu_{i,h} = \sum_{h=t}^{t+(k_2-1)} (r_{i,h})/k_2$, $t = \{1, 2, ..., T - k_2\}$. For example, in the case of the FTSE 100, we have:

[4] World Federation of Exchanges, 2015.

[5] In Appendix 1, we provide the explanatory variables (covariates), parameter definitions and values.

$S_{2,t} = 1$ if $\hat{v}_{1,t} \leq \hat{q}_{0.5}^{(\hat{v}_1)} - k_3$, corresponding to a low volatility market;

$S_{2,t} = 2$ if $\hat{q}_{0.5}^{(\hat{v}_1)} - k_3 < \hat{v}_{1,t} < \hat{q}_{0.5}^{(\hat{v}_1)} + k_3$, for a neutral volatility market; and

$S_{2,t} = 3$ if $\hat{v}_{1,t} \geq \hat{q}_{0.5}^{(\hat{v}_1)} + k_3$, corresponding to a high volatility market;

where $\hat{v}_{1,t}$ is the return volatility for FTSE 100, $\hat{q}_{0.5}^{(\hat{v}_1)}$ is its sample median, and k_3 is a threshold parameter.

Step 2: We split our total sample data of T observations into two segments. In the first segment, we estimate the initial transition probability matrix (TPM) of a prospective trading model, and apply this model in the second segment. The size of the first segment is determined by the number of days that our trading strategy is applied. In this paper, we apply our investment strategy using the one-day ahead decision period for a time horizon of 100 days. In this scenario, based on the highest transition probability in the TPM for the FTSE 100, the trading signal is generated through observation in t , given its previous state, and the combination of states of four explanatory covariates also observed in $t-1$, that we call trading rules, based on Eq.(5.5).

Step 3: We use the t observations to re-estimate the next MTD-Probit trading signal, and repeat the process sequentially, until we reach our trading signals for 100 days of predictions.

Step 4: We record all the returns generated by our combination of covariates or trading rules, and measure total net returns. Mathematically, the returns are determined based on the signal function for the mth MTD-Probit trading rule, $m = 1, ..., M$, given by:

$$R^*_{m,t} = R_{m,t} - R^0_t, \tag{5.8}$$

$$R_{m,t} = I_{m,t} R_t - abs(I_{m,t} - I_{m,t-1}) Tc, \tag{5.9}$$

$$R_t = ln(p_t / p_{t-1}), \tag{5.10}$$

where $R^*_{m,t}$ is the one-day excess return of the mth trading strategy discounting the market benchmark strategy R^0_t, which in our case is the buy-and-hold trading strategy, after accounting for the one-way transaction cost Tc. Furthermore, p_t is the daily closing quote index at time t and $I_{m,t}$ is a variable indicator for the mth MTD-Probit rule, which takes the values 1,0 or -1, if we take a long position, no action or short position, respectively. In this study, selling short (-1) is used here as the reverse of a buy order. Although it is not possible to sell short owing to legal or market restrictions, we follow the approach that it is essential to accurately calculate a total trading rule profitability. Additionally, if our investment rule indicates a non-changing market (no action) we account for no return [6].

[6] We could equally account equal for the overnight cash rate, calculated on the basis, for example, of the "3-month Treasury Bill Yield".

Step 5: For each model set-up, we calculate the percentage success rate (PSR), based on the predictive accuracy of the trading signals generated in the previous steps, as follows:

$$PSR_m = V_m/n, \tag{5.11}$$

where V_m is the number of times that our mth MDT-Probit model estimation matches the real market movement in our $n = 100$ days forecasting horizon.

Step 6: We evaluate the performance of our forecast methodology, using the White (2000) "Bootstrap Reality Check", and the Hansen (2005) data-snooping test. More details of the bootstrap method and tests applied in this study are presented in section 4.

5.3.3 Transaction Costs

In this study, we do not consider transaction costs directly, but make a simple assumption that $Tc = 0$. There is no doubt that an investment rule is profitable only when its profit is greater than any trading costs. However, the recent introduction of a new computational trading floor process and online trading systems has lowered the overall "transactional costs" (see e.g. Bessembinder and Chan, 1995, Mitra , 2010, Bajgrowicz and Scaillet, 2012, and Kuang et al., 2014). Therefore, it is very difficult to choose any previous or recommended one-way transaction cost level.

To minimize the effects of this "somewhat unrealistic assumption" (Bajgrowicz and Scaillet, 2012), we present a break-even transaction costs analysis based on the methodologies of Hsu et al. (2010) and Mitra (2010). Then, we calculate the "potential margins for profitability" (PMP), which is the level of Tc which could offset any foreseen profitability. As proposed the PMP is the break-even transaction cost, which measures the capacity of the trading rule to absorb any transaction costs. It is estimated as follows:

$$PMP = \frac{R_{Tm}}{N_m}, \tag{5.12}$$

where R_{Tm} and N_m are respectively, the total return and the number changing signals generated along the investment period horizon for the mth trading rule. In our investment methodology, the transaction cost should be considered initially when a buy/sell signal generates an investment position, and secondly, when a new signal is generated; requiring a change in the previous investment decision as follows:

$$N_m = \sum_{t=1}^{n} abs(I_{m,t} - I_{m,t-1}). \tag{5.13}$$

5.4 Data-Snooping Bias

Many authors have raised concerns about reusing the same data set to test model forecasting accuracy, as this could generate a data-snooping bias (Lo, 1990; Brock, 1992, Hsu and Kuan, 1999, White, 2000, Hansen, 2005, Hsu and Kuan, 2005, Romano et al., 2005, Park and Irwin, 2010, Day and Lee, 2011, Neuhierl and Schlusche, 2011, Chen et al., 2011, and Yu, 2013). Indeed, the possibility of spurious results is a reasonable assumption since superior profitability could be due to chance rather than to high-performance strategies.

The literature outlines two different approaches to overcome such biases. The first approach is to validate the forecasting results based on an available comparable data set or in the out-of-sample testing (see, e.g. Lo and MacKinlay, 1990). However, such a procedure is not only dependent on the existence of a comparable data set, but it is also highly sensitive to the arbitrary sample splitting choice.

A second approach is to test forecasting performance comparing the weighted distance between two alternative competing strategies. If this pairwise comparisons shows any statistically significant divergence, then we cannot consistently reject the null hypothesis that there is no profitable trading rule.

Nevertheless, the use of this methodology has an important pitfall, since using the same data set for a large number of competing strategies can generate a sequential testing bias. In this case, a null hypothesis is a composite hypothesis of several individual hypotheses, therefore if we are testing each of the models separately (at some level α), then the overall test size increases whenever we test a new hypothesis..

To overcome the sequential test problem, some studies propose new tests to provide a solution to the data-snooping problem. The methodology is based on the "best rule" (Sullivan et al., 1999, White, 2000, Hansen, 2005, and Shynkevich, 2012), verifying whether there is a superior rule within a "universe" of rules that could outperform some benchmark models, for example the buy-and-hold trading strategy or mean zero criterion.

5.4.1 The RC and SPA Tests

In this study, we use the White RC and Hansen SPA tests to provide accurate analysis of the profitability for our MTD-Probit trading rules taking into account data-snooping effects.

On the one hand, White (2000) proposes testing the predictive superiority of a trading rule (model) based on the performance measure relative to the benchmark trading strategy.

Formally, following the literature (see e.g. Lai and Xing, 2008, Hsu et al., 2010, and Metghalchi et al., 2012), let f_m $(m = 1, ..., M)$ denote the excess return of the mth trading rule to the benchmark model or performance measure (White, 2000) and $\varphi_m = \mathrm{E}(f_m)$. The null hypothesis is that there is no superior trading rule in the universe of the M trading rules:

$$H_0 : \underset{1 \leq \mathrm{m} \leq M}{max} \varphi_m \leq 0. \tag{5.14}$$

The rejection of (5.14) implies that at least one of the models has superior performance over the benchmark and is evidence against the EMH. In this context, White (2000) proposes a statistic to test this null hypothesis based on the maximum of the normalized sample average:

$$\overline{V}_n = \underset{1 \leq \mathrm{m} \leq M}{max} \sqrt{n} \bar{f_m}, \tag{5.15}$$

where $\bar{f_m} = \sum_{i=1}^{n} f_{m,i}/n$ with $f_{m,i}$ being the ith observation of f_m and $f_{m,1}, ..., f_{m,n}$ are the computed returns in a sample of n past prices for the mth trading rule. Additionally, the author approximates the sampling distribution of $\overline{V}_n$ [7] by:

$$\overline{V}^*_n = \underset{1 \leq \mathrm{m} \leq M}{max} \sqrt{n}(\bar{f_m} - \varphi_m). \tag{5.16}$$

In this scenario, White (2000) suggests using the Politis and Romano (1994) stationary bootstrap method (SB)[8] to compute the p-values of (5.16), based on the empirical distribution of $\overline{V}_n$, which is obtained with realizations of B bootstrapped samples, $b = 1, ..., B$, of the following statistic:

$$\overline{V}^*_n(b) = \underset{1 \leq m \leq M}{max} \sqrt{n}(\bar{f_m}^*(b) - \bar{f_m}), \tag{5.17}$$

where $\bar{f_m}^*(b) = \sum_{i=1}^{n} f^*_{m,i}(b)/n$ denotes the sample average of the bth bootstrapped sample $\{f^*_{m,1}(b), ..., f^*_{m,n}(b)\}$. White´s reality check test p-value is then obtained comparing $\overline{V}_n$ with the quantiles of the empirical distribution of $\overline{V}^*_n(b)$, computing:

$$\hat{p}_{RC} = \sum_{b=1}^{B} \frac{I_{RC}}{B}, \tag{5.18}$$

where I_{RC} is an indicator function that takes value one if $\overline{V}^*_n(b)$ is higher than $\overline{V}_n$. The null hypothesis is rejected whenever $\hat{p}_{RC} < \alpha$, where α is a given significance level.

On the other hand, Hansen (2005) points out that the RC test has two major limitations as the null distribution is obtained under the "least favorable configuration" [9] and the statistic is not studentized. As result, the author proposes two improvements to produce a more powerful and least conservative test. First, Hansen (2005) proposed the studentization of the White´s RC test statistic on Eq. (5.15):

[7] White (2000) shows in the corollary 2.4 that under a suitable regularity condition, the distribution of $\overline{V}_n$ and $\overline{V}^*_n$ are asymptotically equivalent.

[8] In Appendix 3 we provide an explanation of the SB method. For a more detailed explanation see, e.g. Romano and Wolf (2005).

[9] White (2000) obtains the null distribution based on irrelevant models, i.e. $\varphi_1 = \varphi_2 = ... = \varphi_M = 0$, artificially enhancing the p-values of the RC test (see, e.g. Hsu et al., 2010).

$$\tilde{V_n} = max[\max_{1 \le m \le M} \frac{\sqrt{n}\bar{f_m}}{\hat{\sigma}_m}, 0], \tag{5.19}$$

where $\hat{\sigma}_m^2$ is a consistent estimate of $\sigma_m^2 = var(\sqrt{n}\bar{f_m})$. In this paper, we estimate $\hat{\sigma}_m$ based on the stationary bootstrapped resamples of $\sqrt{n}\bar{f_m}$ (see, e.g. Hansen, 2005 and Hsu and Kuan, 2005).

Secondly, the author suggests that under the null, when there are some $\varphi_m < 0$ and at least one $\varphi_m = 0$, the limiting distribution of (5.16) depends only on the trading rules with zero or higher mean returns. As a result, Hansen´s "superior predictive ability" data-snooping test discards the irrelevant or poor performance models, re-centering the null distribution based on a preset threshold rate given by $-\sqrt{2loglogn}$[10]:

$$\tilde{V_n}^*(b) = max[\max_{1 \le m \le M} \frac{\sqrt{n}\bar{Z_m}^*(b)}{\hat{\sigma}_m}, 0], \tag{5.20}$$

$$\bar{Z_m}^*(b) = \sum_{i=1}^{n} \frac{Z_{m,i}^*(b)}{n}, \tag{5.21}$$

$$Z_{m,i}^*(b) = f_{m,i}^*(b) - \bar{f_m}.\mathrm{I}_{\{\sqrt{n}\frac{\bar{f_k}}{\hat{\sigma_k}} \le -\sqrt{2loglogn}\}}, \tag{5.22}$$

where $\bar{Z_m}^*(b)$[11] is the sample average of the bootstrapped re-centered performance measure $Z_{m,i}^*(b)$, and $\mathrm{I}_{\{.\}}$ is an indicator function taking on the value of one if the condition is satisfied and zero otherwise. In this scenario, the consistent p-values of $\tilde{V_n}$ are determined by the empirical distribution of $\tilde{V_n}^*(b)$, $b = 1, .., B$, and is computed by:

$$\hat{p}_{SPA} = \sum_{b=1}^{B} \frac{I_{SPA}}{B}, \tag{5.23}$$

where I_{SPA} is an indicator function that takes value one if $\tilde{V_n}^*(b)$ is higher than $\tilde{V_n}$. In a similar fashion as in the RC test, the null hypothesis is rejected whenever $\hat{p}_{SPA} < \alpha$.

Hansen (2005) also proposes two additional estimators in order to provide a lower and upper boundary for the consistent p-value of the conventional former test. On the one hand, the lower boundary is based on a stricter configuration that eliminates any negative performance model and is given by:

$$Z_{m,i}^{l*}(b) = f_{m,i}^*(b) - max(\bar{f_m}, 0). \tag{5.24}$$

[10]Hansen´s threshold is motivated by the law of the iterated logarithm. Nonetheless, as pointed out by Hansen (2005), other threshold values can also produce valid results with different p-values in finite samples, for example, Hsu and Kuan (2005) used $n^{\frac{1}{4}}/4$. The log is the natural logarithm.

[11]In this paper, we use the same $\hat{\sigma}_m$ in $\tilde{V_n}$ and $\tilde{V_n}^*(b)$ (see e.g. Shynkevich, 2012).

On the other hand, the upper bound considers the inclusion of the poor and least favorable alternatives as suggested in the RC test:

$$Z^{u*}_{m,i}(b) = f^{*}_{m,i}(b) - \bar{f}_{m}, \tag{5.25}$$

where $Z^{l*}_{m,i}(b) \leq Z^{c*}_{m,i}(b) \leq Z^{u*}_{m,i}(b)$. In the literature, the SPA test given by (5.22) is called the SPA_c and the lower and the upper bounds are referred to as the SPA_l and SPA_u, respectively.

5.5 Empirical Examination

In this section, we provide the empirical evaluation of the MTD-Probit forecast model performance and analyze our data-snooping bias controlled results.

5.5.1 Main Sample Statistics Results

Table 5.1 presents descriptive statistics of the FTSE 100 daily log returns considered in the paper, for a total of n=1037 trading days. The data consists of the adjusted daily closing prices obtained from the Datastream database. The sample used comprises approximately four years of log returns data from the period of January 06, 2009 to December 27, 2012.

From this table, it can be inferred that the mean daily return for the FTSE 100 is 0.026%, which equates to 250 trading days per year, with an approximate average of 6.33% per year. The mean daily return volatility is 1.20% (standard deviation). Additionally, the table also shows that the Index is skewed to the left, which indicates that extreme negative returns are more probable than extreme positive ones. The sample excess kurtosis level reveals that the FTSE 100 return series has fatter tails than the normal distribution, i.e. the low positive and negative returns are more probable. Indeed, the Jarque-Bera portmanteau test (JB) supports the non-normal nature of the sample distribution, as it strongly rejects the null hypothesis of normality at the one percent level.

Regarding the linear time dependence properties, there is non-significant evidence of first-order autocorrelation across the sample, at 1% level or better. Finally, based on the Ljung-Box Q statistics, there is also non-significant autocorrelation, of up to six lags, for some of the Index returns. The null hypothesis of no autocorrelation for all six lags tested is not rejected, at 1% level or better. Although, there is no evidence for autocorrelation in the FTSE 100 Index, this does not mean that the sample returns are independent over time. Indeed, there is the possibility of nonlinear time dependence in the observed data sample. Hence, in this context, the use of the Markov chain test methodology can be an important procedure for forecasting future price behavior.

Table 5.1: FTSE 100 Index Return Descriptive Statistics

Country	UK
No (Obs.)	1037
Mean (%)	0.0253
Max. (%)	5.0323
Min. (%)	-5.4816
S.D. (%)	1.1998
Skewness	-0.1626
Kurtosis	5.0763
ρ(1)	0.008
ρ(2)	-0.013
ρ(3)	-0.071
ρ(4)	0.012
ρ(5)	0.001
ρ(6)	0.009
Q(6)	5.6722
JB	190.85*

Notes: (1)The mean sample log-return (Mean (%)) and the standard deviation.(S.D. (%)) are reported in percentage . (2) JB are the Jarque-Bera test statistics, ρ(n) is the estimated autocorrelation at lag n, and Q(n) are the Ljung-Box-Pierce test statistics for the nth lag. The Ljung-Box Q -statistics p -values are reported with the estimated autocorrelation.

*Statistical Significance at the 1% level.

5.5.2 Results of the Markovian Stock indices Predictions

The results of the MTD-Probit models are presented in Table 5.2 for a total of 144 trading strategies. In this study, we use the closing market prices to calculate the Index log returns (see, e.g. Hsu and Kuan, 2005).

5.5.2.1 Best Performing Trading Rules

In Table 5.2, the first column highlights the top ten most profitable MTD-Probit strategies. The second column reports the strategies mean log return for the sample, where the mean buy-and-hold benchmark return is 0.01742%. This column reports the returns based on five explanatory variables: the return and volatility of the FTSE 100 Index, and a combination of the CAC 40 (France), DAX (Germany), S&P 500 (US) and the NASDAQ (US) indices. Columns 3 and 4 detail the return from the buy and sell trading signals, respectively. In column 5, we also report the $Buy - Sell(\%)$, which is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns[12](see, e.g. Metghalchi et al., 2012).

[12]The t-statistics for the buy-sell mean returns difference are computed according to Brock et al.(1992).

In columns 6 to 8, we report the number of times that our MTD-Probit trading rule estimation matches the real market movement in our investment time horizon of one-hundred days (PSR). The *All*(%), *Buy*(%) and *Sell*(%) are respectively the percentage of the overall, buy and sell correct signals reported in the sample. Additionally, the number of trades for our sample are reported in columns 9 and 10, where $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. In our study, the buy and sell returns were computed without considering the possibility of an additionally risk-free overnight return when the MTD-Probit trading rule indicated the no position (out of the market). Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance.

Inferring from Table 5.2, the best MTD-Probit rules for the FTSE 100 are all statistically significant and the mean log return range is a small interval from 42.21 to 41.48 basis points (bps). Additionally, for the forecast period, the overall buy and sell mean daily returns are significantly different from zero. However, there are non-significant differences between the mean buy and mean sell daily return.

We also observed that the best trading rules consistently show the unique interdependence between the largest European stock markets, namely London and Frankfurt, and some evidence of a spillover effect among the major international financial markets, as evidenced by the S&P 500 and NASDAQ indices.

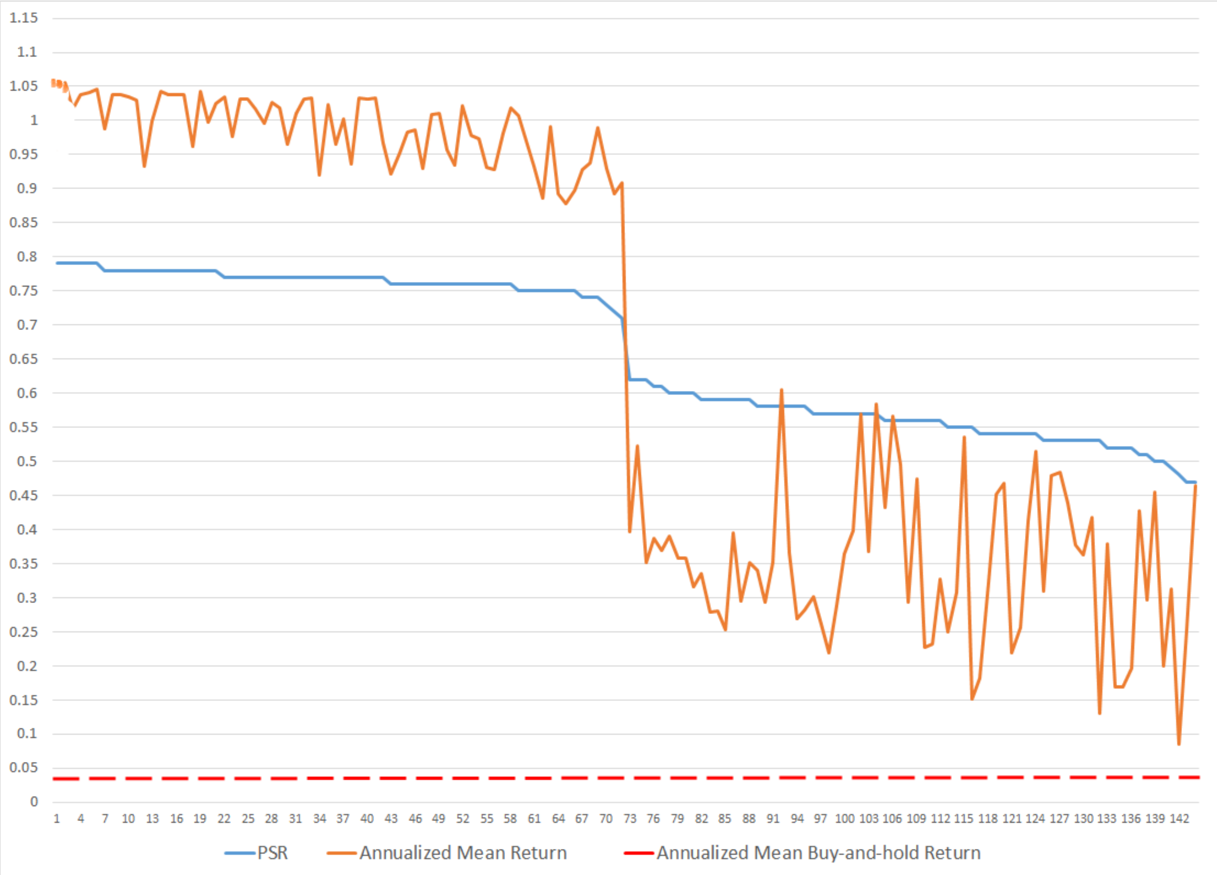
In Figure 1, we illustrate for the 144 MTD-Probit trading strategies the distribution of the MTD-Probit PSR and the annualized mean log return, based on 250 trading days. As a result, our MTD-Probit trading rule matches the real market movement, in our investment time horizon of one-hundred days, in an interval ranging from 79% to 47% of the time.

Table 5.2: The Best 100 Days MTD-Probit Trading Strategies for the FTSE 100

Benchmark=0,01742%	Mean Daily Log Return				PSR			Number of Trades		PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	Buy-Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	(%)
$1, 2, 3, 5, 7 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.4221*	0.4774*	0.4308*	0.0466	79.00	84.78	85.10	49	51	0.767
	(7.76)	(5.78)	(5.82)	(0.30)						
$1, 2, 3, 5, 9 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 5\%$	0.4186*	0.4948*	0.4186*	0.0763	79.00	84.44	87.23	50	50	0.761
	(7.66)	(5.91)	(5.66)	(0.48)						
$1, 2, 3, 6, 8 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 10\%$	0.4174*	0.4824*	0.4261*	0.0563	78.00	84.44	85.11	49	51	0.732
	(7.62)	(5.72)	(5.78)	(0.36)						
$1, 2, 3, 6, 10 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 5\%$	0.4174*	0.4824*	0.4261*	0.0563	78.00	84.44	85.11	49	51	0.732
	(7.62)	(5.72)	(5.78)	(0.36)						
$1, 2, 3, 5, 9 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 5\%$	0.4166*	0.4714*	0.4250*	0.0464	79.00	84.78	85.11	50	50	0.683
	(7.60)	(5.65)	(5.71)	(0.29)						
$1, 2, 3, 5, 9 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.4151*	0.4503*	0.4423*	0.0080	79.00	86.96	82.98	47	53	0.783
	(7.56)	(5.26)	(6.06)	(0.05)						
$1, 2, 3, 6, 7 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 10\%$	0.4148*	0.4592*	0.4422*	0.0170	78.00	86.67	82.98	47	53	0.728
	(7.55)	(5.34)	(6.06)	(0.11)						
$1, 2, 3, 5, 10 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 5\%$	0.4148*	0.4592*	0.4422*	0.0170	78.00	86.67	82.98	47	53	0.728
	(7.55)	(5.34)	(6.06)	(0.11)						
$1, 2, 3, 6, 8 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 5\%$	0.4148*	0.4592*	0.4422*	0.0170	78.00	86.67	82.98	47	53	0.728
	(7.55)	(5.34)	(6.06)	(0.11)						
$1, 2, 3, 6, 10 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 1\%$	0.4148*	0.4592*	0.4422*	0.0170	78.00	86.67	82.98	48	52	0.703
	(7.55)	(5.40)	(6.00)	(0.11)						

Notes: The first column highlights the best 10 performing MTD-Probit strategies, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The $Buy - Sell$(%) is the difference of the mean buy and the mean sell returns. The numbers in parentheses are the standard t-ratios testing the returns significance and the difference of the mean buy and the mean sell returns. In columns 6 to 8, we report the PSR which is the number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, *No.Buy* and *No.Sell* are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" (PMP%) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *Statistical Significance at the 1% level for a two-tailed test.

Figure 5.1: The MTD-Probit PSR and Annualized Mean Return



5.5.3 Robustness Check

The standard statistical t-tests presented in this section have a major weakness since they are formulated based on the stationary, time independent and normally distributed mean returns hypothesis. Nevertheless, asset returns distributions are known to be non-normal, auto-correlated and they have time-varying moments. Furthermore, as pointed out by White (2000), the standard statistical inference based on individual testing understate the possibility of a Type I error when we are choosing the best trading rule. Indeed, in this case the mean return statistical distribution will be affected and the test will be biased towards the rejection of the null hypothesis because of data-snooping (see, e.g. Hsu et al., 2016). In this context, any significant superior performance may be the spurious result of test bias.

In this section, we take this issue into account and provide some robustness results to examine if the presented MTD-Probit trading rules achieve economic and statistical performance.

5.5.3.1 Transaction Costs

It is well known that one of the most common problems in correctly defining the economic performance of any trading rules is related to the size of the transaction costs involved. Indeed, the transaction costs charged to an investor are unknown since these costs depends on many different aspects, such as the type of investor, investment size, and the technological level of the trading floor systems. Indeed, as presented by Shynkevich (2012), the investor may be trading from a relatively low cost of 5 bps (Hsu et al., 2010), for a single trip transaction, to a less conservative assumption of 20 basis points (Shynkevich, 2012).

In our case, the results show that the number of trades and the break-even cost across the sample can substantially change as a function of the covariates that are used in the forecasting process. We observed that the best $PMP\%$ rule is from 78.3 bps to 68.3 bps. These are the boundaries where the evidence of abnormal profitability of the TAI and the EMH rejection should be analyzed. Hence, regarding transaction costs, there is some evidence of abnormal profitability in the use of the MTD-Probit methodology.

5.5.3.2 Results of the Data-snooping Tests

Table 5.3 summarizes the daily and annualized mean return, based on 250 trading days. It equally gives a summary on White and Hansen´s p-values of the best rules in our sample. The RC and SPA test results are presented based on the stationary bootstrapped with $B = 500$ interactions and the geometric distribution parameter set as $q = 0.1$ (see, e.g. Politis and Romano, 1994 and Hansen, 2005).

Table 5.3: The Best Investment Strategy Data-Snooping Results for the FTSE 100 Index

The Best Trading Rule	$B\&H$ %	$An.Ret$ %	$An.Perf$ %	$M.Ret$ %	P_{RC}	SPA_l	SPA_c	SPA_u
$1,2,3,5,7$ $k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	4.35	105.51	101.16	0.4221	0.8016	0.8120	0.8120	0.8120

The $BestTradingRule$ represents the best performing strategies; $B\&H\%$ is the annualized benchmark mean return for the forecast period, based on 250 trading days; $An.Ret\%$ gives the annualized mean return performance without discounting the benchmark return for the period; $An.Perf\%$ is the performance on an annual basis, considering the benchmark return; $M.Ret\%$ is the best rule´s mean log return; finally, PMP is the break-even one day transaction cost. Additionally, P_{RC} is the RC p-value test result. The SPA_l, SPA_c and SPA_u are the lower, consistent and upper SPA p-values, respectively.

Table 5.3 provides a summary of results for the best explanatory variables scenario for the FTSE 100 Index, with the columns described as follows: $BestTradingRule$ represents the best performing strategies; $B\&H\%$ is the annualized benchmark mean return for the forecast period, based on 250 trading days; $An.Ret\%$ gives the annualized mean return performance without discounting the benchmark return for the period; $An.Perf\%$ is the performance on an annual basis, considering the benchmark return; $M.Ret\%$ is the best rule´s mean log return; finally, PMP is the break-even one day transaction cost. Additionally, P_{RC} is the RC p-value test result. The SPA_l, SPA_c and SPA_u are the lower, consistent and upper SPA p-values, respectively.

We observe that the best trading rule results from the interdependence between one of the largest European stock markets and its volatility, namely Frankfurt (DAX Index), and the influence of the spillover of one of the biggest traditional and international financial indexes, the S&P 500 Index.

Additionally, there is some evidence that the MTD-Probit trading rule is capable of consistently and significantly producing superior performance over the buy-and-hold benchmark for the FTSE 100. Indeed, the best MTD-Probit trading strategy has an annualized excess return of 101.16 %, which is more than 24 times its benchmark.

Nevertheless, the data-snooping tests suggest that the best performing rule is not significant for any conventional test level. Indeed, the best RC and SPA p-values are 0.8016 and 0.8120, respectively. Thus, our superior profitability could be due to chance rather than to the existence of high-performing strategies. In this case, the possibility of spurious results is a reasonable assumption.

Under such circumstances, we conclude that there is non-significant evidence of abnormal profitability of the MTD-Probit model, and therefore we cannot reject the weak-form of the efficient market hypothesis (Fama, 1965 and 1970).

5.6 Conclusion

Beyond any doubt, one of the most intricate tasks in finance is forecasting stock market behavior. The main difficulty is choosing a correct model that can be applied in the “real” world.

Usually, the choice of a model might be suggested by the underlying theory. However, often the model is selected based on the concept that it should be capable of reproducing the key characteristics of the time series data. If the researcher believes that financial series display non-random behavior, then forecasts of future price movements should reflect these intrinsic features.

However, given the many available possibilities, the theoretical properties, the different levels of theoretical requirements and the empirical tools to be used, it is no surprise that the use of linear models, as a first approximation can be a reasonable alternative. However, if the financial data series could be a nonlinear class of data that depends on some explanatory variables, then we should initially apply a model that can deal with this type of structure.

This paper makes two main contributions to the literature. With regard to the methodology, we propose a new financial market forecasting approach based on the multivariate Markov chain framework. Moreover, our MTD-Probit model provides a forecasting model that is not only robust, but it is also easy to apply.

Regarding its application, we employ the proposed procedure to obtain new evidence in favor of the EMH hypothesis after data-snooping is controlled. Our empirical results suggest that the MTD-Probit model applied to the FTSE 100 Index cannot significantly out-perform the buy-and-hold benchmark strategy. These results are quite important.

Appendix 1

List of Independent Variables and Parameters

In this appendix we present the list of categorized explanatory variables and parameters used in the MTD-Probit noise estimation model:

Table 5.4: Explanatory Variables and Parameters MTD-Probit Model

Variable Definition	Parameter Number
FTSE Log Return	1
FTSE Log Return Volatility	2
DAX Log Return	3
CAC 40 Log Return	4
S&P 500 Log Return	5
NASDAQ Log Return	6
DAX Log Return Volatility	7
CAC 40 Log Return Volatility	8
S&P 500 Log Return Volatility	9
NASDAQ Log Return Volatility	10
k_1 = interval around the median for the log return	$\{\pm 1\%\}$
k_2 = number of days in the calculation of indices log return volatility	$\{10, 20, 30\}$
k_3 = interval around the log return volatility	$\{\pm 10\%\}$

Appendix 2

Stationary Block Bootstrap Method

The basic idea of the stationary bootstrap method is to construct random data blocks that are independent, yet preserve the time dependence inside each block. The unknown population distribution structure is approximated by block sampling distributions based on a statistical model. As such, the stationary bootstrap methodology provides a re-sampling method which is applicable for weakly-dependent time series, where the pseudo-time series are stationary time series.

The method is based on two basic steps that provide proper consistency and weak convergence properties. Firstly, the original series is re-sampled into a set of b random length overlapping blocks of observations, determined by the realization of a geometric distribution with parameter $q \in (0, 1)$. In this case, the average block size is the inverse of q.

Secondly, the stationary bootstrap method "wraps" the data around in a "circle" to avoid the block end effects (Politis and Romano, 1994, p.1304). The idea is to choose a large enough block length, preferably based on the sample size, so that observations greater than $1/q$ time units apart will be nearly independent.

However, the major difficulty of this method lies in choosing the size of q. Indeed, the size of the block is a controversial topic in the literature (e.g. Sullivan et al., 1999; Hsu and Kuan, 2005; Metghalchi et al., 2012, and Hsu et al., 2010), as a small size will not reproduce the data dependence, and a large value will reduce the statistical efficiency. In this study we adopt what is usually presented in the previous research in this area, and set $q = 0.1$.

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Chapter 6

The Technical Analysis and the Markov Chain Methodology

Abstract

Markov Chain models are applied in many different fields. In this paper, we propose a new multivariate markovian noise-signal reduction instrument, and applied it to study the performance of technical analysis (TA) trading rules. We test our methodology in the FTSE 100 Index for a total of 152,071 trading rules. Our empirical results suggest that the use of the Markov chain methodology can be an important step for the study of TA predictive power.

Keywords: multivariate Markov chains, technical analysis, efficient markets hypothesis, noise-signal reduction.

6.1 Introduction

The widespread use of Technical Analysis (TA) as a leading stock market forecasting instrument (Skynkevich, 2012) is still challenging to the concept of market efficiency. Since the study of Fama (1970), the efficient market hypothesis (EMH) has been one of the most fundamental pillars in modern finance theory. According to the weak-form of the efficient market hypothesis, prices should reflect all available information; therefore, it should not be possible to earn excess returns consistently with any investment strategy that attempts to predict asset price movements based on historical data.

Nevertheless, in recent decades, new empirical evidence has suggested that the stock market could not be efficient and thus, it is possible to obtain abnormal stock returns that are not fully explained by common risk measures. In particular, some authors have addressed the possibility that TA could lead to sustainable profitability (Murphy, 1986; Sweeney,1986,1988; Brown and Jennings, 1989; Brock et al., 1992; Blume et al.,1994; Neely et al., 1997; Gencay,1998,1999; Lo et al., 2000; Griffioen, 2003; Park and Irwin, 2007; Hsu et al., 2010; and Neely and Weller, 2011).

However, as known by financial market professionals, one major difficulty with the use of the TA methodology is to correctly forecast stock price movement signals without being misled by false signals. The trading noises are one of the most challenging problems, since late entries and exit points are responsible for lowering any investment return. Therefore, we believe that any methodology that provide a method to control and filter out false trading signals can provide an important step in the study of the predictive power of TA.

The most common type of false signal are caused by the whipping price effect, which produces a series of small losses and over-trading which jeopardize any profitability. In this context, to control whipsaws losses, traders normally use a series of filters that delay market entries and exits until they validate a buy/sell signal. However, filtering out whipsaws not only delay the start of any trading strategy, thus reducing total gain, but also, in most cases, it is inefficient in dealing with some of the false signals, as in the case of the presence of outliers.

In this paper, we study the TA forecast ability of the FTSE 100 Index, in a new framework. We apply the MTD-Probit model (Nicolau, 2014) as a noise control method for the TA strategies and apply it to forecast the Index. Our main objective is to provide evidence that this methodology not only potentially controls and filters out false trading signals, but also, is an important step for the study of TA predictive power.

We chose the markovian methodology since it is a robust class of stochastic processes which is well known in the literature, and it has applications in several fields. In the financial domain, it is usually used to derive a very simple forecast model, since it does not require any extensive sets of assumptions, such as the data distribution (e.g. normality), or the existence of homoscedasticity in the series under analysis. To the best of our knowledge, this is the first time that the Markov chain methodology is being used in conjunction with technical analysis as part of a stock market investment strategy.

The remainder of this study is organized as follows. The theoretical consideration related to technical analysis and efficient market hypothesis is presented in Section 6.2. In Section 6.3, we present the estimation methodology and detailed descriptions of the trading rules modeling framework which will be used in our study. We present in Section 6.4, the empirical results. Finally, Section 6.5 concludes the paper.

6.2 Technical Analysis and Efficient Market Hypothesis

Over the past years, academic researchers have explored the use of TA as a high-performance method, capable of forecasting financial market securities (see among others Sweeney, 1988; and Brock. et al., 1992; Hudson et al., 1996; Skouras, 2001; Marshall et al., 2010 and Sewell, 2011). Amongst the possible explanations for the superior performance of TA, is the possibility of a nonlinear stochastic dynamic in stock returns (Berchold and Raftery, 2002), as well as some sort of short-run time inefficiency (Timmermann and Granger, 2004).

In this perspective, if markets are inefficient because stock prices do not adjust to new information immediately, then past market variables, such as volume, market Index level and volatility, can be helpful and informative to explain future price movements. Indeed, any superior performance forecasting strategy could be explained as the result of an exploitation of those intrinsic characteristics.

Moreover, if it is supposed that some new information about a security is available to some first “privileged” investors, after it is disseminated to the investment community, then an investor with superior analytical skills could perform better than any other “non-privileged” investor (Treynor and Ferguson, 1985). In this scenario, we suggest, contrary to the EMH advocate, that the use of TA to foresee financial markets can be profitable if trading noise is controlled.

The use of the technical analysis method is probably one of the most popular and old investment tools among practitioners[1], which is mainly used as a complement of fundamental analysis. Formally, TA is a price forecasting and market timing methodology, based on the assumptions that markets move in trends, and that these trends persist, suggesting some sort of serial dependency about the behavior of past prices series. In the TA jargon, market action discounts everything. Nowadays, TA is still a major investment analysis tool among investment funds managers and practitioners. As acknowledged by Menkhoff (2010) in a survey of 692 fund managers in five countries, TA is a highly used methodology and “is obviously in wide-spread and relevant use among fund managers” (p.2573).

However, in the financial literature, the study of performance of TA as a forecasting instrument has had very ambiguous and contradictory results. On the one hand, there is a body of research that validates the market efficiency and presents contrary evidence for the use of TA as a method that could generate abnormal returns, based on publicly available market information (Fama, 1966; Bessembinder and Chan, 1995 and 1998; Allen and Karjalainen, 1999; Ready, 2002; Li and Wang, 2007; and Hoffmann and Shefrin, 2014). On the other hand, several other studies have shown that TA could be a high-performance method capable of analyzing any fundamental stochastic structures presented in financial

[1]The TA principles were established as far back as the late 1800´s.

data series (Sweeney, 1986; Neftci, 1991; Brock et al., 1992; Blume et al., 1994; Sullivan et al., 1999; Lebaron, 1999; Lo et al., 2000; Qi and Wu, 2006; Cheung et al., 2011; Mitra, 2011; Metghalchi et al., 2012; and Shynkevich, 2012).

These main results notwithstanding, one of the most important drawbacks in the use of technical analysis is the existence of false signals that can be present, even in the best-behaved stock price series. Thus, we hope to contribute to the EMH debate by introducing a new TA methodology, based on the Markov chain methodology.

6.3 Methodology

In this section, we present our forecasting methodology based on the new multivariate Markov chain (MMC) method: the MTD-Probit (Nicolau, 2014). We explore a very simple multivariate markovian investment rule and test its capacity as noise control method. Our main hypothesis is that the behavior of security prices and the nature of the successive movements of these prices can be forecast based on past information available and multiple correlated categorical data sequences.

6.3.1 The MTD-Probit Estimation Method

In this paper, we apply a first order MMC model to study the stock market behavior. Formally, we consider a multivariate stochastic Markov process $\{S_{1,t}, ..., S_{s,t}; t = 1, 2, ...\}$ where the present state $S_{j,t}$ $(j = 1, ...s)$ can take values in the finite set $\{1, 2, ..., m\}$. Furthermore, $S_{j,t}$ depends on the previous values of $S_{1,t-1}, ..., S_{j,t-1}, ..., S_{s,t-1}$ and/or some explanatory variables. In this context, a natural model to predict $S_{j,t}$ is based on its transition probabilities:

$$P_j\left(i_0 | \, i_1, ..., i_s\right) := P\left(S_{j,t} = i_0 | \, S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s\right), \tag{6.1}$$

that can be easily estimated through the maximum likelihood estimates (MLE) expression:

$$\hat{P}_j\left(i_0 | \, i_1, ..., i_s\right) = \frac{n_{i_1 i_2 ... i_s i_0}}{\sum_{i_0=1}^{m} n_{i_1 i_2 ... i_s i_0}}, \tag{6.2}$$

where $n_{i_1 i_2 ... i_s i_0}$ is the number of transitions of type $S_{1,t-1} = i_1, ..., S_{s,t-1} = i_s, S_{j,t} = i_0$.

However, modeling these probabilities when s and m are relatively large and the sample size is small or even moderate is unfeasible, as the total number of parameters is $m^s\left(m-1\right)$. In practical terms, this means that the numerator as well as the denominator of Eq.(6.2) may be, in most of cases, zero or very close to zero. As a consequence, the parameters can be neither efficiently estimated nor identified with a finite sample size.

To overcome this problem, Ching et al. (2002 and 2008) considered a simplifying hypothesis, which is, in fact, an extension of Raftery (1985), for modeling high-order

Markov chains. The procedure It involves assuming that the probability $P_j\left(i_0|\, i_1,...,i_s\right) :=$ $P\left(S_{j,t}=i_0|\, S_{1,t-1}=i_1,...,S_{s,t-1}=i_s\right)$ can be written as a linear combination of $\{P_{j1}\left(i_0|\, i_1\right),...,P_{js}\left(i_0|\, i_s$ where $P_{jk}\left(i_0|\, i_k\right) := P\left(S_{j,t}=i_0|\, S_{k,t-1}=i_k\right)$, i.e.:

$$P\left(S_{j,t}=i_0|\, S_{1,t-1}=i_1,...,S_{s,t-1}=i_s\right)=P_j^{MTD}\left(i_0|\, i_1,...,i_s\right):=$$

$$\lambda_{j1}P_{j1}\left(i_0|\, i_1\right)+...+\lambda_{js}P_{js}\left(i_0|\, i_s\right), \tag{6.3}$$

where $\sum_{i=1}^{s}\lambda_{ji}=1$ and

$$0\leq\sum_{k=1}^{s}\lambda_{jk}P_{jk}\left(i_0|\, i_k\right)\leq 1. \tag{6.4}$$

The expression on Eq.(6.3) is called the mixture transition distribution (MTD) model (Raftery, 1985). The MTD model tries to overcome the difficulties for estimated MMC with parsimony and is easier to implement. Indeed. the number of parameters to be estimated is substantially reduced to $m\left(m-1\right)+\left(s-1\right)$ and each additional lag adds only one additional parameter. Nonetheless, there are some difficulties in the MTD parameter estimation process. One of the main challenges in applying this model is linked to the estimation process, the way the nonlinear constraints deal with the numerical optimization and the range of dependence patterns that the model can capture, specially negative partial effects (e.g. Berchtold, 2001, Lèbre and Bourguignon, 2008, Chen and Lio, 2009, and Nicolau, 2014).

However, recently a new MTD estimation process called MTD-Probit (Nicolau, 2014) was proposed. The MTD-Probit model is based on a specification which is completely free from constraints, facilitating the estimation process. Additionally, it has a more accurate specification for $P_j\left(i_0|\, i_1,...,i_s\right)$ which does not alter the consistency of the MLE. More specifically, the MTD-Probit model suggests modeling MMC, as follows:

$$P_j\left(i_0|\, i_1,...,i_s\right)=P_j^{\Phi}\left(i_0|\, i_1,...,i_s\right):=\frac{\Phi\left(\eta_{j0}+\eta_{j1}P_{j1}\left(i_0|\, i_1\right)+...+\eta_{js}P_{js}\left(i_0|\, i_s\right)\right)}{\sum_{i_0=1}^{m}\Phi\left(\eta_{j0}+\eta_{j1}P_{j1}\left(i_0|\, i_1\right)+...+\eta_{js}P_{js}\left(i_0|\, i_s\right)\right)} \tag{6.5}$$

where $\eta_{ji}\in\mathrm{R}(j=1,...,s;i=1,...,s)$ are parameters to be estimated, and Φ is the (cumulative) standard normal distribution function. In this scenario, when $S_{j,t}$ is the dependent variable the likelihood is:

$$logL=\sum_{i_1i_2,...i_si_0}\eta_{i_1i_2...i_si_0}log(P_j^{\Phi}(i_0|\, i_1,...,i_s)), \tag{6.6}$$

and the MLE can be expressed[2] as:

[2]As suggested by Nicolau (2014), we have used the constrained maximum likelihood module in GAUSS software (Aptech Systems, Chandler, Arizona, United States) that allows switching between several algorithms (BFGS, Broyden-Fletcher-Goldfarb-Shanno, DFP, Davidon-Fletcher-Powell, Newton, BHHH, Berndt-Hall-Hall-Hausman, scaled BFGS and scaled DFP) depending on either of three methods of

$$\hat{\eta}_j = argmax_{\eta_{j_1},\eta_{j_2},...\eta_{j_s}} logL \tag{6.7}$$

In addition, the parameters $P_{jk}\left(i_0|\, i_1\right)$, $k = (1, ...s)$ can be consistently estimated in advance through $\hat{P}_{jk}\left(i_0|\, i_1\right) = \frac{n_{i_1 i_0}}{\sum_{i_0=1}^{m} n_{i_1 i_0}}$ where $n_{i_1 i_0}$is the number of transitions of type $S_{k,t-1} = i_1$ to $S_{j,t} = i_0$.

6.3.2 TA Rules Modeling Framework

The main goal of this paper is to enhance the possibility of the TA methodology to predict stock market behavior. In this context, it is crucial to select an appropriate set of technical rules since this is an essential step to ensure properly tested procedures. Therefore, in this paper we adopt three basic rules selection criteria: (1) relevance of the instrument; we chose the most widely tools used in the financial market and in the academic literature; (2) replication capacity; we considered only mathematically well-formulated rules, and (3) analytical appropriateness; we selected the rules that are by construction "Markovian times", as proposed by Neftci (1991). In this scenario, we choose to study technical indicators trading (TAI) rules.

6.3.2.1 Technical Indicators Trading Rules

In the TA methodology there are special kinds of rules based not on the subjective judgment of figures or chart patterns analysis. Instead, they are focused on market variables data transformation such as trade price, volume and volatility, which can easily be quantified and tested (Murphy, 1986). These strategies can be seen as mathematically well-defined methods for foreseeing securities, based only on the past behavior. Indeed, in the case of these rules, study of historical data is enough to identify some aspects of price dynamics that can produce buy or sell signals, which can be used not only to foresee future changes in prices, but also to provide the information needed to create or adjust any market strategy adopted.

In this paper we consider an extensive set of TAI rules, drawn from a wide variety of parametrization specifications that are presented in previous academic studies and also the technical analysis manuals (see e.g. Edwards and Magee, 2012; and Pring, 2014). As acknowledge by Sullivan et al.(1999), the list of trading rules should be "vastly larger than those compiled in previous studies, and we include the most important types of trading rules that can be parsimonious parametrized and that do not rely on "subjective" judgments" (p.1655).

In this context, we choose a broad set of starting parameters that are presented in the financial literature, such as the number of days of the different horizons time measures, the size of the increase or decrease necessary to generate a buy or sell signal, the number

progress: change in function value, number of iterations or change in line search step length.

of days' rate of change in price or volume and overbought/oversold levels. We selected a parameter set that is diversified enough to avoid the type of "survivorship bias" problem related to the best performing historical rules (Sullivan et al., 1999).

Furthermore, since one of the trickiest aspects in technical analysis is the inaccuracy created by short-run false signals we combine TAI strategies, using some complex strategies to confirm an initial trading signal. We want to study multi-indicator trading rules that could help minimize the trading of signal-to-noise and increase profitability (Hsu et al., 2010). We provide an analysis of four complex trading rules. We test the MFI&RSI (Yen and Hsu, 2010), PPO&PVO, PMA&VMA and BBS&RSI.

The list of trading rules is presented in Table 1 and in Appendices 1 and 2, we comprehensively detail how the rules and parameter values used in our analysis were defined. As a result, we select a total of 152,071 TAI trading rules parametrization, based on 36 different sets of simple and complex double-rules, provided by the practitioners and academic mainstream literature in the area (see e.g. Brock et al., 1992; and White, 2000).

Table 6.1: TAI Strategies

TAI Rules	Abbreviation	Number of Rules
Bollinger Bands	BBS-EMA and BBS-SMA	1,890
Commodity Channel Index	CCI	4,080
Chaikin Oscillator	CHO	173
Chaikin Money Flow	CMF	210
Moving Average Convergence Divergence	MACD	9,660
Moving Average Filters based on Price Moving Average Filters based on Volume	PEMA and PSMA VEMA and VSMA	75,918
Money Flow Index	MFI and MFI - Divergence	7,920
Percentage Price Oscillator	PPO	3,479
Percentage Volume Oscillator	PVO and PVO-Divergence	6,958
Rate-of-Change	ROC and ROC Divergence	168
Relative Strength Index	RSI, RSI-Divergence	5,652
Stochastic Oscillator	STO, Fast and Slow STO	1,372
William R%	WRI	280
Complex Rule	BBS&RSI	8,820
Complex Rule	MFI&RSI	7,560
Complex Rule	PEMA&VEMA and PSMA&VSMA	14,452
Complex Rule	PPO&PVO	3,479
	Total Simple and Complex Trading Rules	152,071

6.3.3 Noise Reduction Markov Chain Strategy

This section presents our trading rule methodology used to forecast the FTSE 100 Index. We assume the existence of some sort of serial dependency on prices, which can be seen as a generalization of McQueen and Thorley´s (1991) approach for analyzing stock returns predictability.

6.3.4 Modeling Framework

The integration and globalization of financial markets in the last few decades has increased the interdependence among world stock markets and increased the possibility of mean and volatility spillovers[3]. Indeed, as per recent studies[4], the liberalization of capital movements and advanced computer technology has boosted the co-movements of stock prices among markets (see, e.g. Hamao et al., 1990, Kanas, 1998, Forbes and Ricobon, 2002, Baele, 2005, Christiansen, 2007, Chan et al. 2008, Abou-zaid et al. 2011, Natarajan et al., 2014, and Akca and Ozturk, 2016).

Therefore, it is of fundamental importance in any asset-pricing model, to incorporate the impact of the correlation between stock mean and volatility among financial markets. Indeed, the spillover effects have strong implications for investors´optimal asset allocation, specially in the more capitalized financial markets (e.g. Natarajan et al., 2014).

In this study, we investigate if the MTD-Probit model can be used as noise reduction method to predict the FTSE 100 Index behavior. The FTSE 100 Index, which represents 70 percent of the equity capitalization of all United Kingdom equities, is the most important index in Europe. Indeed, in December 2015, the London Stock Exchange (LSE) was rated as the most capitalized stock exchange in Europe, followed by Frankfurt and Paris and the third largest in the world [5].

We propose evaluating the TA methodology using a very simple approach. We assume that the investors buy or sell the FTSE 100 Index according to a TAI market investment signal that was previously filtered out by a MTD-Probit estimated noise reduction procedure, and liquidates the position only if it has a trend reversal signal, for example, from a buy signal to neutral or sell signal.

Additionally, as a result of the possibility of information transmission among markets, we include the mean and volatility spillover impact from the regional players (Europe aggregated) and global markets as covariates in the estimation process of the FTSE 100 Index. Then, we use the log return and log return volatility of Frankfurt and Paris financial markets represented by the DAX and the CAC 40 indices, respectively to study the regional interdependence of the financial market; and we use the American market based on the S&P 500 and the NASDAQ indices to proxy global market spillover effects.

6.3.5 The MTD-Probit Noise Reduction Forecast Strategy

We propose evaluating a market strategy using a very simple approach. We assume that the investor buys or sells the FTSE 100 Index, according to the trading signal based on the standard TAI model after controlled by the signal provided by the MTD-Probit model.

[3]The co-movements in return and volatility among markets have been commonly termed as mean and volatility spillover, respectively.

[4]For an in-depth review of the literature in the area see Singh et al, 2015.

[5] World Federation of Exchanges, 2015.

We study the proposed strategy under two different investor behavior assumptions: the one-day strategy (ODS) and the trend reversal strategy (TRS). In the first strategy, we assume the naive and costly hypothesis that any signal lasts for a one-day period only. In the second strategy, we consider that the investor liquidates the position, only if it has a trend reversal signal, for example, from a buy signal to a neutral or sell signal.

To sum up, the procedure can be described through the following algorithm:

Step 1: We categorize our data sample. Firstly, we map our indices log return to a first-order three state Markov chain based on an interval around their observed sample median [6]. We consider the following example. Let $r_{1,t}$ be the log return associated with the FTSE 100 and $q_{0.5}^{(1)}$ be the median of the marginal distribution of $r_{1,t}$, i.e. $q_{0.5}^{(1)}$ is such that $P(r_{1,t} \leq q_{0.5}^{(1)}) = 0.5$, and $\hat{q}_{0.5}^{(1)}$ the corresponding sample median. Then, we can map the FTSE 100 as follows:

$S_{1,t} = 1$ if $r_{1,t} \leq \hat{q}_{0.5}^{(1)} - k_1$, corresponding to the bear market;

$S_{1,t} = 2$ if $\hat{q}_{0.5}^{(1)} - k_1 < r_{1,t} < q_{0.5}^{(1)} + k_1$, for a neutral market; and

$S_{1,t} = 3$ if $r_{1,t} \geq \hat{q}_{0.5}^{(1)} + k_1$, corresponding to the bull market.

As such, the FTSE 100 continuous state space is mapped into state space $\{1, 2, 3\}$, which allows us to incorporates the direction of change in the Index returns into the analysis, and its magnitude as a function of parameter k_1(see, e.g. Niederhoffer and Osborne, 1966, and Fielitz and Bhargava, 1973).

Secondly, we map our indices return to short-term volatilities. In this case, we first determine the returns volatilities over a specific time-horizon of k_2 days and compute the median of their marginal distribution. Then, we consider an interval map using the median as a neutral benchmark. This study adopts the sample variance to proxy the return volatility $\hat{v}_{i,z} = \sum_{z=t}^{t+(k_2-1)} (r_{i,z} - \mu_{i,h})^2/k_2$, where $\mu_{i,h}$ is the estimated ith index sample mean for the k_2 time-horizon, that is $\mu_{i,h} = \sum_{h=t}^{t+(k_2-1)} (r_{i,h})/k_2$, $t = \{1, 2, ..., T - k_2\}$. For example, in the case of the FTSE 100, we have:

$S_{2,t} = 1$ if $\hat{v}_{1,t} \leq \hat{q}_{0.5}^{(\hat{v}_1)} - k_3$, corresponding to a low volatility market;

$S_{2,t} = 2$ if $\hat{q}_{0.5}^{(\hat{v}_1)} - k_3 < \hat{v}_{1,t} < \hat{q}_{0.5}^{(\hat{v}_1)} + k_3$, for a neutral volatility market; and

$S_{2,t} = 3$ if $\hat{v}_{1,t} \geq \hat{q}_{0.5}^{(\hat{v}_1)} + k_3$, corresponding to a high volatility market;

where $\hat{v}_{1,t}$ is the return volatility for FTSE 100, $\hat{q}_{0.5}^{(\hat{v}_1)}$ is its sample median, and k_3 is a threshold parameter.

Step 2: We split our T observations into two segments. Then, we use the first t observations to determine the first standard TAI buy, sell or no action signals. The size of t is given by the minimum size that is needed to calculate all the TAI trading rules.

[6] In Appendix 1, we provide the explanatory variables (covariates), parameters definitions and values.

Furthermore, we also use this the first segment to estimate the initial MTD-Probit transition probability matrix (TPM) and generate the trading signal for t+1. The trading signal is determined by the highest transition probability for the FTSE 100, which is estimated based on the combination of states of four explanatory covariates also observed in t, that we call trading rules, and the FTSE 100 previous state.

Step 3: We use the $t+1$ observations to re-estimate the next TAI trading signals and the best MTD-Probit forecasting trading rule signal, and repeat the process sequentially. In this set-up, we take a joint decision to buy or sell based on (1) the signal that is appointed by the TAI under analysis; and (2) the highest probability future state in the Markov chain transition matrix of the Index using the MTD-Probit estimation.

Step 4: We record all the returns generated by our combination of covariates or trading rules, and measure total net returns. Mathematically, the returns are determined based on the signal function for the *mth* MTD-Probit noise reduction trading rule (hereafter, trading rule), $(m = 1, ..., M)$, given by:

$$R^*_{m,t+1} = R_{m,t+1} - R^0_{t+1}, \tag{6.8}$$

$$R_{m,t+1} = I_{m,t+1}R_{t+1} - abs(I_{m,t+1} - I_{m,t})Tc, \tag{6.9}$$

$$R_{t+1} = ln(p_{t+1}/p_t), \tag{6.10}$$

where $R^*_{m,t+1}$ is the one-day excess return of the *mth* trading strategy discounting the market benchmark strategy R^0_{t+1}, which in our case is the buy-and-hold trading strategy, after accounting for the one-way transaction cost Tc. Furthermore, p_t is the daily closing quote index at time t and $I_{m,t+1}$ is a variable indicator for the *mth* MTD-Probit rule, which takes the values 1,0 or -1, if we take a long position, no action or short position in $t+1$, respectively.

In this study, the sell signal (-1) implies short selling. Although it is not possible to sell short owing to legal or market restrictions, we follow the approach that it is essential to accurately calculate a total trading rule profitability. Additionally, if our investment rule indicates a non-change market (no action) we account for no return [7].

Step 5: For each model set-up, we calculate the percentage success rate (PSR), based on the predictive accuracy of the trading signals generated in the previous steps, as follows:

$$PSR_m = V_m/n, \tag{6.11}$$

where V_m is the number of times that our *mth* model estimation matches the real market movement in our forecasting horizon.

[7] We could equally account for the overnight cash rate, calculated on the basis, for example, of the "3-month Treasury Bill Yield".

6.3.6 Transaction Costs

In this study, we do not consider transaction costs directly, but make a simple assumption that $Tc = 0$. There is no doubt that an investment rule is profitable only when its profit is greater than any trading costs. However, the recent introduction of a new computational trading floor process and online trading systems has lowered the overall "transactional costs" (see e.g. Bessembinder and Chan, 1995, Mitra , 2010, Bajgrowicz and Scaillet, 2012, and Kuang et al., 2014). Therefore, it is very difficult to choose any previous or recommended one-way transaction cost level.

To minimize the effects of this "somewhat unrealistic assumption" (Bajgrowicz and Scaillet, 2012), we present a break-even transaction costs analysis based on the methodologies of Hsu et al.(2010) and Mitra (2010). Then, we calculate the "potential margins for profitability" (PMP) which is the level of Tc which could offset any foreseen profitability. The PMP is the break-even transaction cost, which measures the capacity of the trading rule to absorb any transaction costs. It is estimated as follows:

$$PMP = \frac{R_{Tm}}{N_m}, \tag{6.12}$$

where R_{Tm} and N_m are respectively, the total return and the number changing signals generated along the investment period horizon for the mth trading rule. In our investment methodology, the transaction cost depends of the type of market strategy adopted. In the case of ODS is payable twice in each investment decision (round-trip cost), that is:

$$N_m = \sum_{t=1}^{n} 2 * abs(I_{m,t}). \tag{6.13}$$

However, in the TRS case, the transaction cost should be considered initially when a buy/sell signal generates an investment position, and secondly, when a new signal is generated; requiring a change in the previous investment decision as follows:

$$N_m = \sum_{t=1}^{n} abs(I_{m,t} - I_{m,t-1}). \tag{6.14}$$

6.4 Empirical Examination

In this section, we provide the empirical evaluation of the MTD-Probit noise control model applied to the FTSE 100 Index and analyze our results. In this study, we use the market closing prices to calculate the Index log returns (see, e.g. Hsu and Kuan, 2005).

6.4.1 Main Sample Statistics Results

Table 2 contains descriptive statistics of the FTSE 100 daily log returns considered in the paper, for a approximately four trading years. The data consists of the adjusted daily closing prices obtained from the Datastream database. The sample comprises of approximately four years of log returns data from the period of January 06, 2009 to December 27, 2012.

We infer from this table that, the mean daily return for the FTSE 100 is 0.026%, which equates to 250 trading days per year, with an approximate average of 6.33% per year. The mean daily return volatility is 1.20% (standard deviation). Additionally, the table also shows that the Index is skewed to the left, which indicates that extreme negative returns are more probable than extreme positive ones. The sample excess kurtosis level reveals that the FTSE 100 return series has fatter tails than the normal distribution, i.e. the low positive and negative returns are more probable. Indeed, the Jarque-Bera portmanteau test (JB) supports the non-normal nature of the sample distribution, as it strongly rejects the null hypothesis of normality at the one percent level.

Regarding the linear time dependence properties, there is non-significant evidence of first-order autocorrelation across the sample, at 1% level or better. Finally, based on the Ljung-Box Q statistics, there is also non-significant autocorrelation, of up to six lags, for some of the Index returns. The null hypothesis of no autocorrelation for all six lags tested is not rejected, at 1% level or better.

Although, there is no evidence for autocorrelation in the FTSE 100 Index, it does no mean that the sample returns is independent over time. Indeed, there is the possibility of nonlinear time dependence in the observed data sample. Hence, in this context, the use of the Markov chain test methodology can be an important procedure for forecasting future price Behaviour.

Table 6.2: FTSE 100 Index Return Descriptive Statistics

Country	UK
No (Obs.)	1037
Mean (%)	0.0253
Max. (%)	5.0323
Min. (%)	-5.4816
S.D. (%)	1.1998
Skewness	-0.1626
Kurtosis	5.0763
ρ(1)	0.008
ρ(2)	-0.013
ρ(3)	-0.071
ρ(4)	0.012
ρ(5)	0.001
ρ(6)	0.009
Q(6)	5.6722
JB	190.85*

Notes: (1)The mean sample log-return (Mean (%)) and the standard deviation.(S.D. (%)) are reported in percentage . (2) JB are the Jarque-Bera test statistics, ρ(n) is the estimated autocorrelation at lag n, and Q(n) are the Ljung-Box-Pierce test statistics for the nth lag. The Ljung-Box Q -statistics p -values are reported with the estimated autocorrelation.*Statistical Significance at the 1% level.

6.4.2 Empirical Results

In this section, we present the results for the MTD-Probit noise control model of the top 10 performing trading rules based on the PSR criteria.

6.4.2.1 Best Performing MTD-Probit Trading Rules

In Tables 6.3 and 6.4, we present the selected best MTD-Probit model results under the one-day strategy (ODS) and the trend reversal strategy (TRS) criteria, respectively.

In the tables, the first column highlights the top ten most performing MTD-Probit trading strategies, for approximately four trading years (999 trading days). The second column reports the strategies mean log return for the period, where the mean buy-and-hold return is 0.0107%, based on five explanatory variables: the return and volatility of the FTSE 100 Index, and a combination of the CAC 40 (France), DAX (Germany), S&P 500 (US) and the NASDAQ (US) indices. Columns 3 and 4 detail the return from the buy and sell trading signals, respectively.

In columns 5 to 7, we report the number of times that our MTD-Probit trading rule estimation matches the real market movement in our investment time horizon of one-hundred days (PSR). The *All*(%), *Buy*(%) and *Sell*(%) are respectively the percentage of the overall, buy and sell correct signals reported in the sample. Additionally, the number of trades for our sample are reported in columns 8 and 9, where $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. In our study, the buy and sell returns were computed without considering the possibility of an additionally risk-free overnight return when a trading rule indicated the no position (out of the market).

Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance.

Inferring from Table 6.3, the best performing MTD-Probit rules returns for the ODS strategy are all statistically significant and the mean log returns range is a interval ranging from 15.51 to 11.08 basis points (bps). In our investment horizon, those trading rules matches the real market movement in more than half of the time, in an small interval ranging from 55.16% to 54.55%. Table 6.4, we present the results for the TRS strategy. In this case, we observe that the rules return are all statistically significant and the mean log returns range is a interval from 15.51 to 10.70 basis points (bps). Moreover, the MTD-Probit trading rules matches the real market movement in an interval ranging from 55.26% to 54.55%.

In the tables, there are also some evidence that the MTD-Probit trading rule is capable of consistently producing superior performance over the buy-and-hold benchmark for the FTSE 100. Indeed, before adjusting for data-snooping and transaction costs, it was observed that the best performing MTD-Probit trading strategies has a minimum return for the period of 106.89%, which is almost ten times the benchmark, that account approximately 10.73%.

We also observed that the best trading rules consistently show the unique interdependence between the largest European stock markets, namely London, Paris and Frankfurt, and some evidence of a mean and volatility spillover effect among the major international financial markets, as evidenced by the S&P 500 and NASDAQ indices.

6.4.2.2 MTD-Probit Noise Reduction Results

In Tables 6.5 and 6.6, we present the MTD-Probit noise control model results under the ODS and the TRS criteria, respectively. In the tables, the first two column highlights the top 10 most profitable filtered strategies, based on the best $PSR(\%)$ MTD-Probit model signals. In columns 3 and 4, we report the model performance results. In column 3, we report the percentage of the mean return for the MTD-Probit noise control strategies ($Ret.MTDTAI$). Column 4 present the performance of the strategies selected without the MTD-Probit noise control filter ($Ret.TAI$), that is the standard TAI strategies performance results.

In columns 5 and 6, we report the $PSR(\%)$ results. The $\#MTDTAI$ and $\#TAI$ are respectively the overall percentage of noise control correct signals reported during the sample for filtered and standard TAI strategies. Additionally, for each type of trading model, the number of trades and the "potential margins for profitability" (PMP) are presented

Table 6.3: The Top 10 Performing MTD-Probit Trading Strategies for the FTSE 100 Index (ODS)

	Mean Daily Log Return			PSR			Number of Trades		PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	(%)
$1,2,4,6,10 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1334(4.46)*	0.1654(4.20)*	0.1165(2.48)**	55.16	51.43	61.35	546	452	0.1336
$1,2,4,6,7 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.1381(4.62)*	0.1829(4.78)*	0.1145(2.37)**	54.85	48.36	63.80	575	423	0.1382
$1,2,4,5,9 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1326(4.43)*	0.1848(4.77)*	0.1060(2.18)***	54.65	45.90	65.85	596	403	0.1326
$1,2,3,6,9 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.1551(5.20)*	0.2039(5.30)*	0.1297(2.71)*	54.55	47.95	63.60	575	424	01551
$1,2,4,5,7 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 10\%$	0.1126(3.75)*	0.1585(4.11)*	0.0888(1.81)	54.55	45.90	65.64	597	402	0.1126
$1,2,4,6,8 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 1\%$	0.1257(4.20)*	0.1750(4.65)*	0.1004(2.03)***	54.55	45.70	65.85	596	403	0.1257
$1,2,3,6,9 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 10\%$	0.1404(4.69)*	0.2078(5.70)*	0.1082(2.12)***	54.55	43.44	68.10	623	376	0.1404
$1,2,4,5,10 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1108(3.69)*	0.1389(3.40)*	0.0952(2.08)***	54.55	50.61	60.74	545	454	0.1108
$1,2,4,5,8 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1302(4.35)*	0.1650(4.05)*	0.1114(2.43)**	54.55	49.80	61.55	553	446	0.1302
$1,2,4,6,10 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 5\%$	0.1242(4.15)*	0.1648(4.23)*	0.1030(2.14)***	54.55	47.75	63.60	577	421	0.1243

Notes: The first column highlights the top 10 performing MTD-Probit, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The numbers in parentheses are the standard t-ratios testing the returns significance. In columns 5 to 7, we report the PSR which is the, number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" ($PMP\%$) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *Statistical Significance at the 1% level for a two-tailed test.

Table 6.4: The Best Top 10 MTD-Probit Trading Strategies for the FTSE 100 Index (TRS)

	Mean Daily Log Return			PSR			Number of Trades		PMP
Strategies (Parameters)	All (%)	Buy (%)	Sell (%)	All (%)	Buy (%)	Sell (%)	No. Buy	No. Sell	(%)
$1, 2, 4, 6, 10 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1335(4.46)*	0.1654(4.20)*	0.1164(2.48)**	55.26	51.43	61.55	546	452	0.2778
$1, 2, 4, 6, 7 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.1388(4.64)*	0.1829(4.78)*	0.1155(2.39)**	54.95	48.36	64.01	575	423	0.2925
$1, 2, 4, 5, 9 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1326(4.43)*	0.1848(4.77)*	0.1060(2.18)***	54.65	45.90	65.85	596	403	0.2958
$1, 2, 4, 5, 7 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 10\%$	0.1326(4.43)*	0.1848(4.77)*	0.1060(2.18)***	54.55	45.90	65.64	597	402	0.2647
$1, 2, 4, 6, 8 - k_1 = \pm 1\%, k_2 = 20, k_3 = \pm 1\%$	0.1257(4.20)*	0.1750(4.65)*	0.1004(2.03)***	54.55	45.70	65.85	596	403	0.2779
$1, 2, 3, 6, 9 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.1551(5.20)*	0.2039(5.30)*	0.1297(2.71)**	54.55	47.95	63.60	575	424	0.3106
$1, 2, 3, 6, 9 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 10\%$	0.1404(4.69)*	0.2078(5.70)*	0.1082(2.12)***	54.55	43.44	68.10	623	376	0.3323
$1, 2, 3, 6, 7 - k_1 = \pm 1\%, k_2 = 10, k_3 = \pm 1\%$	0.1415(4.73)*	0.1809(4.66)*	0.1206(2.54)**	54.55	49.18	62.37	559	438	0.2977
$1, 2, 3, 6, 9 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 5\%$	0.1070(3.56)*	0.1470(3.89)*	0.0855(1.74)	54.55	47.13	64.21	583	415	0.2491
$1, 2, 4, 5, 10 - k_1 = \pm 1\%, k_2 = 30, k_3 = \pm 1\%$	0.1108(3.69)*	0.1389(3.40)*	0.0952(2.08)***	54.55	50.61	60.74	545	454	0.2166

Notes: The first column highlights the top 10 performing MTD-Probit, based on the log return criteria, while the second column reports the mean return for these strategies. Columns 3 and 4 detail the mean daily return from the buy and sell trading signals, respectively. The numbers in parentheses are the standard *t*-ratios testing the returns significance. In columns 5 to 7, we report the PSR which is the, number of times that our TAI trading rule estimation matches the real market movement for each sub-sample investment time horizon. The *All*(%, *Buy*(% and *Sell*(% are respectively the overall percentage, buy and sell correct signals reported in the sample. Additionally, $No.Buy$ and $No.Sell$ are the total number of buy and sell trades respectively. Finally, in the last column we present the "potential margins for profitability" (PMP%) as suggested by Hsu et al. (2010). That is, the break-even transaction cost values that eliminate any out-performance. *Statistical Significance at the 1% level for a two-tailed test.

in the last columns. These are the $No.MTDTAI$, $No.TAI$, $P.MTDTAI$ and $P.TAI$, respectively.

6.4.2.3 Detailed Empirical Evidence

In Table 6.5 is presented the profitability of the MTD-Probit noise control model for the ODS criteria, where the best 10 rules daily mean returns is an interval ranging from 5.25 bps to 4.38 bps. As observed, the best strategies are based on a trend-following indicators, based on the exponential and simple moving averages (EMA\SMA) of the FTSE 100 price Index volume.

In our case, the EMA\SMA trading strategies uses two volume-based moving averages to generate crossover signals. These crossovers involve the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average.

Table 6.6 highlight the profitability of the MTD-Probit noise control model for the TRS criteria, where the best 10 rules daily mean return is an interval ranging from 9.59 bps to 6.29 bps. We observe, that the best strategies are based on a mix of price and volume TAI strategies. On the one hand, the best predictive rule is based on the volume-based EMA trading rule.

On the other hand, we have the strategies based on price. We have the Percentage Price Oscillator (PPO), Relative Strength Index (RSI), and Williams %R (WRI) strategies. The first strategy (PPO), it is a momentum oscillator that measures the difference between two moving averages as a percentage of the largest moving average. The oscillator moves into positive and negative terrain as a function of the difference between the shorter moving average and the longer moving average.

Additionally, we have the RSI, which is a momentum oscillator that measures the speed and change of price movements, and the WRI. The WRI is an indicator which reflects the level of the stock price relative to the highest high for a look-back period. The indicator oscillates from 0 to -100. Readings from 0 to -20 are considered overbought. Readings from -80 to -100 are considered oversold.

It is important to point-out that the volume-based TAI strategies are present in both tables. This is a suggestive finding. The role of volume to predict stock price Behaviour has been suggested by Blume et al. (1994). For the author, volume may be informative about the process of security returns that cannot be deduced from the price statistic.

6.4.3 Is the MTD-Probit efficient in noise control?

It is well known that one of the most common problems in correctly defining the economic performance of any trading rules is related to the size of the transaction costs involved.

Table 6.5: The Top 10 Performing Noise Reduction Trading Strategies return for the FTSE 100 Index (ODS)

Strategies (Parameters)		Mean Daily Log Return (%)		PSR (%)		Number of Trades		PMP (%)	
MTD	TAI	Ret. MTDTAI	Ret. TAI	#MTDTAI	#TAI	No. MTDTAI	No. TAI	P.MTDTAI	P.TAI
$1,2,3,6,9 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm10\%$	VSMA(5,10,90,0.005,0.005)	0.0525	0.0561	30.03	43.94	481	980	0.1091	0.0572
$1,2,4,6,10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	VSMA(5,10,90,0.005,0.005)	0.0520	0.0561	30.03	43.94	484	980	0.1073	0.0572
$1,2,4,6,8 - k_1 = \pm1\%, k_2 = 20, k_3 = \pm1\%$	VSMA(5,10,90,0.005,0.005)	0.0520	0.0561	29.73	43.94	470	980	0.1105	0.0572
$1,2,4,6,8 - k_1 = \pm1\%, k_2 = 20, k_3 = \pm1\%$	VEMA(5,10,60,0.020,0.020)	0.0498	0.0513	27.43	39.74	417	871	0.1194	0.0588
$1,2,3,6,9 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm10\%$	VEMA(5,10,50,0.015,0.015)	0.0489	0.0452	28.63	40.74	440	904	0.1110	0.0500
$1,2,3,6,9 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VSMA(5,10,90,0.005,0.005)	0.0473	0.0561	29.83	43.94	465	980	0.1017	0.0572
$1,2,4,6,7 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VSMA(5,10,90,0.005,0.005)	0.0470	0.0561	30.03	43.94	469	980	0.1002	0.0572
$1,2,4,6,10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm5\%$	VSMA(5,10,70,0.005,0.005)	0.0467	0.0561	29.93	43.94	482	980	0.0969	0.0572
$1,2,4,6,10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	VEMA(5,12,20,0.015,0.015)	0.0463	0.0452	28.23	40.74	441	904	0.1048	0.0500
$1,2,3,6,9 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VEMA(5,10,50,0.015,0.015)	0.0438	0.0452	28.03	40.74	423	904	0.1035	0.0500

Notes: The first column highlights the top 10 most profitable filtered strategies, based on the best $PSR(\%)$ MTD-Probit model signals. In column 3, we report the percentage of the mean return for the MTD-Probit noise control strategies ($Ret.MTDTAI$). Column 4 present the performance of the strategies selected without the MTD-Probit noise control filter ($Ret.TAI$), that is the standard TAI strategies performance results. In columns 5 and 6, we report the $PSR(\%)$ results. The $\#MTDTAI$ and $\#TAI$ are respectively the overall percentage of noise control correct signals reported during the sample for filtered and standard TAI strategies. Additionally, for each type of trading model, the number of trades and the "potential margins for profitability" (PMP) are presented in the last columns. These are the $No.MTDTAI$, $No.TAI$, $P.MTDTAI$ and $P.TAI$, respectively.

Table 6.6: The Top 10 Performing Noise Reduction Trading Strategies return for the FTSE 100 Index (TRS)

Strategies (Parameters)		Mean Daily Log Return (%)		PSR (%)		Number of Trades		PMP (%)	
MTD	TAI	Ret. MTDTAI	Ret. TAI	#MTDTAI	#TAI	No. MTDTAI	No. TAI	P.MTDTAI	P.TAI
$1, 2, 4, 6, 10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	VEMA(5,12,35,0.0,0.0)	0.0959	0.0461	46.75	43.84	610	428	0.1570	0.1077
$1, 2, 4, 6, 10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	RSI(3,40,60,0.0,0.0)	0.0921	0.0851	47.45	47.95	231	193	0.3982	0.4407
$1, 2, 4, 5, 9 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	RSI(3,40,60,0.0,0.0)	0.0786	0.0851	46.75	47.95	221	193	0.3552	0.4407
$1, 2, 4, 6, 10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	VSMA(5,10,45,0.005,0.005)	0.0755	0.0525	46.45	44.64	591	425	0.1276	0.1235
$1, 2, 4, 5, 10 - k_1 = \pm1\%, k_2 = 30, k_3 = \pm1\%$	RSI(3,40,60,0.0,0.0)	0.0731	0.0851	46.05	47.95	224	193	0.3262	0.4407
$1, 2, 4, 6, 7 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VEMA(5,12,35,0.0,0.0)	0.0700	0.0461	45.45	43.84	588	428	0.1189	0.1077
$1, 2, 3, 6, 9 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	PPO(5,20,5,0.20,0.0)	0.0658	0.0431	44.04	43.94	182	233	0.3668	0.1846
$1, 2, 3, 6, 9 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VEMA(5,12,20,0.15,0.15)	0.0641	0.0721	44.44	45.55	187	372	0.3425	0.1936
$1, 2, 3, 6, 7 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	WRI(5,-75,-20,0.0,0.0)	0.0630	0.0370	45.05	44.04	302	423	0.2083	0.0875
$1, 2, 3, 6, 7 - k_1 = \pm1\%, k_2 = 10, k_3 = \pm1\%$	VSMA(5,10,70,0.025,0.025)	0.0629	0.0406	45.15	44.14	515	516	0.1220	0.0787

Notes: The first column highlights the top 10 most profitable filtered strategies, based on the best $PSR(\%)$ MTD-Probit model signals. In column 3, we report the percentage of the mean return for the MTD-Probit noise control strategies ($Ret.MTDTAI$). Column 4 present the performance of the strategies selected without the MTD-Probit noise control filter ($Ret.TAI$), that is the standard TAI strategies performance results. In columns 5 and 6, we report the $PSR(\%)$ results. The $\#MTDTAI$ and $\#TAI$ are respectively the overall percentage of noise control correct signals reported during the sample for filtered and standard TAI strategies. Additionally, for each type of trading model, the number of trades and the "potential margins for profitability" (PMP) are presented in the last columns. These are the $No.MTDTAI$, $No.TAI$, $P.MTDTAI$ and $P.TAI$, respectively.

Indeed, the transaction costs charged to an investor are unknown since these costs depends on many different aspects, such as the type of investor, investment size, and the technological level of the trading floor systems.

In our case, the results show that the number of trades and the break-even cost across the sample can substantially change as a function of the covariates that are used in the forecasting process. We observed that the MTD-Probit noise control break-even cost ($PMP\%$) of the best rules lies between 39.82 bps to 9.69 bps, which is relatively flexible. Indeed, as presented by Shynkevich (2012), the investor may be trading from a relatively low cost of 5 bps (Hsu et al., 2010), for a single trip transaction, to a less conservative assumption of 20 basis points (Shynkevich, 2012).

Additionally, we verify that there is a reduction in the whipping price effect in the case of the ODS criterion. Indeed, the model is able to filtering out whipsaws reducing the number of trades and consequently the PMP, enhancing the economic performance of the TAI strategies. However, in the case of the TRS criterion is not so clear that there is some improvement in the transaction cost.

We also observe that there are some evidence that the MTD-Probit noise control trading rule is potentially capable of producing superior performance for the FTSE 100. Indeed, before adjusting for transaction costs, it was observed that the best trading strategies have an annualized return of 13.13% (ODS) and 23.97% (TRS), considering 250 trading days per year, which correspond at least more than four times the annualized buy-and-hold benchmark return (2.68%).

Under such circumstances, we conclude that the Markov methodology can provide an important step in the improvement of the economic performance of TAI strategies.

6.5 Conclusion

Since the study of Fama (1970), the efficient market hypothesis has been one of the most fundamental pillars in modern finance theory. According to the hypothesis, prices should reflect all available information, and it should therefore not be possible to earn excess returns consistently from any investment strategy based on historical data. Consequently, the best conditional choice for future prices should be the current price. That is to say, buying and holding the security is the top investment strategy.

Nevertheless, in recent decades, new empirical evidence has suggested that the stock market could not be efficient and thus, is possible to obtain abnormal stock returns that are not fully explained by common risk measures and the possibility that technical analysis could lead to sustainable profitability.

This paper makes two main contributions to the literature. With regard to the methodology, we propose a new Markov chain forecasting procedure. We apply the MTD-Probit model (Nicolau, 2014) as a TA noise reduction method and then compares its results against the traditional TAI trading strategy. To the best of our knowledge, this is the first time that the Markov chain methodology is being used to enhance the use of TA to forecast stock market Behaviour.

Regarding its application, our empirical results provide evidence that the MTD-Probit noise control methodology can potentially control and filter out trading signals, and then improve the economic efficiency of the TA strategies.

Nevertheless, our empirical methodology has an important pitfall, since using the same data set for a large number of competing strategies, can generate a sequential testing bias. In this case, we suggest that the empirical application of the noise control model should take in account data-snooping effects, verifying for example, whether there is a superior rule within a "universe" of rules that could outperform some benchmark models (see, e.g White, 2000, Hansen, 2005 and Hsu and Kuan, 2005). This is an issue that may be worth studying in a future research.

Appendix 1

In this appendix we have summarized the Technical Analysis Indicators used in our study, based on the notations taken from Edwards and Magee (2012) and Pring (2014) and the initial scenario table.

Table 6.7: Technical Analysis Indicators

TAI Rules	Trading Rule Definition
Bollinger Bands - BBS	This is an indicator that uses standard deviations and stock price moving averages to generate buying and selling signal bands. The signal is given by the band's crossover. A bullish crossover occurs when the middle band crosses below the lower standard deviation band. A bearish crossover occurs when the middle band crosses above the higher standard deviation band.
Commodity Channel Index - CCI	The CCI measures the current price level relative to an average price level over a given period of time to generate overbought and oversold signals. The indicator measures the difference between a security's price change and its average price change. As such, high positive readings indicate that prices are well above their average, which is a show of strength. Low negative readings indicate that prices are well below their average, which is a show of weakness. Readings above +100 reflect strong price action that can signal the start of an uptrend. If they fall below -100 it reflects weak price action that can signal the start of a downtrend.
Chaikin Oscillator - CHO	The CHO is an indicator designed to anticipate directional changes in prices by measuring the momentum behind the movements. This oscillator generates signals with crosses above/below the zero line or with bullish/bearish divergences.
Chaikin Money Flow - CMF	This indicator measures the amount of money flow volume over a specific look-back period, typically 20 or 21 days. The resulting oscillator fluctuates above/below the zero line weighing the balance of buying or selling pressure. The CMF usually fluctuates between -.50 and +.50 with zero as the center-line.
Moving Average Convergence-Divergence -MACD	The MACD is a trend-following indicator. It uses the difference between long and short moving averages to measure a momentum. The indicator fluctuates above and below the zero line as the moving averages converge, cross and diverge. Convergence occurs when the moving averages move towards each other. Divergence occurs when the moving averages move away from each other. A 9-day EMA of the MACD line is used as a performance indicator as a signal line to identify market opportunities.
Money Flow Index - MFI	An indicator that uses both price and volume to measure buying and selling pressures. The MFI is positive when the price rises (buying pressure) and negative when the price declines (selling pressure). A ratio of positive and negative money flow is then calculated to create an oscillator that moves between zero and one hundred. As a momentum oscillator, it is used to identify reversals and price extremes with a diversity of signals. There is another version of this indicator, called MFI - Divergence, which compares the cross-over signal generated to buy or sell with its maximum or minimum level and with the price level.
Price Exponential and Simple Moving Average Indicators - PEMA\PSMA	The PEMA\PSMA investment strategy uses two exponential\simple moving averages to generate price crossover signals. These crossovers make the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average. In this paper we use the PEMA\PSMA indicator not only to generate buy and sell signals based on price and volume, but we also use its average, as an indicator of performance and a signal line to identify market opportunities.
Percentage Price Oscillator - PPO	A momentum oscillator that measures the difference between two moving averages as a percentage of the larger moving average. The value of the PPO becomes increasingly positive as the shorter moving average distances itself from the longer moving average reflecting a strong upside momentum. For negative values of the PPO, this indicates that the shorter moving average is below the longer moving average. Increasing negative values indicate that the shorter moving average is distancing itself from the longer moving average, reflecting strong downside momentum.
Percentage Volume Oscillator - PVO	A momentum oscillator for volume. The PVO measures the difference between two volume-based EMA as a percentage of a larger moving average. The PVO is positive when the shorter volume EMA is above the longer volume EMA and negative when the shorter volume EMA is below the longer volume EMA. There is also another type of this indicator called PVO - Divergence, which compares the generated cross-over signal to buy or sell with its maximum or minimum level for a price level.
Rate-of-Change - ROC	This indicator is referred to as Momentum. It is an oscillator that measures the percentage change in stock price from one period to the next. The ROC compares the current price to the price "t" periods ago, and fluctuates above and below the zero line. Moreover, the ROC is used by combining its signal with the divergence in stock price, called ROC - Divergence. In this case a buy(sell) signal is produced if the current ROC value is higher than its previous value, for a lower price.
Relative Strength Index -RSI	A momentum oscillator which measures the speed and change of stock price movements. The RSI oscillates between zero and 100. The indicator is considered overbought when above 70 and oversold when below 30. There is a modification of this indicator called RSI - Divergence, which compares the generated cross-over signal to buy or sell with its maximum or minimum level for some price level.
Stochastic Oscillator - STO	The STO measures the level of the closing stock price relative to the high-low range over a given period of time. When the STO is above 50 the indicator signals that the closing price is in the upper half of the range. In contrast, when it is below 50, this indicates the closing price is in the lower half. A STO reading below 20 signals that the price is near its lowest level for the given time period. However, for high readings (above 80) the rule indicates that the price is near its highest level. There are two other versions of Stochastic Oscillator which use an EMA of the STO to generate cross-over signals to buy or sell. These are the fast and slow STO.
Volume Exponential and Simple Moving Average Indicators -VEMA\VSMA	The VEMA\VSMA investment strategy uses two exponential\simple moving averages to generate volume crossover signals. These crossovers involve the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average. In this paper we use the VEMA\VSMA indicator not only to generate buy and sell signals based on price and volume, but we also use its average, as an indicator of performance and a signal line to identify market opportunities.
Williams %R Indicator - WRI	Technical indicator which reflects the level of the closing stock price relative to the highest high´ for a look-back period. The WRI oscillates from 0 to -100. Readings from 0 to -20 are considered overbought. Readings from -80 to -100 are considered oversold.

Appendix 2

In this appendix we summarize the parameters used in our TAI strategies.

Table 6.8: TAI Parameter Definition

Parameter Definitions

- n= number of days used to calculate the rule
- up = upper thresholds to initiate a position
- low= lower thresholds to initiate a position
- b = band to initiate a position
- s=number of days of the short moving average
- l = number of days of the long moving average
- d=number of days of the second short moving average
- sd= standard deviation multiplier

Trading Rule	Abbreviation	Parameters	
	BBL-PEMA (n,sd,b)	n	3,7,10,12,14,16,20,25,30,35,40,45,50,55,60
Bollinger Bands		sd	0.5,1,1.25,1.5,1.75,2,2.25,2.5,3
	BBL-VEMA (n,sd,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	4,6,8,10,15,20,22,24,26,28,30,35,40,45,50
Commodity Channel Index	CCI (n,up,low)	up	70,75,80,85,90,95,100,110,120,130,140
		low	-70,-75,-80,-85,-90,-95,-100,-110,-120,-130,140
BBS-EMA&RSI	BBL-EMA(n,sd,b,vp) & RSI(n,s,up,low)	s	3,7,10,20,30,40,50
		sd	0.5,1,1.25,1.5,1.75,2,2.25,2.5,3
		b	0,0.01,0.025,0.05,0.10
BBS-SMA&RSI	BBL-SMA(n,sd,b,vp) & RSI(n,s,up,low)	up	70,75,80,85,90,95
		low	5,10,15,20,25,30
Chaikin Oscillator	CHO (s,l)	s	3,7,10,12,14,16,18,20,22,24,26,28,30,35 40,45,50,60,70
		l	5,7,10,12,14,16,18,20,24,26,28,30,35,40,50,60,70
Chaikin Money Flow	CMF (n,b)	n	3,5,7,10,12,14,16,20,22,24,26,28,30,35,40,45,50,55,60,70,75,80,85,90,95,100,120
		b	0,0.01,0.025,0.05,0.10,0.15,0.20,0.25,0.30
		s	3,5,7,10,12,14,16,20,25,30
Moving Average Convergence- Divergence	MACD (s,l,n,b)	l	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50,60,70
		n	3,5,9,12,14,16,20
		b	0,0.005,0.01,0.015,0.02,0.025,0.05,0.10,0.15,0.20
		n	4,6,8,10,12,14,16,20,25,30,35
Money Flow Index	MFI (n,s,up,low)	s	3,5,9,12,14,16,20
		up	60,65,70,75,80,85,90,95
		low	5,10,15,20,25,30,35,40
		n	3,5,10,12,14,16,20,26
MFI&RSI	MFI(n,s,up,low) & RSI(n,s,up,low)	s	3,5,7,9,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50,55,60
		up	60,65,70,75,80,85,90,95
		low	5,10,15,20,25,30,35,40
	PEMA\PSMA (n,l,b)	s	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50
Moving Average Filters		l	10,12,14,16,18,20,22,24,26,28,30,35,40,50,60,70
	VEMA\VSMA (n,l,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,10,12,14,16,20,25,30,35,40,45,50
Percentage Price Oscillator	PPO (n,s,d,b)	s	10,20,24,28,32,40,50,60,70
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,10,12,14,16,20,25,30,35,40,45,50
Percentage Volume Oscillator	PVO (n,s,d,b)	s	10,20,24,28,32,40,50,60,70
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	3,5,7,10,12,14,16,20,25,30
PPO&PVO	PPO(n,s,d,b) & PVO(n,s,d,b)	s	5,10,12,14,16,20,25,30,35,40,45,50
		d	3,5,9,12,14,16,20
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
PEMA&VEMA	PEMA&VEMA (n,l,b)	s	5,10,12,14,16,18,20,22,24,26,28,30,35,40,45,50
		l	10,12,14,16,18,20,2224,26,28,30,35,40,45,50,60,70
PSMA&VSMA	PEMA&VEMA (n,l,b)	b	0,0.01,0.025,0.05,0.10,0.15,0.20
Rate-of-Change	ROC (n,b)	n	5, 10, 12,14,16, 20, 25,30,35,40,45,50
		b	0,0.01,0.025,0.05,0.10,0.15,0.20
		n	5,7,9,10,12,14,16,20,22,24,25,30,45,52
Relative Strength Index	RSI (n,s,up,low)	s	2,4,6,10,12,14,16,20
		up	60,65,70,75,80,85,90
		low	5,10,15,20,25,30,35,40
		n	5, 10, 12, 14,16, 15,20,25
Stochastic Oscillator	STO (n,up,low)	up	5,10,15,20,25,30,35
		low	65,70,75,80, 85,90,95
		n	5, 10,12, 14,16,20,25,30,35
William R%	WRI (n, up, low)	up	-5, -10,-15,-20,-25,-30
		low	-70,-75,-80,-85,-90,-95

Appendix 3

List of Independent Variables and Parameters

In this appendix we present the list of categorized explanatory variables and parameters used in the MTD-Probit noise estimation model:

Table 6.9: Explanatory Variables and Parameters MTD-Probit Model

Variable Definition	Parameter Number
FTSE Log Return	1
FTSE Log Return Volatility	2
DAX Log Return	3
CAC 40 Log Return	4
S&P 500 Log Return	5
NASDAQ Log Return	6
DAX Log Return Volatility	7
CAC 40 Log Return Volatility	8
S&P 500 Log Return Volatility	9
NASDAQ Log Return Volatility	10
k_1 = interval around the median for the log return	$\{\pm 1\%\}$
k_2 = number of days in the calculation of indices log return volatility	$\{10, 20, 30\}$
k_3 = interval around the log return volatility	$\{\pm 10\%\}$

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Chapter 7

Conclusion

The efficient market hypothesis (Fama, 1970) has been one of the most fundamental pillars in modern financial theory. According to the weak-form of the efficient market hypothesis, prices should reflect all available information; therefore, it should not be possible to earn excess returns consistently with any investment strategy that attempts to predict asset price movements based on historical data (Fama, 1965; and Fama & Miler, 1972). Reproducing the words of Fama (1970) : "In short, the evidence in support of the efficient markets model is extensive, and (somewhat unique in economics) contradictory evidence is sparse.".

Nonetheless, the widespread use of technical analysis as a leading stock market forecasting instrument is still challenging the idea of market efficiency. Indeed, in recent decades, empirical studies have provided evidence that the stock market could not be efficient and thus, is possible to obtain abnormal stock returns that are not fully explained by common risk measures. Additionally, over the years, academic research has raised the possibility that technical analysis could be a methodology capable of predicting stock market and lead to sustainable profitability.

Nevertheless, the empirical search for a high-performing forecasting method has been implemented reusing the same data set for a large number of competing strategies. Under such circumstance, the possibility of spurious results is a reasonable assumption since a superior profitability could be due to chance rather than to the existence of high-performance strategy. In this context, any empirical result should take in account data-snooping effects, verifying for example, whether there is a superior rule within a "universe" of best rules that could outperform some benchmark model.

This dissertation makes some contributions to the literature. With regard to the methodology, we propose a new Markov chain time-dependence and time-homogeneity test procedure, based on the MTD-Probit (Nicolau, 2014) and Polansky (2007) models. Furthermore, we use the MTD-Probit model to introduce a new forecasting procedure and a trading noise control method.

Regarding its application, we try to answer a major question: are there some forecast models, based only on the past price movements, which could be used as forecasting methods? That is, can we reject the EMH?

In order to provide some answer, we present some empirical evidence adjusted for data-snooping bias, by applying the White (2000) "Bootstrap Reality Check" and the Hansen (2005) tests. Firstly, we present a new study of the TA rules' profitability, using a unique sample of 152,071 trading rules, in the Portuguese financial market. Our results draw the attention to the importance of control data-snooping to avoid the possibility of spurious results. Indeed, although, there is some "reasonable" evidence that the TA methodology is capable of consistently producing superior profitability, our data-snooping test results discard the existence of high-performance strategies. Under these conditions, we conclude that we cannot reject the EMH in the PSI 20 Euronext Lisbon stock exchange index.

Secondly, we apply our time-dependence and time-homogeneity methodology to explore the EMH hypothesis in the main worldwide stock markets. Our empirical results suggest that the stock market can be efficient. However, the results are inconclusive for the American and the UK financial markets. Indeed, in the Anderson and Goodman (2007) methodology, these markets are first or higher-order time-homogeneous Markov chain process.

We also perform the Markov chain tests on a broad sample of 4,474 stocks and interesting robustness result emerges. The study showed that a lack of full accountability of the interdependence between the time-homogeneity and time-dependence properties can lead a conclusion that a stock market has a higher predictive power than when the time-homogeneity is tested. Additionally, we apply the MTD-Probit model (Nicolau, 2014) to obtain new evidence for the EMH hypothesis in the FTSE 100 Index after data-snooping is controlled. Our empirical results suggest that there is no evidence that is possible to earn excess returns consistently in the FTSE 100 with an investment strategy based on historical data.

Finally, we observe that the use of the Markov chain methodology can be an important step for the study of TA predictive power. Indeed, we provide evidence that the MTD-Probit noise control methodology can potentially control and filter out trading signals, and then improve the economic efficiency of the TA strategies. Nevertheless, the use of this methodology has an important data snooping challenging problem. Indeed, the use of the same data set for a large number of competing strategies, can generate a sequential testing bias. In this case, we suggest that the future research of the empirical application of the noise control model should consider data-snooping effects.

Appendix - Gauss Routines

This appendix contains a set of routines used in the preparation of the PhD dissertation. The routines have been prepared using the GAUSS programming language. GAUSS is a programming language similar to C and Pascal languages and specially designed to work with arrays. The program is marketed by Aptech Systems.

Technical Analysis Routines

BBL

- **Purpose**

  Compute the Bollinger Bands (BBL) indicator. This is an indicator that uses standard deviations and stock price moving averages to generate buying and selling signal bands. The signal is given by the band's crossover. A bullish crossover occurs when the middle band crosses down the lower standard deviation band. A bearish crossover occurs when the middle band crosses up the higher standard deviation band. There are three steps to calculate the BBL bands:

  1. middle band = d-day exponential and simple moving average (EMA/SMA).
  2. upper band = d-day EMA/SMA + (d-day standard deviation (sd_d) times the number of deviations: $sd_d * a$).
  3. lower band = d-day EMA/SMA - $sd_d * a$.

- **Format**

  {midband,upband,lowband,s}=bbl(x,a,d,b)

- **Input**

  - x: closing price vector.
  - a: number of standard deviations.
  - d: number of periods (e.g. trading days) used to compute the BBL bands.
  - b: fixed band or threshold to initiate a trading position.

- **Output**

  - midband: middle band.
  - upband: upper band.
  - lowband: lower band.
  - s: signal vector for buy (1), sell (-1) or no action (0).

CCI

- **Purpose**

  Compute the "Commodity Channel Index" (CCI) indicator. The CCI measures the current price level relative to an average price level over a given period of time to generate overbought and oversold level signals. The indicator measures the difference between a security's price change and its average price change. In this manner, high positive readings indicate that prices are well above their average, which is a show of strength. Low negative readings indicate that prices are well below their average, which is a show of weakness. Readings above +100 reflect strong price action that can signal the start of an uptrend. As it falls below -100 it reflects weak price action that can signal the start of a downtrend.There are three steps to calculate the CCI indicator:

1. Find the Typical Price (TP) =$(z + y + x)/3$.
2. Calculate the Mean Deviation (MD) in two steps. Firstly, subtract the most recent d-period average of the TP from each period's typical price and compute its absolute values. Secondly, sum the absolute values and divide by the total number of periods (d).
3. Compute the CCI = (TP - d-period SMA of TP) /(0.015 $* MD$)).

- **Format**

  {cci,s}=cci(x,y,z,d,up,low)

- **Input**
  - x: closing price vector.
  - y: lowest price vector.
  - z: highest price vector.
  - d: number of periods (e.g. trading days) used to compute the CCI indicator.
  - up: upper CCI limit.
  - low: lower CCI limit.

- **Output**
  - cci: the computed CCI indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

CHO

- **Purpose**

  Compute the "Chaikin Oscillator" (CHO) indicator. The CHO is an indicator designed to anticipate directional changes in prices by measuring the momentum behind the movements. This oscillator generates signals with crosses above/below the zero line or with bullish/bearish divergences. There are four steps to calculate the CHO indicator:

1. Find the Money Flow Multiplier = $[(x - y) - (z - x)]/(z - y)$.
2. Calculate the Money Flow Volume (MFV) = $MFM * v$.

3. Calculate the Accumulation Distribution Line (ADL)= Previous ADL + Current Period's MFV.
4. Compute the Chaikin Oscillator = (d_1-day EMA of ADL) - (d_2-day EMA of ADL) .

- **Format**

  {cho,s}= cho(x,y,z,v,d_1,d_2,b)

- **Input**

  - x: closing price vector.
  - y: lowest price vector.
  - z: highest price vector.
  - v: asset volume vector.
  - d_1: number of periods (e.g. trading days) used to compute the short-run EMA of the CHO indicator.
  - d_2: number of periods (e.g. trading days) used to compute the long-run EMA of the CHO indicator.
  - b: fixed band or threshold to initiate a trading position.

- **Output**

  - cho: the computed CHO indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

CMF

- **Purpose**

  Compute the "Chaikin Money Flow" (CMF) indicator. The CMF measures the amount of money flow volume over a specific look-back period, typically 20 or 21 days. The resulting oscillator fluctuates above/below the zero line weighing the balance of buying or selling pressure. The indicator usually fluctuates between -.50 and +.50 with zero as the center-line. There are three steps to calculate the CMF indicator:

  1. Find the Money Flow Multiplier (MFM) = $[(x-y)-(z-x)]/(z-y)$.
  2. Calculate the Money Flow Volume (MFV) = $MFM * v$.
  3. Compute the d-period CMF = $\sum_d MFV \ / \ \sum_d v$.

- **Format**

  {cmf,s}= cmf(x,y,z,v,d,b)

- **Input**

- x: closing price vector.
- y: lowest price vector.
- z: highest price vector.
- v: asset volume vector.
- d: number of periods (e.g. trading days) used to compute the CMF indicator.
- b: fixed band or threshold to initiate a trading position.

- **Output**
  - cmf: the computed CMF indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

EMA

- **Purpose**

  Compute the "Exponential Moving Average" (EMA). The EMA is used to generate crossover signals. These crossovers involve the comparison between a short moving average and a long moving average. A bullish crossover occurs when the shorter exponential moving average crosses above the longer moving average. A bearish crossover occurs when the shorter moving average crosses below the longer moving average. There are three steps to calculate the EMA:

  1. Calculate the simple moving average (SMA) of d-periods: $\sum_d x$.
  2. Use the SMA as the previous period's EMA in the first calculation.
  3. Determine the EMA Multiplier (m): $(2/(d+1))$.
  4. Compute the EMA: $\{x - EMA(d-1) * m + EMA(d-1)$.

- **Format**

  {ema1,ema2,s}=ema(x,d_1,d_2)

- **Input**
  - x: closing price vector.
  - d_1: number of periods (e.g. trading days) used to compute the short-run EMA.
  - d_2: number of periods (e.g. trading days) used to compute the long-run EMA.

- **Output**
  - ema1: the computed short-run EMA vector.
  - ema2: the computed long-run EMA vector.

– s: signal vector for buy (1), sell (-1) or no action (0).

MACD

- **Purpose**

  Compute the "Moving Average Convergence/Divergence" (MACD) indicator. The MACD is a trend-following indicator. It uses the difference between a long and short moving averages to measure a momentum. The indicator fluctuates above and below the zero line as the moving averages converge, cross and diverge. Convergence occurs when the moving averages move towards each other. Divergence occurs when the moving averages move away from each other. A 9-day EMA of the MACD line is used as an indicator to performance as a signal line to identify market opportunities. There are three steps to calculate the MACD indicator:

  1. Find the MACD Line =(d_1-day EMA - d_2-day EMA).
  2. Calculate the Signal Line (SL)= d_3-day EMA of MACD Line.
  3. Compute MACD Histogram: MACD Line - SL.

- **Format**

  {macd,s}= macd(x,d_1,d_2,d_3,b)

- **Input**

  – x: closing price vector.

  – d_1: number of periods (e.g. trading days) used to compute the short-run EMA of the MACD indicator.

  – d_2: number of periods (e.g. trading days) used to compute the long-run EMA of the MACD indicator.

  – d_3: number of periods (e.g. trading days) used to compute the indicator signal, which is an EMA of the MACD indicator.

  – b: fixed band or threshold to initiate a trading position.

- **Output**

  – macd: the computed MACD indicator vector.

  – s: signal vector for buy (1), sell (-1) or no action (0).

MFI

- **Purpose**

  Compute the "Money Flow Index" (MFI) indicator. The MFI is an indicator that uses both price and volume to measure buying and selling pressures. The MFI is positive when the price rises (buying pressure) and negative when the price declines (selling pressure). A ratio of positive and negative money flow is then calculated to create an oscillator that moves between zero and one hundred. As a momentum oscillator, it is used to identify reversals and price extremes with a diversity of signals. Another version of this indicator exist, called MFI - Divergence, which compares the generate cross-over signal to buy or sell with its maximum or minimum level and with price level. There are three steps to calculate the MFI indicator:

1. Find the Typical Price (TP) =$(z + y + x)/3$.
2. Calculate the Money Flow (MF) = $TP * v$.
3. Compute the Money Flow Ratio (MFR) = (d-period Positive MF)/(d-period Negative MF).
4. Determine the MFI = 100 - 100/(1 + MFR) .

- **Format**

  {mfi,s}= mfi(x,y,z,v,d,up,low)

- **Input**
  - x: closing price vector.
  - y: lowest price vector.
  - z: highest price vector.
  - v: asset volume vector.
  - d: number of periods (e.g. trading days) used to compute the MFI indicator.
  - up: upper MFI limit.
  - low: lower MFI limit.

- **Output**
  - mfi: the computed MFI indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

PPO

- **Purpose**

  Compute the "Percentage Price Oscillator" (PPO) indicator. The PPO is a momentum oscillator that measures the difference between two moving averages as a percentage of the larger moving average. The value of the PPO becomes increasingly positive as the shorter moving average distances itself from the longer moving average reflecting a strong upside momentum. For negative values of the PPO, this indicates that the shorter moving average is below the longer moving average. Increasing negative values indicate that the shorter moving average is distancing from the longer moving average, reflecting strong downside momentum. There are two steps to calculate the PPO indicator:

1. Calculate the PPO:$\{(d_1 - period - EMA(x)) - (d_2 - period - EMA(x))\}/(d_2 - period - EMA(x)) * 100$.
2. Determine the Signal Line: $d_3 - period - EMA(x)$ of PPO.

- **Format**

  {ppo,s}= ppo(x,d_1,d_2,d_3,b)

- **Input**
  - x: closing price vector.
  - d_1: number of periods (e.g. trading days) used to compute the short-run EMA of the PPO indicator.
  - d_2: number of periods (e.g. trading days) used to compute the long-run EMA of the PPO indicator.
  - d_3: number of periods (e.g. trading days) used to compute the indicator signal, which is an EMA of the PPO indicator.
  - b: fixed band or threshold to initiate a trading position.

- **Output**
  - ppo: the computed PPO indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

PVO

- **Purpose**

  Compute the "Percentage Volume Oscillator" (PVO) indicator. The PVO is a version of the PPO, based on the asset volume. There are two steps to calculate the PVO indicator:

  1. Calculate the PVO:$\{(d_1 - period - EMA(v)) - (d_2 - period - EMA(v))\}/(d_2 - period - EMA(v)) * 100$.
  2. Determine the Signal Line: $d_3 - period - EMA$ of PVO.

- **Format**

  {pvo,s}= pvo(x,d_1,d_2,d_3,b)

- **Input**
  - v: asset volume vector.
  - d_1: number of periods (e.g. trading days) used to compute the short-run EMA

of the PVO indicator.

  - d_2: number of periods (e.g. trading days) used to compute the long-run EMA of the PVO indicator.
  - d_3: number of periods (e.g. trading days) used to compute the indicator signal, which is an EMA of the PVO indicator.
  - b: fixed band or threshold to initiate a trading position.

- **Output**
  - pvo: the computed PVO indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

ROC

- **Purpose**

  Compute the “Rate of Change” (ROC) indicator. This indicator is referred to as Momentum. It is an technical analysis indicator that measures the percentage change in stock price from one period to the next. The ROC compares the current price to the price "t" periods ago, and fluctuates above and below the zero line. Moreover, the ROC is used by combining its signal with the divergence in stock price, called ROC - Divergence. In this case a buy(sell) signal is produced if for a lower price, the current ROC value is higher than its previous value. The ROC indicator is:

  1. ROC $=[(x_t - x_{t-d})/x_{t-d}] * 100$.

- **Format**

  {roc,s}= roc(x,d,b)

- **Input**
  - x: closing price vector.
  - d: number of periods (e.g. trading days) used to compute the ROC indicator.
  - b: fixed band or threshold to initiate a trading position.

- **Output**
  - roc: the computed ROC indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

RSI

- **Purpose**

Compute the "Relative Strength Index" (RSI) indicator. The RSI is an indicator which measures the speed and change of stock price movements. The RSI oscillates between zero and 100. The indicator is considered overbought when above 70 and oversold when below 30. There is a modification of this indicator called RSI - Divergence, which compares the generated cross-over signal to buy or sell to its maximum or minimum level for some price level. There are four steps to calculate the RSI indicator:

1. Find the first Average Gain = $\sum_d gains/d$ and the first Average Lost = $\sum_d lost/d$.
2. Calculate the Average Gain(AG) = $[previousAG * d + currentAG * (d-1)]/d$.
3. Calculate the Average Lost(AL) = $[previousAL * d + currentAL * (d-1)]/d$.
4. Compute the RSI =$100 - [100/(1 + RS)]$, where $RS = AG/AL$.

- **Format**

  {rsi,s}= rsi(x,d,up,low)

- **Input**
  - x: closing price vector.
  - d: number of periods (e.g. trading days) used to compute the RSI indicator.
  - up: upper RSI limit.
  - low: lower RSI limit.

- **Output**
  - rsi: the computed RSI indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

STO

- **Purpose**

  Compute the "Stochastic Oscillator" (STO) indicator. The STO measures the level of the stock price close relative to the high-low range over a given period of time. When the STO is above 50 the indicator signals that the closing price is in the upper half of the range. Contrarily, when it is below 50, this indicates the closing price is in the lower half. A STO reading below 20 signals that the price is close to its lowest level for the given time period. However, for high readings (above 80) the rule indicates that the price is close to its highest level. There are two other versions of Stochastic Oscillator which use an EMA of the STO to generate cross-over signals to buy or sell. These are the fast and slow STO. There are two steps to calculate this indicator:

  1. Define the lowest low and highest high for the period d_1, $Lowest{-}y$ and $Highest{-}z$, respectively.
  2. Determine the STO =$(x - Lowest - y)/(Highest - z - Lowes - y) * 100$.

3. Compute the Signal Line (SL) to a $d_2 - period$ simple moving average of STO.

- **Format**

  {sto,s}= sto(x,z,y,d_1,d_2,up,low)

- **Input**
  - x: closing price vector.
  - y: lowest price vector.
  - z: highest price vector.
  - d_1: number of periods (e.g. trading days) used to compute the STO indicator.
  - d_2: number of periods (e.g. trading days) used to compute the SL.
  - up: upper STO limit.
  - low: lower STO limit.

- **Output**
  - sto: the computed STO indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

WRI

- **Purpose**

  Compute the"William %R" (WRI) indicator. The WRI is an indicator which reflects the level of the stock price close relative to the highest high´ for a look-back period. The WRI oscillates from 0 to -100. Readings from 0 to -20 are considered overbought. Readings from -80 to -100 are considered oversold. The WRI indicator is:

  1. Define the lowest low and highest high for the period d, $Lowest-y$ and $Highest-z$, respectively.
  2. Compute the WRI =$(Highest - z - x)/(Highest - z - Lowes - y) * 100$.

- **Format**

  {wri,s}= wri(x,y,z,d,up,low)

- **Input**
  - x: closing price vector.
  - y: lowest price vector.
  - z: highest price vector.
  - d: number of periods (e.g. trading days) used to compute the WRI indicator.

  - up: upper WRI limit.
  - low: lower WRI limit.

- **Output**
  - wri: the computed WRI indicator vector.
  - s: signal vector for buy (1), sell (-1) or no action (0).

Estimation Routines

financial_prediction_MTD_probit

- **Purpose**

  Perform the calculation of the return of the MTD-Probit model.

- **Format**

  {return}= financial_prediction_MTD_probit(p,s,q,log(x))

- **Input**
  - p: MTD-Model estimated probabilities.
  - s: signal vector for buy (1), sell (-1) or no action (0), based on the "trend reversal strategy" (TDS).
  - q: categorization parameter.
  - x: closing price vector.

- **Output**
  - return: the computed strategy return vector.

matrix_financial_MTDTAI_1

- **Purpose**

  Perform the calculation of the return of the MTD-Probit noise reduction model.

- **Format**

  {return}= matrix_financial_MTDTAI_1(p,s,q,log(x))

- **Input**
  - p: MTD-Model estimated probabilities and technical analysis indicators (TAI) parameters .
  - s: signal vector for buy (1), sell (-1) or no action (0), based on the "one-day strategy" (ODS).
  - q: categorization parameter.
  - x: closing price vector.

- **Output**
  - return: the computed strategy return vector.

matrix_financial_MTDTAI_2

- **Purpose**

  Perform the calculation of the return of the MTD-Probit noise reduction model.

- **Format**

  {return}= matrix_financial_MTDTAI_2(p,s,q,log(x))

- **Input**

  – p: MTD-Model estimated probabilities and technical analysis indicators (TAI) parameters .

  – s: signal vector for buy (1), sell (-1) or no action (0), based on the "trend reversal strategy" (TDS).

  – q: categorization parameter.

  – x: closing price vector.

- **Output**

  – return: the computed strategy return vector.

matrix_financial_TAI_1

- **Purpose**

  Perform the calculation of the return of the technical analysis indicators (TAI) strategies.

- **Format**

  {return}= matrix_financial_TAI_1(p,s,q,log(x))

- **Input**

  – p: TAI parameters.

  – s: signal vector for buy (1), sell (-1) or no action (0), based on the "one-day strategy" (ODS).

  – q: categorization parameter.

  – x: closing price vector.

- **Output**

  – return: the computed strategy return vector.

matrix_financial_TAI_2

- **Purpose**

  Perform the calculation of the return of the technical analysis indicators (TAI) strategies.

- **Format**

  {return}= matrix_financial_TAI_2(p,s,q,log(x))

- **Input**

  - p: TAI parameters.
  - s: signal vector for buy (1), sell (-1) or no action (0), based on the "trend reversal strategy" (TDS).
  - q: categorization parameter.
  - x: closing price vector.

- **Output**

  - return: the computed strategy return vector.

Markov Chain Routines

categoriza – Nicolau(2014)

- **Purpose**

  Aggregate a continuous time series process into a discrete state space sequence of finite states.

- **Format**

  {s}= categoriza(r,q)

- **Input**
  - r: nxk vector.
  - q: (m-1)x1 vector, where m is the number of categories.

- **Output**
  - s: nxk vector.

fv_mul_MC_probit (Nicolau, 2014)

- **Purpose**

  Perform the maximum likelihood estimation of the MTD-Probit model.

- **Format**

  fv_mul_mc_probit(b,data)

- **Input**
  - b: vector computed in the "MMC_5_probit" routine.
  - data: data vector.

- **Output**
  - the log likelihood function.

MMC_5_log_HOMC

- **Purpose**

  Compute the maximum log-likelihood estimates for a first-order to the fifth-order Markov chain process.

- **Format**

  {b,fn,cov,t,p_value}= MMC_5_log_HOMC(S,p,&fv_mul_mc_probit)

- **Input**
  - S: vector computed in the “categoriza” routine.
  - p: transition probability matrix (TPM) computed in the “multivariate_Markov_Chain_02” routine.
  - &fv_mul_mc_probit: log-likelihood function computed in the “fv_mul_mc_probit” .

- **Output**
  - b: the estimated parameter vector.
  - fn: function at minimum (the mean log-likelihood).
  - cov: covariance matrix of the parameters.
  - t: the value of the t-test for the significance of the parameters.
  - p_value: p-value of t.

MMC_5_probit

- **Purpose**

  Compute the maximum log-likelihood estimates for the MTD-Probit model with five co-variables.

- **Format**

  {b,fn,cov,t,p_value}= MMC_5_probit(S,p,&fv_mul_mc_probit)

- **Input**
  - S: vector computed in the “categoriza” routine.
  - p: transition probability matrix (TPM) computed in the “multivariate_Markov_Chain_01” routine.
  - &fv_mul_mc_probit: log-likelihood function computed in the “fv_mul_mc_probit” .

- **Output**
  - b: the estimated parameter vector.
  - fn: function at minimum (the mean log-likelihood).
  - cov: covariance matrix of the parameters.
  - t: the value of the t-test for the significance of the parameters.

- p_value: p-value of t.

MMC_5_probit_forecast

- **Purpose**

  Compute the maximum log-likelihood estimates for the MTD-Probit model for a first-order to fifth-order Markov chain process.

- **Format**

  {prob}= MMC_5_probit_forecast(b,p,S)

- **Input**
  - b: the estimated parameter vector computed in the “categoriza” routine.
  - p: transition probability matrix (TPM) computed in the “multivariate_Markov_Chain_01” routine.
  - S: vector computed in the “categoriza” routine.

- **Output**
  - prob: the estimated TPM.

MMC_5_probit_HOMC

- **Purpose**

  Compute the maximum log-likelihood estimates for the MTD-Probit model for a first-order to the fifth-order Markov chain process.

- **Format**

  {b,fn,cov,t,p_value}= MMC_5_probit_HOMC(S,p,&fv_mul_mc_probit)

- **Input**
  - S: vector computed in the “categoriza” routine.
  - p: transition probability matrix (TPM) computed in the “multivariate_Markov_Chain_02” routine.
  - &fv_mul_mc_probit: log-likelihood function computed in the “fv_mul_mc_probit” .

- **Output**
  - b: the estimated parameter vector.
  - fn: function at minimum (the mean log-likelihood).

- cov: covariance matrix of the parameters.
- t: the value of the t-test for the significance of the parameters.
- p_value: p-value of t.

multivariate_Markov_Chain_01 (Nicolau, 2014)

- **Purpose**

  Estimates the transition probability matrix (TPM), the frequency matrix (FM) and the marginal probabilities for a first-order Markov chain process (Ching an Ng, 2006).

- **Format**

  {f,p,x_0}= multivariate_Markov_Chain_01(S)

- **Input**
  - S: vector computed in the "categoriza" routine.

- **Output**
  - f: the estimated FM.
  - p: the estimated TPM.
  - x_0 : the estimated marginal probabilities.

multivariate_Markov_Chain_02

- **Purpose**

  Estimates the transition probability matrix (TPM), the frequency matrix (FM) and the marginal probabilities for a second-order Markov chain process.

- **Format**

  {f,p,x_0}= multivariate_Markov_Chain_02(S)

- **Input**
  - S: vector computed in the "categoriza" routine.

- **Output**
  - f: the estimated FM.
  - p: the estimated TPM.
  - x_0 : the estimated marginal probabilities.

stationary _bootstrap

- **Purpose**

  Compute the “stationary bootstrap” of Politis and Romano (1994).

- **Format**

  {x_new}= stationary_boot(x,q)

- **Input**

  – x: data vector.

  – q: geometric distribution parameter.

- **Output**

  – x_new: the computed bootstrap pseudo vector.

Test Routines

mad

- **Purpose**

  Compute the "Mean Absolute Deviation" (MAD).

- **Format**

  {mad}= mad(x)

- **Input**

  - x: data vector.

- **Output**

  - mad: the compute MAD value.

polansky_homogeneity

- **Purpose**

  Compute the Markov chain time-homogeneity test, based on Polansky (2007) methodology.

- **Format**

  {T_0,T}= polansky_homogeneity (S,min,size, &multivariate_markov_chain_log_HOMC,&MMC
  &teste_bic_aic)

- **Input**

  - S: vector computed in the "categoriza" routine.
  - min: minimum size for the initial transition probability matrix (TPM).
  - size: incremental size parameter.
  - &multivariate_markov_chain_log_HOMC: routine insertion.
  - &MMC_5_log_HOMC: routine insertion.
  - &teste_bic_aic: routine insertion.

- **Output**

  - T_0: the estimated maximum log likelihood for a time-homogeneous Markov chain process.
  - T: the estimated maximum log likelihood for a Time-inhomogeneous Markov chain process, up to six change-points.

polansky_time

- **Purpose**

  Compute the Markov chain time-dependence test, based on the BIC (Schwarz, 1978) criterion and the Polansky (2007) methodology.

- **Format**

  {test_polansky}= polansky_time (S, &multivariate_markov_chain_log_HOMC, &MMC_5_log

- **Input**

  - S: vector computed in the "categoriza" routine.
  - &multivariate_markov_chain_log_HOMC: routine insertion.
  - &MMC_5_log_HOMC: routine insertion.
  - &bic: routine insertion.

- **Output**

  - test_polansky: the time-dependence test result.

test_bic_aic

- **Purpose**

  Compute the Akaike (Akaike, 1975) and the BIC (Schwarz, 1978) criteria applied to the Polansky (2007) methodology.

- **Format**

  {BIC, AIC}= test_bic_aic (S,T,T0)

- **Input**

  - S: vector computed in the "categoriza" routine.
  - T_0: the estimated maximum log likelihood for a time-homogeneous Markov chain process compute in the "polansky_homogeneity" routine.
  - T: the estimated maximum log likelihood for a Time-inhomogeneous Markov chain process compute in the "polansky_homogeneity" routine.

- **Output**

  - BIC: the computed BIC value.
  - AIC: the computed AIC value.

test_Markov_Chain_Time_Dependence_01

- **Purpose**

  Compute the Markov chain time-dependence test, based on the Anderson and Goodman (1957) chi-square methodology. The null hypothesis is zero-order against the alternative of first-order.

- **Format**

  {quitest1,res1}= test_markov_chain_time_01 (S, qua, &mad, &categoriza, &multivariate_markov_chain_01)

- **Input**
  - S: vector computed in the "categoriza" routine.
  - qua: categorization parameter.
  - &mad: routine insertion.
  - &categoriza: routine insertion.
  - &multivariate_markov_chain_01: routine insertion.

- **Output**
  - quitest1: the computed test statistic result.
  - res1: the chi-square distribution test p-value.

test_Markov_Chain_Time_Dependence_12

- **Purpose**

  Compute the Markov chain time-dependence test, based on the Anderson and Goodman (1957) chi-square methodology. The null hypothesis is first-order against the alternative of second or higher-order.

- **Format**

  {quitest1,res1}= test_markov_chain_time_12 (S, qua, &mad, &categoriza, &multivariate_markov_chain_01)

- **Input**
  - S: vector computed in the "categoriza" routine.
  - qua: categorization parameter.
  - &mad: routine insertion.
  - &categoriza: routine insertion.
  - &multivariate_markov_chain_01: routine insertion.

- **Output**
  - quitest1: the computed test statistic result.
  - res1: the chi-square distribution test p-value.

test_Markov_Chain_Time_Homogeneity

- **Purpose**

  Compute the Markov chain time-homogeneity test, based on the Anderson and Goodman (1957) chi-square methodology.

- **Format**

  {quitest1,res1}= test_markov_chain_homogeneity (S, d)

- **Input**
  - S: vector computed in the "categoriza" routine.
  - d: number of sub-samples.

- **Output**
  - quitest1: the computed test statistic result.
  - res1:the chi-square distribution test p-value.

test___Polansky_bootstrap

- **Purpose**

  Compute the Polansky (2007) time-homogeneity test.

- **Format**

  {p_value_bic, p_value_aic}= test_polansky_bootstrap (S,Q,bic /aic, p,&multivariate_Markov_

- **Input**
  - S: vector computed in the "categoriza" routine.
  - Q: estimated change-points computed in the "test_bic_aic" routine.
  - bic/aic: the estimated BIC or AIC computed in the "test_bic_aic" routine.
  - p: the estimated transition probability matrix (TPM) computed in "MMC_5_log_HOMC_5 routine.
  - &multivariate_Markov_Chain_HOMC_5: routine insertion.
  - &MMC_5_log_HOMC_5: routine insertion.

- **Output**
  - p_value_bic: the computed Polansky (2007) BIC p-value.
  - p_value_aic: the computed Polansky (2007) AIC p-value.
  - res1: the chi-square distribution test p-value.