

# Modélisation stochastique et approche fiabiliste pour la sécurité routière des poids lourds articulés

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## Résumé :

L'objectif de cet article est le développement d'une stratégie fiabiliste pour la prévention des risques de renversement et de mise en portefeuille des véhicules, de type poids lourd (tracteur et semi-remorque), en zone accidentogène. Le caractère aléatoire du triplet conducteur-véhicule-infrastructure est intégré à l'aide de variables et processus aléatoires dans un modèle dynamique spécifiquement développé. Une analyse de sensibilité globale ainsi qu'une analyse de fiabilité de type FORM/SORM permettent l'évaluation des probabilité de défaillance et la détection de situations à risque.

## Abstract :

The aim of this paper is to develop a stochastic based approach to prevent rollover and jackknifing risk of articulated heavy vehicle. A specific 6-DOF heavy vehicle model is developed and safety criteria are used. Parameters involved in the model are modeled by random variables or stochastic processes in order to take into account uncertainties. These parameters are deduced from two different sensitivity analysis methods. Then, structural reliability methods FORM/SORM are employed to assess the probability of failure and compared to Monte-Carlo based method.

**Mots clefs :** articulated heavy vehicle ; sensitivity analysis ; reliability analysis

## 1 Introduction

At present time, the truck transport is one of the most important activity of the country's economy. According to the French road safety statistics (*ONISR*) for year 2008 [10], accidents involving heavy vehicles have serious consequences for road users and incidents induce major congestions or damage to the environment or the infrastructure at disproportionate economic costs. The risk of having dead people in accidents involving trucks is multiplied by 2.4 in comparison to the same risk computed for accidents involving only light vehicles, mainly because of the important gross mass difference between light vehicles and trucks. Many real-time active safety systems have been developed to control vehicle stability in dangerous conditions : rollover, jackknifing, roadway departure... : ABS, EPS, RSP, VEST... Nevertheless all these systems don't detect and warn drivers early enough for preventing risky situations. Consequently, the aim is to develop a driving support system, embedded in the heavy vehicle and communicate with on road equipments, to warn about risk's level early enough.

Little work is available in the literature concerning combined probabilistic analysis and vehicle dynamics studies. Is mostly deals with suspensions analysis under uncertain characteristics and loading

[6]. One Icelandic study [15] covers the wind-related accident issue with reliability analysis, but no warning system has been envisaged. More recently, classical reliability methods are deployed in [13, 11] to prevent rollover risk of single heavy vehicle and roadway departure risk of light vehicle.

## 2 Heavy vehicle dynamics model

The framework of this study is focused on an articulated heavy vehicle composed of a rigid tractor with 2 axles and a rigid semitrailer with 3 axles. These two components are linked at the fifth wheel as shown in figure 1. The tractor is the superposition of an unsprung mass (axles and tires) and a sprung mass (cabine and chassis). The trailer is composed of an unsprung mass (axles and tires) and a sprung mass (chassis and trailer). Unsprung and sprung masses are connected together with suspensions. To obtain the dynamics equations of simplified heavy vehicle, the classical Lagrangian formulation is used.

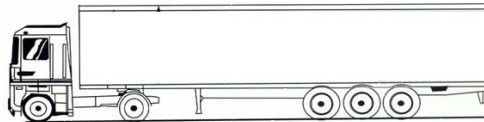


FIGURE 1 – Articulated heavy vehicle

The proposed model is based on 6 degrees of freedom (*DOF*) *yaw-roll* model derived from a 5-DOF [17, 3] with  $q = (x, y, z, \psi, \psi_f)^T$ . The vector equation describing the dynamics of the heavy vehicle is obtained using the Lagrangian formulation. It's of the form :

$$M(q(t), p)\ddot{q}(t) + C(q(t), \dot{q}(t), p)\dot{q} + G(q(t), p, ) = F_g(p, \delta(t)), \quad t \geq 0 \quad (1)$$

where  $F_g$  is the vector of generalized forces,  $M$  is the inertial matrix that is symmetric positive definite,  $C(q, \dot{q})\dot{q}$  is the combined Coriolis and centrifugal forces and  $G$  is the gravity vector. The generalized forces  $F_g$  represent the effect of external forces acting on the vehicle body. These later result from the tire-road interface and suspensions defined in terms of longitudinal and lateral tire forces and vertical forces. Generalized forces depend on steering angle  $\delta$ . All heavy vehicle's parameters are gathered in the vector  $p \in \mathbb{R}^{n_p}$ , where  $n_p = 25$ . Then, we rewrite the vehicle model as a first-order differential equations (ODE) system as :

$$\dot{\mathbf{u}}(t) = f(t, \mathbf{u}(t), p, \delta(t)), \quad t \geq 0 \quad (2)$$

where  $\mathbf{u} = [q^T, \dot{q}^T]^T$  and  $f$  is a function from  $\mathbb{R}_+ \times \mathbb{R}^{12} \times \mathbb{R}^{25} \times \mathbb{R}$  into  $\mathbb{R}^{12}$ .

This equation is solved by using classical Runge-Kutta method of order 4. Such a model easily brings forth useful insights of dynamic phenomena (*yaw-roll*) with fast computation time, compared to multibody approach. The system is simulated and validated in the Matlab environment with a specific ODE Fortran solver. Parameters are obtained from Prosper truck simulator [14]. In practice, heavy vehicle simulation requires only 50 ms compared to a quasi-realtime simulator (Simulink, Prosper, TruckSim...). These improvements allow to embedde our heavy vehicle simulator into sensitivity and reliability algorithms.

## 3 Safety criteria

The framework of this work is focused on two risk situations :

- 1 - the rollover, which is a lateral instability due to a lateral load transfer.
- 2 - the jackknifing, which is a loss of control that causes the rotation of the tractor with respect to the semitrailer.

For each risk we propose and choose a safety criterion to assess and detect dangerous situations. Rollover is a well-known phenomena and its analysis is spread enough in the litterature [8]. Unfortunately, jackknifing risk is a more complicated phenomena and there is few safety criteria.

**Rollover criteria** Rollover is one of the most frequent accidents (20%) and causes significant damages to the vehicles and injuries to its driver and passagers. Several anti-rollover systems and rollover warnings systems were developed to assist and warn the driver [7, 1]. In our study, rollover risk evaluation is based on the maximum of a rollover risk indicator, namely the load transfer ratio (LTR), which corresponds to the load transfer between the left and the right sides of the vehicle. The resulting expression of this indicator is defined as :

$$LTR := \frac{F_{z,l} - F_{z,r}}{F_{z,l} + F_{z,r}} \quad (3)$$

where  $F_{z,l}$  and  $F_{z,r}$  are respectively the left and the right normal forces. In practice, when  $LTR$  is equal to 0, the heavy vehicle has stable roll dynamics. The risk becomes high as this indicator goes towards  $\pm 1$ .

**Jackknifing criterion** Jackknifing is characterized by a loss of stability in the yaw motion of the articulated system. This phenomena is more frequent when the trailer is empty or when the load is badly distributed in the trailer. Theoretically, jackknifing is detected when the relative angle  $\psi_f$  is greater than  $\pi/2$ .

Jackknifing is characterized by the friction indicator [4] :

$$\mu_{min} = \frac{F_y}{\cos(\psi_f)F_z}$$

where  $F_y$  and  $F_z$  are respectively the lateral and normal road forces applied to the heavy vehicle. When  $\mu_{min} \leq 0.1$  the heavy vehicle remains stable.

## 4 Sensitivity analysis

Among all the parameters of the mechanical model, some present a marked random variability. Therefore it is crucial for the credibility of the application to take into account this reality via a suited stochastic modeling. The object of this section is to identify the minimal family of variables that must be considered as random using a sensitivity analysis. We investigate a global sensitivity method is performed by computing the Sobol indices.

The parameters are gathered in the vector  $p \in \mathbb{R}^{n_p}$  modeled as a continuous  $\mathbb{R}^{n_p}$ -valued random variable denoted  $\mathbf{P} = (P_1, \dots, P_{n_p})$  for which the following hypotheses are made :

- (H1) its components are mutually independent,
- (H2) two distributions are alternatively considered for each of its components : a truncated Gaussian distribution and an uniform distribution,
- (H3) all the components of  $\mathbf{P}$  follow simultaneously the same type of distribution,
- (H4) all the components of  $\mathbf{P}$  have the same coefficient of variation.

Each random variable  $P_i$  verifies :

$$\begin{aligned} \mathbb{E}(P_i) &= p_i \\ \sigma(P_i) &= \kappa \mathbb{E}(P_i) = \kappa p_i \end{aligned}$$

where  $\sigma(P_i)$  and  $\kappa$  are respectively the standard deviation and the coefficient of variation of  $P_i$ . In the following, this later is taken equal to 0.01.

In order to take into account perturbations on steering angle  $\delta$ , this one is modeled as bounded stochastic process of the form [11] :

$$\Delta(t) = \delta(t) + \Lambda(t)$$

with :

$$\Lambda(t) = \varepsilon \sin(\nu t + \sigma W(t) + 2\pi\Theta) \quad , \quad t \in \mathbb{R}_+$$

where  $\varepsilon$ ,  $\nu$ ,  $\sigma$  are given real positive constants,  $W$  is a standard real Wiener process and  $\Theta$  is a random variable uniformly distributed on  $[0, 1]$  and independent of  $W$ .

With these stochastic modelings, the response system  $\mathbf{u}$  is now a vector random process  $\mathbf{U}$  which depends on  $(\mathbf{P}, \Delta)$  and the control variable  $r$  becomes a random process  $R$  depending on  $\mathbf{P}$  and  $\Delta$  :

$$R(t) = \mathcal{S}(\mathbf{P}, \Delta(t)) = \mathcal{S}(P_1, \dots, P_{n_p}, \Delta(t)). \quad (4)$$

The Sobol index  $s_i^R$  [16, 12] associated with the random variable  $P_i$  is a deterministic function of  $t$  defined from the conditional expectation  $\mathbb{E}[R|P_i]$ , according to the formula :

$$s_i^R = \frac{\text{Var}(\mathbb{E}[R|P_i])}{\text{Var}(R)} = \frac{V_i}{V} \quad (5)$$

where  $\text{Var}(\cdot)$  denotes the variance operator.

From this definition,  $s_i^R$  ranges from 0 to 1. A small value means that uncertainty on  $P_i$  has few influence on the variability of  $R$  and consequently, in this case,  $P_i$  can be considered as a deterministic parameter. On the contrary if  $s_i^R$  is closed to 1,  $P_i$  must keep its status of random variable.

The sensitivity of  $R$  with respect to  $\Delta$  is estimated from the Iooss's work [9] .

Sobol's indices are computed with a huge sample  $10^6$  to obtain a good first order indices. In fact, sums of Sobol indices are plotted on figure 2 and are closed to 1.

Figures 2(a) and 2(b) show the obtained results for the rollover and jackknifing criteria, using the Sobol sensibility analysis with perturbations on the seering angle. Results exhibit that only 7 parameters  $d_3, l_2, l_3, T_{w,3}, h_2, v_0$  and  $\delta$  are really influent on rollover and only 6 parameters  $d_3, l_2, l_3, h_1, h_2, v_0$ , and  $\delta$  on jackknifing.

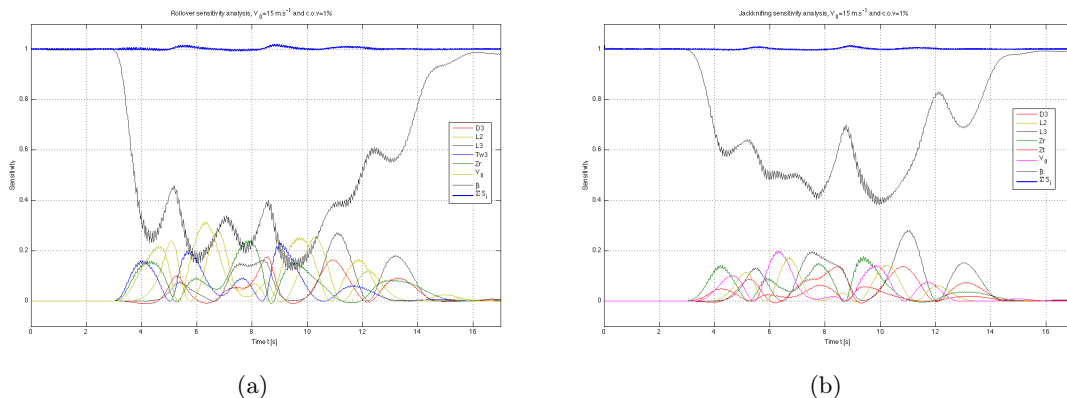


FIGURE 2 – Evolution of first order Sobol indices for rollover criteria (a) and jackknifing criteria(b)

For simplicity and to apply standard reliability analysis, the steering angle perturbation is supposed is not taken into considerations. As a result, the only random parameters considered are at most the six random variables  $P_1 = v_0, P_2 = h_r, P_3 = l_2, P_4 = l_3, P_5 = h_2, P_6 = T_{w,3}$ .

## 5 Reliability analysis

In the following the vector random parameter  $\mathbf{P} = (P_1, \dots, P_6)^T$  is assumed to be defined on the probability space  $(\Omega, \mathcal{F}, P)$ , where  $\Omega$  is a sample space,  $\mathcal{F}$  is  $\sigma$ -algebra on  $\Omega$  and  $P$  is a probability measure on  $\mathcal{F}$ . According to (H1)-(H4) hypotheses (cf. section 4.2), the distribution is known and admits a probability density denoted in that follows  $f_{\mathbf{P}}$ .

Let  $Z$  be the safety margin associated with the control variable  $R$ , such that :

$$Z = r_0 - \max_{t \in T} |R(t)|$$

where  $r_0$  is a given limit value and  $T$  is the observation interval. This real random variable is a function of  $\mathbf{P}$  :

$$Z = G(\mathbf{P})$$

where  $G$  is the limit state function associated with the safety criterion chosen for the study. The main objective of the reliability analysis is then to evaluate the probabilities of failure  $P_f$  defined by :

$$P_f = P(E_f) = \int_{\mathbb{R}^6} f_{\mathbf{P}}(p) \mathbb{1}_{D_f} dp$$

where  $D_f$  is the failure domain and  $f_{\mathbf{P}}$  is the joint probability function of  $\mathbf{P}$

In practice, an exact calculation of  $P_f$  is not possible and a Monte-Carlo procedure must be used (Crude Monte-Carlo, importance sampling, directional sampling...)

FORM approximation consists in linearizing the failure domain  $D_f$  at the design point  $M^*$ . The failure probability  $P_f$  can be approximated by :

$$P_f \approx \Phi(-\beta_{HL})$$

where  $\Phi$  is the one-dimensional standard Gaussian distribution function and  $\beta_{HL}$  is the Hasofer-Lind index [5].

Monte-Carlo method is employed to estimate the "exact" probability of failure. A set of  $10^6$  simulated realizations of  $\mathbf{P}$  is used for an initial velocity ranges from 20 m.s<sup>-1</sup> to 24 m.s<sup>-1</sup>. Each calculation requires about one hour of computations on a standard PC but we have improved the numerical procedure by distributing the computations on a 27-nodes cluster which allows to provide the set of all failure probabilities in one hour. The MatLAB toolbox FERUM [2] has been used to evaluate  $P_f$ . Figure 3 compares the probabilities of failure given by Monte-Carlo method and the FORM/SORM methods.

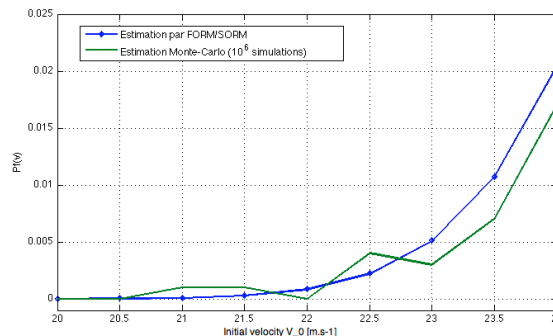


FIGURE 3 – Evolution of probability of failure  $P_f^L$  and  $P_f$  with respect to the initial velocity.

## 6 Conclusion

A sensibility analysis have been presented for a mechanical model describing the dynamics of the articulated heavy vehicle. They have shown that only 6 parameters among the 25 initial parameters are really influent towards rollover and jackknifing safety criteria. An application of structural reliability methods to the proposed mechanical model has shown that the failure probability can be correctly approximated using the classical FORM and SORM methods.

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