Failure of elasto-plastic porous materials due to void shape effects and void growth

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Résumé :

Cette étude utilise le modelé de l'homogénéisation non linéaire du second-ordre (SOM) pour des matériaux poreux (visco)plastique afin d'étudier l'influence du paramètre de Lode et de triaxialité de contraintes sur la rupture ductile de matériaux métalliques. Ce modelé, basé sur la méthode de l'homogénéisation non linéaire du "second-ordre" ou "sécant généralisée", est capable de prendre en compte la forme des pores ellipsoïdales (microstructures particulaires avec une anisotropie macroscopique générale), ainsi que les conditions aux limites tridimensionnelles générales.

Abstract :

This work makes use of the recently proposed second-order nonlinear homogenization model (SOM) for (visco)plastic porous materials to study the influence of the Lode parameter and the stress triaxiality on the failure of metallic materials. This model is based on the "second-order" or "generalized secant" homogenization method and is capable of handling general "ellipsoidal" void shapes (i.e., particulate microstructures with more general orthotropic overall anisotropy) and general three-dimensional loading conditions.

Mots clefs : 3 maximum : Ductile fracture ; Porous materials ; Homogenization

1 Introduction

Failure and ductile fracture of metallic materials has received a lot of attention the last sixty years. One of the main reasons for material failure is the presence or/and nucleation of voids and micro-cracks which tend to evolve in volume fraction, shape orientation as a result of the applied loading conditions. For several years, it was believed that the stress triaxiality, denoted here as X_Σ and defined as the ratio between the mean stress to the von Mises equivalent or effective deviatoric stress, is the main loading parameter that controls ductile fracture. In particular, large amount of experimental data [3, 4] has shown a monotonic decrease of material ductility with the increase of the stress triaxiality. Nonetheless, recent experimental evidence [5, 6] indicate a substantial decrease of the material ductility with decrease of stress triaxiality and certain shear loading conditions. In these studies, it has been identified that a second loading parameter, the Lode parameter, L (or equivalently Lode angle, θ) controls the ductile fracture mechanism at low stress triaxialities. The Lode parameter is a function of the third invariant of the stress deviator and is used to distinguish between the different shear stress states that can be present in a loading history (see details in the following section). In the present work, we make use of the "second-order" model [1] (SOM) to study the influence of the Lode parameter and the stress triaxiality on ductile failure.

2 Modeling and results

We consider a porous material with initially spherical voids subjected to purely triaxial loading conditions with the principal stresses $\sigma_1 = \sigma_{11}$, $\sigma_2 = \sigma_{22}$ and $\sigma_3 = \sigma_{33}$ ($\sigma_{ij} = 0$ for $i \neq j$) being aligned with the laboratory frame axes, $\mathbf{e}^{(1)}$, $\mathbf{e}^{(2)}$ and $\mathbf{e}^{(3)}$, respectively. This allows for the definition of alternative stress measures appropriate for dilatational plasticity, which is the case in the context of porous materials. The three alternative measures are the von Mises equivalent stress (or effective stress), σ_e, the mean stress, σ_m, and the third invariant of the stress deviator J_3 defined all by

$$\sigma_m = \sigma_{kk}/3, \qquad \sigma_e = \sqrt{3\,J_2} = \sqrt{3\,s_{ij}\,s_{ij}/2}, \qquad J_3 = \det(s_{ij}) = \frac{1}{3}\,s_{ij}\,s_{ik}\,s_{jk}, \tag{1}$$

where $s_{ij} = \sigma_{ij} - \sigma_m\,\delta_{ij}$ is the stress deviator. Using these definitions, we can readily define the stress triaxiality, X_Σ, and Lode parameter, L, or equivalently the Lode angle, θ, via the following expressions

$$X_\Sigma = \frac{\sigma_m}{\sigma_e}, \qquad L = -\cos 3\theta = \frac{27}{2}\frac{J_3}{\sigma_e^3}. \tag{2}$$

By definition, the range of values for the X_Σ and L, (or θ) are

$$-\infty < X_\Sigma < \infty, \quad \text{and} \quad -1 \leq L \leq 1 \quad \text{or} \quad 0 \leq \theta \leq \pi/3. \tag{3}$$

Then, relations (2) can be used to express the principal stresses as functions of X_Σ, σ_e and θ, such that

$$\frac{3}{2\sigma_e}\{\sigma_1,\sigma_2,\sigma_3\} = \{-\cos\left(\theta+\frac{\pi}{3}\right), -\cos\left(\theta-\frac{\pi}{3}\right), \cos\theta\} + \frac{3}{2}X_\Sigma\{1,1,1\}. \tag{4}$$

Fig. 1 shows the normalized principal stresses defined in (4) as a function of the Lode parameter

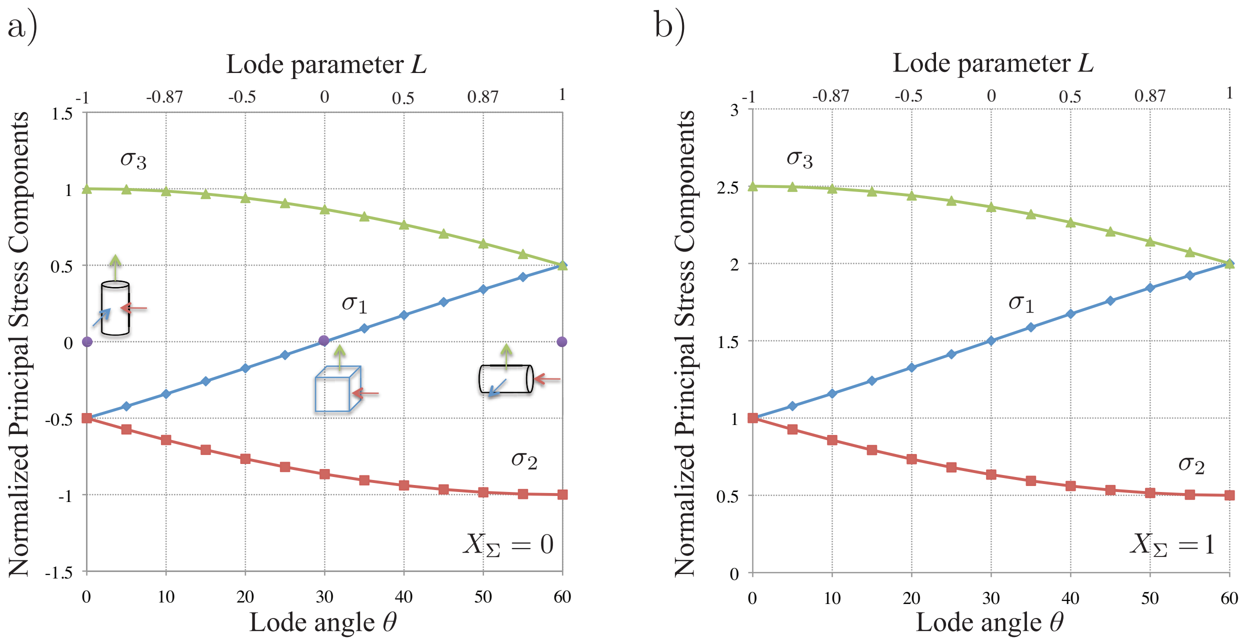

FIGURE 1 – Normalized principal stresses $\frac{3}{2\sigma_e}\{\sigma_1,\sigma_2,\sigma_3\}$, as a function of the Lode angle θ or equivalently the Lode parameter L. Parts (a) and (b) correspond to stress triaxialities $X_\Sigma = 0$ and $X_\Sigma = 1$, respectively.

L and Lode angle θ for (a) $X_\Sigma = 0$ and (b) $X_\Sigma = 1$. It is clear from Fig. 1a that for $L = -1$ or $\theta = 0$, the stress state is axisymmetric with one positive and two negative stresses (axisymmetric tension). On the other end, when $L = 1$ or $\theta = \pi/3$, the stress state is also axisymmetric but with two positive and one negative stresses (biaxial tension with axisymmetric compression). Note that these two different axisymmetric states lead to different evolution of the underlying microstructure and therefore to different overall responses as the deformation progresses. When, $L = 0$ or $\theta = \pi/6$, the stress state is in-plane shear with one stress identically equal to zero (e.g., plane stress state). The rest of the states are between axisymmetric and in-plane shear states. It should be noted that when the stress triaxiality is nonzero then the principal stresses are simply translated by a constant either upwards for $X_\Sigma > 0$, as shown in Fig. 1b for $X_\Sigma = 1$, or downwards for $X_\Sigma < 0$ (not shown here for brevity). Note also that $|X_\Sigma| \to \infty$ and $X_\Sigma = 0$ correspond to purely hydrostatic and purely deviatoric loadings, respectively.

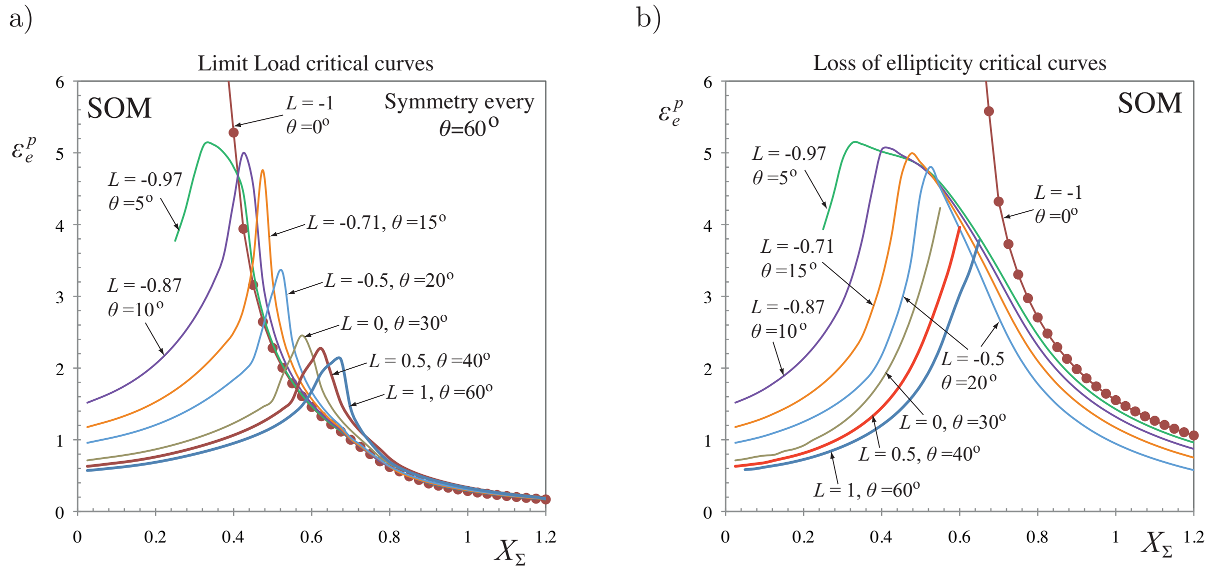

FIGURE 2 – (a) Limit load and (b) loss of ellipticity maps as predicted by the SOM model as a function of the stress triaxiality X_Σ and the Lode parameter L (or θ). The limit load locus is identified with the critical equivalent plastic strain ε_e^p where the overall hardening rate of the porous solid is $H = 0$. The loss of ellipticity locus is identified with the critical equivalent plastic strain ε_e^p where the constitutive equations lose strong ellipticity leading to localized bifurcated solutions and long-wavelength instabilities. The hardening exponent is $N = 0.1$ and the initial porosity $f_0 = 1\%$.

Making use of the SOM model, one may construct failure maps for elasto-plastic porous materials that are subjected to the above-described triaxial loading conditions. In this connection, Fig. 2 shows SOM maps of the critical equivalent plastic strain ε_e^p attained when (a) the limit load (i.e., maximum in the $\sigma_e - \varepsilon_e$ curve or equivalently overall critical hardening rate of the porous solid is $H = 0$) and (b) the conditions for localization [7] and loss of ellipticity (LOE) are reached as a function of the stress triaxiality X_Σ and the Lode parameter L (or Lode angle θ). We find that the overall strain at localization depends strongly on the Lode parameter as well as on the stress triaxiality. According to the SOM model, these effects are due to the collapse of voids at low stress triaxialities where the initially spherical pores tend to become micro-cracks. This void collapse mechanism leads to the formation of shear or dilation localization bands depending on the value of the Lode parameter. On the other hand, at large stress triaxialities the main mechanism of failure is the increase of porosity, which leads to the overall softening of the porous material. The interplay of these two different mechanisms of localization (i.e., void shape change and porosity growth), leads to sharp, high-ductility corners on the localization locus map.

3 Conclusions

The main finding of this work is that failure can occur by two very different mechanisms at high- and low-triaxiality. In agreement with well-established results, at high triaxialities, the model predicts significant *void growth* leading to a softening effect which eventually overtakes the intrinsic strain hardening of the solid material and produces overall softening. Thus, a limit load is reached at a critical strain that decreases with increasing triaxiality and is found to be independent of the Lode parameter. This limit load point is then followed by a significant reduction in the load-carrying capacity of the material and loss of ellipticity at least for negative values of the Lode parameter. On the other hand, at low triaxialities, the model predicts *void collapse* due to an abrupt flattening of the initially spherical voids with *decreasing* porosity, which in turn leads to a sharp drop in the load-carrying capacity of the porous solid. The precise value of the strain at the onset of the instability, which determines the overall ductility of the material, depends on the competition of the hardening produced by the reduction of the porosity and the softening due to the change in shape of the pores, and is highly sensitive to the value of the Lode parameter. Thus, for biaxial tension with axisymmetric compression ($L = 1$), the onset of the limit load instability, as well as the loss of ellipticity shortly thereafter, decreases as the triaxiality is reduced toward zero, while for axisymmetric tension ($L = -1$) no void collapse is possible and therefore no instability is observed for small values of the triaxiality. Moreover, for fixed, small values of the triaxiality ($X_\Sigma < 0.6$), the ductility of the porous material increases as the value of the Lode parameter decreases from $+1$ to -1. In addition, a sharp transition is observed as the failure mechanism switches from void collapse to void growth for intermediate values of the stress triaxiality ($0.3 < X_\Sigma < 0.7$), depending strongly on the value of the Lode parameter leading to high-ductility peaks in the failure maps. In this regard, the theoretical predictions are found to be in qualitative agreement with recent experimental observations by [5] and [6], even though it should be emphasized that the stress and deformation fields are not uniform in these tests and that the values of the triaxiality and Lode parameter are not controlled independently in these experiments. In this sense, the theoretical predictions presented in this work strongly suggest the need for experiments where the fields are made as uniform as possible and the triaxiality and Lode parameter are controlled independently of each other. Finally, results indicating clearly the effect of the evolution of microstructure will be detailed elsewhere.

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