Pseudo Inverse Approach for cold forging processes and its comparison with Adaptive Incremental Approach

F.J. MENGª, A.HALOUANIᵇ, C. LABERGEREª, Y.M. LIᵇ, B. ABBESᵇ, P. LAFONª, Y.Q. GUOᵇ

a. Institut Charles Delaunay LASMIS, Université de Technologie de Troyes
b. Groupe de Recherche en Sciences Pour l'Ingénieur, Université de Reims Champagne Ardenne

Abstract :

Numerical simulation for highly non-linear forming process requires efficient numerical methods. In an optimization loop, the calculation cost is very large, so it is important to choose a good numerical simulation tool to reduce it. In this paper, we propose to simulate a 2D axisymmetric forging process of a wheel by using a pseudo inverse approach and an adaptive incremental approach. These two methods will be compared in terms of computation time and predictability on the form of rough forging. The comparison of the results will show the efficiency and limitations of the Pseudo Inverse Approach.

Résumé :

La simulation numérique des procédés de mise en forme est un problème fortement non-linéaire qui nécessite des méthodes numériques performantes. Les coûts des calculs, souvent importants, rendent difficile l'utilisation d'un outil de simulation numérique dans une boucle d'optimisation. Nous proposons dans ce papier de simuler un procédé 2D axisymétrique de forgeage d'une roue en utilisant une approche pseudo inverse et une approche MEF adaptative incrémentale. Ces deux approches seront ensuite comparées en termes de temps de calcul et prédictibilité sur la forme des bruts de forge. La comparaison avec l'approche adaptive incrémentale montre l'efficacité et les limites de l'Approche Pseudo Inverse.

Keywords : **Metal Forming Processes, Numerical Simulation, Pseudo Inverse Approach, Adaptive Incremental Approach**

1   Introduction

In this paper, we propose to simulate a 2D axisymmetric forging process of a wheel by using a pseudo inverse approach and an adaptive incremental approach. These two methods use different material behaviour models and numerical technologies. The adaptive incremental approach (A.I.A.) is the classical method for the simulation of metal forming processes. It gives good strain and stress estimation, but is time consuming. A new approach called "Pseudo Inverse Approach" (P.I.A.) was proposed by Batoz, Guo et al. [1] for the sheet forming modelling, which keeps the rapidity of the Inverse Approach but gives good stress estimation owing to the loading history consideration. By comparing the results of the two methods, it can be found that the P.I.A. method is very efficient but less accurate than the incremental approach.

2   Methodology of the two numerical approaches

2.1   Pseudo inverse approach

A simplified method called "Inverse Approach" (I.A.) has been developed for the axisymmetrical cold forging modelling. The approach is based on the knowledge of the final part shape. Two basic assumptions are used: the assumption of proportional loading gives an integrated constitutive law without considering the strain path and the plastic flow, and the assumption of tool-billet actions allows one to replace the tool actions by nodal forces without contact treatment [2-3]. These two assumptions make the I.A. calculation very fast. This approach gives fairly good strain estimation but poor stress estimation.
The P.I.A., based on the traditional I.A., is developed to improve the stress estimation. The main

developments in the P.I.A. can be resumed as follows:
- Some intermediates configurations, without contact treatment, are generated for some given punch positions to consider the deformation history.
- For each intermediate configuration, the strain increment is calculated by the inverse method between the previous and actual configurations.
- The meshes at the last and actual steps are independent. So at the beginning of the actual step, a transfer of the strain and stress fields should be carried out between these two meshes for plastic integration scheme.
- An efficient method for plastic integration called "Direct Scalar Algorithm" is developed: the equations in function of unknown stress vectors are transformed into the scalar equations using the notion of the equivalent stress; thus the plastic multiplier $\Delta\lambda$ can be directly obtained without iterations [4].
The P.I.A. is a good numerical tool for the preliminary preform design and optimization for the forging processes.

2.1.1 Large strain calculation

In the I.A., the calculation of large strains is done in one step by directly comparing the initial billet C^0 and the final part C and using the logarithmic strains [2-3]. The same calculation is kept in the P.I.A. but between two successive configurations. For an axi-symmetric problem, it is more convenient to define the strains in the local element system (x, θ, z). The movement of a material point between two successive configurations is expressed by $x^0 = x - u$ where x^0 and x are the initial and final position vectors, u is the displacement vector in the local reference. Then the inverse deformation gradient tensor is defined as follows:

$$\{dx^0\} = \left[\frac{\partial x^0}{\partial x}\right]\{dx\} = \left([I] - \left[\frac{\partial u}{\partial x}\right]\right)\{dx\} = [F]_L^{-1}\{dx\} \tag{1}$$

The inverse Cauchy-Green left tensor in the local reference is defined by:

$$\langle dx^0 \rangle\{dx^0\} = \langle dx \rangle [B_m]^{-1}\{dx\} \;\blacktriangleright\; [B_m]^{-1} = [F]_L^{-T}[F]_L^{-1} \tag{2}$$

The eigenvalues $(\lambda_1^{-2}, \lambda_2^{-2}, \lambda_3^{-2})$ of the tensor $[B_m]^{-1}$ gives the three principal elongations, then the three large logarithmic strains:

$$\{\varepsilon_i\} = \{\ln\lambda_i\} \quad (i = 1,2,3) \tag{3}$$

Finally, these large strains are transformed into the element local reference by:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \cos^2\varphi & 0 & \sin^2\varphi \\ 0 & 1 & 0 \\ \sin^2\varphi & 0 & \cos^2\varphi \\ 2\sin\varphi\cos\varphi & 0 & -2\sin\varphi\cos\varphi \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} \tag{4}$$

where φ is the angle from the local reference to the principal strain reference.

2.1.2 Intermediate configurations and field transfer

The basic idea of the P.I.A is to use several intermediate configurations and to execute an inverse calculation between two successive configurations considering the initial strains and stresses in the last step. These intermediate configurations are determined geometrically then corrected by some iterations of equilibrium.
Let's consider the P.I.A. in two steps (**Figure 1**). For the configuration C^1, the two meshes are independent: M^1_1 is the mesh created then corrected at the end of step 1, M^1_2 is the mesh obtained at each iteration of the step 2. A transfer of the strain and stress fields should be carried out between these two meshes on C^1.

2.1.3 Simplified tool actions

In the forging process, the contact treatment is often unstable and time consuming. In the P.I.A., to avoid the

contact treatment, the tool actions are simply represented by some external nodal forces. At a node, the value of the resultant force is unknown; the direction of this force $\boldsymbol{n}_f$ can be determined by the friction cone and the slide direction:

$$\boldsymbol{n}_f = \frac{1}{\sqrt{1+\mu^2}}(\boldsymbol{n} - \mu \boldsymbol{t}) \tag{5}$$

where $\vec{n}$ is the unit normal vector on the contour, $\vec{t}$ the unit vector of the node slide displacement in the tangent direction of the contour, μ the friction coefficient.

In the iteration loop, the two internal forces at each node k are known. The element internal force vector in the local reference is then given by:

$$W_{\text{int}}^e = \langle u_n^* \rangle 2\pi \iint_{A^e} [B_m]^T \{\sigma\} r dA = \langle u_n^* \rangle \{F_{\text{int}}^e\} \tag{6}$$

Thus the equilibrium condition allows calculating the unknown intensity of the resultant tool action force $\vec{F}$:

$$\{F_{\text{ext}}^k\} - \{F_{\text{int}}^k\} = \begin{Bmatrix} P^k n_r^k \\ P^k n_Z^k \end{Bmatrix}_{\text{ext}} - \begin{Bmatrix} F_r^k \\ F_Z^k \end{Bmatrix}_{\text{int}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{7}$$

where $\langle n_r^k \quad n_Z^k \rangle^T = \vec{n}_f$ represents the direction of the resultant force at the node k.
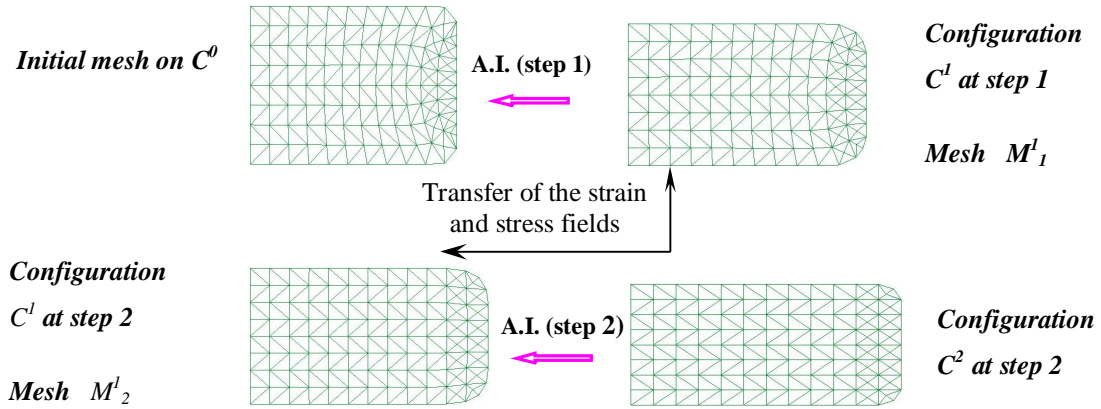

Figure 1. Transfer of Fields between Two Independent Meshes

2.2 FEM adaptive incremental approach

Nowadays, in metal forming processes the classical FE numerical simulation is used to avoid expensive experiments and many research works have been made using this powerful tool [5]. In the laboratory LASMIS of University of Technology of Troyes, extensive works are done since ten years. A specific numerical methodology package based on FEM has been developed to solve elasto- plastic problems with ductile damage in large strains, such as the numerical simulation of metal forming processes. In order to avoid large mesh distortions and achieve properly the solution convergence, the meshing and re-meshing procedure are also included [6-8].

2.2.1 Elasto-plastic constitutive equations

To build the material behaviour model, we should consider coupled multi-physics (plastic strains, hardening and damage) in the forging process. We use an advanced constitutive equations accounting for mixed non-linear isotropic and kinematic hardening strongly coupled with ductile isotropic damage. The coupling between the ductile damage and the elasto- plastic constitutive equations is formulated in the framework of the thermodynamics of irreversible processes together with the Continuum Damage Mechanics (CDM) theory [6-8]. In our study, to simplify the problem and for comparison with the P.I.A., the kinematic hardening is neglected and only three pairs of the internal state variables are used: (ε^e, σ) for the plastic flow; (r, R) for isotropic hardening and (D, Y) for ductile damage. When the current configuration contains some ductile damage, the concept of effective variables based on the hypothesis of total energy equivalence is used.

Accordingly, the fully coupled constitutive equations are summarized as following.

State relation	Evolution equation
$\underline{\sigma} = (1-D)\underline{\underline{\Lambda}} : \underline{\varepsilon}^e$ $R = (1-D^\omega)Qr$ $Y = \dfrac{(1+v)\left[\langle \sigma_I \rangle_+^2 + \langle \sigma_{II} \rangle_+^2 + \langle \sigma_{III} \rangle_+^2\right]}{2(1-D)E}$ $-\dfrac{9v\left[\langle \sigma_H \rangle_+^2\right]}{2(1-D)E} + \dfrac{1}{2}\omega D^{\omega-1}Qr^2$ (Damage)	$f = \dfrac{J_2\underline{\sigma}}{\sqrt{1-D}} - \dfrac{R}{\sqrt{1-D^\omega}} - \sigma_y(T) = 0$ (Von Mises plasticity criteria) $\underline{D}_p = \dot{\delta}\underline{n}$ with $\underline{n} = \dfrac{3}{2\sqrt{1-D}}\dfrac{\underline{\sigma}^{dev}}{J_2\underline{\sigma}}$ $\dot{r} = \dot{\delta}\left(\dfrac{1}{\sqrt{1-D^\omega}} - br\right)$ $\dot{D} = \left(\dfrac{Y-Y_0}{S}\right)^s \dfrac{\dot{\delta}}{(1-D)^\beta}$ $J_2(\underline{\sigma}) = \sqrt{\dfrac{3}{2}\underline{\sigma}^{dev} : \underline{\sigma}^{dev}}$

Where Q is the isotopic hardening modulus; b is the non-liner coefficient; S, s, β and Y_0 are the material coefficients describing ductile damage evolution; $\underline{n}$ the outward normal tensor to the yield surface f=0; σ_y the limit yield stress.

2.2.2 Numerical aspect

The fully coupled thermo-mechanical constitutive equations presented above have been implemented into Abaqus/Explicit FE code using the "VUMAT" user subroutine. A dynamic explicit resolution procedure has been used in order to solve the equilibrium problem [6-8]. Due to the large deformations of the material, the mesh is rapidly distorted. To avoid these distortions, the remeshing procedure must be adapted. By using an adaptive remeshing technique, the mesh is refined in damaged zones and magnified in inactive zones. The remeshing operation is indispensible for geometrically and physically complicated non-linear forming process.

3 Numerical application

In this paper, a 2D axisymmetric forging process is presented and the final shape of the wheel is shown in **Figure 2**. Because of symmetry conditions, only a quarter of the axisymmetric part was modeled. A set of boundary conditions of symmetry were used: the lower horizontal plane is fixed and the upper tool is translated with a constant velocity. The total upper punch travel equals to 62mm.
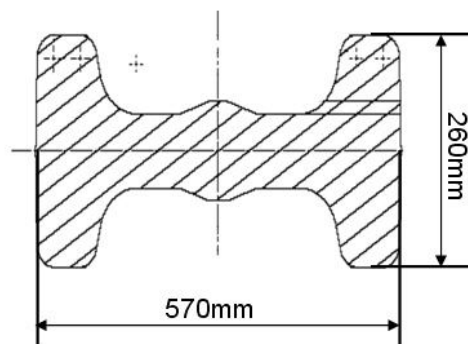
Figure 2. Shape of the final billet

The material of the wheel is aluminium whose mechanical properties are defined as follows: Young's modulus E = 73000 (MPa), Poisson's ratio v = 0.3, friction coefficient between the tool-billet μ=0.05. The material behaviour models used by the two simulation methods are a little different. For the

P.I.A., the Hollomon stress-strain curve is used: $\bar{\sigma} = 170.068(\bar{\varepsilon}^P)^{0.1783}$ MPa. By FEM adaptive incremental approach, an advanced elasto-plastic constitutive equation is used by implementing "VUMAT" user subroutine in the calculation and the parameters using are $\sigma_y=71$(MPa), Q=300(MPa), b=3, S=10000000, s=1, β =5 and Y_0=0. Here we don't take into account the influence of temperature. Although the two methods use different material behaviour model, the final stress-strain curve of the material is almost the same.

3.1 Analysis of the numerical simulation results

3.1.1 Distribution of the equivalent plastic strain and equivalent stress

Figure 3 shows the distributions of the equivalent plastic strain obtained by the P.I.A. and FEM adaptive incremental approach. For the P.I.A., the billet is meshed with 1371 axisymmetric triangle elements. For the method of the FEM adaptive remeshing procedure, the billet is meshed with a quadrilateral element CAX4R and the whole process includes 5 remeshing steps. The initial number of the element is 344 and the final number of the element increase to 2163; the maximum mesh size is 32mm and the minimum mesh size is 0.9mm. With the deformation increasing, the mesh is refined due to the large strain. This adaptive remeshing technology avoids the element distortion during the large deformation metal-forming process.

In **Figure 3,** we also observe that the strain distributions of the two methods are very similar to each other. Both the maximum and minimum values are situated in the same locations. The maximum plastic equivalent strains obtained by the P.I.A. and incremental approach are respectively 0.919 and 0.921.
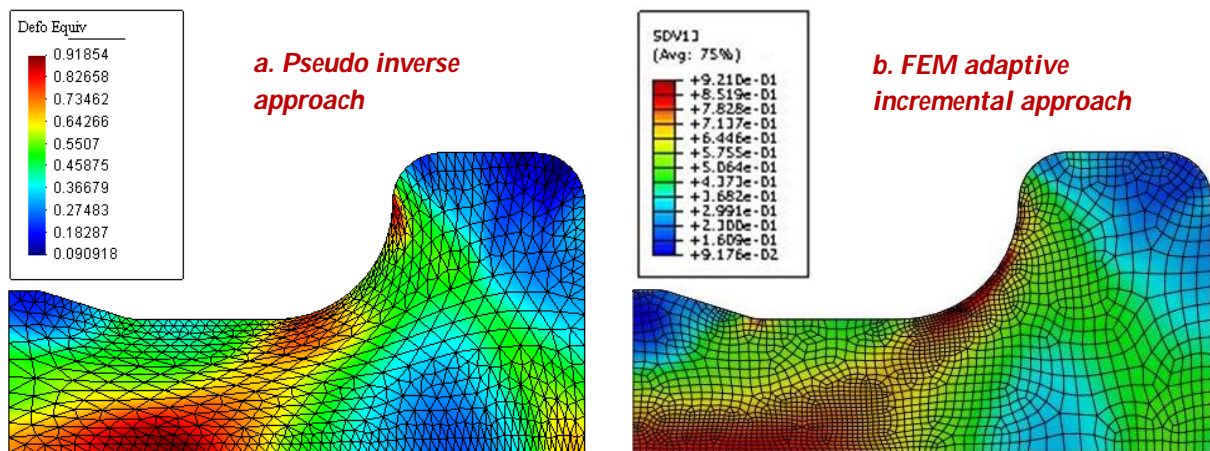

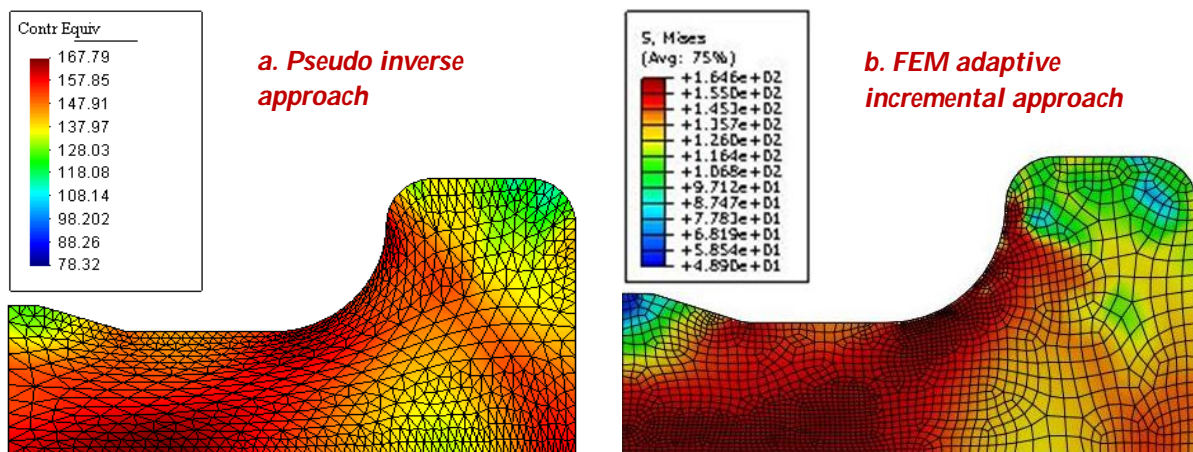Figure 3. Equivalent plastic strain obtained by P.I.A. and A.I.A.

Figure 4. Equivalent stress obtained by P.I.A. and A.I.A.

Figure 4 shows the distributions of the equivalent stress obtained by the P.I.A. and adaptive incremental approach. We note that the two stress distributions are similar. The maximum values are very close to each other (167.8 and 164.6 MPa), but the minimum values are fairly different.

3.1.2 Comparison of the CPU time

The incremental approach is widely used for the forging process modelling and it gives good strain and stress estimation. But compared to the Pseudo Inverse Approach, the incremental approach is very time consuming. The total CPU time used for the simulation of the wheel is 145 s by the incremental approach but only 32 s by P.I.A. (78% of time saving).

In traditional Inverse Approach, two basic assumptions are used: the assumption of proportional loading (for cold forging) gives an integrated constitutive law without considering the strain path neither the visco-plasticity, and the assumption of simplified tool-billet actions allows one to replace the tool actions by nodal forces without contact treatment. These two assumptions make the I.A. calculation very fast. The P.I.A. (or multi-step inverse approach) based on the I.A. keeps the I.A.'s high efficiency but gives good stress estimation owing to the loading history consideration. In the FEM adaptive remeshing method, an advanced elasto-plastic material behaviour model and the face to face contract condition are used, so the whole process is time consuming.

4 Conclusion

By comparing with the results of the two different metal forming simulation methods, we can find that the results of Pseudo Inverse Approach have a good agreement with the results of classical adaptive incremental approach and the pseudo Inverse Approach is proved to be very time saving. So in the case of the complicate tools shape or very large deformation metal forming process, the adaptive incremental approach should be used to get accurate results; in the case of the preliminary preform design and optimization for the forging process modelling, the P.I.A. proves to be very useful and highly efficient.

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