Vibration damping in polygonal plates using the acoustic black hole effect: model based on the image source method

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Abstract:

A method for damping flexural vibrations in thin polygonal plates using the acoustic black hole effect is proposed. Acoustic black holes in thin plates consist of a zone of decreasing thickness in which flexural waves slow down, acting as wave sinks. An acoustic black hole of axisymmetric thickness profile is here tested on different polygonal plates. A parabolic edge is added in order to focus waves into the black hole area. The experimental results show significant reduction of the vibration level. A model based on the image source method provides qualitative agreement with the experiments.

Résumé :

Cet article présente une méthode d'amortissement de vibrations de flexion de plaques polygonales en utilisant l'effet de trou noir acoustique. Ce dernier a lieu lorsqu'une onde de flexion traverse une zone d'épaisseur décroissante, ce qui a pour effet de la ralentir, agissant ainsi comme un piège à ondes. Dans ce travail, différentes plaques polygonales munies d'un trou noir acoustique à symétrie de révolution sont étudiées. Un bord parabolique est également utilisé afin de focaliser les ondes vers le trou noir. Les résultats expérimentaux montrent une réduction importante du niveau vibratoire de la structure. Par ailleurs, un modèle phénoménologique basé sur la méthode des sources image est développé et fournit des résultats en accord qualitatif avec l'expérience.

Keywords: **acoustic black hole; polygonal plate; vibration damping**

1 Introduction

Flexural vibrations are responsible for sound radiation and damage of a large number of thin structures, such as vehicle cabins, industrial liquid containers and satellite panels during launch, for example. The most common methods for passive vibration damping of thin structures rely on surface treatments using polymer-based viscoelastic layers. The main obstacle for improving the efficiency of such damping solutions is the increase of the total weight of the structures due to the added layers.

Recently, attention has been paid to the acoustic black hole effect as a lightweight alternative for vibration damping in thin structures [4–7]. An acoustic black hole in a thin vibrating structure consists of a region of gradually decreasing thickness, typically in the form of a power-law profile. As flexural waves enter such area of the structure, their phase velocity decreases and their wavelength becomes smaller, such that in the ideal case of a thickness profile smoothly decreasing to zero the waves stop propagating and never reflect back. This was first studied by Mironov [8] within the approximation of geometrical acoustics. In practice, a perfectly smooth thickness profile is impossible to achieve, yet it has been proven that the small truncations in the profile can be compensated by using a small amount of damping film in the area of lower thickness [6].

The aim of this paper is to use the acoustic black hole effect for vibration damping in polygonal plates. Since wave propagation takes place in the plane, a two-dimensional acoustic black hole is used, consisting of an axi-symmetric pit of power-law thickness profile. In addition, in order to maximise the effect, the waves are focused towards the region of varying thickness with the aid of a parabolic-shaped edge in its vicinity.

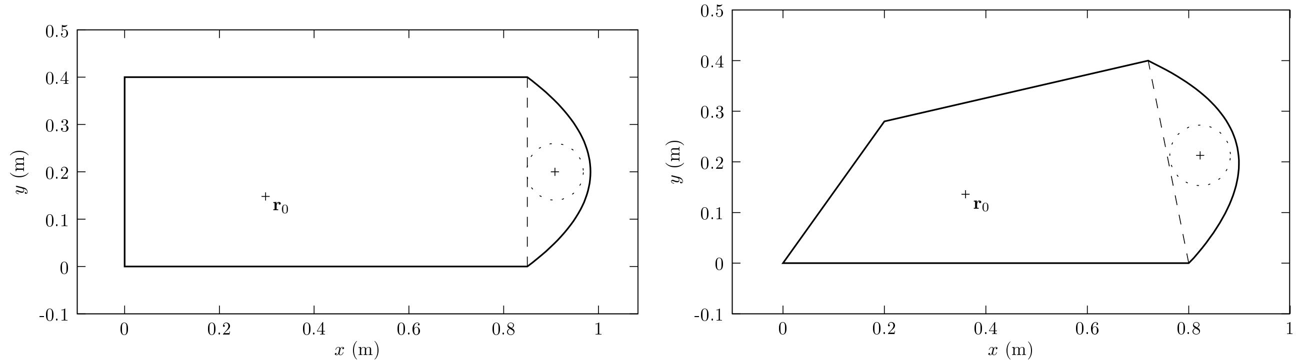
Section 2 of the paper presents the measurements performed on two polygonal plates and in particular a comparison of the driving-point mobilities with and without the acoustic black hole treatment. In sec. 3, a phenomenological model using the image source method is developed as a potential tool for rapid prototyping acoustic black hole damping solutions.

2 Experimental results on polygonal plates

The aim of this section is to show the vibration damping achieved using the acoustic black hole effect. In the following, the driving-point mobility of a polygonal plate with all edges free is compared to the mobility of a plate of same shape with one edge treated with an acoustic black hole extension. The acoustic black hole extension consists of a parabolic-shaped edge intended to focus flexural waves towards the acoustic black hole. A rectangular plate and an irregular polygonal plate, both with and without acoustic black hole extension, are used for the experiments and are depicted in Fig. 1. The dashed line represents the edges of the polygonal plates without black hole treatment. The dotted circle represents the boundary of the acoustic black hole pit, which consists of an axi-symmetric thickness profile of the form

$$h(r) = h_0 \left(\frac{r}{r_b}\right)^2, \tag{1}$$

where h_0 is the thickness of the plate and r_b is the radius of the acoustic black hole.



(a)

(b)

Figure 1: Polygonal plates used for the experiments. The dashed line represents the boundary of the reference plate, without the acoustic black hole treatment. The dotted circle represents the boundary of the acoustic black hole. Distances are in metres.

The plates are made of 1.5 mm-thick aluminium and the black hole profile is carved using high-speed machining, the resulting thickness at the thinest point being of approximately 10 μm. A small amount of damping film is placed in the centre of the pit, as described in Ref. [4]. Driving-point mobilities are acquired using a shaker and an impedance head on point $\mathbf{r}_0$ (see Fig. 1) and shown in Fig. 2. For both plates, the driving-point mobilities show up to 15 dB of reduction of the vibration level. In particular, the resonant behaviour of the response of the plates is significantly reduced. This is due to the fact that the black hole extension acts as an open boundary due to its low-reflecting properties, which dims the interferences with the direct field from the source and the field reflected from the other boundaries.

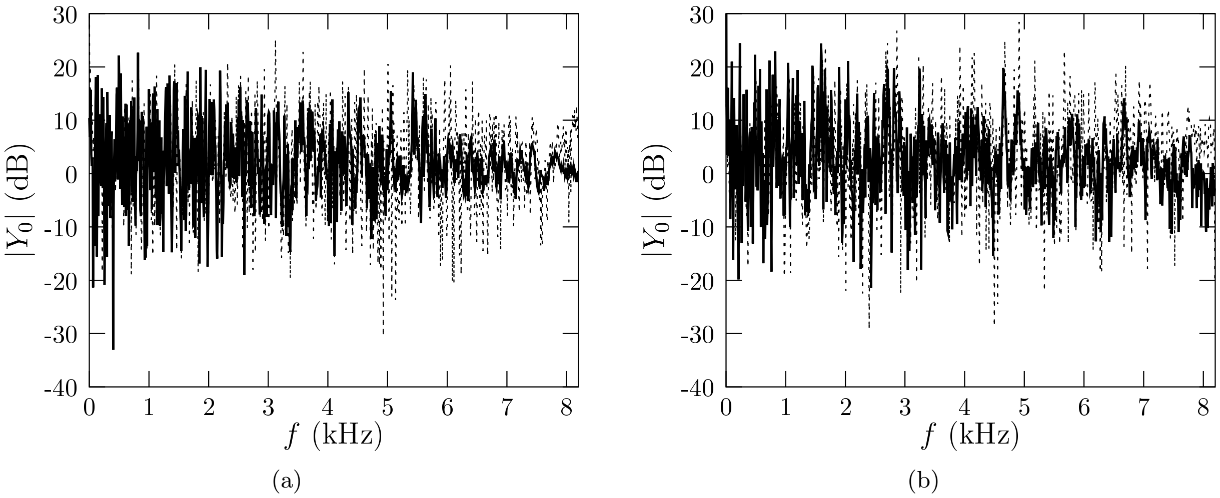


Figure 2: Driving-point mobilities. (a) Rectangular plate; (b) irregular polygonal plate. ······, polygonal plate alone; -----, plate with acoustic black hole treatment.

3 Model using the image source method

The image source method consists in simulating the vibrations of a polygonal domain excited by a point source as the superposition of the vibrations emitted by virtual sources representing successive reflections on the boundaries. Each boundary is described by its reflection properties, which allows the method to compare the effect of different boundary conditions on the overall field of the structure. In the following, the image source method is used as an underlying tool for simulating the vibrations of the polygonal plates studied above, with and without the acoustic black hole treatment. The image source method being restricted to homogeneous domains with polygonal boundaries, the acoustic black hole extension is modelled as an equivalent boundary characterised by its reflection coefficient.

The computation of the flexural vibrations of polygonal plates has been recently studied by the authors using the image source method [1–3]. The main ideas are summarised in the following. Considering a polygonal plate Ω, as depicted in Fig. 3, the transverse displacement Green's function for the flexural vibrations can be written in the form

$$\tilde{G}_\Omega(\mathbf{r},\mathbf{r}_0;k_f) = G_\infty(\mathbf{r},\mathbf{r}_0;k_f) + \sum_{s=1}^{\infty} G_s(\mathbf{r},\mathbf{r}_s;k_f), \tag{2}$$

where $\mathbf{r}_0$ and $\mathbf{r}$ are respectively the source and observer positions and k_f is the flexural wavenumber. The Green's function of the plate arises as a sum of different terms, where G_∞ represents the direct

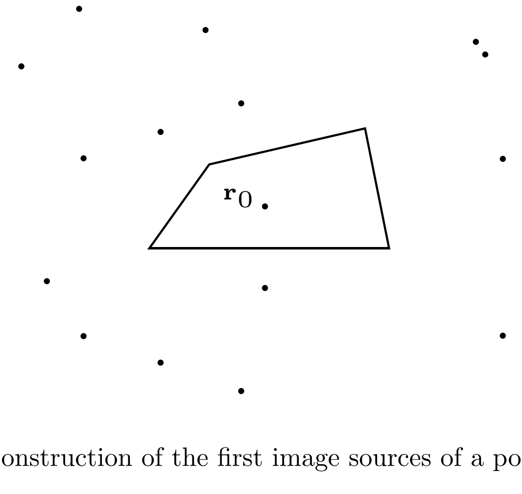


Figure 3: Construction of the first image sources of a polygonal plate.

excitation from the source and the terms G_s represent the contributions of image sources to the field, s being the image source index. Figure 3 shows the construction of the first image sources of the irregular polygonal plate used for the experiments. The image sources are obtained by successive reflections of the original source of the problem, located at $\mathbf{r}_0$.

A one-dimensional model developed in Ref. [4] allows to compute the reflection coefficient of a black hole profile termination as seen from the virtual edge (at 0.08 m from the black hole pit centre). Using the geometrical and material parameters of the manufactured plates, the black hole extension is modelled by a reflection coefficient of 0.01. Fig. 4 shows the computed driving-point mobility of the irregular polygonal plate with all edges free and with the acoustic black hole equivalent edge, using a total of 2934 sources. The figure shows a reduction of 20 in the vibration level by adding the black hole treatment. As in the experimental study, the resonant behaviour of the structure is reduced. From the image source point of view, this can be understood from the fact that the image sources involving one or more reflections on the equivalent black hole edge are weighted by a low reflection coefficient, which diminishes the importance of their contribution.

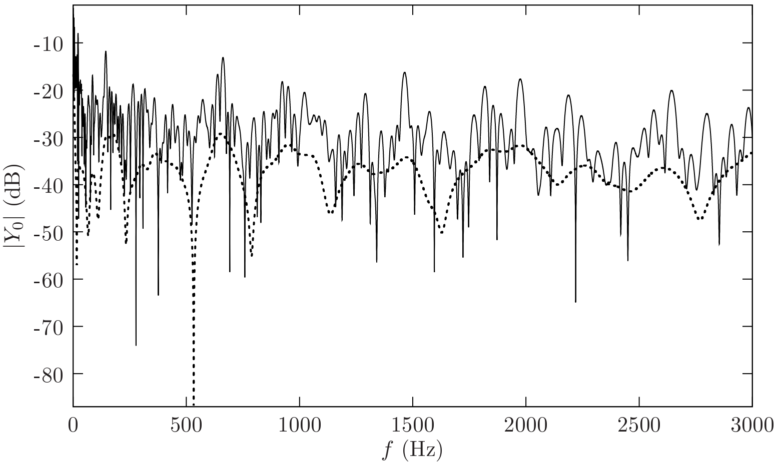


Figure 4: Driving-point mobility of the irregular polygonal plate with and without virtual acoustic black hole edge.

4 Conclusion

In this paper we presented an application of the acoustic black hole effect to the damping of flexural vibrations in polygonal plates. In the studied case, the acoustic black hole consists of a pit of power-law thickness profile acting as a wave sink. Additionally, a parabolic-shaped edge is employed as a means to maximise the effect by focusing waves into the black hole region. By comparing polygonal plates with and without an acoustic black hole treatment, it is seen that the measured driving-point mobilities show a high vibration level reduction. Furthermore, a phenomenological model is developed using the image source method and predicts a similar reduction of the vibration level. The equivalent model by the image source method is a first step towards using it as a tool for rapid prototyping structures with a black hole treatment.

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