Energetically efficient active vibration control of flexible structures

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Résumé :

Dans cet article une stratégie de contrôle appelée " Contrôle semi-actif global " est présentée et validée pour le cas de structures flexibles (poutre encastrée libre). Le but de cette stratégie est d'atteindre les performances d'un contrôleur actif avec une consommation d'énergie réduite comparable à celle nécessaire par les contrôleurs semi-actifs. L'algorithme adopté pour la loi de contrôle est présenté. Le contrôle ISMC (contrôle modal indépendant dans l'espace d'état) est appliqué à la structure pour déterminer la force de contrôle optimale. Les résultats comparatifs de la réponse de la poutre obtenus pour ce type de contrôle et ceux par l'algorithme proposé sont présentés. En effet, on remarque une atténuation des vibrations avec une consommation d'énergie réduite pour le contrôle semi-actif global (régénératif).

Abstract:

In this paper, a control strategy called "Global semi-active control" of flexible structures (cantilever beam) is presented and validated. This strategy aims to achieve potential performance of fully active systems with a reduced energy supply of an amount comparable to this of semi-active strategies. The control approach is presented and the law is offered. Independent Modal Space control (IMSC) study is performed to obtain the optimal force and comparative results of the beam response to the previous type of control and the proposed algorithm are presented. We actually remark the attenuation of the beam tip's displacement versus reduced energy consumption for with global semi-active control (regenerative).

Mots clefs: **active control, energy, semi-active, vibrations.**

Introduction

Traditionally, there were two categories of vibration control defined pertaining to the nature of power flow in dynamic subsystems: passive and active. A third intermediate category is introduced, which will be called regenerative. A regenerative subsystem is the one that is not passive, yet, on average, more energy flows into it than out of it. The concept of regenerative systems for vibration control is based on the self-sustainability from an energetic point of view. i.e. the energy needed to generate the control force is extracted from the system vibrations itself. It is far from being a new concept. The primary focus has been on regenerative automobile suspension systems [1]. For a regenerative vibration control system to be applicable, it must over time absorb more energy from the system than it delivers to it. In other words, it must exhibit positive power absorption averaged over time.

In this work, we present a regenerative control law that we called "Global semi-active law" or "Energetic modal control" as it uses the energy coming from the system vibrations that will be then stored in accumulators for the generation of the control force according to a global semi-active algorithm. The principle of the control strategy is that actuators behave as active ones as long as there is enough energy in accumulators, and as soon as there is a shortage in the available energy they switch to a semi-active law. This control law is applied to flexible structures which are a quite complicated type of systems. For example, because of their multiple modes, they display highly resonant behaviour at or near to their natural frequencies. It is therefore desirable to design a multi-mode controller that can effectively suppress vibrations at and near specific natural frequencies of interest, but does not introduce unwanted vibrations at other natural frequencies (i.e., spill-over). Also, because of the order of these systems is high, the calculation quantity is a burden especially when controlling in real-time. Other desirable requirement for the controller of this kind of structures is employing a minimum number of sensor-actuator pairs using a simple design structure. The principal methods that can be found in the literature for controlling multi-mode vibrations in flexible structures include: positive position feedback control (PPF) [2], independent modal space control (IMSC) [3] and modified independent modal space control (MIMSC) [4] [5]. In IMSC method, the control law is designed in the modal space for each mode independently as it converts a modal matrix at transformation matrix and makes coupled equation of motion with uncoupled equation in modal coordinate system. The traditional problem of flexible structure with a large number of degrees of freedom is then reduced to a set of independent second order systems and so control is easier. Thus, IMSC requires an appreciable less amount of calculation quantity than the coupled control and it also gives a larger choice of control techniques including non linear control. Also, if the number of controlled modes and actuators is the same, controllability is always satisfied and the control spillover will be minimized. But in spite of these many strong points, the number of actuators must be equal to the number of controlled modes for this vibration control algorithm. For these reasons, application field of IMSC is restricted [6] [4] while the Performance Index (PI) is independent of the actuator location. The weak point of IMSC is then that each mode requires its own sensor – actuator pair. So, Baz et al [7] developed a time-sharing technique referred to as Modified Independent Modal Space Control (MIMSC) able to be applied when the number of actuators is less than the controlled modes. Procedure of MIMSC is composed of two steps. First, the system is divided into controlled modes (which are chosen according to their modal energy) and residual ones. Then, each mode is controlled separately and actuators are operated to the modes with the biggest modal energy. The main advantage of MISMC method is the reducibility of the number of actuators, yet it requires a high computation load imposed by the need to calculate and compare the energies in all modes of interest at every time interval. In this work, we will use the IMSC method for the determination of the optimal law that the control algorithm will track. We will also present the simulation results of the beam response in the frequency and time domain.

1 Modal control of flexible structures

Control techniques of flexible structures aim to reduce the system vibrations by the automatic modification of its structural response. The design of the controller (most widely composed of piezoelectric actuators and sensors known for their excellent electromechanical capacities, frequency response characteristics, light weight and low power consumption) is very crucial from the standpoint of performance criterion.
First, we start by modeling our structure for which having an accurate model is necessary to ensure the design of the appropriate controller.

1.1 Finite element formulation

For most structural systems under practical loadings, the vibration response is mainly due to the contribution of certain modes, usually lower order modes which are the most energetic. We will adopt the method of mode superposition to get an approximate reduced order-model system with uncoupled equations of motion in the modal coordinate. First, equations of motion are derived based on Euler-Bernoulli theory. The finite element consists of two nodes with two degree of freedom each. The eigenvalue problem is then solved and the modal vectors with *n* degrees of freedom in the state space are used to decouple the equation of motion which will be written in the following expression:

$$[M]\ddot{q} + [C]\,\dot{q} + [K]\,q = [\mathrm{W}]u + p \tag{1}$$

with:

$[M]$: structural mass matrix,
$[C]$: structural damping matrix ($[C] = \alpha[M] + \beta[K]$ with α and β are Rayleigh mass and stiffness material loss factors),
$[K]$: structural stiffness matrix,
$q,\ \dot{q}, \ddot{q}$: nodal displacement, velocity and acceleration vectors,
$[W]$: actuators location's vector,
u: control force vector,
p : perturbations vector.
$[M]$ and $[K]$ are obtained from the kinetic and potential energies of the beam including the contribution of the piezoelectric patches and sensors.

1.2 Modal decomposition

In modal space, the study is reduced to N modes ($N < n$), where the nodal displacement is approximated by:

$$q = \sum_{i=1}^{N} \phi_i \ r_i \ = [\Phi]\,\{r\} \tag{2}$$

were $\mathrm{r} = \{\mathrm{r}_1, \mathrm{r}_2, \ldots\ldots \mathrm{r}_\mathrm{N}\}^\mathrm{T}$ are the modal coordinates, ϕ_i is the i[th] eigenvector and Φ is the reduced modal matrix. The equation of motion (1) becomes:

$$[M_N]\ddot{q} + \ [C_N]\,\dot{q} + \ [K_N]\,q = \ \Phi^T[W]\,u + \Phi^T\,p \tag{3}$$

with $[\mathrm{M_N}] = \Phi^\mathrm{T}[\mathrm{M}]\Phi$, $[\mathrm{C_N}] = \Phi^\mathrm{T}[\mathrm{C}]\Phi$ and $[\mathrm{K_N}] = \Phi^\mathrm{T}[\mathrm{K}]\Phi$.

Multiplying equation (3) by $[\mathrm{M_N}]^{-1}$, we get the following decoupled form:

$$\ddot{q} + diag(2\xi_i w_i)\dot{q} + \ diag\left(w_i^2\right)q = [L]u + [N]p \tag{4}$$

where w_i and ξ_i are the natural eigenfrequency and damping ratio ($\xi_i = \frac{c_i}{2m_i w_i}$) of the i[th] mode respectively, and $diag(2\xi_i w_i) = [M_N]^{-1}[C_N]$, $diag\left(w_i^2\right) = [M_N]^{-1}[K_N]$, $[L] = [M_N]^{-1}\Phi^T[W]$ and $[N] = [M_N]^{-1}\Phi^T$.

In state space, we introduce the state vector X such as $X = \{q \ \ \dot{q}\ \}^T$, equation (3) is then written as:

$$\dot{X} = [A]\{X\} + [B]u + [B^+]\,p \tag{5}$$

with

$$[A] = \begin{bmatrix} 0 & I \\ -w_i^2 & -2\xi_i w_i \end{bmatrix}, \qquad [B] = \begin{bmatrix} 0 \\ L \end{bmatrix}, \qquad [B^+] = \begin{bmatrix} 0 \\ N \end{bmatrix}$$

where $[A]$ is the state matrix, $[B]$ id the control matrix and $[B^+]$ is the perturbations matrix and I is the identity matrix

1.3 Optimal Independent Modal Space Control

The optimal law that we will use in this paper is obtained by the minimization of a cost criterion $J(u)$ of the form:

$$J(u) = \sum_{i=1}^{N_c} J_i(u) \tag{6}$$

with N_c is the number of the controlled modes and $J_i(u)$, is the modal cost criterion function written as follows:

$$J_i(u) = \int_{t_0}^{t_f} (X^T Q X + \ u^T R u) dt \tag{7}$$

where Q is the positive definite or semi-positive definite weight matrix and R is the positive factor that weights the importance of minimizing the vibration with respect to the control forces. The control force u depends on all the controlled modes which will result in re-coupling equation (4). The main advantage of ISMC method is to avoid re-coupling the system by letting the optimal control force having the following expression [7]:

$$u_{opt} = G\,X = -\,[G_1 \quad G_2]\,\{\mathrm{q} \quad \dot{\mathrm{q}}\}^\mathrm{t} \tag{8}$$

G_1 and G_2 are the gain matrices giving by $G_1 = diag(g_{1i})$ and$G_2 = diag(g_{2i})$, where:

$$g_{1i} = -w_i^2 + w_i\sqrt{w_i^2 + \frac{1}{R}}\,, \quad g_{2i} = \sqrt{-2w_i^2 + \frac{1}{R} + 2w_i\sqrt{w_i^2 + \frac{1}{R}}}$$

This is the optimal law we will track during the optimization problem in the energetic modal control presented in the following section.

2 Energetic modal control

2.1 Set of equations

The objective of this control law is to be able to effectively control the vibrations of a dynamic system with a less amount of power consumption. The controller will have performances similar to those of an active one with the advantage of being energetically independent. Before presenting the control algorithm, some conditions must be pointed out first:

- Being energetically independent implies that our control device must possess storage devices (accumulators), in which energy obtained from the vibrations of the system is stocked. This energy can be increased by using piezoelectric materials known for their capacity of converting mechanical strain into electric power (direct effect of piezoelectricity) and harvesting capacities. We shall notice $E(t), t \in [t_0 : t_f]$, the stored energy, and $E_0 = E(t_0)$, the initial available energy. The storage being physically limited, lower and upper limits for E(t) are needed. This is described as follows: $\mathrm{E_{min}} \leq \mathrm{E(t)} \leq \mathrm{E_{max}}$.
- This condition automatically results in another constraint which is the need of the actuators to be able to store energy even when the accumulators are full. We will assume, for the following that actuators can be temporarily disconnected from their accumulator and work as conventional semi active actuators. This aforementioned constraint can be described by a Boolean function b (such that $\mathrm{b(t)} = 1$ when the accumulators and actuators are connected, otherwise $\mathrm{b(t)} = 0$).
  Actuators we will use for the control are piezoelectric ones. So, the control force corresponds to the voltage applied at the actuator's electrodes. The physical limitations of the piezoelectric actuator result in a threshold voltage $\mathrm{u_{max}}$ not to exceed ($\mathrm{u} \leq \mathrm{u_{max}}$).

Now, our minimization problem can be defined:
(P): minimizing $J(u)$ with constraints:

$$\begin{cases} \dot{X} = AX + Bu \\ E_{min} \leq E(t) \leq E_{max} \\ u \leq u_{max} \quad when\ b(t) = 0 \\ E(t_0) = E_0 \end{cases}$$

The control force to apply to the actuators is proportional to the electric voltage. So, by analogy with the electric power ($P = \dot{E} = \frac{wC_pV^2}{2}$), the power needed to generate the optimal control force can be written as a function of the square of the force, where, w is the radial frequency, C_p , is the piezoelectric capacitance and V is the control voltage [8]. The actuators power which is equal to power delivered from the accumulators can be written as follows:

$$\dot{E} = b.u_{opt}^2 = b\ \ u_{opt}{}^T\ \ G\,X \tag{9}$$

Hamiltonian H resulting from (P) is the following:

$$\begin{aligned} H = {} & X^TQX + u^TRu + \lambda^T\left(AX + Bu - \dot{X}\right) \\ & + \Gamma\left(\dot{E} - b\,u^T\,GX\right) - \gamma_1(E - \mathrm{E_{min}}) + \gamma_2(E - \mathrm{E_{max}}) \\ & - \beta_1(u + \mathrm{u_{max}}) + \beta_2(u - \mathrm{u_{max}}) \end{aligned} \tag{10}$$

with $\lambda, \gamma_i, \beta_i$ and Γ are Lagrange multipliers. Physically, the term Γ represents the power management of the accumulators that will decide either to switch to semi-active law (if there is a shortage of the stored energy) or not. It is then possible, with Euler-Lagrange equations and corresponding edge conditions, to obtain a set of equalities and inequalities governing the optimization problem (P):

$$\begin{cases} \dot{\lambda} = -QX - A^T\lambda - \Gamma\,b\,G^Tu & (11^{**}) \\ \mathrm{u} = -\mathrm{R}^{-1}(B^T\lambda - \Gamma\,b\,G\,X) & (11^{***}) \\ \dot{E} = b\ \ u^T\ \ GX & (11^{****}) \\ \dot{\Gamma} = \gamma_2 - \gamma_1 & (11^{*****}) \end{cases}$$

At instant t, the values of γ_1 and γ_2, relative to the stored energy level, provide the value of Γ in t, and also the value of Γ in $t + dt$, with Γ as we said earlier, related to the accumulator income and outcome power flow. The energetic control law is based on the determination of the value of Γ function of the state vector and the value of the energy available in the accumulator. From equations (11^{***}) and (11^{****}) , the energy management term have the following expression:

$$\Gamma = \frac{\dot{E} + R^{-1}\lambda^T B\, G\, X}{b^2 G^T\, X^T R^{-1}\, GX} \tag{12}$$

2.2 Algorithm of the control strategy

It is possible now to synthesize the conditions seen above for the generation of the energetic modal control algorithm which is primarily based on the level of remaining energy. First, the optimal control force applied to the actuators is calculated (section 1.3). Then, according to the available energy in the accumulators (through the value of the power management term Γ (section 2.1)), optimal force is applied (if the stored energy is sufficient and accumulators keep stocking energy) or semi-active one (shortage of energy) by dissipating the energy.

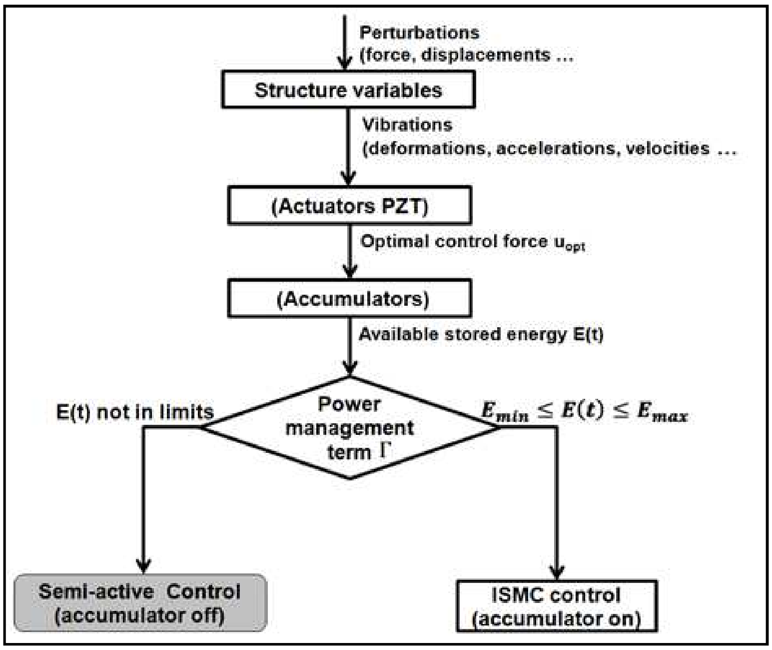


FIG. 1 – Energetic modal control flowchart.

3 Simulations and discussions

In this section, we present some results of the application of the control law we proposed to a cantilever beam subjected to harmonic excitations at the support. Two collocated piezoelectric actuators are bonded near the clamped end and sensors are located near the free end where the vibrations are the most important (as shown in figure.2).

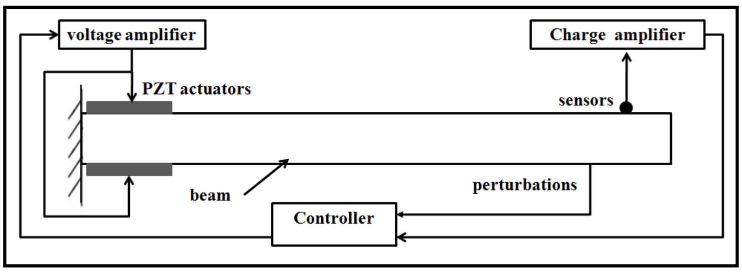
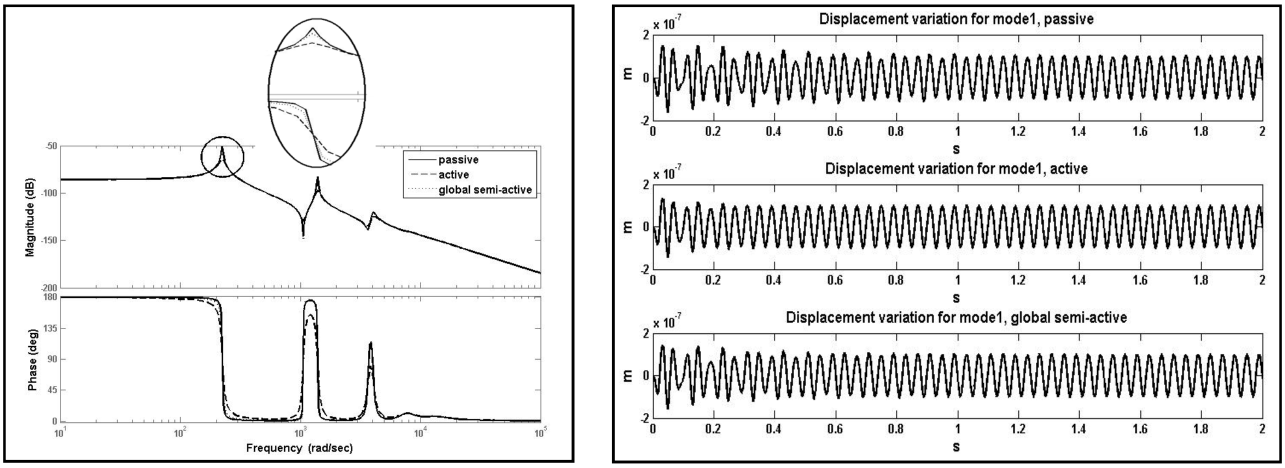


FIG. 2 – Schematic of active vibration control of a smart beam.

The frequency response of the beam for the first three modes shows actually the good performances of our control strategy which are similar to those of the optimal law (ISMC). We as well present the results of the time response of the beam tip displacement (figure.3).



(a) Frequency response (b) Time response of the beam tip's displacement

FIG. 3 – Frequency (a) and time (b) response of the cantilever beam to the 2 types of control (optimal and global semi-active) compared to the uncontrolled response.

In order to clearly evince the good performances of our control law, we present the RMS (Root Mean Square) values of the displacement (Table.1).

RMS	Passive	Global semi-active	Optimal
Displacement (m)	$0.7310e^{-7}$	$\underline{0.7211e^{-7}}$	$0.7099e^{-7}$

Table. 1 – RMS values of the beam tip displacement for the different types of control.

Conclusion

In this paper, an energetic modal control algorithm was developed and presented. It consists on a switching between a semi-active law and an optimal one based on the level of the available energy in accumulators. It was applied to a cantilever beam and results show the good performance of the law versus reduced energy consumption. In further work, we plan to implement piezoelectric patches to extract vibrations energy and convert it to useful energy in order to supply accumulators and so enhance the performances and the energy needs.

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