Gradient constitutive law for brittle or quasi-brittle material with microcracks

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Résumé :

Dans cette communication, nous allons construire un modèle qui prédit la rupture d'une structure contenant une source de concentration de contrainte. Ce modèle établi sur la base d'une méthode d'homogénéisation prend en compte le gradient de la déformation. Nous allons décrire la méthode d'homogénéisation pour établir des relations constitutives du second gradient pour des matériaux hétérogènes. Dans la procédure d'homogénéisation, nous avons contruit une relation du gradient de la déformation á deux dimensions avec des microfissures. Nous prenons comme base physique de notre description de la propagation des fissures dans la structure une énergie potentielle capable de décrire la fracture du matériau fragile. À partir de cette énergie, nous constatons que non seulement la déformation mais aussi son gradient ont une forte influence sur l'évolution de l'endommagement du matériau fragile et quasi-fragile.

Abstract :

In this paper, we have established a fracture model to predict the failure of a structure initiated from a stress concentration source. This model is established on the basis of the homogenization on a microcracked brittle material by taking the strain gradient into account. We first describe a homogenization method with the aim of establishing the strain gradient constitutive relations for heterogeneous materials. In the frame of this homogenization procedure, we have constructed a strain gradient constitutive relation for a bidimentional elastic material with many microcracks by adopting the self-consistent scheme. By assuming a physically realistic resistance curve for microcrack growth, we extend this constitutive law to a strain gradient potential that is capable to describe the fracture of brittle materials. From the expression of this energy potential, we can clearly observe that apart from the strain, the strain gradient has also a strong influence on the damage evolution of brittle or quasi-brittle materials.

Mots clefs : Strain gradient theory ; homogenization, damage, microcracks, brittle materials

1 Introduction

The strain gradient constitutive laws in solids mechanics were introduced by the necessity to describe the size effect observed in micro or macro scales. It is a common belief that the size effect is essentially due to heterogeneities and flaws in materials. The size effect becomes noticeable when the size of these heterogeneities and flaws are of comparable size with that of the structural elements. The size effect is not readily included in classical continuum mechanics frameworks, and researchers often see enriched continuum theories like non-local elasticity (Pijaudier-Cabot and Bazant, 1987) as a replacement for more complicated microscopic and discrete simulations. Under certain conditions, these non-local models can be approximated by the so-called strain gradient elasticity where strain gradient is added to the classical elastic constitutive equations. These gradient approaches are often based on the introduction of length scale effects in elasticity, plasticity or dislocation dynamics by incorporating higher order gradients of strain into the constitutive or evolution equations governing the material description.
In this work, we attempt to demonstrate that the strain gradient plays an important role in fracture behavior of brittle materials. The main objective of this paper is twofold : the establishment of a strain gradient constitutive law for brittle materials with many microcracks by means of homogenization ; the construction of a damage evolution law on the basis of the microcrack growth and its application to the failure prediction of brittle materials under non-singular stress concentration.
We essentially follow the homogenization principle, but in addition, we develop a special procedure with the purpose of transforming the constitutive laws for individual RVEs to those for the continuum. This approach ensures that the obtained gradient constitutive laws are independent of the size of the RVE. After that, we introduce the strains gradient into the damage evolution law on the basis of the microcrack growth. This strain gradient damage model is then implemented into a finite element code. The numerical results on the fracture prediction are compared with experimental data. The comparison clearly demonstrates that the present damage

model can efficiently capture the size effect and fits the test data with a satisfactory accuracy.

2 Creation of strain gradient constitutive relations by homogenization

2.1 Principal hypotheses

1. A RVE is no longer considered an infinitesimal volume. A macroscopic solid Ω is constituted by a finite number M of RVEs $\Omega^{(m)}$. Therefore we assume that :

$$\Omega = \bigcup \Omega^{(m)}, \bigcap \Omega^{(m)} = \varnothing, m = 1...M. \tag{1}$$

2. The higher-order gradients of the macroscopic fields have no influence on the constitutive laws if the material is perfectly homogeneous. In the microscopic scale and inside the RVE, the conventional elasticity theory holds :

$$\begin{cases} \sigma_{ij,j}(y) = 0 \\ \sigma_{ij}(y) = C_{ijpq}(y)u_{p,q}(y) \end{cases} \qquad y \in \Omega^{(m)} \tag{2}$$

where σ, u are respectively the Cauchy stress tensor, the displacement vector at a point in $\Omega^{(m)}$, and C_{ijpq} is the stiffness tensor of the material.
3. We assume that a RVE $\Omega^{(m)}$ is small but not too small, such that the macroscopic fields within the RVE can be represented by a Taylor s expansion, namely,

$$u_i^{(m)}(y) = \overline{u}_{i,j}^{(m)} y_j + \frac{1}{2}\overline{u}_{i,jk}^{(m)} y_j y_k + O(y^3) \qquad y \in \partial\Omega^{(m)} \tag{3}$$

where $\overline{u}_{i,j}^{(m)}, \overline{u}_{i,jk}^{(m)}$ are the values of the displacement derivatives at the geometrical centre of $\Omega^{(m)}$.

2.2 Homogenization on a RVE

Consider a RVE $\Omega^{(m)}$ loaded with a known traction $t_i^{(m)} = \sigma_{ij}n_j$ on its exterior boundary $\partial\Omega^{(m)}$. Let $\delta u_i^{(m)}(y)$ be a virtual kinematically admissible displacement field, as defined in (3) but truncated after the third term. The virtual work of traction $t_i^{(m)}$ on the virtual displacements is :

$$\delta W_u^{(m)} = \int_{\partial\Omega^{(m)}} t_i^{(m)} \delta u_i^{(m)} = \delta\overline{u}_{i,j}^{(m)} \int_{\partial\Omega^{(m)}} t_i^{(m)} y_j dS + \frac{1}{2}\delta\overline{u}_{i,jk}^{(m)} \int_{\partial\Omega^{(m)}} t_i^{(m)} y_j y_k dS \tag{4}$$

Let us define

$$\overline{\sigma}_{ij}^{(m)} = \frac{1}{V^{(m)}} \int_{\partial\Omega^{(m)}} t_i^{(m)} y_j dS \qquad \overline{\sigma}_{ijk}^{(m)} = \frac{1}{2V^{(m)}} \int_{\partial\Omega^{(m)}} t_i^{(m)} y_j y_k dS \tag{5}$$

$\overline{\sigma}_{ij}^{(m)}, \overline{\sigma}_{ijk}^{(m)}$ can respectively be considered as the stress, the double stress in $\Omega^{(m)}$; $V^{(m)}$ is the volume of $\Omega^{(m)}$. We can remark that $\overline{\sigma}_{ij}^{(m)}$ is also the average stress tensor of the RVE. It is independent of the size of the RVE. However, $\overline{\sigma}_{ijk}^{(m)}$ vary with the volume of $\Omega^{(m)}$. Thus (4) becomes :

$$\frac{1}{V^{(m)}} \int_{\partial\Omega^{(m)}} t_i^{(m)} \delta u_i^{(m)} = \delta\overline{u}_{i,j}^{(m)} \overline{\sigma}_{ij}^{(m)} + \delta\overline{u}_{i,jk}^{(m)} \overline{\sigma}_{ijk}^{(m)} \tag{6}$$

The right side of (6) can be regarded as the variation of the average strain energy density of the RVE defined as follows :

$$U^{(m)} = \int_{\Omega^{(m)}} \overline{\sigma}_{ij}^{(m)} d\overline{u}_{i,j}^{(m)} + \int_{\Omega^{(m)}} \overline{\sigma}_{ijk}^{(m)} d\overline{u}_{i,jk}^{(m)} \tag{7}$$

Therefore one can just read (6) as the virtual work principle applied on $\Omega^{(m)}$

$$\delta W_u^{(m)} / V^{(m)} = \delta U^{(m)} \tag{8}$$

The equation (8) is the energy-averaging theorem, known in the literature as the Hill-Mandel condition or macrohomogeneity condition (Hill, 1963 ; Suquet, 1985 ; Fleck and Hutchinson, 1997). It is important to point out that equation (8) is obtained from the homogenization of a single RVE. Therefore, it is only valid for individual RVEs. In the following, we will rewrite it for a linearly elastic continuum.

2.3 Transformation to a macroscopic continuum

According to Mindlin (1965), the second degree strain energy function for a homogeneous and isotropic continuum can be written, using the notation of the present paper, as follows :

$$U = \frac{1}{2}\left(C_{ijpq}u_{i,j}u_{p,q} + l^2 C_{ijkpqr}u_{i,jk}u_{p,qr}\right) \tag{9}$$

where l is a material length-scale which is assumed to be small. C_{ijpq}, C_{ijpqkr} are the stiffness tensors for different degrees of strains. This form of strain energy function can also be found by using the above-mentioned homogenization technique. In fact, for each RVE, we can write the average strain energy density as follows :

$$U^{(m)} = \frac{1}{2}\left(\overline{C}^{(m)}_{ijpq}\overline{u}^{(m)}_{i,j}\overline{u}^{(m)}_{p,q} + \overline{C}^{(m)}_{ijkpqr}\overline{u}^{(m)}_{i,jk}\overline{u}^{(m)}_{p,qr}\right) \tag{10}$$

where $\overline{C}^{(m)}_{ijpq}, \overline{C}^{(m)}_{ijkpqr}$ are the average stiffness tensors of the $m-th$ RVE issued from a homogenization procedure. The length-scale parameter l is incorporated in these tensors for expression simplicity. With $\overline{\sigma}^{(m)}_{ij} = \overline{C}^{(m)}_{ijpq}\overline{u}^{(m)}_{p,q}$, $\overline{\sigma}^{(m)}_{ijk} = \overline{C}^{(m)}_{ijkpqr}\overline{u}^{(m)}_{p,qr}$ the total strain energy in a macroscopic volume $\Omega^{(m)}$ is, according to the hypothesis (1) :

$$\int_\Omega U dV = \sum_{m=1}^{M} V^{(m)}U^{(m)} = \frac{1}{2}\sum_{m=1}^{M} V^{(m)}\left(\overline{u}^{(m)}_{i,j}\overline{\sigma}^{(m)}_{ij} + \overline{u}^{(m)}_{i,jk}\overline{\sigma}^{(m)}_{ijk}\right) \tag{11}$$

where U is the strain energy density in Ω. It is clear that $U \neq U^{(m)}$, as the later depends on the volume and the shape of the RVE. Our objective is to transcribe the summation in (11) into an integral expression in order to extract a true strain energy density. For that purpose, we first consider the integral

$$\int_\Omega u_{i,j}\overline{C}_{ijpq}u_{p,q}dV = \sum_{m=1}^{M}\int_{\overline{\Omega}^{(m)}} u^{(m)}_{i,j}\bar{C}^{(m)}_{ijpq}u^{(m)}_{p,q}dV \tag{12}$$

where $\overline{\Omega}^{(m)}$ represents the homogenized RVE, i.e., when $\overline{\Omega}^{(m)}$ is a mono-connected domain without cavities. $\overline{C}$ is the continuously varying macroscopic stiffness tensor derived from homogenization, $\overline{C}(x) = \overline{C}^{(m)}$ for $x \in \Omega^{(m)}$. With $\overline{I}^{(m)}_{ij} = \frac{1}{V^{(m)}}\int_{\overline{\Omega}^{(m)}} y_i y_j dV$, by substituting (2) into (12), after input in (11) we obtain straightforwardly :

$$\begin{aligned}
\int_\Omega U dV &= \frac{1}{2}\int_\Omega u_{i,j}\overline{C}_{ijpq}u_{p,q}dV + \frac{1}{2}\sum_{m=1}^{M} V^{(m)}u^{(m)}_{i,jk}\left(\overline{C}^{(m)}_{ijkpqr} - \overline{C}^{(m)}_{ijpq}\overline{I}^{(m)}_{kr}\right)\overline{u}^{(m)}_{p,qr} \\
&= \frac{1}{2}\int_\Omega u_{i,j}\overline{C}_{ijpq}u_{p,q}dV + \sum_{m=1}^{M}\int_{\bar{\Omega}^{(m)}} u^{(m)}_{i,jk}\left(\bar{C}^{(m)}_{ijkpqr} - \bar{C}^{(m)}_{ijpq}\bar{I}^{(m)}_{kr}\right)u^{(m)}_{p,qr}dV \\
&= \frac{1}{2}\int_\Omega u_{i,j}\overline{C}_{ijpq}u_{p,q}dV + \int_\Omega u_{i,jk}\left(\bar{C}_{ijkpqr} - \bar{C}_{ijpq}\bar{I}_{kr}\right)u_{p,qr}dV \\
&= \frac{1}{2}\int_\Omega \left(u_{i,j}C_{ijpq}u_{p,q} + u_{i,jk}C_{ijkpqr}u_{p,qr}\right)dV
\end{aligned} \tag{13}$$

with $C_{ijpq} = \bar{C}_{ijpq}$, $C_{ijkpqr} = \bar{C}_{ijkpqr} - \bar{C}_{ijpq}\bar{I}_{kr}$. The strain energy density in the macroscopic continuum is :

$$U = \frac{1}{2}\left(u_{i,j}C_{ijpq}u_{p,q} + u_{i,jk}C_{ijkpqr}u_{p,qr}\right) \tag{14}$$

Then the constitutive relations in the homogenized continuum are :

$$\sigma_{ij} = \frac{\partial U}{\partial u_{i,j}} = C_{ijpq}u_{p,q} \qquad \sigma_{ijk} = \frac{\partial U}{\partial u_{i,jk}} = C_{ijkpqr}u_{p,qr} \tag{15}$$

This transformation procedure can be repeated if higher-order gradients are included in the constitutive law. Equations (14) and (15) are similar to many constitutive laws presented in the literature. However, the relationships in (14) and (15) are issued from the homogenization consideration, therefore benefit from the physical clarity of the method. It is useful to remark that the strain gradient constitutive laws thus obtained are independent of the size and the shape of the RVE.

3 Strain gradient constitutive laws for brittle materials with many micro-cracks

In this part, we will establish a strain gradient constitutive law by using above-described homogenization technique. The idea is, we assume that the constitutive law that we attempt to obtain will have the following form :

$$U = \frac{1}{2}\left(C_{ijpq}\varepsilon_{ij}\varepsilon_{pq} + C_{ijkpqr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right) \tag{16}$$

Consider now a 2-D RVE $\Omega^{(m)}$ containing N cracks. The n-th crack is centered at $y^{(n)}$, $y^{(n)} \in \Omega^{(m)}$, with length $2a$, normal vector $n^{(n)}$ and orientation vector $s^{(n)}$. The remote loading is applied on the boundary of $\Omega^{(m)}$ by :

$$\sigma^o_{ij} = \overline{\sigma}_{ij} + \overline{\sigma}_{ij,k}y_k \qquad \text{or} \qquad \varepsilon^o_{ij} = \overline{\varepsilon}_{ij} + \overline{\varepsilon}_{ij,k}y_k \tag{17}$$

where $\overline{\sigma}_{ij}$ and $\overline{\sigma}_{ij,k}$ are respectively the average macroscopic stress and stress gradient tensors ; and $\overline{\varepsilon}_{ij}$ and $\overline{\varepsilon}_{ij,k}$ are the corresponding average macroscopic strain and strain gradient tensors. The strain energy density (16) can be deduced from homogenization under the remote load (17). We choose the self-consistent scheme (Budiansky and O Connell, 1976) to estimate the tensor C_{ijpq}. Under bi-dimensional loading and with random crack orientations, the self-consistent method leads to the effective stiffness tensor :

$$C_{ijpq} = (1-\pi\rho)C^o_{ijpq} \tag{18}$$

Let us now estimate the stiffness tensors C_{ijkpqr} associated with the strain gradient. We can write the average strain energy density in a RVE with many microcracks as :

$$U^{(m)} = U^0 - \sum_{n=1}^{N}\frac{W^{(n)}}{V^{(m)}} \tag{19}$$

where $W^{(n)}$ is the work of the traction acting on the lips of the n-th crack. U^o is the average strain energy density of the matrix

$$U^o = \frac{1}{2V^{(m)}}\int_{\Omega^{(m)}}(\overline{\varepsilon}_{ij} + \overline{\varepsilon}_{ij,k}x_k)C^o_{ijpq}(\overline{\varepsilon}_{pq} + \overline{\varepsilon}_{pq,r}x_r)dV = \frac{1}{2}\left(\overline{\varepsilon}_{ij}C^o_{ijpq}\overline{\varepsilon}_{pq} + \overline{\varepsilon}_{ij,k}C^o_{ijkpqr}\overline{\varepsilon}_{pq,r}\right) \tag{20}$$

We have $C^o_{ijkpqr} = \overline{I}^{(m)}_{kr}C^o_{ijpq}$ with $\overline{I}^{(m)}_{kr} = \frac{1}{V^{(m)}}\int_{\Omega^{(m)}} x_k x_r dV$. Under the remote load (17), we have the following expression for the traction and the displacement jump between the two crack lips (we don't present here the calculation in details for briefness) :

$$t^{(n)}_i(\eta) = \left(\overline{\sigma}_{ij} + \overline{\sigma}_{ij,k}x^{(n)}_k\right)n^{(n)}_j + \eta\overline{\sigma}_{ij,k}s^{(n)}_k n^{(n)}_j \tag{21}$$

$$\left[u^{(n)}_i(\eta)\right] = \frac{4}{E'}\sqrt{a^{(n)^2} - \eta^2}\left[\left(\overline{\sigma}_{ij} + \overline{\sigma}_{ij,k}x^{(n)}_k\right)n^{(n)}_j + \frac{\eta}{2}\overline{\sigma}_{ij,k}n^{(n)}_j s^{(n)}_k\right] \tag{22}$$

where η is the local coordinate attached to the crack, $\eta \in [-a^{(n)}, a^{(n)}]$ and $E' = \begin{cases} E & \text{for plane stress} \\ \dfrac{E}{1-\mu^2} & \text{for plane strain} \end{cases}$

By substituting (21) and (22) into the second term in the right side of (19), we write the work of the tractions on crack lips on Ω :

$$\sum_{n=1}^{N}\frac{W^{(n)}}{V^{(m)}} = \frac{1}{2V^{(m)}}\sum_{n=1}^{N}\left[\frac{2a^{(n)^2}\pi}{E'}\left(\overline{\sigma}_{ij}\overline{\sigma}_{pq} + \overline{\sigma}_{ij,k}\overline{\sigma}_{pq,r}x^{(n)}_r x^{(n)}_k\right) + \frac{a^{(n)^4}\pi}{4E'}\overline{\sigma}_{ij,k}\overline{\sigma}_{pq,r}s^{(n)}_k s^{(n)}_r\right]n^{(n)}_j n^{(n)}_q \delta_{ip}ds \tag{23}$$

We adopt the conventional hypotheses such that the distributions of the length, orientation and position of the microcracks are uniform and independent; thus we can evaluate all of the quantities in the equation (23). Finaly, the strain energy density with strain gradient can be expressed by the following simple form :

$$U = \frac{1}{2}\left[(1-\pi\rho)C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq} + \frac{\pi\overline{a}^2}{12}C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right] \tag{24}$$

We can observe that this strain energy density is similar to those proposed in earlier literature (Mindlin, 1965 for example). However, (24) benefits from its physical foundation through homogenization of a micro-cracked material.

4 A damage model for elastic micro-cracked materials

In this work, we choose $\overline{a}$ as the inner variable describing the damage state of the material. The generalized thermodynamic force associated with $\overline{a}$ is, according to Lemaitre and Chaboche (1990) :

$$G = -\frac{\partial U}{\partial \overline{a}} = \left(\frac{1}{V^o}C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq} - \frac{1}{12}C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right)\pi\overline{a} \tag{25}$$

G represents the dissipative energy ratio due a to unit increase of $\overline{a}$ in a unit volume. It can be considered as the energy release rate in a more general sense. According to the Griffith criterion, crack propagation occurs when G attains a critic value $G_c(\overline{a})$ or the critical energy release rate, namely :

$$G \leq G_c(\overline{a}) = G_o\overline{a}^\lambda \tag{26}$$

where G_o and λ are real constants that we will determine later. From (25) and (26), we have :

$$g(\overline{a}, \varepsilon_{ij}, \varepsilon_{ij,k}) \equiv \left[\frac{\pi}{G_o}\left(\frac{1}{V^o}C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq} - \frac{1}{12}C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right)\right]^{\frac{1}{\lambda-1}} - \overline{a} \leq 0 \tag{27}$$

Equation (27) represents a crack evolution surface. When $g < 0$, there is no crack growth. By identification of the coefficient , we obtain the following constitutive equations for general loading :

$$U = \begin{cases} \frac{1}{2}\left[(1-\pi\rho)C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq} + \frac{\pi\overline{a}^2}{12}C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}\right] & \text{when} \quad g < 0 \\ \frac{1}{2}C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq} - \frac{1}{2}E^o\varepsilon^2_{max}\frac{\lambda-1}{\lambda+1}\left(\frac{C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq}}{E_o\varepsilon^2_{max}} - \frac{\overline{a}^2_o}{12\rho_o}\frac{C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}}{E^o\varepsilon^2_{max}}\right)^{\frac{\lambda+1}{\lambda-1}} & \text{when} \quad g = 0 \end{cases} \tag{28}$$

In the case where $\overline{a}_o = 0$, this potential is degenerated to a conventional potential similar to the Lennard-Jones potential (1924). The originality of the present model resides in the fact that, on one hand, it is obtained from a homogenization procedure of a cracked material and therefore it is physically reasonable, and on the other hand, the strain gradient is included into this potential. For the numerical modelling, the calculation of the variation of the strain energy density for loading regime is given by :

$$\delta U = \frac{\partial U}{\partial \varepsilon_{ij}}\delta\varepsilon_{ij} + \frac{\partial U}{\partial \varepsilon_{ij,k}}\delta\varepsilon_{ij,k} = (1-d)C^o_{ijpq}\varepsilon_{ij}\delta\varepsilon_{pq} \simeq (1-d)C^o_{ijpq}\varepsilon_{ij}\delta\varepsilon_{pq} \tag{29}$$

with the damage parameter d

$$d = \left(\frac{C^o_{ijpq}\varepsilon_{ij}\varepsilon_{pq}}{E_o\varepsilon^2_{max}} - \frac{\overline{a}^2_o}{12\rho_o}\frac{C^o_{ijpq}\delta_{kr}\varepsilon_{ij,k}\varepsilon_{pq,r}}{E^o\varepsilon^2_{max}}\right)^{\frac{2}{\lambda-1}} \quad \text{with} \quad V^o \equiv \frac{\overline{a}^2_o}{\rho_o} \tag{30}$$

4.1 Comparison of the strain gradient model with experimental data

Li and Zhang (2006) carried out uniaxial tension tests on dog-bone shaped PMMA plates with a central hole. The mechanical characteristics of the material they used were : the elastic modulus E = 3000 Mpa, the Poisson ratio $\nu = 0.36$ and the ultimate tensile stress $\sigma_c = 72MPa$. The section of the specimens was $10 \times 30mm^2$. Central holes were drilled with different diameters, namely, $d = 0.6, 1.2, 2$ and 3 mm. Specimens without holes were also prepared for comparison. These specimens were subjected to an uniaxial tension with a loading rate $v = 5mm/minute$ until failure.

The only unknown parameter in this model is the average spacing between nucleated microcracks represented by $V^o = \frac{\overline{a}_o^2}{\rho_o}$. Figure 1 illustrates the critical fracture loads predicted by the present model for different hole sizes and different values of the parameter V^o. The experimental results are also plotted for comparison. First, we can observe that the present model can describe the size effect observed in the experimentation whatever the value of V^o , i.e., the critical fracture load increases as the hole size decreases. However, we can also notice the very important influence of nucleated microcracks spacing on the prediction accuracy. Roughly speaking, neglecting the strain gradient effect (V^o) will lead to a too conservative fracture prediction. On the contrary, a too large V^o will overestimate the critical fracture loads. For the PMMA samples used in the present study, a value of $V^o \sim 0.2mm^2$ gives a suitable fit to the experimental data.

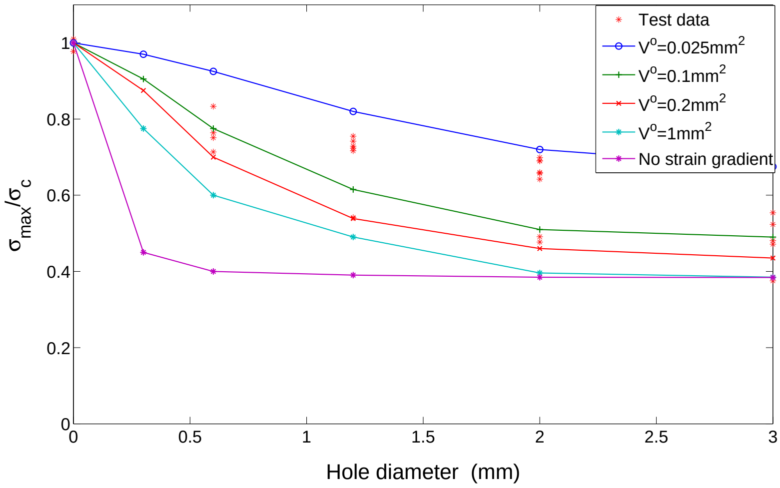

FIG. 1 – Comparison of the predicted fracture loads with the experimental data

5 Discussions and concluding remarks

In this work, we first developed a homogenization procedure in order to establish the strain gradient constitutive relations for a heterogeneous material. When the macroscopic stress or strain gradient cannot be neglected compared to the size of a RVE, the homogenization procedures lead to the gradient constitutive laws in a natural manner. An important feature of the present method is its self-consistency with respect to the selection of the RVE. Once the microstructure of the material is correctly represented in a RVE, the constitutive equations obtained by using the present method are independent of its size and shape.

In the frame of this homogenization procedure, we have constructed a strain gradient constitutive relation for a bidimentional elastic material with many microcracks by adopting the self-consistent scheme. The constitutive equation we obtained shows clearly that the material behaviour depends not only on the crack density, but also on the average crack length associated with the strain gradient.

In order to extend the obtained constitutive equation to a damage evolution law, we propose a resistance curve for the microcrack growth on the basis of experimental observations in brittle materials. This approach leads to an energy potential that is capable to describe the brittle property of fracture. When we incorporate the strain gradient in this potential, we obtain a damage model in which the strain gradient plays a very important role in damage evolution.

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