Dynamic condensation and selective mass scaling in RADIOSS® Explicit

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Abstract :

RADIOSS Explicit is on top of the industrial FE codes which are used for crash simulation since several years, as well as in other application fields such as fluid structure interaction or stamping simulation. One upon the main concerns of RADIOSS development is to improve its performances while providing a high quality of results. Dynamic condensation and selective mass scaling both allow increasing the critical time step without modifying the low response frequencies, neither the rigid body momentum. Thus they are well-suited for the simulation of processes for which the energy content in the high frequency domain is small. The implementation of these 2 methods is discussed. Both the performances and limits of these methods are illustrated through some use cases, issued from crash field.

Keywords: explicit method, time step, dynamic condensation, selective mass scaling

1 Increasing the time step of the explicit method

1.1 Stability of the explicit method

Let the discretized equation of motion

$$M\ddot{u} + C\dot{u} + Ku = F_{ext} \tag{1}$$

The mass matrix M results from assembling the elementary mass matrices

$$M^e = \int_{\Omega^E} \rho < \Phi >^t < \Phi > dV \tag{2}$$

Using the central difference scheme which corresponds to Newmark scheme with γ=1/2 and β=0 – and assuming there is no damping - the current acceleration is calculated at each time step n+1 by solving

$$M\ddot{u}_{n+1} = F_{n+1}^{ext} - F_{n+1}^{\text{int}}\text{, with } F_{n+1}^{\text{int}} = Ku_{n+1} \tag{5}$$

The use of a lumped mass matrix avoids solving simultaneous equations for accelerations in (5).

In the modal basis, the recurrent continuation becomes

$$m\frac{a_{n+1} - 2a_n + a_{n-1}}{\Delta t^2} + ka_n = f_n \tag{7}$$

Let
$$\begin{vmatrix} a_{n+1} \\ a_n \end{vmatrix} = A \begin{vmatrix} a_n \\ a_{n-1} \end{vmatrix} + L, \quad with \quad A = \begin{bmatrix} 2-\omega^2\Delta t^2 & -1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

Where $\omega = \sqrt{\frac{k}{m}}$ is the eigen frequency associated to mode a.

The stability is insured if the spectral radius of A is less than 1 for all modes, let [1,4]

$$\Delta t < \frac{2}{\omega_{\max}} \tag{9}$$

Commercial explicit codes generally provide a way of raising the time step by adding (diagonal) mass to the nodes. Several drawbacks are inherent to this method, as the non conservation of rigid body translational momentum, and the modification of the low frequencies of the system.

1.2 Selective mass scaling

Olovsson and al. [2,3] proposed to add additional terms to the lumped mass matrix, aiming at decreasing the highest frequencies of the system, while affecting the lower frequencies as little as possible.

Starting from

$$\overline{M} = M + \Lambda, \quad with \quad \Lambda = \alpha K \tag{10}$$

For which the eigen frequencies are modified such as

$$\overline{\omega}^2 = \frac{\omega^2}{1+\alpha\omega^2} \tag{11}$$

so that low frequencies are less affected.

Since non linearity would need to compute $\overline{M}$ at each time step, they proposed to assembly Λ from constant elementary matrices such as the rigid translational acceleration of an element is not modified, let for a 4-node shell element

$$\lambda = \frac{m_e}{12}\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \text{ where } m_e \text{ is the total mass of the element.} \tag{12}$$

Thus there is no change in inertia effects on translational rigid body acceleration.

$$\sum_{i,j}\Lambda_{ij}x_j = 0, \forall x \Rightarrow \sum_{i,j} M_{ij}\ddot{u}_j = \sum_{i,j}\overline{M}_{ij}\ddot{u}_j = \sum_i \left(F_i^{ext} - F_i^{\text{int}}\right) \tag{13}$$

The inversion of matrix would not be convenient if it is not constant as in case of non linear stiffness, and would still need to make a lot of operations at each time step for calculating

$$\ddot{u} = \overline{M}^{-1}F \tag{14}$$

Considering that $\overline{M}$ is strictly diagonally dominant, equation (14) is approximated using an iterative resolution method up to some tolerance. Conjugate gradient iterations are made up to

$$\left\|F - \overline{M}\ddot{u}\right\| \le \varepsilon \left\|F - \overline{M}\ddot{u}_0\right\| \tag{15}$$

where the initial solution is computed using the lumped mass matrix

$$\ddot{u}_0 = M^{-1}F \tag{16}$$

1.3 Dynamic condensation

1.3.1 Adaptive meshing

Mesh refinement has been implemented for shell elements, which better allows mapping of state variables than free re-meshing. It is mostly dedicated to stamping simulation and several refinement criteria are available, taking into account the geometry of contacting tools, specially.

A non conforming mesh is maintained and kinematical conditions are imposed at non conforming nodes.

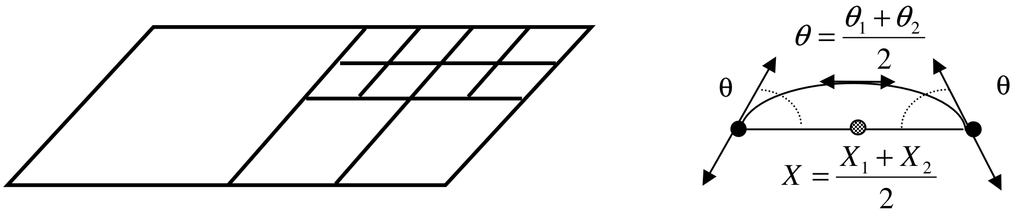


FIG. 1 – Non conforming mesh

It is possible to refine the mesh up to some levels, from time 0.

1.3.2 Dynamic condensation

Let the reduced set of nodes (m) of the coarse mesh. The additional nodes (n) coming from the actual mesh will be condensed onto nodes (n).

Lumped mass $\overline{M}_m$ at the reduced set of nodes is computed by considering the coarse mesh.

Forces computed at condensed nodes are mapped to nodes (m) using the shape functions ϕ_m of the coarse mesh so that the equation of motion at the reduced set of nodes becomes

$$\overline{M}_m \ddot{u}_m = F_m + \sum_n \Phi_m(n) F_n \tag{18}$$

Forces computed at the condensed nodes are still considered as inertia forces

$$\overline{M}_n \left(\ddot{u}_n - \ddot{\overline{u}}_n \right) = F_n, \;\; with \;\; \ddot{\overline{u}}_n = \sum_m \Phi_m(n) \ddot{u}_m \tag{18}$$

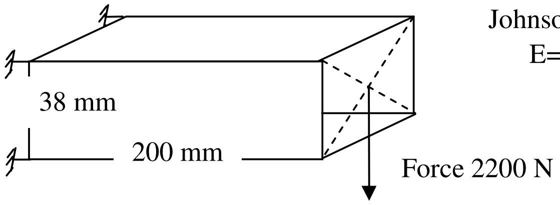
But the lumped mass $\overline{M}_n$ at condensed nodes is set big enough, so that the time step can be computed by only considering the nodes of the coarse mesh. $\overline{M}_n$ is not an absolute mass but it is only relative to the mean displacement $\ddot{\overline{u}}_n$. Therefore, increasing it does not affect global momentum.

On the contrary to selective mass scaling, no matrix inversion is needed with dynamic condensation. But dynamic condensation is available only for 4-node and 3-node shell elements.

2 Use cases

2.1 Box beam

The problem is described at figure 2: a box beam is clamped on one side and the other side is a rigid body upon which a constant force of 2200 N is applied. The section of the beam is square.



Johnson-Cook Elasto-Plastic material law
E=210000 MPa, ρ=0.0078 g mm^{-3}

FIG. 2 – Problem description

The beam is meshed with 4-node shell elements. Due to symmetry, only half of the beam is modelized.

The coarse mesh which is considered for dynamic condensation is shown at figure 3 (deformation at 5 ms is presented). The time step using the coarse mesh is about 0.57 µs, with dynamic condensation as well as without.

The fine mesh is obtained after 2 levels of refinement (each element of the coarse mesh is split into 4x4 elements). The fine mesh is built this way in order to make possible the use of dynamic condensation. But no real adaptive meshing is used since the mesh is refined from time 0 up to the fine mesh.

The deformation of the beam after 5 ms when using the fine mesh and default time step is shown in Figure 3. The results demonstrate to converge to the fine mesh.

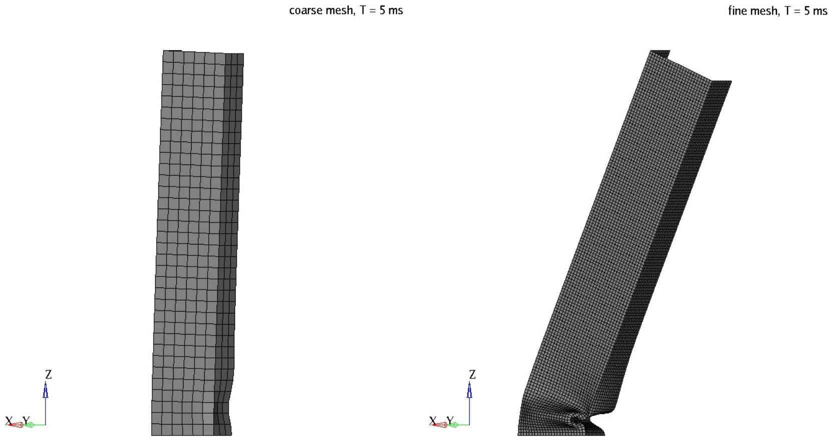
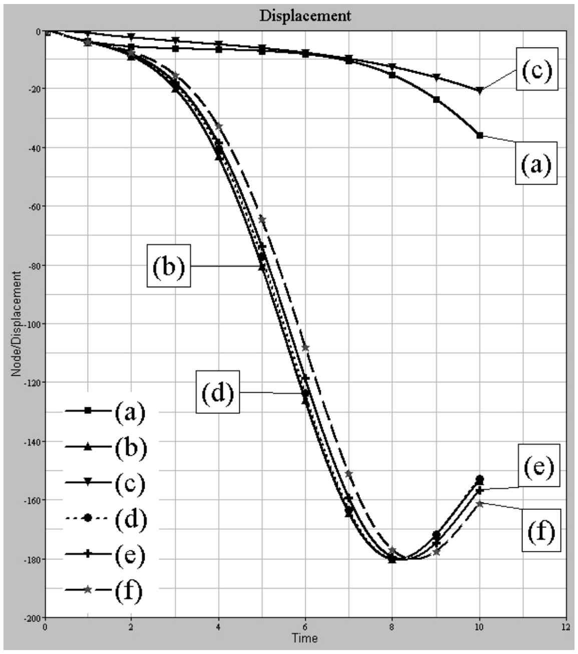


FIG. 3 – Deformations of the coarse mesh and the fine mesh at 5 ms

Figure 4 compares the results which are obtained with the fine mesh depending on the settings which are for the time step.



(a) using coarse mesh and default time step (around 0.57 µs)

(b) using fine mesh and default time step (around 0.15 µs)

(c) using fine mesh and adding mass in order to enforce the time step to 0.57 µs

(d) using fine mesh and dynamic condensation : time step is the same as using the coarse mesh (around 0.57 µs)

(e) using fine mesh and selective mass scaling in order to enforce the time step to 0.57 µs

(f) using fine mesh and selective mass scaling in order to enforce the time step to 1 µs

FIG. 4 – Displacement of the centre of mass of the rigid body along the direction of the applied load

The coarse mesh is clearly not converged, and the results of adding mass on the fine mesh are not better.

The results using dynamic condensation and selective mass scaling with time step imposed to the time step of the coarse mesh are closed to the results of the fine mesh and default time step. There is a little more discrepancy with selective mass scaling and a higher imposed time step (1 µs) but the results are still acceptable.

The computation time and the relative cost of the specific treatments to dynamic condensation and selective mass scaling versus the total computation time are presented at figure 5: dynamic condensation is faster than selective mass scaling, since there is no need to inverse a matrix. The relative cost of iterations of conjugate gradient increases with the imposed time step, since convergence needs more iteration to be achieved.

	(a) Default time step	(d) Dynamic condensation	(e) Selective mass scaling, dt=0.57 µs	(f) Selective mass scaling, dt=1 µs
Number of cycles	74335	17729	17576	10030
Elapsed time	1451 s	385 s	650 s	492 s
Relative cost		6,5%	45%	57%

FIG. 5 – Computation time

2.2 Foam drop test

A Foam block using a visco-elastic law is supported at the bottom by a fixed shell. It is impacted by a rigid sphere with steel material, which gets an initial velocity and is submitted to gravity.

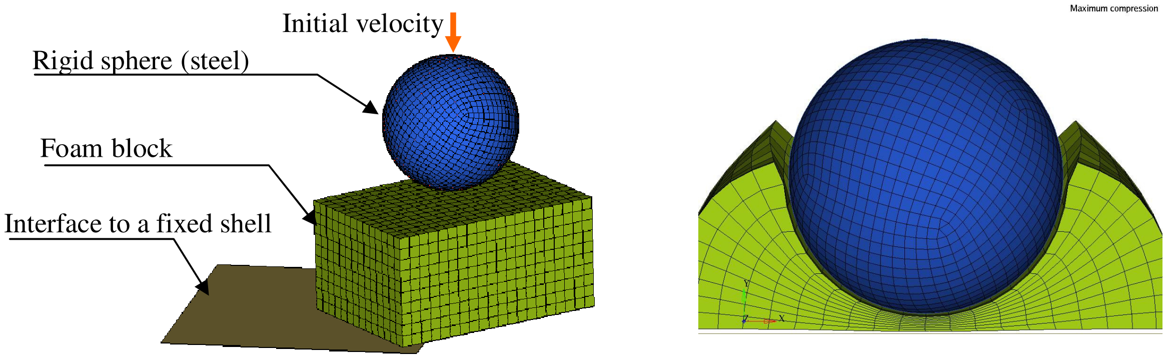


FIG. 9 – Problem description plus deformation at maximum compression (section view)

Figure 10 shows the contact force against the support. One can notice the force peak corresponding to the densification phase of the foam. Results of adding mass and selective mass scaling using an imposed time step equal to 0.5 µs are close to those obtained with default time step. For an imposed time step equal to 5 µs, the behaviour is better represented with selective mass scaling than adding mass.

(a) Default time step
- timestep decreases from 7 µs to 0.2 µs due to a large amount of compression.

(b) Adding mass, dt=0.5 µs
(c) Selective mass scaling dt=0.5 µs
(d) Adding mass, dt=5 µs
(e) Selective mass scaling, dt=5 µs

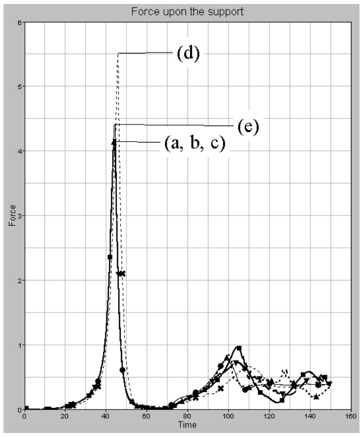


FIG. 10 – Contact force against the support

3 Conclusion

Dynamic condensation and selective mass scaling both allow to increase the time step while getting results closed to those obtained with default time step, at the opposite to adding mass. Despite of the over-cost due to their implementation, they allow to reduce the computation time. Selective mass scaling is more expensive in some cases, but its implementation makes it more general and it is available for all types of elements.

References

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