A priori analysis of subgrid-scale models for shock wave / boundary layer interaction

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Résumé :

Cette étude porte sur l'analyse a priori de modèles LES pour la simulation d'interaction choc/couche limite supersonique. Sur la base de données de simulation numérique directe (DNS), des tests a priori des modèles LES les plus couramment utilisés sont réalisés. La pertinence des diverses approches de filtrage et les diverses voies de modélisation sont discutées afin de déterminer a priori la stratégie de modélisation la plus appropriée.

Abstract :

This study addresses the subgrid-scale modeling issue for large eddy simulation of shock wave / boundary layer interaction. By using a reference flow database, obtained by direct numerical simulation, a priori testing of the most prominent LES models is carried out. The various modelling and filtering approaches are discussed and compared, leading to suggest a priori the most appropriate closure strategy.

Mots clefs : compressible flow, supersonic flow, shock-interactions, turbulence, a priori analysis

1 Introduction

A spatially evolving, supersonic boundary layer flow, interacting with an oblique shock wave, has been computed by using direct numerical simulation (DNS). The free stream Mach number and the momentum thickness Reynolds number (based on free stream conditions) are $M_\infty = 2.25$ and $Re_\theta = 4000$ respectively. For this study, the full simulation of the turbulent flow has been carried out, including the transition region, instead of focusing only on the interaction zone and resorting to alternative recycling/rescaling approaches which are likely to artificially contaminate the observable mecanisms of shock unsteadiness. The numerical procedure relies on i/ 7^{th} order WENO schemes (Weighted Essentially Non-Oscillating) for the convective fluxes, ii/ 4^{th} order compact schemes for viscous terms and iii/ 3^{rd} order Runge-Kutta algorithm for the explicit time integration. Based on the results obtained, the turbulent correlations has been calculated to study the influence of the shock on the structure of turbulence. Previous works [1][2] have already pointed out the similarity between the compressible and the incompressible regime that exist, except within the interaction zone, whereas a new recent study focuses on the relaxation effect on the anisotropy tensor components [3].
Such DNS obviously present a great interest for fundamental studies on the compressible turbulence. However, they cannot be applied for less academic applications, yielding higher Reynolds numbers or more complex geometric features. The large-eddy simulation, which is less computationally expensive, appears thus to be more and more essential to fill the gap with the industrial applications. Most LES models have already been introduced and widely evaluated for incompressible flows. However, they true capabilities still needs more evaluation in many compressible configurations. In addition, for compressible flows, various formulations can be considered and the best choice is still prone to debate.
Based on the DNS database of shock wave/ boundary layer interaction (SWBLI) obtained, a filtering approach and *a priori* analysis of subgrid models are considered to address this issue. The *a priori* analysis basically consists in comparing the subgrid quantities that are estimated by considering an LES model applied on a filtered DNS field with the corresponding quantity that is directly evaluated from the difference between the original DNS field and the filtered field. In this study, the most prominent LES usual models are analysed in order to suggest the best strategy to follow for this challenging flow configuration.

2 A priori analysis of the subgrid tensors

Convoluting the Navier-Stokes equations with a low-pass filter leads to consider the evolution equation for the large resolved scales of the flow. By using the Favre averaging for the velocity components and the temperature gives the possibility to limit the complexity of the equations obtained. However, some unresolved terms are still present. One of the most important, the Reynolds stress tensors, appear in the momentum equation, and are expressed according to :

$$\tau_{ij} = \overline{u'_i u'_j} - \overline{u'_i u'_j} \qquad (1)$$

For this fundamental term, the analysis has been performed at various complementary levels :
- tensorial analysis : in this case, the components of the tensor are directly compared between the LES model and the filtered DNS evaluation by calculating the correlation of each field of component (given by equation 1) directly. These correlations are denoted by $ct11$, $ct12$, $ct13$, $ct22$, $ct23$ et $ct33$ in the following.
- vectorial analysis : in this case, we focus on the components of the gradient of the turbulent stresses $\frac{\partial \tau_{ij}}{\partial x_j}$, denoted by $cv1$, $cv2$ et $cv3$ in the following.
- scalar analysis : in this case, we focus on the result of these stress $u_i \frac{\partial \tau_{ij}}{\partial x_j}$, with a correlation quantity between the DNS based estimate and the LES model one which is denoted by cs.

The estimation of the spatial derivatives is based on the use of Lagrangian interpolants and a *top hat* filtering procedure is build for each possible inhomogeneous direction and arbitrary filter width. The figure 1 illustrates the kind of filtered field that have been obtained in this way. We should note that, a discrimination of the shock region has to be ideally considered. A Ducros filter [4] can be used for example for this purpose.
At first, the Smagorinsky[5], combined with a damping van Driest function, proposed by Piomelli[6] ($0.1 \times \Delta(1 - exp(-(y^+/25)^3)$) have been assessed, by retaining a value of 0.1 for the constant of the model. As it can be checked on figure 2, in spite of a very low level of correlation between the estimated and modeled stress components, the scalar correlation keeps on being rather important (about 50%), at least for the less important filter width considered. This observation are in agreements with the known observed dissipative features of the this model.
The second model considered is the Mixed-Scale Model (MSM). It is designed to combine the advantage of similarity scales (using an additional filter to directly rebuilt the Reynolds stress in function of the lower resolved scales) and the dissipative bahaviour of structural type subgrid models. The figure 2 shows that, for this case, the two types of components of the stress tensor are far better correlated.

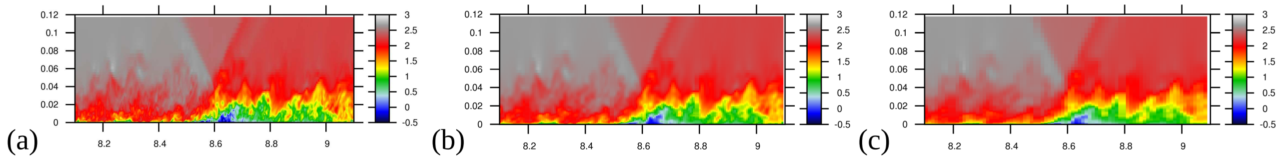

FIG. 1 – **Example of filtered DNS longitudinal velocity fields of supersonic boundary layer at** $Mach = 2.25$ **interacting with an oblique shock for various filtering length : original DNS field (a) ; filter** $\Delta_x = 4$ **and** $\Delta_z = 4$ **(b) ; filter** $\Delta_x = 8$ **and** $\Delta_z = 8$ **(c)**.

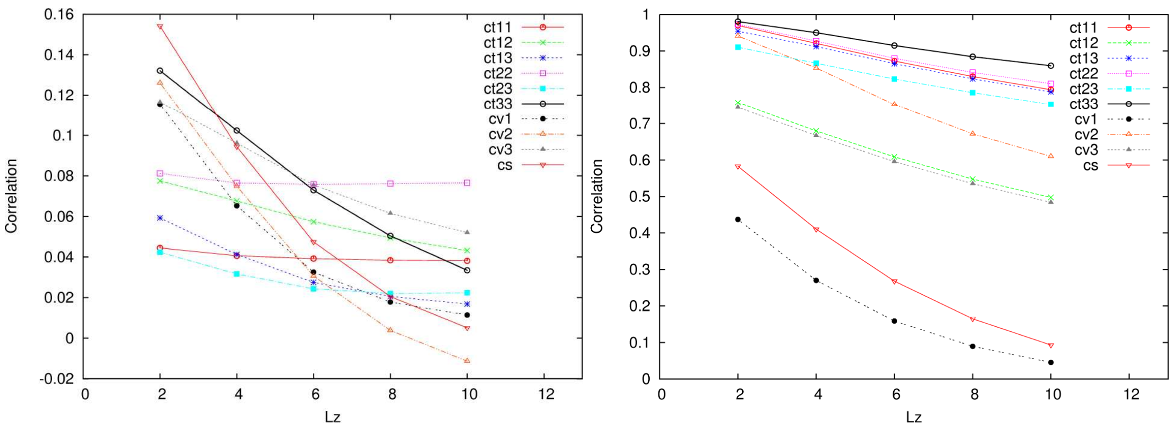

FIG. 2 – **A priori analysis of the Smagorinsky (left) and MSM (right) model for the show wave / turbulent boundary layer interaction : Evolution of the correlation between the subgrid stress components given by the models and directly evaluated from the original DNS fields, in function of the filter width** L_z **(given in number of grid spacings), applied in the spanwise direction** z**.**

3 Perspectives

The final presentation will include additionnal models tested. The representativity of the relative importance of the terms appearing in the energy equation will also be addressed. Finally, the influence of the formulation (macro temperature) retained for the filtered equations will be discussed.

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