Large-eddy simulations of round jets : effects of initial conditions on self-similarity region

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Abstract :

In order to investigate the influence of the initial conditions on the establishment and features of the self-similarity regions of turbulent circular jets, Large-Eddy Simulations are performed using low-dissipation numerical schemes combined with a relaxation filtering as subgrid dissipation. Three jets with the same initial parameters except for the diameters yielding Reynolds numbers of 1800, 3600 and 11000 are first considered. Then two additional jets at Reynolds number 3600 in which modified inflow conditions are specified (namely a thinner shear-layer thickness and a forcing based on lower azimuthal modes, respectively) are calculated. Comparisons of mean and turbulent properties of the flows are shown.

Résumé :

Afin d'étudier les effets des conditions initiales sur l'établissement et les propriétés de la région d'auto-similarité des jets circulaires turbulents, des Simulations des Grandes Echelles sont effectuées à l'aide de schémas numériques peu dissipatifs, en utilisant un filtrage de relaxation de l'énergie turbulente aux plus petites échelles résolues. Trois jets caractérisés par des conditions d'entrée identiques, excepté leurs diamètres définissant des nombres de Reynolds de 1800, 3600 et 11000, sont considérés. Deux autres jets à nombre de Reynolds de 3600, pour lesquels on fait varier les conditions initiales, telles que l'épaisseur de la couche de cisaillement et les propriétés modales des perturbations de vitesse introduites pour exciter les écoulements, sont aussi calculés. Des comparaisons des résultats obtenus pour les champs moyens et turbulents des jets sont montrées.

Mots clefs : large-eddy simulation, jet, turbulence, self-similarity

1 Introduction

The self-similarity region of jet flows has been investigated extensively over the last fifty years. Reference solutions for round jets have for instance been provided by the experimental works of Wygnanski and Fiedler [1], Panchapakesan and Lumley [2], and Hussein *et al.* [3]. Discrepancies between the solutions are however observed, which are expected to result largely from differences in the measurement methods and in the jet initial conditions, the diameter-based Reynolds numbers of the jets in these experiments ranging in particular from 11000 to 100000.
The influence of the Reynolds number on jet flow development has indeed been shown to be significant up to Reynolds numbers around $\mathrm{Re}_D = u_j D/\nu \simeq 10000$, where u_j and D are the jet inlet velocity and diameter, and ν is the kinematic molecular viscosity. It has been examined experimentally especially by Lemieux and Oostuizen [4], Namer and Ötügen [5], Kwon and Seo [6], and Deo *et al.* [7], as well as numerically by Bogey and Bailly [8]. In experiments, however, unlike simulations, the variations of the Reynolds number likely result in modifications of other initial flow conditions, which might have a significant indirect impact on the features of turbulent jet flows.
The properties of the shear layer at the nozzle exit, such as the profile of mean velocity and the turbulence intensities, have thus to be carefully considered. Their effects have been documented in different experiments for circular jets at fixed Reynolds numbers. Ferdman *et al.* [9] reported for instance the influence of nonuniform initial velocity profiles on the downstream evolution of round jets at the Reynolds number of 24000. Raman *et al.* [10] and Antonia *et al.* [11, 12] dealt with jets at the Reynolds numbers of 400000 and 86000, respectively, displaying either transitional or turbulent nozzle-exit boundary layers. Both concluded that jets with initially transitional shear layer develop more rapidly, with shorter core length and higher velocity decay rate, than jets with initially turbulent shear layer. Xu and Antonia [12] moreover noticed that jets approach self-similarity more rapidly when the initial shear layer is laminar.
Given the experimental difficulties, it appears now worthwhile to study the influence of the initial conditions on jet self-similarity using simulations. In computations, the inflow parameters can indeed be changed in-

dependently, and consequently their influence can be distinguished, as it has been demonstrated by previous works [8, 13, 14, 15]. Self-similar jets have for example been calculated by Boersma *et al.* [16], Freund [17] and Uddin and Pollard [18]. In Bogey and Bailly [19], a round jet at a Reynolds number 11000 has also been calculated by a Large Eddy Simulation (LES) using an approach combining low-dissipation schemes [20] and explicit selective filtering to relaxe energy [8]. Jet self-similarity was reached on the computational domain, and described in detail. A very good agreement with the data obtained by Panchapakesan and Lumley [2] at the same Reynolds number was found.
Following this simulation, the influence of the initial conditions on the establishment and features of the self-similarity regions of turbulent circular jets is investigated by performing additional LES. First, in order to deal with the effects of the Reynolds number only, two jets with the same inflow conditions as the jet at $\mathrm{Re}_D = 11000$, but at lower Reynolds numbers 1800 and 3600 are considered. Then, two other jets at $\mathrm{Re}_D = 3600$, in which, respectively, the inflow shear-layer thickness and the forcing procedure used to seed the turbulent transition in the mixing layers are modified with respect to the first jet at the same Reynolds number, are calculated. In this way, the variations of the jet flows with the initial conditions can be explored at a fixed Reynolds number.
In the present paper, the simulation parameters are first briefly described, and jet definitions are given. Vorticity fields are then shown, and some preliminary comparisons between the mean and turbulent properties of the different jets are reported.

2 Simulation parameters

2.1 Numerical procedure

The LES are performed by solving the three-dimensional Cartesian filtered compressible Navier-Stokes equations, using low-dissipation and low-dispersion schemes [20]. Fourth-order 11-point finite differences are implemented for spatial discretization, and a 2nd-order 6-stage low-storage Runge-Kutta algorithm is applied for time integration. Grid-to-grid oscillations are removed every time step by an explicit 2nd-order 11-point filtering of the flow variables, which is designed to damp only the shortest waves discretized. The filtering enables to take into account the effects of the subgrid energy-dissipating scales without affecting significantly the scales accurately resolved. It does not lead in particular to an artificial decrease of the effective Reynolds number of the flow, as it might be the case with eddy-viscosity-based LES modellings [21]. More details on this LES approach based on relaxation filtering (LES-RF) can be found in Bogey and Bailly [8, 19]. Finally, in the present LES, the budgets for the turbulent kinetic energy are also computed directly from the flow-governing equations. All the energy terms including the viscous dissipation and the subgrid dissipation induced by the relaxtion filtering are estimated explicitly [19].

2.2 Jet definition

Five isothermal round jets at Mach number 0.9 and at Reynolds numbers 1800, 3600 and 11000 are simulated on computational domains extending up to 90, 120 and 150 jet radii r_0 in the downstream direction, respectively. A part of the self-similarity region of the jets is thus expected to be calculated.
In the three jets referred to as jetRe1800, jetRe3600 and jetRe11000 in table 1, the jet inflow conditions such as the mean flow profiles, the shear-layer momentum thickness $\delta_\theta = r_0/16$ and the forcing procedure used to seed the turbulent transition in the mixing layers are identical. They have been described in previous papers [15, 19, 22]. Only the diameters of the jets vary so as to yield Reynolds numbers of 1800, 3600 and 11000.
In the jets jetRe3600thin and jetRe3600mode, the Reynolds number is also 3600, but some inflow characteristics are modified with respect to jetRe3600. More precisely, the initial shear layer is thinner in jetRe3600thin with $\delta_\theta = r_0/26$, whereas the inflow forcing used to seed the turbulent transition from random velocity disturbances is based in jetRe3600mode on the two first azimuthal modes $n = 0, 1$ of the jet, instead of modes from $n = 4$ to $n = 15$ in JetRe3600.

Reference	Re_D	$n_x \times n_y \times n_z$	n_t	Tu_j/D	x_c	B	A
jetRe1800	1800	$411 \times 211 \times 211$	1.6×10^6	0.79×10^5	$24.1r_0$	5.8	0.096
jetRe3600	3600	$531 \times 261 \times 261$	1.6×10^6	0.79×10^5	$17.1r_0$	6.1	0.091
jetRe11000	11000	$651 \times 261 \times 261$	2.8×10^6	1.34×10^5	$13.5r_0$	6.4	0.087
jetRe3600thin	3600	$531 \times 261 \times 261$	1.0×10^6	0.48×10^5	$13.7r_0$	6.5	0.084
jetRe3600mode	3600	$531 \times 261 \times 261$	1.0×10^6	0.48×10^5	$14.6r_0$	6.3	0.088

TAB. 1 – Simulation parameters : jet Reynolds numbers, numbers of grid points (n_x, n_y, n_z) and of time steps n_t, simulation times T ; and mean flow characteristics : locations of the end of the potential core x_c, and decay constant B and spreading rate A in the self-similar jet flows.

2.3 Computational parameters

The numerical set-up used for the five LES is the same except for the axial and radial extents which are reduced at lower Reynolds numbers to save computational time. The grids contain from 22 to 44 millions of points, and between 1 and 2.8 millions of time steps have been performed. As reported in table 1, this led to very significant computational times T, which is necessary to obtained converged statistics in the self-similarity region of the jets, in particular for high-order velocity moments and for the energy budgets. More numerical details can be found in a recent paper [19].

3 Results

Mean flow and turbulence properties, including the second-order and third-order velocity moments, and the energy budgets, have been evaluated in the jets from the LES fields. Some results are provided here to give some insight into the influence of the initial conditions.

3.1 Vorticity

A first illustration of the Reynolds number effects is provided by the vorticity fields presented in figure 1. The transitions from laminar shear layers toward fully-developed turbulence can be seen. As the Reynolds number increases, the presence of fine scales is more visible. The initial development of the jet occurs also more rapidly, leading to a decrease of the length of the potential core, in agreement with experimental findings. The potential core lengths x_c, defined by $u_c(x_c) = 0.95u_j$ where u_c is the centerline mean axial velocity, and given in table 1, are consequently $24.1r_0$ for jetRe1800, $17.1r_0$ for jetRe3600 and $13.5r_0$ for jetRe11000.

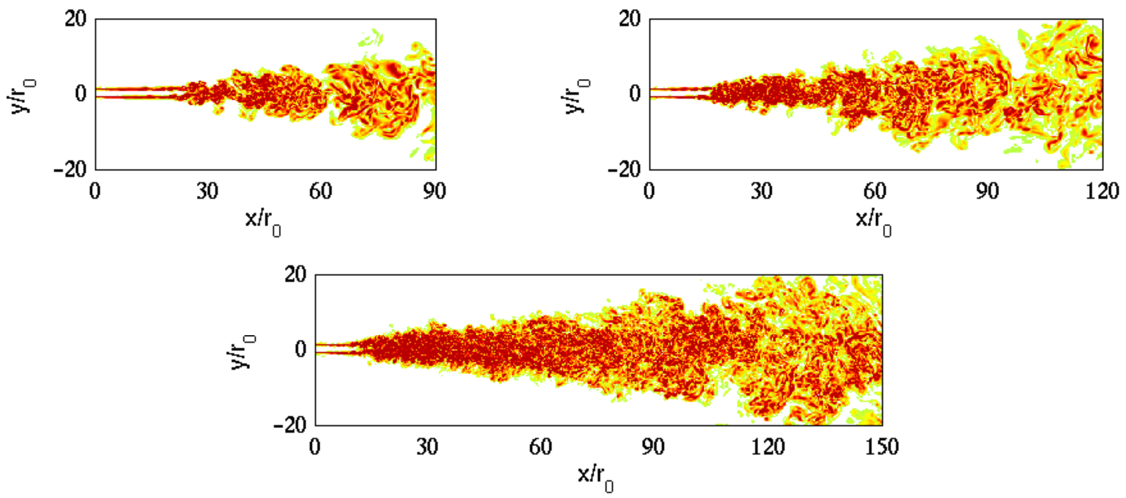

FIG. 1 – Snapshots of vorticity norm normalized by the jet diameter and the centerline mean axial velocity, $|\boldsymbol{\omega}|D/u_c$, in the plane $z = 0$, for : jetRe1800 (top left), jetRe3600 (top right) and jetRe11000 (bottom). The color scale ranges for levels from 0.4 to 2.

The impact of the initial conditions on the three jets at Reynolds number 3600 is also visible on the vorticity fields represented in figure 2. In both jetRethin and jetRemode, vortical structures are generated in the mixing layer more rapidly than in jetRe3600, resulting in potential cores shorter by about $4r_0$ as shown in table 1. It is also interesting to note that the turbulent transition in jetRe3600mode basically differs for the transitions in the other jets. In the former jet, strong persistent axisymmetric vortices can indeed be seen in the jet shear layer, whereas 3-D turbulence appears to dominate in the latter jets.

3.2 Mean flow properties

The effects of the initial conditions of the mean flow development of the jets are studied here from the variations of the centerline mean axial velocity $u_c = [u_x](y = 0)$ and of the jet half-width $\delta_{0.5}$ defined by $[u_x](y = \delta_{0.5}) = u_c/2$, where $[u_x]$ is the mean axial velocity.
The profiles of the inverse of u_c and of $\delta_{0.5}$ determined for jetRe1800, jetRe3600 and jetRe11000 are represented in figure 3. After the transitional region, they seem to vary linearly as expected. In self-preserving round jets, the mean axial velocity evolves indeed as $u_c/u_j = B \times D/(x - x_0)$, where B is the velocity decay constant and x_0 denotes a virtual origin, and the jet half-width can be written as $\delta_{0.5} = A \times (x - x_0)$, where A is the jet spreading rate. The establishment of mean flow self-similarity has then been investigated by evaluating the inverse of the local decay constant and the local spreading rate, which are defined as $1/B' = \mathrm{d}(u_j/u_c)/\mathrm{d}(x/D)$

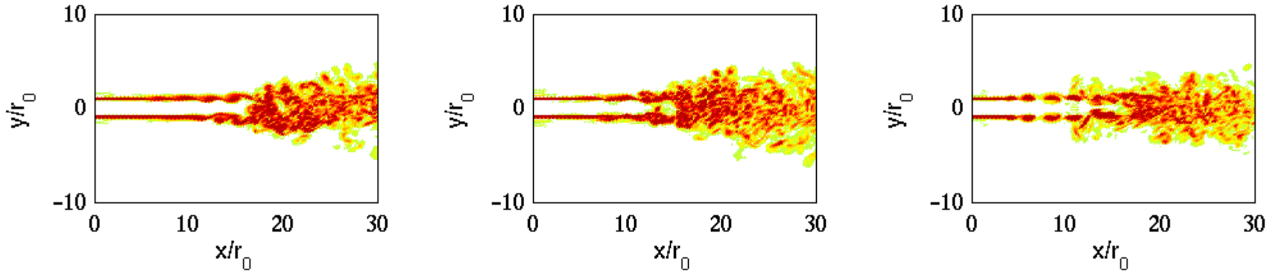

FIG. 2 – Snapshots of vorticity norm normalized by the jet diameter and inflow velocity, $|\boldsymbol{\omega}|D/u_j$, in the plane $z = 0$, for : jetRe3600, jetRe3600thin and jetRe3600mode, from left to right. The color scale ranges for levels from 0.6 to 3.

and $A' = \mathrm{d}\delta_{0.5}/\mathrm{d}x$, respectively. These local mean flow characteristics are not displayed here because of the limited space of the paper, but both are observed to tend to asymptotic values in the downstream direction, which indicate self-similarity and provide constants $B = 5.8$ and $A = 0.096$ for jetRe1800, $B = 6.1$ and $A = 0.091$ for jetRe3600, and $B = 6.4$ and $A = 0.087$ for jetRe11000 in table 1. The mean flow of self-similar round jets therefore develops at a lower rate at higher Reynolds number, in agreement with the recent experimental data obtained for plane jets by Deo *et al.* [7].

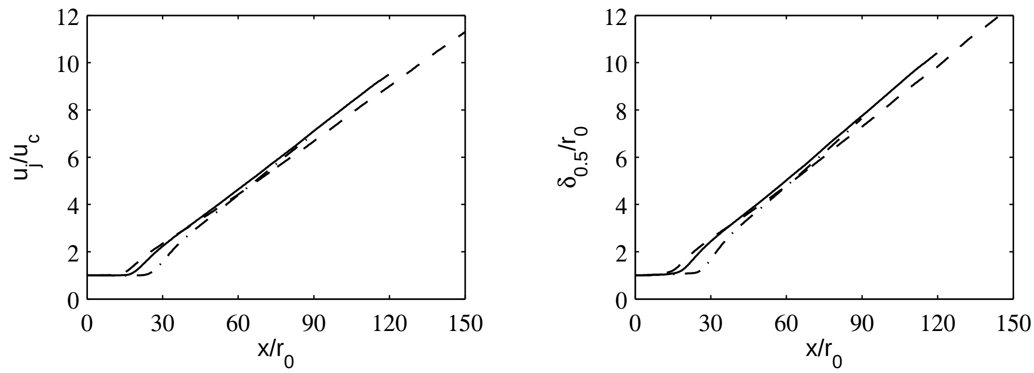

FIG. 3 – Variations of the inverse of centerline mean axial velocity u_j/u_c (left) and of the jet half-width $\delta_{0.5}/r_0$ (right), for : – · – · jetRe1800, ——— jetRe3600, and – – – jetRe11000.

The variations of the inverse of u_c and of δ_δ obtained for the three jets at Reynolds number 3600 are plotted in figure 4. They are found to depend appreciably on the jet initial conditions. In both jetRe3600thin and jetRe3600mode, the mean flow develops indeed, far from the inflow, more slowly than in jetRe3600. These behaviours are supported quantitatively by the values of the constants A and B estimated in the self-similar mean flows, given in table 1. Therefore, in jets at a given Reynolds number, changes in the initial conditions do not only affect the shear layer development, as it could be assumed at first sight, but also the spreading of the self-similar mean flow, several radii downstream of the turbulent transition, which is in agreement with the experimental observations made by Xu and Antonia [12].

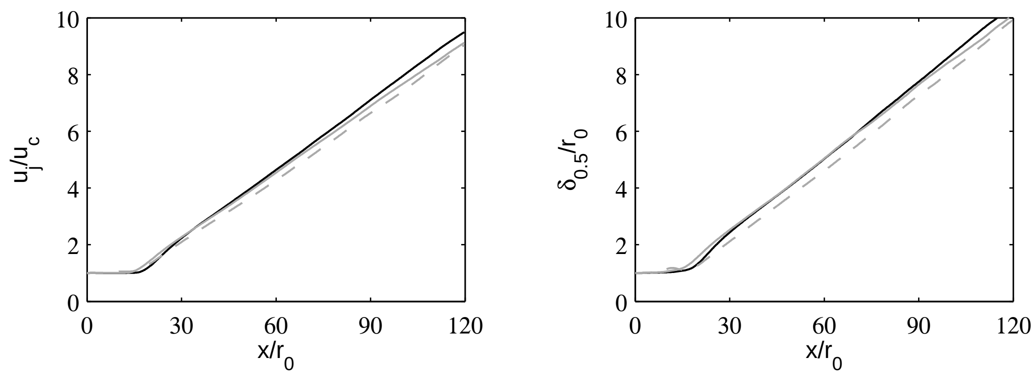

FIG. 4 – Variations of the inverse of centerline mean axial velocity u_j/u_c (left) and of the jet half-width $\delta_{0.5}/r_0$ (right), for : ——— jetRe3600, ——— jetRe3600thin, and – – – jetRe3600mode.

3.3 Turbulent intensities

The centerline profiles of the axial and radial turbulent intensities $[u'u']^{1/2}/u_c$ and $[v'v']^{1/2}/u_c$ are presented in figure 5 for the three jets with same initial conditions but varying Reynolds numbers, and in figure 6 for the three jets at Reynolds number 3600. The axial locations at which the turbulent jet flows achieve self-similarity can thus be determined.
In figure 5, the establishment of self-similarity, obtained when constant values are observed on the jet axis, is shown to occur more slowly at higher Reynolds number, *i.e.* at farther axial distance, in agreement with experimental and numerical results [8, 23]. More quantitatively, self-similarity seems to be reached around $x = 60r_0$ in jetRe1800, $x = 70r_0$ in jetRe3600 and $x = 120r_0$ in jetRe11000.

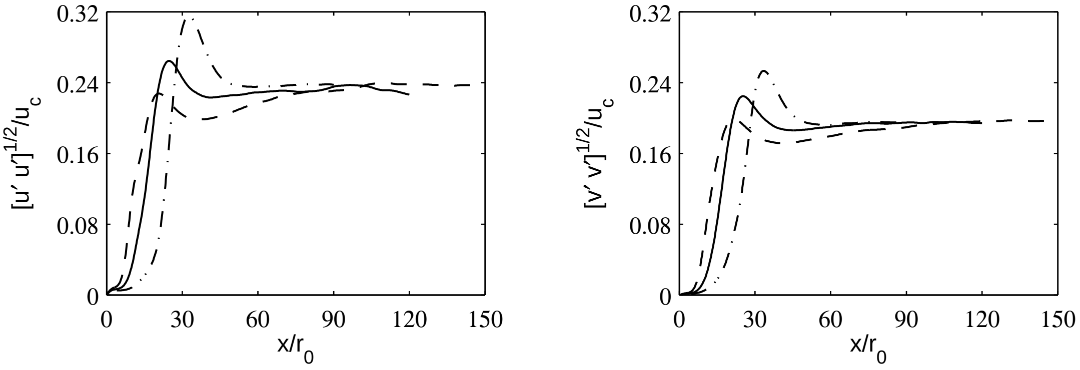

FIG. 5 – Variations along the jet centerline of axial and radial turbulence intensities $[u'u']^{1/2}/u_c$ (left) and $[v'v']^{1/2}/u_c$ (right), for : $- \cdot - \cdot$ jetRe1800, ——— jetRe3600, and $- - -$ jetRe11000.

The distance required to achieve self-similarity in the turbulent jets also appears to depend on the jet initial conditions. As shown in figure 6, the variations of the radial turbulent intensities suggest in particular that self-similarity is reached in jetRe3600mode around $x = 90r_0$, instead of $x \simeq 60r_0$ in jetRe3600 and jetRe3600thin. It is finally interesting to notice that, while the centerline turbulent intensities indicate that self-similarity establishment varies appreciably with the initial conditions and the Reynolds number, they exhibit very similar asymptotic values in the five jets considered. This little influence of the jet inflow conditions on the self-similar fluctuating velocity intensities has been previously mentioned by Xu and Antonia [12].

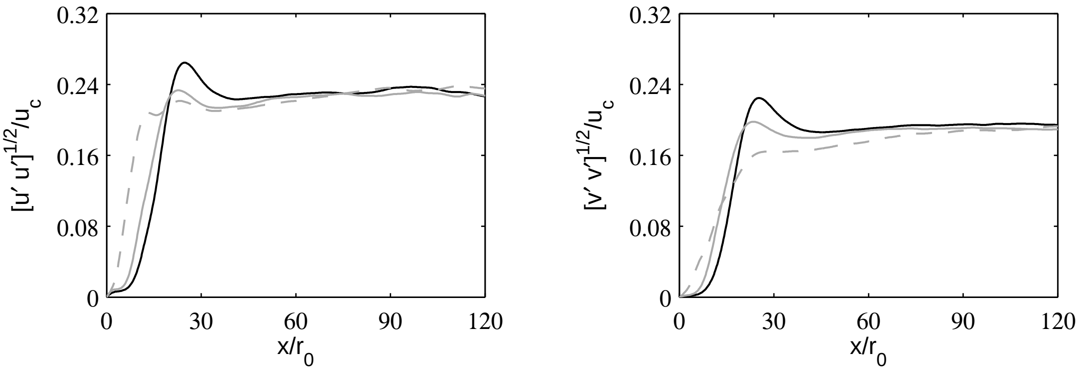

FIG. 6 – Variations along the jet centerline of axial and radial turbulence intensities $[u'u']^{1/2}/u_c$ (left) and $[v'v']^{1/2}/u_c$ (right), for : ——— jetRe3600, ——— jetRe3600thin, and $- - -$ jetRe3600mode.

4 Conclusion

The LES results shown in the present paper provide preliminary information on the influence of the initial conditions on far-field development of round turbulent jets. When they vary, the effects are in particular strong on the characteristics of the self-preserving mean flows, and on the distance required to achieve self-similarity of the turbulent flows. To evidence the rather weak modifications of the turbulence features with the initial conditions more clearly, the simulations for jetRe3600thin and jetRe3600mode will be extended, and further analyses will be proposed. The question whether the effects of the Reynolds number on the self-similar jets could be indirect effects of the variations of the early development of the jet with the Reynolds number will be also addressed.

Acknowledgments

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