Pompeo Della Posta

Central bank intervention, public debt and interest rate target zones

March 3, 2018

Abstract

The euro area crisis has been characterized by speculative attacks reflecting the market fear that some high indebted countries could go bankrupt. What is puzzling, however, is that non-euro area countries with an equally large – and in some cases even larger - public debt-to-GDP ratios have not been subject to attacks. This fact, together with the convex non-linear behavior exhibited by interest rates have been explained by observing that euro area countries could not rely on a lender of last resort, and this made possible the occurrence of self-fulfilling speculative attacks.

The model proposed in this article applies the target zones methodology relative to exchange rates, developed in the late 1980s-early 1990s, to the case of the interest rate, given that public debt stability will only be assured if the former does not exceed a given upper bound. The novelty of the paper is that, by considering the presence of a public debt demand stochastic shock - that may originate from different sources - it is possible to endogenize the determinants of the credibility of the interest rate target zone and, as a result, of public debt stability, something that in a previous paper, still based on the idea of interest rate target zones, had to be taken exogenously. Public debt stability, then, is shown to depend on the potential liquidity that the central bank can create thanks to its role of lender of last resort. Full/partial credibility is obtained when such liquidity is expected to be sufficient/insufficient to absorb completely the demand shock. The expected absence of liquidity determines the non-credibility of the interest rate target, while the expectation of a public debt increase produces a destabilizing non-credibility determining the convex interest rate non-linearity that characterized the euro area crisis.

Keywords: Interest rates target zones, central bank intervention, public debt, foreign debt, exchange rates target zones, speculative attacks

JEL Classification: E65, F34, F36

Pompeo Della Posta
Dipartimento di Economia e Management
Via Ridolfi, 10
56124 Pisa (Italy)

E-mail: pompeo.della.posta@unipi.it

Introduction[1]

After the outbreak of the euro area crisis, it was observed that some countries, like the UK, had not been affected by speculative selling of government bonds, despite the fact that their public debt-to-GDP ratio was as high as that of countries subject to attacks (as, for example, Spain). Euro area countries under attack have also been characterized by a convex non-linear behavior of the interest rate with respect to public debt-to-GDP ratio, having been the interest rates rather low during the initial, 'honeymoon' years of EMU, and jumping up suddenly during the crisis. As for the explanation of the first point, De Grauwe (2012) refers to the fact that a stand-alone country, like the UK, can rely on the potential for debt monetization guaranteed by its national central bank (CB) operating as a lender of last resort. This option, which is essential to reassure the markets and avoid the occurrence of self-fulfilling speculative attacks, was known to be unavailable instead to euro area countries, who cannot rely on the ECB to operate as a lender of last resort, given that the Maastricht Treaty forbids it explicitly to act as such. Since euro area governments cannot rely on domestic central banks (CBs) to monetize the debt, De Grauwe (2012) argued that governments could be considered as issuing, *de facto*, foreign currency denominated debt, differently from what is the case for a stand-alone country like the UK.

The interest rate convex non-linearity could be explained instead by concluding that the crisis was driven by negative self-fulfilling expectations, rather than by

[1] I would like to thank an anonymous referee for providing very useful comments and suggestions. Needless to say, I retain full responsibility for any remaining mistake. I would like to thank also the participants in the seminar held in the Summer 2016 at the University of Victoria (Victoria, BC, Canada), where I presented a different version of this model.

diverging economic fundamentals (De Grauwe and Ji, 2013a). The authors, however, did not provide any formal theoretical explanation for this conclusion.

By adopting a model of heterogeneous markets' expectations about the uncertain level of the country's maximum feasible primary budget surplus, Tamborini (2015) provides an alternative, rigorous but also very intuitive explanation for both puzzles. He shows that the risk premium on public debt rises dramatically and non-linearly when the primary surplus which is necessary to grant debt stabilization approaches its perceived maximum feasibility limits beyond which public debt will not be sustainable anymore.

It would be possible to conclude, then, that it is not the absolute size of public debt that matters for its sustainability, but rather the potential of a country to stabilize it through the intervention of either the monetary authority (De Grauwe and Ji, 2013a) or the fiscal authority (Tamborini, 2015).

Della Posta (2017) uses the exchange rates target zones (TZs) modeling technique and applies it to a case of interest rate targets. He shows that when the interest rate is expected to exceed the target assuring public debt stability, speculation is encouraged. No attacks would take place, however, if either the fiscal authority could avoid the growth of public debt by running a primary surplus, or the CB could operate as a lender of last resort, thereby preventing the interest rate to move beyond its upper threshold. That paper, however, takes exogenously (as Bertola and Caballero, 1992, also do in the different context of exchange rate target zones) the two cases of full or no credibility: it simply concludes that in the case of the CB acting as a lender of last resort or with public debt stability guaranteed by a feasible primary surplus, the interest rate is smoothed, while in the opposite case it follows a non-linear, convex behavior. No explanation is provided of the endogenous reasons determining one case or the other. Moreover, that approach does not allow to consider any intermediate case of partial credibility, precisely because the sources of credibility are taken exogenously. This paper moves beyond those conclusions because it allows to endogenize them. In particular, by introducing a public debt demand stochastic shock v_t, it allows discussing the liquidity problems and to analyze the CB's role of lender of last resort that is subject to the size of the shock. Either outcome, then, is shown to depend on the amount of liquidity that the CB can potentially create, depending on its specific institutional setup. A potential availability of liquidity allows for a large 'honeymoon' effect and 'smooth pasting' (to use the terminology introduced by Krugman, 1991 in his seminal paper) – namely full credibility of public debt stabilization - only when it is sufficient to absorb fully the excess of supply of bonds resulting from the public debt demand stochastic shock v_t. Credibility will only be partial, however, when such a potential liquidity is expected to be insufficient to absorb fully the debt demand shock, while the expected unavailability of liquidity determines the lack of credibility of the interest rate target. Finally, the expectation of a public debt increase – rather than of a full or partial reduction - produces a destabilizing non-credibility determining the convex interest rate non-linearity that characterized the euro area crisis. This is the case in which public debt is *de facto* denominated in a foreign currency, so that the CB reassurance cannot operate, as in the case considered by De Grauwe and Ji (2013a).

This article is structured as follows. Section I reviews briefly the relevant literature on target zones and on the euro area crisis and provides the motivation of the paper. Section II introduces the interest rate on public debt and defines its upper feasibility limit. Section III presents the model to be solved when considering the 'honeymoon' case, in which the interest rate threshold assuring public debt stability is expected not to be overtaken. Section IV considers different degrees of CB credibility in defending the interest rate target zone and therefore in guaranteeing the stability of public debt. Some final remarks close the paper in Section V. The Appendix contains the mathematical steps to be made in order to derive the results of this paper.

I.

Literature review and motivation of the paper

The celebrated ‘honeymoon’ model of exchange rate TZs, presented by Paul Krugman at the end of the 1980s and published eventually in 1991 (Krugman, 1991), generated a vast amount of literature.[2] A summary of the main conclusions obtained in that literature is contained in Della Posta (2017), who applies the approach followed by Krugman (1991) and Bertola and Caballero (1992) to the euro area crisis. In doing so he refers also to Tamborini (2015), who shows how the credibility of the fiscal consolidation required in order to stabilize public debt may be questioned by the feasibility of the primary surplus that a government can run. He argues that the expectation of an upper threshold that limits the social feasibility of the primary surplus – due to the existence of an upper limit on the revenues that a government can excise from its citizens and of a lower limit on the expenditures that it can cut – works exactly as the upper band of a TZs regime. As a matter of fact, the presence of such a limit on the primary surplus also implies that, in order to grant public debt stability, interest rates have to be kept within a given target: a higher interest rate would make public debt unsustainable. A ‘divorce’ which is similar to the one identified by Bertola and Caballero (1992) arises if neither the fiscal authority can run the primary surplus which is required for public debt stability, nor the monetary authority is capable to stabilize the interest rate on public debt because it cannot operate as a lender of last resort. The conditions determining a ‘honeymoon’ or a ‘divorce’, however, are not addressed and are taken, therefore, exogenously (the interest rate target is simply assumed to be credible in one case and non-credible in the other one, without investigating the

[2] Krugman and Miller (1992) contains a first set of important contributions to this literature.

underlying conditions assuring credibility or determining non-credibility). What that paper misses, then, is the explanation of the endogenous mechanism determining one case or the other. Moreover, that approach does not allow to consider any intermediate case of partial credibility, precisely because the sources of credibility are taken exogenously.

The present paper, instead, following the approach taken by Krugman and Rotemberg (1992) (who still refer to an exchange rate target zone) rather than the one taken by Bertola and Caballero (1992), endogenizes those conclusions: by focusing solely on the part taken by the CB – thereby leaving on the background the role of the government's primary surplus - it introduces in the model a public debt demand stochastic shock v_t. The shock may be due, for example, to a recession, to a perturbation in the financial system due to liquidity problems, to contagion or to any other sort of credibility issue determining a "sudden stop". As a matter of fact, this is what append in the euro area after the revelation by the newly elected Greek government, in November 2009, that the previous leader had been cheating on public accounts, so that the actual sizes of both the public deficit and the public debt were much larger both in absolute terms and as a ratio of GDP. The shock propagated soon from Greece to the Southern euro area countries characterized by some other form of intrinsic fragility, like Portugal (whose current account deficit signaled an excess of domestic absorption) or Ireland and Spain (whose bursting of the real estate bubble had obliged the respective governments to step in and increase dramatically their respective public debt in order to avoid the collapse of the banking sector), investing also Italy in 2012.

An additional public debt demand shock, however was also provided by what can be called the "Deauville scare", namely the Franco-German declaration with which, in October 2010, the German chancellor Angela Merkel and the French president

Nicolas Sarkozy, announced the introduction of the principle of the so-called *bail in*. Such an announcement, making fully aware bond holders that the default of any of their debtors would have fallen on their shoulders, scared the market and it is not by chance that after that declaration the euro area crisis definitely took off.

Once a public debt demand stochastic shock of the kind mentioned above hits the economic system, the different possible outcomes are determined by the potential amount of liquidity that the CB is expected to dispose of (once more, leaving aside the role of fiscal consolidation, that will be considered in future research). When the CB can play the role of lender of last resort, the potentially infinite amount of liquidity that it can create is expected to be sufficient to absorb fully the excess of supply of bonds resulting from the assumed demand shock so as to the determine a full public debt stabilization, namely the 'smooth pasting' first identified by Krugman (1991). The degree of credibility that the CB can guarantee depends, then, on the potential liquidity that the CB is expected to be able to dispose of. This paper, however, extending the conclusions obtained by Krugam and Rotemberg (1992) to the case in which public debt is expected to increase, rather than to be reduced, also explains the convex interest rate non-linearity documented by De Grauwe and Ji (2013a) – and that Della Posta (2017) took exogenously. This is the case when public debt is *de facto* denominated in a foreign currency, so that the CB reassurance cannot operate. That is precisely the case considered by De Grauwe and Ji (2013a), which finds in this model, then, a satisfactory formalization.

II.

The interest rate on public debt and its upper threshold

The determinants of the interest rate on public debt are discussed below. This is done by following the approach adopted by Krugman and Rotemberg (1992). As we will see, such a modeling approach allows endogenizing the credibility of the defense of an interest rate target zone.

II.1. The interest rate on public debt

A standard linearized version of the interest rate parity for public debt bonds is as follows:

(1) $$i_t = \bar{r} + RP_t,$$

where:

(2) $$RP_t = \alpha(b + v_t) + \beta \frac{E[di_t]}{dt},$$

so that (assuming for simplicity and without any loss of generality, $\bar{r} = 0$):

(3) $$i_t = \alpha(b + v_t) + \beta \frac{E[di_t]}{dt}.$$

Eq. (1) tells that the nominal interest rate, i_t, can be thought as determined by a riskless reference interest rate, $\bar{r}$ which is assumed to be determined exogenously by the central bank, and by a risk premium, RP_t. In turn, the latter depends on two elements (Eq. 2). The first one, differently from Della Posta (2017), is determined not only by the (constant) supply of public bonds (b),[3] but also by a public debt demand stochastic shock, v_t.[4] The economic intuition suggests that the interest rate increases with the *excess* of supply of public bonds resulting from v_t. As it has been discussed above, it can be thought as resulting from a recession, a shock in the financial system due to

[3] Corsetti *et al.* (2014) consider the public debt-to-GDP ratio – rather than the absolute value of public debt - as a variable affecting positively the interest rate, but in our simplified model the difference is not relevant.

[4] See Krugman and Rotemberg (1992), for the analogue in the case of exchange rate determination, in which a constant money supply and a money demand shock – increasing the excess money supply - jointly determine the exchange rate.

liquidity problems, or a "sudden stop" of the kind of those that have been affecting financial markets in many different instances of currency and financial crises. Such a shock - implying an excess of supply of bonds, reduces their price, with which the interest rate is inversely related. The sensitivity of the interest rate with respect to b and v_t is measured by parameter α. The second part, instead, is the one with self-fulfilling features and captures the interest rate non-linearity documented in the euro area crisis and in many other currency and public debt crises. The lower the expected sustainability of public debt, the higher the expected future variation of the interest rate, that in turn affects directly the current interest rate level with a weight given by parameter β.

II.2. The interest rate upper threshold on public debt

The central bank defends either explicitly or implicitly an upper threshold for the nominal yield on public bonds guaranteeing public debt sustainability. The concern of the ECB for the interest rate and public debt stability of euro area countries is proved by the repeated declarations of its president, Mr. Mario Draghi, to justify the bank's intervention precisely with the need to reduce the high interest rates – unjustified by the state of economic fundamentals - that were threatening public debt sustainability and in the end undermined the orderly running of monetary policy. Needless to say, in this case no lower interest rate threshold exists, since public debt would only benefit of low interest rates.[5] The value taken by i_t can be identified, then, as follows:

$$i_t = \bar{\imath}^* \text{ if } i_t \geq \bar{\imath}^*$$

$$i_t = \tilde{\imath}_t \text{ if } i_t < \bar{\imath}^* \tag{4}$$

where $\bar{\imath}^*$, $\tilde{\imath}_t$ and i_t represent respectively the upper central bank's thresholds for the interest rate, the interest rate that would obtain when it fluctuates within the announced

[5] Thanks to an anonymous referee for pointing this aspect to my attention.

upper limit, and the interest rate prevailing in case no commitment is taken by the central banks.

The value of $\bar{\iota}^*$ may be thought as implicitly determined by a public debt solvency equation, as it will be discussed below.

The standard public debt dynamics, namely the continuous time variation of the public debt-to-GDP ratio, db_t, in the case in which both the fiscal and the monetary authority can intervene to stabilize public debt, is as follows:[6]

(5) $$db_t = (f_t - m_t)dt + (i_t - g_t)b_t dt$$

The term f_t is the primary public deficit-to-GDP ratio (namely e_t - t_t, where e_t is government's fiscal expenditure-to-GDP and t_t are government revenues-to-GDP). The term m_t is the rate of money creation that allows to reduce public debt and g_t is the rate of growth of nominal GDP. The term $(i_t - g_t)b_t$ is therefore the service on the debt as a ratio of GDP.

In order to have debt stabilization, namely $db_t = 0$ in equation (5), it must be the case that:

(6) $$m^* - f^* = (i^* - g^*)\bar{b},$$

where the symbol * refers to long term values and $\bar{b}$ is a steady state value of the public debt-to-GDP ratio.

---

[6] The deterministic version of the stability condition reported in equation (1) above can be derived easily by considering the dynamic equation of public debt:

$$\frac{dB_t}{dt} = (E_t - T_t) - M_t + i_t B_t$$

where B_t is the steady state level of public debt, E_t is constant level of government expenditure, T_t is the constant level of taxation - so that $(E_t - T_t)$ is the constant primary deficit F_t, M_t is the level of money supply, and i_t is the constant nominal interest rate to service the public debt. From the equation above, by dividing through by the nominal GDP, thereby considering the public debt/GDP ratio Equation (5) above is obtained.

As it is clear from the equation above, the larger $i^*$and the smaller g^*, the larger the required central bank intervention, m^*, and the lower the fiscal deficit-to-GDP ratio, f^*, will have to be in order to stabilize public debt.

By rearranging equation (6), then, and by assuming $f^* = 0$ so as to concentrate on the role of the central bank (as De Grauwe and Ji, 2013a implicitly also do), we have that:

(7) $$i^* = g^* + \frac{m^*}{\bar{b}}$$

Where i^* indicates the long term interest rate that assures a steady state public debt-to-GDP ratio, $\bar{b}$, for a given long term rate of money growth, m^* and a long term GDP rate of growth, g^*.

In principle, a stand-alone central bank will be able to increase infinitely the rate of growth of money. If that is the case, m^* is unbounded and can match any increase of $\bar{b}$, so as to leave unchanged the interest rate i^* granting public debt stability.

However, it is reasonable to consider, for future use, also the case in which the central bank may have an upper constraint for money growth that will not want to overtake (mainly for the fear of the inflationary consequences that an excessive money creation would produce), and that we can indicate with $\bar{m}^*$. In turn, then, $\bar{m}^*$ determines $\bar{\imath}^*$ (for given g^* and $\bar{b}$), which is the maximum long run interest rate that would grant public debt sustainability and that the central bank is targeting. From the definitions given above, then, it follows that for the sustainability of $\bar{b}$ to be possible (given g^*), it must be that $m^* \leq \bar{m}^*$, thereby implying that $i^* \leq \bar{\imath}^*$.

So, the $\bar{\imath}^*$ that the central bank can credibly defend is determined by equation (7') below:

(7') $$\bar{\imath}^* = g^* + \frac{\bar{m}^*}{\bar{b}} \ .$$

By considering equation (1), equation (7') implies:

(1'') $$\overline{RP}^* = g^* + \frac{\bar{m}^*}{\bar{b}} - \bar{r}^* ,$$

where $\overline{RP}^*$ reflects the highest level of the risk premium that a central bank can stand for given $\bar{m}^*$, $\bar{r}^*$, g^*in order to keep $\bar{b}$ at its steady state value.[7] The variable $\bar{r}^*$ is the lowest possible long run interest rate level that the central bank can choose (that might coincide with the zero lower bound, but that might even be negative) while still assuring public debt stability.

II.3. The interest rate target model

As we have seen above, the central bank stabilizes public debt in the steady state for a given maximum feasible level of $\bar{m}^*$, for g^* and for $\bar{\iota}^*$. Such a stabilization, however, does not rule out other variations of public debt.

As a matter of fact, while the supply of public debt is assumed to be constant and is represented by *b*, a public debt demand stochastic shock can cause an excess of public debt supply. The public debt demand shock is assumed to follow a driftless Brownian motion:[8]

(8) $$dv_t = \sigma dz$$

The parameter σ represents the instantaneous standard deviation of the Brownian motion and the term *dz* is the Brownian motion variation which is so characterized:

(9) $$dz = \chi\sqrt{dt},$$

where χ is a random variable which is independently, identically and normally distributed, with 0 mean and variance equal to 1, and *dt* is an infinitesimal time variation.

Let us consider the equation determining the value of the interest rate that would prevail within a TZ ($\tilde{\iota}_t$):

[7] Notice that with a little model modification, the argument would not change if we reasoned directly in terms of the risk premium, $\overline{RP}^*$, rather than in terms of the interest rate.
[8] Krugman and Rotemberg (1992) consider instead a Brownian motion with drift. In this context, the latter would not add any further insight and we can therefore avoid its use here.

(3') $$\tilde{\imath}_t = \alpha(b + v_t) + \beta \frac{E[di_t]}{dt}$$

III.

Feasibility of central bank intervention: the 'honeymoon'.

In order to solve Equations (3'), given (8) and (9), let us follow the TZ literature by assuming a generic functional form for the $\tilde{\imath}_t$ as a function of (b+v_t). We will then use it in order to have a closed form solution of the differential Eq. (3'). The simplest and more obvious functional form that we can assume for $\tilde{\imath}_t$ is as follows:

(10) $$\tilde{\imath}_t = f(b + v_t)$$

We can now use this equation to calculate the expected interest rate variation. In order to do this, let's expand our (stochastic) equation in a Taylor-type series, by calculating Ito's differential:

(11) $$d\tilde{\imath}_t = f'(b + v_t)E(dv_t) + \frac{1}{2}f''(b + v_t)E(dv_t)^2$$

From the definition of dv_t in (8), it turns out that $(dv_t)^2 = \sigma^2\chi^2 dt.$ By considering expected values and by dividing by the infinitesimal temporal variation, we obtain Ito's Lemma:

(12) $$\frac{E(d\tilde{\imath}_t)}{d_t} = \frac{1}{2}f''(b + v_t)\sigma^2,$$

given that $\frac{E(dvt)}{dt} = 0$ and $\frac{E(dvt)^2}{dt} = \sigma^2$.

By replacing (12) into (3) we have:

(13) $$\tilde{\imath}_t = \alpha(b + v_t) + \frac{\beta\sigma^2}{2}f''(b + v_t)$$

This is a differential equation of the second order whose generic solution (given that we have assumed no lower bound) is of the kind:

(14) $$\tilde{\imath}_t = f(b+v_t) = \alpha(b+v_t) + Ae^{\lambda v_t}.$$

By taking a second order derivative with respect to v_t we have:

(15) $$f''(b+v_t) = \lambda^2 Ae^{\lambda v_t},$$

so that, by replacing it into Eq. (13) above, we obtain:

(16) $$\tilde{\imath}_t = \alpha(b+v_t) + \beta\frac{\sigma^2}{2}\lambda^2 Ae^{\lambda v_t}.$$

By comparing (16) with (14), it turns out that:

$$Ae^{\lambda v_t} = \lambda^2\beta\frac{\sigma^2}{2}Ae^{\lambda v_t},$$

namely:

$$Ae^{\lambda v_t}\left(\lambda^2\beta\frac{\sigma^2}{2} - 1\right) = 0\,.$$

This means that we have two roots of the characteristic equation, that are:

(17) $$\lambda_{1,2} = \pm\sqrt{2/\beta\sigma^2}$$

The general solution, then, will be:

(18) $$\tilde{\imath}_t = f(b+v_t) = \alpha(b+v_t) + Ae^{\lambda v_t}$$

In order to determine *A* and close the model in the case in which the CB operates as a lender of last resort, we can use the 'smooth pasting' condition, that was also used to close the first generation of TZs models, initiated by Krugman (1991) and showing the stabilizing effects of the imposition of fluctuation bands on exchange rates. This is realized by a tangency condition.

Intuitively, the expectation that the current interest rate, $\tilde{\imath}_t$, will not be allowed to exceed the upper level $\bar{\imath}^*$, is based on the expectation that the more the public debt demand shock, v_t, hits the economy (thereby increasing the excess of supply of public debt), the more the supply of public debt (b) subject to an interest rate service will be absorbed by the CB's debt monetization. Such an intervention will prevent the excess of public debt supply that, if happening, would drive the price of public bonds down, and the interest rate on them up. This will only be possible, however, if the CB is able to operate as a lender of last resort, demanding the excess of public debt issued by the government and undesired by the market. If that is the case, when $\tilde{\imath}_t$ reaches $\bar{\imath}^*$, the former is expected not to overtake the latter, therefore remaining within the band, no matter whether b_t will be increasing or decreasing: this can only happen if a tangency condition occurs, as in Figure 2.

In this case, the upper threshold of the interest rate operates as a reflecting barrier, rather than as an absorbing one.

So, let us impose the 'smooth pasting' condition by taking the FOC of Eq. (18):

$$\frac{d\tilde{\imath}_t}{dv_T} = \alpha + \lambda A e^{\lambda v_T} = 0,$$

where v_T is the value of the public debt demand shock at which $\tilde{\imath}_t$ reaches $\bar{\imath}^*$, and from which it follows that:

(19) $$A = \frac{-\alpha e^{-\lambda v_T}}{\lambda} < 0.$$

By replacing (19) into (18) it turns out that when the interest rate moving within a credible TZ (that is indicated with $\tilde{\imath}_t^C$) reaches $\bar{\imath}^*$, we have:

(20) $$\tilde{\imath}_t^C(v_T^C) = \bar{\imath}^* = \alpha(b + v_T^C) - \frac{\alpha}{\lambda}$$

So that:

(21) $$v_T^C = \frac{\bar{\imath}^*}{\alpha} + \frac{1}{\lambda} - b,$$

Where v_T^C is the largest public debt demand shock that can be resisted when a credible interest rate target is in place, namely the value of v_t at which $\tilde{\imath}_t^C = \bar{\imath}^*$. We know also that if the interest rate is not subject to the expectation of any interest rate targeting (I identify it with i_t^{FF} in Figure 2), when reaching the upper edge it will take the following value:

(22) $$i_t^{FF}(\bar{v}) = \bar{\imath}^* = \alpha(b + \bar{v}),$$

from which we have: $\bar{v} = \frac{\bar{\iota}^*}{\alpha} - b$, where $\bar{v}$ is the highest level of the public debt demand shock still granting public debt sustainability when no interest rate targeting is undertaken, namely the level of v_t at which $i_t^{FF} = \bar{\iota}^*$.

By equating Eq. (20) to Eq. (22), it is immediate to conclude, then, that:

$$v_T^C - \bar{v} = \frac{1}{\lambda} \tag{23}$$

The difference between v_T^C and $\bar{v}$, which is given by $\frac{1}{\lambda} = \sqrt{\frac{\beta\sigma^2}{2}}$, represents what Krugman defined – in the different context of exchange rates TZs - as the 'honeymoon' effect. It tells us by how much the excess of public debt supply can increase – namely it tells us what is the largest public debt demand stochastic shock that can be resisted - while keeping $\tilde{\iota}_t^C \leq \bar{\iota}^*$ in the case in which the CB enjoys full credibility and the 'smooth pasting' condition applies, thereby allowing the upper interest rate band to act as a reflecting barrier. It will become clear in a while what the size of the 'honeymoon effect' depends upon, which is the novelty of this paper: we will have been able, then, to endogenize the credibility of the interest rate target assuring public debt stability.

All of this would only apply if the CB has, and/or is expected to have, an unlimited room for buying the excess of public debt which is undesired by the market, so that the interest rate will not be allowed to exceed its stability threshold. This condition realized both during the initial, 'honeymoon' years of EMU, and after the celebrated 'whatever it takes' Draghi speech.

What follows, by taking the direction given by Krugman and Rotemberg (1992) in the different context of exchange rates TZs, highlights the role that the availability of potential liquidity plays in determining the credibility of the CB in acting as a lender of last resort, thereby endogenizing the result that Della Posta (2017) had to take exogenously.

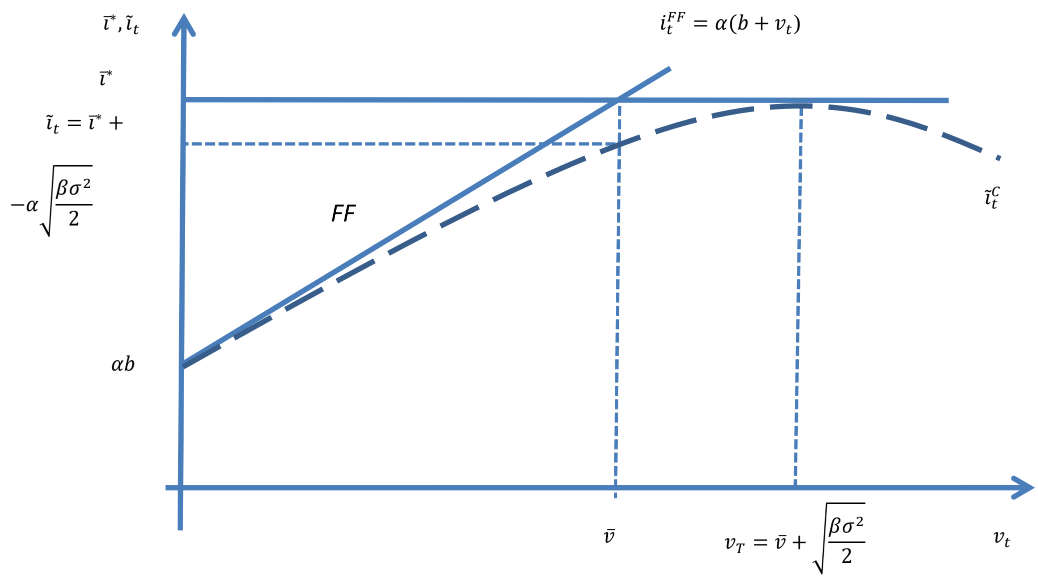

Figure 1: The interest rate 'smooth pasting' condition.

IV.

Potential liquidity, public debt stability and speculative attacks

A public debt default (as a consequence of the interest rate exceeding its stability threshold, $\bar{\imath}^*$) can only be avoided if the CB is in a position to buy the debt issued by the government that the market at some point may decide not to rollover. As observed by De Grauwe (2012) and by De Grauwe and Ji (2013a), this is certainly possible in the case of a stand-alone CB, which can print money at its own will and can be thought, then, as having the potential for an unlimited supply of liquidity.[9]

Even in that case, however the presence of foreign currency denominated public debt (together with CB's preferences about the effects of a public debt default as opposed to preferences for an inflationary spike, to be discussed below), may impair the CB commitment to avoid a debt default and its resulting credibility, unless it has the

[9] Normally, an anti-inflationary and independent central bank will avoid printing money when that may cause inflation, but it is difficult to imagine that a monetary authority which is caught between the need to avoid inflation and the need to avoid the bankruptcy of her own country will prefer the second option.

availability of a sufficiently large quantity of foreign reserves.[10] CB's reserves can be either 'real', namely physically present with the CB, or (in this case too) 'virtual' or "potential", if it is known and expected that some Special Drawing Rights or international loans will be available in case of need. In what follows, however, the case of the availability of foreign reserves will not be considered, in order to prove the main results of the paper in a more direct way.

We can consider at least three different cases, ranging from the most stable one - in which a stand-alone CB, who is in charge of the conduct of monetary policy within a domestic country, guarantees the full repayment of public debt - to the most unstable one, in which the market expects no CB intervention to stabilize public debt (because the CB cannot operate as a lender of last resort, being the public debt denominated, formally or *de facto*, in a foreign currency of which the CB has no availability). The intermediate

[10] Baksai *et al.* (2012) discuss thoroughly the role played by foreign currency denominated debt and by foreign reserves. Foreign reserves, however, are also relevant for guaranteeing public debt stability when the latter is not denominated in foreign currency. Their required size ranges from the value corresponding to 3 months of imports, to the size of short term *foreign* debt (this is the so-called Guidotti-Greenspan-IMF rule and the adjective *foreign* refers to the part of the debt which is in the hands of foreign residents, which is usually considered as more volatile than the debt which is owned by domestic residents), or to a more general ratio of 20% of the monetary aggregate M2 (see IMF, 2011 and Greenspan, 1999). It has to be acknowledged, then, that the exact size of foreign reserves guaranteeing the repayment of public debt, is far from univocally established, and it varies depending on the status of the country under scrutiny (emerging market, low income country or advanced economy) and "on a variety of [other] factors: macro-economic fundamentals; the exchange rate regime; the quality of private risk management and financial sector supervision; and the size and currency composition of the external debt." (Fischer 2001, reported by Rodrik (2006) in discussing the social costs of the accumulation of foreign reserves). An often cited theoretical model of the optimal size of foreign reserves is Jeanne and Ranciere (2005).

situation, as we will see, will be the one in which the stand-alone central bank may choose optimally not to monetize the public debt fully in order to avoid a too high inflation rate.

Moreover, public debt, b, is assumed to be either fully denominated in a domestic currency, or fully denominated in a foreign currency. Mixed compositions of the public debt (partly denominated in a domestic currency and partly denominated in a foreign currency) could also be considered, but once more, while certainly enriching the analysis, that exercise would not add too much to the main conclusions of the paper.

The different cases are analyzed in the next paragraph.

IV.1. Interpreting 'smooth pasting': full credibility of the central bank as a lender of last resort

As we have observed above, the 'smooth pasting' condition only obtains if the CB is expected to have the potential to provide enough liquidity to absorb the excess of bonds supply resulting from the occurrence of a stochastic shock hitting the demand of public debt. The role of potential liquidity has remained hidden in the background so far, and did not play any role in Della Posta (2017).

When public debt is denominated in domestic currency ($b = b_D$) and we are in the case of a stand-alone central bank (namely a central bank which is in charge of the conduct of monetary policy within its country), there is always the possibility to repay the debt thanks to money creation, as suggested by De Grauwe (2012) and De Grauwe and Ji (2013a, 2013b), so the potential liquidity which is available to the CB is virtually unlimited.

In such a case, then, considering the public debt stability condition described by Equation (6) (in which we take $f^* = 0$), we have that $b_D = \bar{b} = \frac{m^*}{\bar{\imath}^* - g^*}$, so that there is a full guarantee that the public debt will be honored.

From Eq. (20) above, we know already that, with 'smooth pasting', namely with a fully credible interest rate target, when $\tilde{\imath}_t^C$ touches $\bar{\imath}^*$ we have that $\tilde{\imath}_T^C(v_T^C) = \bar{\imath}^* = \alpha(b + v_T^C) - \frac{\alpha}{\lambda}$.

We also know that, after the shock, the excess of public debt supply will be limited to v_T^C (with which we indicate the largest public debt demand shock that can be resisted while guaranteeing public debt sustainability in the case of a credible TZ). This is due to the fact that the CB will have bought back the existing supply of bonds, *b*, thanks to the available potential liquidity which is assumed to be sufficient to guarantee the repayment of *b*. After that intervention, the interest rate will not be targeted anymore, and it will follow a free float path (I indicate it with $i_t^{F'F'}$ and it is described by Eq. (24) below). $i_t^{F'F'}$ is parallel to i_t^{FF}, that we have seen in Figure 1 above and that is described by Eq. (22). When $i_t^{F'F'}$ reaches $\bar{\imath}^*$, it takes the value:

(24) $$i_t^{F'F'}(v_T^C) = \bar{\imath}^* = \alpha v_T^C.$$

A standard arbitrage argument (also used by Krugman and Rotemberg (1992) in the different context of exchange rates TZs), suggests that there cannot be any discrete jump in the interest rate when moving from one regime to the other, namely from Eq. (20) to Eq. (24). At point C in Figure 2, then, the two curves need to take the same value, so that by equating them it follows that:

(25) $$b = \frac{1}{\lambda}.$$

Notice that by using Equation (25) into (21) above, we obtain the same result as in Equation (24), thereby confirming the coherence of the approach that has been followed.

By considering Eq. (23) above, this gives us an extremely important and in fact a key result, namely that:

$$v_T^C - \bar{v} = b.$$

This allows us to provide a deeper interpretation of Eq. (23): in the case of 'smooth pasting', the size of the 'honeymoon' $(v_T^C - \bar{v})$, is determined by the size of the domestic debt that can be guaranteed by the CB. In other words, the 'honeymoon effect' is not a gift from heaven, but it is something that is tied down by the availability of the potential liquidity for debt stabilization: the larger is the latter, the larger the 'honeymoon'.

In the case of a public debt which is fully denominated in domestic currency, then, it must be that:

(26) $$v_T^C - \bar{v} = b_D = \frac{m^*}{\bar{\iota}^* - g^*}.$$

Namely:

(26') $$\frac{m^*}{\bar{\iota}^* - g^*} = \frac{1}{\gamma} = \sqrt{\beta\sigma^2/2} = (v_T^C - \bar{v}).$$

The size of the 'honeymoon' $(v_T^C - \bar{v})$, obtained in Equation (23) above, then, has been endogenized, and now we know that it is given by the amount of liquidity which is potentially available to the CB, whose value, in the case that we are considering, is $b_D = \frac{m^*}{\bar{\iota}^* - g^*}$.

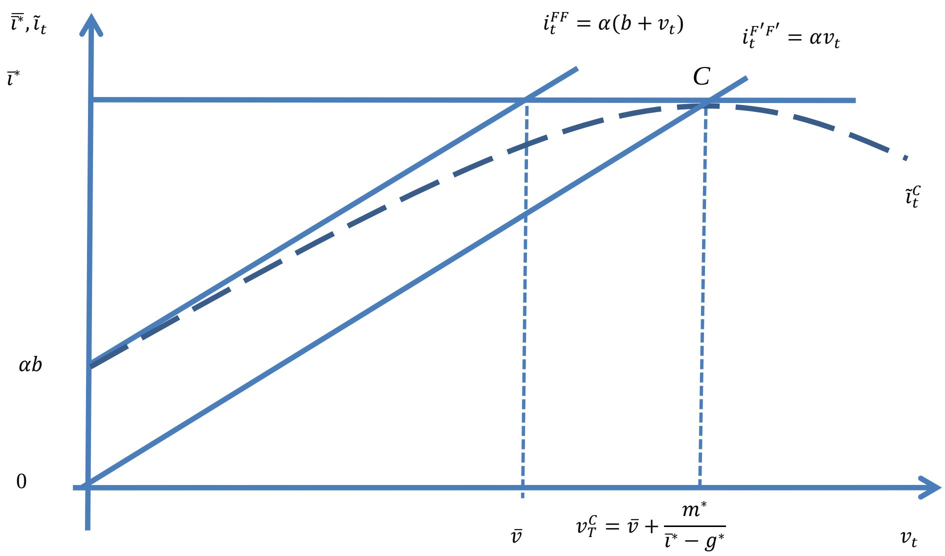

Figure 2: A credible target zone, producing the result of interest rate 'smooth pasting'.

IV.2. Partial credibility of the central bank as a 'lender of last resort'

Let us follow now a similar approach, but let us consider the case in which the CB is not expected to have at its disposal (or it is not expected to be willing to use it) enough liquidity to cover the excess of debt supply. If this is the case, the maximum amount of potential CB liquidity is $\bar{m}^* < m^*$, as discussed in Paragraph II.2. above.

This means, then, that $\frac{\bar{m}^*}{\bar{\iota}^* - g^*}$ is only sufficient to cover a fraction ρ (with $0 < \rho < 1$) of public debt. So, we have that $\frac{\bar{m}^*}{\bar{\iota}^* - g^*} = \rho b_D$ where $\frac{\bar{m}^*}{\bar{\iota}^* - g^*}$ is the amount of capitalized money creation that covers only partially the amount of public debt. In other words, in response to a speculative attack, the CB is assumed to be able to buy back only a fraction $\frac{\frac{\bar{m}^*}{\bar{\iota}^* - g^*}}{b_D} < 1$ of public debt so that after the speculative attack there is a fraction of it which is uncovered and on which, if the market does not desire to roll it over, there will be a default.

After some calculations (see Appendix A1) it is possible to obtain, then, that:

(27) $$v_T^C = v_T^{PC} + \left(b_D - \frac{\bar{m}^*}{\bar{\iota}^* - g^*}\right) > v_T^{PC}.$$

The lower size of public debt that can be monetized reduces the size of the 'honeymoon': the limited amount of available potential liquidity only allows absorbing an equally limited public debt demand shock, so that the 'honeymoon' becomes smaller.

It should be noted, as Krugman and Rotemberg (1992) already observed in the exchange rates TZs analogue, that the smooth pasting condition does not play any role in this case: the interest rate reaches the top of the margin without any tangency condition (see Figure 3).

It is worth underlying also that in this paper it has been possible to associate explicitly to each different quantity of potentially available liquidity its corresponding size of ‘honeymoon’, something that in the different Krugman and Rotemberg (1992) exchange rate TZ case was only implicitly derived, given the slightly different setup.

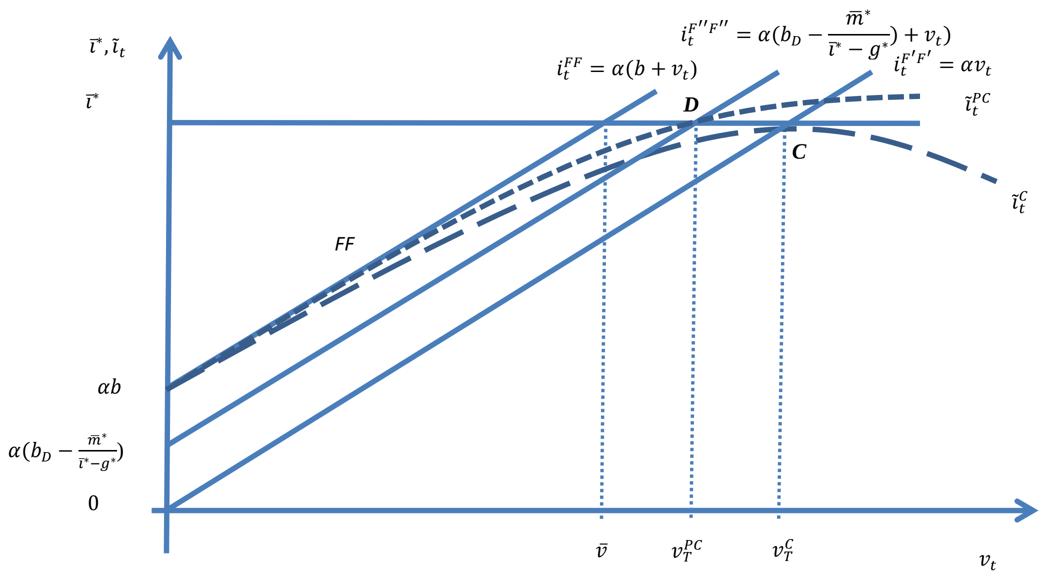

Figure 3: A partially credible interest rate target zone (determining $\tilde{\imath}_t^{PC}$), with no ‘smooth pasting’ and a lower ‘honeymoon’, to be compared with a fully credible one, determining $\tilde{\imath}_t^C$.

IV.3. Non-credibility of the central bank as a ‘lender of last resort’.

Let us consider now the case, suggested by De Grauwe (2012) and De Grauwe and Ji (2013a, 2013b) in which the public debt of a euro area country, although denominated in euros, is *de facto* uncovered, since the ECB has to abide by the no-bail out rule contained in the Maastricht Treaty. In such a case, then, it is as if public debt was denominated in a foreign currency, which is under the control of a ‘foreign’ CB that does not accept to take any responsibility for the public debt stability of any single EMU member country.

In the case of a speculative attack, then, the whole of the public debt will remain uncovered, and therefore will be subject to a potential default.

This means that (see Appendix A2 for the derivation of this result):

(28) $$v_T^{NC} = \bar{v}$$

In the case of a non-credible interest rate target, then, as the economic intuition would also suggest, the upper edge of the band would play an irrelevant role, being just ignored by the markets.

IV.4. Destabilizing non-credibility and convex interest rate non-linearity

Applying directly to interest rate TZs and public debt the non-credibility situation represented by Krugman and Rotemberg (1992) in the different context of exchange rates TZs, however, would not allow to account for the destabilizing expectations determining the convex interest rate non-linearity identified by De Grauwe and Ji (2013a) in the euro area crisis and also discussed and explained by Tamborini (2015) by referring to the role played by fiscal policies.

As a matter of fact, while Krugman and Rotemberg (1992) assume a constant money supply, Bertola and Caballero (1992), consider the probability that the breaching of the exchange rates TZ will determine an increase of money supply, that was instead kept constant within the TZ. This assumption allows them to obtain an exchange rate convex non-linearity.

In the case of public debt crisis that we are considering in this model, the same limitation would result, if we assumed the size of *b* as constant. Should the approaching of $\bar{\iota}^*$ be accompanied instead by an expected increase of public debt we would have, then, a different closing condition and an effect which is the opposite of the 'honeymoon'. In fact, this may well be the case in reality because of the expected increase of the interest rate service on it, because of a higher expected fiscal deficit that would add to it, or even because of the fear of a dissolution of the euro area and a return to devalued legacy currencies that would imply a higher public debt to be repaid.

As a result, we will have that (see the derivation of this result in Appendix A3):

(29) $$v_T^X = \bar{v} - (b^X - b) < \bar{v}.$$

Figure 4 shows this last ‘divorce’ case, together with the ‘honeymoon’ case.

The situation that has been analyzed in this paragraph, in which the CB is not in a position to operate as a lender of last resort and public debt is expected to increase, is precisely the one that characterized the euro area during the crisis and before the ‘whatever it takes’ Draghi speech. As a matter of fact, as argued by De Grauwe (2012), the public debt of euro area countries could be considered as denominated in a foreign currency, precisely because the euro area governments issuing it could not rely on a domestic CB operating as a lender of last resort and monetizing it, in case of need. [11]

[11] Just to provide another example, it can be observed that this was also the case of Argentina, whose public debt-to-GDP ratio was only 55 percent, namely well below the corresponding value of many euro area countries and also below the stability requirement reported in the Maastricht Treaty! The problem of Argentina, though, was that, in the attempt to reassure the markets, its public debt was almost all issued in foreign currency: as it became clear at a later stage, the market could not be reassured in the absence of foreign reserves capable to guarantee the repayment of the debt and the negative expectations on the explosion of public debt did the rest.

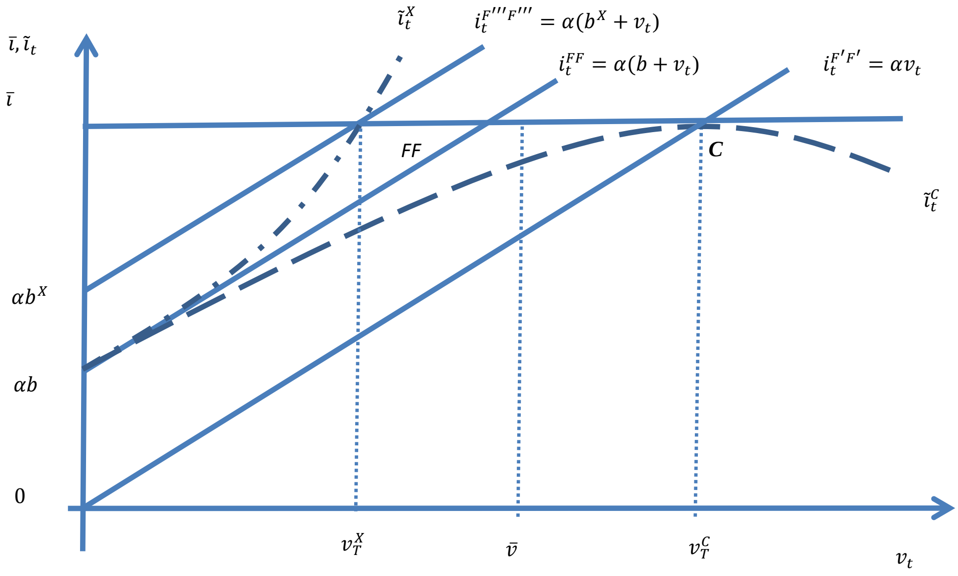

Figure 4: A non-credible target zone with the expectation of a public debt increase and the 'divorce' effect, and a credible target zone with the 'honeymoon' effect.

V.

Concluding remarks

Exchange rate TZs modelling is applied to the case of an interest rate TZs in order to analyze and interpret the euro area crisis as a different paper had done already. The present model, however, considers explicitly a stochastic shock hitting the demand of public debt and highlights the fact that a CB playing the role of lender of last resort guarantees the liquidity of the public debt which is denominated in domestic currency and reassures the markets (thereby avoiding the occurrence of self-fulfilling speculative attacks), because of the unlimited possibility it has to print money. This approach, then, allows to endogenize the stability of public debt, identifying the conditions that need to be satisfied in order to have full/partial/no credibility.

It turns out that any credibility bonus (what Krugman, 1991, dubbed as 'honeymoon' effect), depends on the size of the potential liquidity which is available with the CB: the larger the possibility for a CB to dispose freely - or with the lowest possible number of constraints - of liquidity, the larger the size of the 'honeymoon' effect.

I have also shown that the convex interest rate non-linearity that has characterized the euro area crisis emerges when considering the destabilizing effects of the expectation of a public debt increase, due, for example, to the fear of a dissolution of the euro area and a return to (devalued) legacy currencies that would imply an increase of the public debt to be repaid.

References

Baksay, G., F. Karvalits and Z. Kuti (2012), "The impact of public debt on foreign exchange reserves and central bank profitability: the case of Hungary", p. 179-191 in Bank for International Settlements (eds.), *Fiscal policy, public debt and monetary policy in emerging market economies*, vol. 67, Bank for International Settlements

Bertola, G., and R. Caballero (1992), "Target Zones and Realignments", *The American Economic Review, 82*(3), pp. 520-536. Retrieved from http://www.jstor.org/stable/2117319.

Corsetti, G., K. Kuester, A. Meier, and G. J. Müller (2014), "Sovereign risk and belief-driven fluctuations in the euro area", *Journal of Monetary Economics,* Vol. 61, Pages 53–73, January.

De Grauwe, P. (2012), "The Governance of a Fragile Eurozone" *Australian Economic Review,* 45 (3), pp. 255–68.

De Grauwe, P. and Y. Ji (2013a), "Self-fulfilling crises in the Eurozone: An empirical test", *Journal of International Money and Finance* - The European Sovereign Debt Crisis: Background & Perspective, Vol. 34, April, pp. 15–36.

De Grauwe, P. and Y. Ji (2013b), "From panic-driven austerity to symmetric macroeconomic policies in the Eurozone", *Journal of Common Market Studies,* Vol.51, Annual Review, p. 31-41.

Della Posta (2017), "Interest rate targets and speculative attacks on public debt", *Macroeconomic Dynamics,* (forthcoming).

Fischer, S. (2001), *Opening Remarks at the IMF/World Bank International Reserves: Policy Issues Forum,* Washington DC, April 28. (http://www.imf.org/external/np/speeches/2001/042801.htm).

Greenspan, A. (1999), "Currency Reserves and Debt" *Remarks Before the World Bank Conference on Recent Trends in Reserves Management*, Washington, D.C., April 29, 1999.

IMF (2011), *Assessing Reserve Adequacy,* Prepared by Monetary and Capital Markets, Research, and Strategy, Policy, and Review Departments in consultation with other departments, Approved by Reza Moghadam, Jonathan D. Ostry and Robert Sheehy, February 14, 2011

Jeanne, O., and R. Ranciere (2005), "The Optimal Level of International Reserves for Emerging Market Economies: Formulas and Applications," *IMF Research Department,* May.

Krugman, P. (1991), "Target Zones and Exchange Rate Dynamics", *The Quarterly Journal of Economics,* Vol. 106, No. 3. (August), pp. 669-682.

Krugman, P. and J. Rotemberg (1992), "Speculative Attacks on Target Zones", in Krugman, P. and M. Miller (eds), *Exchange Rate Targets and Currency Bands*, pp. 117-132, Cambridge Univesity Press.

Kugman, P. and M. Miller (eds) (1992), *Exchange Rate Targets and Currency Bands*, Cambridge Univesity Press.

Rodrik, D. (2006), “The Social Cost of Foreign Exchange Reserves”, *International Economic Journal, Vol. 30, No. 3, pp.253-266*

Tamborini, R. (2015), “Heterogeneous Market Beliefs, Fundamentals and the Sovereign Debt Crisis in the Eurozone”, *Economica*, Volume 82, Issue s1, December 2015, Pages 1153–1176, DOI: 10.1111/ecca.12155.

Appendix

A1. Partial credibility of the central bank acting as a ‘lender of last resort’

Let us consider the case in which public debt is still fully denominated in domestic currency, but it is known (given its relative preferences relative respectively to default on public debt and inflation spike and given the trade-off between the two objectives), that at some point the CB will prefer to choose a public debt default, rather than an excessive inflation rate within the country. This means that the CB will not be willing to print money beyond the upper limit $\overline{m}^* < m^*$, discussed in Paragraph II.2. above.

This means, then, that $\frac{\overline{m}^*}{\overline{\iota}^*-g^*}$ is only sufficient to cover a fraction ρ (with $0 < \rho < 1$) of public debt. So, we have that $\frac{\overline{m}^*}{\overline{\iota}^*-g^*} = \rho b_D$ where $\frac{\overline{m}^*}{\overline{\iota}^*-g^*}$ is the amount of capitalized money creation that covers only partially the amount of public debt. In other words, in response to a speculative attack, the CB is assumed to be able to buy back only a fraction $\frac{\frac{\overline{m}^*}{\overline{\iota}^*-g^*}}{b_D} < 1$ of public debt so that after the speculative attack there is a fraction of it which is uncovered and on which, if the market does not desire to roll it over, there will be a default. This will be equal to:

$$b' = (1-\rho)b_D = b_D - \frac{\overline{m}^*}{\overline{\iota}^*-g^*}.$$

After the speculative attack, then, the untargeted interest rate $i_t^{F''F''}$(which is parallel to the initial i_t^{FF} and is represented in Figure 4) is a function of the remaining fraction of the foreign denominated debt that could not be guaranteed ($b_D - \frac{\bar{m}^*}{\bar{\iota}^* - g^*}$). At point *D*, when hitting $\bar{\iota}^*$, such an equation will take the value:

(A1) $$i_t^{F''F''}(v_T^{PC}) = \bar{\iota}^* = \alpha[b_D - \frac{\bar{m}^*}{\bar{\iota}^* - g^*} + v_T^{PC}].$$

The interest rate moving within the floating band in the case of partial credibility of the TZ (that is indicated with $\tilde{\iota}_t^{PC}$), instead, is equal to $\bar{\iota}^*$ when v_t takes the value v_T^{PC}, namely:

(A2) $$\tilde{\iota}_t^{PC}(v_T^{PC}) = \bar{\iota}^* = \alpha(b_D + v_T^{PC}) + Ae^{\lambda v_T^{PC}}$$

We need to calculate, then, the trajectory of $\tilde{\iota}_t^{PC}$ so that, when the demand shock to public debt reaches the v_T^{PC}, the interest rate will be exactly on the $i_t^{F''F''}$ line. This is the untargeted interest rate in the case in which the CB has intervened by monetizing only the part of public debt which is able to cover. Also in this case, we have to rule out any discrete jumps that, if occurring, would violate the no-arbitrage condition. By equating Eq. (A1) to Eq. (A2), then, we have:

$$\alpha \frac{\bar{m}^*}{\bar{\iota}^* - g^*} + Ae^{\lambda v_T^{PC}} = 0$$

That is:

(A3) $$\frac{\alpha \frac{\bar{m}^*}{\bar{\iota}^* - g^*}}{e^{\lambda v_T^{PC}}} = -A > 0.$$

From Eq. (A3) we conclude that the constant term *A* is still negative, as in the case with full credibility. However, while in the case of full CB's credibility considered above (with $b = b_D = \frac{m^*}{\bar{\iota}^* - g^*}$), we had that b was fully guaranteed and covered by reserves, now we know that $b = b_D > \frac{\bar{m}^*}{\bar{\iota}^* - g^*}$. Given this result and the fact that with full credibility we had that $b = v_T^C - \bar{v}$, it turns out that:

$$v_T^C = \frac{m^*}{\bar{\iota}^* - g^*} + \bar{v} > \frac{\bar{m}^*}{\bar{\iota}^* - g^*} + \bar{v} = v_T^{PC}.$$

As a matter of fact, by comparing the value of $i_t^{F'F'}$ when hitting $\bar{\iota}^*$ (Eq. 24 above) with the corresponding value of $i_t^{F''F''}$ (A1), we have Equation (27) in the text:

(27) $$v_T^C = v_T^{PC} + \left(b_D - \frac{\bar{m}^*}{\bar{\iota}^* - g^*}\right) > v_T^{PC}.$$

A2. Non-credibility of the central bank acting as a 'lender of last resort'.

Let us consider now the case in which the CB does not act as a lender of last resort, so that in the case of a speculative attack public debt will not be guaranteed, being subject, then, to the risk of a potential default.

The interest rate on public debt in this non-credibility case ($\tilde{\iota}_T^{NC}$), will reach $\bar{\iota}^*$ when v_t takes the value v_T^{NC}. We have, then, that:

(A4) $$\tilde{\iota}_T^{NC}(v_T^{NC}) = \bar{\iota}^* = \alpha(b_F + v_T^{NC}) + Ae^{\lambda v_T^{NC}}.$$

The value of the interest rate immediately after the attack, when the shock of size v_T^{NC} realizes, will be determined by an equation which is characterized by the same initial supply of public debt, which coincides with the one determining i_t^{FF}, since no reserves are available to reduce the supply of debt by buying it back, so that:

(A5) $$i_t^{FF}(v_T^{NC}) = \bar{\iota}^* = \alpha(b_F + v_T^{NC}).$$

By comparing Eq. (A4) with Eq. (A5) it turns out that:

(A6) $$Ae^{\lambda v_T^{NC}} = 0,$$

implying clearly that $A = 0$. This implies Equation (28) above.

A3. Destabilizing non-credibility and convex interest rate non-linearity

Finally, let us consider the expectation of a public debt increase as a result of the breaching of the interest rate TZ (as De Grauwe and Ji (2013a) and as Tamborini (2015) also do). When v_t takes the value v_T^X (the size of the shock on public debt at which the interest rate reaches its upper level in the case in which public debt is expected to increase), we will have that $\tilde{\imath}_t^X = \bar{\imath}^*$. The value of the interest rate at the top of the floating band, then, will take the value:

(A7) $$\tilde{\imath}_t^X(v_T^X) = \bar{\imath}^* = \alpha(b + v_T^X) + Ae^{\lambda v_T^X}$$

The value of the untargeted interest rate when the public debt demand shock v_T^X occurs ($i_t^{F'''F'''}(v_T^X)$), will be instead:

(A8) $$i_t^{F'''F'''}(v_T^X) = \bar{\imath}^* = \alpha(b^X + v_T^X),$$

where $b^X > b$ indicates the larger expected level of public debt after the abandonment of the interest rates TZ.

By comparing those two equations we have:

$$Ae^{\lambda v_T^X} = \alpha(b^X - b),$$

namely:

(A9) $$\frac{\alpha(b^X - b)}{e^{\lambda v_T^X}} = A > 0.$$

Contrary to the previous cases, then, given that *A* takes a positive value, the interest rate curve within the band will be convex.

In order to calculate the size of the ‘divorce effect’ emerging in this case, let us compare the value of v_T^X with the value of $\bar{v}$.

From Eq. (A8) it turns out that:

(A10) $$v_T^X = \frac{\bar{\imath}^*}{\alpha} - b^X.$$

From Eq. (25), instead, we have:

(A11) $$\bar{v} = \frac{\bar{\imath}^*}{\alpha} - b.$$

As a result we obtain Equation (29) in the paper.