Assessment of velocity fields through open-channels: creation of empirical laws

Evaluation des champs de vitesses dans un canal ouvert: création de lois empiriques

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RESUME

L'exploitation des réseaux d'assainissement exige désormais une connaissance fine des flux polluants véhiculés. Dans ce contexte, connaître la distribution spatiale des vitesses dans la section étudiée rend l'estimation du débit plus réaliste. L'article défendu ici présente deux relations mathématiques simples permettant d'établir une cartographie des vitesses dans une section donnée, pour un écoulement développé. L'une, issue des équations de Navier-Stokes, requiert la connaissance de la vitesse de cisaillement et la position de la vitesse maximale quand l'autre utilise des paramètres empiriques issus de traitement statistiques. Les deux sont validées sur des cas expérimentaux.

ABSTRACT

The exploitation of the sewers networks requires now a fine knowledge of conveyed polluting flows. In this context, to know the spatial distribution velocities in the studied section returns the estimate of the flow more realistic. The article defended here presents two mathematical relations making it possible to establish a cartography of the velocity in a cross section, for a full-developed flow. One, resulting from the Navier-Stokes equations, requires the shear velocity and the maximal velocity position, when the other uses empirical parameters resulting from statistics treatment. Both are validated on experimental cases.

KEY WORDS

Laws, open-channels flow, velocity fields

INTRODUCTION

Since about fifteen years now, the lawful constraints on the sewer networks were developed. Indeed the January 1992 national water policy law transposed from the May 1991 European Community Directive stipulates that any town, producing a daily pollutant load of more than 900 kg, has to be equipped with a wastewater collection network. Moreover, according to the decree of the 22nd December, relating to the self survey, in order to better evaluate the depollution rate of a system, it is necessary to install, today, measuring equipment able to estimate the flows moving towards the plants or the discharge system. Besides the fact that the usual hypothesis about spatial homogeneity for flow rates and pollutant loads is false (Wohrle and Brombach, 1991), the dual behaviour of some wastewater collection networks linked to the rain water catching gives difficult the flow rates measurement. A better assessment of the spatial distribution of the velocities in a cross section is, therefore, one way to improve the flow acquisition, but also pollutant fluxes. The present paper presents two empirical laws to calculate, for a fully developed flow in a given cross section, the spatial distribution of the velocity.

1 METHODOLOGIC APPROACH

1.1 Simple model based on Navier-Stokes equations

The Reynolds Averaged Navier-Stokes (RANS) momentum equations can be written as:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\right) \quad (1)$$

where U_i are the mean velocity in the x (streamwise), y (lateral) and z (vertical) directions, P is the pressure, ρ is the fluid density and $\overline{u_i u_j}$ are the components of the Reynolds stress tensor. In a steady, uniform fully developed turbulent flow, equation (1) in the streamwise (x-) direction can be written:

$$V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} = gS_0 + \frac{\partial}{\partial y}\left(-\overline{uv}\right) + \frac{\partial}{\partial z}\left(-\overline{uw}\right) + \nu\frac{\partial^2 U}{\partial y^2} + \nu\frac{\partial^2 U}{\partial z^2} \quad (2)$$

where g is the acceleration due to gravity, S_0 the energy slope ($S_0 = \sin\theta$). Yang et al. (2004) have reported that in the near bed region, the vertical gradient ($\frac{\partial .}{\partial z}$) should be dominating and the horizontal gradient ($\frac{\partial .}{\partial y}$) could be neglected. Therefore equation (2) in the near bed region can be written as follows:

$$W\frac{\partial U}{\partial z} = gS_0 + \frac{\partial}{\partial z}\left(-\overline{uw}\right) + \nu\frac{\partial^2 U}{\partial z^2} \quad (3)$$

Nezu et al. (1989) have highlighted the pattern of secondary currents in open channels, the transverse flow (V>0) near the free surface is directed from the side wall towards the channel centre and this current flows down (W<0) along the channel centre from the free surface to the bottom. This means, that in the channel centre, there is only the vertical mean velocity that contributes to the secondary current. In order to assess such constraint, the vertical velocity must satisfy 3 conditions: W equals to 0 at the bed and free surface level, and W negative in the channel centre.

After integration and simplification, by using the no-slip boundary condition at $\xi = \xi_0 = \frac{z_0}{h}$, where z0 is the roughness length of the surface, the vertical distribution of the velocity in the channel central region and out of the inner region (z/h > 0.2) can be determined by:

$$\frac{U}{u_*} = \frac{\alpha}{\kappa} \frac{1}{(0.5\xi^2 + \xi + C(A))} \left(0.25\xi^2 + \xi + C(A) \ln \frac{\xi}{\xi_0} \right) \quad (4)$$

where κ is the Von-Karman constant, u* the shear velocity, $\alpha = \frac{ghS}{u_*^2} - 1$, $\xi = \frac{z}{h}$ relative distance from the bottom and C(A) a parameter in function of Ar (the aspect ratio) that can be evaluated as follows:

$$C(A) = D.\left(\xi_{dip}\right)^E \quad (5)$$

where D = 9.3 and E = 1.7 are two constants deduced from the analytical resolution of equation (4). When ξ_{dip} is unknown, we can use the sigmoid relation developed by Morgan and al (1975) with adjusted parameters. In the inner region, the logarithmic law remains valid.

1.2 Simple model based on fitted empirical parameters

The following approach comes within the framework of the development of a new sensor measuring flow rates in sewers. First, a transductor, using ultrasonic signal, gives a Doppler response transformed in a velocity profile in a cross-section. Then, as each velocity measurements have identified positions, it could be interesting to insert this chord in the whole velocity field and to deduct from this the flow rate. The challenge is clearly here to know this field only from the hydraulic characteristics, i. e. the geometry channel and the water level.

1.2.1 First step: use numerical modelling

The method starts with the numerical simulations of different flows in several classical sections (in sewer networks): rectangular, circular and egg-shape. The table 1 shows the list of all the simulations realised with the Fluent software.

Shape	rectangular	circular	egg-shape
Dimensions (mm)	width x height : 600 x 800, 1000 x 500, 2000 x 500	diameter: 500,1000,1500, 2000	height: 1000,1500,2000
Filling	20%, 30%, 40%, 50%, 70%, 90%	20%, 30%, 40%, 50%, 70%, 80%	20%, 30%, 40%, 50%, 70%, 80%
Mean Velocity (m/s)	0,25-0,5-0,75-1,00-1,25-1,5-2	0,25-0,5-0,75-1,00-1,25-1,5-2	0,25-0,5-0,75-1,00-1,25-1,5-2

Table 1 : Fluent simulation framework

The first work were the definition of the modelling way. The first problem with the open channels remains the free surface and the anisotropy of the turbulence. The many studies realised (Larrarte and al 2005) showed that the best way to have a good agreement with the reality (Bonakdari and al 2006)) were the V.O.F method (diphasic model) combined with an R.S.M. model for the turbulence (Bardiaux and al 2006).

1.2.2 Second step: Have a representative field of the type of channel

The library of velocity fields obtained, for each kind of channel allows us to generate a representative field. After having transformed the fields within a dimensionless channel (the position is now given by ξ and the relative distance from the wall $\frac{y}{B}$) divided each local velocity by the associated mean velocity, the local values for the medium field come are the average of all the local velocities for this position.

1.2.3 Last step: fit an algebraic formulation to the medium field

The formulation chosen here is clearly based on the known description of classical velocity fields in open-channels (Nezu (1993)). End, logarithmic (for the "vertical " behaviour) and power (for the horizontal one) laws have been crossed. Moreover, in the aim of taking into account the dip-phenomenon, a trigonometric one has been added. At last, to limit the asymmetric effects, we use absolute value. Then, the formulation proposed is the following one (equation (6)):

$$\frac{U}{U_{mean}}=\left[\left|a.\ln\left(\xi\right)+b+c.\sin\left(d.\xi\right)\right|+e.\left(\frac{y}{B}\right)^{f}\right].\left(\frac{y}{B}\right)^{g} \quad (6)$$

where a, b, c, d, e, f, g are empiric parameters stemmed to the fitting, depending on the water level.

2 RESULTS

2.1 Circular open-channel:

2.1.1 Description of the data

The data presented in the figure 1 correspond to a flow through a circular pipe (Q=11.7l/s and diameter=123 mm, water level=123.5 mm) (Knight and Sterling (2000))

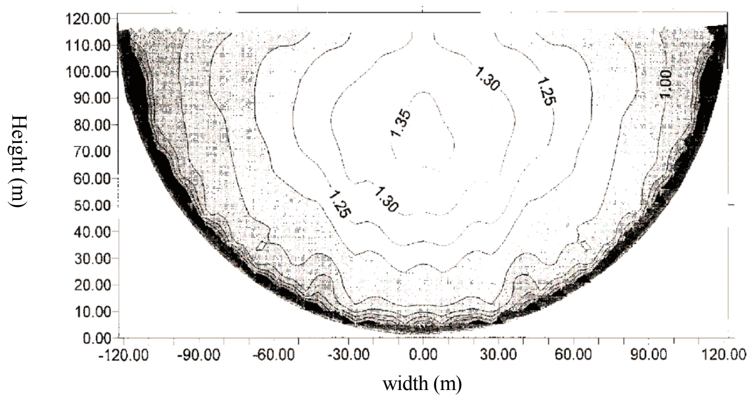


Figure 1 : Isolevels of velocity in a circular pipe (Knight and Sterling (2000))

2.1.2 Profiles on the central axis

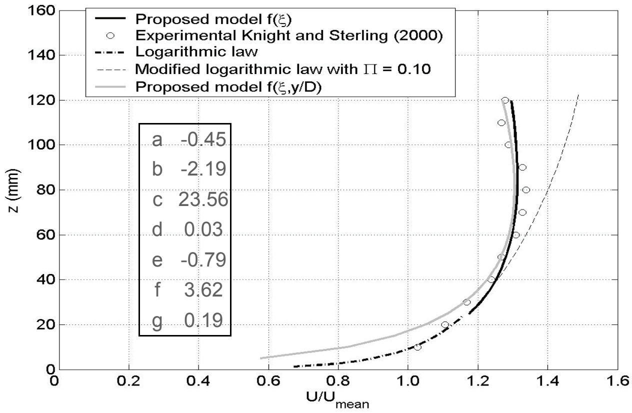


Figure 2 : Velocity profiles on central axis

The two approaches present a good agreement with the experimental data in the central region according the figure 2. In fact, unlike Coles relation, the two models are able to simulate the dip-phenomenon on the right position.

2.1.3 *Velocity distribution in the section*

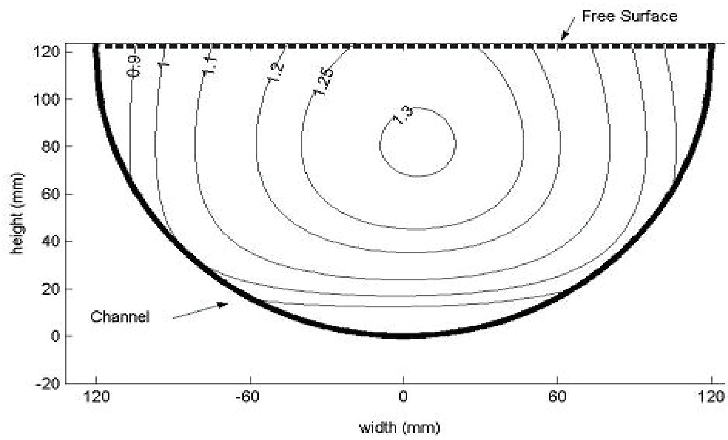


Figure 3 : Isolevel of $\frac{U}{U_{mean}}$ from eq(6)

The figures 3 and 4 show the ability of the equation (7) to simulate the velocity field on around 80% (less than 5%) of the section with a good agreement, but around the wall the model is inadequate. The right position of the Dip-phenomenon is clearly on the figures 2 and 3, despite of a weaker maximal velocity.

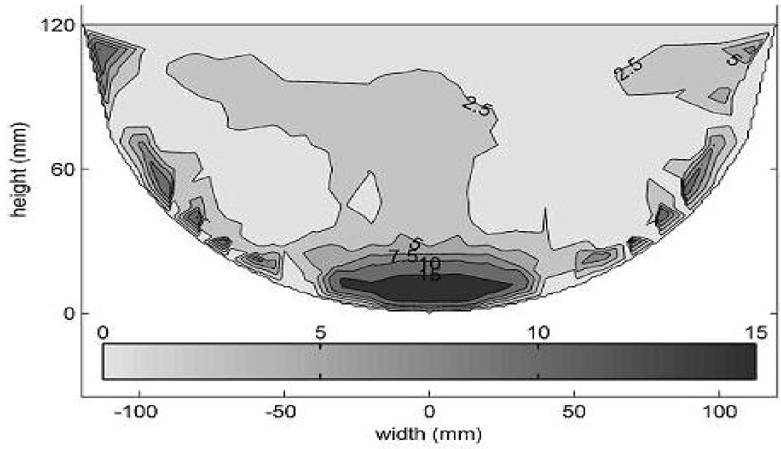


Figure 4 : Spatial distribution of the error (%)

2.2 Experimental site in a sewer network

The experimental site (figure 6) is located in an area called Cordon Bleu, a few kilometres upstream the treatment plant on the main sewer line of the city of Nantes (France). In the egg-shape pipe, the mean velocity was 0.88 m/s, the water height 1.20 m. No measurement could be realised over the walkway but a wide of flow conditions have been experimentally investigated (Larrarte (2006)).

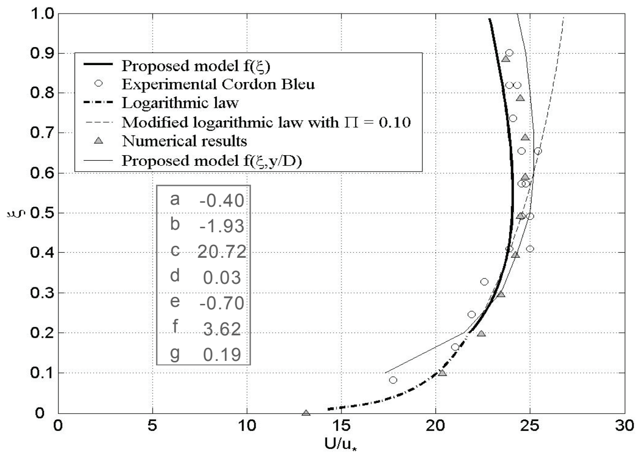
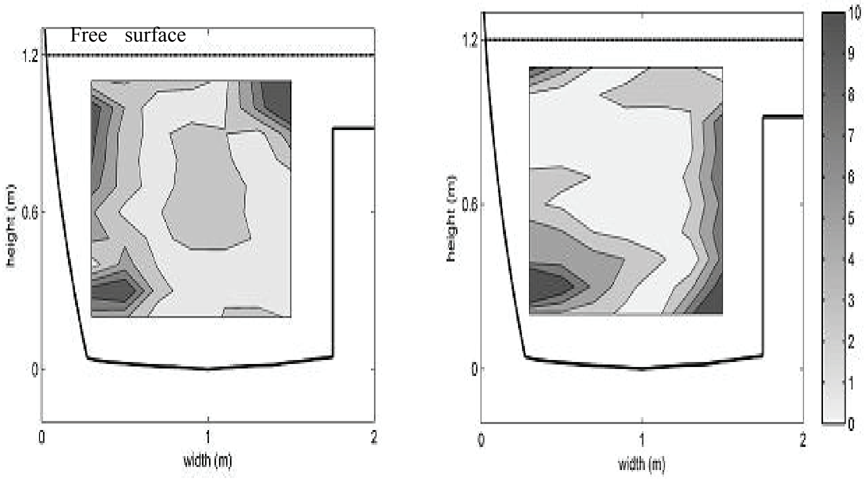


Figure 5 : Velocity profiles on central axis



a) b)

Figure 6 : spatial distribution of the error (%) a) from eq(4) and b) from eq(6)

The figures 5 and 6 raise, as in 3.1, the good description in the central region, less than 5 %. But, the walk-away generate special flow (re-circulations) which are not correctly reproduced.

3 CONCLUSION

The two models presented here allow a good representation of a velocity field in a fully developed flow. The advantage is to be independent of the measurement conditions. Indeed, geometry and water level are sufficient to reach an assessment of the flow rate. Thus, if a sensor gives a partial chord (range limit), it is possible to build up the whole field and estimate the flow rate. Furthermore, if the sensor has to be fasten sideways, the localisation of the sampled volume can be used even if the maximal velocity area is not scanned.

Certainly the nearness of the walls remains a problem but this is not the area, which contributes the most to the flow rate. But the main work to add is the treatment of the walk-away area. Indeed, local deformation of the velocity field cannot be simulated with the expressions (4) and (6). Data acquisition on a new physical model will help us to improve this.

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