

Simulation of subcritical flow at a combining junction

Simulation de l'écoulement fluvial à travers une confluence

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RESUME

Une simulation numérique unidimensionnelle de l'écoulement écoulement fluvial, à surface libre, à travers une confluence est décrite. Dans les branches, le modèle mathématique appliqué est celui de Saint Venant, alors que pour la jonction; différents modèles basés sur la conservation d'énergie ou de la quantité de mouvement existent. Dans cette étude, nous décrivons le traitement des conditions aux limites internes. La méthode des caractéristiques est couplée avec quatre modèles différents, pour la simulation de l'écoulement fluvial traversant une jonction, qui sont : le modèle d'égalité de hauteurs, et les modèles de Gurram, Hsu et Shabayek. Ces modèles ont été comparés à des données expérimentales disponibles et entre eux à travers des problèmes d'écoulement stationnaire et transitoire. Cette observation montre un déficit du modèle de l'égalité auprès des autres modèles dans la simulation de l'écoulement fluvial à nombre de Froude élevé à l'aval de la jonction ($F_d \approx 0.7$).

ABSTRACT

A one-dimensional numerical simulation for subcritical flow in open channels network combining junction is considered and described allowing a comprehension of water flow modelling throughout a junction. In channel branches, the mathematical model used is the Saint Venant hyperbolic system, while for the junction; different models of equations based on momentum or energy conservation exist. In this study, we describe the treatment of internal boundary conditions. The characteristics method is coupled with four different combining models, for the simulation of subcritical flow through junction, which are : the equality of water stages, Gurram model, Hsu model and Shabayek model. These models are compared to available experimental data and to each other in steady and transient flow problems. This investigation shows a deficit for the equality model in relation to the other models in the simulation of high Froude number subcritical flow downstream the junction ($F_d \approx 0.7$).

KEYWORDS

1D ; junction models ; combining ; open channel flow ; subcritical ; steady and transient

1 INTRODUCTION

In the design of flood-control channels, one of the most important hydraulic problems is the analysis of the flow conditions at open channel junctions. Typical examples of these junctions are encountered in urban water network, irrigation and drainage canals, and natural river systems.

For the simulation of the one-dimensional shallow water flow through a junction, the whole system is considered as a set of branches in which the Saint Venant equations are applied and linked by different hydraulic models of junction. The hydraulic conditions at a junction can be modeled by the mass conservation equation and either the energy conservation equation or the momentum equation.

The first question to ask when modeling open channel flow through a junction is : which model is more appropriate for flow simulation through the junction ? Akan and Yen (1981) show that when the flow through the junction is subcritical, the energy equation can be approximated by the water stages equality. Several applications like the One-Dimensional Hydrodynamic Model from Environment Canada (1988), the Mike 11 and Mouse model from the Danish Hydraulic Institute (1999), the Canoe model from Sogreah and Insavalor (2001) and InfoWorks from Wallingford Software (2006) use the water stages equality at the junction.

The objective of the present work is to show that, the use of the water stages equality model largely used in the classical hydraulic engineering codes is not always suitable and should be replaced, in serious situations, by a model based on momentum conservation. To this end, the deviations between the results obtained with the water stages equality model and with the three other remainders are shown in the simulation of steady and unsteady flow through the junction. Roe's first order explicit scheme is used to solve the Saint Venant equations in branches. For the junction, the four models of : equality of water stages, Gurrum et al. (1997), Hsu et al. (1998) and Shabayek et al. (2002) coupled with the method of characteristics are solved by using Newton-Raphson iterations.

In section 2, a bibliographical study of 1D junction models is presented followed by a comparison of the four junction models with available experimental data. Section 3 and 4 briefly describe the numerical method, the treatment of internal and external boundary conditions and the coupling of the flow internal variables model with junction equations. Section 5 is devoted to the numerical results and discussions.

2 1D JUNCTION MODELS

2.1 Bibliographical study

Previous studies on combining open channel flows proposed theoretical approaches based on mass and momentum conservation, which allow solving the upstream-to-downstream depth ratio. Taylor (1944) presented the first study on simple junction flow, and referred to the complexity of the problem. A second systematic study to simple junction flow was presented by Webber and Greated (1966). Modi et al. (1981) investigated open channel combinations using a conformal mapping approach based on a complex variable theory and therefore did not account for energy losses. Best and Reid (1984) analyzed experimentally the geometry of the separation zone at sharp edged open channel junctions. In discussing the experimental approach of Best and Reid, Hager (1987) introduced a simple model in which the pressure distribution on the lateral sidewall and the lateral momentum contribution were taken into account. Ramamurthy et al. (1988) studied the combining open channel flow at right angled junction on the basis of momentum transfer from the lateral branch to main channel branch.

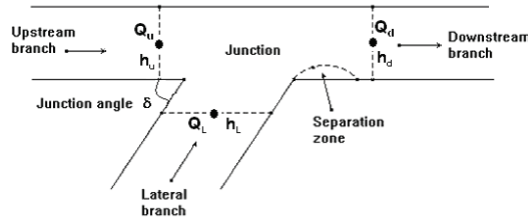


Figure 1 : junction problem.

The most recent equations for a subcritical junction, based on momentum approach, are the equations studied by Gurrarn et al. (1997), Hsu et al. (1998) and Shabayek et al. (2002). In order to give an idea of the structure of each model, we consider a network composed of three rectangular branches linked by one junction (figure 1). We denote branch u, branch L and branch d, the upstream, lateral and downstream branches to the junction, respectively. The water depths at upstream, lateral and downstream points to the junction are denoted by h_u , h_L and h_d . $Y_u = h_u / h_d$ and $Y_L = h_L / h_d$ are the upstream-to-downstream depth ratio and the lateral-to-downstream depth ratio in the junction, $q_u = Q_u / Q_d$ and $q_L = Q_L / Q_d$ the upstream-to-downstream and lateral-to-downstream discharge ratio, δ is the junction angle and F_d the Froude number at the downstream point of the junction.

The easiest model is the equality of water stages, which, as mentioned in the introduction, is used by several applications in the hydraulic engineering. This model allows findings for every depth downstream to the junction, the upstream and lateral depths by assuming depth equalities ($h_u = h_L = h_d$).

Gurrarn et al. (1997) and Hsu et al. (1998) assumed equal-width junction flows ($B_u = B_L = B_d$) and equality of the upstream and lateral depths ($h_u = h_L$). Gurrarn et al. (1997) studied the characteristics of the lateral flow and the flow contraction in the tail water channel and determines expressions for the momentum correction coefficients and the lateral wall pressure force. The nonlinear equation derived by Gurrarn involves $Y_u, q_u, q_L, F_d, \delta$ and has the following form : $\varphi(Y_u, q_u, q_L, F_d, \delta) = 0$. Hsu et al. (1998) applied overall mass and energy conservation to the junction and momentum conservation to two control volumes in the junction and computed an energy loss coefficient as well as the depth ratio. Hsu's nonlinear model involves, in addition to Gurrarn's parameters, correction factors and takes the following form : $\varphi(Y_u, q_u, F_d, \delta, \alpha, \beta) = 0$, where α and β are the energy and momentum correction factors. The mean values of α and β are 1.27 and 1.12, respectively.

Most recently, Shabayek et al. (2002) developed a one-dimensional theoretical model providing the necessary interior boundary equations for combining subcritical open channel junctions. The main advantage of this model is that it does not assume equal upstream depths. The dynamic treatment of the junction is so consistent with that of the reaches in a network model. The model is based on applying the momentum principle together with mass continuity through the junction. Shabayek et al. (2002) constructed an analytical approach for solving the upstream-to-downstream depth ratio $Y_u = h_u / h_d$ and the lateral-to-downstream depth ratio $Y_L = h_L / h_d$ in the junction.

Shabayek model consist of two big nonlinear equations having the following makeup : $\varphi^1(h_d, F_d, Y_u, Y_L, q_u, q_L, L_1, B_d, w_1, w_2, S_0, K', C) = 0$ and $\varphi^2(h_d, F_d, Y_u, Y_L, q_u, q_L, L_1, L_2, B_d, w_1, w_2, S_0, K', K, C) = 0$ where B_u , B_L and B_d are the widths in upstream, lateral and downstream branches to the junction. $w_1 = B_u / B_d$ and $w_2 = B_L / B_d$ are the upstream-to-downstream and lateral-to-downstream width ratios. S_0 is the longitudinal slope of the junction. C is the Chezy nondimensional coefficient, L_1 and L_2 are the outer lengths of the two control volumes. K' is the interfacial shear coefficient and K the

separation zone coefficient. The value of K^* and K are given by : $K^* = -0.0015 \delta + 0.3$ and $K = 0.0092 \delta - 0.1855$.

2.2 Experimental investigation in steady state

In this subsection, the experimental results obtained by Hsu et al. (1998a, 1998b) and Webber and Greated (1966) are used for the verification of recent previously discussed models. Hsu et al. (1998b) conducted experiments in a rectangular flume with a horizontal bed. The main and lateral branches were 6 and 1.5 m long, respectively. The branches width is 0.155 m with a junction angle fixed at 90°. In Hsu et al. (1998a), the main and lateral branches were 12 and 4m long, respectively. The width of the branches are 0.155 m with a junction angle being 30°, 45° and 60°. Webber and Greated (1966) branch was 0.127 m wide with a junction angle being 30° and 90°. In all tests, the Chezy nondimensional coefficient is equal to 17.

In table 1, we calculated the relative errors (in L^1 norm) between experiments and upstream-to-downstream depth ratios (Y_u) evaluated in function of the upstream-to-downstream discharge ratios (q_u) with respect to four junction angles 30°, 45°, 60° and 90°. On the other hand, in table 2, we calculated errors between experiments and upstream-to-downstream depth ratios (Y_u) evaluated according to the downstream Froude number (F_d) with respect to the following two cases : junction angles 30° and upstream-to-downstream discharge ratio 0.2, junction angles 90° and upstream-to-downstream discharge ratio 0.6.

| | E | G | H | S |
|-------|-----|------|----|------|
| Angle | | | | |
| 30° | 10% | 4% | 1% | 0.9% |
| 45° | 11% | 4% | 1% | 0.9% |
| 60° | 13% | 4% | 2% | 2% |
| 90° | 16% | 3.6% | 2% | 1% |

Table 1 : Errors due to the comparison of junction models with respect to Hsu (1998a, 1998b) data.

| | E | G | H | S |
|-------|------|-------|------|-------|
| F_d | | | | |
| 0.24 | 2.5% | 0.9% | 1.1% | 0.32% |
| 0.38 | 7% | 3.2% | 1.4% | 0.35% |
| 0.5 | 12% | 3.66% | 5.1% | 2% |
| 0.6 | 15% | 3.7% | 8% | 4.7% |

Table 2 : Errors due to the comparison of junction models with respect to Webber and Greated (1966) data.

E = equality, G = Gurrum, H = Hsu, S = Shabayek

One notices according to tables 1 and 2, that the influence of the downstream Froude number on the errors is more significant than the junction angle. The increasing of the error due to the increase in the downstream Froude number is more important than the variation of the error due to the augmentation of the junction angle.

In any case, the Shabayek model gives the best agreement to the experimental data. The agreement with the Gurrum model is very satisfactory in some cases and acceptable in other cases, while for the Hsu model, the accord in all cases is satisfactory. The most important notice is that the equality of water stages model gives the greatest errors in all cases compared to the experimental results. However, when $F_d < 0.38$, the error remains acceptable (less than 7 %).

3 ANALYSIS OF THE SOLUTION IN BRANCHES

It is generally accepted that the unsteady flow of water in a wide channel of slowly varying cross-section with a sufficiently gentle bottom slope can be described by the one-dimensional shallow water or the so-called Saint Venant equations. For flow in a prismatic channel of rectangular cross-section, the system has the following conservative form :

$$U_t + F(U)_x = G(U)$$

$$\text{with } U = \begin{pmatrix} A \\ Q \end{pmatrix}; F(U) = \begin{pmatrix} Q \\ Q^2 / A + 0.5gA^2 / B \end{pmatrix}; G(U) = \begin{pmatrix} 0 \\ gA(S_o - S_f) \end{pmatrix}$$

where U is the flow vector, $F(U)$ the flux vector and $G(U)$ the source vector. Q is the flow discharge, A wet section, B the channel bottom width and g the acceleration due to gravity, S_o bed slope and S_f friction term.

In order to solve the Saint Venant hyperbolic system, we applied a first order Upwind explicit scheme. An upwind treatment of the source term was performed following the lead of Bermudez and Vazquez (1994). The numerical method takes the following form :

$$U_i^{n+1} = U_i^n - \Delta t / \Delta x (\tilde{F}_{i+1/2}^n - \tilde{F}_{i-1/2}^n) + \Delta t G_i^n; \Delta x \text{ grid space}; \Delta t \text{ time step}$$

The flux at the interfaces ($\tilde{F}_{i+1/2}^n$) is obtained by applying the Riemann solver of Roe (1981).

4 TREATMENT OF THE BOUNDARY CONDITIONS

It is known in hydraulic engineering that the numerical scheme is only defined for interior points. A general method used for finding the solution at the boundary is the method of characteristics. Since the flow is subcritical in all channels, one physical boundary condition must be specified at each boundary, and the other condition is calculated from the method of characteristics. In this paper, we followed the work of Garcia-Navarro and Savirón (1992). At the upstream end of the branches preceding the junction, the water discharge is specified and the depth is calculated from the C^- (backward) characteristic, while at the downstream end of the branch following the junction, the water depth is specified and the discharge is calculated from the C^+ (forward) characteristic.

To find the solution at the internal points enclosing the junction, we have to find six unknowns : three discharges Q_u , Q_L and Q_d and three water levels h_u , h_L and h_d . Therefore, we need to have six equations. The first three equations are derived from the three characteristics coming from the branches solutions. They can have the

following expressions : $Q_R^{n+1} = (K_R^n B_R) h_R^{n+1} + C_R^n$ (for $R = u, L, d$). K and C depends of the flow variables at instant $t = n$. The fourth equation corresponds to the mass conservation at the junction : $Q_d^{n+1} = Q_u^{n+1} + Q_L^{n+1}$ and the two other equations come from one of the different junction models.

The system written with the water stage equality is linear, and therefore it is not difficult to resolve, while the system obtained with the other models is nonlinear. Its resolution is realized by the Newton-Raphson method.

5 NUMERICAL RESULTS AND DISCUSSIONS

In this section we will give two examples for the purpose of comparing the four junction models. As seen in section 2, the downstream Froude number has greater influence than the junction angle. For this reason, we investigated two hydraulic problems with constant junction angle of 45° . The first one is characterized by a high downstream Froude number ($F_d \approx 0.7$). A steady and a transient case were investigated. In the second problem, the downstream Froude number is lower than

0.3. In the steady and the transient state, the initial conditions are taken as a uniform state in all branches and for all models. Thirty uniformly distributed cells were used in these computations.

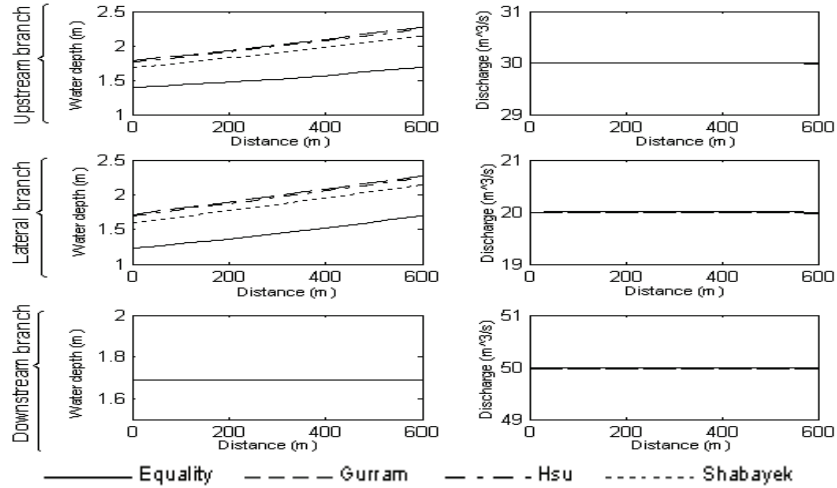


Figure 2 : steady solution for example 1.

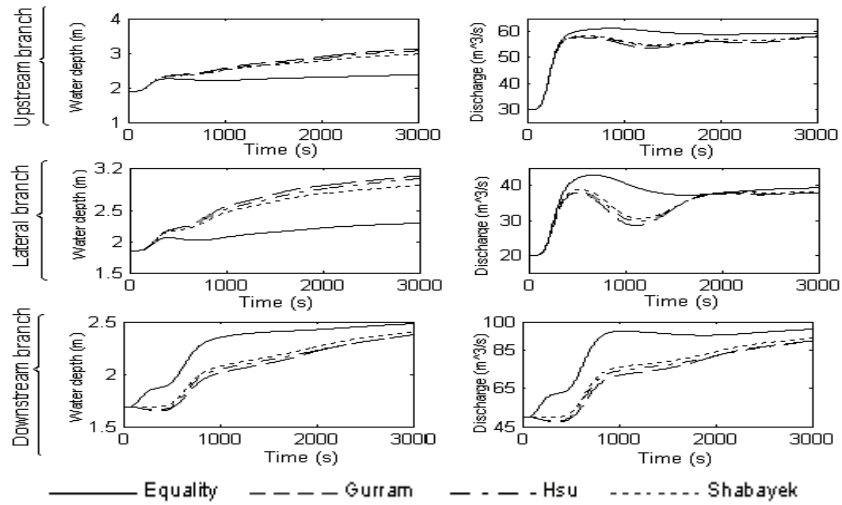


Figure 3 : Time history at the midpoint of each branch for example 1.

5.1 Example 1

This example is composed of three rectangular branches of equal width, length and slope. The junction Chezy coefficient is 83 (used by Shabayek model) and the properties of the branches are listed in table 3. We investigated the steady case and an unsteady case. The steady solution is ensured by fixing constant inflows discharges ($Q = 30 \text{ m}^3/\text{s}$ for the upstream branch and $Q = 20 \text{ m}^3/\text{s}$ for the lateral branch). A constant depth ($h = 1.69 \text{ m}$) is imposed at the downstream of the branch following the junction. In the unsteady case, we increase abruptly the inflow discharges from the value of 30 to 60 (m^3/s) in the upstream branch and 20 to 40

(m³/s) in the lateral branch, and we keep the outflow depth constant in the downstream waterway.

| Branch | Length (m) | Slope | Width (m) | Manning coefficient |
|--------|---------------|-------|--------------|------------------------|
| u | 600 | 0.001 | 10 | 0.0141 |
| L | 600 | 0.001 | 10 | 0.0138 |
| d | 600 | 0.001 | 10 | 0.0125 |

Table 3 : Branches properties for example 1.

The steady state profile of the flow variables is illustrated in figure 2. Here, the downstream Froude number is large ($F_d \approx 0.7$). Obviously, the solution obtained by the equality model is different from the other three models. A deviation is observed in the modeling of the upstream and lateral water depths, while for the other flow lines, all results coincide. A more significant deviation appears in the transient case of this problem. The depth and discharge time profiles simulated at the midpoint of each branch are shown in figure 3. Now, results are more expressive, in all branches a serious deviation between equality model and conservation model results emerge. Analyzing the flow variable's deviation (of all branches) according to Shabayek results, we obtain : through the use of the equality model, depth and discharge deviations of 20% and 25%, respectively, whereas, the depth and discharge deviations vary between 1 to 4% and 1 to 5%, respectively, through the use of the conservation models.

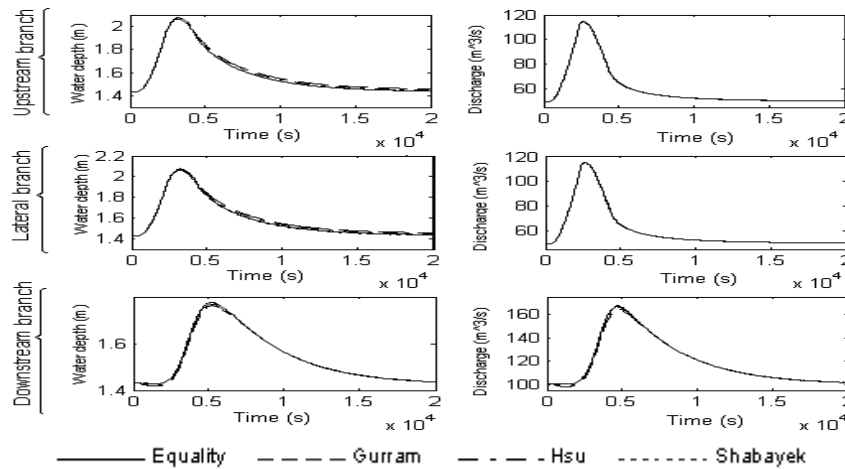


Figure 4 : Time evolution at the midpoint of each branch for example 2.

5.2 Example 2

In this example, we have tested a transient case. The branch width downstream to the junction is equal to the sum of the branch's widths upstream to the junction. The properties of the branches are listed in table 4 and the Chezy coefficient is 42 (for the Shabayek model).

The initial condition is a uniform flow with depth $h = 1.42$ m and discharge $Q = 50$ m³/s in upstream and lateral branches and $Q = 100$ m³/s in the downstream branch.

In the simulation, numerical results computed by the different junction models were very close. This can be seen in figure 4, where the time history of the water depth and

discharge are presented at the midpoint of each branch. In this problem, the downstream Froude number is low ($F_d < 0.35$) which reflects the fact that the four-junction models give close results.

These computational examples, supported by the investigation in section 2, prove that in high Froude number situations (downstream the junction) the equality model does not transmit the correct information to internal boundaries. A better transmission is achieved by a conservation model.

| Branch | Length (m) | Slope | Width (m) | Manning coefficient | Unsteady external boundary conditions |
|--------|---------------|--------|--------------|------------------------|------------------------------------------------------------------------------------------------------------------------|
| u, L | 5000 | 0.0002 | 50 | 0.025 | $Q_{m^3/s} = \begin{cases} 0.05t + 50 & t \leq 2000 \\ -0.05t + 250 & 2000 < t \leq 4000 \\ 50 & t > 4000 \end{cases}$ |
| d | 5000 | 0.0002 | 100 | 0.025 | Uniform flow |

Table 4 : Branches properties for example 2.

6 CONCLUSION

In this study, we evaluated four junction models with the experimental data of Hsu et al. (1998a, 1998b) and Weber and Greated (1966). The equality model was acceptable for low Froude number (F_d) downstream the junction. Afterwards, these models were introduced to simulate water flow in conduits by coupling branches at a combining junction. Two steady and transient problems were computed using the four methods. For $F_d \approx 0.7$, a serious deviation between equality model and conservation models results was noticed in the transient case. Indeed, for a low F_d the equality of water stages represents a suitable approximation of the energy equation and led, in our numerical instigation, to very close results to the others. However, it is better to use one of the simple models based on momentum conservation. The follow-up to this study will consist in comparing one-dimensional models with two-dimensional numerical results.

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