Modélisation des profils tridimensionnels de vitesse et de turbulence pour améliorer l'instrumentation des réseaux d'assainissement

Model for the numerical simulation of 3D turbulent flow to improve sewer net instrumentation

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RESUME

L'objectif de ce travail est l'interpolation d'un champ de vitesse à partir de données discrètes fournies par un vélocimètre Doppler. Pour cela une modélisation précise de l'écoulement est indispensable. Ainsi ce travail porte sur la description d'un modèle RSM en vue de la modélisation 3D des écoulements turbulents en canal rectangulaire à surface libre. Nous avons implémenté ce modèle en utilisant la méthode des éléments finis de Galerkine pour écrire un code sous Matlab. Les courants secondaires du champ moyen ont été mis ainsi en évidence à travers une section tranversale ainsi que leur sensibilité aux parois. Ils ne peuvent être modélisés qu'en tenant compte de l'anisotropie de la turbulence. D'où l'intérêt d'un modèle complet comme le RSM.

ABSTRACT

The aim of this work is the interpolation of a mean flow stream from discreet data supplied by a velocimeter Doppler. That is why a precise flow modelling in needed. This work deals so with the description of a Reynolds stress model for the numerical simulation of uniform 3D turbulent open-channel flows. The finite element method (Galerkine's method) is used for the numerical solution (with Matlab) of the flow equations and transport equations of the Reynolds stress components. It is found that both production terms by anisotropy of Reynolds normal stress and by Reynolds shear stress contribute to the generation of secondary currents. The results presented here are the calculation for full section flow in a rectangular channel, it is a step in the overall project.

KEY WORDS

Open channel flow, reynold stress model, secondary currents, turbulent.

INTRODUCTION

A great number of natural receiving waters are polluted because of urban net discharges. The protection of these natural sites needs a real time management of urban sanitation net. This is the general context of the RITEAU project, whose aim is the conception of a sensor which is able to measure both flow and suspended material concentration at the same time. SHU laboratory is taking part in this project, has been asked for 3D hydraulic modelling and experiments in laboratory.

This main aim of this study is the modelling of 3D turbulent open channel flows. The anisotropy of the turbulence, brought about the wall and the free surface, is known to generate secondary currents in open channel flows. k-ε model is isotropic closure so it cannot be used here. The RSM model has been implemented, directly solving Reynolds stresses transport equations thanks to the finite element method (Galerkine method). This code has been specially designed for sewer net conduit flow modelling.

1   TURBULENT FLOW MODELLING

1.1   Bibliography

Clarification of secondary currents is essential for the understanding of 3D turbulent flow structure. Secondary currents are defined as currents which occur in a plane normal to the local primary flow axis. These secondary currents have a great effect on the mean flow stream. There are two kinds, the first one, which is brought about by the channel's curves, is not considered here. The second one, called 'secondary currents of Prandtl's second kind as pointed out by Rodi (1986) is due to the anisotropy of turbulence and consists of large opposed spin vortices.

The secondary currents cause the 'Dip phenomenon'. The maximum velocity is not at the free surface but below where z/H ≈ 80%. The k-ε model does not allow to get this phenomenon. Furthermore the walls geometry (solid boundaries) and the free surface influence the second currents.

Figure 1 : Secondary currents	Figure 2 : Secondary currents and angle effects. (Nezu and Rodi 1985)	Figure 3 : Illustration of Dip phenomenon
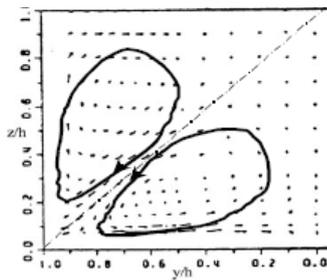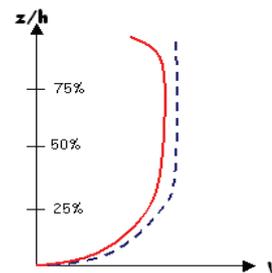
The issue at the core of this modelling is the turbulent closure model to solve the Reynolds averaged equations. A common model is the k-ε one which was originally developed by Jones and Launder (1972) for the modelling of homogeneous isotropic turbulence. This model does not correctly predict near-wall flow and the Dip phenomenon.

An other range of models deals with the transport of all Reynolds-stress components in addition to Reynolds averaged equations like the RSM developed by Hanjalić and Launder (1972). In that way, the anistropy of turbulence is taken into account. Moreover the RSM requires the ε equation. Here is what Christophe BAILLY wrote about it (2003)[1]: "These second order models have never had the development of the models k-ε, in particular because of the number of constants to be determined […]. There is no standard model.". That is why a RSM based on former works is developed here.
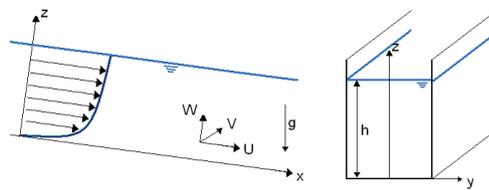
The steady flow is studied in an X infinite rectangular channel. So all x spatial derivatives are ignored except for the mean pressure; there is charge loss. In that way, the mean velocity stream is calculated in a cross section thanks to a 2D mesh.

Figure 4 : the channel

Reynolds averaged equations:

DP/dx = cst :
charge loss

$$\frac{\partial \overline{V}}{\partial y} + \frac{\partial \overline{W}}{\partial z} = 0$$

$$\rho \overline{V} \frac{\partial \overline{U}}{\partial y} + \rho \overline{W} \frac{\partial \overline{U}}{\partial z} = -\frac{\partial \overline{P}}{\partial x} + \mu \frac{\partial^2 \overline{U}}{\partial y^2} + \mu \frac{\partial^2 \overline{U}}{\partial z^2} - \rho \frac{\partial \overline{uv}}{\partial y} - \rho \frac{\partial \overline{uw}}{\partial z}$$

$$\rho \overline{V} \frac{\partial \overline{V}}{\partial y} + \rho \overline{W} \frac{\partial \overline{V}}{\partial z} = \mu \frac{\partial^2 \overline{V}}{\partial y^2} + \mu \frac{\partial^2 \overline{V}}{\partial z^2} - \rho \frac{\partial \overline{vv}}{\partial y} - \rho \frac{\partial \overline{vw}}{\partial z}$$

$$\rho \overline{V} \frac{\partial \overline{W}}{\partial y} + \rho \overline{W} \frac{\partial \overline{W}}{\partial z} = \mu \frac{\partial^2 \overline{W}}{\partial y^2} + \mu \frac{\partial^2 \overline{W}}{\partial z^2} - \rho \frac{\partial \overline{vw}}{\partial y} - \rho \frac{\partial \overline{ww}}{\partial z}$$

It is the appearance of the Reynolds stress that makes the problem so difficult. To calculate it, it is necessary to close Reynolds averaged equations. Reynolds stress transport equations are obtained by subtracting Reynolds averaged equations to Navier Stockes equations and then multiplying by turbulent velocity and averaging.

There are numerous degrees of freedom due, in particular, to turbulent velocity triple correlations. The following part deals with models for the different term of RSM equations, the aim is to reduce the number of unknowns.

1.1.1 Modelling Turbulent Diffusive Transport

Here the diffusion of Reynolds stresses is modelled using Daly and Harlow triple correlations model.

$$\overline{u_i u_j u_k} = -Cs\frac{k}{\varepsilon}(\overline{u_i u_l}\frac{\partial \overline{u_j u_k}}{\partial x_l} + \overline{u_j u_l}\frac{\partial \overline{u_i u_k}}{\partial x_l} + \overline{u_k u_l}\frac{\partial \overline{u_i u_j}}{\partial x_l})$$ This model looks like

Boussinesq approximation that assumes that the principal axes of the Reynolds stress tensor are coincident with those of the mean strain rate, applied for triple turbulent velocity correlation tensor.

Boussinesq: $-\overline{u_i u_j} = \nu_t\left(\frac{\partial \overline{U_i}}{\partial x_j} + \frac{\partial \overline{U_j}}{\partial x_i}\right)$ with $\nu_t = C_\mu\frac{k^2}{\varepsilon}$, $\frac{k}{\varepsilon}\overline{u_i u_l} \approx \frac{k^2}{\varepsilon} \equiv \nu_t$ shows the

similarities between the two models.

1.1.2 Modelling the Dissipation Rate

$$\overline{u_i\frac{\partial \tau_{jk}}{\partial x_k}} = \frac{\partial}{\partial x_k}\left(\mu\frac{\partial\left(\overline{u_i u_j}\right)}{\partial x_k}\right) - 2\mu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} \quad with \quad 2\mu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \varepsilon_{ij} = \frac{2}{3}\delta_{ij}\rho\varepsilon$$

The scalar dissipation rate, ε, is computed with a model transport equation similar to the one that is used in the standard k-ε model.

$$\frac{\partial \varepsilon}{\partial t} + \rho\overline{U}_k\frac{\partial \varepsilon}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\left(\mu + \frac{\mu_t}{\sigma_t}\right)\frac{\partial \varepsilon}{\partial x_k}\right) + \frac{\varepsilon}{k}\left(C_{1\varepsilon}\frac{P_{kk}}{2} - C_{2\varepsilon}\rho\varepsilon\right)$$

1.1.3 Modelling the Pressure-Strain Term

The pressure-strain term plays a pivotal role in modelling the turbulent flow. The following decomposition is used:

$$-\overline{u_i\frac{\partial P'}{\partial x_j} - u_j\frac{\partial P'}{\partial x_i}} = -C_1\rho\frac{\varepsilon}{k}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}k\right) - C_2\left(P_{ij} - C_{ij}\right) + \phi_{ij,wall}$$

$\Phi_{ij, wall}$ redistributes turbulent kinetic energy among the Reynolds stresses. Here is the model proposed par Speziale *et al* [6].

$$\phi_{ij,wall} = \alpha_0\varepsilon b_{ij} + \alpha_1\varepsilon(b_{ik}b_{jk} - \frac{1}{3}b_{mn}b_{nm}\delta_{ij}) + \alpha_2 k S_{ij} + \alpha_3 P_k b_{ij}$$

$$+ \alpha_4 k(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}) + \alpha_5 k(b_{ik}W_{jk} + b_{jk}W_{ik})$$

$$b_{ij} = \frac{R_{ij}}{k} - \frac{2}{3}\delta_{ij} : \text{anisotropy tensor}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \text{ : rate of strain tensor} \qquad W_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \text{ : rotation tensor}$$

P_k : rate of production of turbulent energy

$\alpha_0 = -3.4, \ \alpha_1 = 4.2, \ \alpha_2 = 0.8 - 1.3(b_{mn}b_{nm})^{1/2}, \ \alpha_3 = -1.8, \ \alpha_4 = 1.25 \text{ and } \alpha_5 = 0.4$

Speziale's model transports anisotropy tensor and so is necessary to generate second currents vortices. Because it is very difficult, as the Speziale model has not yet been totally implemented in order to see if the last term $\Phi_{ij, \ wall}$, the more complicate, is indispensable.

Reynolds stresses transport equation follows:

$$\rho(1+C_2)\frac{\partial\left(\overline{u_i u_j} U_k\right)}{\partial x_k} = Cs\frac{k}{\varepsilon}(\overline{u_i u_l}\frac{\partial\overline{u_j u_k}}{\partial x_l} + \overline{u_j u_l}\frac{\partial\overline{u_i u_k}}{\partial x_l} + \overline{u_k u_l}\frac{\partial\overline{u_i u_j}}{\partial x_l})$$

$$+ (C_2-1)\rho\sum_k(\overline{u_j u_k}\frac{\partial\overline{U_i}}{\partial x_k} + \overline{u_i u_k}\frac{\partial\overline{U_j}}{\partial x_k}) + C_1\rho\frac{\varepsilon}{k}\overline{u_i u_j} + 2\mu\Delta(\overline{u_i u_j})$$

1.2   Flow conservation

The overall continuity equation is added to impose flow (Q), to calculate charge loss (dP/dx) that has been considered as a constant in the whole channel. Kang and Choi 2006 [4] impose charge loss writing dP/dx = g.I with g gravitational acceleration and I the channel slope. Maybe Q is imposed that way considering that dP/dx $\equiv$ Q².

The overall equation is not integrated at each node but on the whole section every iteration. Here is the equation: $\int \overline{U} dS = Q$.

2   FINITE ELEMENT MODELING: CALCULUS CODE CREATION

2.1   Presentation

In the present study, the finite element (Galerkine 's method) is used to solve Reynolds averaged and Reynolds stresses transport equations. Because the system is non-linear, the numeric scheme is iterative by substitution. The code has been optimised by reducing the calculus number for each iteration (using preintegration). Moreover Matlab is a powerful tool for matrix manipulations, that is why logical operators (if, for, while…) have been replaced by matrix operations as much as possible.

This work has not been completed. One step has been to work with full section flow to get rid of free surface influence and to verify that the result given is symmetric. This is what is presented here.

2.2 Modelling results

2.2.1 Laminar flow

To test the part of the code which calculates mean velocities, the laminar flow has been modelled; the 6 Reynold stresses are equivalent to zero. The water depth is 10m, channel width is 1m, slope is 0, stream-wise mean velocity is 1 m.s^{-1}, and Q = 10 m^3.s^{-1}. The channel is narrow to get rid of ceiling and bottom roughless wall effect.
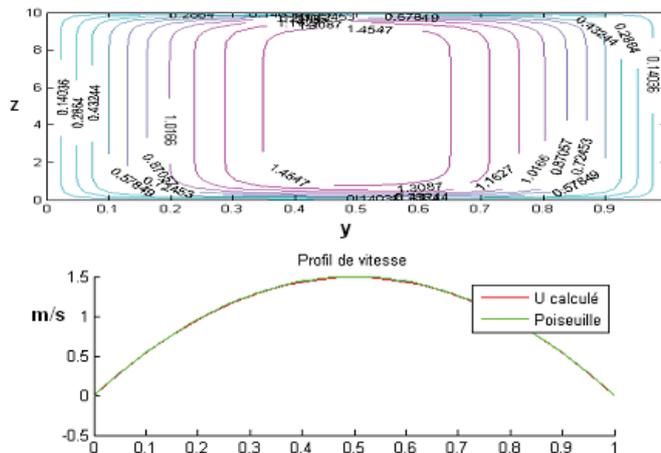
V and W are also modelled but the result is close to 0 ($<<10^{-300}$).

Here results are compared with Poiseuille mean velocity profile. The correlation between the two curves is very good.

Figure 1 : Laminar flow modelling stream

2.2.2 Turbulent flow

Here is turbulent flow modelling, the water depth is 4m, channel width is 4m, slope is 0, stream wise mean velocity is 1 m.s^{-1}, and Q = 16 m^3.s^{-1}.
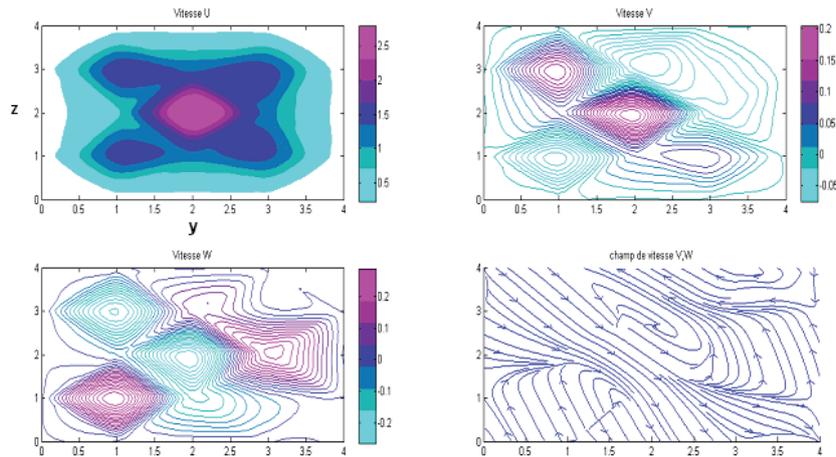
Figure 6 : Turbulent flow modelling, mean

The mean stream obtained is clearly not symmetric but close to it. U looks a lot like the laminar solution. Without free surface there is no Dip phenomenon, so calculated profile and average value appear to be quite good.

According to Nezu and Nakagawa [5], magnitude of V and W is only about 2 % of the maximum stream wise velocity. Results are thus coherent. Secondary currents form is not good, calculations do not end up converging. Moreover Speziale model's $\Phi_{ij, wall}$ has no physical meaning but is a mathematic function that reproduce secondary currents. This term seems to be necessary.
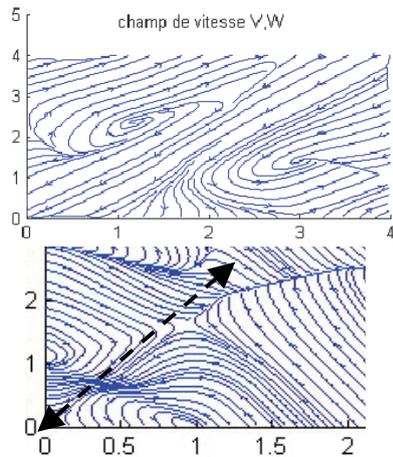
On this other modelling, secondary currents are better, opposed spin vortices can be seen but the angle effect is not visible. On the following figure, the angle effect appears to be good, that is to say vortices close to the bottom corner are divided along the angle bisector.

About double turbulent velocity correlations, shown below, overall values are quite good according to the Kang and Choi article [4]. Nevertheless, vw > 0 it should be the other way around. As a matter of fact, Reynolds tensor terms not from diagonal are negative like the results for uv and uw. Here the symmetry is quite good.

Figures 7-8 : Opposed spin vortices illustration
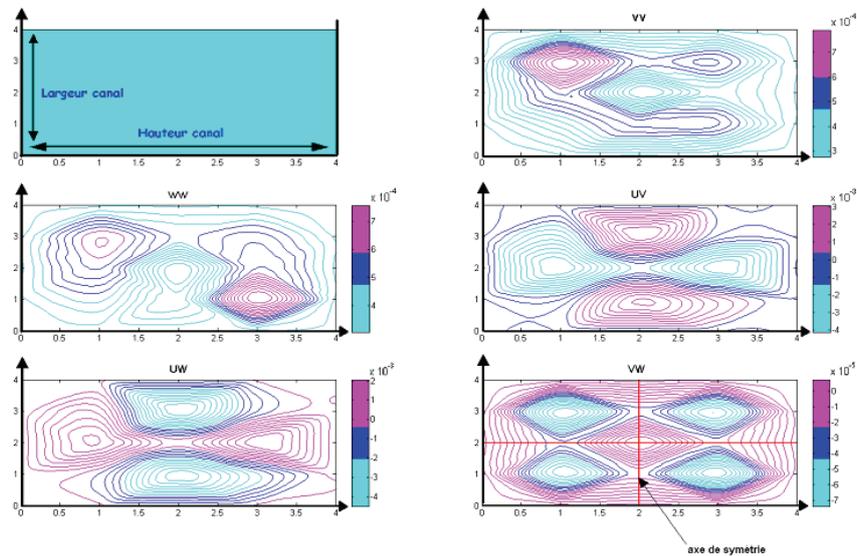
Figure 9 : Turbulent flow modelling, double turbulent velocity correlations

In the aim of understanding the difficulties there are for calculation, here an analyse on the studied dynamic system.

There are three kinds of dynamic system. The first one takes place on an attractor such as the Poiseuille flow. These systems are very stable and always converge to the same final stage. On the contrary, chaotic systems are very unstable and almost impossible to foresee as they are sensitive to all governing factors. A tiny change of the initial conditions changes the behaviour of the system completely.

The last kind is composed of a chaos order transition system which gives rise to strange attractors. It is greatly detailed and complex in order to manage a large energy contribution. For instance, the flow studied has a high Re which implies high kinetic energy. At the beginning, this energy is consumed in a 'common' way thanks to transport and viscosity dissipation but it is not enough and the energy balance is not obtained. As the fluid's behaviour takes place on a complex structure, secondary currents are generated (see 1.2). These secondary currents consist of large opposed spin vortices. In this way, the system consumes more energy and is energy balanced. Actually the studied flow is complex structure hydrodynamic system, close to chaotic, which implies difficulties to converge and high sensibily to boundary conditions.

CONCLUSION

The aim of the present study was precise modelling of 3D turbulent flow for the conception of a sensor. The RSM model is the only one taking into account anisotropy of turbulence that generates secondary currents and the Dip phenomenon. On one hand, a very precise RSM has been developed, on the other hand the problem has been simplified so that the calculations are lead into 2D mesh. The RSM equation has been rewritten with a finite element formulation, then integrated and implemented. The computed mean flow and turbulent velocity correlations were compared with Kang and Choi's results. Model predictions are quite good but not totally successfully because of convergence problems. Inner secondary currents are reproduced but significant improvements will be made when the Speziale model is properly implemented.

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